USC FBE APPLIED ECONOMICS WORKSHOP
presented by Li, Hao
FRIDAY, Oct. 26, 2007
1:30 pm – 3:00 pm, **Room: HOH-706**

Delay in Strategic Information Aggregation

Ettore Damiano
University of Toronto

Li, Hao
University of Toronto

Wing Suen
The University of Hong Kong

October 24, 2007

Abstract. We study a model of collective decision making in which divergent preferences of the agents make information aggregation impossible in a single round of voting. With costly delay, we show that repeated voting can help the agents reach a mutually preferred decision, even though there is no new direct information about the decision between two rounds of voting. An increase in the cost of delay can improve the efficiency of information aggregation, and hence the ex ante welfare of the agents involved, by encouraging the agents to be more forthcoming with their private information in the initial rounds of voting. Allowing an additional round of voting in case of disagreements can similarly improve the ex ante welfare when there is an intermediate degree of conflict, but reduces the welfare otherwise. With sufficiently many rounds of voting allowed, the equilibrium play of the repeated voting game involves gradually increasing concessions.

Notes. This is a preliminary draft. Please do not circulate without permission. Suen and Li thank Guanghua School of Management of Beijing University, and especially the Department of Applied Economics and its chair Hongbin Cai, for their hospitality and research support during their visit. Li wishes to thank the Marshall School of Business of the University of Southern California for hosting his sabbatical when part of the research for this paper is done.

1. Introduction

Individuals may disagree with one another when they have different preferences or when they have different private information. Often, it is difficult to distinguish between these two types of disagreement because divergent preferences provide incentives for individuals to distort their information. Even though they may share a common interest in some states had the individuals known each other's private information, the strategic distortion of information can still cause disagreement in these states. If disagreements lead to delay in making decisions, then it often seems that any decision is better than no decision and costly delay. For example, people complain about legislative deadlock and the delay costs it entails. We argue in this paper, however, that institutionalized delay in the decision making process can serve a useful purpose. In the context of a stylized model of repeated voting, the prospect of costly delay induces the parties to be more accommodating in the initial rounds of voting and enhances information aggregation. As a result, a greater delay cost between two rounds of voting can lead to an improvement in the ex ante welfare of the decision makers.

The problem of disagreement that we study resembles but is not identical to a pure bargaining problem. The two decision makers in our model have private information about which is the appropriate alternative to adopt. If they could perfectly aggregate their private information, there are some states of the world in which they still would disagree because of divergent preferences but there are also some states of the world in which they would agree. Therefore the role of delay described in this paper is different from that in a pure bargaining model (Stahl 1972; Rubinstein 1982). In the Stahl-Rubinstein bargaining model, the trade off between getting a bigger share of the pie but at a later date helps pin down a unique solution to the bargaining problem which is plagued by multiple equilibria in a one-shot model. One feature of the Stahl-Rubinstein model is that delay does not occur in equilibrium: the mere possibility of incurring delay cost helps prompt the two bargaining parties to reach an agreement in the first round. In our setting, the one-round voting game has a generically unique equilibrium, and the repeated voting game entails a different outcome with delay occurring in equilibrium.

There are numerous extensions to the Stahl-Rubinstein model that can generate delay as part of the equilibrium outcome. One strand of this literature relies on asymmetric information about the size of the pie that is being divided.[1] In a model of strikes, for example, a firm knows its own profitability but the firm's unionized workforce does not. Strike or delay is a signaling device in the sense that the willingness to endure a longer work stoppage can credibly signal the firm's low profitability and help it to arrive at a more favorable wage bargain. In this type of signaling models, each agent's gains from trade at a given price depend only on his own private information. In our model, disagreement over the alternatives is not a pure bargaining issue, because individuals in our model would sometimes agree on which is the best alternative had they known the true state. Put differently, voting outcomes in our setup determine the size as well as the division of the pie. We show that delay can play a constructive role in overcoming disagreement improving the ex ante welfare of all individuals.

Our paper is also related to the literature on debates (Austen-Smith 1990; Austen-Smith and Feddersen 2006; Ottaviani and Sorensen 2001) and voting (Li, Rosen and Suen 2001) in committees.[2] Models of debate typically analyze repeated information transmission as cheap talk, while we emphasize the role of delay cost in multiple rounds of voting. Our setup is the closest to the Li, Rosen and Suen paper. The focus there is on the impossibility of efficient information aggregation. Here, we have intentionally skirted issues such as quality of private signals and the trade-off between making the two different types of errors. We focus instead on how delay and multiple voting rounds can help improve information aggregation in a simple environment.

Section 2 introduces the information structure and the preferences of two individuals in a symmetric strategic information aggregation problem with two alternatives. There is one disagreement state in which each of the two individuals prefers his own favorite alternative

[1] See, for example, Chatterjee and Samuelson (1987), Cho (1990), Cramton (1992), and Kennan and Wilson (1993). There are also bargaining models that generate equilibrium delay under complete information, through commitment to not accepting offers poorer than past rejected ones (Freshtman and Seidmann 1993), simultaneous offers (Sakovic 1993), and multi-lateral negotiations (Cai 2000).

[2] Coughlan (2000) investigates conditions under which jurors vote their signals and their information is efficiently aggregated in a model where a mistrial leads to a retrial by a new independent jury. He does not consider the issues of delay that are the focus of the present paper.

if they knew this was the true state, and one agreement state for each alternative, with the degree of conflict between the two individuals captured by the prior probability of disagreement state. Each individual is either a "extremist," whose private signal is that the state is the agreement state corresponding to his favorite alternative, or a "moderate," who knows only that the the state is either the other agreement state or the disagreement state. As a benchmark for models with delay, we first consider a game with a single round of voting between the two alternatives, in which two agreeing votes lead to the agreed alternative being chosen and disagreeing votes result in a coin flip between the two alternatives. The equilibrium outcome is generically either an efficient aggregation of the private information through informative voting when the degree of conflict is low, or otherwise a coin flip due to uninformative voting. In the latter case, we show that without the possibility of delay, there is no incentive compatible outcome that Pareto dominates the coin flip.

Section 3 allows the two individuals to vote for a second time after paying a delay cost, if they disagree in the first round of voting. There is a unique equilibrium outcome, in which the option of voting again in case of disagreement makes the voting by the moderate types less informative in the first round when the degree of conflict is low, but has the opposite effect when the degree of conflict is high. In the latter case, the softening of the positions taken by the moderate types improves information aggregation in the first round voting. It turns out that the effect of delay cost on the ex ante welfare of the decision makers is non-monotone. Small delay cost does not help resolve disagreement while large delay cost facilitates good decisions at too high a price. There is an intermediate range of delay cost that improves the ex ante welfare of decision makers over a single round of voting.

In section 4 we extend the analysis to games with a finite deadline that may involve more than two rounds of voting, and a game without a deadline so that a disagreement always leads to another round of voting. We show that a longer deadline can increase the ex ante welfare when the degree of conflict is at intermediate levels but otherwise decreases the welfare. An implication is that the "optimal deadline" from the ex ante point of view is positive for intermediate degrees of conflict but zero when the degree of conflict is either too low or too high. In the game without a deadline, as in the game with possibly two rounds of voting, an increase in the delay cost leads to a softening of the positions taken by the moderate types for all levels of conflict. However, unlike the game with possibly two rounds of voting, when the deadline is sufficiently long, his willingness to compromise depends negatively on the degree of conflict. As a result, the equilibrium play has the intuitive feature of gradually increasing concessions: the moderate types start out with a tough position, and each disagreement resulting from both individuals voting for their favorite alternatives leads to a less pessimistic belief that the state is the disagreement state, and hence to a softening of his position. Section 5 concludes the paper with some discussion of interesting issues that remain to be investigated.

2. The Model

2.1. Information and preferences

Two players have to make a public choice between two alternatives, c and p. For convenience, we call one player a conservative (player C) and the other a progressive (player P). There are three possible states of the world: R, M, and L. The corresponding prior probabilities are denoted π_R, π_M, and π_L. Our mode is intentionally symmetric; we therefore assume that $\pi_R = \pi_L = \pi$ and $\pi_M = 1 - 2\pi$. The information structure is as follows: the conservative is able to distinguish whether the state is R or not, while the progressive is able to distinguish whether the state is L or not. Such information is private and unverifiable.

The relevant payoffs for the two players are summarized in the following table:

	L	M	R
c	$(1-2\Delta, 1-2\Delta)$	$(1, 1-2\Delta)$	$(1, 1)$
p	$(1, 1)$	$(1-2\Delta, 1)$	$(1-2\Delta, 1-2\Delta)$

In each cell of this table, the first entry refers to the payoff to the conservative and the second entry refers to the payoff to the progressive. We normalize the gain from making the preferred decision to 1 and let the payoff from making the less preferred decision be $1-2\Delta$. The parameter $\Delta > 0$ is the error cost, or the cost of making the wrong decision. In state R both players prefer c to p, and in state L both prefer p to c. The two players' preferences are different when the state is M: the conservative prefers c while the progressive prefers p. Thus in our model there are elements of both common interest and conflict between these two players.

A conservative who knows that the state is R prefers c to p, and is referred to as an "extreme conservative." A progressive who knows that the state is L prefers p to c, and is called an "extreme progressive."[3] The preference between c and p of a conservative who knows that the state is either L or M depends on the relative likelihood of these states. Let γ denote his belief that the state is M. We have

$$\gamma = \frac{\pi_M}{\pi_L + \pi_M} = \frac{1-2\pi}{2\pi}.$$

If the conservative could dictate the outcome, he strictly prefers c to p if and only if

$$\gamma + (1-\gamma)(1-2\Delta) > \gamma(1-2\Delta) + (1-\gamma),$$

which is equivalent to $\gamma > \frac{1}{2}$. We designate him as a "moderate conservative." Similarly, because of the assumption that $\pi_R = \pi_L$, a progressive with the signal that the state is not L also has belief γ that the state is M. He strictly prefers p to c if and only if $\gamma > \frac{1}{2}$. We designate him as a "moderate progressive." In our later analysis of games with repeated voting, given that there is no exogenous new information, the designations of extremists and moderates do not change.

We note that γ can be interpreted as the ex ante degree of conflict. When γ is high, a moderate player perceives that his opponent is likely to have different preferences regarding the correct decision to be chosen. In the following analysis of games with repeated voting, the belief of the moderate types, and hence the degree of conflict, changes endogenously as voting progresses.

[3] In our model an extremist is informed of the Pareto optimal decision, rather than someone with a large bias for his favorite choice that may not be overcome with contrary evidence as in many models of strategic communication or voting.

2.2. Single round of voting

We first consider a benchmark game with one round of voting. In this game, each player votes c or p simultaneously. If the votes agree, that alternative is implemented. If the votes disagree, the decision is made by flipping a coin (i.e., c or p is each adopted with probability $\frac{1}{2}$). Note that the payoff from flipping a coin is $1 - \Delta$.

We say that player C votes "according to his preference" if he votes c, and that he votes "against his preference" if he votes p. Similarly, player P votes according to his preference if he votes p, and he votes against his preference if he votes p. For the extreme conservative, since the state is known to be R, his dominant strategy is to vote c. Similarly, the extreme progressive's dominant strategy is to vote p. Let $x_C \in [0, 1]$ be the probability that the moderate conservative votes c. Similarly, let x_P be the probability that that the moderate progressive votes p. We interpret x_C and x_P a measure of how "tough" the moderates are playing. A higher value of x_C or x_P means that they are playing more like the extreme types, and thus voting less informatively.

The moderate conservative's expected payoff from voting c is

$$\gamma(x_P(1 - \Delta) + (1 - x_P)) + (1 - \gamma)(1 - \Delta);$$

and his expected payoff from voting p is

$$\gamma(x_P(1 - 2\Delta) + (1 - x_P)(1 - \Delta)) + (1 - \gamma).$$

If $\gamma > \frac{1}{2}$, then it is a dominant strategy for the moderate conservative to vote c. A similar reasoning suggests that the moderate progressive has a dominant strategy to propose p (i.e., $x_C = x_P = 1$). The equilibrium outcome is that player C and player P always disagree, and the decision is always determined by flipping a coin. If $\gamma < \frac{1}{2}$, then the dominant strategy for the moderate players is to vote against his preferences (i.e., $x_C = x_P = 0$). In states L and R, such equilibrium voting strategies lead to the correct decision being made. In state M, the decision is determined by flipping a coin. Finally, if $\gamma = \frac{1}{2}$, both the moderate conservative and the moderate progressive are indifferent between voting c

and voting p. Any $(x_C, x_P) \in [0,1]^2$ constitutes an equilibrium. As is the case throughout the paper, we focus on symmetric equilibria with $x_C = x_P = x_0$.[4]

Let $V_0(\gamma)$ be the equilibrium payoff to the moderate types in the game with one round of voting. The above analysis leads to:

$$V_0(\gamma) = \begin{cases} 1 - \Delta\gamma & \text{if } \gamma \in [0, \frac{1}{2}), \\ \in [1-\Delta, 1 - \frac{1}{2}\Delta] & \text{if } \gamma = \frac{1}{2}, \\ 1 - \Delta & \text{if } \gamma \in (\frac{1}{2}, 1]. \end{cases} \tag{1}$$

Similarly, let $W_0(\gamma)$ be the equilibrium payoff to the extreme types. Note that although the extreme conservative has belief 0 that the state is M, his equilibrium payoff depends on the moderate progressive's belief γ. We have

$$W_0(\gamma) = \begin{cases} 1 & \text{if } \gamma \in [0, \frac{1}{2}), \\ \in [1-\Delta, 1] & \text{if } \gamma = \frac{1}{2}, \\ 1 - \Delta & \text{if } \gamma \in (\frac{1}{2}, 1]. \end{cases} \tag{2}$$

Comparing the two payoff functions, we have $V_0(1) = W_0(1)$ and $V_0(\gamma) \leq W_0(\gamma)$ for all γ.

The equilibrium outcome in this game of a single round of voting is discontinuous at $\gamma = \frac{1}{2}$. Information aggregation is efficient for $\gamma < \frac{1}{2}$ as the two players vote according to their signals, while for $\gamma > \frac{1}{2}$ there is no information aggregation at all. At $\gamma = \frac{1}{2}$, there is a continuum of equilibria that bridge the discontinuity: the equilibrium payoff to the moderate players V_0 ranges between $1 - \frac{1}{2}\Delta$ and $1 - \Delta$ and the payoff to the extreme players W_0 ranges between $1 - \Delta$ to 1, both decreasing in how tough the moderate types are playing in equilibrium.

2.3. The impossibility of information aggregation

When $\gamma > \frac{1}{2}$, the equilibrium of the one round voting game leads to a rather undesirable outcome, as both players would strictly prefer adopting c in state R (and p in state L) to deciding by coin flipping if they knew the true state. This result depends on our assumption

[4] The game with a single round of voting provides the simplest case that asymmetric equilibria exist in our symmetric model. See section 5 for some brief discussion on this issue.

about the structure of the voting game. We can obtain different outcomes by changing the rules of the voting game. For example, suppose the rule is that player C is the decisive voter. Then it is still a Nash equilibrium for player C to vote c and for player P to vote p regardless of their private information. The ex ante equilibrium payoff of the conservative (before observing his private information) is $1 - 2\pi_L\Delta$ while the ex ante payoff of the progressive is $1 - 2(\pi_R + \pi_M)\Delta$. In this case, changing the rule of the voting game benefits the conservative player but hurts the progressive player compared to our benchmark game in the previous subsection. However the unweighted sum of equilibrium payoffs for these two players remains the same as in the benchmark case.

More generally, we can ask if it is possible to improve on the benchmark outcome using some other mechanism without side transfers. By the revelation principle, it suffices to consider any direct mechanism which satisfies the incentive compatibility constraints for truthful reporting of private signals. In a truth-telling equilibrium, the true state can be recovered from the reports submitted by the conservative and progressive players. For example, if both player C and player P report that they are moderate types, then the true state must be M. Let q_R, q_M, and q_L be the probabilities of implementing alternative c when the true state is R, M, and L, respectively. Let $\tilde{q}$ be the probability of implementing c when the reports are inconsistent, that is, when both C and P report that they are extreme types. The incentive constraints can be written as:

$$
\begin{aligned}
& q_R + (1-q_R)(1-2\Delta) \geq q_M + (1-q_M)(1-2\Delta), \\
& q_L(1-2\Delta) + (1-q_L) \geq q_M(1-2\Delta) + (1-q_M), \\
& \gamma(q_M + (1-q_M)(1-2\Delta)) + (1-\gamma)(q_L(1-2\Delta) + (1-q_L)) \\
& \qquad \geq \gamma(q_R + (1-q_R)(1-2\Delta)) + (1-\gamma)(\tilde{q}(1-2\Delta) + (1-\tilde{q})), \\
& \gamma(q_M(1-2\Delta) + (1-q_M)) + (1-\gamma)(q_R + (1-q_R)(1-2\Delta)) \\
& \qquad \geq \gamma(q_L(1-2\Delta) + (1-q_L)) + (1-\gamma)(\tilde{q} + (1-\tilde{q})(1-2\Delta)).
\end{aligned} \tag{3}
$$

The first inequality is the incentive constraint for the extreme conservative. The left side of this inequality is his expected payoff if he reports the truth. If he lies and reports that he is a moderate instead, since the moderate progressive truthfully reports his type, the state is taken to be M and the extreme conservative's payoff is given by the right side of the inequality. The other three inequalities are the truth-telling conditions for the extreme progressive, the moderate conservative and the moderate progressive respectively, and can be understood in an similar manner.

We argue that when $\gamma > \frac{1}{2}$, $q_R = q_M = q_L$ in any incentive compatible direct mechanism. To see this, note that the incentive constraints for the extremists (the first two inequalities of (3)) imply that $q_R \geq q_M \geq q_L$. The incentive constraints for the moderates (the last two inequalities of (3)) imply

$$(1-\gamma)(\tilde{q} - q_L) \geq \gamma(q_R - q_M);$$
$$(1-\gamma)(q_R - \tilde{q}) \geq \gamma(q_M - q_L).$$

Adding these two inequalities gives

$$(1-\gamma)(q_R - q_L) \geq \gamma(q_R - q_L),$$

which is inconsistent with $\gamma > \frac{1}{2}$ unless $q_R - q_L = 0$. This shows that $q_R = q_M = q_L$.

