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## Net Neutrality on the Internet: A Two-sided Market Analysis\*

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### Abstract

We discuss the benefits of net neutrality regulation in the context of a *two-sided market* model in which platforms sell Internet access services to consumers and may set fees to content and applications providers “on the other side” of the Internet. When access is monopolized, we find that generally net neutrality regulation (that imposes zero fees “on the other side” of the market) increases total industry surplus compared to the fully private optimum at which the monopoly platform imposes positive fees on content and applications providers. Similarly, we find that imposing net neutrality in duopoly increases total surplus compared to duopoly competition between platforms that charge positive fees on content providers. We also discuss the incentives of duopolists to collude in setting the fees “on the other side” of the Internet while competing for Internet access customers. Additionally, we discuss how price and non-price discrimination strategies may be used once net neutrality is abolished. Finally, we discuss how the results generalize to other two-sided markets.

Key words: net neutrality, two-sided markets, Internet, monopoly, duopoly, regulation, discrimination

JEL Classification: L1, D4, L12, L13, C63, D42, D43

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## 1. Introduction

The Internet is the primary global network for digital communications. A number of different services are provided on the Internet, including e-mail, browsing (using Internet Explorer, Firefox, Opera, or others), peer-to-peer services, Internet telephony (Voice over Internet Protocol “VOIP”), and many others. A number of different functions/applications run on top of the Internet browser, including information services (Google, Yahoo, MSN), display of images, transmission of video and others. Since the inception of the Internet, information packets are transported on the Internet under “net neutrality,” a regime that does not distinguish in terms of price between bits or packets depending on the services that these bits and packets are used for, and also does not distinguish in price based on the identities of the uploader and downloader.

As video services and digital distribution of content over the Internet are growing, Internet broadband access providers AT&T, Verizon and a number of cable TV companies, have recently demanded additional compensation for carrying valuable digital services. Ed Whitacre, AT&T’s CEO has been recently quoted in BusinessWeek referring to AT&T’s Internet infrastructure: “Now what they would like to do is use my pipes free, but I ain’t going to let them do that because we have spent this capital and we have to have a return on it.”<sup>1</sup> Of course no one is using the Internet for free, since both sides of an Internet transfer pay.<sup>2</sup> AT&T’s president, together with Verizon and cable TV companies, are asking for the abolition of “net neutrality.” AT&T and Verizon and some cable companies would like to abolish the regime of net neutrality and substitute for it a pricing schedule where, besides the basic service for transmission of bits, there will be additional charges by the Internet operator for services applied to the originating party (such as Google, Yahoo, or MSN). The access network operators also have reserved the right to charge differently based on the identity of the provider even for the same type of packets, for example charge more Google than Yahoo for the same transmission.

In abolishing net neutrality, the telephone and cable companies are departing from the “end-to-end principle” that governed the Internet since its inception. Under the end-to-end principle, computers attached to the Internet sending and receiving information packets did not need to know the structure of the network and could just interact end-to-end. Thus there could be innovation “at the edge” of the network without interference from the network operators.<sup>3</sup> The way the Internet has operated

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<sup>1</sup> Interview of Ed Whitacre, BusinessWeek November 7, 2005.

Q. How concerned are you about Internet upstarts like Google (GOOG), MSN, Vonage, and others?  
A. How do you think they’re going to get to customers? Through a broadband pipe. Cable companies have them. We have them. Now what they would like to do is use my pipes free, but I ain’t going to let them do that because we have spent this capital and we have to have a return on it. So there’s going to have to be some mechanism for these people who use these pipes to pay for the portion they’re using. Why should they be allowed to use my pipes?  
The Internet can’t be free in that sense, because we and the cable companies have made an investment and for a Google or Yahoo! (YHOO) or Vonage or anybody to expect to use these pipes [for] free is nuts!

<sup>2</sup> See Economides (2008).

<sup>3</sup> See Cerf (2006a, b) for a detailed explanation of this argument.

this far is a radical departure from the operating principles of traditional digital electronic networks that predated it, such as Compuserve, Prodigy, AOL, AT&T Mail, MCI Mail, and others. These older electronic networks were centralized with very little functionality allowed at the edge of the network.

From an economics point of view, the departure from net neutrality regulation will have six consequences. First, it will introduce on the Internet *two-sided pricing* where a transmission company controlling some part of the Internet (here last mile access) will charge a fee to content or application firms “on other side” of the network which typically did not have a contractual relationship with it. Second, it will introduce prioritization which may enhance the arrival time of information packets that originate from paying content and application firms “on the other side,” and may degrade the arrival time of information packets that originate from non-paying firms. In fact, the present plans of access providers are to create a “special lane” for the information packets of the paying firms while restricting the lane of the non-payers without expanding total capacity. By manipulating the size of the paying firms’ lane, the access provider can guarantee a difference in the arrival rates of packets originating from paying and non-paying firms, even if the actual improvement in arrival time for paying firms’ packets is not improved over net neutrality. Third, if the access providers choose to engage in identity-based discrimination, they can determine which one of the firms in an industry sector on the other side of the network, say in search, will get priority and therefore win. This can easily be done by announcing that prioritization will be offered to only one of the search firms, for example the one that bids the highest. Thus, the determination of the winner in search and other markets “on the other side” will be in hands of the access providers. This can create very significant distortions since the surplus “on the other side” of the Internet is a large multiple of the combined telecom and cable TV revenue from residential Internet access.<sup>4</sup> Fourth, new firms with small capitalization (or those innovative firms that have not yet achieved significant penetration and revenues) will very likely not be the winners of the prioritization auction. This is likely to reduce innovation. Fifth, the access networks can favor their own content and applications rather than those of independent firms. Finally, since the Internet consists of a series of interconnected networks, any one of these networks, and not just the final consumer access ones, can, in principle, ask content and application providers for a fee. This can result in multiple fees charged on a single transmission and lead to a significant reduction of trade on the Internet,<sup>5</sup> similar to the reduction of trade in medieval times when the weakening of the state power of the Roman Empire allowed multiple fees to be collected by many independent city powers along a trading route.

In this paper, we deal primarily with the first issue in the previous paragraph

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<sup>4</sup> See Economides (2008) for a more detailed discussion of this issue.

<sup>5</sup> The imposition of multiple margins by independent producers of complementary goods was first discussed by Cournot (1838). In Cournot’s setup, there are two complementary components that can be combined in fixed proportions to produce a composite good. In the setup, each component is produced by a single firm, *i.e.*, we have two independent monopolists. In a second setup, both components are produced by the same firm (integrated monopoly). He showed that the price of the composite good will be higher with independent monopolists than with integrated monopoly. This is because each of the independent monopolists does not take fully into account the effect of his price increase on the market. This has been called “double marginalization.”

by formally building a model of a two-sided market. In terms of potential welfare reduction as a result of the departure from net neutrality, we model the first effect above which results has the least reduction in total surplus compared with net neutrality. The second issue above, manipulation of the prioritized lanes and reduction of the “standard” lane is likely to result in further degradation of consumers’ surplus. The third and fourth issues above are also likely to reduce surplus as the winners “on the other side” will be determined by the access networks and not by their innovative products or services. Similarly, the fifth issue (network favoring own content) is likely to distort competition and reduce surplus. The same is true for the sixth issue which can result in multiple fees on a transmission. Even though we make the best possible case for total surplus to increase when departing from net neutrality (by not focusing on factors two to six that are likely to reduce surplus), we find that typically total surplus decreases, both in monopoly and duopoly when we depart from net neutrality.

We explicitly model the Internet broadband market as a two-sided network consisting of broadband users on one side and content and applications providers on the other side. Prices imposed on both sides have direct implications on the number of broadband consumers as well as on the number of active providers of content and applications. We discuss the incentives of a monopoly broadband Internet access network, starting from net neutrality that is, starting from a zero fee to content providers to initiate a positive fee to the content and applications side of the market besides the price it charges to users/subscribers. We show that while a monopoly broadband Internet access network has an incentive to charge a positive fee to content providers, typically an increase of such a fee above zero decreases total surplus. It is in fact total surplus maximizing for the platform to subsidize content providers. This is not surprising given the two-sided nature of the Internet market. We further show that generally net neutrality increases total surplus compared to duopoly competition between platforms that would impose positive fees on content providers.

Despite a considerable literature discussing the rights and legal issues of net neutrality and its abolition, the literature on economic analysis of this issue is thin. In relation to the second issue above, the prioritization of information packets, two papers have emerged. In a paper relating to the establishment of multiple “lanes” or quality options for application providers, Hermalin and Katz (2007) analyze a model where net neutrality is equivalent to a single product (quality) requirement. The effect of restricting the product-line is that low valuation application providers get excluded, medium valuation providers purchase higher and more efficient qualities and high valuation application providers purchase lower valuation and less efficient qualities. The impact on total surplus is ambiguous, but the set of applications available is reduced.<sup>6</sup> Focusing on congestion, Cheng, Bandyopadhyay and Guo (2007) model two content providers who can avoid congestion by paying ISPs for preferential access.<sup>7</sup> They find that abolishing net neutrality will benefit ISPs and hurt content providers. Depending on parameter values, consumers are either unaffected or better off. Social welfare increases when net neutrality is abandoned and one content provider pays for access, but remains unchanged when both content providers pay.

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<sup>6</sup> Hermalin and Katz (2007) do not address the issue of the reduction of the “standard” lane for Internet access that is likely to reduce consumers’ welfare.

