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Managing Conflicts in Relational Contracts*

Jin Li[†]
Northwestern University

Niko Matouschek[‡]
Northwestern University

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Abstract

A worker interacts repeatedly with a manager who is privately informed about the opportunity costs of paying him. The worker therefore cannot distinguish non-payments that are efficiency enhancing from those that are rent extracting. The optimal relational contract generates periodic conflicts during which effort and expected profits decline gradually but recover instantaneously. To manage a conflict, the manager uses a mix of informal promises and formal commitments that evolves with the duration of the conflict. Liquidity constraints limit the manager's ability to manage conflicts but may also induce the worker to respond to a conflict by providing more effort rather than less.

Keywords: relational contracts, imperfect monitoring

JEL classifications: C73, D23, D82, J33

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[†]Kellogg School of Management, jin-li@kellogg.northwestern.edu.

[‡]Kellogg School of Management, n-matouschek@kellogg.northwestern.edu.

1 Introduction

Relational contracts often suffer from conflicts during which workers punish managers for broken promises. A common cause for such conflicts is disagreement over the availability and efficient use of funds. In a typical conflict of this sort, workers demand that a bonus be paid while managers insist that the necessary funds are either non-existent or better used on something else, such as an exceptional investment opportunity.

One source for disagreements over the availability and efficient use of funds is asymmetric information. In general, managers are better informed than workers about the challenges and opportunities that their firms face. As a result, managers often have private information about the opportunity costs of paying their workers. The aim of this paper is to explore optimal relational contracts in such a setting. For this purpose, we examine the repeated relationship between a manager and a worker in which the manager's opportunity costs of paying the worker are stochastic and privately observed by her. We show that in the optimal relational contract, the manager promises a bonus if opportunity costs are low but none if they are high. Conflicts therefore arise whenever opportunity costs are high and the manager does not pay the bonus. To manage these conflicts, the manager relies on a combination of informal promises and formal commitments that evolves with the duration of the conflict. Because of the manager's actions, effort and expected profits decline during a conflict only gradually and then recover instantaneously. The same pattern is repeated over time. The relationship between the manager and the worker therefore never terminates, nor does it reach a steady state. Instead, it cycles indefinitely.

The Lincoln Electric Company provides an example of the type of situation that motivates this paper. In the early 1990s, Lincoln Electric was a leading manufacturer of welding machines that was well-known for its promise to share a significant fraction of profits with its factory workers. In 1992 Lincoln's U.S. business had generated a significant profit and as a result its U.S. workers expected to be paid their bonus. Mounting losses in its recently acquired foreign operations, however, more than wiped out U.S. profits. This presented CEO Donald Hastings with a dilemma: "*Our 3,000 U.S. workers would expect to receive, as a group, more than $50 million. If we were in default, we might not be able to pay them. But if we didn't pay the bonus, the whole company might unravel*" (Hastings 1999, p.164). To prevent the company from unraveling, Hastings decided to borrow $52.1 million and pay the bonus.

Why would Hastings have to take the seemingly inefficient step of borrowing money to pay the bonus? After all, the bonus was explicitly a "cash-sharing bonus" and U.S. workers had a long history of accepting fluctuations in the bonus in response to fluctuations in U.S. profits. The reason, it seems, was that U.S. workers were unable to observe foreign losses and therefore did not know whether U.S. profits really were needed to cover those losses. This explains why shortly after he paid the bonus, Hastings also "[...] *instituted a financial education program so that employees would understand that no money was being hidden from them* [...]" (Hastings 1999, p.172).

The Lincoln Electric case illustrates the issues that arise if a manager is privately informed about the opportunity costs of paying her worker. In such a setting, if the manager does not pay a bonus, the worker cannot observe her motives. Is the manager not paying the bonus because it is more efficient to spend resources on something else, as she claims? Or is she just making up an excuse to extract some of the worker's rents? To keep the manager honest, the worker must then punish her whenever she does not pay a bonus. As a result, the manager faces a trade-off between the current benefits of adapting bonus payments to their opportunity costs and the future costs the worker inflicts on her if she does not pay a bonus. In short, the manager faces a trade-off between the benefits of adaptation and the costs of conflicts.

To explore this trade-off, we examine a firm that consists of a risk neutral owner-manager and a risk neutral but liquidity constrained worker. Output and effort are observable but not contractible. At the beginning of every period the manager offers the worker a contractible wage and a non-contractible bonus. After accepting the offer, the worker decides on his effort level. Effort is continuous and imposes a cost on the worker. Finally, output is realized and the manager decides how much to pay the worker. So far this is a relational contracting model with public information that is well understood (MacLeod and Malcomson 1989). The only change we make to this standard model is to assume that the manager's opportunity costs of paying the worker are stochastic and privately observed by her. In particular, just before the manager decides how much to pay the worker, she observes whether the firm has been hit by a shock - in which case opportunity costs are high - or not - in which case they are low.

In this setting, the manager could avoid conflicts altogether by promising to pay the same bonus whether opportunity costs are high or low. This is essentially what CEO Hastings did at Lincoln Electric when he decided to pay the bonus even though he had to borrow the funds to do so. In our setting, however, this is never optimal. Instead, the manager always promises a positive bonus if opportunity costs are low and none if they are high. The benefits of adaptation therefore always outweigh the costs of conflicts.

To see how conflicts evolve over time, consider an arbitrary sequence of shock and no-shock periods. In any period in which the firm is hit by a shock, expected profits in the next period will be strictly lower, unless expected profits are already at their lower bound, in which case they stay there. And in any period in which the firm is not hit by a shock, expected profits will jump back to their upper bound in the next period. Expected profits therefore cycle indefinitely and the relationship never terminates. These cycles differ in length depending on the number of consecutive shock periods the firm experiences. They all, however, follow the same pattern in which downturns are gradual and recoveries instantaneous.

To understand what generates this pattern, consider a typical conflict. In particular, consider a period in which expected profits are at their upper bound and suppose the firm is then hit by shocks in a large number of consecutive periods. As we just saw, expected profits will then decline gradually until they bottom out at their lower bound. And once they have bottomed out at their lower bound, expected profits will stay there until the next no-shock period. We can now divide this conflict into three phases that differ according to the actions the manager takes to manage the conflict.

In the initial phase of the conflict, the manager promises the worker a larger and larger bonus but she does not commit to a wage. The worker accepts the manager's offers but provides less and less effort. Initially, the manager therefore relies only on informal promises to slow down the worker's effort reductions.

In the intermediate phase of the conflict, the manager complements the informal promise to pay a no-shock bonus with a formal commitment also to pay a wage. Both the no-shock bonus and the wage increase throughout this phase of the conflict. The worker accepts the manager's offers and always provides the same effort level. Notice that the manager's commitment to pay a wage is costly, since she has to pay the wage even if the firm is hit by a shock. In the intermediate phase of the conflict, however, effort is so low that it is more efficient to commit to a wage than to tolerate further effort reductions.

In the final phase of the conflict, the bonus, the wage, and effort stay constant until a period in which the firm is not hit by a shock. In that period the manager finally pays the promised bonus and expected profits return to their upper bound. Once a conflict hits rock bottom, therefore, the manager makes no more changes to the compensation package. Instead, she simply waits for the next no-shock period to revive her relationship with the worker.

A key assumption in our model is that the firm is not liquidity constrained, that is, the manager can always pay the worker any positive amount, even if the opportunity costs of doing so may be high. In our main extension we relax this assumption. We show that liquidity constraints limit the manager's ability to manage conflicts, which slows down recoveries and may lead to termination. They can also, however, induce the worker to respond to a conflict by providing more effort rather than less. Essentially, the worker understands that more effort relaxes the firm's liquidity constraint which, in turn, allows the manager to pay him a larger bonus.

To illustrate the role of liquidity constraints in managing relational contracts, we return to the Lincoln Electric case. In early 1993, a few months after he had borrowed the necessary funds to pay his workers, CEO Hastings realized that European losses would once again wipe out U.S. profits. The covenants in the debt that he took on the previous year, however, prevented him from again borrowing the necessary funds to pay the bonus:

"*The way I saw it, we had two choices: we could resort to massive layoffs and cut executive salaries to save money, or we could make extraordinary efforts to increase revenues and profits. I never seriously considered the first option.* [...] *Our longtime covenant with our workers guaranteed them a least 30 hours of work per week. Downsizing could only result in deterioration of morale, trust, and productivity. It's bad long-term business.* [...]

So rather than downsize, we turned to our U.S. employees for help. I presented a 21-point plan to the board that called for our U.S. factories to boost production dramatically [...]. *'We blew it,' I said* [to the U.S. employees]. *'Now we need you to bail the company out. If we violate the covenants, banks won't lend us money. And if they don't lend us money, there will be no bonus in December'*" (Hastings 1999, pp. 171-172).

According to Hastings, his "*statement appealed not only to* [the U.S. workers'] *loyalty but also to what James F. Lincoln called their 'intelligent selfishness'*" (Hastings 1999, p.172). And, apparently, it worked:

"*Thanks to the Herculean effort in the factories and in the field, we were able to increase revenues and profits enough in the United States to avoid violating our loan covenants*" (Hastings 1999, p.178).

As a result, Hastings was able to renew the covenants which, in turn, allowed him to once again borrow the necessary funds and pay the bonus. In line with the reasoning that we sketched above, therefore, Lincoln Electric's U.S. workers increased their efforts to relax the firm's liquidity constraints which, in turn, allowed Hastings to pay their bonus.