When $\gamma > \frac{1}{2}$, the expected payoff of the conservative from any incentive compatible mechanism, in which $q_R = q_M = q_L = q$, is given by

$$1 - 2(q\pi_L + (1-q)(1-\pi_L))\Delta;$$

while the payoff of the progressive is

$$1 - 2(q(1-\pi_R) + (1-q)\pi_R)\Delta.$$

The conservative's payoff is increasing in q while the progressive payoff is decreasing in q. The Pareto frontier under any incentive compatible mechanism is linear. Since the two players are ex ante symmetric, with $\pi_R = \pi_L$, it is natural to focus on the mechanism with $q = \frac{1}{2}$, which is equivalent to our benchmark one-round voting game. The above analysis thus suggests that the one-round voting game is a relevant benchmark because no incentive compatible mechanism without side transfers can Pareto-improve on the outcome of the benchmark game.[5]

[5] This result does not depend on the symmetry assumption that $\pi_L = \pi_R$. No information aggregation is possible for all π_L and π_R as long as both are less than π_M.

3. Repeated Voting with Possibly Two Rounds

In this section, we consider a game with possibly two rounds of simultaneous voting. Each player, C or P, can vote for either c or p in each round. If the two votes in the first round are both c or both p, then that alternative is implemented immediately and the game ends. If the two votes differ in the first round, then each player have to incur a delay cost $d > 0$ and the two of them will vote again in the second round. If the two votes agree then the decision is made according to the votes; otherwise, the decision is made by a fair coin toss.

The cost of delay is modeled as as an additive fixed cost in this paper. Such cost may reflect the time and expenses of setting up a second round of meeting and negotiations. An alternative way to model delay cost is to apply a multiplicative discount factor to the payoffs if the decision is implemented in the second round. In this case, delaying a preferred decision is more costly than delaying an inferior decision. Consequently the analysis of the discounting case is slightly more cumbersome than the fixed cost case. We therefore adopt the more transparent assumption of fixed delay cost. The basic insights of the paper do not depend on which of these two assumptions is used.

When the delay cost is large, the inferior alternative can be better than the preferred alternative with delay. In that case, even an extreme conservative would prefer to vote p if he knows that the progressive will vote p. The strategic situation is analogous to a "battle-of-the-sexes" game and the main economic issue is that of coordination to one of the two asymmetric outcomes (all voting for p or all voting for c) to avoid the large delay cost. Of course these two outcomes cannot be Pareto ranked. Our main concern in this paper instead, is to study whether and how delay can improve the payoff of both players. To this effect we focus on equilibria in which the strategies of the extreme conservative and the extreme progressive are symmetric to each other, and the strategies of the moderate conservative and the moderate progressive are symmetric to each other. We leave the discussion of asymmetric equilibria to section 5.

3.1. Characterization of equilibrium play

In the two-round voting game, there are two possible kinds of disagreement in the first round voting: the conservative votes c and the progressive votes p ("regular disagreement"); and the conservative votes p and the progressive votes c ("reverse disagreement"). The updating of beliefs of the moderate types upon these two types of disagreement depends on the equilibrium strategies adopted by the players. Suppose in equilibrium the extreme progressive votes p with probability 1, while the moderate progressive votes p with probability x_1. Then, upon a regular disagreement, the moderate conservative would revise his belief that the state is M downward to

$$\gamma' = \frac{\gamma x_1}{\gamma x_1 + 1 - \gamma} \leq \gamma,$$

unless $\gamma = 1$ and $x_1 = 0$, in which case the Bayes' formula does not apply. From the discussion of the previous section, in the final round the extreme types vote according to their preferences while the equilibrium strategies of the moderate players are given by:

$$x_0(\gamma') = \begin{cases} 0 & \text{if } \gamma' \in [0, \frac{1}{2}), \\ \in [0,1] & \text{if } \gamma' = \frac{1}{2}, \\ 1 & \text{if } \gamma' \in (\frac{1}{2}, 1]. \end{cases}$$

Note that $\gamma' = \frac{1}{2}$ for any $\gamma \geq \frac{1}{2}$ if $x_1 = (1-\gamma)/\gamma$, in which case the continuation play is not unique. Upon a reverse disagreement, the moderate conservative updates his belief to 1, except when $x_1 = 1$, because he can exclude the possibility that the state is L. In the final round, the moderate players choose $x_0(1) = 1$, and the game ends with the decision made by a coin flip.

Given this assumed strategy profile, the payoff of the moderate conservative from voting c is

$$V_1^c(\gamma, x_1) = \gamma(x_1(-d + V_0(\gamma')) + (1 - x_1)) + (1-\gamma)(-d + V_0(\gamma')), \tag{4}$$

for all γ and x_1 such that γ' is defined and is not equal to $\frac{1}{2}$. Let $V_1^c(1,0) = 1$,[6] and for any $\gamma \geq \frac{1}{2}$ and $x_1 = (1-\gamma)/\gamma$, let $V_1^c(\gamma, (1-\gamma)/\gamma)$ represent a continuum of the continuation payoffs, each of which corresponds to a value of $V_0(\frac{1}{2}) \in [1-\Delta, 1-\frac{1}{2}\Delta]$. The payoff of the moderate conservative from voting p is

$$V_1^p(\gamma, x_1) = \gamma[x_1(1-2\Delta) + (1-x_1)(-d + V_0(1))] + (1-\gamma) \tag{5}$$

[6] When the Bayes' rule does not apply, by construction the choices of the out-of-equilibrium beliefs do not affect the values of $V_1^c(1,0)$ and $V_1^p(\gamma, 1)$, which are determined by continuity.

for all $x_1 < 1$ with $V_1^p(\gamma, 1) = \gamma(1-2\Delta) + 1 - \gamma$. Let $\hat{x}_1(\gamma)$ be the x_1 that solves $V_1^c(\gamma, x_1) = V_1^p(\gamma, x_1)$, if it exists. We have the following result.[7]

Proposition 1. *In the two-round voting game, there exists an equilibrium in which in the first round the extreme types vote according to their preferences with probability 1 and the moderate types vote according to their preferences with probability $x_1(\gamma)$ given by*

$$x_1(\gamma) = \begin{cases} 0 & \text{if } \gamma \in [0, G_1^1), \\ \min\{\hat{x}_1(\gamma), 1\} & \text{if } \gamma \in [G_1^1, P_1), \\ (1-\gamma)/\gamma & \text{if } \gamma \in [P_1, Q_1), \\ \min\{\hat{x}_1(\gamma), 1\} & \text{if } \gamma \in [Q_1, 1]; \end{cases}$$

with $G_1^1 = d/(2d+\Delta)$, $P_1 = \max\{\frac{1}{2}, 3d/(4d+\Delta)\}$, and $Q_1 = (3d+\Delta)/(4d+2\Delta)$.

Proof. We will first verify the incentives for the moderate types. There are four cases.
(i) Consider first the case in which $x_1(\gamma) = 0$ and $\gamma' = 0$. For this to be an equilibrium, a moderate conservative must prefer voting p to voting c. Since $V_0(1) = 1-\Delta$ and $V_0(0) = 1$, the condition $V_1^c(\gamma, 0) \le V_1^p(\gamma, 0)$ is equivalent to $\gamma \in [0, G_1^1]$.
(ii) Next, consider the case in which upon a regular disagreement the updated belief γ' belongs to $(0, \frac{1}{2})$. In this case, $V_0(\gamma') = 1 - \Delta\gamma'$, so the $\hat{x}_1(\gamma)$ that solves the indifference condition $V_1^c(\gamma, x_1) = V_1^p(\gamma, x_1)$, and is given by:

$$\hat{x}_1(\gamma) = \frac{\gamma(d+\Delta) - (1-\gamma)d}{2d\gamma}. \tag{6}$$

When $\gamma \ge G_1^1$, we have $\hat{x}_1(\gamma) \ge 0$. When $\gamma < P_1$, we have $\hat{x}_1(\gamma) < (1-\gamma)/\gamma$, so the resulting posterior γ' is indeed less than than $\frac{1}{2}$. Moreover $\hat{x}_1(\gamma) \ge 1$ is equivalent to $V_1^c(\gamma, 1) \ge V_1^p(\gamma, 1)$, in which case $x_1(\gamma) = 1$ is an equilibrium.
(iii) In the third case, $x_1(\gamma) = (1-\gamma)/\gamma$ so that $\gamma' = \frac{1}{2}$. When $\gamma \in [P_1, Q_1)$, there exists a continuation value $V_0(\frac{1}{2}) \in (1-\Delta, 1-\frac{1}{2}\Delta]$ such that the indifference condition $V_1^c(\gamma, (1-\gamma)/\gamma) = V_1^p(\gamma, (1-\gamma)/\gamma)$ is satisfied.

[7] The equilibrium play characterized by the following proposition is generically unique subject the symmetry restriction and the restriction on the voting behavior of the extreme players. Since the present game with possibly two rounds of voting is a special case of repeated voting games with a finite deadline, we leave the proof for uniqueness to the next section.

(iv) When the updated belief γ' upon a regular disagreement is in $(\frac{1}{2}, 1]$, we have $V_0(\gamma') = 1 - \Delta$. In this case, the solultion $\hat{x}_1(\gamma)$ to the indifference condition $V_1^c(\gamma, x_1) = V_1^p(\gamma, x_1)$ for the moderate conservative is given by:

$$\hat{x}_1(\gamma) = \frac{\gamma(d+\Delta) - (1-\gamma)(d+\Delta)}{2d\gamma}. \tag{7}$$

For $\gamma > Q_1$, we have $\hat{x}_1(\gamma) \geq (1-\gamma)/\gamma$, so that γ' indeed exceeds $\frac{1}{2}$. Moreover $\hat{x}_1(\gamma) \geq 1$ is equivalent to $V_1^c(\gamma, 1) \geq V_1^p(\gamma, 1)$, in which case $x_1(\gamma) = 1$ is an equilibrium.

To complete the proof, we need to verify that the extreme conservative has no incentive to deviate to voting p. When the moderate progressive votes p with probability x_1, the extreme conservative's payoff from voting c is:

$$W_1^c(\gamma, x_1) = x_1(-d + W_0(\gamma')) + (1 - x_1),$$

and his payoff from voting p is:

$$W_1^p(\gamma, x_1) = x_1(1 - 2\Delta) + (1 - x_1)(-d + W_0(1)).$$

If $x_1 = 0$, the extreme conservative clearly prefers voting c (with an immediate payoff of 1) to voting p (with a payoff of $-d + W_0(\gamma)$). If $x_1 > 0$, we have $V_1^c(\gamma, x_1) \geq V_1^p(\gamma, x_1)$, or

$$\gamma[x_1(-d + V_0(\gamma') - 1 + 2\Delta) + (1 - x_1)(1 + d - V_0(1))] + (1-\gamma)(-d + V_0(\gamma') - 1) \geq 0.$$

The second term of the above expression is strictly negative. Since $V_0(\gamma') \leq W_0(\gamma')$ for all γ' and $V_0(1) = W_0(1)$, the above inequality implies that

$$x_1(-d + W_0(\gamma') - 1 + 2\Delta) + (1 - x_1)(1 + d - W_0(1)) > 0,$$

which is equivalent to $W_1^c(\gamma, x_1) > W_1^p(\gamma, x_1)$. *Q.E.D.*

We say that the moderate conservative "compromises" if he votes c with probability 0. Similarly, the moderate progressive compromises if he votes p with probability 0. In the one-round voting game, the moderate players compromise if and only if $\gamma < \frac{1}{2}$. In the equilibrium of the two-round voting game, the moderate players compromise in the first round of voting if and only if $\gamma < G_1^1$. Since G_1^1 is strictly lower than $\frac{1}{2}$, the moderate players are initially less likely to compromise than they are in the benchmark one-round voting game. More generally, Proposition 1 shows that $x_1(\gamma) \geq x_0(\gamma)$ for any fixed $\gamma < \frac{1}{2}$, and the opposite is true for $\gamma > \frac{1}{2}$. In other words, introducing the possibility of one more round of voting tends the make the moderate players tougher in equilibrium if the degree of conflict is low ($\gamma < \frac{1}{2}$), but it will make the moderate players less tough if the degree of conflict is high ($\gamma > \frac{1}{2}$). The reason for the different implications to the equilibrium voting behavior depending on γ, is that delay is unlikely when the degree of conflict is low so the possibility of re-voting makes the moderate types want to get their own way, while delay is likely with a high degree of conflict so re-voting makes the moderates more willing to comprise.

The opportunity of voting again for a second time in case of disagreement also means that the moderate types can learn from the voting outcome in the first round, even though no exogenous new information arrives between the two rounds. We may think of learning in our model as represented by the moderate types' vote switching between the two rounds. To this end, it is helpful to classify the different types of equilibrium behavior described in Proposition 1 according to the first round voting strategy of the moderate players and how they vote in the second round upon a regular disagreement. When $\gamma \in [0, G_1^1)$, the moderate players compromise in both rounds.[8] We call this a "compromise equilibrium." When $\gamma \in [G_1^1, P_1)$, the moderate conservative randomizes in the first round but switches to a compromise vote (i.e., voting p for sure) upon a regular disagreement.[9] We call this a "switching equilibrium." When $\gamma \in [P_1, Q_1)$, the moderate conservative randomizes in the first round. Upon a regular disagreement, his updated belief is $\gamma' = \frac{1}{2}$, so he continues to randomize in the second round. We call this a "random switching equilibrium." When $\gamma \in [Q_1, 1]$, the moderate conservative randomizes in the first round, but never compromises in

[8] More precisely, the moderate type is ready to comprise again upon a regular disagreement, which can happen only off the equilibrium path.

[9] If $P_1 = \frac{1}{2}$, or equivalently, if $\Delta > 2d$, the moderate conservative actually votes c with probability 1 for $\gamma \in [d/\Delta, \frac{1}{2})$. Upon a regular disagreement, the updated belief stays at $\gamma < \frac{1}{2}$ and the moderate types switch to a comprise in the final round.

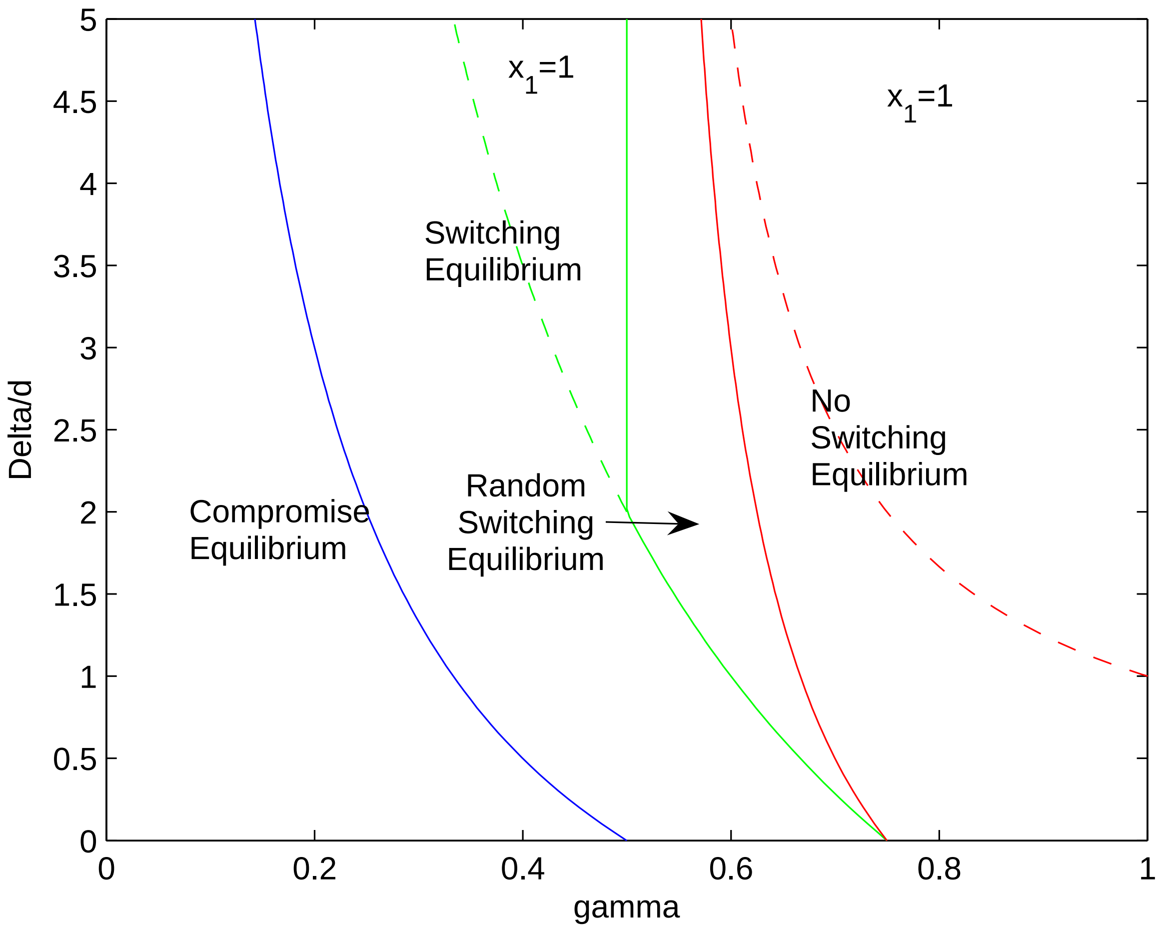

Figure 1

the next round upon a regular disagreement.[10] We call this a "no switching equilibrium."

Which type of equilibria obtains depends only on γ and Δ/d. Figure 1 summarizes these results by partitioning the parameter space into four regions with the different types of equilibria. Note that $x_1(\gamma)$ is not monotone in γ due to the discontinuity in the equilibrium play $x_0(\gamma)$ in the second round voting at $\frac{1}{2}$. In a switching equilibrium or a no switching equilibrium, $x_1(\gamma)$ is increasing in γ, but in a random switching equilibrium, the value of $x_1(\gamma)$ must be such that the updated belief γ' is fixed at $\frac{1}{2}$. In the latter case, the

[10] When $(d + \Delta)/(2\Delta) < 1$, or equivalently, when $\Delta > d$, the moderate conservative actually votes c with probability 1 for $\gamma \in [(d + \Delta)/(2\Delta), 1]$. The updated belief upon a regular disagreement stays at $\gamma > \frac{1}{2}$ and there is no comprise in the final round. In this case, there is a persistent disagreement between the two players.

higher the γ, the lower must be $x_1(\gamma)$. In this case, the moderate players are less tough as the degree of conflict increases. However, in spite of the non-monotonicity of $x_1(\gamma)$, the equilibrium updated belief γ' is increasing in the prior belief γ, with γ' being a constant in $[P_1, Q_1)$, so that a higher degree of conflict in the first round voting in equilibrium results a higher degree of conflict in the second round.