<sup>7</sup> Also see Jamison and Hauge (2007).

The reason consumer surplus may increase is that it is always the more profitable content provider that pays for access, and hence gets preferential treatment. This benefits consumers of the more profitable content provider because congestion is reduced. Consumers of the less profitable content provider that does not pay for preferential access lose, since congestion costs increase. They also find that incentives for the broadband provider to expand its capacity are higher under net neutrality regulation since more capacity leads to less congestion. Because congestion decreases, Internet services become more valuable (to the benefit of ISPs). If net neutrality is abolished, their model predicts reduced investment incentives due to congestion becoming less of a problem.<sup>8</sup>

In contrast to the above literature, we focus on the issue of two-sided pricing made possible by the abolishment of net neutrality regulation. Hence, our paper is closely related to the literature on two-sided markets, mainly Armstrong (2006).<sup>9</sup> We have structured our paper in the following way. We first present and evaluate the impact of net neutrality regulation in a monopoly model in section 2. In section 3, we extend the monopoly model to a duopoly setting with multi-homing content providers. We conclude in section 4.

## 2. **Platform Monopoly**

We start with a platform monopoly model of a two-sided market. A platform (say a telephone company, such as AT&T) sells broadband Internet access to consumers at a subscription price  $p$  and possibly collects a fee  $s$  from each content or application provider to allow the content to reach the consumer. We assume that the platform monopolist (and later in the paper, duopolists) only offer linear fee contracts, *i.e.*, they do not offer quantity discounts and do not offer take-it-or-leave-it contracts with lump sum fees. We further abstract from the full complexity of the Internet, which consists of many interconnected networks, and assume that the

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<sup>8</sup> Additionally, Chen and Nalebuff (2007) analyze competition between complements and briefly touch upon the issue of net neutrality. Some services that are offered by an ISP may also be offered over the Internet (such as Vonage or Skype). There is a concern that the ISP would like to disrupt the quality of the services of its competitors to further its own product. However, the authors show that this would not be profit maximizing in their model since a monopolist ISP benefits from valuable complements such as VOIP services (a higher price for internet access could be charged instead of trying to force consumers to its own VOIP service). Hogendorn (2007) analyze the differences between open access and net neutrality and emphasizes that they are different policies that may have different implications. Hogendorn interprets net neutrality in a slightly different way than what most of the literature has. Open access refers to allowing intermediaries access to conduits (so intermediaries such as Yahoo can access conduits such as AT&T at a nondiscriminatory price), while net neutrality is interpreted to mean that content providers have unrestricted access to intermediaries (so Yahoo can not restrict which content providers can be reached through its portal). Under net neutrality, a smaller number of intermediaries enter the market due to decreased profits. Open access on the other hand increases entry of intermediaries since they now have free access to conduits. In general, the author finds that open access is not a substitute for net neutrality regulation. Finally, Economides (2008) discuss several possible price discrimination strategies that may become available if network neutrality is abolished. He presents a brief model, showing that total surplus may be lower when the platform imposes a positive fee on an application developed for it. This is because the fee raises the marginal cost of the application and hence also its price.

<sup>9</sup> See also Caillaud and Jullien (2003), Hagiu (2006) and Rochet and Tirole (2003, 2006).

networks that lie between the access provider and content provider are passive.<sup>10</sup> Finally, we assume that the cost of providing the platform service is  $c$  per consumer.

## 2.1 Consumers

Consumers are interested in accessing the Internet to reach search engines (e.g. Google), online stores (e.g. Amazon), online auctions (e.g. eBay) and online video, audio, still pictures, and other content. Consumers are differentiated in their preferences for Internet access. A consumer  $i$ 's location (type)  $x_i$  indexes his/her preference for the Internet, so that consumers with a lower index place a higher value on the service. Consumers pay a transportation cost equal to  $t$  per unit of distance "traveled."<sup>11</sup> We assume that consumers are uniformly distributed on the interval  $x \in [0,1]$  with the platform located at  $x = 0$ . Consumer  $i$ 's utility is specified as

$$u_i = v + bn_{cp} - tx_i - p \quad (1)$$

where  $v > c$  is an intrinsic value a consumer receives from connecting to the Internet irrespective of the amount of content,<sup>12</sup>  $b$  is the marginal value that a consumer places on an additional content provider on the Internet, and  $n_{cp}$  is the number of content providers that are active.

## 2.2 Content Providers

Content providers rely on advertising revenue per consumer,  $a$ , to generate revenue. We assume that content providers are uniformly distributed on the unit interval and have a unit mass. We make the simplifying assumption that content providers are independent monopolists, each in its own market, and therefore do not compete with each other. Each content provider then earns  $an_c$ , where  $n_c$  is the number of consumers paying the platform for access to content providers. Thus,  $a$  is the value to a content provider of an additional consumer connected to the Internet.

Content providers are heterogeneous in terms of fixed costs of coming up with a business idea and setting up their business. A content provider indexed by  $j$  faces a fixed cost of  $fy_j$ , where  $y_j$  is the index the content provider's location on the unit interval.<sup>13</sup> Marginal costs for serving advertisements to consumers are taken to be zero. Each content provider may have to pay the platform a lump sum fee equal to  $s$

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<sup>10</sup> As noted earlier, if the in-between networks also attempted to charge a fee to content providers, there is the possibility of high prices because of double or multiple marginalization.

<sup>11</sup> Assume that the market is not covered and demand is differentiable.

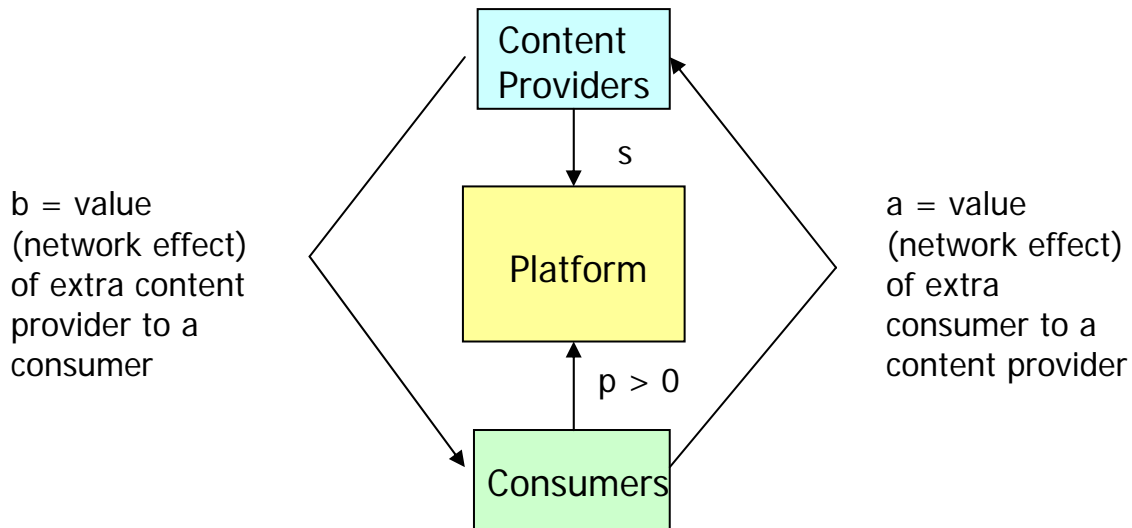
<sup>12</sup> Such benefit may arise from Internet-enabled services that do not crucially depend on the number of other Internet subscribers or availability of content. An example may be television services bundled with Internet access.

<sup>13</sup> We assume that the "market is not covered" in the sense that some content providers will always have fixed costs so high that they decide not to enter the market. Further, we assume that demand for access to consumers is differentiable.

to gain access to users. This fee is assumed to be the same for all content providers and it is set by the platform. Thus, a content provider  $j$ 's profit is<sup>14</sup>

$$\pi_j = an_c - s - fy_j \quad (2)$$

As noted above, net neutrality regulation equals the case where  $s$  is equal to zero. Figure 1 shows the interaction between consumers and content providers through the platform.



