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Political Competition between Differentiated Candidates

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Abstract

We introduce a framework of electoral competition in which voters have general preferences over candidates' immutable characteristics (such as gender, race or previously committed policy positions) and flexible policy positions. Candidates are uncertain about the distribution of voter preferences and choose policy positions to maximize their winning probability.

We characterize a property of voter utility functions ("uniform candidate ranking", UCR) that captures a form of separability between fixed characteristics and policy. When voters have UCR preferences, candidates' equilibrium policies converge in any strict equilibrium. In contrast, notions like competence or complementarity lead to non-UCR preferences and policy divergence. In particular, we introduce two new classes of models that, respectively, contain the probabilistic voting model and the classical spatial Downsian model as special cases and in which there is a unique equilibrium that features policy divergence (except in those special cases known from previous literature).

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1 Introduction

The political competition model introduced by Downs (1957) analyzes a setup in which two candidates choose a platform from a set of feasible policies that is a one-dimensional set, such as the interval $[0, 1]$, and where all voters have single-peaked preferences over this policy space. If candidates are ex-ante identical and purely office-motivated, they propose identical policies to voters, namely the one that maximizes the utility of the median voter. The question we address in this paper is whether policy convergence is a robust feature of political competition if we admit a more general policy space and, in particular, more general voter preferences, but otherwise keep Downs's assumption of two office-motivated candidates who compete under plurality rule.

To answer our question, we introduce a model where candidates have some unchangeable characteristics like a candidate's previous experience, gender or race. On other policy issues, candidates are flexible, and they are willing to use these positions as tools to maximize the probability of getting elected. Fixed positions may also stand for positions on core issues in which a candidate cannot credibly promise to implement a policy that differs from his preferred one. Thus, while our candidates are formally "office-motivated" in that they choose flexible positions so as to maximize their probability of winning, our model can still capture substantial "policy motivation". A candidate may ultimately care about being elected because his core convictions (as captured by fixed positions) differ from those of his competitor. By interpreting (some) fixed characteristics as already committed policy positions, our model provides a middle ground between Downsian models, in which candidates are free to choose any position, and the citizen candidate model in which no commitment is possible.

Voters' preferences defined over the candidates' vectors of characteristics and policies are completely general. In particular, we do not require that preferences are separable across characteristics and issues; fundamentally, this is the main departure from the existing literature. The distribution of voter preferences depends on a state variable that is unknown to candidates at the time they choose their positions.

Our first and very straightforward result, Theorem 1, shows that differentiated fixed characteristics are a necessary condition for (generic) equilibria with policy divergence to arise, even if we admit arbitrary voter preferences. Intuitively, without fixed characteristics, the candidates' payoffs on the main diagonal of the payoff matrix (i.e., if both candidates choose the same policy) are equal to $1/2$. Since we have a constant sum game, the winning probabilities in *any* pure strategy equilibrium must be $1/2$ for each candidate (and, generically, this can only be the case if the candidates choose the same policy). This argument, however, breaks down if candidates have differentiated fixed characteristics. In that case, it is possible that candidates choose different policies in an equilibrium in which one candidate's winning probability is strictly less than 50%, because candidates cannot perfectly copy their opponent: Even if a candidate chooses the same policy platform as his opponent, the existence of fixed characteristics implies that many or all voters can still have strict preferences for one of the candidates. Hence, there is no guarantee that imitating the opponent increases the winning probability of the candidate who has the lower winning probability in an equilibrium with policy divergence.

This insight raises the question whether there is a class of voter preferences for which policy convergence is still guaranteed (in equilibrium), even if candidates have differentiated fixed characteristics. We find such a general property of voter preferences, which we call *uniform candidate ranking* (UCR). UCR does not impose any restrictions on voter preferences if candidates choose different policies, but if the two candidates choose the same policy platform, a UCR voter always prefers the same candidate. That is, suppose that, due to the difference in fixed characteristics, a voter prefers Candidate 0 to Candidate 1 if both propose policy a; then a UCR voter also prefers Candidate 0 to Candidate 1 if both propose policy a'. Since every voter votes the same, whether both candidates choose a or both choose a', the UCR assumption implies that the diagonal entries of the payoff matrix must be identical.[1] Using this observation and the fact that the game is a zero sum game, Theorem 3 shows that there is policy convergence in any strict Nash equilibrium of a voting game with UCR preferences and ex-ante non-identical candidates.

Are UCR preferences a *necessary* condition for equilibrium policy convergence? Absent additional conditions, we cannot expect any assumption on individual preferences to be necessary. For example, if citizens with non-UCR preferences are never pivotal, then the violation of UCR would not matter for equilibrium convergence. The same is true if UCR is violated for some policies that are sufficiently undesirable for most voters. However, if we endow an arbitrary individual with non-UCR preferences, then there exists a voting game where all other voters have UCR preferences and the unique strict Nash equilibrium has policy divergence (Theorem 4). As an alternative approach to show that UCR is ”close” to a necessary condition, Section 5 shows a class of models in which equilibrium policies converge if and only if preferences are UCR.

Most preferences found in models in the literature — such as the one-dimensional Downsian model, the Downsian model with uncertainty about the median, the Downsian model with valence, or the probabilistic voting model — are additively separable between fixed characteristics and flexible issues and can easily be seen to satisfy UCR. While Theorem 2 shows that the class of UCR preferences is more general than the class of additively separable preferences, it is also easy to find natural circumstances in which voters have non-UCR preferences, and where policies diverge in a robust pure strategy equilibrium.

In particular, we present two classes of models that generalize the two probably most widely used models in the literature, in a way that captures the notions of complementarity and competence, respectively. In Section 5, we consider a generalization of the classical probabilistic voting model (PVM), in which groups are identified as voters who have the same “economic” preferences (i.e., preferences over pledgeable policies), but within a group, voters may differ with respect to what Persson and Tabellini (2000), p. 52 refer to as “ideology.” They write that “one way to motivate [ideology] is to think about a second policy dimension, orthogonal to fiscal policy, in which candidates cannot make credible commit-

[1] Of course, in contrast to the case with identical fixed characteristics, the winning probabilities on the main diagonal do not have to be $1/2$.

ments, but set an optimal policy after the election according to their ideology." Rather than modeling the second policy dimension explicitly, most papers in the probabilistic voting literature operationalize the notion of ideology by adding an additive ideology shock to the economic preferences.

We explicitly model the relevant policy space as two-dimensional: In one dimension, candidates are exogenously fixed while they can choose their policy position in the other dimension. If indifference curves are exact circles in this two-dimensional space, then the fixed and the flexible dimension are completely independent of each other in the voters' utility functions. In this case, the model has a unique equilibrium with convergence that corresponds to the equilibrium of the standard PVM. In contrast, elliptical indifference curves with the major axis located in a southwest–northeast direction capture a notion of complementarity between fixed and flexible dimension, in the sense that a voter's ideal policy on the flexible policy dimension is increasing in the candidate's position on the fixed dimension. For example, suppose that two presidential candidates differ in how open they are to international security cooperation, and also by how much cooperation/opposition these candidates would get from international actors. Candidates are fixed in this dimension, but they can choose the size of their proposed military spending. In this context, it is not implausible that a voter's ideal defense budget depends on the candidate's identity (i.e., his fixed characteristics).

With elliptical preferences, the model still has a unique equilibrium, but one that features policy divergence.[2] Specifically, the candidate with a higher fixed characteristic chooses a higher position on the flexible policy dimension than his opponent.

In Section 6, we develop a second class of models in which we explore the notion of competence in a public finance framework.[3] A standard interpretation of the classical Downsian model is one in which policy proposals correspond to uniform tax rates, and where tax revenues are used to finance a public good. Voters differ in their preferences over the size of government, founded ultimately either on differences in tastes or on differences in income. In this setup, it is well-known that both candidates will propose the preferred policy of the median voter in equilibrium.

A more-or-less implicit assumption in the classical Downsian model is that only total tax revenue, but not the identity of the office-holder, matters for public good production. We depart from this assumption by endowing each candidate with his own production function that captures how he can transform tax revenue into public goods. Moreover, we assume that one candidate has lower fixed cost, but also a lower marginal productivity than his competitor (and therefore has a production advantage when tax rates are low, while his competitor produces more public goods when tax rates are high).

[2]From a technical point of view, the model shows the surprising usefulness of Theorem 3 in a setting with non-UCR preferences. Specifically, we show that the policy space can be deformed in a way that voter preferences are UCR in the deformed policy space. We then apply Theorem 3 to show that the equilibrium in the deformed space is unique and features convergence. Re-transformation of the policy space then shows that the equilibrium in the actual policy space is still unique, but features divergence.

[3]This section comprises results from and supersedes our previous working paper Krasa and Polborn (2009b).

Whenever a pure strategy equilibrium exists in our model (and we provide sufficient conditions for this to be true), it is unique and features policy divergence. Moreover, the equilibrium is characterized by a cutoff voter who is indifferent between both candidates. Each candidate chooses a platform that maximizes the cutoff voter's utility. Thus, our model captures the central intuitions from the standard model that candidates compete for the support of some moderate voter type, and that they do so by proposing the policy that is optimal for this closely contested voter type. Yet, there are two crucial differences between the cutoff voter in our taxation model and the standard median voter. First, the location of the cutoff voter is determined by candidate technology and voter preferences (basically, it is the voter type that is indifferent between candidates if both candidates choose their optimal policy for this type, respectively). However, in contrast to the standard spatial model, the location of the cutoff voter is independent of the distribution of voter preferences — in general, the cutoff voter differs from the expected (or median) median voter in the economy. This also implies that one of the candidates can be more likely to win the election than his competitor.

The two new classes of non-UCR models capture the natural notions of complementarity and of competence differences and are thus of direct substantive interest. They also provide us with tractable models in which purely office-motivated candidates choose divergent policy platforms — in contrast to the standard model in which office motivated candidates have a strong incentive for platform convergence. One of the most popular models used to explain policy divergence within the standard spatial framework assumes that candidates are policy-motivated, i.e., candidates are willing to lower their chance of winning in the election in exchange for being able to implement a particular policy in case they win. Thus, the reader may ask whether we need an explanation other than policy motivation for policy divergence, and whether our model is empirically distinguishable from the model with policy-motivation.

Concerning the first question, we do not see our assumption of "office-motivation" as diametrically opposed to policy-motivation. In fact, it is quite plausible that candidates are policy-motivated in some issues, but these issues are captured as "fixed positions" in our framework. Candidates use the positions on the *remaining* issues as tools to get elected —either because they care about the material aspects of the office (classical office-motivation), or because they care primarily about the implementation of their core convictions. Explaining policy divergence *on flexible issues* in this framework is useful, because by focusing on the standard model of policy motivation, we may miss other interesting and relevant reasons why divergence arises in practice. In particular, in our model, divergence may be a strategy that maximizes a candidate's probability of winning, and thus would not have to be interpreted as an indication that the candidate is policy-motivated.

Related to the second question, the candidates' incentives that generate policy divergence differ between our model and the standard model with policy-motivated candidates. These different incentives can be used to generate testable predictions that allow to empirically discriminate between the two models. In the standard spatial model, there are costs and benefits of policy divergence. By choosing a platform farther away from his opponent's, a candidate trades off an increased utility from policy if he wins against a lower chance of winning. In our model, candidates are assumed to maximize the probability of winning, and in some situations, this will induce them to choose positions that diverge from their opponent's equilibrium position. Thus, changes in the environment that affect the costs and benefits (e.g., an increase in the wage of the office-holder) should affect policy positions in the Downsian model, but not in ours. Similarly, the cost of policy divergence (in terms of reduction of the winning probability) is affected by the quality of information about the median voter's preferred position. Better and more easily available opinion polls should translate into smaller policy divergence in the standard model. In contrast, the extent of equilibrium divergence in Sections 5 and 6 is independent of the uncertainty about voter preferences, and thus of the availability and quality of opinion polls.

2 Previous Literature

The platform choice of candidates for political office is one of the major areas of interest in formal models of politics. There is a huge literature on the topic of policy convergence or divergence in one-dimensional models (or models with one policy dimension and one valence dimension). For excellent reviews of this area, see, e.g., Osborne (1995) and Grofman (2004).

There is a large literature that tries to explain, within the Downsian model, the empirical observation that candidates often propose considerably divergent policies. Candidates may diverge even though this decreases their winning probability, because they care about the implemented policy (see, e.g., Wittman (1983), Calvert (1985), Roemer (1994), Martinelli (2001), Gul and Pesendorfer (2009)). In contrast, in our model, divergence may increase a candidate's probability of winning.

Some models obtain policy divergence with office-motivated candidates in a one-dimensional setting with incomplete information among voters about candidate characteristics (e.g. Callander (2008)) or among candidates about the position of the median voter (Castanheira (2003), Bernhardt, Duggan, and Squintani (2006)). Another branch of literature on divergence with office motivation, which is less directly related to our paper, explains policy divergence as entry deterrence by two dominant parties (e.g., Palfrey (1984), Callander (2005)).

Both the literature on candidates with valence (e.g. Ansolabehere and Snyder (2000), Groseclose (2001)) and the probabilistic voting literature (e.g., Hinich (1978), Lindbeck and Weibull (1987), Lindbeck and Weibull (1993), Coughlin (1992), Dixit and Londregan (1995), Banks and Duggan (2005)) share with our paper the feature that voters care both about canddiates' unchangeable characteristics and their flexible policy positions. However, voter preferences in all these papers satisfy our UCR-property and thus, by Theorem 3, any pure strategy equilibrium in these models features convergence.

Krasa and Polborn (2010) analyze a model with office-motivated candidates in which both fixed characteristics and flexible positions are binary and voters have an additively separable utility function. The main focus of Krasa and Polborn (2010) is to characterize voter preference distributions for which candidates have "majority-efficient" positions, and under which conditions candidates choose majority-efficient positions in settings where those exist (a position on flexible issues is majority-efficient if there is no other position that a majority of voters would prefer from that candidate). Since additive voter preferences satisfy our UCR condition, any equilibrium "divergence" in Krasa and Polborn (2010) is in mixed strategies only. In contrast, in Sections 5 and 6 of the present paper, we show that divergence can arise in a strict pure strategy Nash equilibrium when voter preferences are of the non-UCR type.

Finally, there are a few dispersed papers in the literature in which voters are endowed with non-UCR preferences and in which a pure strategy equilibrium thus (can) feature policy-divergence. For example, Adams and Merrill (2003) analyze a model in which voters have, in addition to preferences over policy positions from the $[0, 1]$ interval, "non-policy preferences" over the two candidates, which corresponds to different fixed positions in our setting. They assume that voters may abstain due to being almost indifferent between candidates, or due to "alienation" (if their preferred candidate does not provide them with sufficient utility). While there is still policy convergence in this model if voters only abstain from indifference (see also Erikson and Romero (1990)), they show that abstention from alienation may provide an incentive for strong divergence. We show that abstention due to alienation leads to non-UCR preferences, which is the fundamental reason for divergence in Adams and Merrill (2003). Similarly, in a variation of their basic probabilistic voting model of redistribution between different economic groups, Dixit and Londregan (1996) show that, if candidates differ in how well they can transfer resources to different interest groups, then they usually propose different transfers. Finally, Soubeyran (2009), Krasa and Polborn (2009a) and Jensen (2009) analyze settings in which candidates differ in their ability to implement certain policies. In all of these papers, the focus is on the particular application, while our main interest here is to understand which general properties of voter utility functions drive policy convergence or divergence results.

3 The Model

Two candidates, $j = 0, 1$, compete in an election. Candidates are office-motivated and receive utility 1 if elected, and utility 0 otherwise, independent of the implemented policy. Candidate j has fixed characteristics $c_j \in C$, which we also call his *type*. If elected, Candidate j implements a policy position $a_j \in A$.

Uncertainty about voter preferences is described by a probability space $(\Omega, \mathfrak{O}, \mu)$: A state $\omega \in \Omega$ determines voters' preferences over $C \times A$, and μ is the probability distribution of these "preference shocks", while $\mathfrak{O}$ is the set of measurable events. In particular, let P_r be the set of preferences on $C \times A$. Then the preferences of voter $\ell \in \mathfrak{L} = \{1, \ldots, L\}$ in state $\omega \in \Omega$ are $\succeq_\omega^\ell \in P_r$.[4]

[4]More formally, let $\mathfrak{P}_r$ be a σ-algebra of measurable subsets of P_r then voter ℓ's random preferences are given by a measurable function $t_\ell \colon \Omega \to P_r$. For example, if C and A are finite then P_r is finite. In this case, $\mathfrak{P}_r$ is the set of all subsets of P_r, and

The timing of the game is as follows:

Stage 1 Candidates $j = 0, 1$ simultaneously announce policies $a_j \in A$. A mixed strategy by Candidate j consists of a probability distribution σ_j over A.