The equilibrium value of $x_1(\gamma)$ is weakly increasing in the error cost Δ and weakly decreasing in the delay cost d (both within each type of equilibrium and across different types). In other words, when the delay cost d is low, or when the cost of implementing the inferior decision Δ is high, the moderate players will behave in a tougher way by voting more like extreme players. In the limit, when delay cost d is very small, the equilibrium outcome in the two-round voting game is a persistent disagreement in both rounds of voting.

3.2. Welfare analysis

Let $V_1(\gamma)$ denote the expected payoff of the moderate conservative in the two-round voting game when his belief at the beginning of the first round is given by $\gamma = \pi_M/(\pi_L + \pi_M)$. Similarly, let $W_1(\gamma)$ be the expected payoff of the extreme conservative. The ex ante welfare of player C is

$$U_1(\gamma) = (\pi_L + \pi_M)V_1(\gamma) + \pi_R W_1(\gamma) = \frac{1}{2-\gamma}V_1(\gamma) + \frac{1-\gamma}{2-\gamma}W_1(\gamma). \tag{8}$$

We first compare U_1 with the equilibrium expected payoff U_0 of the one-round voting game of section 2.

When the degree of conflict between the conservative and the progressive is low, more specifically, when $\gamma < \frac{1}{2}$, a moderate player always yields to his opponent in the one-round voting game, and the resulting equilibrium is Pareto efficient. Adding the possibility of another round of voting cannot improve the ex ante welfare of the players. Indeed, it is easy to show that neither a moderate type nor an extreme type can benefit from having an additional round of voting. Consider the moderate player first. Suppose $x_1(\gamma) < 1$. Then

$$\begin{aligned} V_1(\gamma) &= \gamma[x_1(\gamma)(1-2\Delta) + (1-x_1(\gamma))(-d+V_0(1))] + (1-\gamma) \\ &\le \gamma(1-\Delta) + (1-\gamma) = V_0(\gamma). \end{aligned}$$

Suppose $x_1(\gamma) = 1$. Then

$$V_1(\gamma) = -d + V_0(\gamma) < V_0(\gamma).$$

Next, consider the extreme player. His expected payoff in the two-round game is

$$W_1(\gamma) = x_1(\gamma)(-d + W_0(\gamma')) + (1 - x_1(\gamma)).$$

In the one-round voting game, $W_0(\gamma) = 1$ for $\gamma < \frac{1}{2}$. Hence, $W_1(\gamma) \leq W_0(\gamma)$.

When the degree of conflict is high ($\gamma > \frac{1}{2}$), however, the players cannot both do better than flipping a coin with one round of voting. In any equilibrium of the one-round voting game, the probability of implementing the preferred decision cannot exceed $\frac{1}{2}$. Allowing a second round of voting can potentially improve their ex ante welfare as the moderate player is less tough to his opponent (i.e., $x_1(\gamma) \leq x_0(\gamma)$). Therefore the probability of implementing the preferred alternative in states R and L can be higher than $\frac{1}{2}$. To see this, note that in state R, it is always an extreme conservative (who votes c in both periods) that meets a moderate progressive. Any disagreement between the two players in the first round is a regular disagreement, and we can calculate the overall probability of implementing c (in either round). This is given by

$$q_R = \begin{cases} \frac{1}{2}x_1(\gamma) + 1 - x_1(\gamma) & \text{in no switching equilibrium,} \\ x_1(\gamma)(\frac{1}{2}x_0(\frac{1}{2}) + 1 - x_0(\frac{1}{2})) + 1 - x_1(\gamma) & \text{in random switching equilibrium,} \\ 1 & \text{in switching equilibrium.} \end{cases}$$

Similarly, the overall probability of implementing c in state L is

$$q_L = \begin{cases} \frac{1}{2}x_1(\gamma) & \text{in no switching equilibrium,} \\ \frac{1}{2}x_1(\gamma)x_0(\frac{1}{2}) & \text{in random switching equilibrium,} \\ 0 & \text{in switching equilibrium.} \end{cases}$$

Thus in these three types of equilibria, we have

$$q_R > \frac{1}{2} > q_L$$

if $x_1(\gamma) < 1$. From the analysis in section 2.3 we know that no incentive compatible mechanism without side transfers can implement a social choice function with $q_R > q_L$.

This result does not apply to our two-round voting game, because we allow the possibility that players may have to pay an additional cost d, which is dissipated as delay rather than transferred to the other player. Thus, it is the possibility of budget breaking that is responsible for potentially improving the equilibrium outcome.[11] Formally, we have the following proposition.

PROPOSITION 2. *For each $\gamma \in (\frac{1}{2}, 2-\sqrt{2})$, and only for these values of γ, there exists an interval of values for Δ/d such that $U_1(\gamma) > U_0(\gamma)$.*

The proof of Proposition 2 is in the Appendix, where we give the exact lower bound and upper on Δ/d as functions of γ such that the ex ante payoff of each player is strictly higher in an equilibrium of the two-round voting game than in the equilibrium of the single-round game. Intuitively, welfare gains from introducing the second round of voting exist only when the degree of conflict is not too high. In particular, when γ is close to 1 so that there is little room for a first-round comprise or subsequent voting switching, the impact of adding the second round is that the players incur the cost of delay the eventual disagreement. More generally, Proposition 2 shows that when the degree of conflict is sufficiently high ($\gamma > 2-\sqrt{2}$), the improvement in the quality of the decision is outweighed by the direct cost of delay. Figure 2 provides a graphical depiction of the region of parameter space for which welfare is higher under the two-round game than under the benchmark one-round game. Note that this region of welfare improvement contains a subset of each region in the parameter space corresponding to a no switching equilibrium, a random switching equilibrium, and a switching equilibrium. In other words, for each of these three types of equilibria, there are values of γ, Δ, and d such that delay increases the welfare of each player.

It is clear from Figure 2 that the cost of delay has a non-monotone effect on players' welfare. Fix γ at some value between $\frac{1}{2}$ and $2-\sqrt{2}$. When d is very low, the equilibrium is a no switching equilibrium with $x_1(\gamma) = 1$. Since there is complete disagreement between the players in both rounds, the only effect of introducing an additional round of voting

[11] Our idea of using delay as a mechanism to improve collective decision making shares the same logic as in Holmstrom's (1982) model of moral hazard in teams and Myerson and Satterthwaite model of bilateral trading with asymmetric information.

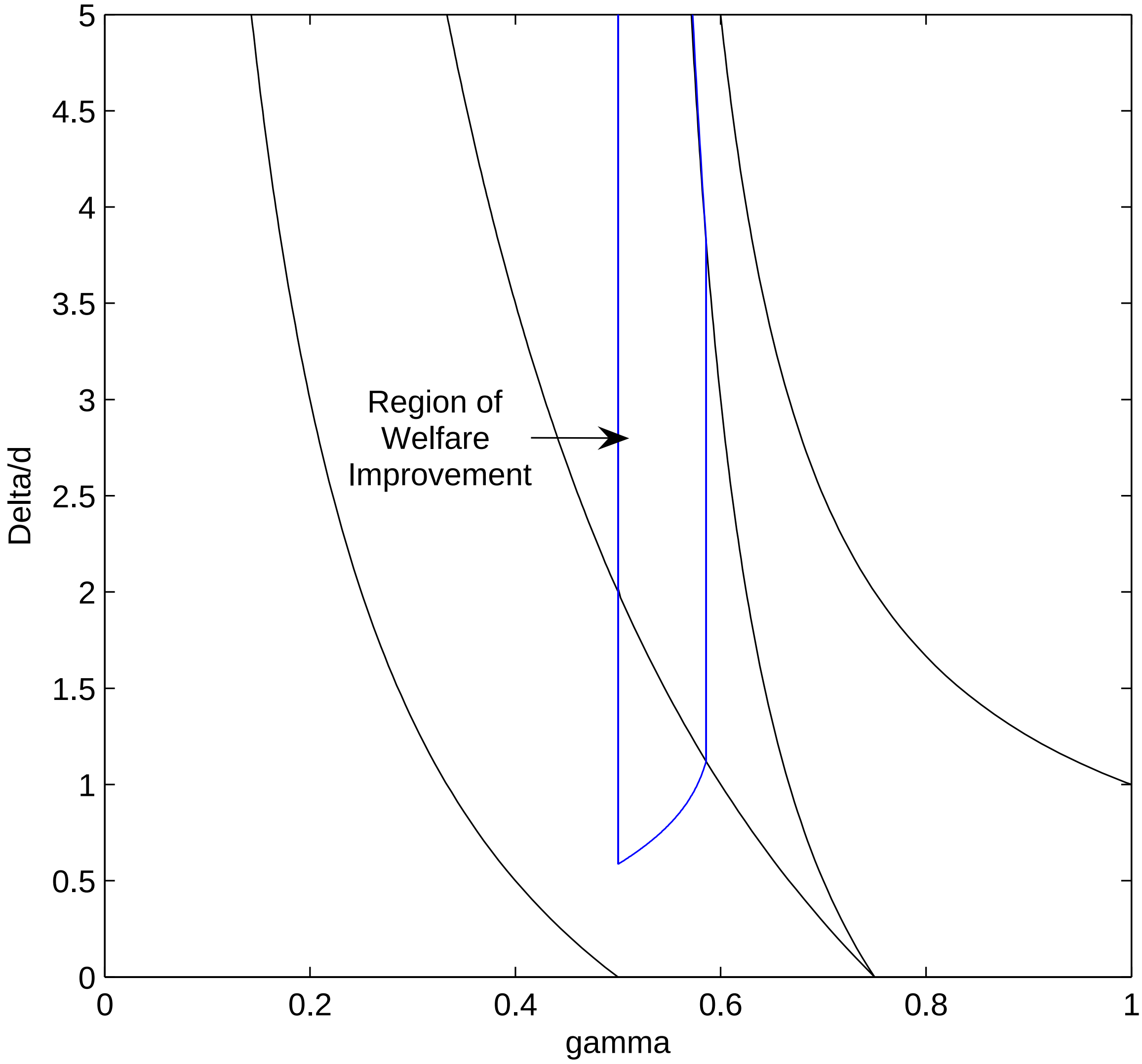

Figure 2

is that players have to pay the delay cost. As d increases, $x_1(\gamma)$ eventually becomes strictly less than 1. A lower $x_1(\gamma)$ is beneficial to the players when the state is L or R because it increases the chance of arriving at the preferred decision in these states. When d is sufficiently large, the equilibrium is a switching equilibrium and the probability of reaching the preferred decision in states L and R is equal to 1. However, the probability of incurring the delay cost, given by

$$(\pi_L + \pi_R)x_1(\gamma) + \pi_M \left[x_1(\gamma)^2 + (1 - x_1(\gamma))^2\right],$$

is increasing in d as $x_1(\gamma)$ becomes very small, so that the total delay cost necessarily increases. Beyond a certain point, the direct cost of delay outweighs the benefits from improving the quality of decisions, and the two-round voting game yields lower ex ante welfare than the benchmark one round voting game. Nevertheless, Proposition 2 shows that there is an intermediate range of d such that introducing the possibility of delay will strictly improve the welfare of the players.

4. Repeated Voting with More than Two Rounds

Consider a repeated voting game with a deadline T; we allow T to be infinity. By "deadline" we mean the maximum number of rounds in which the two players can vote again in case of disagreement. If in any round of voting, the votes of the two players agree, then that decision is implemented and the payoffs are realized. Each additional round of voting is associated with an additional delay cost of d to each player. We index the voting rounds in the reverse order, with the first round being round T, and the final round being round 0. The game considered in section 3 is a special case, with $T = 1$.

A non-terminal history in this game at some round of voting consists of the first moves by nature, which determine the permanent type of each player, and subsequent disagreeing votes cast by the two players. An information set for a player of a given type is a collection of all histories that begin with the same nature's move determining the player's type and share the same sequence of disagreeing votes. A strategy of a player is a sequence of randomizations over the two votes for each of his information sets, and a belief system is a sequence of probability measures over histories for each information set. A sequential equilibrium is a profile of strategies and a belief system that are sequentially rational and consistent (Kreps and Wilson, 1982). This seems very complicated, but note that the only unobserved component of a terminal history that affects the payoff to each player is the permanent type of his opponent, or equivalently, whether the state is M or not. We therefore restrict equilibrium strategies of each player such that the vote cast at all information sets in a given round of voting depends only on his belief that the state is M, and simultaneously restrict the belief system to the collection of beliefs of each player at each information set that the state is M. The notions of sequential rationality and consistency can be applied in a straightforward fashion to the restricted strategy profiles and belief systems. We refer to the resulting solution concept as "equilibrium." We look for equilibria such that in each round of voting on and off the equilibrium path: (i) the extreme types always vote according to their preferences; and (ii) for each pair of information sets of the moderate types that share the same sequence of disagreeing votes, the two types have identical beliefs about the state being M and vote according to their preferences with the same probability. Note that on the equilibrium path, the notion of consistency in the definition of sequential equilibrium implies that, if the moderate types vote according to their preference with the same probability then they have the same belief about the state being M after any observed sequence of disagreeing votes.

4.1. Repeated voting with a finite deadline

We will establish existence and uniqueness of equilibrium for the game with a finite deadline T by backward induction. Backward induction is equivalent to mathematical induction on the deadline T, as a repeated voting game with a finite deadline T and initial belief of the moderate types that the state is M with probability γ can be alternatively viewed as the equilibrium continuation game with $T + 1$ rounds of voting remaining and belief γ for a game with a longer deadline.

Let t be the number of rounds of voting remaining before the final round, including the current round; we refer to the current round as round-t voting. Denote as $x_t(\gamma)$ the common equilibrium probability of the moderate types voting according to their preferences in a round-t voting when the common belief of the moderate types is that the state is M with probability γ. Let $V_t(\gamma)$ and $W_t(\gamma)$ be the corresponding equilibrium payoffs for the moderate types and the extreme types. The induction hypothesis is that for any $t = T - 1, \ldots, 0$ and for any prior belief γ of the moderate types that the state is M, in the repeated voting game with a deadline t: (i) there is a unique equilibrium, except when $t < \Delta/(2d)$ and $\gamma = \frac{1}{2}$, in which case there is a continuum of equilibria satisfying $x_\tau(\frac{1}{2}) = 1$ for all $\tau = t, \ldots, 1$; (ii) $V_t(\gamma)$ is a continuous and piecewise linear function of γ, except when $t < \Delta/(2d)$ and $\gamma = \frac{1}{2}$, in which case $V_t(\frac{1}{2})$ takes any value on the interval $\left[\lim_{\gamma \downarrow \frac{1}{2}} V_t(\gamma), \lim_{\gamma \uparrow \frac{1}{2}} V_t(\gamma)\right]$; and (iii) $V_t(1) = W_t(1)$, and $V_t(\gamma) \leq W_t(\gamma)$ for all γ, except when $t < \Delta/(2d)$ and $\gamma = \frac{1}{2}$, in which case $V_t(\frac{1}{2}) \leq W_t(\frac{1}{2})$ for any selection of the equilibria. Note from equations (1) and (2) the above properties are satisfied for $T = 0$.

Given the induction hypothesis, we write $V_t^c(\gamma, x_t)$ as the expected payoff to the moderate conservative from voting c in round t, when the belief that the state is M is γ and when the moderate progressive votes p with probability x_t. We have:

$$V_t^c(\gamma, x_t) = \gamma(x(-d + V_{t-1}(\gamma')) + (1 - x_t)) + (1 - \gamma)(-d + V_{t-1}(\gamma')) \tag{9}$$

for all γ and x_t such that $\gamma' = \gamma x_t/(\gamma x_t + 1 - \gamma)$ is defined and is not equal to $\frac{1}{2}$. Next, let $V_t^p(\gamma, x_t)$ be the corresponding expected payoff to the moderate conservative from voting p, given by

$$V_t^p(\gamma, x_t) = \gamma[(x_t(1 - 2\Delta) + (1 - x_t)(-d + V_{t-1}(1))] + (1 - \gamma_t) \tag{10}$$

for any $x_t < 1$. Since $V_{t-1}(\gamma')$ is piecewise linear, we can write it as

$$V_{t-1}(\gamma') = a_{t-1} - b_{t-1}\gamma'.$$

Applying the Bayes' formula, we have $V_t^c(\gamma, x_t) - V_t^p(\gamma, x_t)$ is given by

$$\gamma[x_t(-d + a_{t-1} - b_{t-1} - 1 + 2\Delta) + (1 - x_t)(1 + d - V_{t-1}(1))] + (1 - \gamma)(-d + a_{t-1} - 1).$$

Note that $V_t^c(\gamma, x_t) - V_t^p(\gamma, x_t)$ is decreasing in x_t if

$$u_{t-1} \equiv (1 + d - V_{t-1}(1)) + (1 + d - a_{t-1}) + (b_{t-1} - 2\Delta) > 0.$$

Also, if in equilibrium $x_t \in (0, 1)$ round-t voting, we must have $V_t^c(\gamma, x_t) = V_t^p(\gamma, x_t)$, which implies

$$x_t = \frac{\gamma(1 + d - V_{t-1}(1)) - (1 - \gamma)(1 + d - a_{t-1})}{\gamma u_{t-1}}.$$

Substituting this value of x_t into $V_t^c(\gamma, x_t)$ and setting $V_t(\gamma) = a_t - b_t\gamma$, we obtain a pair of difference equations:

$$\begin{aligned} a_t &= 1 - \frac{(1 + d - V_{t-1}(1) - 2\Delta)(1 + d - a_{t-1})}{u_{t-1}}, \\ b_t &= 2\Delta + \frac{(1 + d - V_{t-1}(1) - 2\Delta)(b_{t-1} - 2\Delta)}{u_{t-1}}. \end{aligned} \tag{11}$$

To complete the induction proof for the existence and uniqueness of equilibrium, we need two preliminary results that can be established by separate induction arguments. For the first result, let s be the smallest integer that is strictly greater than Δ/d; note that $s \geq 1$. We have:

Lemma 1. *(i) For any $t \leq s-1$, in any equilibrium $x_t(1) = 1$ and $V_t(1) = 1 - \Delta - td$; (ii) for any $t \geq s$, in any equilibrium $x_t(1) \in (0,1)$ and $V_t(1)$ satisfies*

$$1 - V_t(1) = \frac{(1 + d - V_{t-1}(1))^2}{2(1 + d - V_{t-1}(1) - \Delta)};$$

(iii) $V_t(1) > 1 - 2\Delta$ for all $t \leq s-2$ and $V_t(1) < 1 - 2\Delta$ for all $t \geq s-1$; (iv) $\{V_t(1)\}$ is a monotonically decreasing sequence, with the limit $V_\infty(1) = 1 - \Delta - \sqrt{d^2 + \Delta^2}$.