2 Related Literature

There is a large literature that examines relational contracts both between and within firms; see MacLeod (2007) and Malcomson (forthcoming) for recent reviews. Our paper contributes to the branch of this literature that studies the actions managers can take to sustain relational contracts better, such as the timing of payments (MacLeod and Malcomson 1998), the design of explicit contracts (Baker, Gibbons, and Murphy 1994, Che and Yoo 2001), the allocation of ownership rights (Baker, Gibbons, and Murphy 2002, Rayo 2007), the differential treatment of workers (Levin 2002), the grouping of tasks (Mukherjee and Vasconcelos 2011), and others. In contrast to these papers, our focus is on how to manage conflicts once they arise, rather than on how to prevent them in the first place.

A closely related model is the second part of Levin (2003), which examines the optimal relational contract when a manager cannot observe effort but does have private information about a worker's performance. In this setting, the manager faces a similar trade-off between the benefits of adaptation and the costs of conflicts as in our model. In contrast to our model, however, the optimal relational contract is stationary and takes the form of a termination contract. The key difference between the models is that in our settings transfers are inefficient when the firm is hit by a shock while they are always efficient in Levin (2003). Moreover, in Levin (2003) effort is privately observed by the worker, while it is publicly observed in our setting. In Section 7 we discuss why these two differences imply that termination contracts are not optimal in our setting.

Two other closely related papers are Yared (2010) and Englmaier and Segal (2011). Both papers allow for inefficient transfers and assume one-sided private information. Yared (2010) characterizes the optimal relational contract between an aggressive country that seeks concessions and a non-aggressive country with private information about the costs of concessions. Englmaier and Segal (2011) study the relationship between a worker and a manager who is privately informed about the costs of transfers. They focus on a particular class of relational contracts and examine the role of unions in mitigating conflicts.

Our paper also contributes to the recent and growing literature on dynamics within relational contracts. Chassang (2010) studies a model of exploration with private information and shows that the relationship is path-dependent and can settle in different long-run equilibria. Fong and Li (2010) study a moral hazard problem in which the worker has limited liability and explore patterns of the worker's job security, pay level, and the sensitivity of pay to performance. Padro i Miquel and Yared (2011) examine a political economy model and study the likelihood, duration, and intensity of war. Thomas and Worrall (2010) examine a partnership game with perfect information and two-sided limited liability. They show that the relationship becomes more efficient over time as the division of future rents becomes more equal. Dynamics also arise in models of relational contracts in which agents have private and fixed types; see for example, Halac (forthcoming), Watson (1999, 2002), and Yang (2011). In these papers, dynamics arise when the principal updates her beliefs about the agent's type.

In terms of its analytical structure, our model is related to the literature of dynamic games with hidden information; see for example Abdulkadiroglu and Bagwell (2010), Athey and Bagwell (2001, 2008), Athey, Bagwell and Sanchirico (2004), Hauser and Hopenhayn (2008), Mobius (2001), and for surveys, Mailath and Samuelson (2006) and Samuelson (2006). Most of this literature examines settings with symmetric players, multi-sided private information, and without monetary transfers. Our model instead explores a setting with one-sided private information and inefficient transfers. As a result, first best is not possible in our model. We discuss the reason for this difference in more detail in Section 7.

Our model is also related to the literature on dynamic contracting between banks and privately informed entrepreneurs (DeMarzo and Fishman 2007, DeMarzo and Sannikov 2006, Biais et al. 2007, and Clementi and Hopenhayn 2006). In contrast to this literature, we focus on a setting in which long-term contracts are not feasible. The availability of long-term contracts is crucial for our results. Indeed, we show in Section 7 that if the long-term contracts were feasible, the parties could approximate first best.

Finally, since the efficiency of transfers depends on the state of the world, our model is related to the large literature on risk sharing. Kocherlakota (1996) and Ligon, Thomas, and Worrall (2002) explore efficient risk sharing between risk averse agents when information is public and commitment is limited. Hertel (2004) examines the case with two-sided asymmetric information without commitment. Thomas and Worrall (1990) study a one-sided asymmetric information problem with commitment. This literature typically assumes that the player's endowments are exogenously given and path-independent. In our model, instead, output depends on the worker's effort and thus on how it was divided in the past.

3 The Model

A firm consists of a risk neutral owner-manager and a risk neutral but liquidity constrained worker. The manager and the worker are in an infinitely repeated relationship. Time is discrete and denoted by $t = \{1, 2, ..., \infty\}$.

At the beginning of any period t the manager makes the worker an offer. The offer consists of a contractible commitment to pay wage $w_t \geq 0$ and a non-contractible promise to pay bonuses $b_{s,t}$ and $b_{n,t}$, where s and n stand for "shock" and "no-shock." The worker either accepts the offer or rejects it. We denote the worker's decision by d_t, where $d_t = 0$ if he rejects the offer and $d_t = 1$ if he accepts it. If the worker rejects the offer, the manager and the worker realize their per period outside options $\underline{\pi} > 0$ and $\underline{u} > 0$ and time moves on to period $t + 1$.

If, instead, the worker accepts the manager's offer, he next decides on his effort level $e_t \geq 0$. Effort is costly to the worker and we denote his effort costs by $c(e_t)$. The cost function is strictly increasing and convex with $c(0) = c'(0) = 0$ and $\lim_{e \to \infty} c'(e) = \infty$. After the worker provides effort e_t, the manager realizes output $y(e_t)$. The output function is strictly increasing and concave with $y(0) = 0$. Effort e_t, effort costs $c(e_t)$, and output $y(e_t)$ are observable to both parties but not contractible. We denote the first best effort level that maximizes $y(e) - c(e)$ by e^{FB} and assume that $y(e^{FB}) - c(e^{FB}) > \underline{\pi} + \underline{u}$. The relationship is therefore productive, provided that the worker puts in enough effort.

After the manager realizes output $y(e_t)$, she privately observes the state of the world $\Theta_t \in \{s, n\}$, where, as mentioned above, s and n stand for "shock" and "no-shock." The states are drawn independently across time from a binary distribution. The probability with which a shock state occurs is given by $\theta \in (0, 1)$. The state of the world determines the opportunity cost of paying the worker: if the firm is not hit by a shock, paying the worker an amount of $w + b$ costs the manager $w + b$; if, instead, the firm is hit by a shock, paying the worker $w + b$ costs the manager $(1 + \alpha)(w + b)$, where $\alpha \in (0, \infty)$. We do not model explicitly why opportunity costs may be high. As discussed above, however, managers do sometimes face high opportunity costs of paying their workers. This may be the case, for instance, because they need to borrow money to make their payments, as in the Lincoln Electric case.

After the manager observes the state of the world, she pays the worker the wage w_t and a bonus $b_t \geq 0$. Since the promised bonus is not contractible, the payment b_t can be different from the promises $b_{n,t}$ and $b_{s,t}$.

Finally, at the end of period t, the manager and the worker observe the realization x_t of a public randomization device. The existence of a public randomization device is a common assumption in the literature and is made to convexify the set of equilibrium payoffs. The timing is summarized in Figure 1.

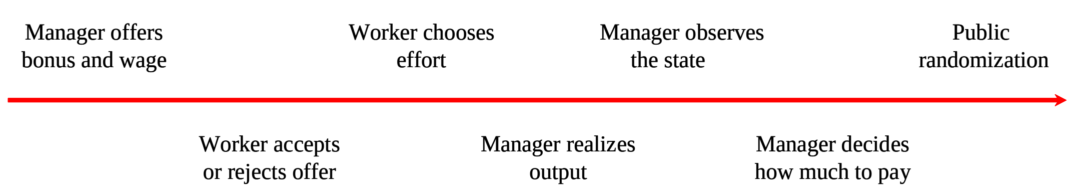


Figure 1: Timeline

The manager and the worker share the same discount factor $\delta \in (0, 1)$. At the beginning of any period t, their respective expected payoffs are therefore given by

$$\pi_t = (1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}\mathrm{E}\left[d_\tau y\left(e_\tau\right) - \left(1+1_{\{\Theta_\tau = s\}}\alpha\right)\left(w_\tau + b_\tau\right) + (1-d_\tau)\underline{\pi}\right]$$

and

$$u_t = (1-\delta)\sum_{\tau=t}^{\infty}\delta^{\tau-t}\mathrm{E}\left[d_\tau\left(w_\tau + b_\tau - c\left(e_\tau\right)\right) + (1-d_\tau)\underline{u}\right].$$

Note that we multiply the right-hand side of each expression by $(1-\delta)$ to express profits and payoffs as per period averages.

We follow the literature on imperfect public monitoring and define a "relational contract" as a pure-strategy Perfect Public Equilibrium (henceforth PPE) in which the manager and the worker play public strategies and, following every history, the strategies are a Nash Equilibrium. Public strategies are strategies in which the players condition their actions only on publicly available information. In particular, the manager's strategy does not depend on her past private information. We consider only pure strategies to avoid issues of measurability which arise because the action spaces of the manager and the worker are a continuum. Notice that when the parties use pure strategies, there is no need to consider private strategies since every sequential equilibrium outcome is also a PPE outcome (Mailath and Samuelson 2006).

Formally, let $h_{t+1} = \{w_\tau, b_{n,\tau},\, b_{s,\tau},\, d_\tau,\, e_\tau,\, b_\tau,\, x_\tau\}_{\tau=1}^{t}$ denote the public history at the beginning of any period $t+1$ and let H_{t+1}denote the set of period $t+1$ public histories. Note that $H_1 = \Phi$. A public strategy for the manager is a sequence of functions $\{W_t, B_{s,t},\, B_{n,t},\, B_t\}_{t=1}^{\infty}$, where $W_t : H_t \to [0,\infty)$, $B_{s,t} : H_t \to [0,\infty)$, $B_{n,t} : H_t \to [0,\infty)$, and $B_t : H_t \cup \{w_t, b_{s,t},\, b_{n,t},\, d_t,\, e_t, \theta_t\} \to [0,\infty)$. Similarly, a public strategy for the worker is a sequence of functions $\{D_t,\, E_t\}_{t=1}^{\infty}$, where $D_t : H_t \cup \{w_t, b_{s,t},\, b_{n,t}\} \to \{0,1\}$ and $E_t : H_t \cup \{w_t, b_{s,t},\, b_{n,t},\, d_t\} \to [0,\infty)$.