**Figure 1:** Interaction of consumers with content providers and vice versa through the platform

### 2.3 Demand

In this two-sided market, demand for content depends on the expected amount of content provided since more consumers will connect to the network if more expected content is available. Additionally, the provision of content depends on expected the number of consumers. That is, when the expected number of consumers is  $n_c^e$  and the expected number of content providers is  $n_{cp}^e$ , the marginal consumer,  $x_i$ , indifferent between subscribing to the Internet and staying out is

$$x_i = n_c = \frac{v + bn_{cp}^e - p}{t} \quad (3)$$

while the marginal content firm,  $y_i$ , indifferent between being active and staying out of the market is

<sup>14</sup> Alternatively, one can specify the fee to the platform to be proportional to the number of platform customers,  $\pi_j = an_c - sn_c - fy_j$ . The qualitative results of our main specification go through in this alternative specification.



$$y_i = n_{cp} = \frac{an_c^e - s}{f}. \quad (4)$$

At fulfilled expectations equilibrium, each side of the market anticipates correctly its influence on the demand of the other side, and therefore  $n_c^e = n_c$  and  $n_{cp}^e = n_{cp}$ . Thus, at fulfilled expectations, the number of consumers and active content providers is given by the solution to the simultaneous equation system (3) and (4) at fulfilled expectations which is

$$n_c(p, s) = \frac{f(v-p) - bs}{ft - ab} \quad \text{and} \quad n_{cp}(p, s) = \frac{a(v-p) - ts}{ft - ab}.^{15} \quad (5, 6)$$

## 2.4 Monopoly Platform Optimum

Consider first the monopoly platform private optimum under which platform is free to set both subscription price  $p$  and fee to content providers  $s$ . The platform faces the problem of choosing  $p$  and  $s$  to maximize

$$\Pi(p, s) = (p - c)n_c(p, s) + sn_{cp}(p, s). \quad (7)$$

Because the two markets provide complementary products, the monopolist finds an inverse relationship between  $p$  and  $s$ ; that is, maximizing with respect to  $p$  results in a smaller  $p$  when  $s$  is larger, and maximizing with respect to  $s$  results in a smaller  $s$  when  $p$  is larger. Specifically, the optimal  $p$  for the monopolist given  $s$ , defined by  $\frac{\partial \Pi}{\partial p} = 0$ , is given by

$$p(s) = \frac{f(v+c) - (a+b)s}{2f}, \quad (8)$$

and the optimal  $s$  for the monopolist given  $p$ , defined by  $\frac{\partial \Pi}{\partial s} = 0$  is

$$s(p) = \frac{av + bc - (a+b)p}{2t}. \quad (9)$$

Solving the above two equations simultaneously gives the consumers' subscription price and the fee charged to the content providers that maximize the platform's profits:<sup>16,17</sup>

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<sup>15</sup> We check later to ensure that under our assumptions  $n_c \in [0, 1]$  and  $n_{cp} \in [0, 1]$  at equilibrium.

<sup>16</sup> To satisfy the second order conditions,  $-\frac{2f}{(ft - ab)} < 0$  and  $\frac{4ft - (a+b)^2}{(ft - ab)^2} > 0$ , we assume

$4ft - (a+b)^2 > 0$ , which implies  $ft - ab > 0$ .

<sup>17</sup> Since  $p^M - c = \frac{(v-c)(2ft - ab - a^2)}{4ft - (a+b)^2} > 0$ , the price consumers pay,  $p^M$ , is above marginal cost if

$$p^M = \frac{(2ft - ab)(v + c) - b^2c - a^2v}{4ft - (a + b)^2} \quad \text{and} \quad s^M = \frac{(a - b)f(v - c)}{4ft - (a + b)^2}. \quad (10, 11)$$

The superscript “ $M$ ” indicates the fully private optimum where both  $p$  and  $s$  are chosen by the monopoly platform. The participation levels are:

$$n_c^M = \frac{2f(v - c)}{4ft - (a + b)^2} \quad \text{and} \quad n_{cp}^M = \frac{(a + b)(v - c)}{4ft - (a + b)^2},^{18} \quad (12)$$

and the monopoly platform’s profits are

$$\Pi^M = \frac{f(v - c)^2}{4ft - (a + b)^2}. \quad (13)$$

The access platform benefits from additional content (since additional content increases the willingness to pay of its subscribers) but does not receive the full benefit of the content increase. Therefore the platform cannot fully internalize the network effects of content and charges a positive price to content providers. The platform service provider sets a positive fee to content providers for accessing users if  $a > b$ . This means that, if content providers value additional consumers higher than consumers value additional content providers, the platform will charge content providers a positive price for accessing consumers. One may argue that consumers have become more valuable to content providers lately, so that there are higher incentives for a platform, such as AT&T, to seek ways to be able to charge content providers for access to users. In some other networks, for example, in the network of a game platform/console (such as the Sony PlayStation platform) and games (software), the platform similarly collects a fee from independent game developers.

It is worth noting that in some two-sided markets, a firm on the other side of the market may value an additional platform consumer less than a platform consumer values an additional firm on the other side of the market, that is,  $a < b$ . For example, a Windows application (not sold by Microsoft) may value less an additional Windows purchaser than this consumer values the existence of this additional application. When this is true, the platform will subsidize the firms on other side of the market to increase their number and more fully internalize the externality. Thus, operating systems companies typically subsidize developers of applications by embedding in the operating systems subroutines that are valuable to applications developers, but not directly valuable to users.<sup>19,20</sup> Another example is the interaction between a credit

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$2ft - a(a + b) > 0$  and above 0 if  $2ft(v + c) - (a + b)(av + bc) > 0$ . Although a negative price might not be implementable, the platform may tie other products with the offer for Internet access and thereby in effect achieve a negative price. See Amelio and Jullien (2007).

<sup>18</sup> To ensure that the market is not covered on either side, we impose

$4ft - (a + b)^2 - (a + b)(v - c) > 0$  and  $4ft - (a + b)^2 - 2f(v - c) > 0$ , *i.e.*, that the differentiation parameters  $f$  and  $t$ , are sufficiently high.

<sup>19</sup> Also see Economides and Katsamakas (2006a, b) for a deeper discussion of this issue and a contrast with practices in open source operating systems.

card platform network (such as VISA), a credit card issuing bank, and consumers. Some consumers who pay their monthly balances in full are effectively subsidized by the issuing banks by receiving airline miles and other perks while the issuers collect fees from the merchants. In this case, the value of an extra consumer to the issuing bank exceeds the value of an extra issuing bank to a consumer, *i.e.*,  $b > a$ .<sup>21</sup> Thus, in summary, we have shown:

***Proposition 1:*** *An unconstrained profit-maximizing platform charges a positive fee to the other side of the market if and only if content providers value additional consumers higher than consumers value additional content providers.*

## 2.5 Monopoly Under Network Neutrality Regulation

Now consider the optimal choices of the monopoly platform provider under net neutrality regulation, that is, when by regulation  $s = 0$ . The platform's objective is now to maximize

$$\Pi^{NN} = (p - c)n_c, \quad (14)$$

which gives the equilibrium price  $p^{NN} = \frac{v+c}{2}$ .<sup>22</sup> Equilibrium participation levels are

$$n_c^{NN} = \frac{f(v-c)}{2(ft-ab)} \quad \text{and} \quad n_{cp}^{NN} = \frac{a(v-c)}{2(ft-ab)}, \quad (15, 16)$$

and platform profits are

$$\Pi^{NN} = \frac{f(v-c)^2}{4(ft-ab)}. \quad (17)$$

## 2.6 Social Optimum

In this subsection, we solve for prices  $p$  and  $s$  that maximize total surplus defined as

$$TS(p, s) = \Pi(p, s) + CS_c(p, s) + \Pi_{cp}(p, s), \quad (18)$$

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<sup>20</sup> Also note that in some two sided markets the organizing networks have arbitrarily set the fee between different network firms without allowing the market to set a positive or negative fee across them according to specific circumstances. This is the case in the Visa and MasterCard networks of acquiring and issuing banks. These networks have set a fixed percentage fee between an acquiring and an issuing bank on the dollar value of transactions without regard to the specific market position of each pair of such banks. See Economides (2007) and Rochet and Tirole (2003).

<sup>21</sup> In this case, we place the consumers at the top of Figure 1 and the credit card issuing banks at the bottom.

<sup>22</sup> The second order condition  $-\frac{2f}{ft-ab} < 0$  is satisfied if  $ft - ab > 0$ . In addition, we need to impose

that  $2(ft - ab) - f(v - c) > 0$  and  $2(ft - ab) - a(v - c) > 0$  to ensure that the markets are not covered.

where  $\Pi(p, s)$  are platform profits,

$$CS_c(p, s) = \int_0^{n_c(p, s)} (v + bn_{cp}(p, s) - tx - p) dx \quad (19)$$

is consumer surplus and

$$\Pi_{cp} = \int_0^{n_{cp}(p, s)} (an_c(p, s) - fy - s) dy, \quad (20)$$

is the sum of content providers' profits.

Maximizing total surplus,<sup>23</sup> a planner chooses

$$p^* = \frac{ftc - b(a+b)c - a(a+b)v}{ft - (a+b)^2} < c \quad \text{and} \quad s^* = -\frac{bf(v-c)}{ft - (a+b)^2} < 0. \quad (21)$$

This results in maximized total surplus

$$TS(p^*, s^*) = \frac{f(v-c)^2}{2(ft - (a+b)^2)}. \quad (22)$$

***Proposition 2a:*** *A total surplus maximizing planner/regulator in the two-sided market with network effects chooses below-cost pricing in both markets.*

***Proposition 2b:*** *A total surplus maximizing planner/regulator in a two-sided market with network effects constrained to marginal cost pricing in the subscription market chooses below-cost pricing in the content market.*<sup>26</sup>

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<sup>23</sup> The second order conditions,  $-\frac{f(ft - a^2 - 2ab)}{(ft - ab)^2} < 0$ ,  $-\frac{f(ft - b^2 - 2ab)}{(ft - ab)^2} < 0$  and

$\frac{ft - (a+b)^2}{(ft - ab)^2} > 0$ , are satisfied if  $ft > (a+b)^2$ , which we assume to be the case. Further, we impose

$ft - f(v-c) - (a+b)^2 > 0$  and  $ft - (a+b)(v-c) - (a+b)^2 > 0$  to ensure that the market is not covered at optimum.

<sup>24</sup> These inequalities are implied by  $v > c$ .