The proof of Lemma 1 is in the appendix. A key to the proof is that once the moderates' belief that the state is M becomes 1, it will remain 1 for the remainder of the game. This allows to use an induction argument independent of the establishment of equilibrium existence and uniqueness for the whole game. From the proof we immediately obtain that $x_t(1)$ is strictly decreasing in t for $t \geq s-1$. Thus, in the unique equilibrium continuation play after a reverse disagreement in the previous round, the moderate types randomize between the two votes if sufficiently many rounds of voting remain ($t \geq s$), but become tougher as the deadline approaches and eventually revert to voting according to their preferences.[12] Finally, note that since $W_0(1) = V_0(1)$ and $x_t(1) > 0$ for all t, a straightforward induction argument establishes that $W_t(1) = V_t(1)$ for all t.

For Lemma 2 below, we study the difference equations (11), using the sequence $\{V_t(1)\}$ characterized by Lemma 1. From equation (1), there are two sets of initial conditions: $a_0 = 1$ and $b_0 = \Delta$, corresponding to $\gamma < \frac{1}{2}$ in the final round; and $a_0 = 1 - \Delta$ and $b_0 = 0$, corresponding to $\gamma > \frac{1}{2}$. The proof is in the appendix.

Lemma 2. *(i) $u_t > 0$ for all t; (ii) $b_t \geq 0$ for all t; (iii) $a_t - \frac{1}{2}b_t < 1 - \Delta$ for all $t \geq s$; and (iv) $1 + d - a_t \geq 0$ for all $t \geq s$.*

[12] Thus, when the deadline is long enough, the equilibrium behavior of the moderate types when $\gamma = 1$ is different from that of the extreme types, who have the same belief.

Now we are ready to complete the induction argument. Proposition 3 below is the main characterization result in repeated games with a finite deadline.

PROPOSITION 3. *For any finite deadline T, there is a unique equilibrium, except when $T < \Delta/(2d)$ and $\gamma = \frac{1}{2}$, in which case there is a continuum of equilibria with $x_t(\frac{1}{2}) = 1$ for all $t = T, T-1, \dots, 1$.*

PROOF. First we extend the payoff functions $V_T^c(\gamma, x_T)$ and $V_T^p(\gamma, x_T)$, given by equations (9) and (10) respectively with $t = T$. Let $V_T^c(1,0) = 1$, and $V_T^p(\gamma,1) = \gamma(1-2\Delta)+(1-\gamma)$. For each $\gamma \in [\frac{1}{2}, 1]$, we let $V_T^c(\gamma, (1-\gamma)/\gamma)$ be multi-valued and recursively given by $2(1-\gamma)(V_{T-1}(\frac{1}{2}) - d) + (2\gamma - 1)$.

Next, we claim that the necessary and sufficient conditions for equilibrium in the repeated game with deadline T are: there is an equilibrium with $x_T = 0$ if and only if $V_T^c(\gamma, 0) \leq V_T^p(\gamma, 0)$; there is an equilibrium with $x_T \in (0,1)$ if and only if $V_T^c(\gamma, x_T) = V_T^p(\gamma, x_T)$, or, there exists a value of $V_T^c(\gamma, (1-\gamma)/\gamma)$ equal to $V_T^p(\gamma, (1-\gamma)/\gamma)$ in the case of $T < \Delta/(2d)$, $\gamma \in [\frac{1}{2}, 1)$ and $x_T = (1-\gamma)/\gamma$; and there is an equilibrium with $x_T = 1$ if and only if $V_T^c(\gamma, 1) \geq V_T^p(\gamma, 1)$. The necessity of the condition for each of the three cases above is a straightforward application of the one-deviation principle, if the Bayes' rule applies. Further, when the Bayes' rule does not apply, by construction the choices of the out-of-equilibrium beliefs do not affect the values of $V_T^c(1,0)$ and $V_T^p(\gamma, 1)$. To establish the sufficiency of the conditions, we only need to show that in each case the extreme conservative does not want to deviate from voting c in round-T voting. This follows from part (iii) of the induction hypothesis and an identical argument as in the proof of Proposition 1.

Define $D_T(\gamma, x_T) = V_T^c(\gamma, x_T) - V_T^p(\gamma, x_T)$. Then, for any γ, D_T is a function of x_T, with the proviso that D_T is multi-valued at $x_T = (1-\gamma)/\gamma$ for any $\gamma \geq \frac{1}{2}$, as $V_T^c(\gamma, (1-\gamma)/\gamma)$ is multi-valued. We claim that for any fixed $\gamma \in [0,1]$, D_T is a piece-wise linear, continuous and strictly decreasing function of x_T, except at $x_T = (1-\gamma)/\gamma$ for $T < \Delta/(2d)$ and $\gamma \in [\frac{1}{2}, 1)$ when D_T is multi-valued; $D_T(\gamma, (1-\gamma)/\gamma) = \left[\lim_{x_T \downarrow (1-\gamma)/\gamma} D_T(\gamma, x_T), \lim_{x_T \uparrow (1-\gamma)/\gamma} D_T(\gamma, x_T)\right]$ for any $\gamma \in (\frac{1}{2}, 1)$; and $D_T(\frac{1}{2}, 1) = \left[-d, \lim_{x_T \uparrow 1} D_T(\frac{1}{2}, x_T)\right]$. Given the definitions of V_T^c and V_T^p, and part (i) and part (ii) of the induction hypothesis, we only need to show that D_T is strictly decreasing in x_T except at $x_T = (1-\gamma)/\gamma$ for $T < \Delta/(2d)$ and $\gamma \in [\frac{1}{2}, 1)$, which follows from part (i) of Lemma 2.

It follows from the above properties of D_T and the equilibrium conditions that: if $D_T(\gamma, 0) \leq 0$, then there is a unique equilibrium with $x_T = 0$; if $D_T(\gamma, 0) > 0$, and either $D_T(\gamma, 1) < 0$ when $\gamma \neq \frac{1}{2}$ or $\max D_T(\frac{1}{2}, 1) < 0$, then there is a unique equilibrium with $x_T \in (0, 1)$; if $D_T(\gamma, 1) \geq 0$ when $\gamma \neq \frac{1}{2}$ then there is a unique equilibrium with $x_T = 1$; and if $\max D_T(\frac{1}{2}, 1) > 0$, or equivalently if $\gamma = \frac{1}{2}$ and $\Delta > 2Td$, then there is a continuum of equilibria with $x_T = 1$, each associated with a value of $V_0(\frac{1}{2})$ or equivalently a value of $x_0(\frac{1}{2})$ satisfying that $D_T(\frac{1}{2}, 1) \geq 0$. *Q.E.D.*

The model of repeated voting games with finite deadlines and the above characterization result allow us to analyze the "deadline effect": does extending the deadline by one more round of voting in case of disagreement improve or reduce the ex ante payoff of the two players? To answer this question, we first note that the deadline effect is negative for if the degree of conflict as measured by γ is extreme, in the sense that for all finite deadline T any possibility of re-voting is worse than flipping a coin. At one extreme, for $\gamma = 1$, from Lemma 1 we have $W_T(1) = V_T(1) < V_0(1) = W_0(1) = 1 - \Delta$ for all finite deadline T.[13] At the other extreme, since the equilibrium outcome in the one-round voting game (with $T = 0$) is efficient for $\gamma < \frac{1}{2}$, we have $V_T(\gamma) < 1 - \Delta\gamma = V_0(\gamma)$ for the moderate types while $W_T(\gamma) \leq 1 = W_0(\gamma)$. Given that the deadline effects are negative for extreme values of γ , it is natural to consider γ just above $\frac{1}{2}$. As suggested by Proposition 2, if the deadline effect is positive, then it is likely to be the most pronounced for γ just above $\frac{1}{2}$. For the following proposition, let $U_T(\gamma)$ be the ex ante equilibrium payoff for each player in the repeated game with deadline T and prior belief of the moderate types that the state is M with probability γ. Define

$$Q_T = \frac{(T+2)d + \Delta}{2(T+1)d + 2\Delta};$$

[13] This is perhaps not surprising, but note that $V_T(1)$ and $W_T(1)$ decrease with T in spite of the fact that $x_T(1)$ weakly decreases with T. That is, despite of softening positions by the moderate types, a longer deadline only worsens the situation for the two players when the prior level of conflict is sufficiently high.

note that it is strictly greater than $\frac{1}{2}$ and coincides with the value given in Proposition 1 when $T = 1$.[14]

Proposition 4. *If* $\gamma \in (\frac{1}{2}, Q_T]$ *and* $2 \leq T < \Delta/(2d)$*, then* $U_T(\gamma) > U_{T-1}(\gamma)$.

Proof. Since $T - 1 < T < \Delta/(2d)$, by Proposition 3 there is a continuum of equilibria at $\gamma = \frac{1}{2}$ in the repeated voting game with deadline $T - 1$, each of which is associated with a probability $x_0(\frac{1}{2})$ of the moderate types voting according to preferences in the final round. We argue that in the repeated voting game with deadline T, there is an interval of values of γ, which is given by $(\frac{1}{2}, Q_T]$, such that $x_T(\gamma) = (1-\gamma)/\gamma$, the updated belief for the moderate types upon a regular disagreement is $\frac{1}{2}$, and each γ on this interval is associated with a value of $x_0(\frac{1}{2})$. Given this equilibrium, the expected payoff from voting c in round T for a moderate conservative with belief γ is:

$$\begin{aligned} V_T^c(\gamma, (1-\gamma)/\gamma) &= 2(1-\gamma)\Big(- d + V_{T-1}\Big(\frac{1}{2}\Big)\Big) + (2\gamma - 1) \\ &= 2(1-\gamma)\Big(- Td + 1 - \frac{1}{2}\Delta - \frac{1}{2}x_0\Big(\frac{1}{2}\Big)\Delta\Big) + (2\gamma - 1), \end{aligned}$$

where the second equality follows from induction. The expected payoff from voting p in round T is

$$\begin{aligned} V_T^p(\gamma, (1-\gamma)/\gamma) &= 2(1-\gamma)(1-\Delta) + (2\gamma - 1)(-d + V_{T-1}(1)) \\ &= 2(1-\gamma)(1-\Delta) + (2\gamma - 1)(-Td + 1 - \Delta), \end{aligned}$$

where the second equality follows Lemma 1 because $T < \Delta/(2d) \leq s - 1$. Note that $V_T^p(\gamma, (1-\gamma)/\gamma)$ is decreasing in γ, which implies that a greater γ in the interval $(\frac{1}{2}, Q_T]$ is associated with a greater $x_0(\frac{1}{2})$ and hence a lower value of $V_0(\frac{1}{2})$. Solving for $x_0(\frac{1}{2})$ from $V^c(\gamma, (1-\gamma)/\gamma) = V^p(\gamma, (1-\gamma)/\gamma)$ gives

$$x_0\Big(\frac{1}{2}\Big) = \frac{\gamma}{1-\gamma}\frac{Td + \Delta}{\Delta} - \frac{3Td}{\Delta}. \tag{12}$$

[14] For $T = 1$, we already know from Proposition 2 the necessary and sufficient conditions for $U_1(\gamma) > U_0(\gamma)$. The argument below for Proposition 4 applies intact for the case of $T = 1$, but when $T = 1$ and $\Delta/(2d)$ arbitrarily close to 1, Q_1 is greater than $2 - \sqrt{2}$. This is consistent with Figure 2 which shows that when $\Delta/(2d)$ is approaching 1 from above, the region of γ such that $U_1(\gamma) > U_0(\gamma)$ is a sub-interval of $(\frac{1}{2}, Q_1]$.

For γ greater than but arbitrarily close to $\frac{1}{2}$, we have that $x_0(\frac{1}{2}) = 1 - 2Td/\Delta$. At the other end, the upper bound Q_T of the interval is determined by the largest value of $x_0(\frac{1}{2})$, which from the proof of Proposition 3 satisfies $V^c_{T-1}(\frac{1}{2}, 1) = V^p_{T-1}(\frac{1}{2}, 1)$. This yields $x_0(\frac{1}{2}) = 1 - 2(T-1)d/\Delta$. Substituting this value of $x_0(\frac{1}{2})$ into equation (12) gives the largest value of γ for which $x_T(\gamma) = (1-\gamma)/\gamma$. Straightforward calculations show that the value is Q_T as defined above.

For any $\gamma \in (\frac{1}{2}, Q_T]$, the equilibrium payoff to the moderate types in the repeated voting game with deadline T is given by $V_T(\gamma) = V^p_T(\gamma, (1-\gamma)/\gamma)$. The equilibrium payoff to the extreme types is given by

$$\begin{aligned} W_T(\gamma) &= x_T(\gamma)\Big(- d + W_{T-1}\Big(\frac{1}{2}\Big)\Big) + (1 - x_T(\gamma)) \\ &= \frac{1-\gamma}{\gamma}\Big(- Td + W_0\Big(\frac{1}{2}\Big)\Big) + \frac{2\gamma-1}{\gamma} \\ &= \frac{1-\gamma}{\gamma}\Big(- Td + 1 - x_0\Big(\frac{1}{2}\Big)\Delta\Big) + \frac{2\gamma-1}{\gamma}, \end{aligned}$$

where we have used the fact that $x_T(\gamma) = (1-\gamma)/\gamma$, and that $x_{T-1}(\frac{1}{2}) = \ldots = x_1(\frac{1}{2}) = 1$. After substituting the expression for $x_0(\frac{1}{2})$ given in equation (12) into $W_T(\gamma)$ and applying a version of equation (8) for T, we have for any $\gamma \in (\frac{1}{2}, Q_T]$,

$$\begin{aligned} U_T(\gamma) =& \frac{1}{2-\gamma}(2(1-\gamma)(1-\Delta) + (2\gamma - 1)(-Td + 1 - \Delta)) \\ &+ \frac{1-\gamma}{2-\gamma}\left(\frac{1-\gamma}{\gamma}(2Td+1) - (Td + \Delta) + \frac{2\gamma-1}{\gamma}\right). \end{aligned} \tag{13}$$

It is straightforward to see that $U_T(\gamma)$ is increasing T if and only if $\gamma < 2 - \sqrt{2}$. Finally, given the definition of Q_T, we can verify that if $T \geq 2$ and hence $\Delta/(2d) > 2$, then $Q_T < 2 - \sqrt{2}$. *Q.E.D.*

If we define the "optimal deadline" as the number of re-voting rounds that maximizes the ex ante welfare of the players, then the above proposition implies that the optimal deadline is not only positive for intermediate values of γ, as shown in Proposition 2, but also no smaller than $\Delta/(2d)$.[15] Thus, if the delay cost d is small, or if the error cost Δ is

[15] For $T > \Delta/(2d)$, we can show that for γ just above $\frac{1}{2}$, $V_T(\gamma) > V_0(\gamma)$ if $T < \Delta/d$, but the opposite is true if $T > \Delta/d$. This follows from the observation that in equilibrium the moderate types randomize between c and p in round T voting, implying $V_T(\gamma) = V^p_T(\gamma, x_T)$ with x_T arbitrarily close to $x_T(\frac{1}{2})$, and the characterization of $V_{T-1}(1)$ from part (iii) of Lemma 1. Since $W_T(\gamma) \geq V_T(\gamma)$ and since $W_0(\gamma) = V_0(\gamma)$, we have $U_T(\gamma) > U_0(\gamma)$. However, we do not know if how $U_T(\gamma)$ compares with $U_{T+1}(\gamma)$ for $T > \Delta/(2d)$.

large, it is likely that extending the deadline by one more round in case of disagreement is beneficial. Note that from equation (13) in the proof, the ex ante payoff $V_T(\gamma)$ of the moderate types is decreasing in T, while the payoff $W_T(\gamma)$ to the extreme types is increasing in T, with the latter dominating in the average payoff $U_T(\gamma)$.

As in the repeated game with possibly two rounds analyzed in section 3, when the deadline T is relatively short, the discontinuity in the equilibrium play in the final round generates an interval values of γ on which $x_T(\gamma)$ is decreasing. However, when the deadline is sufficiently long, we expect the impact of the discontinuity in the final round play to dissipate, as suggested by Proposition 3 because the equilibrium is unique when the deadline T exceeds $\Delta/(2d)$. In fact, part (iv) of Lemma 2 implies that $x_T(\gamma)$ is increasing in γ when $T \geq s$.[16] Since the updated belief γ' after a regular disagreement satisfies

$$\frac{\gamma'}{1-\gamma'} = \frac{\gamma}{1-\gamma} x_T(\gamma) = \frac{\gamma(1+d-V_{T-1}(1)) - (1-\gamma)(1+d-a_{T-1})}{(1-\gamma)u_{T-1}},$$

which is increasing in γ, we expect the equilibrium play in repeated voting games with sufficiently long deadlines to have the feature that in the initial rounds of voting the moderate types are increasingly more willing to comprise. However, such a conclusion would require us to argue that $x_t(\gamma)$ changes little if t is sufficiently large. In fact, once the deadline is near, $x_t(\gamma)$ is no longer monotone even though the updated belief continues to decrease. Further, Lemma 1 shows that $x_t(1)$ is weakly decreasing in t. Since the updated belief always stays at 1, it is not true that the players are increasingly more willing to comprise in the initial rounds when they are almost certain that the state is the disagreement state. In the next subsection we analyze a repeated voting game without a deadline T in which these reasons for the non-monotonicity of the equilibrium play disappear.

4.2. Repeated voting without deadline

Consider a game in which players can vote repeatedly for an indefinite number of rounds as long as they do not agree. We will first construct a stationary equilibrium and then argue

[16] Moreover, part (iii) of Lemma 2 states that $a_t - \frac{1}{2}b_t < 1 - \Delta$ for all $t \geq s$, which implies that $V_t(\gamma) = a_t - b_t\gamma < 1 - \Delta = V_0(\gamma)$ for $\gamma > \frac{1}{2}$, and therefore the moderate types are worse off with any deadline longer than s compared to a single round of voting.

that it is unique. For each belief $\gamma \in [0, 1]$ that the moderate types hold regarding the disagreement state M, we denote as $x(\gamma)$ the equilibrium probability that the moderate types vote according to their preferences.[17] Upon a regular disagreement, the moderate players' belief that the state is M is revised downward to

$$\gamma' = \frac{\gamma x(\gamma)}{\gamma x(\gamma) + 1 - \gamma},$$

if it is not the case that $\gamma = 1$ and $x(1) = 0$. Upon a reverse disagreement, on the other hand, the moderates are sure that his opponent is a moderate and therefore his belief that the state is M jumps to 1, unless $x(\gamma) = 1$.