We define an "optimal relational contract" as a PPE with payoffs that are not Pareto-dominated by any other PPEs. Note that when the discount factor is sufficiently small, the only relational contract is a trivial one in which the parties forever take their outside options. To make the analysis more interesting, we assume that the parties are sufficiently patient so that a non-trivial relational contract exists. Our objective is to characterize the set of optimal relational contracts.

4 Preliminaries

In this section we use the technique developed by Abreu, Pearce, and Stacchetti (1990) to characterize the PPE payoff set. In the first sub-section we show that we can restrict attention to the PPE frontier and then derive its basic properties. In the second sub-section we then provide the recursive formulation of the PPE frontier.

4.1 PPE Frontier

We denote the set of PPE payoffs by E. Each payoff pair $(\pi, u) \in E$ is supported by a set of actions and continuation payoffs. Suppose for now that the worker accepts the manager's offer. We then need to specify the worker's effort e, the wage w, the bonuses b_s and b_n, and the associated continuation payoffs. It is without loss of generality that if either party deviates publicly, the continuation payoffs are given by the outside options $\underline{\pi}$ and $\underline{u}$. If, instead, neither party deviates publicly, the continuation payoffs are given by (π_s, u_s) and (π_n, u_n). To support (π, u) as a PPE payoff, it is necessary and sufficient that (i.) the set of actions and continuation payoffs are feasible, including the restriction that the continuation payoffs are again in E, (ii.) neither party can benefit from deviating to other actions, and (iii.) the weighted average of current and continuation payoffs are equal to (π, u).

Before we characterize the payoff set E, it is useful to derive a few results that will simplify the analysis. For this purpose, we define the payoff frontier as

$$u(\pi) \equiv \sup\{u' : (\pi, u') \in E\}.$$

We can now state our first lemma.

LEMMA 1. *The PPE payoff set E is compact. Let $\overline{\pi}$ be the manager's maximum PPE payoff. The PPE payoff set E is then given by*

$$E = \{(\pi', u') : \pi' \in [\underline{\pi}, \overline{\pi}], u' \in [\underline{u}, u(\pi')]\}.$$

Moreover,

$$u(\overline{\pi}) = \underline{u}.$$

A key implication of this lemma is that the payoff set E is fully characterized by its frontier $u(\pi)$. There are two reasons for this. First, taking the outside options forever is a PPE that gives both parties their minimax payoffs. Second, since there is a public randomization device any payoff below the frontier and above the outside options can be obtained by randomization. Randomization, however, is not needed to support payoffs on the frontier, as shown in the next lemma.

LEMMA 2. *Any payoff pair on the frontier $(\pi, u(\pi))$ can be supported by pure actions in the stage game. Moreover, for all $\pi \in [\underline{\pi}, \overline{\pi}]$, the PPE frontier $u(\pi)$ is concave and differentiable and its derivative satisfies $-1 < u'(\pi) \leq -1/(1 + \alpha\theta)$.*

This lemma states several properties of the PPE frontier that we will need later on. First, the PPE frontier is differentiable mainly because there is a continuum of effort levels. If effort levels were discrete, the PPE frontier would not be differentiable. Second, the PPE frontier is everywhere downward sloping because the wage is contractible. If the manager were not able to contractually commit to a wage, the PPE frontier could have an upward sloping portion. Finally, the fact that the slope of the PPE frontier is strictly larger than -1 implies that the parties can never achieve first best as we will see below.

The next lemma shows that as long as neither party deviates publicly, the continuation payoff of any payoff pair on the frontier must also be on the frontier. In other words, the optimal relational contracts are sequentially optimal.

LEMMA 3. *Consider a payoff pair $(\pi, u(\pi))$ on the PPE frontier. Let (π_s, u_s) and (π_n, u_n) be the associated continuation payoffs following the shock and no-shock state respectively. Then*

$$u_s = u(\pi_s) \quad \text{and} \quad u_n = u(\pi_n).$$

Essentially, since the worker's actions are publicly observable, it is not necessary to punish him by moving below the PPE frontier. This feature of our model is similar, for instance, to Spear and Srivastava (1987) and the first part of Levin (2003). In contrast, joint punishments are necessary in models with two-sided private information, such as Green and Porter (1984), Athey and Bagwell (2001), and the second part of Levin (2003).

Recall from Lemma 2 that the public randomization device is not needed for payoffs on the PPE frontier. Lemma 3 then implies that the optimal relational contracts can be played out without the public randomization device. To simplify the exposition, we refer to an optimal relational contract as one in which no public randomization device is used.

4.2 Recursive Formulation

To support each PPE payoff, a number of constraints have to be satisfied. In this section we first list a subset of these constraints and then combine and simplify them. At the end of this section we then show that the PPE frontier is characterized by a maximization problem subject to these constraints.

Consider a payoff pair $(\pi, u(\pi))$ on the frontier with associated stage game actions e, w, b_s, and b_n, and the continuation payoffs $(\pi_s, u(\pi_s))$ and $(\pi_n, u(\pi_n))$. For the actions and continuation payoffs to support $(\pi, u(\pi))$, we need the following constraints to hold:

First, the promise-keeping condition for the manager requires that the manager's payoff π is equal to the weighted sum of her current and continuation payoffs, that is,

$$\pi = \theta \left\{ (1-\delta) \left[y\left(e\right) - (1+\alpha)(w+b_s) \right] + \delta \pi_s \right\} + (1-\theta) \left\{ (1-\delta) \left[y\left(e\right) - w - b_n \right] + \delta \pi_n \right\}. \quad (\text{PK}_\text{M})$$

Second, the self-enforcing constraints require that the continuation payoffs remain in the PPE payoff set. In particular, we need

$$\pi_s \geq \underline{\pi} \quad (\text{SE}_\text{S})$$

and

$$\pi_n \leq \overline{\pi}. \quad (\text{SE}_\text{N})$$

Third, the wage must be non-negative, that is,

$$w \geq 0. \quad (\text{NN}_\text{W})$$

And finally, the manager's truth-telling constraint in a no-shock period requires that

$$\delta \left(\pi_n - \pi_s\right) \geq (1-\delta) \left(b_n - b_s\right). \quad (\text{TT}_\text{N})$$

To further simplify these constraints, we first need to establish the next lemma which shows that the manager always pays a weakly larger bonus if the firm is not hit by a shock.

LEMMA 4. *The no-shock bonus is always weakly larger than the shock bonus, that is,* $b_n(\pi) \geq b_s(\pi)$ *for all* $\pi \in [\underline{\pi}, \overline{\pi}]$.

We can now prove the next lemma which will allow us to eliminate the shock and no-shock bonuses from the above constraints.

LEMMA 5. *For any payoff pair* $(\pi, u(\pi))$ *on the frontier, there exists a set of actions and continuation payoffs supporting it such that (i.) the manager's truth-telling constraint in a no-shock period* TT_N *is binding and (ii.) the shock bonus* $b_s(\pi)$ *is zero.*

For the first part of the lemma notice that since the PPE frontier is concave we can increase the worker's expected payoff by reducing the distance between π_n and π_s. And we can do so until the manager's truth-telling constraint binds. For the second part of the lemma, consider a payoff pair on the frontier that is supported by actions that include a strictly positive shock bonus. Specifically, suppose that the payoff pair is supported by some $\widehat{w} \geq 0$, $\widehat{b}_s > 0$, and $\widehat{b}_n \geq \widehat{b}_s$. Now consider alternative actions for which the shock bonus is zero but wages are given by $\widehat{w} + \widehat{b}_s$ and the no-shock bonus is given by $\widehat{b}_n - \widehat{b}_s$. It can be shown that since the original relational contract satisfies all the necessary constraints for the payoffs to be on the frontier, so does this alternative one. And of course their payoffs are identical.

Finally, using Lemmas 2-5, we can represent the PPE frontier recursively. For this purpose, consider any bounded function f with support $[\underline{\pi}, \overline{\pi}]$, in which $\overline{\pi}$ is arbitrarily chosen. We then define the operator T as follows: for all $\pi \in [\underline{\pi}, \overline{\pi}]$,

$$Tf(\pi) = \max_{e,w,\pi_s,\pi_n} (1-\delta)\left[y(e) - c(e)\right] + \theta\delta\left[\pi_s + u(\pi_s)\right] + (1-\theta)\delta\left[\pi_n + u(\pi_n)\right] - (1-\delta)\theta\alpha w \quad (1)$$

subject to

$$\pi = (1-\delta)\,y(e) + \delta\pi_s - (1-\delta)(1+\theta\alpha)\,w, \quad (\mathrm{PK_M})$$

$$\pi_s \geq \underline{\pi}, \quad (\mathrm{SE_S})$$

$$\pi_n \leq \overline{\pi}, \text{ and} \quad (\mathrm{SE_N})$$

$$w \geq 0. \quad (\mathrm{NN_W})$$

Notice that in this maximization problem we are not choosing b_s and b_n and we are not listing the manager's truth-telling constraint ($\mathrm{TT_N}$). The reason is that we have solved the $\mathrm{TT_N}$ constraint for b_n and then substituted it into the $\mathrm{PK_M}$ constraint. Moreover, we set b_s equal to zero which, as we just saw, is without loss of generality.

The next lemma states that the joint payoff on the PPE frontier is equal to the maximal joint PPE payoff - expressed as the weighted sum of current and continuation joint payoffs - subject to the constraints discussed above.