<sup>25</sup> In our case, with  $a > b$ , clearly  $s^* < 0 < s^M$ . But, even in industries where  $a < b$  and the platform monopolist subsidizes the other side of the market we have  $s^* < s^M < 0$ , that is, the monopolist subsidizes the other side of the market less than the regulator would because the monopolist does not fully internalize the network externality from the availability of more complementary goods on the other side of the market. In general, the unregulated monopolist will impose a higher fee to the other side of the market than the regulated monopolists,  $s^* < s^M$ , when  $ft > a(a+b)^2 / (a+3b)$ , that is, when there is sufficiently high differentiation among the consumers and the content firms.

<sup>26</sup> Choosing  $s$  to maximize  $TS(c, s)$  gives  $s^{**} = -\frac{b(a^2 + ft)(v-c)}{t(ft - 2ab - b^2)} < 0$  since  $ft > (a+b)^2$ . The

**Proposition 2c:** *A total surplus maximizing planner/regulator in a two-sided market constrained to marginal cost pricing in the content market chooses below-cost pricing in the subscription market.*<sup>27</sup>

Because of network effects arising from the complementarity of the content and Internet subscription market, to internalize the externality of content on subscribers and of subscribers to content, the planner sets a negative fee to content providers  $s^* < 0$  and a subscription price below its marginal cost  $p^* < c$ . The fact that the planner subsidizes content providers suggests that net neutrality (where  $s$  is set to zero) may also result in higher surplus than the private optimum. The fact that  $s^*$  is negative does not prove net neutrality will achieve a higher surplus than the private optimum because  $s^*$  resulted from the *unconstrained* maximization of total surplus for a planner. To see if net neutrality is better in terms of total surplus than the private optimum, we need to take into consideration the fact that the monopolist is maximizing profits by choosing price  $p^M$ , while  $s^*$  was calculated based on the planner choosing  $p^*$ . Thus, we need to define total surplus under the maintained condition that, whatever the level of  $s$ , the monopolist chooses price  $p$  to maximize its profits. The planner then optimizes this constrained total surplus function, and considers if setting  $s = 0$  (that is, imposing net neutrality) improves over the fully private solution. This is done in the next section.

## 2.7 Welfare Implications of Imposing Net Neutrality

In this subsection we examine the welfare implications of imposing net

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maximized surplus is  $TS(c, s^{**}) = \frac{(ft + a^2)(c - v)^2}{2t(ft - 2ab - b^2)}$ . The sufficient condition for a maximum is  $-\frac{t(ft - 2ab - b^2)}{(ft - ab)^2} < 0$ .

<sup>27</sup> Choosing  $p$  to maximize  $TS(p, 0)$  gives  $p^{**} = \frac{(ft - ab)c - a^2v - abv}{ft - 2ab - a^2} < c$  since  $ft > (a + b)^2$ . The

maximized surplus is  $TS(p^{**}, 0) = \frac{ft(c - v)^2}{2(ft - a(a + ab))}$ . The sufficient condition for a maximum is

$-\frac{f(ft - 2ab - a^2)}{(ft - ab)^2} < 0$ . Comparing  $TS(c, s^{**})$  with  $TS(p^{**}, 0)$  we have that

$TS(c, s^{**}) - TS(p^{**}, 0) = -\frac{(a^4 + 2a^3b - b^2ft)(v - c)^2}{2t(a^2 + 2ab - ft)(2ab + b^2 - ft)} > 0$  if  $ft > \frac{a^3}{b^2}(a + 2b)$ . The

percentage gains in total surplus in our model when going from marginal cost pricing on one side of the market and optimality on the other to full optimality are  $\frac{TS(p^*, s^*) - TS(c, s^{**})}{TS(p^*, s^*)} = \frac{a^2(a + b)^2}{ft(ft - 2ab - b^2)} > 0$

and  $\frac{TS(p^*, s^*) - TS(p^{**}, 0)}{TS(p^*, s^*)} = \frac{b^2}{ft - 2ab - a^2} > 0$ . The percentage gain in total surplus of optimality over

net neutrality is  $\frac{TS(p^*, s^*) - TS(p^{NN}, 0)}{TS(p^*, s^*)} = \frac{a^4 - 2ab^3 + ft(3b^2 + ft) + a^2(b^2 + 2ft)}{4(ft - ab)^2}$ .

neutrality in two ways. First, starting with a regime of net neutrality, we examine the incentive of the platform to set a small positive fee to the content providers and the effects of such an action to total industry surplus. To assess these, we examine the incremental change in platform profits and total industry surplus as the fee charged to content providers increases from zero to a small positive value. Of course, this is done under the maintained assumption that the monopoly platform chooses the subscription price  $p(s)$  to maximize its profits. Second, we examine the changes in welfare that occur when moving from a privately optimal  $p$ , given  $s = 0$ , to the full private optimum ( $p^M$  and  $s^M$ ).

Thus, we first define total surplus under the restriction that the monopolist, given  $s$ , will set his optimal price for subscription  $p(s)$ , as defined in equation (6a), that is, we define the constrained total surplus function  $TS(p(s), s)$ . We then evaluate the derivatives of the monopolist's profits and total surplus  $TS(p(s), s)$  with respect to the fee  $s$  at 0.

The monopolist's incentive to increase the fee to content providers from zero to a small positive value is

$$\left. \frac{d\Pi}{ds} \right|_{\frac{\partial \Pi}{\partial p}=0} \Big|_{s=0} = \left. \frac{d\Pi(p(s), s)}{ds} \right|_{s=0} = \frac{(a-b)(v-c)}{2(ft-ab)}, \quad (23)$$

which is positive when  $a > b$ . The incentive of a planner to increase the fee to the content providers from zero to a small positive value taking into account that the monopolist chooses subscription price  $p(s)$  is

$$\left. \frac{dTS}{ds} \right|_{\frac{\partial \Pi}{\partial p}=0} \Big|_{s=0} = \left. \frac{dTS(p(s), s)}{ds} \right|_{s=0} = \frac{(v-c)(a(a^2 - ab + 2b^2) + (a-3b)ft)}{4(ft-ab)^2}, \quad (24)$$

which is negative provided that  $a < 3b$  and  $ft$  is sufficiently large, *i.e.*, if the consumers and content providers are sufficiently differentiated. We also require concavity of  $TS(p(s), s)$ , for which it is sufficient that  $a < 2b$ .<sup>28</sup> Thus, for  $b < a < 2b$  and  $ft$  sufficiently large, starting from a zero fee under net neutrality, the incentives of the platform and society go in opposite directions: the monopolist's incentive is for the platform to charge a positive fee to content providers, while the social incentive is for the platform to subsidize content providers. It follows that net neutrality ( $s = 0$ ) is better for society than the profit maximizing solution of the monopoly platform, which implies a positive fee to content providers ( $s^M > 0$ ).

***Proposition 3a:*** *Starting from the net neutrality regime of a zero fee to*

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<sup>28</sup> Note that  $\frac{d^2 TS(p(s), s)}{ds^2} = \frac{a(a-2b)(a+b)^2 - (a^2 - 6ab - 3b^2)ft - 4(ft)^2}{4f(ft-ab)^2} < 0$  provided that  $a < 2b$  and  $ft$  sufficiently large.

content providers, a platform monopolist choosing optimally his subscription price would like to marginally increase the fee to content providers above zero.

**Proposition 3b:** *Starting from the net neutrality regime of a zero fee to content providers and facing a platform monopolist that chooses the subscription price, a total surplus maximizing planner/regulator will choose to marginally decrease the fee to content providers below zero.*

We have shown that a regulator/planner setting a fee  $s$  to content providers (expecting the platform monopolist to set his profit-maximizing subscription price  $p(s)$ ) will choose a negative fee  $s$ , *i.e.*, will subsidize the content providers. We now calculate this fee,  $s^{***}$  and the subscription price  $p^{***} = p(s^{***})$  the monopolist chooses given this fee. Maximizing the constrained total surplus function  $TS(p(s), s)$  with respect to  $s$  we find

$$s^{***} = -\frac{f(a(a^2 - ab + 2b^2) + (a - 3b)ft)(c - v)}{(a^2 - 6ab - 3b^2)ft + 4f^2t^2 - a(a - 2b)(a + b)^2} \quad (25)$$

and the corresponding monopolist's subscription price

$$p^{***} = \frac{a^2(cft + b^2(2c + v)) + a(2bft(2c + v) - 2cb^3) - a^4v - ft(3b^2c - 2ft(c + v))}{(a^2 - 6ab - 3b^2)ft + 4f^2t^2 - a(a - 2b)(a + b)^2}. \quad (26)$$

The fee  $s^{***}$  to content providers is negative provided that  $a < 3b$  and  $ft$  is sufficiently large, which we have assumed earlier.<sup>29</sup> Given that the  $s^{***}$  is negative, the platform profits from consumers cover the subsidy to content providers if:

$$(ft)^2(3a^2 - 10ab - 9b^2 + 4ft) - a(a + b)(a + b)(a^2 - 3ab + 4b^2) + (a - 3b)(a + 4b)ft > 0 \quad (27)$$

which is true for sufficiently large  $ft$ .<sup>30</sup> Thus, the platform's overall profits are positive even when, following the regulator's orders, the platform provides subsidy  $-s^{***}$  to the other side of the market.

**Proposition 4:** *A total surplus maximizing planner/regulator, facing a platform monopolist that chooses subscription price, will choose a below-cost fee to content providers, *i.e.*, will subsidize content providers. Even paying the below-cost*

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<sup>29</sup> For  $s^{***} < 0$  it is sufficient to have  $a(a(a - b) + 2b^2) + ft(a - 3b) < 0$  which is implied by  $a < 3b$  and  $ft$  sufficiently large.