Stage 2 State $\omega \in \Omega$ is realized and each citizen votes for his preferred candidate, or abstains when he is indifferent.[5]

Candidate j wins the election if he receives more votes than his opponent. In case of a tie between the candidates, each wins with probability 1/2. Let $W^j(\omega, a_0, a_1)$ denote Candidate j's winning probability in state ω, given policies a_0 and a_1. Formally, $W^0(\omega, a_0, a_1) = \xi(\nu(\omega, a_0, a_1))$, where $\xi(x < 0) = 0$, $\xi(0) = 1/2$ and $\xi(x > 0) = 1$; and $\nu(\omega, a_0, a_1) = \#\left\{\ell \;\middle|\; (c_0, a_0) \succeq_\omega^\ell (c_1, a_1)\right\} - \#\left\{\ell \;\middle|\; (c_1, a_1) \succeq_\omega^\ell (c_0, a_0)\right\}$.

4 Convergence and Divergence of Equilibrium Policies

4.1 A General Convergence Result without Fixed Characteristics

Our first result shows that, for arbitrary voter preferences, if candidates' fixed characteristics coincide, then any generic pure strategy equilibrium displays policy convergence. Note that Theorem 1 is a characterization result and does not provide conditions under which a strict Nash equilibrium exists. Indeed, since our framework is very general, necessary and sufficient conditions for equilibrium existence are hard to obtain. Nevertheless, we know that Theorem 1 is not vacuous as there are classes of voter preferences, such as the Downsian model or the probabilistic voting model, in which a strict equilibrium is known to exist. The main usefulness of Theorem 1 is therefore that it tells modelers that, as long as we assume that candidates are identical, *no* utility functions for voters will be able to generate equilibrium divergence.

Theorem 1 *Suppose that $c_0 = c_1$.*

1. *If there exists a pure strategy Nash equilibrium (a_0, a_1) with $a_0 \neq a_1$, then (a_0, a_0) and (a_1, a_1) are also pure strategy Nash equilibria.*
2. *If there exists a strict Nash equilibrium (a_0, a_1) then $a_0 = a_1$ and this strict Nash equilibrium is the unique Nash equilibrium (pure or mixed).*

measurability means that the set of all states ω that is mapped into one particular preference ordering is measurable.

[5]If a voter has a strict preference, then it is a weakly dominant strategy to vote for the preferred candidate. If a voter is indifferent, he could in principle vote for any candidate or abstain. We assume that he abstains, which is quite natural (e.g., in the presence of even very small voting costs), and also allows us to easily model a random number of voters $L(\omega) \leq L$ by simply by modeling $L - L(\omega)$ voters as indifferent between all policies, so that they will abstain no matter what policies the candidates choose.

Divergent pure strategy equilibria cannot be unique, as long as candidates' fixed characteristics do not differ: Whenever they exist, there is also an equilibrium with policy convergence; moreover, any policy divergence is weak in the sense that candidates do not strictly prefer the particular platform they choose. Thus, our result generalizes the convergence results familiar from the Downsian model to a setup with multiple issues and uncertainty about preferences. In the Downsian model under certainty both candidates choose the policy that is most preferred by the median voter. If the position of the median voter is uncertain, then candidates converge on the "median median," that is, there is no other position that would make a majority better off in a majority of states. The intuition of the median voter theorem continues to hold for general preferences: In an equilibrium, no other position can make a majority of voters better off in a majority of states. The reason is that, if such a policy position existed, then either candidate could deviate to it, thereby increasing his winning probability to more than $1/2$.

Theorem 1 is related to Theorem 7.1 in Austen-Smith and Banks (2005). In a setting with certainty about the preference distribution of voters, they show that a pair of platforms (a_0, a_1) is an equilibrium if and only if a_0 and a_1 are both policies that cannot be blocked by a decisive coalition (i.e., in the case of plurality rule, that are Condorcet winners). In many frameworks, there is (at most) one Condorcet winner, in which case convergence arises trivially. However, even if this is not the case, Theorem 1 shows that divergent equilibria can neither be strict nor unique.

4.2 UCR Preferences

We now turn to the more relevant case that candidates' fixed characteristics differ, and analyze under which conditions there is policy convergence in those issues that candidates are free to choose. In this section, we identify a condition on voter preferences called uniform candidate ranking (UCR). In Section 4.3, we show that UCR preferences are sufficient for equilibrium policies to (generically) converge, and that they are, in a certain sense, also necessary for convergence results.

We start with the definition of UCR preferences. Suppose that both candidates choose the same policy $a \in A$. We say that a voter has *uniform candidate ranking (UCR)* preferences if a voter's preferences for the candidates are independent of a. For example, suppose that $C = A = \{0, 1\}$. Preferences are therefore defined on $\{0, 1\} \times \{0, 1\}$, where the first coordinate is the candidate's fixed characteristic and the second one the policy issue. A UCR voter prefers $(0, 0)$ to $(1, 0)$ if and only if he also prefers $(0, 1)$ to $(1, 1)$.

Definition 1 *Preferences $\succeq$ on $C \times A$ allow for a* ***uniform candidate ranking (UCR)*** *if, for all $c_0, c_1 \in C$ and all $a, a' \in A$,*

$$(c_0, a) \succeq (c_1, a) \text{ if and only if } (c_0, a') \succeq (c_1, a'). \tag{1}$$

Models in which candidates have no fixed characteristics (e.g., the standard one-dimensional Downsian model) automatically satisfy Definition 1. Also, a model with a one-dimensional policy space and random candidate valences satisfies UCR, as does a model with uncertainty about the preferred position of the median voter (as well as valence). Likewise, voter preferences in the probabilistic voting model (see, e.g., Lindbeck and Weibull (1987), Lindbeck and Weibull (1993), Coughlin (1992)) satisfy UCR.

For example, consider a model with stochastic valence: In state $\omega = (\omega_0, \omega_1)$, voter θ's utility from Candidate 0 is given by $\omega_0 - (a_0 - \theta)^2$, while his utility from Candidate 1 is given by $\omega_1 - (a_1 - \theta)^2$. Clearly, when $a_0 = a_1$, the voter strictly prefers Candidate 0 if and only if $\omega_0 > \omega_1$. Since this preference is independent of the particular policy $a_0 = a_1$, UCR is satisfied.

While UCR preferences are prevalent in the literature, there are circumstances in which "natural" preferences violate UCR. For example, suppose that a candidate's fixed characteristics capture his competence in implementing different policies. Specifically, suppose that the fixed characteristic is whether or not a candidate has served in the military, while the policy issue is whether or not to go to war with some other country. It is conceivable that a voter considers the candidate who has served in the military as a better leader for the country during a war, while preferring his opponent with a civilian background if there is peace. Formally, such a voter could have the preference $(1, 1) \succ (0, 0) \succ (1, 0) \succ (0, 1)$, that is, prefers most to go to war with a leader with military experience, while the second best option is not to go to war and have a leader with civilian background, which again is better than both "mixed" policy vectors. These preferences violate UCR, because the voter's preferred candidate changes from the situation that both propose to go to war to another one in which both propose peace.

We now characterize the set of utility functions that represent UCR preferences.

Theorem 2 *Let A and C be separable metric spaces, and let C be compact. Then the following statements are equivalent:*

1. *Rational (i.e., complete and transitive) and continuous[6] preferences $\succeq$ on $C \times A$ satisfy UCR.*
2. *The preferences $\succeq$ can be described by a continuous utility function $u(c, a) = g(f(c), a)$ where $f : C \to Y \subset \mathbb{R}$ is continuous, and $g : Y \times A \to \mathbb{R}$ is continuous and strictly monotone in $y \in Y$.*

We can interpret $f(c)$ as the voter's ranking of the candidates' fixed characteristics — a higher value of $f(c)$ indicates that the voter ranks the candidate higher, since g is strictly monotone in $f(c)$. Thus, a voter's preferences satisfy UCR if and only if there is such a ranking that is independent of policy a.

If the utility function is additively separable across A and C, i.e., $u(c, a) = u_C(c) + u_A(a)$, then Theorem 2 immediately implies that preferences satisfy UCR. Suppose, for example, that $C \subset \mathbb{R}$ and that $A = \prod_{i=1}^{I} A_i$ (i.e., there are I different issues). Thus, a candidate's policy can be written as $a = (a_1, \ldots, a_I)$, and the "weighted issue preferences" of Krasa and Polborn (2010), can be represented by

[6] Note that continuity is automatically satisfied if A and C are finite.

the additively separable utility function

$$u(a,c) = \lambda_C|c-\theta_C| + \sum_{i=1}^{I} \lambda_i|a_i-\theta_i|. \tag{2}$$

Parameters θ and λ can be interpreted as ideal positions and weights that measure the relative importance of the fixed and selectable issue, respectively.[7] Another class of preferences with additively separable utility function are those where indifference curves are circles around an ideal point θ. While additive separability guarantees that UCR holds, the following example shows that it is not a necessary condition.

Example 1 Let $c_0 = 0$, $c_1 = 1$, and assume that there is only one binary policy issue, i.e., $A = \{0,1\}$. The voter's preference is $(0,0) \succ (0,1) \succ (1,1) \succ (1,0)$. Clearly, UCR is satisfied, as Candidate 0 is always preferred to Candidate 1. However, these preferences cannot be represented by an additively separable utility function $u_C(c)+u_A(a)$. In particular, $(0,0) \succ (0,1)$ would imply $u_A(0) > u_A(1)$. However, $(1,1) \succ (1,0)$ implies $u_A(1) > u_A(0)$, a contradiction. ■

4.3 Convergence and Divergence

The following Theorem 3 again considers the topic of convergence, but in contrast to Theorem 1, it allows for candidates' fixed positions to differ and focuses on the case that all voters have UCR preferences for a.e. realization of $\omega \in \Omega$. Under these conditions, there is policy convergence in all strict Nash equilibria. Moreover, if a strict Nash equilibrium exists, then it is unique.

Theorem 3 *Suppose that all voters have UCR preferences for a.e. realization of $\omega \in \Omega$ (c_0 and c_1 are arbitrary, in contrast to Theorem 1).*

1. *There is policy convergence in any strict Nash equilibrium (a_0, a_1), i.e. $a_0 = a_1$.*
2. *If there exists a strict Nash equilibrium then it is the unique Nash equilibrium (pure or mixed).*

The intuition for Theorem 3 is as follows. Suppose both candidates choose the same policy a. Since voters have UCR preferences, the winning probabilities do not change if both candidates switch to a'. This means that the entries on the diagonal of the payoff matrix (i.e., where $a_0 = a_1$) are identical, though

[7] Implicitly, separability of preferences is assumed in several internet-based political comparison programs. For example, smartvote.ch (a cooperation project of several Swiss universities) collects the political positions of candidates in national elections by asking candidates a number of yes/no questions on different political issues. Voters can answer the same questions on a website (and also choose a weight for each issue) and are given a list of those candidates who agree with them most. Similar programs exist for the U.S. (http://www.myspace.com/mydebates), Germany (http://www.wahl-o-mat.de), Austria (http://www.wahlkabine.at/) and the Netherlands (http://www.stemwijzer.nl/).

not necessarily equal to $1/2$. Suppose, by way of contradiction, that a strict Nash equilibrium (a_0, a_1), with $a_0 \neq a_1$, exists. This would require that Candidate 0 strictly prefers his payoff in (a_0, a_1) to his payoff in (a_1, a_1), i.e., the payoff that he could obtain by deviating to a_1. Similarly, Candidate 1 strictly prefers his payoff in (a_0, a_1) to his payoff in (a_0, a_0). However, since the candidates play a constant sum game and the payoffs in (a_0, a_0) and (a_1, a_1) are equal because of UCR, we get a contradiction.

One of the very few models with an equilibrium in which office-motivated candidates choose divergent platforms is Adams and Merrill (2003). Our results indicate that this must be due to non-UCR preferences in their model. Voters in their model have additively separable preferences that incorporate both a (continuous) policy issue and partisan preferences (akin to "fixed characteristics" in our terminology). Specifically, consider the following example.

Example 2 There is one fixed characteristic, which Adams and Merrill (2003) refer to as partisanship, and a one-dimensional policy variable in $[0, 1]$. A citizen's type is of the form (P, θ), where $P \in \{D, R\}$ denotes the partisan preference, and θ the most preferred policy. Utility of type (D, θ) from Candidate (D, x) is $B - |\theta - x|$ and $-|\theta - x|$ from Candidate (R, x). Similarly, type (R, θ) also has θ as ideal point, but gets a utility benefit of B from the Republican candidate. However, this "utility function" is not a standard utility function in the sense that it completely describes behavior. In particular, they assume that citizens abstain (i) if the utility difference between candidates is below a threshold ("abstention from indifference"), or (ii) if the utility from the preferred candidate is below some threshold T ("abstention from alienation"). While the model of Erikson and Romero (1990) has only the first effect and generates equilibrium convergence, the second effect may lead to (effective) preferences violating UCR. To see this, consider only the second effect, and define effective voter preferences of a Democratic partisan (D, θ) given policy platforms x_D and x_R as

$$\begin{aligned} D \succ R &\iff B - |x_D - \theta| > -|x_R - \theta| \text{ and } B - |x_D - \theta| > T \\ R \succ D &\iff B - |x_D - \theta| < -|x_R - \theta| \text{ and } -|x_R - \theta| > T \\ D \sim R &\iff B - |x_D - \theta| \leq T \text{ and } -|x_R - \theta| \leq T \end{aligned}$$

In order to have some participation, $B \geq T$ and in order for the alienation constraint to matter $B \leq T + 0.5$. To see that these preferences violate UCR, consider a Democratic partisan with an ideal policy point of $\theta = 0$. If both candidates were to propose the same policy $x_D = x_R = 0$, then $D \succ R$ (i.e., the voter votes for D). If, instead, $x = 0.5$ then $D \sim R$, because the voter is alienated and therefore abstains. Thus, these preferences violate UCR.[8] ■

Theorem 3 indicates that we must focus on non-UCR preferences in order to generate policy divergence. In fact, it is easy to find such voting games.

[8] Since voters in Erikson and Romero (1990) and Adams and Merrill (2003) only fulfill transitivity for strict preferences, our theorems do not apply directly. However, from comparing the two models, it is clear that the violation of UCR in Adams and Merrill (2003) drives the divergence result.

Example 3 There are two candidates $c_G \neq c_B$ and two policies, a_G, a_B, where a_G is interpreted as focusing spending on national security (guns), while a_B corresponds to focusing on healthcare or schooling (butter). Candidate 0 is knowledgeable about national security issues, while Candidate 1's expertise is on social policies. Thus, it is reasonable to assume that there are the following types of voters:

Type G: $(c_G, a_G) \succ (c_B, a_B) \succ (c_G, a_B) \succ (c_B, a_G)$.

Type B: $(c_B, a_B) \succ (c_G, a_G) \succ (c_B, a_G) \succ (c_G, a_B)$.

Thus, type G voters prefer "guns" to "butter", and also have a preference for competent policy implementation, i.e., they prefer policies implemented by the candidate who has the corresponding expertise. Type B voters prefer "butter" to "guns", and also seek competence in policy implementation. Let the number of citizens of each type be given by $n_G(\omega)$ and $n_B(\omega)$, respectively, where $\omega \in \Omega$ reflects uncertainty about the distribution of preferences. Then the number of voters in state ω is given by

	(c_B, a_G)	(c_B, a_B)
(c_G, a_G)	$n_G(\omega) + n_B(\omega), 0$	$n_G(\omega), n_B(\omega)$
(c_G, a_B)	$n_G(\omega), n_B(\omega)$	$0, n_G(\omega) + n_B(\omega)$

If $\mu(\{\omega | n_G(\omega) > n_B(\omega)\}) > 0$ and $\mu(\{\omega | n_G(\omega) < n_B(\omega)\}) > 0$, then it follows immediately that (c_G, a_G), (c_B, a_B) is the unique Nash equilibrium.[9] ■

Are UCR preferences a *necessary* condition for equilibrium policy convergence? An assumption that imposes restrictions on individual preferences only, such as UCR, cannot always be necessary for a particular result. For example, if citizens with non-UCR preferences are never pivotal, then the violation of UCR would not matter for equilibrium convergence. The same is true if UCR is violated for some policies that are sufficiently undesirable for most voters. In view of this, it is obvious that no property imposed solely on citizens' preferences can be simultaneously necessary and sufficient for policy convergence. However, Theorem 4 shows that even if there is just one voter with non-UCR preferences, then there are always some voting games in which everyone else has UCR preferences, but that have a strict equilibrium with policy divergence. This is completely analogous to the well-known condition of single-peaked preferences in a one-dimensional policy space. If all voters have single-peaked preferences, the existence of a Condorcet winner is guaranteed. However, while a Condorcet winner can still exist when some voters don't have single-peaked preferences, it is also possible to construct examples in which only one voter violated single-peakedness and no Condorcet winner exists.