As in the case of finite deadlines, we construct an equilibrium by first considering the equilibrium play when the moderate players believe that the state is M with probability 1. It is straightforward to see that in any equilibrium $x(1) \in (0, 1)$. If the moderate progressive votes p with probability 1, then for the moderate conservative the outcome from voting c would be delay forever, which is strictly worse than conceding by voting p; and if the moderate progressive votes c with probability 1, then voting c would bring an immediate agreement and strictly dominate voting p. Neither case can be an equilibrium. It then follows from the indifference condition that

$$V(1) = x(1)(-d + V(1)) + (1 - x(1)) = x(1)(1 - 2\Delta) + (1 - x(1))(-d + V(1)).$$

Solving these two equations gives a unique pair of equilibrium values

$$\begin{aligned} V(1) &= 1 - \Delta - \sqrt{d^2 + \Delta^2}, \\ x(1) &= \frac{-d + \Delta + \sqrt{d^2 + \Delta^2}}{2\Delta}. \end{aligned} \tag{14}$$

We note that $V(1) < 1 - 2\Delta$, and it is the same value identified in part (iv) of Lemma 1, where we have used the different notation of $V_\infty(1)$.

Next, consider the equilibrium play when $\gamma = 0$. Since the moderate conservative believes that the state is L and his opponent (who is an extreme progressive) votes p, voting p to obtain the preferred decision is strictly better than voting c. Thus, we have

[17] For notational brevity, we will drop the subscripts ∞ from all variables.

$x(0) = 0$ and $V(0) = 1$. Given this, when γ is positive but sufficiently small the moderate players vote against their preferences with probability 1. To see this, note that $x(\gamma) = 0$ implies that the updated belief upon a regular disagreement is $\gamma' = 0$. Therefore, the payoff to a moderate conservative from voting c is

$$V^c(\gamma, 0) = \gamma + (1-\gamma)(-d + V(0)),$$

and his payoff from voting p is

$$V^p(\gamma, 0) = \gamma(-d + V(1)) + (1-\gamma).$$

Voting p is strictly preferred to voting c if and only if

$$\gamma < \frac{d}{(1 + d - V(1)) + d} \equiv G^1.$$

Therefore, when $\gamma < G^1$, in equilibrium the moderate conservative votes p and his corresponding equilibrium payoff takes the linear form of

$$V(\gamma) = 1 - (1 + d - V(1))\gamma. \tag{15}$$

Conversely, when $\gamma > G^1$, in any equilibrium we have $x(\gamma) > 0$.

We refer to the interval $[0, G^1]$ as the "compromise region." For γ just above G^1, it is natural to conjecture that the equilibrium $x(\gamma)$ is such that the one-step updated belief γ' falls into the compromise region. We may then try to identify some one-step interval $[G^1, G^2]$, and so on. This conjecture turns out to be correct. That is, there exists an infinite sequence, $G^0 < G^1 < G^2 < \ldots$, with $G^0 = 0$ and $\lim_{k\to 1} G^k = 1$, such that if $\gamma \in (G^k, G^{k+1}]$ for $k = 1, 2, \ldots$, then $x(\gamma)$ is such that $\gamma' = \gamma x(\gamma)/(\gamma x(\gamma) + 1 - \gamma) \in (G^{k-1}, G^k]$. Furthermore, we conjecture that the value function is piecewise linear of the form $V(\gamma) = a^k - b^k\gamma$ for $\gamma \in (G^k, G^{k+1}]$, with $a^0 = 1$ and $b^0 = 1 + d - V(1)$ from equation (15). Let $\gamma \in (G^k, G^{k+1}]$ for $k \geq 1$. Given the conjecture, the expected payoff to the moderate conservative from voting c is

$$\begin{aligned} V^c(\gamma, x) &= (\gamma x + 1 - \gamma)(-d + a^{k-1} - b^{k-1}\gamma') + \gamma(1-x) \\ &= (\gamma x + 1 - \gamma)(-d + a^{k-1}) - \gamma x b^{k-1} + \gamma(1-x). \end{aligned}$$

The payoff from voting p is

$$V^p(\gamma, x) = \gamma[x(1-2\Delta) + (1-x)(-d+V(1))] + (1-\gamma).$$

The moderate conservative is indifferent between voting c and voting p when x is given by

$$x(\gamma) = \frac{\gamma(1+d-V(1)) - (1-\gamma)(1+d-a^{k-1})}{\gamma((1+d-V(1)) + (1+d-a^{k-1}) + (b^{k-1}-2\Delta))}. \tag{16}$$

Substituting the above into the expression for V^c, we obtain $V(\gamma) = a^k - b^k\gamma$, where

$$\begin{aligned} a^k &= 1 - \frac{(1+d-a^{k-1})(b^0-2\Delta)}{b^0 + (1+d-a^{k-1}) + (b^{k-1}-2\Delta)}, \\ b^k &= 2\Delta + \frac{(b^0-2\Delta)(b^{k-1}-2\Delta)}{b^0 + (1+d-a^{k-1}) + (b^{k-1}-2\Delta)}. \end{aligned} \tag{17}$$

Equation (17) is a pair of difference equations for the sequence $\{(a^k, b^k)\}$. We have the following preliminary result regarding (17).

LEMMA 3. *(i)* $a^k \leq 1$ *and* $b^k > 2\Delta$ *for all* k*; (ii) both* a^k *and* b^k *are decreasing in* k*; and (iii)* $\lim_{k\to\infty} a^k = 1 + \Delta - \sqrt{d^2+\Delta^2}$ *and* $\lim_{k\to\infty} b^k = 2\Delta$.

The proof of the lemma is in the appendix. Note that by part (ii) of the lemma, the implied value function $V(\gamma)$ for the moderate types is convex. We are now ready to state our main result.

PROPOSITION 5. *There exists an infinite sequence,* $G^0 < G^1 < G^2 < \ldots$*, with* $G^0 = 0$ *and* $\lim_{k\to 1} G^k = 1$*, such that if* $\gamma \in [G^0, G^1]$*, then* $x(\gamma) = 0$*; if* $\gamma \in (G^k, G^{k+1}]$ *for* $k = 1, 2, \ldots$*, then* $x(\gamma)$ *satisfies* $\gamma x(\gamma)/(\gamma x(\gamma) + 1 - \gamma) \in (G^{k-1}, G^k]$.

PROOF. Starting with $G^1 = d/(b^0 + d)$, we define $G^2, G^3, \ldots$, recursively using the relation:

$$\frac{G^{k+1}x(G^{k+1})}{G^{k+1}x(G^{k+1}) + 1 - G^{k+1}} = G^k.$$

Using the equilibrium $x(\cdot)$ function given by equation (16), the above can be rewritten as:

$$G^{k+1} = \frac{(1+d-a^{k-1}) + G^k(b^0 + b^{k-1} - 2\Delta)}{b^0 + (1+d-a^{k-1}) + G^k(b^{k-1}-2\Delta)}. \tag{18}$$

We note that G^1 is strictly between 0 and 1. Since $a^{k-1} \leq 1$ and $b^{k-1} > 2\Delta$, an induction argument establishes that $G^k \in (0,1)$ for all $k \geq 1$. Furthermore, subtracting G^k from both sides of (18), we obtain

$$G^{k+1} - G^k = \frac{(1+d-a^{k-1})(1-G^k) + (b^{k-1}-2\Delta)G^k}{(1+d-a^{k-1}) + b^0 + (b^{k-1}-2\Delta)G^k} > 0.$$

Since G^k is an increasing and bounded sequence, it has a limit value, which is found to be equal to 1.

Fix any $\gamma \in (G^k, G^{k+1}]$. By the Bayes' rule, we have

$$\frac{\gamma'}{1-\gamma'} = \frac{\gamma}{1-\gamma} x(\gamma),$$

where $x(\gamma)$ is given by (16). It follows that that γ' is increasing in γ. Since the sequence $\{G^k\}$ is defined in such a way that $\gamma' = G^k$ if $\gamma = G^{k+1}$ and $\gamma' = G^{k-1}$ if $\gamma = G^k$, for any $\gamma \in (G^k, G^{k+1}]$ we must have $\gamma' \in (G^{k-1}, G^k]$.

Lastly, we verify that the extreme types have no incentive to deviate by voting against his preferences. To this end, let $W(\gamma)$ be the value function of the extreme conservative when the moderate type is having belief γ that the state is M. For $\gamma \in [G^0, G^1]$, since his opponent is choosing $x(\gamma) = 0$, the extreme conservative's payoff from voting c is 1, while his payoff from voting p is $-d + W(1) < 1$. Therefore, the extreme conservative indeed votes c, implying that $W(\gamma) = 1 > V(\gamma)$. For $\gamma = 1$, since his opponent is choosing p with probability $x(1)$, the extreme conservative is indifferent between voting c and voting p, and his payoff is $W(1) = V(1)$. Finally, for $\gamma \in (G^k, G^{k+1}]$, $k = 1, \ldots$, since the moderate conservative is randomizing between voting c and voting p, we have $V^c(\gamma, x(\gamma)) = V^p(\gamma, x(\gamma))$. This indifference condition can be written as

$$\gamma \left[x(\gamma)(-d + V(\gamma') - 1 + 2\Delta) + (1 - x(\gamma))(1 + d - V(1))\right] + (1-\gamma)(-d + V(\gamma') - 1) = 0,$$

where $\gamma' = \gamma x(\gamma)/(\gamma x(\gamma) + 1 - \gamma) \in (G^{k-1}, G^k]$. Since the last term is strictly negative, this above inequality implies that the expression in the square bracket is strictly positive. Note that we have already shown $W(1) = V(1)$. Thus, if $W(\gamma') > V(\gamma')$, then

$$x(\gamma)(-d + W(\gamma') - 1 + 2\Delta) + (1 - x(\gamma))(1 + d - W(1)) > 0,$$

which is equivalent to the condition that the extreme conservative strictly prefers voting c to voting p. That $W(\gamma) > V(\gamma)$ for all $\gamma < 1$ can be established by induction, as follows. We already know $W(\gamma) = 1 > V(\gamma)$ for $\gamma \in [G^0, G^1]$. For any $\gamma \in [G^k, G^{k+1}]$ and $k = 1, \ldots$, with $\gamma' = \gamma x(\gamma)/(\gamma x(\gamma) + 1 - \gamma) \in (G^{k-1}, G^k]$, we have

$$\begin{aligned} W(\gamma) &> (\gamma x(\gamma) + 1 - \gamma)(-d + W(\gamma')) + \gamma(1 - x(\gamma)) \\ &> (\gamma x(\gamma) + 1 - \gamma)(-d + V(\gamma')) + \gamma(1 - x(\gamma)) \\ &= V(\gamma), \end{aligned}$$

where the first inequality follows from the fact that $x(\gamma) < \gamma x(\gamma) + 1 - \gamma$ and the second inequality follows from the induction hypothesis. *Q.E.D.*

The equilibrium stated in Proposition 5 can be described easily. In each round of voting, there are four possible outcomes: immediate agreement on c, immediate agreement on p, a regular disagreement, or a reverse disagreement. We interpret a reverse disagreement as a breakdown of the negotiation process. Once a reverse disagreement occurs, it is revealed that what is a good decision for one player is necessarily an inferior decision for another player. The voting game then evolves just like an battle-of-the-sexes game with attrition, in which each moderate player chooses the stationary strategy represented by $x(1)$ until they reach a decision. Upon a regular disagreement, on the other hand, the moderate player is more convinced that he is playing against an extreme type. The extreme type will continue to vote according to his preference, but the moderate player will soften his position as $x(\gamma') < x(\gamma)$. In a sense, the negotiation between the two players is making progress, because the probability of choosing the mutually preferred decision rises if the state is L or R. Moreover, for any γ bounded away from 1, it only takes a finite number of rounds of regular disagreement before the moderate player yields to his opponent completely by switching to vote against his his preference (i.e., $x(\gamma) = 0$), provided there is no breakdown of negotiation before that. Once the game reaches this stage, there is either agreement on the mutually preferred decision, or the negotiation breaks down and the two moderate players engage in a war of attrition by adopting the strategy of voting according to his preference with probability $x(1)$.

The equilibrium constructed in Proposition 5 is unique subject to a weak continuity restriction. More specifically, the function $x(\gamma)$ given in the proposition is the unique continuous function that gives the equilibrium probability of voting according their preferences by the moderate types for each $\gamma \in [0,1]$. To see this, first we argue that in any equilibrium the moderate types must vote against their preferences with probability 1 if their belief γ is in $[0, G^1]$. To see this, note that regardless of the continuation plays, $V^c(\gamma, x) < V^p(\gamma, x)$ for all x if γ is sufficiently small, implying that in any equilibrium $x(\gamma) = 0$ for γ sufficiently small, while $V^c(\gamma, 1) < V^p(\gamma, 1)$ for $\gamma < G^1$, implying that in any equilibrium $x(\gamma)$ is bounded away from 1. Then, if it is not true that $x(\gamma) = 0$ for all $\gamma \in [0, G^1]$, there would be some $\tilde{\gamma} \in (0, G^1]$ such that the equilibrium play of the moderate types is given by $\tilde{x} \in (0,1)$ followed by the moderate types voting against their preferences with probability 1 upon a regular disagreement, which makes it impossible to satisfy the indifference condition $V^c(\tilde{\gamma}, \tilde{x}) = V^p(\tilde{\gamma}, \tilde{x})$. Next, fix any $k \geq 1$ and consider the interval $(G^k, G^{k+1}]$. For any γ on this interval such that the equilibrium probability x of voting according to their preferences satisfies that the updated belief γ' upon a regular disagreement $\gamma x/(\gamma x + 1 - \gamma)$ is smaller than or equal to G^k, by induction there is a unique continuation value $V(\gamma')$ as given by the proposition. Thus, we must have $x = x(\gamma)$, because for each $\gamma' \leq G^k$ and the corresponding $k' \leq k-1$, only $x(\gamma)$ simultaneously satisfies the equilibrium condition of $V^c(\gamma, x) = V^p(\gamma, x)$ and the Bayes' rule $\gamma' = x/(\gamma x + 1 - \gamma)$. Finally, we can rule out by contradiction the possibility that there is a subset of $(G^k, G^{k+1}]$ of a positive measure such that for each γ on this subset the equilibrium x is such that the updated belief γ' is greater than G^k. By the restriction of continuity of x as a function of γ, the infimum of this subset $\underline{\gamma}$ also has the property that the corresponding equilibrium $\underline{x}$ is such that the updated belief $\underline{\gamma}'$ is greater than or equal to G^k. Clearly, $\underline{x} < 1$; otherwise, we would have $V(\underline{\gamma}) = V(\underline{\gamma}) - d$, which is impossible. It then follows that $G_k \leq \underline{\gamma}' < \underline{\gamma}$, which contradicts the continuity restriction at $\underline{\gamma}$ since we have already shown that for any $\gamma \in [G_k, \underline{\gamma})$ the equilibrium play is given by $x(\gamma)$ defined in Proposition 5.

We can also show that the equilibrium constructed in Proposition 5 is the limit of the equilibria established in Proposition 3 as the deadline T becomes arbitrarily large. To see this, first note that by Proposition 3, when T is sufficiently large, there is a unique equilibrium. Next, from Lemma 1 we have that $x_T(1) < 1$, and since $\{V_T(1)\}$ converges to $V(1)$, the sequence $\{x_T(1)\}$ converges to $x(1)$ given in (14). At the other end, the comprise region for a finite deadline T is determined by

$$V_T^c(\gamma, 0) - V_T^p(\gamma, 0) = \gamma + (1-\gamma)(V_{T-1}(0) - d) - (\gamma(V_{T-1}(1) - d) + (1-\gamma)),$$

where $V_{T-1}(0) = 1$ following from a straightforward induction argument that $x_t(0) = 0$ for all t. We have $x_T(\gamma) = 0$ for all γ such that $V_T^c(\gamma, 0) \leq V_T^p(\gamma, 0)$, or

$$\gamma \leq \frac{d}{1 + 2d - V_{T-1}(1)} \equiv G_T^1.$$

Since $V_{T-1}(1)$ is arbitrarily close to $V(1)$, we have G_T^1 converging to G^1 as T converges to infinity. A straightforward induction argument then shows that for each $k = 1, \ldots,$ when T is sufficiently large, we can define G_T^{k+1} and $x_T(\gamma)$ for any $\gamma \in (G_T^k, G_T^{k+1}]$, such that $\lim_{T\to\infty} G_T^{k+1} = G^{k+1}$ and $x_T(\gamma) = x(\gamma)$, establishing convergence.

Due to the stationarity of the equilibrium strategy, comparative statics analysis is cleaner and more intuitive when there is no deadline. The following proposition establishes as the delay cost increases, or equivalently, as the error cost Δ decreases, equilibrium voting by the moderate types becomes less tough for any degree of conflict except for when they are certain what the state is. In spite of the softening of positions, however, the moderate types are unambiguously worse off because the direct impact of a greater delay cost dominates.

Proposition 6. *As d increases, G^k strictly increases for each $k \geq 1$, and $V(\gamma)$ and $x(\gamma)$ strictly decrease for all $\gamma \in (0, 1)$.*

The proof the above result is in the appendix. As in the game with $T = 1$ in section 3, the overall impact of an increase in the delay cost d turns out to be non-monotone. The reason is that the effect on the payoffs of the extremes is non-monotone. For any $\gamma \in [G^k, G^{k+1}]$, $k = 0, 1, \ldots,$ the equilibrium payoff $W(\gamma)$ to the extreme types can be written as

$$1 - (x(\gamma^k) + x(\gamma^k)x(\gamma^{k-1}) + \ldots)d,$$

where $\gamma^k = \gamma$, and for each $l = k-1, \ldots, 1$,

$$\gamma^l = \frac{\gamma^{l+1}x(\gamma^{l+1})}{\gamma^{l+1}x(\gamma^{l+1}) + 1 - \gamma^{l+1}}$$

is the updated belief from γ^{l+1}. Note that the term in the bracket is the same as

$$\sum_{l=0}^{k}(1 - x(\gamma^{k-l}))\Pi_{m=0}^{l-1}x(\gamma^{k-m}),$$

which is the expected number of voting rounds before a decision is made. Since by Proposition 6 each $x(\gamma^k)$ is decreasing in d, there is less expected delay when the delay cost is higher. However, the direct positive impact of the delay cost means that the expected cost of equilibrium delay can be non-monotone. Thus, $W(\gamma)$ can actually increase in the delay cost d, which means that the delay cost may have non-monotone effects on the ex ante payoff $U(\gamma)$.[18] It is more likely that an increase in the delay cost raises the ex ante payoff $U(\gamma)$ for intermediate values of γ, as the payoff $V(\gamma)$ to the moderate types receives a greater weight in the ex ante welfare.