<sup>30</sup> The condition can be reformulated as

$4(ft)^3 + (3a^2 - 10ab - 9b^2)(ft)^2 - a(a + b)(a - 3b)(a + 4b)ft - a(a + b)a(a + b)(a^2 - 3ab + 4b^2) > 0$  or  $A(ft)^3 + B(ft)^2 - C(ft) - D > 0$  with  $A = 4 > 0$ . Hence, for  $ft$  large enough the expression is positive.

fee, the platform makes positive profits.

We can also explicitly compare prices, equilibrium participation levels, and surplus distribution across a setting where the platform is free to set both  $s$  and  $p$ , and a setting of net neutrality regulation where  $s$  is constrained to equal zero. Starting with net neutrality, consider the impact of removing net neutrality regulation *i.e.*, compare results from above with the results from the privately optimal solution. In what follows, we also assume that  $a > b$ . The difference in equilibrium price to consumers and fee to content providers as we go away from net neutrality are

$$\Delta p = p^M - p^{NN} = -\frac{(a-b)(a+b)(v-c)}{2(4ft - (a+b)^2)} < 0, \quad (28)$$

$$\Delta s = s^M - s^{NN} = s^M = \frac{(a-b)f(v-c)}{4ft - (a+b)^2} > 0, \quad (29)$$

while the difference in equilibrium participation levels are

$$\Delta n_c = n_c^M - n_c^{NN} = f(v-c)\left(\frac{2}{4ft - (a+b)^2} - \frac{1}{2(ft-ab)}\right) > 0, \quad (30)$$

$$\Delta n_{cp} = n_{cp}^M - n_{cp}^{NN} = (v-c)\left(\frac{a+b}{4ft - (a+b)^2} - \frac{a}{2(ft-ab)}\right) < 0. \quad (31)$$

The platform's equilibrium profits are of course higher when it is unconstrained:

$$\Delta \Pi = \Pi^M - \Pi^{NN} = f(v-c)^2\left(\frac{1}{4ft - (a+b)^2} - \frac{1}{4(ft-ab)}\right) > 0. \quad (32)$$

Total consumer surplus and content provider profits are under private optimum

$$CS_c^M = \frac{2f^2t(v-c)^2}{(4ft - (a+b)^2)^2} \quad \text{and} \quad \Pi_{cp}^M = \frac{(a+b)^2 f(v-c)^2}{2(4ft - (a+b)^2)^2} \quad (33, 34)$$

and under net neutrality regulation

$$CS_c^{NN} = \frac{f^2t(v-c)^2}{8(ab-ft)^2} \quad \text{and} \quad \Pi_{cp}^{NN} = \frac{a^2 f(v-c)^2}{8(ab-ft)^2}. \quad (35, 36)$$

The change in consumer surplus when net neutrality regulation is removed is then<sup>32</sup>

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<sup>31</sup> This is implied by  $2ft - a(a+b) > 0$  which is implied by  $ft > (a+b)^2$  that was assumed for the second order conditions of the unconstrained total surplus optimization.

<sup>32</sup> Note that  $\frac{16}{(4ft - (a+b)^2)^2} - \frac{1}{(ft-ab)^2} = \frac{(a-b)^2(4(ft-ab) + (4ft - (a+b)^2))}{(4ft - (a+b)^2)^2(ft-ab)^2} > 0$  since  $4ft - (a+b)^2 > 0$ .



$$\Delta CS_c = CS_c^M - CS_c^{NN} = \frac{1}{8} f^2 t (v-c)^2 \left( \frac{16}{(4ft - (a+b)^2)^2} - \frac{1}{(ft - ab)^2} \right) > 0 \quad (37)$$

and the change in content provider profits

$$\Delta \Pi_{cp} = \Pi_{cp}^M - \Pi_{cp}^{NN} = \frac{1}{8} f (v-c)^2 \left( \frac{4(a+b)^2}{(4ft - (a+b)^2)^2} - \frac{a^2}{(ft - ab)^2} \right) < 0. \quad (38)$$

We now calculate the change in total surplus that occurs when net neutrality regulation is removed. Total surplus under the private optimum is

$$TS^M = \frac{f(12ft - (a+b)^2)(v-c)^2}{2(4ft - (a+b)^2)^2} \quad (39)$$

and under net neutrality regulation is

$$TS^{NN} = \frac{f(v-c)^2(a^2 - 2ab + 3ft)}{8(ft - ab)^2}. \quad (40)$$

The change in total surplus is then

$$\Delta TS = TS^M - TS^{NN} = \frac{f(v-c)^2}{8} \left( \frac{4(12ft - (a+b)^2)}{(4ft - (a+b)^2)^2} - \frac{(a^2 - 2ab + 3ft)}{(ft - ab)^2} \right) < 0, \quad (41)$$

which is negative provided that  $a < 5b$  and  $ft$  is sufficiently large.<sup>34</sup> Thus, removing net neutrality regulation decreases social welfare.

***Proposition 5:*** *Comparing net neutrality and the monopolist platform's choice, we find that total surplus is higher in net neutrality, the content sector has higher profits at net neutrality, and the platform and consumers are better off at monopoly.*

It is interesting that consumers' surplus is higher at monopoly while total surplus is higher at net neutrality. At monopoly, the consumers benefit from a lower subscription price since the monopolist has incentives to attract more consumers to generate extra revenue from charging content providers. Although charging content providers leads to lower content provision, the direct effects of a lower subscription price dominates. In contrast, total surplus takes into account the profits of content providers which are significantly higher under net neutrality. Thus, despite

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<sup>33</sup> This is implied by  $2ft - a(a+b) > 0$  which is implied by  $ft > (a+b)^2$  that was assumed for the second order conditions of the unconstrained total surplus optimization.

<sup>34</sup> Under assumptions  $a > b$  and  $ft - (a+b)^2 > 0$ , for the condition  $\Delta TS < 0$  to hold, it is sufficient that  $4(a-5b)(ft)^2 + b(a^2 + 23ab + 3b^2)ft - a(a+b)^2(a^2 + ab + 2b^2) < 0$ , which holds for large enough  $ft$  and  $a < 5b$ .

consumers' surplus being lower at net neutrality, total surplus is higher.

## 2.8 Summary of Results for Platform Monopolist

We have showed that for a wide range of parameter values, the private and social incentives to set a positive fee to content providers diverge. A private monopolist has an incentive to set a positive fee, while a social planner prefers a negative fee. Additionally, implementing net neutrality regulation is beneficial for total welfare. We have also compared a privately optimal solution where the monopolist is free to set price to consumers and content providers to the outcome where a zero fee to content providers is imposed. The comparison showed that removing net neutrality regulation will lead to an increase in the fee content providers must pay for access and hence less content provided. The price consumers pay for Internet access decreases, so more consumers purchase Internet access, but they have access to less content. In the aggregate, consumers and the platform are better off and content providers worse off. However, total welfare is reduced, so net neutrality regulation is beneficial for society.

## 3. Duopoly Platforms with Multi-homing Content Providers

We now extend our model to duopoly competition between two platforms with multi-homing content providers. We assume that consumers single-home, that is each consumer buys Internet access from one platform only. Content and applications providers, however, are assumed to multi-home, *i.e.*, they sell through both platforms, paying the fees charged by platforms. As in monopoly, we assume that platforms offer only linear subscription prices and content provider fees.

Content providers value consumers to the extent that they are willing to pay both platforms to reach all consumers instead of only paying one platform and reaching a subset of the consumers (only the consumers subscribing to that platform). In other words, each (atomistic) content provider decides to join each platform independently of joining the other.

### 3.1 Consumers

There are two platforms (1 and 2) located at  $x = 0$  and  $x = 1$ . We assume that each platform offers the same intrinsic benefit  $v$  to consumers. Given an expected number of content providers  $n_{cpk}^e$  in each platform  $k$ ,  $k \in \{1, 2\}$ , the marginal consumer, indifferent between buying from platform 1 or 2, is located at  $x_i$  that obeys

$$v + bn_{cp1}^e - tx_i - p_1 = v + bn_{cp2}^e - t(1 - x_i) - p_2. \quad (42)$$

Assuming full market coverage,<sup>35</sup> sales of the two platforms are

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<sup>35</sup> We get very similar results when the market is not fully covered. See a more detailed discussion in the Appendix.

$$n_{c1} = \frac{1}{2} - \frac{b(n_{cp2}^e - n_{cp1}^e) - (p_2 - p_1)}{2t} \quad \text{and} \quad n_{c2} = 1 - n_{c1}. \quad (43, 44)$$

### 3.2 Content Providers

Content providers are defined as in the monopoly model above, that is, they are heterogeneous with respect to fixed costs of setting up shop. The expected number of consumers able to reach each content provider is  $n_{ck}^e$ , if the content provider buys access from platform  $k$ ,  $k \in \{1, 2\}$ . Total revenue for each content provider is  $an_{ck}^e$ .