[9]Note that we can easily add more voter types to Example 3 without immediately affecting the equilibrium. Even adding an arbitrary number of partisans (who vote for one candidate irrespective of the candidate's policy) preserves (c_G, a_G), (c_B, a_B) as the unique Nash equilibrium, as long as type G and B voters remain pivotal with positive probability. If the probability that type G and B voters are pivotal is zero, then any combination of strategies is an equilibrium.

Theorem 4 *Let $\succeq$ be some arbitrary non-UCR preferences on $C \times A$. Then there exists a voting game with the following property:*

1. *One citizen has preferences $\succeq$ and all other citizens have UCR preferences.*
2. *There exists a strict Nash equilibrium with policy divergence. Further, this is the unique Nash equilibrium (pure or mixed).*

The key for proving Theorem 4 is to make the voter with non-UCR preferences pivotal. Since we do not assume that A consists of only two policies, we must also ensure that the policies for which UCR is violated are sufficiently desirable to citizens that candidates want to use them. The detailed construction of the voting games is in the Appendix.

Another way of showing that UCR is "close" to a necessary condition for policy convergence is to restrict attention to a parametrized class of voting games, and prove that UCR is necessary and sufficient for convergence within this class. We choose this approach in the following sections.

5 A Generalized Probabilistic Voting Model

5.1 The Classic Model with Microfoundation

In the classical probabilistic voting model (PVM), groups are identified as voters with the same "economic" preferences. However, voters within the same group may vote for different candidates because of what Persson and Tabellini (2000), p. 52 refer to as "ideology." They write that "one way to motivate [ideology] is to think about a second policy dimension, orthogonal to fiscal policy, in which candidates cannot make credible commitments, but set an optimal policy after the election according to their ideology." Rather than modeling the second policy dimension explicitly, they operationalize this idea by adding an additive ideology shock to the economic preferences.[10]

Our objective in this subsection is to formulate a model that takes this notion of a fixed second policy dimension seriously. In the following section, we consider a model in which indifference curves can take any elliptical form. However, to see the relationship with the classic PVM, we start with the special cases of Euclidean preferences in a two-dimensional policy space, i.e., circular indifference curves.

Suppose that voters have one of finitely many policy ideal points θ_j, $j = 1, \ldots, J$. Let λ_j be the fraction of voters with ideal point θ_j, which we assume to be deterministic. Voters with policy preference θ_j are differentiated with respect to their ideal point on the fixed issue. We assume that the the distribution of ideal points on fixed issues, ε, for voters in group j depends on state ω, and is given by $F_j(\varepsilon - \omega)$

[10] An example of particular voter preferences generating such an additive shock are provided in Persson and Tabellini (2000), Section 3.8 on pp. 64-65.

or $f_j(\varepsilon - \omega)$, where F_j and f_j are a cdf and its corresponding pdf. That is, ω is a shift parameter that affects the preferences of all voters. As in the general model, ω is distributed according to a probability distribution μ.

We use the following assumption in the current and the following subsection.

Assumption 1

f_j is continuously differentiable.

ω has a distribution with strictly positive on its support, which is a non-empty interval.

The median and the mode of the distribution of each ε_j is obtained at 0, i.e., $F_j(0) = 0.5$ and $f'_j(0) = 0$, for all $j = 1, \ldots, J$.

The first two items are fairly innocuous technical assumptions. The third one, which assumes that the median ideology shock is the same for all groups, is made for convenience, in particular for stating second order conditions. Since our results are not knife-edge cases, it is clear that this condition could be relaxed at the expense of more cumbersome algebra. Also, note that the assumption is weaker than symmetry of f_j.

If preferences are Euclidean then type j with ideal point ε on the fixed issue prefers Candidate 0 to Candidate 1 if and only if

$$(\varepsilon_j - c_0)^2 + (\theta_j - a_0)^2 < (\varepsilon_j - c_1)^2 + (\theta_j - a_1)^2. \tag{3}$$

(3) is equivalent to

$$\varepsilon_j < \frac{1}{2}\left[c_0 + c_1 + \frac{(a_1 - a_0)(a_1 + a_0 - 2\theta_j)}{c_1 - c_0}\right]. \tag{4}$$

Thus, for a given value of ω, the fraction of voters who support Candidate 0 is given by

$$\sum_{j=1}^{J} \lambda_j F_j\left(\frac{1}{2}\left[c_0 + c_1 + \frac{(a_1 - a_0)(a_1 + a_0 - 2\theta_j)}{c_1 - c_0}\right] - \omega\right).$$

Clearly, this fraction is continuous and decreasing in ω, and goes to 0 for $\omega \to \infty$, while it goes to 1 for $\omega \to -\infty$. Thus, for any pair of policies (a_0, a_1), there exists a critical value $\omega^*(a_0, a_1)$ such that the election ends in a tie if $\omega = \omega^*(a_0, a_1)$. If $\omega < \omega^*$ then Candidate 0's win because his vote share strictly exceeds 50%. The reverse is true, i.e., candidate 1 wins, if $\omega \geq \omega^*$.

Furthermore, it must be true that each candidate maximizes his vote share in the critical state ω^*. If this was not true for Candidate 0, say, then he could simply increase his vote share in state ω^* and thus win for sure in all states ω in a neighborhood of ω^*; moreover, since Candidate 0 also wins for all lower states, his winning probability must increase by this deviation. The argument for Candidate 1 is analogous.

Thus, formally, Candidate 0 solves

$$\max_{a_0} \sum_{j=1}^{J} \lambda_j F_j \left(\frac{1}{2} \left[c_0 + c_1 + \frac{(a_1 - a_0)(a_1 + a_0 - 2\theta_j)}{c_1 - c_0} \right] - \omega^* \right), \tag{5}$$

while Candidate 1 solves

$$\min_{a_1} \sum_{j=1}^{J} \lambda_j F_j \left(\frac{1}{2} \left[c_0 + c_1 + \frac{(a_1 - a_0)(a_1 + a_0 - 2\theta_j)}{c_1 - c_0} \right] - \omega^* \right), \tag{6}$$

where ω^* is the realization at which the candidates' winning probabilities are 0.5, i.e.,

$$\sum_{j=1}^{J} \lambda_j F_j \left(\frac{1}{2} \left[c_0 + c_1 + \frac{(a_1 - a_0)(a_1 + a_0 - 2\theta_j)}{c_1 - c_0} \right] - \omega^* \right) = 0.5, \tag{7}$$

where a_0 and a_1 solve (5) and (6), respectively.

The first order conditions of (5) and (6) are

$$\sum_{j=1}^{J} \lambda_j f_j \left(\frac{1}{2} \left[c_0 + c_1 + \frac{(a_1 - a_0)(a_1 + a_0 - 2\theta_j)}{c_1 - c_0} \right] - \omega^* \right) \frac{\theta_j - a_0}{c_1 - c_0} = 0; \tag{8}$$

$$-\sum_{j=1}^{J} \lambda_j f_j \left(\frac{1}{2} \left[c_0 + c_1 + \frac{(a_1 - a_0)(a_1 + a_0 - 2\theta_j)}{c_1 - c_0} \right] - \omega^* \right) \frac{\theta_j - a_1}{c_1 - c_0} = 0. \tag{9}$$

If $a_0 = a_1$ then (8) holds if and only if (9) holds. immediately imply that any solution has the property that $a_0 = a_1$. Substituting $a_0 = a_1$ into (7) implies

$$\sum_{j=1}^{J} \lambda_j F_j \left(\frac{c_0 + c_1}{2} - \omega^* \right) = 0.5, \tag{10}$$

Since the median of the distributions of ε_j is obtained at 0, equation (10) implies that $\omega^* = (c_0 + c_1)/2$. This, $a_0 = a_1$ and (8) implies that

$$\sum_{j=1}^{J} \lambda_j f_j(0)(\theta_j - a) = 0, \tag{11}$$

Clearly, there always exists a unique value of a that solves (11).

The second order conditions of (5) and (6) are

$$\sum_{j=1}^{J} \lambda_j \left(\frac{(\theta_j - a_0)^2}{c_1 - c_0} \frac{f'(\cdot)}{f(\cdot)} - 1 \right) < 0;$$

$$\sum_{j=1}^{J} \lambda_j \left(\frac{(\theta_j - a_1)^2}{c_1 - c_0} \frac{f'(\cdot)}{f(\cdot)} - 1 \right) < 0.$$

At $a_0 = a_1$ the second order conditions reduce to the single condition

$$\sum_{j=1}^{J} \lambda_j \left(\frac{(\theta_j - a_0)^2}{c_1 - c_0} \frac{f'(0)}{f(0)} - 1 \right) < 0. \tag{12}$$

Since $f'(0) = 0$, condition (12) is satisfied. Thus, $a_0 = a_1$ is a local, strict equilibrium. Sufficient conditions for global optimality are difficult to state, as the left hand side of (12) can be positive if f' is evaluated far enough away from 0. However, if we restrict a to be from a sufficiently small interval $[\underline{\alpha}, \bar{\alpha}]$ that contains $a_0 = a_1$, then the local equilibrium that we identified is also guaranteed to be a global equilibrium in the restricted game. Theorem 3 therefore implies (corresponding to standard results for the standard PVM with additive ideology shocks) that a_0, a_1 is the unique Nash equilibrium, pure or mixed, of the restricted game.

Theorem 5 *Suppose that Assumption 1 is satisfied. Then there exists $\underline{\alpha}_i < a_i < \bar{\alpha}_i$ such that a_0, a_1 is a Nash equilibrium if the candidates' strategy spaces are given by $[\underline{\alpha}_i, \bar{\alpha}_i]$, $i = 0, 1$. There is policy convergence in the Nash equilibrium: $a_0 = a_1$. Moreover, there does not exist any other local pure strategy Nash equilibrium.*

As in the standard PVM, the intervals $[\underline{\alpha}_i, \bar{\alpha}_i]$ becomes larger (or global), if the type distribution is more spread out, i.e., if f' stays small if we move away from zero. Of course, if $f' \equiv 0$ (i.e., if the distribution is uniform) then the equilibrium is always global.

5.2 General Spatial Preferences

We now consider preferences for which indifference curves are ellipses rather than circles. Intuitively, indifference curves that are circles capture preferences where the ideal policy a is independent of the fixed characteristic c. In contrast, consider, for example, elliptical indifference curves for which the major axis is the 45 degree line. This corresponds to a situation where the fixed characteristic and the policy are complements in the following sense: The voter's ideal policy on the flexible policy dimension is the higher, the higher the candidate's position on the fixed dimension is.

For example, consider the following situation: The fixed characteristic measures the general attitude of the candidate towards cooperation with foreign governments in solving international problems. A candidate who favors broad international cooperation and consensus building in international organizations would be denoted as (say) a low type on this dimension, while a candidate who prefers a unilateral approach and does not care much about the international opinion would be a high type. Candidates are fixed to their respective (different) positions in that dimension. This assumption appears to be reasonable, as it is probably very difficult to credibly commit to a particular foreign policy "attitude".

There is a second dimension that is more concrete and where candidates can commit to a particular position. For concreteness, think of this dimension as the defense budget. It is quite plausible that the type of the executive (i.e., the position of a candidate in the first dimension) influences a voter's preferences over policy in the second dimension; for example, a voter may prefer that a more assertive candidate has a higher (or lower) defense budget than a more cooperative type. In the first case, we would say that characteristic and policy are complements, in the second case, they are substitutes. Both cases imply that a voter's indifference curves are not circles but rather could be captured by ellipses whose major axis is not exactly horizontal or vertical.[11]

Before we proceed, it is useful to conceptually differentiate between the shape of the indifference curves and correlation in the distribution of ideal points. So far, we have argued that it is plausible that a single voter's preferences over fixed characteristics and flexible policies display complementarity or substitutability. This effect influences the shape of indifference curves. Conceptually different from this is correlation in the distribution of ideal points in both dimensions. For example, it may be the case that many voters who have a preference for "tough-talking" executives also have, on average, a higher ideal point on the defense budget. Thus, if we were to plot voter ideal points in a $c - a$-diagram, these ideal points might display positive correlation. Whether or not there is correlation in ideal points does not affect our theory much, so we do not need to take a position on this question.

Consider the preferences depicted in Figure 1 where the two parameters κ_1 and κ_2 determine the shape of the indifference curves. In particular, as depicted in the graph, κ_1 determines the ratio of the two axes, while κ_2 determines the angle of rotation. Clearly, any preferences with elliptical indifference curves can be represented by the ideal point (ε, θ), κ_1 and κ_2. In particular, letting $\kappa_1 = 1$ produces standard Euclidean preferences, reducing the model to the standard PVM.

More formally, let

$$M = \begin{pmatrix} 1 & \kappa_2 \\ -\kappa_1\kappa_2 & \kappa_1 \end{pmatrix} \tag{13}$$

For $x \in [0, 1]^2$ define the norm $||x||_M = ||Mx||_2$, where $|| \cdot ||_2$ denotes the Euclidean norm. Let (ε, θ) be a voter's ideal point. Then

$$(c, a) \succeq^{\varepsilon,\theta} (c', a') \text{ if and only if } ||(c, a) - (\varepsilon, \theta)||_M \leq ||(c', a') - (\varepsilon, \theta)||_M. \tag{14}$$

[11] As a related example where complementarity between a candidate's type and the policy choice is plausible, consider the following example: Suppose candidates differ with respect to their beliefs about the possibility of rehabilitating criminal convicts. While a low c candidate believes that rehabilitation is often effective, a high c candidate believes that it does not. Consequently, if the tough politician is in power, criminals will remain more or less unreformed (whether or not rehabilitation is in principle possible). Suppose that a corresponds to the amount of money spent on building and maintaining prisons (not including any rehabilitation expenses). Then, independent of their ideal point, voters would want the candidate who does not believe in rehabilitation to build more prisons, since absent rehabilitation efforts, this is the better choice than releasing prisoners early because of a lack of space in prisons. In contrast, if the executive believes in and funds rehabilitation programs, additional prison space is less useful, and the voter would prefer a lower a.

It is easy to check that indifference curves are of the form

$$(c-\varepsilon, a-\theta)\begin{pmatrix} 1+\kappa_1^2\kappa_2^2 & \kappa_2(1-\kappa_1^2) \\ \kappa_2(1-\kappa_1^2) & \kappa_1^2+\kappa_2^2 \end{pmatrix}\begin{pmatrix} c-\varepsilon \\ a-\theta \end{pmatrix} = \bar{u} \tag{15}$$

The eigenvectors of the matrix in (15) are $(-\kappa_2, 1)$ and $(1, \kappa_2)$ with associated eigenvalues $\kappa_1^2(1+\kappa_2^2)$ and $1+\kappa_2^2$. Thus, as indicated in Figure 1 indifference curves are elliptical, with the main axes given by the above eigenvectors, and the ratio of the length of the axes is κ_1.

The case depicted in the left panel where the major axis has positive slope, corresponds to the case where a voter's optimal level of a increases with c. We say that c and a are complements. If in contrast, the slope of the major axis is negative, we say that c and a are substitutes. Formally, if $u(c, a) = -\|(c, a) - (\varepsilon, \theta)\|_M^2$ represents the preferences, then

$$\frac{\partial^2 u(c, a)}{\partial c \partial a} = -2(1-\kappa_1^2)\kappa_2. \tag{16}$$

For $\kappa_1 > 1$ and $\kappa_2 > 0$ as in the graph, the sign of the cross derivative is positive, indicating complements.