5. Discussion

There are a number of unresolved issues that are of immediate concern. One is whether the optimal deadline is always finite. We know already that the optimal deadline is zero both for $\gamma < \frac{1}{2}$ (because the equilibrium outcome in the one-round voting is Pareto-efficient), and for γ close to 1. Moreover, numerical calculations suggest that the ex ante equilibrium payoff $U(\gamma)$ in the repeated voting game with $T = \infty$ is greater than the value $U_0(\gamma)$ in the game with $T = 0$ only for all γ just above $\frac{1}{2}$. Our conjecture is that the optimal deadline is between $\Delta/(2d)$ and Δ/d, so that the optimal deadline T is large when d is small, but is finite for any fixed Δ/d. Another question is whether for sufficiently long deadlines, a further increase in the deadline always increases $x_T(\gamma)$ for γ small but decreases $x_T(\gamma)$ for γ large. From the analysis in section 3, we know this is true when T increases from 0 to 1. Further, the comprise interval of γ values in which $x_T(\gamma) = 0$ shrinks with T while $x_T(1)$ weakly decreases with T. Our conjecture is that for $T \geq s$, there exists a threshold value which also depends on T, such that $x_T(\gamma)$ increases with T for γ below the threshold and decreases with T above the threshold.

[18] This is confirmed by explicit calculations; details are available upon request.

Below we briefly discuss two perhaps less immediate, but not less interesting, issues that arise in the two-round voting model of section 3. We have done some work on the issues but plan to investigate them further.

5.1. Asymmetric equilibria

The four types of equilibria describe in section 3 do not exhaust all possible equilibria of the two-round voting game. Unlike the single-round voting game analyzed in section 2.2, which generically has no asymmetric equilibria because each type has a dominant strategy, the possibility of re-voting allows the two players to coordinate by having one of them play tougher than the other in the first round and then switch the roles in the second round. As a result, asymmetric equilibria generally exist alongside the symmetric equilibria identified in Proposition 1. Here, we give a catalog and a brief discussion of all the possible asymmetric equilibria that can exist. Throughout the discussion, we assume that player P plays tougher than C in the first round; for each type of asymmetric equilibrium constructed below, there is another one in which the roles of the two players are switched.

The first type of asymmetric equilibrium can be called a "yielding equilibrium." Suppose both the moderate and extreme progressive vote p in round one. Then, when the delay cost is sufficiently large, it can be a best response for the moderate conservative to yield by voting p as a pure strategy. In terms of our notation in section 2.2, $x_P = 1$ and $x_C = 0$ in a yielding equilibrium. To have this as an equilibrium, first we need $d > 2\Delta$; otherwise, the extreme conservative would prefer to vote p as well in the first round, as the payoff from voting p is $1 - 2\Delta$ and the payoff from c is $-d + 1$. (The incentive for the extreme progressive is satisfied as he gets the maximum payoff of 1 by voting p.) Second, we need $\gamma > d/(2d + \Delta)$ (which is equal to G_1^1), in order for the moderate progressive to prefer p to c, or

$$\gamma + (1 - \gamma)(-d + 1) > \gamma(1 - d - \Delta) + (1 - \gamma).$$

Third, to figure out the incentive of the moderate conservative, we need to consider two cases of what happens when he deviates and votes c in the first round: if $\gamma > \frac{1}{2}$, the optimal continuation play is vote c, because the moderate progressive will switch to c in the second round while the extreme progressive will stick to p; if $\gamma < \frac{1}{2}$, the optimal continuation play is to vote p. The payoff to the moderate conservative from voting p is always $\gamma(1-2\Delta)+(1-\gamma)$. Thus, if $d > \Delta/2$, then an equilibrium of this type exists if $\gamma < (d+\Delta)/3\Delta)$, while if $d < \Delta/2$, an equilibrium of this type exists if $\gamma < d/\Delta$. These conditions can be summarized as

$$d < 2\Delta \quad \text{and} \quad \frac{d}{2d+\Delta} < \gamma < \min\left\{\frac{d}{\Delta}, \frac{d+\Delta}{3\Delta}\right\}.$$

The second type of asymmetric equilibrium may be called a "partial yielding equilibrium." In this equilibrium, both moderate types randomize in the first round, but the moderate progressive is tougher than the other. Specifically, consider $1 > x_P > (1-\gamma)/\gamma > x_C > 0$; note that this requires $\gamma > \frac{1}{2}$. Upon a regular disagreement, the moderate progressive switches his vote to c (because his updated belief of the state being M falls below $\frac{1}{2}$) while the moderate conservative continues to vote c (because his updated belief stays above $\frac{1}{2}$). Consider the incentive of the moderate conservative. If he votes c, he gets:[19]

$$\gamma(x_P(-d+1)+(1-x_P))+(1-\gamma)(-d+1-\Delta).$$

If he votes p, he gets:

$$\gamma(x_P(1-2\Delta)+(1-x_P)(-d+1-\Delta))+(1-\gamma).$$

Since the moderate C is indifferent between voting p and voting c, we have

$$x_P = \frac{\gamma(d+\Delta)-(1-\gamma)(d+\Delta)}{\gamma(2d-\Delta)}.$$

We need: $x_P > 0$, which requires $2d > \Delta$; $x_P > (1-\gamma)/\gamma$, which requires $\gamma > 3d/(4d+\Delta)$ (and this is equal to P_1); and $x_P < 1$, which requires $\gamma < (d+\Delta)/3\Delta$. For the moderate progressive, his payoff from voting p is

$$\gamma(x_C(-d+1-2\Delta)+(1-x_C))+(1-\gamma)(-d+1),$$

[19] In the symmetric switching equilibrium, the payoff after the regular disagreement by the moderate types would be $-d+1-\Delta$ because C is also switching, and this is where the difference with the symmetric equilibrium comes from. A similar comment applies to the payoff from voting p for the moderate progressive below.

while his payoff from voting c is

$$\gamma(x_C(1-2\Delta)+(1-x_C)(-d+1-\Delta))+(1-\gamma).$$

Indifference requires

$$x_C=\frac{\gamma(d+\Delta)-(1-\gamma)d}{\gamma(2d+\Delta)}.$$

We want to make sure: $x_C>0$, which requires $\gamma>d/(2d+\Delta)$ and is satisfied because $\gamma>\frac{1}{2}$; and $x_C<(1-\gamma)/\gamma$, which requires $\gamma<(3d+\Delta)/(4d+2\Delta)$ (and this is equal to Q_1). These conditions can be summarized as

$$2d>\Delta \quad \text{and} \quad \frac{3d}{4d+\Delta}<\gamma<\min\left\{\frac{d+\Delta}{3\Delta},\frac{3d+\Delta}{4d+2\Delta}\right\}.$$

The third possible type of asymmetric equilibrium is a "random yielding equilibrium." In this type of equilibrium, $x_P=1$ and $x_C=(1-\gamma)/\gamma$ (which again requires $\gamma>\frac{1}{2}$). Upon a regular disagreement, the moderate progressive randomizes in the second round, voting p with probability $x_0\in[0,1]$, while the moderate conservative continues to vote c because his belief is unchanged. The payoff to the moderate conservative from voting c in the first round is

$$\gamma(-d+x_0(1-\Delta)+(1-x_0))+(1-\gamma)(-d+1-\Delta),$$

and his payoff from voting p is $\gamma(1-2\Delta)+(1-\gamma)$. For the moderate conservative to be indifferent between the two votes, with x_0 varying from 0 to 1, we need

$$\gamma(\Delta-d)<(1-\gamma)(d+\Delta)<\gamma(2\Delta-d),$$

which is satisfied if and only if $d<2\Delta$, $\gamma>(d+\Delta)/\Delta$ and $\gamma<\min\{(d+\Delta)/(2\Delta),1\}$. For the moderate progressive, the payoff from voting c in the first round is

$$\gamma(x_C(1-2\Delta)+(1-x_C)(-d+1-\Delta))+(1-\gamma),$$

while the payoff from voting p is

$$\gamma(x_C(-d+x_0(1-\Delta)+(1-x_0)(1-2\Delta))+(1-x_C))+(1-\gamma)(-d+x_0(1-\Delta)+(1-x_0)).$$

Substituting $x_C = (1-\gamma)/\gamma$ into the above two expressions and comparing them, we have that the moderate progressive prefers p to c if and only if $\gamma > (3d+\Delta)/(4d+\Delta)$. Summarizing, the necessary and sufficient conditions for this type of equilibria are

$$d < 2\Delta \quad \text{and} \quad \max\left\{\frac{d+\Delta}{3\Delta}, \frac{3d+\Delta}{4d+2\Delta}\right\} < \gamma < \min\left\{\frac{d+\Delta}{2\Delta}, 1\right\}.$$

Since our game in section 3 is symmetric, an asymmetric equilibrium exists only with one-sided vote-switching in the second round after a regular disagreement by the two moderate types in the first round. It follows that the above are the only three types of asymmetric equilibria that exist generically.[20] We have yet to investigate whether a simple average of asymmetric equilibrium plays always improves on the symmetric equilibrium outcome we have constructed. Our conjecture is that any welfare improvement arises because asymmetric plays avoid the costly simultaneous vote-switching after a regular disagreement that occurs in a symmetric switching equilibrium. Another way to understand any potential welfare improvement is that asymmetric plays allow players to trade votes inter-temporally, and thus circumvent the constraint of no side transfers in our model. This then appears to be another mechanism for delay to improve strategic information aggregation, different from the one identified in symmetric equilibrium plays in which delay improves welfare by breaking the "incentive budget." Finally, any asymmetric play still requires at least some coordination such as pre-play public randomizations. Whether such coordination is realistic in applications is an issue that needs to be addressed.

5.2. Optimal delay mechanism

We have presented the idea that delay can be used as a mechanism to improve collective decision making. As shown in section 2.3, without delay there is no incentive compatible outcome that beats flipping a coin when γ is greater than $\frac{1}{2}$, while Proposition 2 shows that with delay there can be Pareto improving equilibrium outcome for γ just above $\frac{1}{2}$. A

[20] The fourth candidate for asymmetric equilibria, which is given by $x_P = (1-\gamma)/\gamma$ and $x_C = 0$ with random one-sided switching, does not exist generically. This is because only off the equilibrium path can the moderate conservative randomize in the second round after a regular disagreement in the first round. The indifference condition for the moderate progressive cannot be satisfied generically.

natural question is whether there are other mechanisms using delay that are even better for the players. This question bears resemblance to the problem of "dynamic mechanism design" in the existing literature (Dewatripont 1988; Laffont and Tirole 1988). Although unlike the existing models in the present setup we do not allow side transfers, it is still useful to relate to the literature in the three different commitment environments that have been used. Note that in discussing the different commitment environments, we implicitly assume that the two players have full commitment power regarding the implementation the pre-specified decision before delay and the imposition of delay; it is only whether or not they have any commitment power regarding how the decision is made after delay that differentiates the three commitment environments.[21] For the discussion in this subsection, we assume that $\gamma > \frac{1}{2}$.

Under full commitment, the two players can adopt any direct mechanism that maps two reports of the signals to a probability of choosing c without delay, a probability of choosing p without delay, and the remaining probability of imposing the delay cost on each player and getting a continuation outcome. Under full commitment, the continuation outcome associated with any pair of reports can be an arbitrary mixture between c and p. Since each player only cares about the total probability that each alternative is chosen and the probability of paying the delay cost, the revelation principle allows us to represent any delay mechanism by pairs of the total probability that a particular alternative is chosen and the probability of delay. As in section 2.3, we denote as q_R, q_M, and q_L the total probabilities of implementing alternative c when the inferred true state is R, M, and L, respectively, and denote as $\tilde{q}$ the total probability of implementing c when the reports are inconsistent. Let the corresponding probabilities of delay be η_R, η_M, η_L and $\tilde{\eta}$. For simplicity, we restrict our attention to symmetric mechanisms, with $q_R = 1 - q_L = q$, $q_M = \tilde{q} = \frac{1}{2}$, and $\eta_R = \eta_L = \eta$. The incentive constraint for truthful reporting is

$$q + (1 - q)(1 - 2\Delta) - \eta d \geq 1 - \Delta - \eta_M d$$

[21] There is also a literature on renegotiation in "static" mechanism design problems, where the issue is whether or not there is any commitment regarding implementation of the specified outcome after the mechanism is played (i.e., at the ex post stage), or regarding the play of the mechanism itself before the mechanism is played (i.e., at the interim stage). See Maskin and Moore (1999), and Segal and Whinston (2002) for renegotiation under complete information, and Holmstrom and Myerson (1983) under incomplete information.

for the extremists, and

$$\gamma(1-\Delta-\eta_M d)+(1-\gamma)(q+(1-q)(1-2\Delta)-\eta d)$$
$$\geq \gamma(q+(1-q)(1-2\Delta)-\eta d)+(1-\gamma)(1-\Delta-\tilde{\eta} d)$$

for the moderate types. The objective for this design problem is to maximize $U(\gamma)$, where $V(\gamma)$ and $W(\gamma)$ are given by the left-hand-side of the above incentive constraint for the moderates and for the extremists, respectively. It is straightforward to see that the first best is achieved with $q=1$, $\eta=\eta_M=0$ and $\tilde{\eta}=1$, if and only if $\gamma \leq (d+\Delta)/(d+2\Delta)$. If $\gamma > (d+\Delta)/(d+2\Delta)$, in any optimal mechanism, the incentive constraint for the moderates binds, with $\eta_M=0$ and $\tilde{\eta}=1$, and q and η satisfying

$$q+(1-q)(1-2\Delta)-\eta d = 1-\Delta+\frac{1-\gamma}{2\gamma-1}d. \tag{19}$$

Thus, the same optimal value for $U(\gamma)$ can be achieved by delay mechanisms that involve either delay when the state inferred from the reports is an agreement state R or L, or a probability of implementing the agreement that is strictly less than 1, or both. Moreover, this value is strictly increasing in the delay cost d until the first best is achieved. Finally, there is always an optimal delay mechanism with $\eta=0$: since the right-hand-side of (19) reaches the minimum of $1-\Delta$ when $\gamma=1$, in which case (19) is satisfied with $q=\frac{1}{2}$ and $\eta=0$, if for some $\gamma<1$ there exist $\hat{q}$ and $\hat{\eta}>0$ such that (19) holds, then there exists $q\in(\frac{1}{2},\hat{q})$ such that (19) holds with q and $\eta=0$. Thus, an optimal delay mechanism may not involve delay on the equilibrium path at all.

A commitment environment that is less stringent, and perhaps more realistic, referred to as "commitment through renegotiation" in the literature (Dewatripont 1988), leads to the following mechanism design problem in our setup. As in the case of full commitment, each mechanism under consideration consists of a message space for each player, corresponding to the two possible private signals they can have, and an outcome function that maps each pair of reports to a probability of choosing c without delay, a probability of choosing p without delay, and the remaining probability of imposing the delay cost on each player and getting a continuation outcome. Unlike in the case of full commitment, where each continuation outcome can be an arbitrary randomization between the two alternatives, here on the truth-telling equilibrium path it has to be renegotiation-proof, which translates into a requirement that the two players choose the mutually preferred alternative if it exists. Using the same notation as in the full commitment environment analyzed above, and again restricting to symmetric mechanisms, we can write one implication of the renegotiation-proof constraint as $q \geq \eta$. For example, when the state is R, following any delay the continuation probability choosing c will be renegotiated to 1, and therefore the total probability of choosing c is greater than or equal to the probability that delay occurs. Another implication of renegotiation-proofness is more subtle: players must not benefit by deviating and anticipating the renegotiation after inconsistent reports.[22] For example, the two reports are inconsistent when the moderate conservative misreports his type and when the other player is an extreme progressive reporting truthfully. In this case, the continuation outcome after delay will be renegotiated to choosing p with probability 1, as the progressive knows the state is L. Thus, the truth-telling condition for the moderates becomes:

$$\begin{aligned}&\gamma(1-\Delta-\eta_M d)+(1-\gamma)(q+(1-q)(1-2\Delta)-\eta d)\\&\qquad\geq \gamma(q+(1-q)(1-2\Delta)-\eta d)+(1-\gamma)((1-\tilde{\eta})(1-\Delta)+\tilde{\eta}(1-d)).\end{aligned}$$

The strongest incentive for truthful reporting is therefore provided by either $\tilde{\eta}=0$ if $\Delta \geq d$, or $\tilde{\eta}=1$ if $\Delta < d$. In the former case, it follows from the two incentive constraints for truth-telling that any optimal renegotiation-proof mechanism must have $\eta_M = 0$ and $\eta = \frac{1}{2}$, implying that the optimal mechanism is equivalent to flipping a coin. In the latter case, it is straightforward to show that the first best can be achieved if and only if $\gamma \leq d/(d+\Delta)$, which is smaller than the threshold $(d+\Delta)/(d+2\Delta)$ under full commitment. If $\gamma > d/(d+\Delta)$, in any optimal mechanism $\eta_M = 0$, and q and η satisfy

$$q+(1-q)(1-2\Delta)-\eta d = 1-\Delta+\frac{1-\gamma}{2\gamma-1}(d-\Delta).$$

As in the case of full commitment, if the above is satisfied by some $\hat{q}$ and $\hat{\eta} > 0$, there is always $q \in (\frac{1}{2}, \hat{q})$ that satisfies the above with $\eta = 0$, and thus the constraint $q \geq \eta$ imposed by renegotiation-proofness following two consistent reports does not bind. Comparing

[22] The continuation outcome specified in the mechanism after inconsistent reports and delay is not relevant, as it will not occur in any truth-telling equilibrium of a renegotiation-proof mechanism.

the above to (19) we see that reducing the commitment power of the players by allowing renegotiation mitigates but does not destroy the effectiveness of delay as a device to improve strategic information aggregation, so long as the delay cost is large relative to the error cost.

Finally, the no-commitment environment is in fact the one we have implicitly assumed in the two-period repeated voting game in section 3. As in the dynamic mechanism design literature, a particular voting game is considered, in which two agreeing votes lead to the agreement being implemented immediately while disagreeing votes lead to a costly delay and re-voting. This game does not achieve either the optimality under full-commitment or the optimality under commitment through renegotiation, because the equilibrium probability of delay is strictly positive in state M. However, this is not the only game possible under no commitment. An alternative game is where each player can vote c or p, with an agreement leading to the immediate implementation of the agreed decision and a disagreement leading to the imposition of the delay cost and a coin toss (as opposed to re-voting in our game). Our preliminary calculations show that the equilibrium outcome of this this alternative game can ex ante dominate the equilibrium of the two-round repeated voting game considered in section 3. We could also consider other games, compare them in terms of equilibrium ex ante payoffs to the players, and perhaps identify the "optimal delay game."[23] However, since the revelation principle does not apply in this environment, we have yet to find a systematic way of going forward.

[23] In another repeated voting game, following a reverse disagreement a coin flip decides the outcome without delay. This variation may lead to both a greater willingness to comprise and a higher disagreement payoff, and thus a greater ex ante equilibrium payoff for the two players.