Platform  $k$  collects a fee  $s_k$  from each content provider to allow access to its users. Thus, a content provider  $j$ 's profit from selling through platform  $k$  is

$$\pi_{jk} = an_{ck}^e - s_k - fy_j. \quad (45)$$

Each content provider with  $\pi_{jk} \geq 0$  sets up its business, pays platform  $k$  for access to its consumers and makes non-negative profits from sales to those consumers. Thus, the marginal content firm indifferent between being active and staying out of the market is

$$n_{cpk} = \frac{an_{ck}^e - s_k}{f}, \quad k \in \{1, 2\} \quad (46, 47)$$

Since consumers single-home, content providers can only reach each platform's consumers by buying access from that platform.<sup>36</sup>

### 3.3 Demand

At the fulfilled expectations equilibrium, each side of the market anticipates correctly its influence on the demand of the other side and therefore  $n_{ck}^e = n_{ck}$  and  $n_{cpk}^e = n_{cpk}$ ,  $k \in \{1, 2\}$ . Thus, at fulfilled expectations, the number of consumers and active content providers is given by the solution to the simultaneous equation system of (43, 44) and (46, 47) which is

$$n_{c1} = \frac{1}{2} + \frac{b(s_2 - s_1) + f(p_2 - p_1)}{2(ft - ab)} \quad \text{and} \quad n_{c2} = \frac{1}{2} - \frac{b(s_2 - s_1) + f(p_2 - p_1)}{2(ft - ab)}, \quad (48, 49)$$

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<sup>36</sup> A "competitive bottleneck" arises as there is no competition for content providers due to the fact that they make a decision to join one platform independent of the decision to join the other. This phenomenon is common in, for example, competing mobile telecommunications networks (receivers join one network but callers may call to all networks) and newspapers (a consumer may subscribe to only one newspaper but advertisers may advertise in all newspapers). See Armstrong (2006).

$$n_{cp1} = \frac{a(b(s_1 + s_2) + f(t + p_2 - p_1)) - (a^2b + 2fts_1)}{2f(ft - ab)} \quad \text{and} \quad (50)$$

$$n_{cp2} = \frac{a(b(s_1 + s_2) + f(t + p_1 - p_2)) - (a^2b + 2fts_2)}{2f(ft - ab)}. \quad (51)$$

### 3.4 Unrestricted Duopoly Equilibrium

When the duopoly platforms are free to set prices to both consumers and content providers, platform  $k$  maximizes

$$\Pi_k(p_1, p_2, s_1, s_2) = (p_k - c)n_{ck} + s_k n_{cpk}, \quad (52, 52)$$

$k = 1, 2$ , resulting in equilibrium prices

$$p_1^D = p_2^D = t + c - \frac{a^2 + 3ab}{4f} \quad \text{and} \quad s_1^D = s_2^D = \frac{a - b}{4}. \quad (54, 55)$$

The firms split the market on the consumer side and profits are

$$\Pi_1^D = \Pi_2^D = \frac{4ft - (a + b)^2 + 4(ft - ab)}{16f}. \quad (56, 57)$$

### 3.5 Duopoly Under Network Neutrality Regulation

Under net neutrality regulation,  $s_1 = s_2 = 0$ , and the duopolists set independently their prices to consumers to maximize

$$\Pi_1 = (p_1 - c)n_{c1} \quad \text{and} \quad \Pi_2 = (p_2 - c)n_{c2} \quad (58)$$

with respect to  $p_1$  and  $p_2$  respectively, resulting in equilibrium prices of

$$p_1^{DNN} = p_2^{DNN} = t + c - \frac{ab}{f}. \quad (59)$$

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<sup>37</sup> The second order conditions are  $-\frac{f}{ft - ab} < 0$ ,  $-\frac{(2ft - ab)}{f(ft - ab)} < 0$  and

$\frac{(4ft - (a + b)^2) + 4(ft - ab)}{4(ab - ft)^2} > 0$  and are satisfied since we have assumed  $4ft - (a + b)^2 > 0$ .

<sup>38</sup> Note that equilibrium platform prices given  $s_1$  and  $s_2$  are

$$p_1(s_1, s_2) = t + c - \left( \frac{3ab + (2a + b)s_1 + (a - b)s_2}{3f} \right), \quad p_2(s_1, s_2) = t + c - \left( \frac{3ab + (2a + b)s_2 + (a - b)s_1}{3f} \right).$$

The firms split the market equally on the consumer side and their profits are

$$\Pi_1^{DNN} = \Pi_2^{DNN} = \frac{1}{2} \left( t - \frac{ab}{f} \right). \quad (60)$$

### 3.6 Welfare Implications of Imposing Network Neutrality in Duopoly

In this section we proceed as in monopoly by first looking at incentives to set a positive fee to content providers and then making point-to-point comparisons between the duopoly equilibrium outcome under net neutrality regulation ( $s_1 = s_2 = 0$ ) and under no regulation.

We start by comparing the private and social incentives to set a positive fee to content providers. The individual incentive for a platform (either 1 or 2) to increase its fee to content providers from zero to a small positive value when the opponent is charging a zero fee is

$$\begin{aligned} \frac{d\Pi_1 \Big|_{\frac{\partial \Pi_1}{\partial p_1} = \frac{\partial \Pi_2}{\partial p_2} = 0}}{ds_1} \Big|_{s_1 = s_2 = 0} &= \frac{d\Pi_1(p_1(s_1, s_2), p_2(s_1, s_2))}{ds_1} \Big|_{s_1 = s_2 = 0} = \\ \frac{d\Pi_2 \Big|_{\frac{\partial \Pi_1}{\partial p_1} = \frac{\partial \Pi_2}{\partial p_2} = 0}}{ds_2} \Big|_{s_1 = s_2 = 0} &= \frac{d\Pi_2(p_1(s_1, s_2), p_2(s_1, s_2))}{ds_2} \Big|_{s_1 = s_2 = 0} = \frac{a-b}{3f} > 0 \end{aligned} \quad (61)$$

We define total surplus ( $TS$ ) as consisting of consumer surplus

$$CS = \int_0^{n_{c1}} (v + bn_{cp1} - tx - p_1) dx + \int_{n_{c1}}^1 (v + bn_{cp2} - t(1-x) - p_2) dx, \quad (62)$$

the sum of platform profits,

$$\Pi_1 = (p_1 - c)n_{c1} + s_1 n_{cp1}, \quad \Pi_2 = (p_2 - c)n_{c2} + s_2 n_{cp2} \quad (63, 64)$$

and total content provider profits

$$\Pi_{cp} = \int_0^{n_{cp1}} (an_{c1} - s_1 - fy) dy + \int_0^{n_{cp2}} (an_{c2} - s_2 - fy) dy. \quad (65)$$

Starting with a regime of net neutrality, we examine the incentive of each duopolist to set a small positive fee to content providers and the effects of such an action to total industry surplus. To assess these, we examine the incremental change in a duopolist's profits and in total industry surplus as the fee charged by this

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<sup>39</sup> The second order condition,  $-\frac{f}{ft - ab} < 0$ , is satisfied since we have assumed throughout  $ft - ab > 0$ .

duopolist to content providers increases from zero to a small positive value. Of course, the total surplus comparison is done under the maintained assumption that the duopolists choose their equilibrium subscription prices  $p_1(s_1, s_2), p_2(s_1, s_2)$ . The derivatives of constrained total surplus  $TS(p_1(s_1, s_2), p_2(s_1, s_2), s_1, s_2)$  with respect to the fees  $s_1$  and  $s_2$  respectively, evaluated at  $s_1 = s_2 = 0$  are<sup>40</sup>

$$\begin{aligned} \frac{dTS}{ds_1} \Big|_{\substack{\frac{\partial \Pi_1}{\partial p_1} = \frac{\partial \Pi_2}{\partial p_2} = 0 \\ s_1 = s_2 = 0}} &= \frac{dTS(p_1(s_1, s_2), p_2(s_1, s_2))}{ds_1} \Big|_{s_1 = s_2 = 0} = \\ \frac{dTS}{ds_2} \Big|_{\substack{\frac{\partial \Pi_1}{\partial p_1} = \frac{\partial \Pi_2}{\partial p_2} = 0 \\ s_1 = s_2 = 0}} &= \frac{dTS(p_1(s_1, s_2), p_2(s_1, s_2))}{ds_2} \Big|_{s_1 = s_2 = 0} = -\frac{b}{2f} < 0. \end{aligned} \quad (66)$$

Hence, as in monopoly, in duopoly, if  $a > b$ , social and private incentives are in opposite directions. Social incentives are to reduce the fees to content providers below zero, while each duopolist has an incentive to increase its fee to content providers above zero if the rival has a zero fee. Therefore net neutrality is desirable from social perspective but undesirable for each duopolist.

***Proposition 6a:*** *Starting from the net neutrality regime of a zero fee to content providers by platform duopolists, each duopolist would like to marginally increase its fee to content providers above zero.*

***Proposition 6b:*** *Starting from the net neutrality regime of a zero fee to content providers and facing platform duopolists that choose subscription prices non-cooperatively, a total surplus maximizing planner will choose to marginally decrease the fee to content providers below zero.*

A planner, anticipating the duopolists subscription equilibrium prices, chooses negative fees to content providers,  $s_1 = s_2 = -\frac{b}{2} < 0$ , to maximize the constrained total surplus function  $TS(p_1(s_1, s_2), p_2(s_1, s_2), s_1, s_2)$ . Imposition of these fees results in duopoly equilibrium subscription prices  $p_1 = p_2 = t + c - \frac{ab}{2f}$ . Even paying the subsidy to content providers, the profits of the duopoly platforms are positive at the resulting equilibrium,  $\Pi_1 = \Pi_2 = \frac{2ft - (2ab + b^2)}{4f} > 0$ .