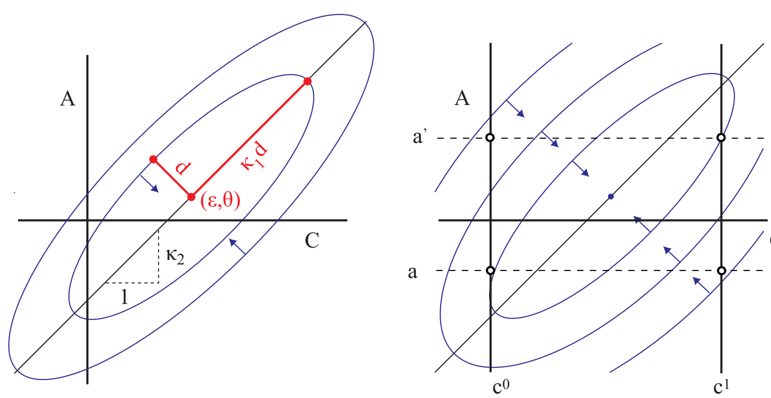

Figure 1: Elliptical Preferences and Violation of UCR

We next prove that UCR is violated for these preferences. The violation of UCR can most easily be seen in Figure 1 (and the argument can clearly be formalized). If both candidates select policy a then the voter prefers Candidate 0. If, instead, both candidates select policy a' then the voter prefers Candidate 1. The only elliptical preferences that satisfy UCR are those for which the major or minor axis is horizontal, i.e., where $\kappa_2 = 0$. Such preferences are given by a utility function $u(c, a) = -k^2(c-\varepsilon)^2 - (a-\theta)^2$. In this case, $u(c, a) \geq u(c', a)$ if and only if $u(c, a') \geq u(c', a')$.

Directly analyzing the voting game with elliptical indifference curves would be very complicated. Thus, we transform the policy space such that preferences become Euclidean (in the transformed model) and thus satisfy UCR. Theorem 3 can then be used to identify possible equilibria and to prove uniqueness of equilibrium.

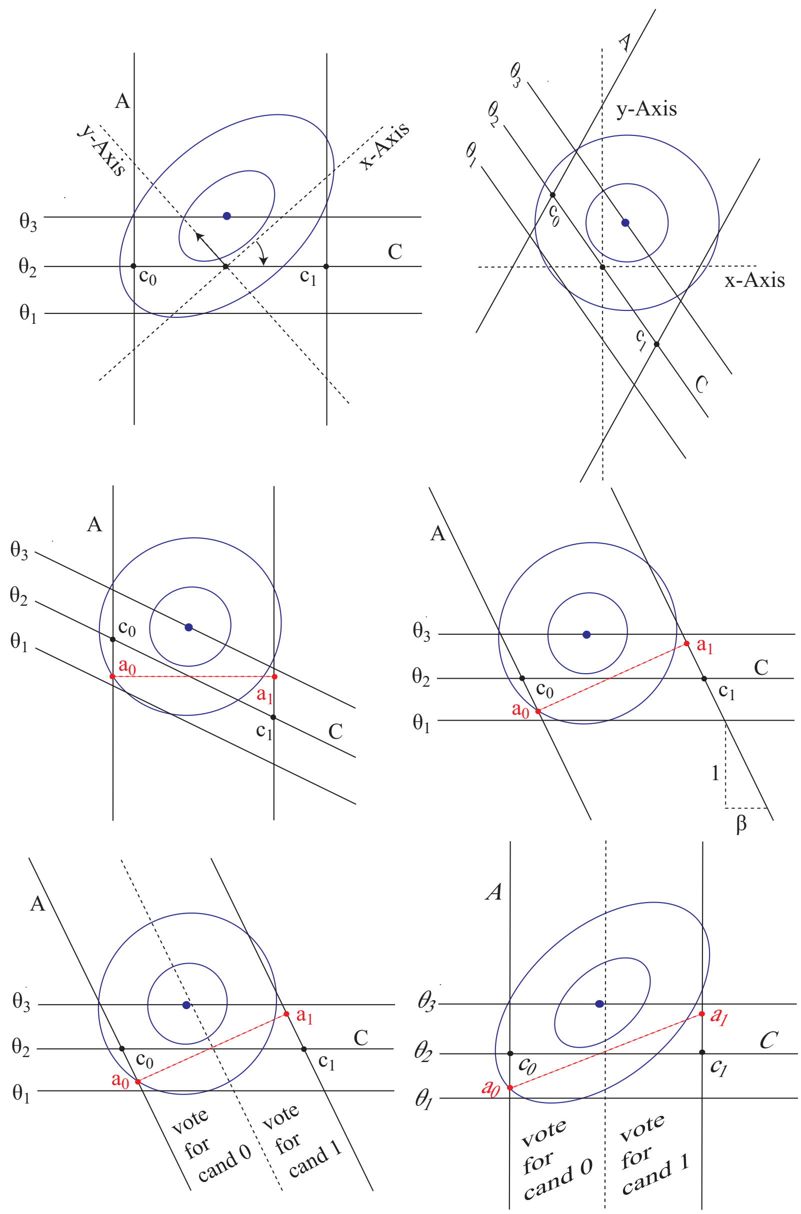

Figure 2: Transforming Elliptical Preferences to Euclidean Preferences and Equilibrium

We now explain how to analyze equilibria by using Figure 2. The detailed mathematical arguments can be found in the Appendix. The top left panel of Figure 2 depicts the original model. In the standard PVM, individuals with the same θ are interpreted as a "group" that has the same "economic" interests (i.e., ideal value of policy). Members of the same group differ only in their "ideological" preferences captured by ε (i.e., their ideal value of the fixed position). In PVMs, it is standard to consider finitely many "groups" (each with a continuous, possibly group-specific distribution of ideology), and we adopt the same approach. In Figure 2, there are three "groups" with policy ideal points θ_1, θ_2 and θ_3, and the indifference curves of one particular type with a policy ideal point of θ_3. We apply a linear transformation (given by matrix M in (13) above) to the top left panel. As indicated, the x and y-axes coincide with the directions of the major and minor axes of the ellipses. We apply a rotation, indicated by the curved clockwise arrow, and at the same time we stretch along the y-axis as indicated by the straight arrow pointing northwest until indifference curves become circles. The result of applying M is depicted in the top right-panel. Note that the x and y-axes are now horizontal and vertical, while the locus of voter types as well as the set of feasible policy are skew and no longer form a right angle (because of the stretching).

It is more convenient to analyze the model in the two positions depicted in the middle panels. Both are obtained by applying rotations to the top right panel. In the middle left panel, the candidates' sets of feasible policies are vertical lines (and the indifference curves are circles and therefore satisfy UCR). As a consequence, Theorem 3 applies that in any strict Nash equilibrium equilibrium policies must be identical, i.e., $a_0 = a_1$. If an equilibrium exists, then second order conditions guarantee strictness, just like in the standard PVM. Thus, if an equilibrium exists, it must also be unique.

Existence can be shown most easily using the right-middle panel. This corresponds to the PVM from the previous section, except that the candidates' feasible policy lines are skew. As indicated in the graph, the slope of the policy lines is given by $1/\beta$, where

$$\beta = \frac{\kappa_2(1-\kappa_1^2)}{\sqrt{1+\kappa_1^2\kappa_2^2}}. \tag{17}$$

Note that β has exactly the opposite sign of (16). Thus, if c and a are complements as in Figure 2, then $\beta < 0$.

If the main axes of the ellipses in the original mode are horizontal or vertical, i.e., if $\kappa_2 = 0$, or if indifference curves are circles at the outset, i.e., $\kappa_1 = 1$, then $\beta = 0$. In this case, the two middle panels are identical, and as a consequence, $a_0 = a_1$, i.e., there is policy convergence.

Now return the the case where $\beta \neq 0$. As we rotate the graph from the middle left panel to the middle right, the condition $a_0 = a_1$ becomes

$$\tilde{a}_1 = \tilde{a}_0 - \frac{\beta}{\beta^2+1}(\tilde{c}_1 - \tilde{c}_0), \tag{18}$$

where the tilde above each parameter indicates that coordinates are with respect to that in the middle right panel. Equation (18) and Figure 2 imply, $\tilde{a}_0 \neq \tilde{a}_1$ (note that Theorem 3 does not apply in the right-middle panel, since the candidates' feasible policy lines are skew).

To determine the necessary and sufficient conditions for equilibrium we proceed as in the previous section, except that we need to adjust for the fact that the feasible policy lines are skew. The resulting first order conditions are

$$\sum_{j=1}^{J}\lambda_j f_j\left(\frac{\tilde{c}_0+\tilde{c}_1}{2\sqrt{1+\kappa_1^2\kappa_2^2}}-\omega^*\right)\left[\frac{\beta}{2}+(1+\beta^2)\frac{\tilde{\theta}_j-\tilde{a}_0}{\tilde{c}_1-\tilde{c}_0}\right]=0; \tag{19}$$

$$\sum_{j=1}^{J}\lambda_j f_j\left(\frac{\tilde{c}_0+\tilde{c}_1}{2\sqrt{1+\kappa_1^2\kappa_2^2}}-\omega^*\right)\left[-\frac{\beta}{2}+(1+\beta^2)\frac{\tilde{\theta}_j-\tilde{a}_1}{\tilde{c}_1-\tilde{c}_0}\right]=0. \tag{20}$$

The second order conditions, detailed in (54) and (55) are of the form

$$\sum_{j=1}^{J}\lambda_j\left(\frac{\Gamma_i(\tilde{c}_0,\tilde{c}_1,\tilde{a}_0,\tilde{a}_1,\theta_j)}{2(1+\beta^2)(\tilde{c}_1-\tilde{c}_0+\beta(\tilde{a}_1-\tilde{a}_0)}\frac{f'(\cdot)}{f(\cdot)}-1\right)<0.$$

where Γ_i is a function of the indicated variables, and the candidate $i=0,1$. One can check that, at the solution of the first-order conditions, $f'(\cdot)=0$. Hence, the second-order conditions are satisfied, and we have again at least a local equilibrium.

Finally, we transform the policy space back into its original form. This process is depicted in the bottom panel of Figure 2. After the transformation, policies still differ. The line separating supporters for candidates 0 and 1 is vertical as in the standard model. Recall that ω^* is determined such that the winning probabilities are 0.5. After the transformation, the condition is identical to (10) in the previous section, and hence $\omega^*=(c_0+c_1)/2$. The first order conditions (19) and (20) change to

$$\sum_{j=1}^{J}\lambda_j f_j(0)\left[\frac{\beta}{2}+\frac{(1+\beta^2)\kappa_1(1+\kappa_2^2)}{1+\kappa_1^2\kappa_2^2}\frac{\theta_j-a_0}{c_1-c_0}\right]=0. \tag{21}$$

$$\sum_{j=1}^{J}\lambda_j f_j(0)\left[-\frac{\beta}{2}+\frac{(1+\beta^2)\kappa_1(1+\kappa_2^2)}{1+\kappa_1^2\kappa_2^2}\frac{\theta_j-a_1}{c_1-c_0}\right]=0. \tag{22}$$

Condition (18) changes to

$$a_1-a_0=-\frac{\beta(1+\kappa_1^2\kappa_2^2)}{(1+\beta^2)\kappa_1(1+\kappa_2^2)}(c_1-c_0), \tag{23}$$

Thus, if $\beta<0$, which is the case of complementarity between fixed characteristics and policy depicted in Figure 2, then $a_0<a_1$ as the right-hand side of (23) is positive in the case. If, instead, $\beta>0$ then fixed characteristics and policy are substitutes, and $a_0>a_1$.

The arithmetic average $\bar{a}=(a_0+a_1)/2$ of policies a_0 and a_1 has no direct substantive significance in our model (in particular, it is not the *expected* policy, as the candidates' winning probabilities are usually different). However, we can use $\bar{a}$ to show uniqueness of local equilibria as follows. Add equations (21) and (22) to get

$$\sum_{j=1}^{J}\lambda_j f_j(0)\left[\frac{2(1+\beta^2)\kappa_1(1+\kappa_2^2)}{1+\kappa_1^2\kappa_2^2}\frac{\theta_j-\bar{a}}{c_1-c_0}\right]=0. \tag{24}$$

Since the coefficient of $\frac{\theta_j - \bar{a}}{c_1 - c_0}$ is strictly positive, (24) simplifies to

$$\sum_{j=1}^{J} \lambda_j f_j(0)(\theta_j - \bar{a}) = 0, \tag{25}$$

which is identical to (11) (replacing a by $\bar{a}$). Thus, $\bar{a}$ is exactly the same as the equilibrium policy in a Euclidean model where $\kappa_1 = 1$ or $\kappa_2 = 0$.[12]

We now summarize our results. It should be noted that the requirements for existence in Theorem 6 mirror those in Theorem 5 and thus correspond to those for the standard PVM.

Theorem 6 *Suppose that Assumption 1 is satisfied and that preferences are given by* (14). *Then there exists* $\underline{\alpha}_i < a_i < \bar{\alpha}_i$ *such that* a_0, a_1 *is a Nash equilibrium if the candidates' strategy spaces are given by* $[\underline{\alpha}_i, \bar{\alpha}_i]$, $i = 0, 1$. *Equilibrium policies are given by* (23). *There is policy divergence, i.e.,* $a_0 \neq a_1$, *unless indifference curves are circles or the major axis is horizontal or vertical. Moreover, there does not exist any other local pure strategy Nash equilibrium.*

5.3 Comparison of the Classic and the General Spatial Models

One of the main points of interest of the standard PVM is to determine which features of the distribution of voter preferences influence the equilibrium policy. The central finding of the PVM is that the equilibrium policy maximizes a weighted sum of the voters' economic (i.e., non ideological) utilities, $-(\theta_j - a)^2$, where the weights of group j in the maximization problem is determined both by the group size λ_j, and by how many members of group j can be moved easily, which is determined by $f_j(0)$. The same determinants influence equilibrium policy in the general spatial model. In particular, policy $\bar{a}$ solves exactly the same optimization problem, and existence of equilibrium can be proved along the same lines as in the standard model (once the economy is transformed as explained in the previous section).

The key difference between the classical and the general spatial models is that, in the classical model, both candidates solve the same optimization problem and thus their equilibrium policies coincide. In contrast, the optimization problems of the two candidates differ with general preferences, resulting in policy divergence. Policy divergence increases in the ex-ante difference between candidates. In practice, the ex-ante differences between candidates may increase if parties are more polarized on the dimension captured by the fixed characteristic, c. In contrast, the difference between candidates' fixed characteristics are irrelevant for policy choice in the standard model.

The model with general elliptical preferences also indicates another aspect in which the standard PVM produces special results. Consider the effect of a change in the voters' preference distribution over the fixed characteristic, say, an ideology shift in ω that favors the Democratic candidate. In a

[12] In fact, $\bar{a}$ is uniquely determined, and this fact can be used to provide an alternative proof for uniqueness of a local equilibrium.

standard PVM, this shift does not affect the equilibrium policies that both candidates choose (and since both choose the same position, it also does not affect the expected policy). The only effect of a change in the electorate's distribution of ideologies is a change in the winning probabilities of the Democratic and Republican candidates. In contrast, when indifference curves are elliptical, then a change in the ideological distribution of the electorate also affects the expected flexible policy.

6 The Tax Model

In Section 5, we presented a model with non-UCR preferences that captures the notion of complementarity between a fixed and a flexible policy dimension. In the present section, we analyze a model in which candidates differ in their competence to implement certain policies. We do this for two reasons.

First, the model provides a setting in which voters' non-UCR preferences arise naturally as an "indirect" utility function derived from standard economic preferences in combination with differential candidate abilities. In contrast, the elliptical preferences in Section 5, while more general than the standard Euclidean preferences, were imposed directly.

Second, there are also technical differences between the two models. The model in Section 5 has a two-dimensional type space of voters. As a consequence, equilibrium existence requires, just as the standard PVM, sufficiently large "preference diversity" among voters along the exogenously fixed dimension. In contrast, the model of this section has (effectively) a one-dimensional voter type space. Thus, it is very close to the classical spatial framework, and also very tractable.