LEMMA 6. *For all $\pi \in [\underline{\pi}, \overline{\pi}]$, the joint payoff $\pi + u(\pi)$ satisfies*

$$\pi + u(\pi) = T(\pi + u(\pi)).$$

Recall that an optimal relational contract is a PPE whose payoffs are on the PPE frontier. It is therefore characterized by the constrained maximization problem (1). Since in that problem we have set $b_s = 0$, from now on an optimal relational contract refers to one in which the manager does not pay a bonus in a shock period.

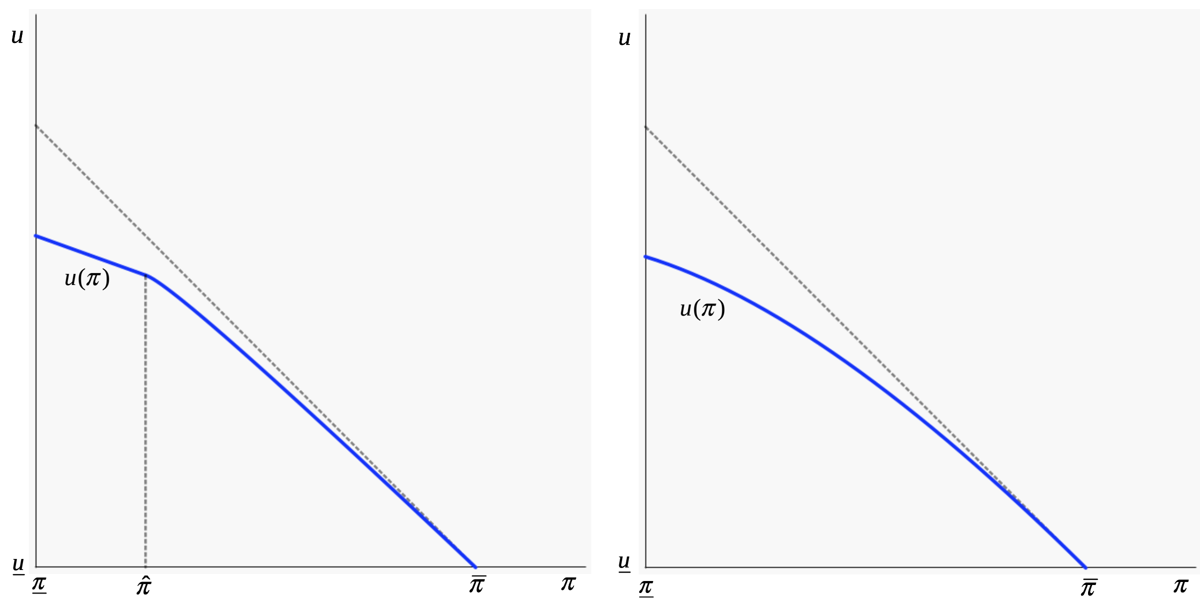
Notice that the above lemma specifies a necessary condition for the joint payoff on the PPE frontier, not a sufficient one. In general, T can have multiple fixed points; see for example, Baker, Gibbons and Murphy (1994) for an illustration. The multiplicity arises, in part, because $\overline{\pi}$ appears in the constraint $\mathrm{SE_N}$. As a result, T is no longer a contraction mapping. It is immediate, however, that even if T has multiple fixed points, the joint payoff on the PPE frontier is the largest one.

5 The Optimal Relational Contract

In this section we first characterize the optimal relational contract and then trace out its evolution over time.

5.1 Characterizing the Optimal Relational Contract

The optimal relational contract is determined by PPE frontier $u(\pi)$. Figures 2a and 2b illustrate some of the key properties of the PPE frontier that we derived in the previous section and that will be relevant for the characterization of the optimal relational contract.



Figures 2a and 2b: The PPE Frontier

In both figures, the PPE frontier is differentiable, concave, and its slope satisfies $-1 < u'(\pi) \leq -1/(1+\alpha\theta)$. The two figures differ, however, in terms of the critical level of expected profits $\widehat{\pi}$, which is defined by

$$\widehat{\pi} = (1-\delta)y(\widehat{e}) + \delta\underline{\pi}, \tag{2}$$

where $\widehat{e}$ is the unique effort level that solves

$$\frac{c'(\widehat{e})}{y'(\widehat{e})} = \frac{1}{1+\alpha\theta}. \tag{3}$$

Notice that $\widehat{\pi}$ can be larger than $\underline{\pi}$, as in Figure 2a, or it can be smaller, as in Figure 2b. If $\widehat{\pi}$ is larger than $\underline{\pi}$, then the PPE frontier is linear with slope $u'(\pi) = -1/(1+\alpha\theta)$ for all $\pi \in [\underline{\pi}, \widehat{\pi}]$ and it is strictly concave for all $\pi \in (\widehat{\pi}, \overline{\pi}]$. If, instead, $\widehat{\pi}$ is smaller than $\underline{\pi}$, then the PPE frontier is strictly concave for all $\pi \in [\underline{\pi}, \overline{\pi}]$.

Under the optimal relational contract, the parties' payoffs move along the PPE frontier. How they do so, and what effort, bonus, and wage decisions support these payoffs, is determined by the solution to the problem stated in Lemma 6 and is characterized in the next proposition. The proposition also shows that for any level of expected profits π, there is a unique optimal relational contract.

PROPOSITION 1. *For any level of expected profits π, there exists a unique optimal relational contract that gives the worker $u(\pi)$. Under the optimal relational contract:*

(*a.*) *Effort $e^*(\pi)$ is given by the unique effort level e that solves*

$$\frac{c'(e)}{y'(e)} = -u'(\pi) \text{ for all } \pi \in [\underline{\pi}, \overline{\pi}].$$

(*b.*) *Wages are given by*

$$w^*(\pi) = \max\left[\frac{\widehat{\pi} - \pi}{(1-\delta)(1+\alpha\theta)},\ 0\right].$$

(*c.*) *The no-shock bonus $b_n^*(\pi)$ is given by*

$$b_n^*(\pi) = \frac{\delta}{1-\delta}\left(\overline{\pi} - \pi_s^*(\pi)\right) > 0.$$

(*d.*) *If the firm is hit by a shock, the continuation profit $\pi_s^*(\pi)$ satisfies*

$$\pi_s^*(\underline{\pi}) = \underline{\pi} \text{ and } \pi_s^*(\pi) < \pi \text{ for all } \pi \in (\underline{\pi}, \overline{\pi}]$$

(*e.*) *If the firm is not hit by a shock, the continuation profit is given by*

$$\pi_n^*(\pi) = \overline{\pi} \quad \text{for all } \pi \in [\underline{\pi}, \overline{\pi}].$$

We will discuss this proposition in reverse, starting with Parts (d.) and (e.) and then working our way up. Parts (d.) and (e.) show that the optimal continuation profits are strictly smaller if the firm is hit by a shock than if it is not, that is, $\pi_s^*(\pi) < \pi_n^*(\pi)$ for all $\pi \in [\underline{\pi}, \overline{\pi}]$. The optimal relational contract therefore punishes the manager if she does not pay a bonus and rewards her if she pays the no-shock bonus $b_n^*(\pi)$. This suggests that $b_n^*(\pi)$ is not just weakly positive, as we know from the previous section, but strictly so.

Part (c.) shows that this is indeed the case. Notice that the expression for the optimal no-shock bonus follows immediately from substituting the zero shock bonus and the optimal continuation profits in the manager's truth-telling constraint (TT_N).

The manager therefore always makes the bonus payments contingent on their opportunity costs. Doing so does lead to conflicts that lower expected profits. And since joint surplus $\pi + u(\pi)$ is increasing in expected profits, it also destroys joint surplus. In our setting, however, the manager's current benefits of adapting bonuses to their opportunity costs always outweigh the future costs that the worker inflicts on her if she does not pay a bonus.

Consider next the expression for wages in Part (b.). To interpret this expression, recall that $\widehat{\pi}$ can be larger than $\underline{\pi}$, as in Figure 2a, or smaller, as in Figure 2b. In particular, it follows from (2) and (3) that there exists a critical value of the size of the shock α such that $\widehat{\pi} < \underline{\pi}$ if and only if α is above the critical value. From Part (b.) we then have the intuitive result that the manager never commits to wages if paying the worker in a shock period is sufficiently costly. If, instead, α is below the critical value, the manager does commit to strictly positive wages when expected profits are sufficiently low, that is, when $\pi \leq \widehat{\pi}$.

Finally, Part (a.) shows that the optimal effort level is determined by the slope of the PPE frontier. Since the PPE frontier is concave, effort is monotonically increasing in expected profits. Notice, however, that even when expected profits are at their upper bound $\overline{\pi}$, effort is strictly less than first best. The parties are therefore never able to achieve first best. We will return to this issue in Section 7.

5.2 The Dynamics of the Optimal Relational Contract

We can now use the characterization of the optimal relational contract to trace out its evolution over time. We focus on the case in which the manager pays wages at least sometimes, that is, we focus on $\widehat{\pi} > \underline{\pi}$. Once we understand this case, the evolution of the optimal relational contract when the manager never pays wages will be immediate.

Consider first the evolution of expected profits. It follows from the proposition that if the firm is hit by a shock in one period, expected profits in the next period will be strictly lower, unless expected profits are already at their lower bound $\underline{\pi} > 0$, in which case they stay there. And if the firm is not hit by a shock in one period then expected profits in the next period will be at their upper bound $\overline{\pi}$. Expected profits therefore cycle indefinitely and the relationship never terminates. These cycles differ in length depending on the number of consecutive shock periods the firm experiences. They all, however, follow the same pattern in which downturns are gradual and recoveries instantaneous. This is also illustrated in Figure 3 which plots expected profits for an arbitrary sequence of shock periods - indicated by red squares - and no-shock periods - indicated by blue dots.