Appendix

A. Proof of Proposition 2

We consider each type of equilibrium separately.

(i) Consider first a no switching equilibrium. Since $x_1(\gamma) > 0$, the moderate conservative's equilibrium payoff is given by $V_1^c(\gamma, x_1(\gamma))$, or

$$V_1(\gamma) = \gamma[x_1(\gamma)(-d+1-\Delta) + (1-x_1(\gamma))] + (1-\gamma)(-d+1-\Delta).$$

An extreme conservative always votes c and receives a payoff of

$$W_1(\gamma) = x_1(\gamma)(-d+1-\Delta) + (1-x_1(\gamma)).$$

Using equation (8), we can write the payoff difference between $U_1(\gamma)$ and the benchmark payoff of $U_0(\gamma) = 1-\Delta$ for $\gamma > \frac{1}{2}$ as:

$$U_1(\gamma) - U_0(\gamma) = \frac{1-\gamma}{2-\gamma}(-d) + \frac{1}{2-\gamma}(x_1(\gamma)(-d) + (1-x_1(\gamma))\Delta).$$

After substituting the equilibrium value of $x_1(\gamma)$ given by (7) into this expression, we obtain:

$$U_1(\gamma) - U_0(\gamma) = \frac{d}{2\gamma(2-\gamma)}\left(-(2\gamma-1)\delta + 2(1-\gamma)\delta + 1 - 4\gamma + 2\gamma^2\right),$$

where $\delta \equiv \Delta/d$. This is a quadratic expression in δ, and it is non-negative if:

$$\delta \in \left[\frac{1-\gamma-\sqrt{\gamma(4\gamma^2-9\gamma+4)}}{2\gamma-1}, \frac{1-\gamma+\sqrt{\gamma(4\gamma^2-9\gamma+4)}}{2\gamma-1}\right].$$

The intersection between this region and the region $\gamma \in [Q_1, 1]$ that supports a no switching equilibrium is

$$\delta \in \left(\frac{3-2\gamma}{2\gamma-1}, \frac{1-\gamma+\sqrt{\gamma(4\gamma^2-9\gamma+4)}}{2\gamma-1}\right]; \quad \gamma \le 2-\sqrt{2}.$$

(ii) Next, consider a random switching equilibrium. Again, using $V_1^c(\gamma, x_1)$, we get

$$U_1(\gamma) = \frac{\gamma}{2-\gamma}\left[x_1(\gamma)\Big(-d+V_0\Big(\frac{1}{2}\Big)\Big)+(1-x_1(\gamma))\right]+\frac{1-\gamma}{2-\gamma}\Big(-d+V_0\Big(\frac{1}{2}\Big)\Big)$$
$$+\frac{1-\gamma}{2-\gamma}\left[x_1(\gamma)\Big(-d+W_0\Big(\frac{1}{2}\Big)\Big)+(1-x_1(\gamma))\right].$$

Since $x_1(\gamma) = (1-\gamma)/\gamma$, substituting in the above expression the equilibrium value of $V_0(\frac{1}{2})$ that solves the indifference condition $V_1^c(\gamma,(1-\gamma)/\gamma) = V_1^p(\gamma,(1-\gamma)/\gamma)$ for a random switching equilibrium (with the corresponding value of $W_0(\frac{1}{2})$), we obtain:

$$U_1(\gamma) - U_0(\gamma) = \frac{d}{\gamma(2-\gamma)}\left(\gamma^2 - 4\gamma + 2\right).$$

So, $U_1(\gamma) \geq U_0(\gamma)$ if and only if $\gamma \leq 2-\sqrt{2}$. The region of welfare improvement that satisfies the condition $\gamma \in [P_1, Q_1)$ for a random switching equilibrium is therefore

$$\delta \in \left[\frac{3-2\gamma}{\gamma}, \frac{3-2\gamma}{2\gamma-1}\right]; \quad \gamma \leq 2-\sqrt{2}.$$

(iii) In a switching equilibrium, we use $V_1^c(\gamma, x_1(\gamma))$ for the moderate type and $W_1^c(\gamma, x_1(\gamma))$ for the extreme type. After subtracting the benchmark payoff $1-\Delta$, we get

$$U_1(\gamma) - U_0(\gamma) = \frac{\gamma}{2-\gamma}(x_1(\gamma)(-d)+(1-x_1(\gamma))\Delta)+\frac{1-\gamma}{2-\gamma}(-d+\Delta)$$
$$+\frac{1-\gamma}{2-\gamma}(x_1(\gamma)(-d+\Delta)+(1-x_1(\gamma))\Delta).$$

Use the equilibrium value of $x_1(\gamma)$ given by (6), we obtain:

$$U_1(\gamma) - U_0(\gamma) = \frac{d}{2\gamma(2-\gamma)}\left(-\gamma^2\delta^2 + 4\gamma(1-\gamma)\delta + 1 - 4\gamma + 2\gamma^2\right).$$

This is a quadratic expression in δ, and is non-negative when

$$\delta \in \left[\frac{2(1-\gamma)-\sqrt{6\gamma^2-12\gamma+5}}{\gamma}, \frac{2(1-\gamma)+\sqrt{6\gamma^2-12\gamma+5}}{\gamma}\right].$$

The intersection between this region and the region that is consistent with the condition $\gamma \in (\frac{1}{2}, P_1)$ for a switching equilibrium is:

$$\delta \in \left[\frac{2(1-\gamma)-\sqrt{6\gamma^2-12\gamma+5}}{\gamma}, \frac{3-2\gamma}{\gamma}\right); \quad \gamma \leq 2-\sqrt{2}.$$

The union of these three regions of strict welfare improvement identified for the three types of equilibria is

$$\frac{\Delta}{d} \in \left(\frac{2(1-\gamma) - \sqrt{6\gamma^2 - 12\gamma + 5}}{\gamma}, \frac{1 - \gamma + \sqrt{\gamma(4\gamma^2 - 9\gamma + 4)}}{2\gamma - 1} \right); \quad \gamma \in \left(\frac{1}{2}, 2 - \sqrt{2} \right).$$

B. Proof of Lemma 1

Observe that starting from $\gamma = 1$, it is sufficient to check the incentives of the moderate types for equilibrium play.

(i) If $s = 1$, then we are already done; suppose that $s \geq 2$. Suppose further that $x_\tau(1) = 1$ for all $\tau = t-1, \ldots, 0$; consider $\tau = t$. By induction we have $V_{t-1}(1) = 1 - \Delta - (t-1)d$. This implies that $V_t^c(1, x_t) - V_t^p(1, x_t)$ is strictly decreasing in x_t, and that $V_t^c(1,1) - V_t^p(1,1) = \Delta - td$, which is nonnegative because $t \leq s-1$. Thus, there is a unique equilibrium for $\tau = t$ with $x_t(1) = 1$. It follows that $V_t(1) = V_{t-1}(1) - d = 1 - \Delta - td$, completing the proof of (i). Note that $V_t(1) > 1 - 2\Delta$ for all $t \leq s-2$ and $V_{s-1}(1) < 1 - 2\Delta$.

(ii), (iii) The same argument as in (i) establishes that $V_s^c(1, x_t) - V_s^p(1, x_s)$ is strictly decreasing in x_s, and that $V_s^c(1,1) - V_s^p(1,1) = \Delta - sd < 0$. Note that $V_s^c(1,0) - V_s^p(1,0) > 0$. Thus, there is a unique equilibrium with $x_s(1) \in (0,1)$ that satisfies that $V_s^c(1, x_s(1)) = V_s^p(1, x_s(1))$, implying that

$$x_s(1) = \frac{1 + d - V_{s-1}(1)}{2(1 + d - V_{s-1}(1) - \Delta)},$$

where $V_{s-1}(1) = 1 - \Delta - (s-1)d$ by (i). Direct substitution completes the argument for (ii) when $t = s$. Note that

$$V_s(1) = V_s^p(1, x_s(1)) = x_s(1)(1 - 2\Delta) + (1 - x_s(1))(V_{s-1}(1) - d) < 1 - 2\Delta$$

because $V_{s-1}(1) = 1 - \Delta - (s-1)d$ and $x_s(1) < 1$. Now, suppose that (ii) holds for all $\tau = t-1, \ldots, s$ and $V_\tau(1) < 1 - 2\Delta$; consider $\tau = t$. Since $V_{t-1}(1) < 1 - 2\Delta$, we can verify that $V_t^c(1, x_t) - V_t^p(1, x_t)$ is strictly decreasing in x_t, and that $V_t^c(1,1) - V_t^p(1,1) < 0$. It then follows that there is a unique equilibrium for $\tau = t$ with $x_t(1) \in (0,1)$ satisfying that

$V_t^c(1, x_t(1)) - V_t^p(1, x_t(1)) = 0$. The induction argument is completed by solving for $x_t(1)$ and substituting into $V_t(1)$, and by noting that

$$V_t(1) = V_t^p(1, x_t(1)) = x_t(1)(1 - 2\Delta) + (1 - x_t(1))(V_{t-1}(1) - d) < 1 - 2\Delta$$

because $V_{t-1}(1) < 1 - 2\Delta$ by the induction hypothesis and $x_t(1) < 1$.

(iv) For $t \leq s - 1$, since $V_t(1) = 1 - \Delta - td$ we have $V_t(1) < V_{t-1}(1)$. For $t \geq s$, since

$$1 - V_t(1) = \frac{(1 + d - V_{t-1}(1))^2}{2(1 + d - V_{t-1}(1) - \Delta)}.$$

we have that $V_t(1) < V_{t-1}(1)$ is equivalent to

$$1 - V_{t-1}(1) < \Delta + \sqrt{d^2 + \Delta^2}.$$

We want to show by induction that the above is true for all $t \geq s$. First, note that since $V_{s-1} = 1 - \Delta - (s - 1)d$ and since $(s - 1)d \leq \Delta$, the claim is true for $t = s$. Next, suppose that the claim is true for $t - 1$; we want to show that it holds for t. This is equivalent to

$$\frac{(1 + d - V_{t-1}(1))^2}{2(1 + d - V_{t-1}(1) - \Delta)} < \Delta + \sqrt{d^2 + \Delta^2}.$$

By taking derivatives, we can show that the left-hand-side of the above inequality is strictly increasing in $1 - V_{t-1}(1)$ because $V_{t-1}(1) < 1 - 2\Delta$ for all $t \geq s + 1$. Since by the induction hypothesis $1 - V_{t-1}(1) < \Delta + \sqrt{d^2 + \Delta^2}$, the maximum of the left-hand-side of the above inequality can be shown to be strictly less than $\Delta + \sqrt{d^2 + \Delta^2}$, completing the induction. Direct calculation yields the limit $V(1) = 1 - \Delta + \sqrt{d^2 + \Delta^2}$.

C. Proof of Lemma 2

(i) First note that $u_0 = 2d > 0$: for $\gamma < \frac{1}{2}$ we have $a_0 = 1$ and $b_0 = \Delta$, and for $\gamma > \frac{1}{2}$ we have $a_0 = 1 - \Delta$ and $b_0 = 0$.

Next, consider the case $t \leq s - 1$. We can combine the two equations of (11) to obtain:

$$1 + d - a_t + b_t = d + \frac{(1 + d - a_{t-1} + b_{t-1})(1 + d - V_{t-1}(1))}{u_{t-1}}. \qquad \text{(A.1)}$$

For $t \leq s-1$, from Lemma 1 we have $V_t(1) = 1 - td - \Delta$. Adding $(1 + d - V_t(1)) - 2\Delta$ to the left side and adding $(t+1)d - \Delta$ to the right side of (A.1), we get

$$u_t = 2(t+1)d + \frac{(td + \Delta)(-td + \Delta)}{u_{t-1}}.$$

Since $\Delta - td > 0$ for $t \leq s-1$, if $u_{t-1} > 0$, then $u_t > 0$. This induction argument establishes that $u_t > 0$ for all $t \leq s-1$.

Let $w_t = 1 + d - a_t + b_t$. Since $u_t < w_t$ for $t \leq s-1$, the fact that $u_t > 0$ for $t \leq s-1$ implies $w_t > 0$ for $t \leq s-1$. We now rewrite equation (A.1) as

$$w_t = d + \frac{w_{t-1}(1 + d - V_{t-1}(1))}{w_{t-1} + (1 + d - V_{t-1} - 2\Delta)}. \tag{A.2}$$

Since $V_{s-1}(1) < 1 - 2\Delta$ and $V_t(1)$ decreases with t by Lemma 1, we have $1 + d - V_{t-1}(1) - 2\Delta > 0$ for all $t \geq s$. An induction argument on equation (A.2) then establishes that $w_t > 0$ for all $t \geq s$, and thus $u_t = w_t + (1 + d - V_t(1) - 2\Delta) > 0$.

(ii) We claim that $\frac{1}{2}b_t \geq 0$ and $1 + d - a_t + \frac{1}{2}b_t > 0$ for all t. This is true for $t = 0$ as either $a_0 = 1$ and $b_0 = \Delta$, or $a_0 = 1 - \Delta$ and $b_0 = 0$.

Next, consider the case $t \leq s$. In this case, $V_{t-1}(1) = 1 - (t-1)d - \Delta$. We can rewrite (11) as

$$\begin{aligned} 1 + d - a_t + \frac{1}{2}b_t &= d + \frac{\frac{1}{2}b_{t-1}\Delta + \left(1 + d - a_{t-1} + \frac{1}{2}b_{t-1}\right)td}{u_{t-1}}, \\ \frac{1}{2}b_t &= \frac{\left(1 + d - a_{t-1} + \frac{1}{2}b_{t-1}\right)\Delta + \frac{1}{2}b_{t-1}td}{u_{t-1}}. \end{aligned} \tag{A.3}$$

An induction argument establishes the claim for all $t \leq s$.

For $t \geq s+1$, rewrite (11) as

$$\begin{aligned} 1 + d - a_t + \frac{1}{2}b_t &= d + \frac{w_{t-1}\Delta + \left(1 + d - a_{t-1} + \frac{1}{2}b_{t-1}\right)(1 + d - V_{t-1}(1) - 2\Delta)}{u_{t-1}}, \\ \frac{1}{2}b_t &= \frac{w_{t-1}\Delta + \frac{1}{2}b_{t-1}(1 + d - V_{t-1}(1) - 2\Delta)}{u_{t-1}}. \end{aligned}$$

From part (i) we have that $w_{t-1} \geq 0$ and $1 + d - V_{t-1}(1) - 2\Delta \geq 0$ for $t \geq s$. An induction argument on the above difference equations then establishes the claim for all $t \geq s+1$.

(iii) For $t \leq s-1$, since $td - \Delta \leq 0$, the first equation of (A.2) implies that

$$1 + d - a_t + \frac{1}{2}b_t \geq d + \frac{\frac{1}{2}b_{t-1}\Delta + \left(1 + d - a_{t-1} + \frac{1}{2}b_{t-1}\right)td}{1 + d - a_{t-1} + b_{t-1}} \geq (t+1)d.$$

For $t = s-1$, this inequality implies $1 + d - a_{s-1} + \frac{1}{2}b_{s-1} \geq sd \geq \Delta$.

Now, for $t = s$, we can use the first equation of (A.3) to get

$$1 + d - a_s + \frac{1}{2}b_s \geq d + \frac{w_{s-1}\Delta + (1 + d - V_{s-1}(1) - 2\Delta)\Delta}{u_{s-1}} = d + \Delta.$$

An induction argument on the first equation of (A.3) then establishes that $1+d-a_t+\frac{1}{2}b_t \geq d + \Delta$ for all $t \geq s$.

(iv) First, using the second equation of (A.2), we have

$$\frac{1}{2}b_s \leq \frac{\left(1 + d - a_{s-1} + \frac{1}{2}b_{s-1}\right)\Delta + \frac{1}{2}b_{s-1}sd}{1 + d - a_{s-1} + b_{s-1}} \leq sd.$$

Recall that from (iii) we have $1+d-a_s+\frac{1}{2}b_s \geq d+\Delta$. Adding these two inequalities gives

$$1 + d - a_s \geq \Delta - (s-1)d > 0.$$

Now, subtract the second equation of (A.3) from the first to get

$$1 + d - a_t = d + \frac{(1 + d - a_{t-1})(1 + d - V_{t-1}(1) - 2\Delta)}{u_{t-1}}.$$

Since $1+d-V_{t-1}(1)-2\Delta \geq 0$ for $t \geq s$, an induction argument establishes that $1+d-a_t \geq 0$ for all $t \geq s$.

D. Proof of Lemma 3

(i) For $k = 0$, we have $a^0 = 1$ and $b^0 = d + \Delta + \sqrt{d^2 + \Delta^2} > 2\Delta$. Next, if $a^{k-1} \leq 1$ and $b^{k-1} > 2\Delta$, the two fractions that appear in the difference equation (17) are both positive. Hence $a^k \leq 1$ and $b^k > 2\Delta$ by induction.

(ii) For the monotonicity of b^k, we can subtract b^{k-1} from both sides of the second equation in (17) to get:

$$b^k - b^{k-1} = -\frac{1 + d - a^{k-1} + b^{k-1}}{b^0 + 1 + d - a^k + b^k - 2\Delta}(b^{k-1} - 2\Delta) < 0.$$

To establish the monotonicity of a^k, we use induction. First, it is easy to see that $a^1 < a^0 = 1$. Next, assume that $a^{k-1} < a^{k-2}$. We can write:

$$
\begin{aligned}
a^k - a^{k-1} &= \frac{(1+d-a^{k-2})(b^0-2\Delta)}{b^0+1+d-a^{k-2}+b^{k-2}-2\Delta} - \frac{(1+d-a^{k-1})(b^0-2\Delta)}{b^0+1+d-a^{k-1}+b^{k-1}-2\Delta} \\
&< \frac{(1+d-a^{k-2})(b^0-2\Delta)}{b^0+1+d-a^{k-2}+b^{k-2}-2\Delta} - \frac{(1+d-a^{k-1})(b^0-2\Delta)}{b^0+1+d-a^{k-1}+b^{k-2}-2\Delta} \\
&< \frac{(1+d-a^{k-2})(b^0-2\Delta)}{b^0+1+d-a^{k-2}+b^{k-1}-2\Delta} - \frac{(1+d-a^{k-2})(b^0-2\Delta)}{b^0+1+d-a^{k-2}+b^{k-2}-2\Delta} \\
&= 0,
\end{aligned}
$$

where the first inequality follows from $b^{k-1} < b^{k-2}$ and the second inequality follows from the induction hypothesis and the fact that the second term is decreasing in a^{k-1}.
(iii) Solving for the steady state version of the difference equation (17), we obtain the steady state values $a^\infty = 1+\Delta-\sqrt{d^2+\Delta^2}$ and $b^\infty = 2\Delta$. By the monotonicity of a^k and b^k, these steady state values are also the limit values of the sequence $\{(a^k, b^k)\}$.