6.1 Description of the Model

In this model, Candidate j's proposed policy a_j is a tax rate. Each candidate's fixed characteristic $c_j = (c_{F,j}, c_{M,j})$ describes his ability to provide public goods using tax revenue, where $c_{F,j}$ is the candidate's fixed cost of running the government, and $c_{M,j}$ the marginal product of providing the public good. Formally, let $\bar{m}$ be average income, assumed to be non-random. Then the level of public good g_j provided by Candidate j is given by $g_j = c_{M,j}(a_j\bar{m} - c_{F,j})$. We analyze situations in which Candidate 0 has an advantage with respect to fixed costs, while his opponent has the advantage of having a higher marginal product of providing the public good, i.e., $c_{F,0} < c_{F,1}$ and $c_{M,0} < c_{M,1}$. Thus, Candidate 0 is better at running a small government, while Candidate 1 would be preferable for large government expenditures.

Voters have preferences over consumption bundles that consist of one private good x and one public good g.[13] There is a continuum of citizens whose types are given by (θ, m), where θ is the preference type and m is income. Preferences over public and private good consumption are $u_\theta(x, g) = x + h(\theta)w(g)$,

[13] One can think of the private good x as a composite good. It is easy to generalize our model to one where individuals have (possibly different) utility functions over several private goods, as long as these private utility functions are homothetic.

where $h(\cdot)$ is strictly increasing and positive, and $w(\cdot)$ is strictly increasing and strictly concave. This generates the following preferences for political candidates and their platforms, defined over $C \times A$:

$$(c_F, c_M, a) \succeq (c'_F, c'_M, a') \text{ if and only if } u_\theta\Big(x(1-a), c_M(a\bar{m} - c_F)\Big) \geq u_\theta\Big(x(1-a'), c'_M(a'\bar{m} - c'_F)\Big) \quad (26)$$

The preferences in (26) violate UCR. To see this, suppose first that $c_F < c'_F$ and $c_M < c'_M$. If $a = c'_F/\bar{m}$ then any voter prefers the candidate with the lower fixed costs: Consumption $x(1-a)$ is the same with both candidates, but public good provision is zero with the candidate who has the higher fixed costs and strictly positive with his opponent. Thus, $(c_F, c_M, a) \succ (c'_F, c'_M, a)$. Now suppose instead that $a' > (c'_F - c_F)/(c'_M - c_M)$. Then the candidate with (c'_F, c'_M) provides strictly more of the public good for the same level of taxes. Thus, $(c_F, c_M, a') \prec (c'_F, c'_M, a')$, which contradicts UCR.

The distribution of voters is given by a cdf $F_\omega(\theta, m)$, where $\omega \in \Omega$ is uncertainty that is revealed to all parties ex-post. We denote the distribution of ω by μ, and, as stated above, we assume that average income, $\bar{m}$ is independent of ω, i.e., $\bar{m} = \int m\, dF_\omega(\theta, m)$ for every ω.

6.2 Discussion of Model Assumptions

The assumption that different candidates have different production possibilities appears eminently reasonable. Economists agree that workers or firms differ in their productivities, and this fact is evident as output can easily be measured in many private sector occupations. In contrast, the "output" of politicians in terms of public good production is significantly more difficult to measure, and thus it is tempting to use expenditures on inputs as a proxy measure for the quantity of the public good supplied. However, in reality, citizens derive utility, for example, from the quality of education in state schools and not *per se* from the money spent on education. Thus, when two competing candidates propose to spend the same amount of money on schools, this does not mean that both of them would produce the same quality of service for citizens if elected. Our model formalizes this notion.

There are several different interpretations of the candidates' differentiated production possibilities. First, there is a widespread notion that Republicans have an advantage when it comes to running a small government. For example, Egan (2008) demonstrates that Republicans have a long-run public opinion advantage over Democrats on the issue of "taxes", while simultaneously a majority of people say that they trust Democrats more than Republicans on large expenditure issues such as education and health care. Of course, it is not straightforward to interpret what these opinion poll results actually mean, as revenues and expenditures are two sides of the same coin.[14] Our preferred interpretation of these opinion polls is therefore that (many) people think that the advantage of a Republican government is that it is better in taking care of taxpayer dollars by trimming government spending to a minimum, a task in

[14] Possibly, (some) people just want to say that they would most like to have a Democratic (i.e., large) level of spending on issues such as education and health care, while being only lightly taxed (as under Republicans). Of course, such a "can I have my cake and eat it, too"-attitude would not be a meaningful political preference in a world of limited resources.

which Democrats may be hampered, for example by their connections to unions of government workers. On the other hand, Democrats are preferable for delivering a high level of public good service.

A difference between political parties can also arise as a consequence of specialization on different policy areas: Republicans may be specialized in the efficient provision of services such as law enforcement that are "basic" in the sense that every government – whether Democratic or Republican – has to provide them, while the Democrats' efficiency advantage lies in the provision of "optional" services (i.e., services that could, but need not be provided by the government) such as, for example, government provision of health care.

Alternatively, suppose that learning-by-doing increases the incumbent's marginal productivity over his challenger's one. However, incumbency also leads to entrenchment, so if the next office holder were charged with reducing bureaucracy and government spending, it may well be the case that the challenger is better able to achieve this objective.

Finally, it is useful to interpret the specific way how we model the difference in production possibility sets. Fundamentally, we want our model to capture the notion that a low level of output is cheaper for society if produced by Candidate 0 than by Candidate 1, while the reverse holds for a high level of output. Our approach that distinguishes candidates by their "fixed costs" and "marginal productivities" is a simple way to achieve this objective. However, while marginal productivity is readily interpretable, it is probably less useful to think of "fixed cost" literally as fixed expenditures for government services that do not create any useful output, but rather as a shift parameter.[15]

6.3 Characterization of Equilibria

We start by providing some intuition for the equilibrium of the game, and will then proceed to a more formal statement of our results. Tax revenue under Candidate j's plan is $a_j\bar{m}$. The net revenue after fixed costs will generate $c_{M,j}(a_j\bar{m} - c_{F,j})$ units of the public good. For $a_j \geq c_{F,j}/\bar{m}$, define

$$W_j(a_j) = w(c_{M,j}(a_j\bar{m} - c_{F,j})). \tag{27}$$

This is the common component of utility from the public good. Remember, however, that citizens also differ in the term $h(\theta)$ that multiplies $W_j(a_j)$.

Using the functions $W_0(a)$ and $W_1(a)$, we can provide an intuition for the nature of the equilibrium. Remember that $c_{F,0} < c_{F,1}$ so that $W_0(a) > W_1(a)$ for low levels of a. The larger marginal productivity of Candidate 1 implies that W_1 increases more steeply than W_0, and the two curves intersect at a unique tax rate $\bar{a}$ (as depicted in Figure 3, which we will use further to discuss the equilibrium).

[15] For an analogous interpretative problem, consider the macroeconomic consumption function $C = a + bY$. While b is naturally interpreted as the marginal propensity to consume, it is not useful to think of a as "the amount that society would consume if there was no economic activity."

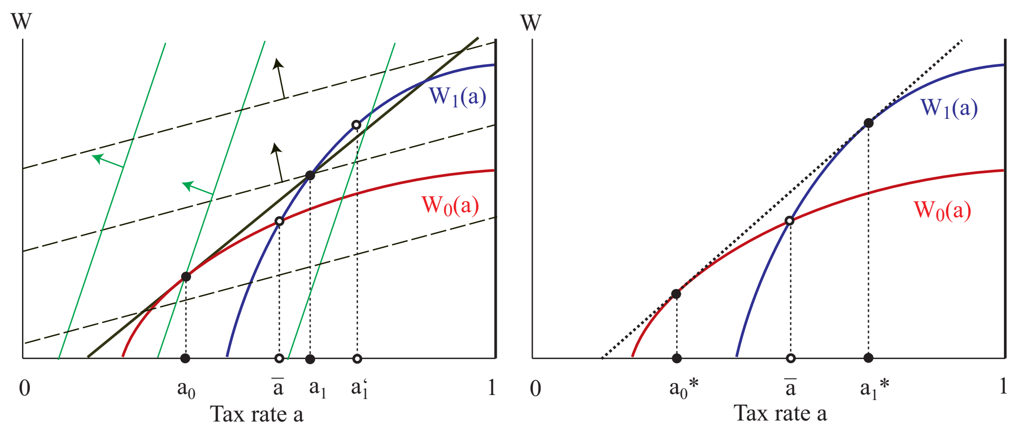

Figure 3: Characterization of Equilibrium Policies a_0^*, a_1^*

Consider first what would happen if both candidates were to propose the same tax rate a: If $a < \bar{a}$, then *all* voters would prefer Candidate 0 over Candidate 1 (as he produces more of the public good, with no difference in the level of taxation). Conversely, if both candidates propose the same tax rate $a > \bar{a}$, then all voters would prefer Candidate 1. Thus, there cannot be an equilibrium in which both candidates propose the same tax rate. More generally, there also cannot be a pure strategy equilibrium in which both candidates propose different tax rates a_0, a_1 where either $a_0, a_1 < \bar{a}$ or $a_0, a_1 > \bar{a}$. For example, if $a_0, a_1 < \bar{a}$, then Candidate 0 could win all votes (with certainty) by selecting $a_0 = a_1$. Thus, in any pure strategy equilibrium, each candidate must locate in a region where he dominates his competitor.

To gain more insight into the properties of equilibrium, consider the preferences of voters. While the type space of voters is two-dimensional, there is a one-dimensional sufficient statistic similar to a marginal rate of substitution that determines the voter's choice. The utility of voter type (θ, m) if Candidate j is elected is

$$(1 - a_j)m + h(\theta)w(c_{M,j}(a_j\bar{m} - c_{F,j})). \tag{28}$$

Thus, indifference curves in the (a, W) space of Figure 3 are linear and upward-sloping with slope

$$\frac{dW}{da} = \frac{m}{h(\theta)} \equiv \tau(m, \theta). \tag{29}$$

Clearly, the better-direction is to the Northwest in Figure 3. Voters with a higher income m, or those with a low preference parameter θ for the public good, have relatively steep indifference curves, while those with low m and/or high θ have relatively flat indifference curves.

Consider a situation in which candidates propose tax rates $a_0 < \bar{a} < a_1$; see the left panel of Figure 3. A voter with the solid (steep) indifference curves strictly prefers Candidate 0, while voters with the dashed and flatter indifference curves prefer Candidate 1. The third line represents the indifference curves of a voter who is indifferent between the two candidates. Note that the situation depicted in the left panel is not an equilibrium: If Candidate 1 deviates to a_1', then the previously indifferent voter is now solidly in the camp of Candidate 1 as well as voters with slightly steeper preferences who previously preferred Candidate 0.

Consider the line that connects $(a_0, W_0(a_0))$ and $(a_1, W_1(a_1))$, which is the indifference curve of the voter who is just indifferent between the two candidates. Given a_1, the objective of Candidate 0 is to choose a_0 such that this line is as flat as possible, as this maximizes the set of types who vote for him. Conversely, the objective of Candidate 1 is to make the line as steep as possible. If the indifference curve of a voter who is indifferent between candidates 0 and 1 is tangent to $W_0(a_0^*)$ and $W_1(a_1^*)$ (as indicated in the right panel), then Candidate 0 cannot make the line connecting $(a_0, W_0(a_0))$ and $(a_1, W_1(a_1))$ flatter, and Candidate 1 cannot make it steeper, and thus both candidates maximize the set of types who vote for them, respectively. We refer to the indifferent voter in the right panel as the "cutoff voter", and to the slope of his indifference curve as τ^*.

We still need to consider under which conditions (a_0^*, a_1^*) is in fact an equilibrium, and under which conditions it is the unique equilibrium. Let us start with the first question: Are there any deviations from (a_0^*, a_1^*) that would increase the winning probability of the deviating candidate? By the argument given above, it is geometrically clear that we have at least a local equilibrium. For example, a small deviation by Candidate 0 (in either direction) will not change the fact that those voters with the flattest indifference curves prefer Candidate 1, and leads to a cutoff voter with a steeper indifference curve so that the set of voters who support Candidate 0 is a strict subset of the set of voters who previously supported Candidate 0. However, it is in principle conceivable that a *large* deviation such that, after the deviation, voters with flat indifference curves vote for Candidate 0 and those with steep indifference curves vote for Candidate 1, is profitable. For this to happen, it would have to be possible for one of the candidates to "outflank" his opponent on that side on which his opponent is stronger. In real life, it often appears implausible that, for example, a Democrat can win an in such a way that he attracts all right-wingers by "out-Republicaning" the Republican candidate. In Theorem 7 below, we provide a condition that prevents such outflanking, and we discuss this condition in more detail below.

As for the second question of uniqueness, it is clear that without uncertainty about the type distribution, there are generically multiple equilibria, because one of the candidates wins with certainty (and possibly by choosing any of some set of policies), and his opponent receives a payoff of 0 no matter which policy he chooses. Assumption 2 below is a sufficient condition that guarantees that there is sufficient uncertainty about the distribution of voters so that both candidates have a strictly positive probability of winning, and that any policy change matters with positive probability for the outcome of the election. As we will show, this guarantees the uniqueness of equilibrium.

Assumption 2 *Let τ_ω be the median of the distribution of $m/h(\theta)$ in state ω and denote the cdf of this distribution by Φ. Then Φ is strictly monotone on* $[W_0'(1), W_1'(c_{F,1}/\bar{m})]$.[16]

[16]Note that we could also assume that Φ has a density that is strictly positive on $[W_0'(1), W_1'(c_{F,1}/\bar{m})]$.

With a slight abuse of terminology, we talk about a median voter τ_ω in each state ω, and a "median median", τ_m, i.e. the median of the distribution of τ_ω. Assumption 2 ensures that any slope between $W_0'(1)$ and $W_1'(c_{F,1}/\bar{m})$ could be the median in some state ω. If a candidate deviates, he loses voter types τ^* and some voters close to τ^*. By Assumption 2, these types could be pivotal for the election outcome, so that his winning probability strictly decreases.

Theorem 7 below shows formally that an equilibrium exists and is unique. In equilibrium, Candidate 0 proposes a strictly lower tax rate and public good provision than Candidate 1. Voter behavior is characterized through a set of cutoff voter types who are indifferent between both candidates. Both candidates appeal to the cutoff voter by choosing his most preferred policy. Most voters have a strict preference for one of the two candidates. High voter types (i.e., those with high income and/or a low preference for public goods) strictly prefer Candidate 0, while low voter types strictly prefer Candidate 1.

Theorem 7 *Suppose that Assumption 2 is satisfied. Let a_0^*, a_1^* be the unique solution of*

$$W_0'(a_0^*) = W_1'(a_1^*) = \frac{W_1(a_1^*) - W_0(a_0^*)}{a_1^* - a_0^*}. \tag{30}$$

Then (a_0^*, a_1^*) is a local equilibrium.

1. *Suppose that $W_0(1) \leq W_1(a_1^*)$ and that $a_0^*\bar{m} \leq b_1$ then (a_0^*, a_1^*) is also a global equilibrium. and (a_0^*, a_1^*) is the unique Nash equilibrium (pure or mixed) of the voting game.*
2. *In equilibrium both candidates maximize the utility of voters (m, t) for whom $m/h(\theta) = \tau^*$, where $\tau^* = W_0'(a_0^*) = W_1'(a_1^*)$. All voters (m, t) with $m/h(\theta) > \tau^*$ strictly prefer Candidate 0, while all voters (m, t) with $m/h(\theta) < \tau^*$ strictly prefer Candidate 1.*
3. *In equilibrium Candidate 0's proposed tax rate and public good provision is strictly less than that of Candidate 1.*

It is useful to start with two comments about the assumptions in Theorem 7. First, note that a_0^* and a_1^* are *defined as* the unique solution of an equation system that depends only on exogenous parameters. Thus, the assumptions that guarantee that the equilibrium is global does not "rely on assuming that an equilibrium exists." Further, the assumption is considerably stronger than necessary since it ensures that the equilibrium is global independently of the distribution of the voters. For example, one could alternatively prove that if the winning probabilities given a_0^* and a_1^* are not too far from 0.5, then (a_0^*, a_1^*) is a global equilibrium.

The fact that both candidates maximize the utility of a[17] cutoff voter τ^* is reminiscent of the equilibrium in standard Downsian model where both candidates choose their policies to maximize the utility

[17]In the following, we will refer to the cutoff voter in singular (i.e., as *one* cutoff voter type). The reason is that, while there are many (m, θ) voter types that are cutoff voters, individuals' behavior in our model depends only on τ, and all cutoff voters have the same τ. It is thus justified to consider all of the cutoff voters as effectively *one* type.

of the median voter. However, the similarity stops here: In the equilibrium of the standard model (with policy convergence), *all* voters are indifferent between the candidates, while in the present model, only a small set of voters (of measure 0) is indifferent between both candidates. Thus, our model preserves the Downsian notion that candidates in an election campaign fiercely contest a relatively small set of swing voters, but avoids the (rather counterfactual) prediction that this implies that *all* voters are (almost) indifferent between candidates.