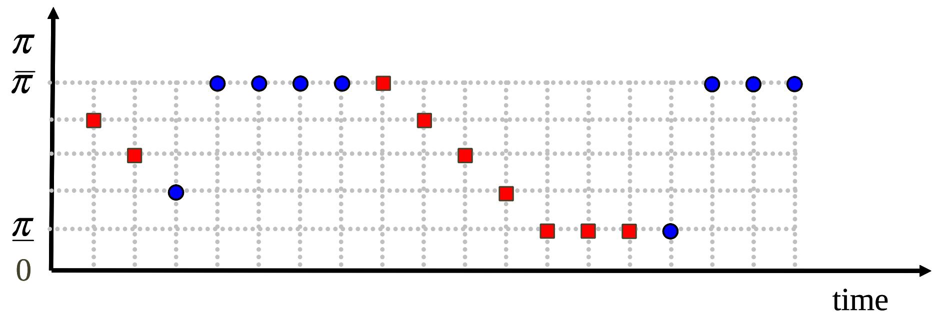


Figure 3: Evolution of Expected Profits

To understand what generates the evolution of expected profits, consider a typical conflict. In particular, consider a period in which expected profits are at their upper bound $\overline{\pi}$ and suppose the firm is then hit by shocks in a large number of consecutive periods. As we just saw, expected profits will then decline gradually until they bottom out at their lower bound $\underline{\pi}$. Once expected profits have bottomed out at their lower bound, they will stay there until the next period in which the firm is not hit by a shock. We can now divide this conflict into three phases: the initial phase, which consists of the periods for which $\pi \in [\widehat{\pi}, \overline{\pi}]$, the intermediate phase, which consists of the periods for which $\pi \in (\underline{\pi}, \widehat{\pi})$, and the final phase, which consists of the periods for which $\pi = \underline{\pi}$. In terms of Figure 2a, the initial phase consists of periods in which expected profits are on the strictly concave segment of the PPE frontier, the intermediate phase consists of periods in which expected profits are on the linear segment, and the final phase consists of periods in which expected profits are at their lower bound.

In the initial phase of the conflict, the manager promises the worker a larger and larger no-shock bonus but she does not commit to a wage. The worker accepts the manager's offers but provides less and less effort.

In the intermediate phase of the conflict, the manager complements the informal promise to pay a no-shock bonus with a formal commitment also to pay a wage. Both the no-shock bonus and the wage increase throughout this phase of the conflict. The worker accepts the manager's offers and always provides the same, strictly positive effort level $\widehat{e}$. Committing to a wage is of course costly, since the manager has to pay the wage even if the firm is hit by a shock. In the intermediate phase of the conflict, however, effort is so low that committing to a wage is less costly than further effort reductions would be.

In the final phase of the conflict, the no-shock bonus and the wage stay constant at their maximized levels and effort stays constant at $\hat{e}$. This final phase of the conflict continues until the parties reach a period in which the firm is not hit by a shock. In that period the manager finally pays the promised no-shock bonus and expected profits return to their upper bound $\overline{\pi}$. The evolution of bonuses, wages, and effort during this conflict are also illustrated in Figure 4, where once again red squares indicate shock periods and blue dots indicate no-shock periods.

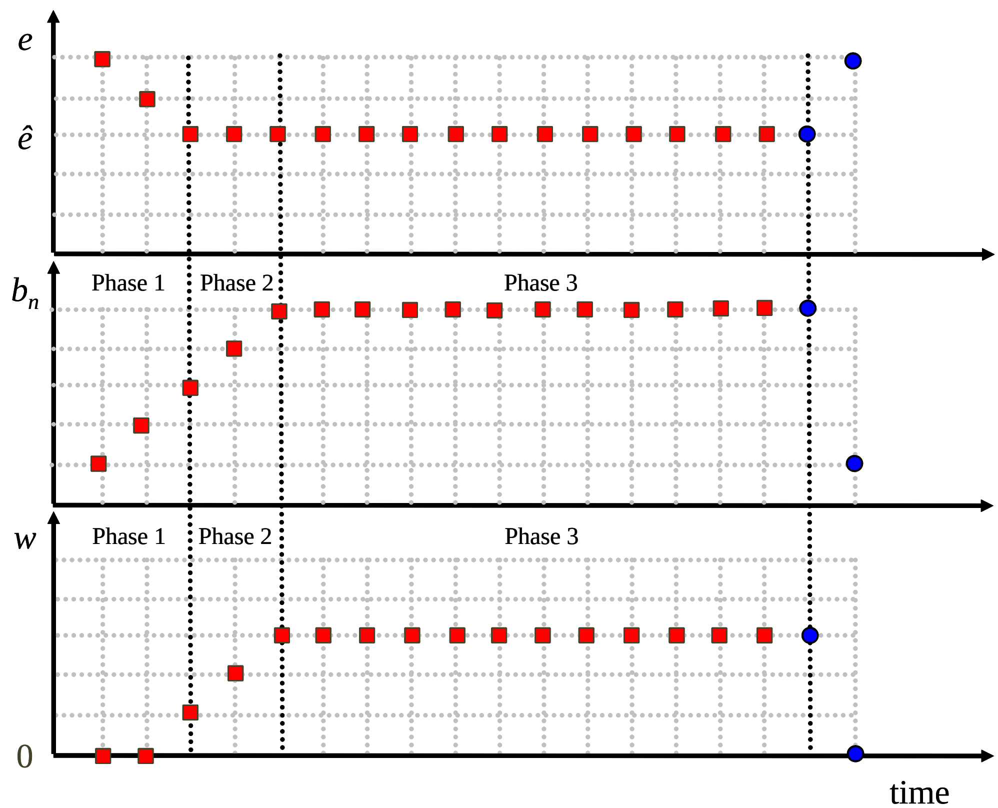


Figure 4: Evolution of Effort, Bonuses, and Wages

The three phases of a conflict therefore differ according to the actions the manager takes to manage the conflict. In the initial phase, the manager relies only on informal promises to slow down the worker's effort reductions. In the intermediate phase, the manager then depends on both informal promises and formal commitments to halt further effort reductions. And in the final phase, the manager makes no more changes to the compensation package. Instead, she simply waits for the next no-shock period to revive her relationship with the worker.

6 Allowing for Liquidity Constraints

The Lincoln Electric case that we discussed in the Introduction suggests that liquidity constraints can have significant effects on managers' ability to manage conflicts. In this section we explore this issue by allowing for the firm to be liquidity constrained.

Specifically, we now assume that if the firm realizes output $y(e)$ and is not hit by a shock, the manager can pay the worker at most $(1+m)\,y(e)$, where the parameter $m \geq 0$ captures the extent to which the firm is liquidity constrained. Liquidity constraints make the PPE frontier non-differentiable and thus substantially complicate the characterization of the optimal relational contract. To make the analysis more tractable, we now assume that the size of the shock α is equal to infinity. The manager therefore cannot pay the worker whenever the firm is hit by a shock. An immediate implication of this assumption is that wages and the shock bonus are always equal to zero.

Recall that without liquidity constraints the entire PPE frontier can be sustained by pure strategies and termination never occurs in equilibrium. The next lemma shows that if the firm is liquidity constrained this need no longer be the case.

LEMMA 7. *There exists a critical level of expected profits $\pi_0 \in [\underline{\pi}, \overline{\pi})$ such that for all $\pi \geq \pi_0$ the PPE frontier $u(\pi)$ is supported by pure actions and for all $\pi < \pi_0$ it is supported by randomization. Specifically, for any $\pi < \pi_0$ the manager and the worker randomize between terminating their relationship and playing the strategies that deliver expected payoffs π_0 and $u(\pi_0)$.*

In Appendix B we provide a sufficient condition for $\pi_0 > \underline{\pi}$. In that case, the PPE frontier is supported by randomization for sufficiently small levels of expected profits. We will see below that this implies that the relationship is certain to terminate after a finite number of periods. If, instead, $\pi_0 = \underline{\pi}$, the entire PPE frontier can once again be supported by pure strategies and the relationship never terminates.

We can now state our next proposition which characterizes the optimal relational contract when the firm is liquidity constrained.

PROPOSITION 2. *Suppose that the firm is liquidity constrained. Then, in an optimal relational contract:*

(a.) Effort $e^*(\pi)$ is non-monotonic in expected profits π. In particular, $e^*(\pi)$ increases in π for all $\pi \in [\pi_0, \pi_1] \cup [\pi_2, \overline{\pi})$ but decreases in π for all $\pi \in (\pi_1, \pi_2)$, where *π_1 and π_2 are defined in Appendix B and where $\pi_0 \leq \pi_1 \leq \pi_2$.*

$(b.)$ *The no-shock bonus* $b_n^*(\pi)$ *is given by*

$$b_n^*(\pi) = \frac{\delta}{1-\delta}\left(\pi_n^*(\pi) - \pi_s^*(\pi)\right) > 0.$$

$(c.)$ *If the firm is hit by a shock, the continuation profit* $\pi_s^*(\pi)$ *satisfies*

$$\pi_s^*(\pi_0) = \underline{\pi} \text{ and } \pi_s^*(\pi) < \pi \text{ for all } \pi \in (\pi_0, \overline{\pi}].$$

$(d.)$ *If the firm is not hit by a shock, the continuation profit* $\pi_n^*(\pi)$ *satisfies*

$$\begin{aligned} \pi_n^*(\pi) &= \overline{\pi} && \text{for all } \pi \in [\pi_1, \overline{\pi}] \text{ and} \\ \pi < \pi_n^*(\pi) &< \overline{\pi} && \text{for all } \pi \in [\pi_0, \pi_1]. \end{aligned}$$

The proposition shows that liquidity constraints affect the manager's ability to manage relational contracts in three main ways. First, they limit her ability to induce instantaneous recoveries. To see this, consider a period in which the firm is not hit by a shock. Without liquidity constraints, expected profits π then immediately return to their upper bound $\overline{\pi}$. This is also the case if the firm is liquidity constrained provided that expected profits are sufficiently large. If expected profits are quite low, however, then it takes at least two periods for expected profits to return to their upper bound. Essentially, for the worker to agree to move to the equilibrium in which expected profits are at their upper bound, the manager has to compensate him for the corresponding loss in his payoff. If the firm is not liquidity constrained, the manager can do so with a single, large bonus payment. But if the firm is liquidity constrained, the manager may have to spread the bonus payment over multiple periods. Liquidity constraints, therefore, slow down the recovery from sufficiently severe conflicts.