***Proposition 7:*** *A total surplus maximizing planner, facing platform duopolists that choose their subscription prices based on the planner's choice of a fee to content providers, will choose a below-cost fee to content providers. Even paying the below-*

<sup>40</sup> The constrained total surplus function  $TS(p_1(s_1, s_2), p_2(s_1, s_2), s_1, s_2)$  is concave under the assumptions  $a^4 + ft(5b^2 - 18ft) - ab^2(15a + 4b) - a(a - 32b)ft < 0$  and  $a^4 - 2ab^2(3a + 2b) - (a^2 - 14ab - 5b^2)ft - 9f^2t^2 < 0$ . In addition to ensure that the market is not covered on the content providers side we assume that  $a + b - 2f > 0$ .

cost fee, the duopolists make positive profits.

We now consider the incentives of a duopolist to increase its fee to content providers given a possibly positive fee by its competitor. We evaluate

$$\frac{d\Pi_1 \Big|_{\frac{\partial \Pi_1}{\partial p_1} = \frac{\partial \Pi_2}{\partial p_2} = 0}}{ds_1} \Big|_{s_1=0} = \frac{d\Pi_1(p_1(s_1, s_2), p_2(s_1, s_2))}{ds_1} \Big|_{s_1=0} = \frac{(a-b)}{3f} - \frac{(a-b)^2 s_2}{9f(ft-ab)} \quad (67)$$

and therefore,

$$\begin{aligned} \frac{d\Pi_1 \Big|_{\frac{\partial \Pi_1}{\partial p_1} = \frac{\partial \Pi_2}{\partial p_2} = 0}}{ds_1} \Big|_{s_1=0} - \frac{d\Pi_1 \Big|_{\frac{\partial \Pi_1}{\partial p_1} = \frac{\partial \Pi_2}{\partial p_2} = 0}}{ds_1} \Big|_{s_1=s_2=0} &= \frac{d\Pi_1(p_1(s_1, s_2), p_2(s_1, s_2))}{ds_1} \Big|_{s_1=0} \\ - \frac{d\Pi_1(p_1(s_1, s_2), p_2(s_1, s_2))}{ds_1} \Big|_{s_1=s_2=0} &= -\frac{(a-b)^2 s_2}{9f(ft-ab)} < 0. \end{aligned} \quad (68)$$

Thus, for  $a > b$ , platform 1 has a lower incentive to set a positive fee to content providers if platform 2 quotes a positive fee to content providers. Imposing net neutrality on platform 1's competitor will strengthen platform 1's incentives to increase the fee to content providers. Thus, the incentive of a duopolist to depart from net neutrality is higher when the opponent observes net neutrality and not when the opponent charges a positive fee to content providers. Conversely, an action by duopolists to depart simultaneously from net neutrality is not supported by individual non-cooperative incentives and therefore if it occurs, it arouses the suspicion of collusion on the content side of the market. We discuss collusion in one side of the market with competition on the other side of the market in the next section.

***Proposition 8:*** *The incentive of a duopolist to increase its fee to content providers above zero decreases as the rival duopolist charges a higher fee.*

We now do a point-to-point comparison between unconstrained duopoly and the market equilibrium under net neutrality. As in the monopoly model, we compare changes in price to consumers and fees to content providers when moving from a regime with net neutrality to a regime of no regulation. Since the market is covered in both regimes, consumer participation does not change. The differences in equilibrium prices to consumers and fees to content providers are

$$\Delta p_1 = p_1^D - p_1^{DNN} = \Delta p_2 = p_2^D - p_2^{DNN} = -\frac{a(a-b)}{4f} < 0, \quad (69)$$

$$\Delta s_1 = s_1^D - s_1^{DNN} = \Delta s_2 = s_2^D - s_2^{DNN} = s_1^D = s_2^D = \frac{a-b}{4} > 0, \quad (70)$$

and the difference in content provider participation is

$$\Delta n_{cp1} = n_{cp1}^D - n_{cp1}^{DNN} = \Delta n_{cp2} = n_{cp2}^D - n_{cp2}^{DNN} = -\frac{(a-b)}{4f} < 0. \quad (71)$$

The differences in consumer surplus, platform profits and content provider profits are

$$\Delta CS = CS^D - CS^{DNN} = \frac{(a-b)^2}{16f} > 0, \quad (72)$$

$$\Delta \Pi_1 = \Pi_1^D - \Pi_1^{DNN} = \Delta \Pi_2 = \Pi_2^D - \Pi_2^{DNN} = -\frac{(a-b)^2}{16f} < 0, \quad (73)$$

and

$$\Delta \Pi_{cp} = \Pi_{cp}^D - \Pi_{cp}^{DNN} = -\frac{(a-b)(3a+b)}{16f} < 0. \quad (74)$$

Total welfare is reduced when net neutrality regulation is removed since

$$\Delta TS = TS^D - TS^{DNN} = -\frac{(a-b)(3a+b)}{16f} < 0. \quad (75)$$

Thus, under no regulation, competition for consumers is more intense since profits from content providers can be competed away. As a result, consumers enjoy lower prices and are better off under no regulation than under net neutrality. Net neutrality regulation relaxes price competition, leading to higher profits for platforms. Platforms are better off under net neutrality, which is the opposite to what was the case in the monopoly model.<sup>41</sup>

***Proposition 9:*** *Comparing unconstrained duopoly with duopoly under net neutrality, we find that total surplus is higher in net neutrality and the content sector and the platforms have higher profits. The consumers are worse off under net neutrality.*

### 3.7 Collusion on Fees to Content Providers

As we have shown, duopolist platforms like the net neutrality regime because it allows them to charge higher subscription prices. However, the individual incentive of each firm is to increase its fee to content providers and depart from net neutrality provided that the opponent stays at net neutrality. Therefore, in a two-strategies game where each duopolist can set  $s_i^{DNN} = 0$  or the non-cooperative equilibrium fee  $s_i^D$ , both firms choose  $s_i^D$  leading to a prisoners' dilemma equilibrium with lower profits for both platforms than when both play  $s_i^{DNN} = 0$ . We show below that collusion between the platforms will also result in zero fees to content providers if the platforms are constrained to choose non-negative fees.

Suppose that the duopolists first collude on fees to content providers, *i.e.*, set cooperatively  $s_1$  and  $s_2$  to maximize joint profits  $\Pi_1 + \Pi_2$ , and then set subscription

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<sup>41</sup> The result that platforms are better off under net neutrality is partly due to the simplifying assumption that the market is always covered on the consumer side. We show in the appendix that relaxing this assumption and allowing the market for subscription to the Internet not to be fully covered at equilibrium leads to the conclusion that platforms are worse off under net neutrality regulation.



fees non-cooperatively.<sup>42</sup> Given subscription fees  $s_1$  and  $s_2$ , the non-cooperative equilibrium subscription prices are

$$p_1(s_1, s_2) = \frac{b(s_2 - s_1) - a(3b + 2s_1 + s_2)}{3f} + t + c, \quad (76)$$

$$p_2(s_1, s_2) = \frac{b(s_1 - s_2) - a(3b + 2s_2 + s_1)}{3f} + t + c. \quad (77)$$

Substituting these in joint profits  $\Pi_1 + \Pi_2$  and maximizing with respect to  $s_1$  and  $s_2$  we find that the joint profit maximizing fee for the platforms is zero:

$s_1^{DCO} = s_2^{DCO} = 0$ . Therefore, the firms cannot improve over net neutrality if they collude.

***Proposition 10:*** *Duopolists colluding in setting fees to content providers while competing non-cooperatively in subscription prices will choose zero fees if they are constrained not to choose non-negative fees. Thus, the duopolists cannot improve over net neutrality by cooperating in linear fees to content providers.*

### **3.7 Summary of Results for Platform Duopoly**

Extending the monopoly model to a duopoly setup, we showed that most of our results are robust to the introduction of competition between platforms.<sup>43</sup> In platform duopoly, we find that for a wide range of parameter values, the private and social incentives to set a positive fee to content providers diverge. A social planner would prefer a negative fee, while competing duopolists would like to choose a positive fee. Hence, net neutrality regulation is beneficial for social welfare even when some competition is present in the platform market. Comparisons between outcomes under the private equilibrium with two-sided pricing and the private equilibrium under net neutrality regulation indicated that a removal of net neutrality regulation would lead to lower subscription price for consumers, but less content available due to an increase in fees to content providers. Content providers are worse off, but consumers better off in the aggregate. Social welfare is reduced supporting the result that net neutrality regulation is good for total welfare.

## **4. Concluding Remarks**

We developed a model of a two-sided market to assess the potential benefits of the Internet departing from “net neutrality” whereby broadband Internet access providers (telephone and cable TV companies) do not charge a positive fee to content and applications providers. We explicitly allowed monopoly and duopoly access providers to charge a positive fee to content and applications providers. We

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<sup>42</sup> Consumers and content providers form expectations and make their decisions subsequently.

<sup>43</sup> This echoes earlier theoretical evidence suggesting that introducing competition in a two-sided market does not necessarily lead to a pricing structure that is closer to the socially optimal one. See, for example, Wright (2004) and Armstrong (2006).

contrasted this with a setup where a regulator chooses the fee to content providers to maximize total surplus taking into account the pricing of a monopolist or duopolists in the consumer subscription side of the market. We showed that, under these conditions, the regulator will choose a negative fee to content providers while a monopolist or duopolists choose positive fees. We also showed that society is better off in terms of total surplus at net neutrality rather than either the monopolist's or duopolists' choices of positive fees to content providers.