Policy-motivation is probably the most widely-accepted explanation in the standard model for the widespread observation that policies diverge. While this model and ours both lead to a prediction of policy divergence, there is a subtle difference in the prediction of candidates' reaction to *changes* in the perceived distribution of voter preferences. Suppose, for example, that voters develop a stronger preference for public goods (i.e., there is a first-order stochastic dominance shift in the distribution of τ_ω). Such a change in the distribution has no effect on the candidates' equilibrium policies in our model, but changes the winning probability in favor of Candidate 1. In contrast, a shift of the preference distribution has a significant effect on the policy platforms chosen by policy-motivated candidates in the standard model. For example, if the shift in preferences is due to a common shock to utility functions that affects both voters and candidates in the same way,[18] then both candidates' platforms shift in parallel by the same amount. Even if only voter, but not candidate, preferences shift, both platforms generically shift in the same direction. Finally, with only office-motivated candidates, a change in the expected position of the median would translate only in a change of the equilibrium policies of candidates, without affecting their equilibrium winning probabilities – exactly the opposite from the effect in our model.

The rigidity of candidates is a novel and, at least in some cases, appealing feature of our model. Consider, for example, the present political positioning of the Republican party which suffered a severe defeat in the 2006 and 2008 elections. It is relatively clear that the distribution of U.S. voter preferences has shifted in a way that favors the Democrats. However, Congressional Republicans in early 2009 have almost unanimously opposed President Obama's economic stimulus package that was supported by a clear supermajority of voters. As argued above, this contradicts the prediction of a standard model with identical party capabilities and policy motivated candidates. In contrast, in our model, Republicans cannot successfully imitate the Democrats' policy position; sticking with their previous platform and hoping for a reversal of the preference shift is the best (from an electoral perspective) that Republicans can do in our model.

Another significant difference between the standard model and ours is that the two candidates' winning probability and expected vote shares can differ substantially in equilibrium. Moreover, the equilibrium is an ex-post stable situation in the sense that, even knowing the realization of ω, no candidate could increase his vote share by choosing a different position. In contrast, any standard model with convergence leads to either equal vote shares between candidates, or one of the candidates winning all votes

[18]For example, suppose that some external threat arises and increases the preferred amount of defense spending (interpreted as the policy variable in a standard one-dimensional model) for all voters and candidates.

(say, in a model with convergence and ex-ante uncertain valence). In standard models with divergence (say, with policy-motivated candidates), ex-post vote shares may differ substantially, but at least one candidate could increase his vote share ex-post, and possibly ex-ante.

7 Conclusion

In this paper, we develop a model of candidate competition that is more general than the previous literature on this subject, as we allow for voters to care about both the candidates' fixed characteristics and their chosen policy platforms in an arbitrary way. The framework thus contains all existing frameworks of candidate competition — such as the spatial model or the probabilistic voting model – as special cases.

The main contribution of the model is twofold. First, it enhances our understanding of what drives certain features of equilibrium in existing models of candidate competition, notably policy convergence. Specifically, we show that competition by (also) office-motivated candidates does not produce policy convergence in isolation. Rather, this conclusion follows from the interplay of office motivation and a certain "independence" of fixed characteristics and flexible policy positions in the voters' utility functions. We formalize this form of "independence" by identifying the class of UCR preferences for which equilibrium policy convergence arises even when candidates differ in fixed characteristics (Theorem 3). Conversely, Theorem 4 shows that UCR preferences are also, in a certain sense, necessary for convergence: Even if only one voter has non-UCR preferences, there exists a voting game in which the unique and strict Nash equilibrium features policy divergence.

For the most general setup, we obtain characterization results — they tell us how an equilibrium looks like or cannot look like *if it exists*. Since our model contains a very general class of models, including some for which no pure strategy equilibrium exists, it is effectively impossible to identify necessary and sufficient conditions that guarantee existence of a pure strategy strict equilibrium within the general framework. Nevertheless, we know from previous literature that an equilibrium exists for several subclasses such as the one-dimensional spatial model and the probabilistic voting model. Thus, our characterization results are not vacuous, and they help us to understand *why* policy convergence obtains in these models.

The second major contribution of our paper is to identify interesting classes of models in which voters have non-UCR preferences. For instance, scenarios in which a candidate's competence in a policy area affects the voter's preferred policy from the candidate naturally give rise to non-UCR preferences. The two models that we present capture the notions of complementarity between fixed and flexible positions, and of competence differentials. These models arise naturally as generalizations of the probabilistic voting model and the one-dimensional spatial model, respectively. Moreover, the generalized models are essentially as tractable as the corresponding special cases in that there is (at least under certain additional, relatively mild conditions) a unique and strict Nash equilibrium that can easily be characterized. However, we show that the equilibrium of the game between the candidates features policy convergence *only* in those special cases analyzed in the previous literature, while generically, there is policy divergence in equilibrium. Also, comparative statics effects (i.e., which primitives influence equilibrium policy choice, and which ones do not) differ substantially between the generalized models and those special cases on which the previous literature has focused.

Our results, in particular for the class of models where voters have non-UCR preferences, open several interesting avenues for future research. First, one can focus more closely on particular applications, such as we do in Krasa and Polborn (2009a), where we formalize the notion of issue-ownership, first informally formulated by Petrocik (1996) in the political science literature. Second, one can analyze how differences between candidates in what Stokes (1963) calls “valence issues” (i.e., where all voters prefer a more productive candidate) interact with what he calls “position issues” (i.e., social policy questions such as abortion in which voter preferences for positions are split). Third, one can analyze the question of candidate selection in more detail. In the present paper, candidates are exogenously endowed with certain fixed characteristics. It may be interesting to add a prior stage to the game where candidates are chosen by parties and their members from two, possibly distinct, sets of available candidates. Interesting questions include how party members, who arguably are primarily interested in policy rather than in winning, choose among potential candidates knowing that these candidates will then go on and choose a policy for the general election in a way to maximize their respective probability of winning.

8 Appendix

Proof of Theorem 1. If $a_0 = a_1$, then $c_0 = c_1$ and reflexivity of preferences imply that all voters are indifferent between the candidates. Thus, the winning probabilities are 0.5. Let (a_0, a_1) be a Nash equilibrium. If Candidate j's payoff were strictly less than 0.5 in this equilibrium, then Candidate j could increase the payoff to 0.5 by using the same policy as the other agent. However, since $W^1(\omega, a_0, a_1) = 1 - W^0(\omega, a_0, a_1)$ this implies $\int W^j(\omega, a_0, a_1)\, d\mu(\omega) = 0.5$, i.e., in equilibrium (a_0, a_1) each candidate's winning probability is 0.5.

We now prove that (a_1, a_1) is Nash equilibrium. Suppose by way of contradiction that there exists a deviation $\tilde{a}_i$ that makes Candidate i strictly better off. If $i = 0$ then Candidate 0 would have used $\tilde{a}_0$ against a_1 thereby increasing his payoff, resulting in a winning probability that is strictly greater than 0.5. This contradicts the assumption that (a_0, a_1) is a Nash equilibrium (as the candidates' winning probability in (a_0, a_1) is 0.5). Thus, we can assume that $i = 1$, i.e., $\tilde{a}_1$ played against a_1 results in a ex-ante winning probability that is strictly greater than 0.5. However, $c_0 = c_1$ implies that $W^0(\omega, a_0, a_1) = W^1(\omega, a_1, a_0)$. Thus, $0.5 < \int W^1(\omega, a_1, \tilde{a}_1)\, d\mu(\omega) = \int W^0(\omega, \tilde{a}_1, a_1)\, d\mu(\omega) \leq 0.5$, where the last inequality follows since (a_0, a_1) is a Nash equilibrium with winning probabilities 0.5. This contradiction proves that (a_1, a_1) is a Nash equilibrium. Similarly, it follows that (a_0, a_0) is a Nash equilibrium.

Now suppose that (a_0, a_1) is a strict Nash equilibrium. If $a_0 \neq a_1$ then the previous argument implies that (a_0, a_0) is also a Nash equilibrium resulting in the same winning probability, which contradicts the assumption that (a_0, a_1) is strict. Thus, $a_0 = a_1 = \bar{a}$. Suppose by way of contradiction that there exists another pure strategy Nash equilibrium (a', a'), where $a' \neq \bar{a}$ (because of the first part of the proof we can assume that both candidates use the same strategy). Since the equilibrium $(\bar{a}, \bar{a})$ is strict we get $0.5 = \int W^0(\omega, \bar{a}, \bar{a})\, d\mu(\omega) > \int W^0(\omega, a', \bar{a})\, d\mu(\omega)$. Thus, $W^0 + W^1 = 1$ implies $\int W^1(\omega, a', \bar{a})\, d\mu(\omega) > 0.5$. Hence, (a', a') is not a Nash equilibrium since there exists a profitable deviation for Candidate 1, a contradiction.

Finally, suppose that there exists a mixed strategy equilibrium. Without loss of generality we can assume that Candidate 0 mixes with strictly positive probability. The argument in the previous paragraph implies that $\int W^1(\omega, a, \bar{a})\, d\mu(\omega) \geq 0.5$ for all $a \in A$, and that the inequality is strict for $a \neq \bar{a}$. Similarly, $\int W^0(\omega, \bar{a}, a)\, d\mu(\omega) \geq 0.5$ for all $a \in A$. The first inequality and the fact that Candidate 0 mixes imply that by choosing $a_1 = \bar{a}$ with probability 1, Candidate 1 gets a winning probability that is strictly greater than 0.5. The second inequality implies that Candidate 0's winning probability must be at least 0.5. Thus, the winning probabilities add to a number strictly greater than 1, a contradiction. Hence, there does not exist a mixed strategy equilibrium. ■

Proof of Theorem 2. We start by proving that statement 2 implies statement 1. Since f and g are continuous, the implied preferences are continuous. In remains to prove that UCR holds. Let $(c, b) \geq$

(c', b). Then $g(f(c), b) \geq g(f(c'), b)$. Since g is strictly monotone in the first argument this implies $f(c) \geq f(c')$. Again, strict monotonicity implies $g(f(c), b') \geq g(f(c'), b')$, which implies $(c, b') \geq (c', b')$, i.e., UCR holds.

We now prove that statement 1 implies statement 2. Define preferences $\geq^C$ on C as follows: $c \geq^C c'$ if there exists $a \in A$ with $(c, a) \geq (c', a)$. Note that these preferences are well defined. In particular, the ability to uniformly rank candidates in state ω implies that $(c, a') \geq (c', a')$ for any $a' \in A$. Further preferences $\geq^C$ are complete since $\geq$ are complete and therefore either $(c, a) \geq (c', a)$ or $(c', a) \geq (c, a)$ must be satisfied. In the first case $c \geq^C c'$ while in the second case $c' \geq^C c$. Transitivity of $\geq^C$ follows also immediately from transitivity of $\geq$. In particular, suppose that $c \geq^C c'$ and $c' \geq^C c''$. Then for any $a \in A$ we get $(c, a) \geq (c', a)$ and $(c', a) \geq (c'', a)$. Thus, $(c, a) \geq (c'', a)$, which implies that $c \geq^C c''$.

Since C is a separable metric space and since preferences are continuous, there exists a continuous utility function f that describes preferences $\geq^C$, i.e., $f(c) \geq f(c')$ if and only if $c \geq^C c'$. Let $Y = f(C)$ and $c, c' \in f^{-1}(y)$ for some $y \in Y$. We now define preferences on $Y \times A$ as follows: $(y, a) \geq' (y', a')$ if and only if there exist $c \in f^{-1}(y)$ and $c' \in f^{-1}(y')$ with $(c, a) \geq (c', a')$.

To show that these preferences are well defined, let $\hat{c} \in f^{-1}(y)$ and $\hat{c}' \in f^{-1}(y')$. We must show that $(\hat{c}, a) \geq (\hat{c}', a')$. $f(c) = f(\hat{c})$ and $f(c') = f(\hat{c}')$ and the fact that f is a utility function for $\geq^C$ implies that $(c, a) \sim (\hat{c}, a)$ and $(c', a') \sim (\hat{c}', a')$. Thus, $(\hat{c}, a) \sim (c, a) \geq (c', a') \sim (\hat{c}', a')$.

Completeness of preferences $\geq'$ follows immediately from completeness of $\geq$. To prove transitivity, let $(y, a) \geq' (y', a')$ and $(y', a) \geq' (y'', a')$. This implies $(c, a) \geq (c', a')$ and $(\hat{c}', a') \geq (c'', a'')$, where $c \in f^{-1}(y)$, $c', \hat{c}' \in f^{-1}(y')$ and $c'' \in f^{-1}(y'')$. Since $c', \hat{c}' \in f^{-1}(y')$ we get $(c', a') \sim (\hat{c}', a')$. Thus, transitivity of $\geq$ implies $(c, a) \geq (c'', a'')$, and therefore $(y, a) \geq' (y'', a'')$.

Next, we show continuity of $\geq'$. Let (y_i, a_i), $i \in \mathbb{N}$ be a sequence with limit (y, a), and let $(\bar{y}, \bar{a}) \in Y \times A$, such that $(y_i, a_i) \geq' (\bar{y}, \bar{a})$ for all $i \in \mathbb{N}$. We must show that $(y, a) \geq' (\bar{y}, \bar{a})$.

For each $i \in \mathbb{N}$ let $c_i \in f^{-1}(y_i)$. Since C is compact, there exists a subsequence c_{i_k}, $k \in \mathbb{N}$ that converges. Let $c = \lim_{k \to \infty} c_{i_k}$. Continuity of f implies $f(c) = \lim_{k \to \infty} f(c_{i_k}) = \lim_{k \to \infty} y_{i_k} = y$. Since $(y_{i_k}, a_{i_k}) \geq' (\bar{y}, \bar{a})$ if follows that $(c_{i_k}, a_{i_k}) \geq (\bar{c}, \bar{a})$ for some $\bar{c} \in f^{-1}(\bar{y})$. Continuity of preferences $\geq$ implies that $(c, a) \geq (\bar{c}, \bar{a})$. Hence $(y, a) \geq' (\bar{y}, \bar{a})$.

Similarly, it follows that if $(y_i, a_i) \leq' (\bar{y}, \bar{a})$ for all $i \in \mathbb{N}$ then $(y, a) \leq' (\bar{y}, \bar{a})$. Thus, preferences $\geq'$ are continuous.

Next, note that preferences $\geq'$ are strictly monotone in y. In particular, let $(y, a), (y', a) \in Y \times A$ with $y > y'$. Let $c \in f^{-1}(y)$ and $c' \in f^{-1}(y')$. Because f is a utility function describing preferences on C it follows that $c >^C c'$. This, in turn implies $(c, a) > (c', a)$, and therefore $(y, a) >' (y', a)$.

Because $Y \times A$ is again a separable metric space, and the preferences $\geq'$ on $Y \times A$ are continuous, there exists a utility function g that describes preferences $\geq'$. Strict monotonicity of preferences in y implies that g is strictly monotone in y. Finally, $u(a) = g(f(c), a)$ is a continuous utility function that describes preferences $\succeq$. ■

Proof of Theorem 3. Suppose by way of contradiction that there exists a strict Nash equilibrium (a_0, a_1) with $a_0 \neq a_1$. Then

$$\int W^0(\omega, a_0, a_1)\, d\mu(\omega) > \int W^0(\omega, a_1, a_1)\, d\mu(\omega), \tag{31}$$

$$\int W^1(\omega, a_0, a_1)\, d\mu(\omega) > \int W^1(\omega, a_0, a_0)\, d\mu(\omega). \tag{32}$$

Next, note that if preferences are UCR then $(c_0, a) \succeq^\ell_\omega (c_1, a)$ if and only if $(c_0, a') \succeq^\ell_\omega (c_1, a')$ for any citizen ℓ and for any state $\omega \in \Omega$. Thus, citizens' voting behavior is the same if both candidates choose a or if both choose a'. Thus, the winning probabilities do not change for candidates $j = 1, 2$, i.e.,

$$W^j(\omega, a, a) = W^j(\omega, a', a'), \text{ for all } a, a' \in A. \tag{33}$$

(31), (33) , and the fact that $W^0 + W^1 = 1$ imply

$$\int W^1(\omega, a_0, a_1)\, d\mu(\omega) < \int W^1(\omega, a_1, a_1)\, d\mu(\omega) = \int W^1(\omega, a_0, a_0)\, d\mu(\omega), \tag{34}$$

But (34) contradicts (32). Thus, in any strict Nash equilibrium $a_0 = a_1 = a$.