As mentioned above, liquidity constraints may also force the manager to accept termination as part of the optimal relational contract. To see this, consider a period in which expected profits are at their upper bound and suppose the firm is then hit by shocks in a large number of consecutive periods. As in the model without liquidity constraints, expected profits will then gradually decline over time. And if $\pi_0 = \underline{\pi}$ expected profits will again bottom out at their lower bound $\underline{\pi}$, where they will stay until a period in which the firm is not hit by a shock. If $\pi_0 > \underline{\pi}$, however, the manager and the worker will eventually reach a period in which $\pi_s^*(\pi) \leq \pi_0 < \pi$. At that point it will take at most two more consecutive shock periods for the relationship to terminate.

To get an intuition for why liquidity constraints may make termination necessary, recall that if expected profits are low, the continuation profits in a no-shock period $\pi_n^*(\pi)$ are small. The manager's reward for paying $b_n^*(\pi)$ is therefore limited. To induce the manager to continue to be truthful, the worker then needs to increase the manager's punishment for not paying $b_n^*(\pi)$.

And when expected profits are already small, the only way to do so is to increase the threat of termination.

While liquidity constraints limit the manager's ability to manage conflicts, they can also induce the worker to respond to a conflict by providing more effort rather than less. This will be the case, in particular, if $\pi_2 > \pi_1$, a sufficient condition for which we provide in Appendix B. The worker may increase his effort during a conflict because he understands that this relaxes the firm's liquidity constraint which, in turn, allows the manager to pay him a larger bonus if the firm is not hit by another shock. As discussed in the Introduction, this reasoning is broadly consistent with the experience at Lincoln Electric.

In summary, liquidity constraints limit the manager's ability to manage conflicts which slows down recoveries and can lead to termination. Liquidity constraints, however, can also induce the worker to respond to a conflict by providing more effort rather than less.

7 Discussion

In this section we revisit key features and assumptions of our model and examine them in more detail. We focus on our main model without liquidity constraints.

7.1 Termination and Suspension Contracts

The optimal relational contract in our setting cannot be implemented by termination contracts. As discussed in Section 2, this is in contrast to related models such as the second part of Levin (2003) in which termination contracts are optimal (see also Fuchs 2007). There are two main reasons for this difference. First, in our setting, the worker's effort is publicly observable. This implies, in particular, that in equilibrium the worker is never punished and the continuation payoffs always stay on the frontier. In contrast, in the second part of Levin (2003), joint punishment is necessary and the continuation payoffs fall below the frontier. The outside option can then help to sustain the continuation payoff. Second, in our setting transfers are inefficient whenever the firm is hit by a shock. This friction implies that transfers cannot be used to divide surplus without affecting its size. In particular, in our model the PPE frontier is concave and not a negative 45 degree line as it is in Levin (2003). To support any payoff on the PPE frontier, it is therefore not possible to have the payoff itself, its continuation payoffs, and its outside options on the same line. As a result, termination contracts are not optimal.

In another well-known type of relational contract, the parties alternate between cooperation and punishment phases. We refer to such contracts as "suspension contracts" since cooperation is only suspended and not terminated. Suspension contracts are of course familiar from Green and Porter (1984). And in the context of employment relationships they are examined in Englmaier and Segal (2011) and Gary-Bobo and Jaaidane (2011).

In our setting, a suspension contract requires the parties to start out cooperating, with the worker providing a high level of effort. If the manager ever does not pay a bonus, the parties switch to punishment for a fixed number of periods. During the punishment phase the worker provides low effort and the manager pays no bonus. After that, the parties revert back to the cooperation phase.

Suspension contracts are not optimal in our setting. The reason, in part, is that effort costs are strictly convex. To see this, take any suspension contract and augment it by increasing the effort level that is associated with the punishment phase. One can then lengthen the duration of the punishment phase to ensure that the manager's truth-telling constraint is still satisfied. Since effort costs are convex, this augmented suspension contract makes the worker better off and is thus more efficient. Suspension contracts are therefore not optimal in our setting.

7.2 The Failure to Achieve First Best

We show in Appendix C (Proposition C1) that the Folk Theorem holds in our setting. Specifically, we show that as the discount factor δ goes to one, the limit set of the PPE payoff contains the interior of the set of feasible payoffs. Joint surplus therefore converges towards first best as the parties become increasingly patient. It is important to note, however, that as long as the discount factor δ is strictly less than one, joint surplus is strictly less than first best. For any $\delta < 1$ the optimal relational contract is therefore inefficient. This is in contrast to related repeated games with imperfect public monitoring such as Athey and Bagwell (2001) in which first best can be achieved for a range of discount factors.

The reason for the parties' inability to achieve first best is that the worker can never be sure that opportunity costs are low. The fact that the worker can never be sure that opportunity costs are high, in contrast, does not matter for the parties' inability to achieve first best. To see this, suppose that whenever the firm is not hit by a shock, there is some probability $p \in [0, 1)$ with which it becomes publicly known that the firm's opportunity costs are low. And whenever the firm is hit by a shock, there is some probability $q \in [0, 1)$ with which it becomes publicly known that the firm's opportunity costs are high. If $p = q = 0$, this model is the same as our main model. And if either p or q were equal to one, the state would be publicly observed and there would be no need for the manager to be punished on the equilibrium path. We discuss this public information benchmark in the next section. In Appendix C (Proposition C2) we show that in the setting in which $p \in [0,1)$ and $q \in [0,1)$, first best can be achieved for sufficiently high discount factors if and only if $p > 0$. Essentially, when $p > 0$, the manager does not pay the worker when the firm is hit by a shock but promises him a larger bonus in the next period in which it is publicly observed that the firm's opportunity costs are low. Since the occurrence of such an event is publicly observable, the manager's promise is credible and first best is feasible.

Firms that ask their workers to accept cuts to their compensation often open their books to prove that those cuts really are necessary (recall, for instance, the Lincoln Electric case in the Introduction; see also Englmaier and Segal 2011). The above argument suggests that firms should not only open their books during hard times, in the hope of avoiding worker punishments. Instead, it may be even more important for firms to keep their books open during good times, so as to make punishments less costly.

7.3 Benchmarks: Public Information and Long-term Contracts

A first benchmark against which one can compare our model is one in which shocks are publicly observed. In Appendix C (Proposition C3) we examine this case and characterize the PPE payoff set and the associated optimal relational contracts. Relative to our model, there are three key differences. First, when information is public, expected profits, effort, and joint surplus weakly increase over time. Second, they reach their highest achievable levels with probability one and then stay there forever. And third, if the manager and the worker are patient enough, those highest achievable levels are equal to first best. The evolution of the relationship between the manager and the worker therefore depends crucially on whether shocks are publicly observed.

A second benchmark against which one can compare our model is one in which the manager can commit to a long-term contract. To explore this benchmark, suppose that in any period t the manager first observes the state $\Theta_t \in \{n, s\}$ and then makes an announcement $m_t \in \{n, s\}$ about the state. Suppose also that before the first period, the manager can commit to a contract that, for any period t, maps her announcements $(m_1, m_2, ..., m_t)$ into the bonus b_t that she has to pay the worker at the end of period t.

A long-term contract does not allow the parties to achieve first best. It does, however, allow them to approximate first best. To see this, let $\tau(t)$ denote the number of consecutive periods immediately preceding t in which the manager did not pay the worker. Now consider a contract with three features. First, the contract asks the worker to provide first best effort in all periods. If the worker ever does not provide first best effort, the manager will never again pay him. Second, the contract specifies that if, in period t, the manager announces that the firm has not been hit by a shock, she pays the worker a bonus

$$b_t(\tau(t)) = \left(1 + \frac{1}{\delta} + \frac{1}{\delta^2} + ... + \frac{1}{\delta^{\tau(t)}}\right)\left(\underline{u} + c\left(e^{FB}\right)\right).$$

And third, the contract specifies a number $T \geq 1$ that determines how much the manager has to pay the worker whenever she announces that the firm has been hit by a shock. In particular, if, in period t, the manager announces that the firm has been hit by a shock and if $\tau(t) < T$, then the manager does not have to pay the worker. If, however, $\tau(t) = T$, then the manager has to pay the worker a bonus $b_t(T)$.

In Appendix C (Proposition C4) we show that under such a contract, the worker always provides first best effort and the manager always announces the state truthfully. Essentially, under this contract the manager has to pay the worker $\left(\underline{u} + c\left(e^{FB}\right)\right)$ per period, independent of her announcements. By lying about the state, the manager can therefore affect the timing of payments but not their net present value.

This contract does not achieve first best since it induces inefficient payment whenever the firm is hit by shocks in T consecutive periods. By agreeing to a large T, however, the parties can come arbitrarily close to achieving first best. The evolution of the relationship between the manager and the worker therefore depends crucially on whether the manager is able to commit to a long-term contract. And as we saw above, it also depends on whether shocks are privately observed.