E. Proof of Proposition 6

Let $s^k = 1+d-a^k$; $w^k = 1+d-a^k+b^k-2\Delta$; and $u^k = b^0+w^k$. Note that $b^0 = 1+d-V(1)$, and therefore $\partial b^0/\partial d = 1 + d/\sqrt{d^2+\Delta^2}$. We first establish a series of claims.
Claim 1: w^k is decreasing in k. To prove it, combine equations (11) to obtain

$$w^k = d + \frac{(b^0-2\Delta)w^{k-1}}{b^0+w^{k-1}}. \tag{A.4}$$

The derivative of the right-hand-side with respect to w^{k-1} is positive. So $w^{k-1} < w^{k-2}$ implies $w^k < w^{k-1}$. Now,

$$w^1 - w^0 = d - \frac{(1+d-a^0+b^0)w^0}{b^0+w^0} = \frac{-b^0(b^0-2\Delta)}{2b^0+d-2\Delta} < 0.$$

So an induction argument establishes the claim.
Claim 2: w^k is increasing in d for each k. Take derivative of equation (A.4) to get

$$\frac{\partial w^k}{\partial d} = 1 + \frac{w^{k-1}(w^{k-1}+2\Delta)}{(b^0+w^{k-1})^2}\frac{\partial b^0}{\partial d} > 0,$$

and

$$\frac{\partial w^k}{\partial w^{k-1}} = \frac{b^0(b^0-2\Delta)}{(b^0+w^{k-1})^2} > 0.$$

Now,

$$\frac{\mathrm{d}w^k}{\mathrm{d}(d)} = \frac{\partial w^k}{\partial d} + \frac{\partial w^k}{\partial w^{k-1}} \frac{\mathrm{d}w^{k-1}}{\mathrm{d}(d)}.$$

An induction argument establishes that $\mathrm{d}w^k/\mathrm{d}(d) > 0$ if we can show that $\mathrm{d}w^0/\mathrm{d}(d) > 0$, which is true because $w^0 = d + b^0 - 2\Delta$ is increasing in d.

Claim 3: b^0/u^k is decreasing in d for each k. Write $\omega^k = w^k/(b^0 + w^k)$. Use equation (A.4) for w^k to write:

$$\omega^k = \frac{d + \omega^{k-1}(b^0 - 2\Delta)}{b^0 + d + \omega^{k-1}(b^0 - 2\Delta)}.$$

The partial derivative $\partial\omega^k/\partial d$ has the same sign as $b^0 + (2\omega^{k-1}\Delta - d)(\partial b^0/\partial d)$. By Claim 1, ω^k is decreasing in k. Therefore, this expression is greater than $b^0 + (2\omega^\infty\Delta - d)(\partial b^0/\partial d)$, which is positive, where $\omega^\infty = \frac{1}{2} - \frac{1}{2}\Delta/(1 + d - V(1) - \Delta)$ is the limit value of ω^k as k goes to infinity. It is also easy to see that ω^k is increasing in ω^{k-1}. So the proof is complete when we show $\mathrm{d}\omega^0/\mathrm{d}(d) > 0$. Since

$$\omega^0 = \frac{b^0 + d - 2\Delta}{2b^0 + d - 2\Delta} = \frac{2d - \Delta + \sqrt{d^2 + \Delta^2}}{3d + 2\sqrt{d^2 + \Delta^2}},$$

we have

$$\frac{\mathrm{d}\omega^0}{\mathrm{d}(d)} = \frac{2d + \Delta + 3\sqrt{d^2 + \Delta^2}}{(3d + 2\sqrt{d^2 + \Delta^2})^2} \frac{\Delta}{\sqrt{d^2 + \Delta^2}} > 0.$$

Claim 4: $(b^0 - 2\Delta)/u^k$ is increasing in d for each k. To prove it, let $y^k = w^k + 2\Delta$. Write the difference equation for w^k in the form:

$$\frac{y^k}{b^0 - 2\Delta + y^k} = \frac{(d + 2\Delta)u^{k-1} + (b^0 - 2\Delta)(y^{k-1} - 2\Delta)}{(b^0 + d)u^{k-1} + (b^0 - 2\Delta)(y^{k-1} - 2\Delta)}.$$

Let $\theta^k = (b^0 - 2\Delta)/u^k = 1 - y^k/u^k$. Then the above equation can be transformed into:

$$\theta^k = \frac{b^0 - 2\Delta}{d + 2b^0 - 2\Delta - b^0\theta^{k-1}}.$$

It is clear that $\partial\theta^k/\partial\theta^{k-1} > 0$. Moreover, $\partial\theta^k/\partial d$ has the same sign as:

$$-(b^0 - 2\Delta) + (d + 2\Delta - 2\Delta\theta^{k-1})\frac{\partial b^0}{\partial d}.$$

By Claim 1, θ^k is increasing in k, so the above expression is greater than

$$-(b^0 - 2\Delta) + (d + 2\Delta - 2\Delta\theta^\infty)\frac{\partial b^0}{\partial d} > 0,$$

where $\theta^\infty = \frac{1}{2} - \frac{1}{2}\Delta/(1+d-V(1)-\Delta)$ is the limit value of θ^k as k goes to infinity. So an induction argument will establish the monotonicity of θ^k with respect to d if we can show $\mathrm{d}\theta^0/\mathrm{d}(d) > 0$. Now, $\theta^0 = (b^0 - 2\Delta)/(2b^0 + d - 2\Delta)$. Taking derivative with respect to d gives

$$\frac{\mathrm{d}\theta^0}{\mathrm{d}(d)} = \frac{2d - \Delta + 3\sqrt{d^2+\Delta^2}}{(3d + 2\sqrt{d^2+\Delta^2})^2}\frac{\Delta}{\sqrt{d^2+\Delta^2}} > 0.$$

Claim 5: b^k is increasing in d for each k. We can write the difference equation for b^k as:

$$b^k = 2\Delta + \theta^{k-1}(b^{k-1} - 2\Delta),$$

implying that

$$\frac{\mathrm{d}b^k}{\mathrm{d}(d)} = (b^{k-1} - 2\Delta)\frac{\mathrm{d}\theta^{k-1}}{\mathrm{d}(d)} + \theta^{k-1}\frac{\mathrm{d}b^{k-1}}{\mathrm{d}(d)}.$$

From Claim 4 we already have that $\mathrm{d}\theta^{k-1}/\mathrm{d}(d) > 0$. Moreover, $\mathrm{d}b^0/\mathrm{d}(d) > 0$. So an induction argument shows that $\mathrm{d}b^k/\mathrm{d}(d) > 0$ for each k.

Claim 6: a^k is decreasing in d for each k. To prove it, write the difference equation for a^k as:

$$a^k = 1 - \theta^{k-1}(1 + d - a^{k-1}).$$

Thus,

$$\frac{\mathrm{d}a^k}{\mathrm{d}(d)} = -\theta^{k-1} - (1 + d - a^{k-1})\frac{\mathrm{d}\theta^{k-1}}{\mathrm{d}(d)} + \theta^{k-1}\frac{\mathrm{d}a^{k-1}}{\mathrm{d}(d)}.$$

Since $\mathrm{d}a^0/\mathrm{d}(d) = 0$, an induction argument establishes that $da^k/d(d) \le 0$ for each k.

Claim 7: s^k is increasing in d for each k. Since $s^k = 1 + d - a^k$, this claim follows immediately from Claim 6.

Claim 8: s^k/b^0 is increasing in d for each k. To prove it, write the difference equation for a^k as: $s^k/b^0 = (d/b^0) + \theta^{k-1}(s^{k-1}/b^0)$. Note that $s^0/b^0 = d/b^0$ is increasing in d. Also, θ^{k-1} is increasing in d. So an induction argument establishes the claim.

Claim 9: $s^k/(b^0 - \Delta)$ is increasing in d for each k. To prove it, write the difference equation for a^k as:

$$\frac{s^k}{b^0 - \Delta} = \frac{d}{b^0 - \Delta} + \theta^{k-1}\frac{s^{k-1}}{b^0 - \Delta}.$$

Note that $s^0/(b^0 - \Delta) = d/(b^0 - \Delta)$ is increasing in d. So an induction argument establishes the claim.

Claim 10: $(b^k - \Delta)/(b^0 - \Delta)$ is decreasing in d for each k. Write the difference equation for b^k as:

$$\frac{b^k - \Delta}{b^0 - \Delta} = \frac{\Delta}{b^0 - \Delta} + \frac{(b^0 - 2\Delta)((b^{k-1} - \Delta) - \Delta)}{(b^0 - \Delta) + (1 + d - a^{k-1}) + (b^{k-1} - \Delta)}.$$

Define $\rho^k = (b^k - \Delta)/(b^0 - \Delta)$ and $\sigma^k = (1 + d - a^k)/(b^0 - \Delta)$. Then we can write

$$\rho^k = (1 - \theta^{k-1}) + \frac{b^0 - 2\Delta}{b^0 - \Delta} \frac{\rho^{k-1}}{1 + \sigma^{k-1} + \rho^{k-1}}.$$

Note that

$$\begin{aligned}
\frac{\partial \rho^k}{\partial \rho^{k-1}} &= \frac{b^0 - 2\Delta}{b^0 - \Delta} \frac{1 + \sigma^{k-1}}{(1 + \sigma^{k-1} + \rho^{k-1})^2} > 0, \\
\frac{\partial \rho^k}{\partial \sigma^{k-1}} &= -\frac{b^0 - 2\Delta}{b^0 - \Delta} \frac{\rho^{k-1}}{(1 + \sigma^{k-1} + \rho^{k-1})^2} < 0, \\
\frac{\partial \rho^k}{\partial \theta^{k-1}} &= -\frac{\Delta}{b^0 - \Delta} < 0, \\
\frac{\partial \rho^k}{\partial b^0} &= -\frac{\Delta}{(b^0 - \Delta)^2} \frac{1 + d - a^{k-1} + \Delta}{u^{k-1}} < 0.
\end{aligned}$$

Now,

$$\frac{\mathrm{d}\rho^k}{\mathrm{d}(d)} = \frac{\partial \rho^k}{\partial b^0}\frac{\partial b^0}{\partial d} + \frac{\partial \rho^k}{\partial \theta^{k-1}}\frac{\mathrm{d}\theta^{k-1}}{\mathrm{d}(d)} + \frac{\partial \rho^k}{\partial \sigma^{k-1}}\frac{\mathrm{d}\sigma^{k-1}}{\mathrm{d}(d)} + \frac{\partial \rho^k}{\partial \rho^{k-1}}\frac{\mathrm{d}\rho^{k-1}}{\mathrm{d}(d)}.$$

Since $\partial b^0/\partial d > 0$, $\mathrm{d}\theta^{k-1}/\mathrm{d}(d) > 0$ (by Claim 4), $\mathrm{d}\sigma^{k-1}/\mathrm{d}(d) > 0$ (by Claim 9), and $\mathrm{d}\rho^0/\mathrm{d}(d) = 0$. An induction argument establishes that $\mathrm{d}\rho^k/\mathrm{d}(d) < 0$ for each k.

Claim 11: s^k/u^k is increasing in d for each k. To show it, divide both the denominator and numerator or s^k/u^k by $b^0 - \Delta$ to get:

$$\frac{s^k}{u^k} = \frac{\sigma^k}{1 + \sigma^k + \rho^k}.$$

Since Claim 9 shows that σ^k is increasing in d and Claim 10 shows that ρ^k is decreasing in d, we have that s^k/u^k is increasing in d.

Now we are ready to show G^k is increasing in d for each k. Rewrite the difference equation for G^k as:

$$\frac{G^{k+1}}{1 - G^{k+1}} = \frac{s^{k-1}}{b^0} + \frac{u^{k-1}}{b^0} \frac{G^k}{1 - G^k}.$$

From Claim 3 and Claim 8 we have that both s^k/b^0 and u^k/b^0 are increasing in d. It is also clear that G^{k+1} is increasing in G^k. Finally, $G^1 = d/(b^0 + d)$ is increasing in d. So an induction argument establishes the first part of the proposition.

To establish the second part of the proposition, let $\tilde{d} > d$. Denote the sequence of threshold values of γ corresponding to $\tilde{d}$ as $\{\tilde{G}^k\}$, and denote the corresponding sequence of coefficients of the payoff function V as $\{(\tilde{a}^k, \tilde{b}^k)\}$. Suppose that $\gamma \in [G^k, G^{k+1})$ while $\gamma \in [\tilde{G}^{\tilde{k}}, \tilde{G}^{\tilde{k}+1})$. Then

$$\tilde{a}^{\tilde{k}} - \tilde{b}^{\tilde{k}}\gamma < a^{\tilde{k}} - b^{\tilde{k}}\gamma \leq a^k - b^k\gamma,$$

where the first inequality follows from Claims 5 and 6 and the second inequality follows from the convexity of $V(\gamma)$. Thus, $V(\gamma)$ is decreasing in d.

Finally, for any γ, let

$$x^k(\gamma) = \frac{b^0}{u^{k-1}} - \frac{1-\gamma}{\gamma}\frac{s^{k-1}}{u^{k-1}}.$$

Since $x^k(G^{k+1}) = x^{k+1}(G^{k+1})$, and since

$$\frac{\partial x^k(\gamma)}{\partial \gamma} = \frac{1}{\gamma^2}\frac{s^{k-1}}{u^{k-1}} < \frac{1}{\gamma^2}\frac{s^k}{u^k} = \frac{\partial x^{k+1}(\gamma)}{\partial \gamma}$$

by Claim 11, we have $x^k(\gamma) \geq x^{k+1}(\gamma)$ for all $\gamma \leq G^{k+1}$. Iterating the argument establishes that $x^k(\gamma) \geq x^{\tilde{k}}(\gamma)$ for all $\gamma \leq G^{k+1}$ and all $\tilde{k} \geq k$. The same argument to prove that $x^k(\gamma) \geq x^{\tilde{k}}(\gamma)$ for all $\gamma \geq G^k$ and all $\tilde{k} \leq k$. Combining these two results, we have $x^k(\gamma) \geq x^{\tilde{k}}(\gamma)$ for all $\tilde{k}$ if $\gamma \in [G^k, G^{k+1})$. Now, for any $d' > d$, denote the corresponding equilibrium strategy as $\tilde{x}(\gamma)$, and define $\tilde{x}^k(\gamma)$ analogously. Then, for any $\gamma \in [G^k, G^{k+1})$,

$$x(\gamma) = x^k(\gamma) \geq x^{\tilde{k}}(\gamma) > \tilde{x}^{\tilde{k}}(\gamma) = \tilde{x}(\gamma),$$

where the first inequality follows because $\gamma \in [G^k, G^{k+1})$, and the second inequality comes from Claims 3 and 11. Thus, $x(\gamma)$ is decreasing in d for all γ.

References

Austen-Smith, David, “Information Transmission in Debate,” *American Journal of Political Science* 34(1), 1990, pp. 124–152.

Austen-Smith, David and Timothy J. Feddersen, “Deliberation, Preference Uncertainty, and Voting Rules,” *American Political Science Review* 100(2), May 2006, pp. 209–217.

Cai, Hong-bin, “Delay in Multilateral Bargaining under Complete Information,” *Journal of Economic Theory* 93, pp. 260-276.

Chatterjee, Kalyan and Larry Samuelson, “Bargaining with Two-sided Incomplete Information: An Infinite Horizon Model with Alternating Offers,” *Review of Economic Studies* 54(2), April 1987, pp. 175–192.

Cho, In-Koo, “Uncertainty and Delay in Bargaining,” *Review of Economic Studies* 57(4), October 1990, pp. 575–595.

Cho, In-Koo and David Kreps, “Signalling Games and Stable Equilibria,” *Quarterly Journal of Economics* 102, pp. 179–221.

Coughlan, Peter J., “In Defense of Unanimous Jury Verdicts: Mistrials, Communication, and Strategic Voting,” *American Political Science Review* 94(2), June 2000, pp. 375–393.

Cramton, Peter C., “Strategic Delay in Bargaining with Two-Sided Uncertainty,” *Review of Economic Studies* 59(1), January 1992, pp. 205–225.

Dewatripont, Mathias, “Commitment Through Renegotiation-Proof Contracts with Third Parties,” *Review of Economic Studies* 55(3), July 1988, pp. 377–389.

Fershtman, Chaim and Daniel Seidmann, “Deadline Effects and Inefficient Delay in Bargaining with Endogenous Commitment,” *Journal of Economic Theory* 60, 1993, pp. 306–321.

Kennan, John and Robert Wilson, “Bargaining with Private Information,” *Journal of Economic Literature* 31(1), March 1993, pp. 45–104.

Holmstrom, Bengt, “Moral Hazard in Teams,” *Bell Journal of Economics* 13(2), Autumn 1982, pp. 324–340.

Holmstrom, Bengt and Roger Myerson, “Efficient and Durable Decision Rules with Incomplete Information,” *Econometrica* 51, pp. 1799–1819.

Kreps, David and Robert Wilson, “Sequential Equilibrium,” *Econometrica*, 54, 1982, pp. 863–894.

Laffont, Jean-Jacques and Jean Tirole, “The Dynamics of Incentive Contracts,” *Econometrica* 56 (5), September 1988, pp. 1153–1175.

Li, Hao, Sherwin Rosen and Wing Suen, “Conflicts and Common Interests in Committees,” *American Economic Review* 91(5), December 2001, pp. 1478–1497.

Maskin, Eric and John Moore, “Implementation and Renegotiation,” *Review of Economic Studies* 66, pp. 39–56.

Myerson, Roger and Mark Satterthwaite, “Efficient Mechanisms for Bilateral Trading,” *Journal of Economic Theory* 29(2), April 1983, pp. 265–281

Ottaviani, Marco and Peter Sorensen, “Information Aggregation in Debate: Who Should Speak First?” *Journal of Public Economics* 81, 2001, pp. 393–421.

Rubinstein, Ariel, “Perfect Equilibrium in a Bargaining Model,” *Econometrica* 50, 1982, pp. 97–109.

Sakovics, Jozsef, ”Delay in Bargaining Games with Complete Information,” *Journal of Economic Theory* 59, 1993, pp. 78–95.

Segal, Ilya and Michael Whinston, “The Mirrlees Approach to Mechanism Design and Renegotiation (with an Applications to Hold-up and Risk-Sharing,” *Econometrica* 70, pp. 1–45.

Stahl, Ingolf, *Bargaining Theory*, Stockholm: Economics Research Institute, Stockholm School of Economics, 1972.