As noted in the introduction, the economics literature on net neutrality regulation is still in early stages. Further rigorous economic analysis is needed on issues such as the impact of net neutrality regulation on innovation among content providers, on non-linear platform pricing to content providers, and on congestion and broadband penetration are important aspects that require further study. We believe that it is important for future studies to account for the two-sided nature of the market. One-sided analysis of two-sided markets may easily lead to incorrect conclusions.<sup>44</sup>

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<sup>44</sup> See Wright (2004).

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## 6. Appendix

### Duopoly Model with Demand Expansion Effects on the Consumer Side

Here we consider the model of duopoly under the assumption that the market on the consumers side is not covered, *i.e.*, we account for demand expansion effects on the consumers side as we already do on the content provider side. We show that our main conclusions do not change under this scenario.

In contrast to the duopoly model presented above where the platforms were located at the end points of the unit interval over which consumers are uniformly distributed, we here locate the platforms at a distance  $d < \frac{1}{2}$  from the endpoints. We assume that  $d$  and  $t$  are large enough so that the market is never covered and that the platforms compete for consumers located between them. Hence, there will be 3 marginal consumers denoted  $x_1$ ,  $x_2$  and  $x_3$ . The consumer located at  $x_1$  is indifferent between buying from platform 1 and staying out of the market. The consumer located at  $x_2$  is indifferent between the two platforms and the consumer located at  $x_3$  is indifferent between staying out of the market and buying from platform 2. Given our utility specification, the locations of these indifferent consumers are given by

$$\begin{aligned} x_1 &= d - \frac{v + bn_{cp1}^e - p_1}{t} \\ x_2 &= \frac{1}{2} - \frac{b(n_{cp2}^e - n_{cp1}^e) - (p_2 - p_1)}{2t} \\ x_3 &= (1 - d) + \frac{v + bn_{cp2}^e - p_2}{t} \end{aligned}$$

and demand on the consumer side is  $n_{c1} = x_2 - x_1$  and  $n_{c2} = x_3 - x_2$ . The content provider side remains the same as in section 3.

At the fulfilled expectations equilibrium we can obtain expressions for the number of active consumers and content providers as functions of all four prices. These are

$$\begin{aligned} n_{c1}(p_1, p_2, s_1, s_2) &= \frac{2ab(2bs_1 + f(2p_1 - t + 2dt - 2v)) + ft(b(-3s_1 + s_2) + f(-3p_1 + p_2 + t - 2dt + 2v))}{4a^2b^2 - 6abft + 2f^2t^2} \\ n_{cp1}(p_1, p_2, s_1, s_2) &= \frac{-2fs_1t^2 + 2a^2b(2p_1 - t + 2dt - 2v) + at(b(3s_1 + s_2) + f(-3p_1 + p_2 + t - 2dt + 2v))}{4a^2b^2 - 6abft + 2f^2t^2} \\ n_{c2}(p_1, p_2, s_1, s_2) &= \frac{2ab(2bs_2 + f(2p_2 - t + 2dt - 2v)) + ft(b(s_1 - 3s_2) + f(p_1 - 3p_2 + t - 2dt + 2v))}{4a^2b^2 - 6abft + 2f^2t^2} \\ n_{cp2}(p_1, p_2, s_1, s_2) &= \frac{-2fs_2t^2 + 2a^2b(2p_2 - t + 2dt - 2v) + at(b(s_1 + 3s_2) + f(p_1 - 3p_2 + t - 2dt + 2v))}{4a^2b^2 - 6abft + 2f^2t^2} \end{aligned}$$

Consumer surplus is

$$CS = \int_{x_1}^d (v + bn_{cp1} - t(d-x) - p_1)dx + \int_d^{x_2} (v + bn_{cp1} - t(x-d) - p_1)dx \\ + \int_{x_2}^{(1-d)} (v + bn_{cp2} - t((1-d)-x) - p_2)dx + \int_{(1-d)}^{x_3} (v + bn_{cp2} - t(x-(1-d)) - p_2)dx$$

and content provider profits are

$$\Pi_{cp} = \int_0^{n_{cp1}} (an_{c1} - s_1 - fy)dy + \int_0^{n_{cp2}} (an_{c2} - s_2 - fy)dy.$$

Total surplus is defined as the sum of consumer surplus, platform profits and content provider profits.

We first solve for equilibrium prices and fees in the unrestricted duopoly equilibrium. Platform k choose prices and fees to maximize

$$\Pi_k(p_1, p_2, s_1, s_2) = (p_k - c)n_{ck}(p_1, p_2, s_1, s_2) + s_k n_{cpk}(p_1, p_2, s_1, s_2)$$

resulting in symmetric equilibrium prices of

$$p_1^D = p_2^D = \frac{ab(8b^2c + ft(-22c - 9t + 18dt - 18v)) + 4a^2b(t - 2dt + 2v) + a^2(3ft(-t + 2dt - 2v) + 4b^2(2c + t - 2dt + 2v)) + 2ft(-3b^2c + 2ft(3c + t - 2dt + 2v))}{8ab(a+b)^2 - 2(3a^2 + 20ab + 3b^2)ft + 20f^2t^2}$$

$$s_1^D = s_2^D = \frac{(a-b)f(4ab - 3ft)(2c + (2d-1)t - 2v)}{8ab(a+b)^2 - 2(3a^2 + 20ab + 3b^2)ft + 20f^2t^2}.^{45}$$

Under net neutrality regulation ( $s_1 = s_2 = 0$ ), equilibrium subscription prices are obtained by each platform setting price to maximize

$$\Pi_k(p_1, p_2, 0, 0) = (p_k - c)n_{ck}(p_1, p_2, 0, 0)$$

resulting in a symmetric subscription prices of

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<sup>45</sup> The second order conditions are  $f(\frac{1}{ab-ft} + \frac{2}{2ab-ft}) < 0$ ,  $\frac{t(3ab-2ft)}{(ab-ft)(2ab-ft)} < 0$ , and  $\frac{(3ft-4ab)(4ab(a+b)^2 - 3(a^2+6ab+b^2)ft + 8f^2t^2)}{4(ab-ft)^2(ft-2ab)^2} > 0$ . To satisfy the second order

conditions we need to impose  $ft - 2ab > 0$  and

$4ab(a+b)^2 - 3(a^2+6ab+b^2)ft + 8f^2t^2 > 0$ , that is, that the heterogeneity parameters are large enough.

$$p_1^{DNN} = p_2^{DNN} = \frac{ft(-3c - t + 2dt - 2v) + 2ab(2c + t - 2dt + 2v)}{8ab - 5ft}.^{46}$$

We now compare the unconstrained duopoly and the market equilibrium under net neutrality. Through rather tedious calculations, one can show that for a sufficiently large transportation cost parameter, the differences in equilibrium prices to consumers and fees to content providers are

$$\begin{aligned}\Delta p_1 &= p_1^D - p_1^{DNN} = \Delta p_2 = p_2^D - p_2^{DNN} < 0, \\ \Delta s_1 &= s_1^D - s_1^{DNN} = \Delta s_2 = s_2^D - s_2^{DNN} > 0.\end{aligned}$$

and the differences in consumer and content provider participation are

$$\begin{aligned}\Delta n_{c1} &= n_{c1}^D - n_{c1}^{DNN} = \Delta n_{c2} = n_{c2}^D - n_{c2}^{DNN} > 0, \\ \Delta n_{cp1} &= n_{cp1}^D - n_{cp1}^{DNN} = \Delta n_{cp2} = n_{cp2}^D - n_{cp2}^{DNN} < 0.\end{aligned}$$

The differences in consumer surplus, platform profits and content provider profits are

$$\begin{aligned}\Delta CS &= CS^D - CS^{DNN} > 0, \\ \Delta \Pi_1 &= \Pi_1^D - \Pi_1^{DNN} = \Delta \Pi_2 = \Pi_2^D - \Pi_2^{DNN} > 0, \\ \Delta \Pi_{cp} &= \Pi_{cp}^D - \Pi_{cp}^{DNN} < 0. \\ \Delta TS &= TS^D - TS^{DNN} < 0.^{47}\end{aligned}$$

Under no regulation, competition for consumers is more intense since profits from content providers can be competed away. As a result, consumers enjoy lower prices and are better off under no regulation than under net neutrality. Platforms are also better off under no regulation. This is the opposite result to that of the case when the market was covered. This is the case because of profits from more consumers entering the market. Content providers are worse off, and total welfare is reduced.

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<sup>46</sup> The second order conditions  $f\left(\frac{1}{ab - ft} + \frac{2}{2ab - ft}\right) < 0$  are satisfied for  $ft - 2ab > 0$ .

<sup>47</sup> Total welfare is reduced when net neutrality regulation is removed if  $3a - 23b < 0$  and differentiation parameters  $f$  and  $t$  are large enough so that  $8a^2b(3a^4 + 18a^3b + 18a^2b^2 + 54ab^3 + 11b^4)ft + (39a^3 - 31a^2b + 491ab^2 + 21b^3)f^3t^3 + 5(3a - 23b)f^4t^4 < 16a^3b^2(a + b)^2(a^2 + ab + 2b^2) + a(9a^4 + 133a^3b + 48a^2b^2 + 730ab^3 + 76b^4)f^2t^2$ .