8 Conclusions

In a well-known article in *The New Yorker*, Stewart (1993) describes the upheavals at the investment bank Credit Suisse First Boston (CSFB) after consecutive years of disappointing bonus payments. Problems started in 1991 when traders demanded that management pay them a higher bonus. Management, however, stood firm, insisting that a higher bonus was not justified because of the need to "*build capital.*" To appease the traders, management then simply "*promised that 1992 would be different - that salaries and bonuses would again be competitive.*" Traders were forthcoming in expressing their disappointment but their retaliations were limited. The traders' behavior changed the following year, however, when bonus payments were once again below expectations. This time "*many traders seemed to drag their heels, further depressing the firm's earnings*" and "*defections* [...] *increased as soon as First Boston actually began paying bonuses.*" This response forced management to adapt its compensation policy by formally committing to "*guaranteed pay raises*," in some cases as much as 100%.

At the heart of the conflict at CSFB was uncertainty, and possibly private information, about the opportunity costs of bonus payments. In particular, while there was no disagreement about the traders' performance, there was disagreement about the extent to which bonus payments should be contingent on the need to "build capital." The aim of this paper was to explore the conflicts that arise in such a setting. In our model, it is optimal for the manager to make payments contingent on their opportunity costs, even though this makes conflicts inevitable. As in the CSFB example, the manager responds to such conflicts by adapting compensation to their duration, moving from the informal - promising that 1992 will be different - to the formal - committing to guaranteed pay raises. Because the manager responds to a conflict by changing the compensation she offers the worker, conflicts evolve gradually. This is again illustrated in the CSFB example in which traders did not switch from cooperation to punishment abruptly. Instead, the relationship deteriorated gradually in response to repeated disagreements about bonus pay. Finally, in our setting, expected profits cycle indefinitely. The relationship between the manager and the worker therefore never terminates, nor does it reach a steady state. This, of course, is in contrast to the CSFB example where many traders did leave. Termination, however, can also arise in our setting once we allow for the firm to be liquidity constrained.

To discuss the empirical implications and testability of our model, it is useful to note that the basic structure of the stage game is closely related to a "trust game." In a standard trust game, the "proposer" first decides on the size of a monetary gift that he makes to the "responder." The gift is then increased by some amount after which the responder decides how much to give back to the proposer. One can therefore view our model as an infinitely repeated trust game in which the responder faces shocks to the costs of giving back. There is an extensive literature in experimental economics that examines trust games. This suggests that one could test our model in a laboratory setting. Two predictions, in particular, are clear cut. First, the evolution of trust - as measured by the size of the gift - depends crucially on whether shocks are publicly observed or not. If shocks are publicly observed, trust increases over time until it tops out at some level. If shocks are instead privately observed, trust evolves through booms and busts. Second, the long-term prospects of a relationship depend on whether the responder is liquidity constrained. If the responder is not liquidity constrained, the relationship continues forever. But if she is liquidity constrained, it is certain to terminate eventually.

We have cast our model in the context of an employment relationship. The main ingredients of the model - repeated interaction, limited commitment, and inefficient transfers - are also relevant in many other economic settings. One example is the lending relationship between an entrepreneur and an investor who are not able to commit to long-term contracts. The entrepreneur can have private information about her marginal value of money and the investor can adjust his future financing terms based on the payment history of the entrepreneur. Another example is that of long-term and informal supplier relationships in which buyers face shocks to their ability to pay their suppliers. In 1995, for instance, Continental Airlines was close to bankruptcy and its "*most pressing need was to shore up its cash position. The airline* [...] *was only able to make its January 1995 payroll when* [its CEO] *Bethune successfully begged Boeing to return cash deposits on aircraft whose delivery he had deferred*" (Frank 2009). A final example involves the informal insurance relationships among farmers in developing countries. There is some evidence that the farmers' income is private information; see, for example, Kinnan (2011). While most of the literature has focused on moral hazard and insurance issues separately, our model suggests that these issues are related since insurance decisions affect future production choices.

9 Appendix A: Main Model

In this appendix we prove the lemmas and proposition in Sections 4 and 5 which analyze our main model. In the first part , we formally list all of the constraints to support $(\pi, u(\pi))$ as a payoff on the PPE frontier. In the second part, we prove the equivalent of Lemma 1-6 in the preliminary section in the sense we consider a relaxed program that ignores the worker's incentive compatibility (IC) constraints. And in the third part, we check the worker's ICs are satisfied and prove Proposition 1.

9.1 List of Constraints

Recall that E is the set of PPE payoffs. Consider a payoff pair $(\pi, u) \in E$ and the associated e, w, b_s, b_n, (π_s, u_s) and (π_n, u_n). To support (π, u) as a PPE payoff, we need three sets of constraints: (a.) *feasibility*: the set of actions and continuation payoffs are feasible, including the restrictions that the continuation payoffs are again in E, (b.) *No deviation*: the players cannot benefit from deviating to other actions, and (c.) *Promise-keeping*: (π, u) is equal to the weighted average of current and continuation payoff.

9.1.1 Feasibility

For the actions to be feasible, the base wage and bonuses need to be non-negative and so is the effort level. Specifically, we need

$$b_s \geq 0, \quad (\text{NN}_\text{S})$$

$$b_n \geq 0, \quad (\text{NN}_\text{N})$$

$$w \geq 0, \text{ and} \quad (\text{NN}_\text{W})$$

$$e \geq 0. \quad (\text{NN}_\text{e})$$

For the continuation payoffs to be feasible, the continuation payoffs need also be PPE payoffs:

$$(\pi_s, u_s) \in E \text{ and} \quad (\text{SE}_\text{S})$$

$$(\pi_n, u_n) \in E. \quad (\text{SE}_\text{N})$$

9.1.2 No Deviation

For the parties not to deviate, we need to consider two types of deviations: off-schedule and on-schedule. Off-schedule deviations are those that can be publicly observed. If an off-schedule deviation occurs, there is no loss of generality in assuming that the parties will permanently break up the relationship by taking their outside options, as this is the worst possible equilibrium that give each party its minimax payoff. Here, the manager deviates off-schedule when he fails to pay a bonus equalling either b_s or b_n. When this occurs, the manager's continuation payoffs will be $\underline{\pi}$.

To prevent the manager from off-schedule deviations, it suffices that her loss in future continuation payoff exceeds her maximum possible current gain from deviating. The manager's current gain from deviation is maximized when she pays zero bonus. This gives us the non-reneging constraints

$$\delta\pi_s - \delta\underline{\pi} \geq (1-\delta)(1+\alpha)\, b_s \tag{NR$_\mathrm{S}$}$$

and

$$\delta\pi_n - \delta\underline{\pi} \geq (1-\delta)\, b_n. \tag{NR$_\mathrm{N}$}$$

For the worker, he deviates off-schedule when he does not put in effort e. When this occurs, the worker will receive zero bonus and that his continuation payoff will be $\underline{u}$. By deviating away from e, the worker gains most by putting in zero level of effort. Therefore, to prevent the worker from off-schedule deviation, the worker's payoff from putting in zero effort and receiving the base wage and a continuation payoff of $\underline{u}$ must be smaller than his equilibrium payoff. In other words,

$$\delta\underline{u} + (1-\delta)w \leq u. \tag{IC$_\mathrm{W}$}$$

In addition to off-schedule, there are on-schedule deviations, which are those privately observed by the parties. Since only the manager has private information, there are two types of on-schedule deviations. First, the manager pays b_s in a no-shock state. Second, she pays b_n in a shock state. To prevent the manager from paying out b_s in a no-shock state, we need

$$\delta(\pi_n - \pi_s) \geq (1-\delta)(b_n - b_s). \tag{TT$_\mathrm{N}$}$$

Similar, to prevent the manager from paying out b_n in a shock state, we need

$$\delta(\pi_n - \pi_s) \leq (1+\alpha)(1-\delta)(b_n - b_s). \tag{TT$_\mathrm{S}$}$$

9.1.3 Promise-Keeping

Finally, the consistency of the PPE payoff decomposition requires that the players' payoffs are equal to the weighted sum of current and future payoffs. Specifically, we have

$$\pi = \theta\left((1-\delta)\left(y(e) - (1+\alpha)(w+b_s)\right) + \delta\pi_s\right) + (1-\theta)\left((1-\delta)\left(y(e) - w - b_n\right) + \delta\pi_n\right), \tag{PK$_\mathrm{M}$}$$

and

$$u = \theta((1-\delta)(w+b_s) + \delta u_s) + (1-\theta)\left((1-\delta)(w+b_n) + \delta u_n\right) - (1-\delta)\, c(e). \tag{PK$_\mathrm{W}$}$$

9.2 Preliminary Lemmas

For the analysis below, we will first prove all of the results by ignoring IC_W. In other words, the analysis can be thought of as dealing with a model in which the worker's effort is contractible. We check at the end of the analysis (in Part 4) that IC_W holds for all optimal relational contracts for this case.