Next, we prove uniqueness of the Nash equilibrium (a, a). First, suppose that there exists another pure strategy Nash equilibrium (a_0, a_1). Since the Nash equilibrium (a, a) is strict, it follows that $a_0, a_1 \neq a$. Further, $\int W^1(\omega, a, a)\, d\mu(\omega) > \int W^1(\omega, a, a_1)\, d\mu(\omega)$ and $\int W^0(\omega, a, a)\, d\mu(\omega) > \int W^0(\omega, a_0, a)\, d\mu(\omega)$. Since $W^0 + W^1 = 1$ we get

$$\int W^0(\omega, a, a)\, d\mu(\omega) < \int W^0(\omega, a, a_1)\, d\mu(\omega); \text{ and} \tag{35}$$

$$\int W^1(\omega, a, a)\, d\mu(\omega) < \int W^1(\omega, a_0, a)\, d\mu(\omega). \tag{36}$$

(35), (36) and the fact that (a_0, a_1) is a Nash equilibrium implies

$$\int W^0(\omega, a, a)\, d\mu(\omega) < \int W^0(\omega, a, a_1)\, d\mu(\omega) \leq \int W^0(\omega, a_0, a_1)\, d\mu(\omega); \tag{37}$$

$$\int W^1(\omega, a, a)\, d\mu(\omega) < \int W^1(\omega, a_0, a)\, d\mu(\omega) \leq \int W^0(\omega, a_0, a_1)\, d\mu(\omega). \tag{38}$$

Since $W^0 + W^1 = 1$, adding (37) and (38) yields a contradiction. Thus, the Nash equilibrium is unique among all pure strategy equilibria. The remainder of the proof, that there is no mixed strategy equilibrium, is identical to the last step in the proof of Theorem 1. ■

Proof of Theorem 4. Since u is non-UCR, there exist fixed characteristics c_0, c_1 and policies $\bar{a}$, $\bar{a}'$ such that $u(c_0, \bar{a}) > u(c_1, \bar{a})$ and $u(c_0, \bar{a}') \leq u(c_1, \bar{a}')$. First, suppose that $u(c_0, \bar{a}') < u(c_1, \bar{a}')$, $u(c_0, \bar{a}) > u(c_1, \bar{a}')$ and $u(c_0, \bar{a}') > u(c_1, \bar{a})$.

Let $\xi_A \colon A \to \mathbb{R}$ with $\bar{a} = \arg\max_{a \in A} \xi_A(a)$ and $\bar{a}' = \arg\max_{a \in A, a \neq \bar{a}} \xi_A(a)$. Let $\xi'_A \colon A \to \mathbb{R}$ with $\bar{a}' = \arg\max_{a \in A} \xi'_A(a)$ and $\bar{a} = \arg\max_{a \in A, a \neq \bar{a}'} \xi_A(a)$. Further, let $\xi_C \to \mathbb{R}$ be a function with $\xi_C(c_0) > \xi_C(c_1)$.

Consider the following utility functions: $u_1(c,a) = \frac{\xi_C(c) - \xi_C(c_1)}{1 + \xi_C(c_0) - \xi_C(c_1)} + \frac{\xi_A(a) - \xi_A(\bar{a}')}{\xi_A(\bar{a}) - \xi_A(\bar{a}')}$, $u_2(c,a) = \frac{\xi_C(c) - \xi_C(c_1)}{1 + \xi_C(c_0) - \xi_C(c_1)} + \frac{\xi'_A(a) - \xi'_A(\bar{a})}{\xi'_A(\bar{a}) - \xi'_A(\bar{a})}$, and $u_3(c,a) = -\xi_C(c)$.

Theorem 2 immediately implies that these three utility functions describe UCR preferences. Let $\Omega = \{\omega_1, \omega_2\}$, where $\mu(\{\omega_i\}) > 0$ for both states $i = 1, 2$. In state ω_1 there is one voter with utility u_1, one with utility u_2 and one with utility u and two with utility u_3. In state ω_2 there are two voters with utility u_1, one with utility u and two with utility u_3. In state ω' there is one voter with utility u_1, one with utility u_2 and one with utility u and two with utility u_3. Let $n_1(\omega)$ and $n_2(\omega)$ denote the number of voters with utility u_1 and u_2, respectively in state ω. Then the number of votes received by the candidates are as as follows:

	$(c_1, \bar{a})$	$(c_1, \bar{a}')$
$(c_0, \bar{a})$	3,2	$n_1(\omega) + 1, n_2(\omega) + 2$
$(c_0, \bar{a}')$	$n_2(\omega) + 1, n_1(\omega) + 2$	2,3

Thus, $\bar{a}$ is a strictly dominant strategy for Candidate 0. The unique Nash equilibrium is therefore (a, a').

Next, suppose that $u(c_0, \bar{a}') < u(c_1, \bar{a}')$, $u(c_0, \bar{a}) > u(c_1, \bar{a}')$ and $u(c_0, \bar{a}') < u(c_1, \bar{a})$. Then the number of voter each candidates receives are:

	$(c_1, \bar{a})$	$(c_1, \bar{a}')$
$(c_0, \bar{a})$	3,2	$n_1(\omega) + 1, n_2(\omega) + 2$
$(c_0, \bar{a}')$	$n_2(\omega), n_1(\omega) + 3$	2,3

Thus, $\bar{a}$ remains strictly dominant for Candidate 0, and $(\bar{a}, \bar{a}')$ is still the unique Nash equilibrium. If $u(c_0, \bar{a}') < u(c_1, \bar{a}')$, $u(c_0, \bar{a}) < u(c_1, \bar{a}')$ and $u(c_0, \bar{a}') < u(c_1, \bar{a})$ or $u(c_0, \bar{a}') < u(c_1, \bar{a}')$, $u(c_0, \bar{a}) < u(c_1, \bar{a}')$ and $u(c_0, \bar{a}') > u(c_1, \bar{a})$, then the above argument applies if we relabel the candidates, i.e., replace c_0 by c_1 and c_1 by c_0.

Now suppose that $u(c_0, \bar{a}') = u(c_1, \bar{a}')$, $u(c_0, \bar{a}) > u(c_1, \bar{a}')$ and $u(c_0, \bar{a}') > u(c_1, \bar{a})$. Noting that in the case of indifference, a voter abstains, we get

	$(c_1, \bar{a})$	$(c_1, \bar{a}')$
$(c_0, \bar{a})$	3,2	$n_1(\omega) + 1, n_2(\omega) + 2$
$(c_0, \bar{a}')$	$n_2(\omega) + 1, n_1(\omega) + 2$	2,2

As $n_1(\omega_2) = 0$, and $n_2(\omega_2) = 2$ Candidate 1 wins in state ω_2 and loses in state ω_1 when strategies

(a, a') are played. Thus, (a, a') is the unique Nash equilibrium if $\mu(\{\omega_2\}) > 0.5$. All other cases are symmetric. ■

8.1 Derivation of Equations Used in Section 5.2

To solve for the equilibrium we proceed as follows. We first transforming the coordinates using M, which results in indifference curves that are circles. We then rotate the coordinates such that the fixed characteristic, c, is again on the horizontal axis. This can be done by applying the matrix

$$O = \begin{pmatrix} \frac{1}{\sqrt{1+\kappa_1^2\kappa_2^2}} & -\frac{\kappa_1\kappa_2}{\sqrt{1+\kappa_1^2\kappa_2^2}} \\ \frac{\kappa_1\kappa_2}{\sqrt{1+\kappa_1^2\kappa_2^2}} & \frac{1}{\sqrt{1+\kappa_1^2\kappa_2^2}} \end{pmatrix} \tag{39}$$

Note that

$$O \cdot M \cdot \begin{pmatrix} c \\ 0 \end{pmatrix} = \begin{pmatrix} c\sqrt{1+\kappa_1^2\kappa_2^2} \\ 0 \end{pmatrix}, \text{ and } O \cdot M \cdot \begin{pmatrix} 0 \\ a \end{pmatrix} = \frac{a}{\sqrt{1+\kappa_1^2\kappa_2^2}} \begin{pmatrix} \kappa_2(1-\kappa_1^2) \\ \kappa_1(1+\kappa_2^2) \end{pmatrix} \tag{40}$$

For any c and a let

$$\xi_C(c) = c\sqrt{1+\kappa_1^2\kappa_2^2}, \text{ and } \xi_A(a) = \frac{a\kappa_1(1+\kappa_2^2)}{\sqrt{1+\kappa_1^2\kappa_2^2}} \tag{41}$$

Let $\tilde{c}_i = \xi(c_i)$, for candidates $i = 0, 1$, and $\tilde{a} = \xi_A(a)$. Define β by (17). Then (40) implies that we have a new voting game in which Candidate i can choose policies $(\tilde{c}_i + \beta\tilde{a}, \tilde{a})$, and voters have Euclidean preferences over $(\tilde{c}, \tilde{a})$.

Voter type $(\varepsilon_j, \theta_j)$ in the original voting game, corresponds to type $(\tilde{\varepsilon}_j + \beta\tilde{\theta}_j, \tilde{\theta}_j)$ in the transformed game, where $\tilde{\varepsilon} = \xi_C(\varepsilon)$ and $\tilde{\theta}_j = \xi_A(\theta_j)$. In the transformed game indifferences curves are circles. Thus, $(\varepsilon_j, \theta_j)$ preferes Candidate 0 to Candidate 1 if and only if

$$(\tilde{\varepsilon}_j + \beta\tilde{\theta}_j - \tilde{c}_0 - \beta\tilde{a}_0)^2 + (\tilde{\theta}_j - \tilde{a}_0)^2 > (\tilde{\varepsilon}_j + \beta\tilde{\theta}_j - c_1 - \beta\tilde{a}_1)^2 + (\tilde{\theta}_j - \tilde{a}_1)^2. \tag{42}$$

(42) is equivalent to

$$\tilde{\varepsilon}_j < \frac{1}{2}\left[\frac{(\tilde{c}_1 + \beta\tilde{a}_1)^2 - (\tilde{c}_0 + \beta\tilde{a}_0)^2 + \tilde{a}_1^2 - \tilde{a}_0^2 - 2\tilde{\theta}_j(\tilde{a}_1 - \tilde{a}_0)}{\tilde{c}_1 - \tilde{c}_0 + \beta(\tilde{a}_1 - \tilde{a}_0)} - 2\beta\tilde{\theta}_j\right],$$

which implies

$$\varepsilon_j < \frac{1}{2\sqrt{1+\kappa_1^2\kappa_2^2}}\left[\frac{(\tilde{c}_1 + \beta\tilde{a}_1)^2 - (\tilde{c}_0 + \beta\tilde{a}_0)^2 + \tilde{a}_1^2 - \tilde{a}_0^2 - 2\tilde{\theta}_j(\tilde{a}_1 - \tilde{a}_0)}{\tilde{c}_1 - \tilde{c}_0 + \beta(\tilde{a}_1 - \tilde{a}_0)} - 2\beta\tilde{\theta}_j\right].$$

The optimization problems of candidates 0 and 1 are therefore

$$\max_{\tilde{a}_0} \sum_{j=1}^{J} \lambda_j F_j \left(\frac{1}{2\sqrt{1+\kappa_1^2\kappa_2^2}} \left[\frac{(\tilde{c}_1+\beta\tilde{a}_1)^2-(\tilde{c}_0+\beta\tilde{a}_0)^2+\tilde{a}_1^2-\tilde{a}_0^2-2\tilde{\theta}_j(\tilde{a}_1-\tilde{a}_0)}{\tilde{c}_1-\tilde{c}_0+\beta(\tilde{a}_1-\tilde{a}_0)} - 2\beta\tilde{\theta}_j \right] - \omega^* \right), \quad (43)$$

$$\min_{\tilde{a}_1} \sum_{j=1}^{J} \lambda_j F_j \left(\frac{1}{2\sqrt{1+\kappa_1^2\kappa_2^2}} \left[\frac{(\tilde{c}_1+\beta\tilde{a}_1)^2-(\tilde{c}_0+\beta\tilde{a}_0)^2+\tilde{a}_1^2-\tilde{a}_0^2-2\tilde{\theta}_j(\tilde{a}_1-\tilde{a}_0)}{\tilde{c}_1-\tilde{c}_0+\beta(\tilde{a}_1-\tilde{a}_0)} - 2\beta\tilde{\theta}_j \right] - \omega^* \right). \quad (44)$$

In equilibrium a_0 and a_1 must satisfy (43) and (44) and ω^* must solve

$$\sum_{j=1}^{J} \lambda_j F_j \left(\frac{1}{2\sqrt{1+\kappa_1^2\kappa_2^2}} \left[\frac{(\tilde{c}_1+\beta\tilde{a}_1)^2-(\tilde{c}_0+\beta\tilde{a}_0)^2+\tilde{a}_1^2-\tilde{a}_0^2-2\tilde{\theta}_j(\tilde{a}_1-\tilde{a}_0)}{\tilde{c}_1-\tilde{c}_0+\beta(\tilde{a}_1-\tilde{a}_0)} - 2\beta\tilde{\theta}_j \right] - \omega^* \right) = 0.5. \quad (45)$$

Let

$$k(\tilde{a}_0,\tilde{a}_1) = \frac{(\tilde{c}_1+\beta\tilde{a}_1)^2-(\tilde{c}_0+\beta\tilde{a}_0)^2+\tilde{a}_1^2-\tilde{a}_0^2-2\tilde{\theta}_j(\tilde{a}_1-\tilde{a}_0)}{\tilde{c}_1-\tilde{c}_0+\beta(\tilde{a}_1-\tilde{a}_0)}, \text{ and } K = \frac{1}{2\sqrt{1+\kappa_1^2\kappa_2^2}}$$

Then the first order conditions are

$$\sum_{j=1}^{J} \lambda_j f_j \left(K(k(\tilde{a}_0,\tilde{a}_1) - 2\beta\tilde{\theta}_j) - \omega^* \right) K \left[\frac{-2\beta(\tilde{c}_0+\beta\tilde{a}_0) - 2\tilde{a}_0 + 2\tilde{\theta}_j + k(\tilde{a}_0,\tilde{a}_1)}{\tilde{c}_1-\tilde{c}_0+\beta(\tilde{a}_1-\tilde{a}_0)} \right] = 0; \quad (46)$$

$$-\sum_{j=1}^{J} \lambda_j f_j \left(K(k(\tilde{a}_0,\tilde{a}_1) - 2\beta\tilde{\theta}_j) - \omega^* \right) K \left[\frac{-2\beta(\tilde{c}_1+\beta\tilde{a}_1) - 2\tilde{a}_1 + 2\tilde{\theta}_j + k(\tilde{a}_0,\tilde{a}_1)}{\tilde{c}_1-\tilde{c}_0+\beta(\tilde{a}_1-\tilde{a}_0)} \right] = 0. \quad (47)$$

Suppose $\tilde{a}_0$ and $\tilde{a}_1$ satisfy condition (18) discussed in the main text. If, in addition $\tilde{a}_0$ and $\tilde{a}_1$ satisfy (46) then $\tilde{a}_0$ and $\tilde{a}_1$ also satisfy (47). Substituting (18) into (46) yields

$$\sum_{j=1}^{J} \lambda_j f_j \left(\frac{\tilde{c}_0+\tilde{c}_1}{2\sqrt{1+\kappa_1^2\kappa_2^2}} - \omega^* \right) \frac{(1+\beta^2)}{2\sqrt{1+\kappa_1^2\kappa_2^2}} \left[\frac{\beta}{2} + (1+\beta^2)\frac{\tilde{\theta}_j-\tilde{a}_0}{\tilde{c}_1-\tilde{c}_0} \right] = 0, \quad (48)$$

which, using the definition of $\tilde{c}_i$, $\tilde{a}_i$ and $\tilde{\theta}_i$ is equivalent to

$$\sum_{j=1}^{J} \lambda_j f_j \left(\frac{c_0+c_1}{2} - \omega^* \right) \left[\frac{\beta}{2} + (1+\beta^2)\frac{\xi_A(\theta_j-a_0)}{\xi_C(c_1-c_0)} \right] = 0. \quad (49)$$

(45) simplifies to

$$\sum_{j=1}^{J} \lambda_j F_j \left(\frac{c_0+c_1}{2} - \omega^* \right) = 0.5, \quad (50)$$

which implies that $\omega^* = (c_0+c_1)/2$.

Continuity of (49) in a_0 immediately implies that there exists a solution. To get a_1 we use the fact that $\tilde{a}_0 = \xi_A(a_0)$ and then apply (18) to get $\tilde{a}_1$. Finally, $a_1 = \xi_A^{-1}(\tilde{a}_1)$, implies condition (23).

Next, we derive the second order condition. The derivative of the left-hand side of (46) with respect to $\tilde{a}_0$ is

$$\begin{aligned}\sum_{j=1}^{J} \lambda_j f_j' \left(K(k(\tilde{a}_0,\tilde{a}_1) - 2\beta\tilde{\theta}_j) - \omega^*\right) K \left(\frac{\partial k(\tilde{a}_0,\tilde{a}_1)}{\partial \tilde{a}_0}\right)^2 \\ + \sum_{j=1}^{J} \lambda_j f_j \left(K(k(\tilde{a}_0,\tilde{a}_1) - 2\beta\tilde{\theta}_j) - \omega^*\right) K \left(\frac{\partial^2 k(\tilde{a}_0,\tilde{a}_1)}{\partial^2 \tilde{a}_0}\right).\end{aligned} \tag{51}$$

Next,

$$\frac{\partial^2 k(a_0,a_1)}{\partial^2 a_0} = -\frac{2(1+\beta^2)}{\tilde{c}_1 - \tilde{c}_0 + \beta(\tilde{a}_1 - \tilde{a}_0)} + \frac{1+\beta}{\tilde{c}_1 - \tilde{c}_0 + \beta(\tilde{a}_1 - \tilde{a}_0)} \frac{\partial k(\tilde{a}_0,\tilde{a}_1)}{\partial \tilde{a}_0} \tag{52}$$

At any critical value of a_0, (46) must be satisfied. Thus,

$$\sum_{j=1}^{J} \lambda_j f_j \left(K(k(\tilde{a}_0,\tilde{a}_1) - 2\beta\tilde{\theta}_j) - \omega^*\right) K \frac{1+\beta}{\tilde{c}_1 - \tilde{c}_0 + \beta(\tilde{a}_1 - \tilde{a}_0)} \frac{\partial k(\tilde{a}_0,\tilde{a}_1)}{\partial \tilde{a}_0} = 0. \tag{53}$$

If $\hat{a}_0$ and $\hat{a}_1$ satisfy (18) then $\hat{a}_0 > \hat{a}_1$ and

$$\tilde{c}_1 - \tilde{c}_0 + \beta(\hat{a}_1 - \hat{a}_0) = \frac{1}{1+\beta^2}(\tilde{c}_1 - \tilde{c}_0) > 0.$$

Hence there exists $\underline{a} < \hat{a}_1 < \hat{a}_0 < \bar{a}$ such that $\tilde{c}_1 - \tilde{c}_0 + \beta(\tilde{a}_1 - \tilde{a}_0) > 0$ for any $\tilde{a}_0, \tilde{a}_1 \in [\underline{a}, \bar{a}]$.

As a consequence, (51), (52), and (53) imply that the second order condition is

$$\sum_{j=1}^{J} \lambda_j K \left[f_j' \left(K(k(\tilde{a}_0,\tilde{a}_1) - 2\beta\tilde{\theta}_j) - \omega^*\right) \left(\frac{\partial k(\tilde{a}_0,\tilde{a}_1)}{\partial \tilde{a}_0}\right)^2 - f_j \left(K(k(\tilde{a}_0,\tilde{a}_1) - 2\beta\tilde{\theta}_j) - \omega^*\right) \frac{2(1+\beta^2)}{\tilde{c}_1 - \tilde{c}_0 + \beta(\tilde{a}_1 - \tilde{a}_0)} \right] < 0,$$

which is equivalent to

$$\sum_{j=1}^{J} \lambda_j \left[\frac{f_j' \left(K(k(\tilde{a}_0,\tilde{a}_1) - 2\beta\tilde{\theta}_j) - \omega^*\right)}{f_j \left(K(k(\tilde{a}_0,\tilde{a}_1) - 2\beta\tilde{\theta}_j) - \omega^*\right)} \frac{\left(-2\beta(\tilde{c}_0 + \beta\tilde{a}_0) - 2\tilde{a}_0 + 2\tilde{\theta}_j + k(\tilde{a}_0,\tilde{a}_1)\right)^2}{\left(2(1+\beta^2)\right)\left(\tilde{c}_1 - \tilde{c}_0 + \beta(\tilde{a}_1 - \tilde{a}_0)\right)} - 1 \right] < 0, \tag{54}$$

where the last inequality holds since $\tilde{c}_1 - \tilde{c}_0 + \beta(\tilde{a}_1 - \tilde{a}_0) > 0$.

Similarly, the second order condition for (44) is

$$\sum_{j=1}^{J} \lambda_j \left[\frac{f_j' \left(K(k(\tilde{a}_0,\tilde{a}_1) - 2\beta\tilde{\theta}_j) - \omega^*\right)}{f_j \left(K(k(\tilde{a}_0,\tilde{a}_1) - 2\beta\tilde{\theta}_j) - \omega^*\right)} \frac{\left(-2\beta(\tilde{c}_0 + \beta\tilde{a}_1) - 2\tilde{a}_1 + 2\tilde{\theta}_j + k(\tilde{a}_0,\tilde{a}_1)\right)^2}{\left(2(1+\beta^2)\right)\left(\tilde{c}_1 - \tilde{c}_0 + \beta(\tilde{a}_1 - \tilde{a}_0)\right)} - 1 \right] < 0. \tag{55}$$

Both second order conditions are satisfied at the solutions of the first order conditions. In particular,

$$f_j' \left(K(k(\tilde{a}_0,\tilde{a}_1) - 2\beta\tilde{\theta}_j) - \omega^*\right) = f_j' \left(\frac{c_0 + c_1}{2} - \omega^*\right) = f_j'(0) = 0,$$

which implies that the left-hand sides of (54) and (55) are $-\sum_{j=1}^{J}\lambda_j < 0$. Thus, we have a local equilibrium that is strict.

We next show that (a_0, a_1) is characterized by (18) and (49) is the unique equilibrium pure or mixed.

In particular, we change coordinates, by using the orthogonal matrix

$$D = \begin{pmatrix} \frac{1}{\sqrt{1+\beta^2}} & -\frac{\beta}{\sqrt{1+\beta^2}} \\ \frac{\beta}{\sqrt{1+\beta^2}} & \frac{1}{\sqrt{1+\beta^2}} \end{pmatrix} \tag{56}$$

Because D is a rotation, the indifference curves of voters remain circles. In the previous voting game, policy choices where on lines of the form $(\tilde{c}_i + \beta a, a)$. Now note that after applying D the lines on which policies are chosen are vertical. Next, (18) implies

$$D \cdot \left[\begin{pmatrix} \tilde{c}_1 + \beta\tilde{a}_1 \\ \tilde{a}_1 \end{pmatrix} - \begin{pmatrix} \tilde{c}_0 + \beta\tilde{a}_0 \\ \tilde{a}_0 \end{pmatrix} \right] = D \cdot \begin{pmatrix} \frac{1}{1+\beta^2}(\tilde{c}_1 - \tilde{c}_0) \\ -\frac{\beta}{1+\beta^2}(\tilde{c}_1 - \tilde{c}_0) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1+\beta^2}} \\ 0 \end{pmatrix}.$$

Thus, both candidates choose the same policy in the transformed voting game. Further, the second order conditions imply that the equilibrium is strict. Since preferences are circles, they are UCR. As a consequence, Theorem 3 implies that the equilibrium in the transformed voting game is unique. Hence $(\tilde{a}_0, \tilde{a}_1)$ is the unique equilibrium in the voting game with fixed positions $\tilde{c}_i$ and feasible policy lines $(\tilde{c}_i + \beta a, a)$.

We next show that the arithmetic mean of the candidates' policies $\bar{a} = (a_0 + a_1)/2$ is independent of κ_1 and κ_2. In particular, using (18) to substitute a_0 for a_1 in (47) yields

$$\sum_{j=1}^{J} \lambda_j f_j \left(\frac{\tilde{c}_0 + \tilde{c}_1}{2\sqrt{1 + \kappa_1^2\kappa_2^2}} - \omega^* \right) \frac{(1+\beta^2)}{2\sqrt{1 + \kappa_1^2\kappa_2^2}} \left[-\frac{\beta}{2} + (1+\beta^2)\frac{\tilde{\theta}_j - \tilde{a}_1}{\tilde{c}_1 - \tilde{c}_0} \right] = 0. \tag{57}$$

Adding (48) and (57) yields

$$\sum_{j=1}^{J} \lambda_j f_j \left(\frac{\tilde{c}_0 + \tilde{c}_1}{2\sqrt{1 + \kappa_1^2\kappa_2^2}} - \omega^* \right) \frac{(1+\beta^2)^2}{2\sqrt{1 + \kappa_1^2\kappa_2^2}} \left[\frac{2\tilde{\theta}_j - (\tilde{a}_0 + \tilde{a}_1)}{\tilde{c}_1 - \tilde{c}_0} \right] = 0. \tag{58}$$

Substituting $\bar{a}$ for $(a_0 + a_1)/2$, applying functions ξ_A and ξ_C, and eliminating constants, (58) simplifies to

$$\sum_{j=1}^{J} \lambda_j f_j \left(\frac{c_0 + c_1}{2} - \omega^* \right) \left[\frac{\theta_j - \bar{a}}{c_1 - c_0} \right] = 0. \tag{59}$$

Thus, the solution $\bar{a}$ of (59) is independent of κ_1 and κ_2.

8.2 Proofs for the Tax Model

Proof of Theorem 7. We first show that condition (30) is necessary. As argued in the text, an individual voter (θ, m)'s expected utility if Candidate j is elected is given by (28); using this and (27), it follows that the citizen prefers candidate j to candidate k if

$$(a_k - a_j)m \geq h(\theta)\left[W(a_k) - W(a_j)\right] \tag{60}$$

If $a_0 = a_1 = t$ then candidate j wins if $f_j(t\bar{m} - b_j) > f_k(t\bar{m} - b_k)$ and there is a tie if $f_j(a_j\bar{m} - b_j) = f_k(a_k\bar{m} - b_k)$.

Now suppose that $a_0 \neq a_1$. Define

$$S(a_0, a_1) = \frac{W(a_1) - W(a_0)}{a_1 - a_0} \tag{61}$$

Then $S(a_0, a_1)$ is the cutoff type who is indifferent between the candidates. First, suppose that in an equilibrium $a_0 < a_1$. Then all types $\tau > S(a_0, a_1)$ vote for Candidate 0. Since the distribution Φ of the median τ_ω is strictly monotone, it follows that Candidate 0 must choose a_0 to maximize the set of all citizens (θ, m) that satisfy (60) for $j = 0$, $k = 1$. This is achieved by selecting a_0 to maximize $S(a_0, a_1)$ for given a_1. Similarly, it follows that a_1 must minimize $S(a_0, a_1)$. In contrast, if $a_0 > a_1$ then a_0 must minimize $S(a_0, a_1)$ while a_1 maximizes $S(a_0, a_1)$.

Thus, a necessary condition for a Nash equilibrium with $0 < a_0 \neq a_1$ is that the first order condition, $\frac{\partial S(a_0,a_1)}{\partial a_j} = 0$ is satisfied for $j = 0, 1$. This, however, is exactly condition (30)

First, note that a_j with $W_j(a_j) < W_k(a_j)$ cannot be an equilibrium since Candidate k could get 100 percent of the votes by deviating to $a_k = a_j$. Further, if $W_j(a_j) = W_k(a_k)$ then the fact that $W_j'(a) \neq W_k'(a)$ for any a implies that the first order condition is violated and hence at least one of the candidates does not maximize his vote share. Thus, it follows that $a_0 < a_1$.

We now take the second order condition:

$$\frac{\partial S(a_0, a_1)}{\partial a_0} = \frac{W_0''(a_0)}{a_1 - a_0} - \frac{2W_0'(a_0)}{(a_1 - a_0)^2} + \frac{2(W_1(a_1) - W_0(a_0))}{(a_1 - a_0)^3}; \tag{62}$$

$$\frac{\partial S(a_0, a_1)}{\partial a_1} = -\frac{W_1''(a_1)}{a_1 - a_0} - \frac{2W_1'(a_1)}{(a_1 - a_0)^2} + \frac{2(W_1(a_1) - W_0(a_0))}{(a_1 - a_0)^3}; \tag{63}$$

For any point choice of a_j that satisfies the first order condition, (30) must hold. Inserting (30) into (62) and (63) implies

$$\frac{\partial S(a_0, a_1)}{\partial a_0} = \frac{W_0'(a_0)}{a_1 - a_0}, \text{ and } \frac{\partial S(a_0, a_1)}{\partial a_1} = -\frac{W_1'(a_1)}{a_1 - a_0} \tag{64}$$

Since $a_0 < a_1$, (64) and $W'' < 0$ implies that $\frac{\partial S(a_0,a_1)}{\partial a_0} < 0$ for any a_0 that satisfies the first order condition. Thus, a_0 maximizes $S(a_0, a_1)$. Similarly, $W'' < 0$ implies $\frac{\partial S(a_0,a_1)}{\partial a_0} > 0$ and hence a_1 minimizes $S(a_0, a_1)$ as required. Thus, (a_0, a_1) is a local equilibrium.

We now show that the equilibrium is global under the assumption of the Theorem, i.e., it should not be optimal for Candidate 0 to deviate to $a_0' > a_1^*$ or for Candidate 1 to deviate to $a_1' < a_0^*$. Suppose that Candidate 1 chooses $a_1' < a_0^*$. To cover fixed costs, $a_1'\bar{m} \geq c_{F,1}$. However, by assumption, $a_0^* \leq \bar{m}c_{F,1}$, which implies $a_1' \geq a_0^*$, a contradiction. Now suppose that Candidate 0 deviates. Then $W_0(a_0') \leq W_0(1) \leq W_1(a_1^*)$. This and $a_0' > a_1^*$ implies that all citizens prefer Candidate 1's a_1^*. Hence, candidate 0's deviation is not optimal.

Let $\tau^* = W_0'(a_0^*) = W_1'(a_1^*)$. Then it is immediate that all voters with $m/h(\theta) > \tau^*$ strictly prefer Candidate 0, while all voters (m, t) with $m/h(\theta) < \tau^*$ strictly prefer Candidate 1. Also, since $W_0'(\bar{a})$ / $W_1'(\bar{a})$ it follows that $a_0^* < \bar{a} < a_1^*$. Hence $W_0(a_0^*) < W_1(a_1^*)$ and strict monotonicity of w implies $f(\bar{m}a_0^* - b_0) < f(\bar{m}a_1^* - b_1)$, i.e., Candidate 1's production exceeds that of Candidate 0.

We now prove uniqueness. We have already shown that $a_0 = a_0^*$ and $a_1 = a_1^*$ is a necessary condition for pure strategy equilibria. Now suppose that (σ_0, σ_1) is a mixed strategy equilibrium where σ_i is Candidate i's probability distribution over tax rates. Suppose that one candidate, say Candidate 1, strictly mixes in equilibrium. If Candidate 0 plays a_0^*, then all types with $\tau \geq \tau^*$ vote for Candidate 0 if Candidate 1 chooses a_1^*, and the set of voters supporting Candidate 0 is strictly larger when $a_1 \neq a_1^*$. Thus, by Assumption 2, Candidate 0's winning probability is strictly larger than in equilibrium (a_0^*, a_1^*). Since Candidate 0's winning probability using σ_0 against σ_1 must be at least as large as that using a_0^* against σ_1, it follows that Candidate 0's winning probability in equilibrium (σ_0, σ_1) is strictly larger than in equilibrium (a_0^*, a_1^*). By an analogous argument, Candidate 1's could always deviate to a_1^*, so that his winning probability in the mixed strategy equilibrium must be at least as large as in the equilibrium (a_0^*, a_1^*). However, this is a contradiction since the sum of the winning probabilities must be 1. ■

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