LEMMA 1'. *The PPE payoff set E is compact. Let $\overline{\pi}$ be the maximum PPE payoff of the manager. The PPE payoff set E is given by*

$$E = \{(\pi', u') : \pi' \in [\underline{\pi}, \overline{\pi}], u' \in [\underline{u}, u(\pi')]\}.$$

In addition,

$$u(\overline{\pi}) = \underline{u}.$$

Proof: First, note that $(\underline{\pi}, \underline{u})$ is in the PPE payoff set, enforced by that the parties taking their outside options in each period. Note also that $\underline{\pi}$ is the manager's minmax payoff and $\underline{u}$ is the worker's minmax payoff, it follows that any PPE payoff must give the manager at least $\underline{\pi}$ and the worker at least $\underline{u}$. It is then immediate that the bonus payment of the managers in any PPE is bounded above, and consequently, the worker's effort is also bounded above. In other words, we can restrict the actions of the manager and the worker to compact sets. Standard argument then implies that the PPE payoff set E is compact, and

$$u(\pi) = \max\{u, (\pi, u) \in E\}.$$

Now to see that $u(\overline{\pi}) = \underline{u}$, suppose to the contrary that $u(\overline{\pi}) > \underline{u}$. Note that $(\overline{\pi}, u(\overline{\pi}))$ is an extremal point of the PPE, so it is sustained by pure action in period 1. Let $e(\overline{\pi})$ be the worker's effort associated with $\overline{\pi}$ in period 1. Now by increasing $e(\overline{\pi})$ to $e(\overline{\pi}) + \varepsilon$ for small enough ε and keep everything else the same. This change results in a strategy that is also a PPE. But the new PPE gives the manager a higher payoff than $\overline{\pi}$. This contradicts the definition of $\overline{\pi}$.

Now the availability of the public randomization device implies that any payoff on the line segment between $(\underline{\pi}, \underline{u})$ and $(\overline{\pi}, \underline{u})$ can be supported as a PPE payoff. It then follows that any payoff (π, u') can be obtained from the randomization between $(\pi, \underline{u})$ and $(\pi, u(\pi))$ for all $u' \in [\underline{u}, u(\pi)]$. Finally, since each player can choose to take its outside option, any PPE payoff pair must give the manager at least $\underline{\pi}$ and the worker at least $\underline{u}$. This finishes the proof. ■

The next lemma corresponds to Lemma 2 in the text except that it does not contain $u' > -1$, which will be proved later.

LEMMA 2'. *For all $\pi \in [\underline{\pi}, \overline{\pi}]$, the following properties hold. (i.): $(\pi, u(\pi))$ can be supported by pure actions in the stage game (other than taking the outside option), (ii.): u is concave, (iii.): u is differentiable with $u'(\pi) \leq -1/(1+\alpha\theta)$.*

Proof: Part (ii.) follows directly from the availability of the public randomization device.

Part (i.) results from the following two steps. In Step 1, we show that, for any $\pi_1 < \pi_2$, if both $u(\pi_1)$ and $u(\pi_2)$ can be sustained by pure actions in the stage game other than taking the outside option, then $u(\pi)$ can also be sustained by pure actions for any $\pi \in (\pi_1, \pi_2)$. In Step 2, we show that $u(\underline{\pi})$ is supported by a pure action in the stage game without taking the outside option.

To prove Step 1, it suffices to show that for any $\pi_1 < \pi_2$, if both $u(\pi_1)$ and $u(\pi_2)$ can be sustained by pure actions in the stage game (other than the outside option), then for any $\pi = \rho\pi_1 + (1-\rho)\pi_2$ for some $\rho \in (0,1)$, we can support $(\pi, \rho u(\pi_1) + (1-\rho)u(\pi_2))$ with pure actions. To do this, suppose $e_i, w_i, b_{s_i}, b_{n_i}, \pi_{s_i}, \pi_{n_i}$, $i = 1, 2$ are the actions and continuation payoffs associated with π_1 and π_2. Define e be the effort level such that

$$y(e) = \rho y(e_1) + (1-\rho)y(e_2).$$

Since y is increasing and concave,

$$y(\rho e_1 + (1-\rho)e_2) \geq \rho y(e_1) + (1-\rho)y(e_2).$$

Therefore, by the monotonicity of y, we have

$$e \leq \rho e_1 + (1-\rho)e_2.$$

Also, let

$$\begin{aligned} w &= \rho w_1 + (1-\rho)w_2; \\ b_s &= \rho b_{s_1} + (1-\rho)b_{s_2}; \\ b_n &= \rho b_{n_1} + (1-\rho)b_{n_2}; \\ \pi_s &= \rho \pi_{s_1} + (1-\rho)\pi_{s_2}; \\ \pi_n &= \rho \pi_{n_1} + (1-\rho)\pi_{n_2}. \end{aligned}$$

One can check that this set of e, w, b_s, b_n, π_s, and π_n supports a PPE that gives the manager π and the worker at least $\alpha u(\pi_1) + (1-\alpha)u(\pi_2)$. Therefore, $u(\pi)$ can be sustained by pure actions.

For Step 2, define π_0 as the smallest payoff of the manager such that $u(\pi_0)$ is sustained by pure action. If $\pi_0 > \underline{\pi}$, then on the one hand, for $\pi \in (\underline{\pi}, \pi_0)$, we have $u(\pi) < u(\pi_0)$. This is because

$u(\pi)$ is sustained by randomization (and that $u(\underline{\pi}) = \underline{u} < u(\pi_0)$), so its value falls in $(\underline{\pi}, u(\pi_0))$. On the other hand, consider the actions and continuation payoffs that supports $u(\pi_0)$, keep everything except increases the wage by small enough ε. It can be checked that this change implements a PPE payoff of $(\pi_0 - (1+\alpha\theta)\varepsilon, u(\pi_0)+\varepsilon)$. By the definition of u, this implies that

$$u(\pi_0 - (1+\alpha\theta)\varepsilon) \geq u(\pi_0) + \varepsilon.$$

By taking ε to 0, we then have that

$$u'_{-}(\pi) \leq -\frac{1}{1+\alpha\theta} < 0,$$

where $u'_{-}(\pi)$ is the left derivative of u at π. This implies that $u(\pi_0) < u(\pi_0 - (1+\alpha\theta)\varepsilon)$ for small enough ε, which contradicts the above. Therefore, we must have $\pi_0 = \underline{\pi}$, so Part (ii.) is proved.

For Part (iii.), consider $\pi < \overline{\pi}$. Let e be the worker's effort that supports $u(\pi)$. By increasing effort to $e+\varepsilon$ for small enough ε, we can support $(\pi + y(e+\varepsilon) - y(e), u(\pi) - (c(e+\varepsilon) - c(e)))$ as a PPE payoff. By the definition of u, this implies that

$$u(\pi + y(e+\varepsilon) - y(e)) \geq u(\pi) - (c(e+\varepsilon) - c(e)).$$

Sending ε to zero, we obtain that

$$-\frac{c'(e)}{y'(e)} \leq u'_{+}(\pi).$$

By the concavity of u, we have $u'_{+}(\pi) \leq u'_{-}(\pi)$, which is smaller than $-1/(1+\alpha\theta)$ by above. Therefore,

$$\frac{c'(e)}{y'(e)} \geq \frac{1}{1+\alpha\theta},$$

so $e > 0$. Now, by keeping everything the same except by lowering the effort for small enough ε, the same argument as above shows that $u'_{-}(\pi) \leq -c'(e)/y'(e)$.

This implies that $u'_{+}(\pi) = u'_{-}(\pi)$, and, thus, u is differentiable. And since we have $u'_{-}(\pi) \leq -1/(1+\alpha\theta)$, by above, Part (iii.) is proved. ■

LEMMA 3'. *Consider a payoff pair, $(\pi, u(\pi))$, on the PPE frontier. Let (π_s, u_s) and (π_n, u_n) be the associated continuation payoffs following the shock and no-shock state respectively. Then*

$$\begin{aligned} u_s &= u(\pi_s); \\ u_n &= u(\pi_n). \end{aligned}$$

Proof: Suppose to the contrary that $u_s < u(\pi_s)$. Now keep the same actions and continuation payoffs as $(\pi, u(\pi))$ except change the continuation payoff (π_s, u_s) to $(\pi_s, u_s + \varepsilon)$ for some small positive ε. This change does not violate any of the constraints, and therefore the new payoff pair from this change is also a PPE payoff. This new payoff pair again gives the manager the same payoff of π. But the worker's payoff has increased to $u(\pi)+\delta(1-\theta)\varepsilon$, violating the definition of $u(\pi)$. Identical argument shows that $u_n = u(\pi_n)$. ■

LEMMA 4'. *The no-shock bonus is always weakly larger than the shock bonus, that is,* $b_n(\pi) \geq b_s(\pi)$ *for all* $\pi \in [\underline{\pi}, \overline{\pi}]$.

Proof: By combining the truth-telling constraints both in the shock and no-shock state, we have

$$(1+\alpha)(1-\delta)(b_n - b_s) \geq \delta(\pi_n - \pi_s) \geq (1-\delta)(b_n - b_s).$$

This implies $b_n - b_s \geq 0$. ■

LEMMA 5'. *For any payoff pair* $(\pi, u(\pi))$ *on the frontier, there exists a set of actions and continuation payoffs supporting it such that (i.) the manager's truth-telling constraint in a no-shock period* $\mathrm{TT_N}$ *is binding and (ii.) the shock bonus* $b_s(\pi)$ *is zero.*

Proof: For Part (i.), consider, to the contrary that $\delta(\pi_n - \pi_s) > (1-\delta)(b_n - b_s)$ for some π. Let

$$\begin{aligned}\pi_s' &= \pi_s + \theta\varepsilon;\\ \pi_n' &= \pi_n - (1-\theta)\varepsilon.\end{aligned}$$

for some small ε (while keeping w, b_s, b_n and e). This change keep all of the constraints satisfied, and in addition, the new set of continuation payoffs satisfy

$$\theta\pi_s' + (1-\theta)\pi_n' = \theta\pi_s + (1-\theta)\pi_n.$$

This implies that the worker's payoff under the new continuation payoffs is

$$\begin{aligned}&(1-\delta)(w + \theta b_s + (1-\theta)b_n - c(e)) + \delta(\theta u(\pi_s') + (1-\theta)u(\pi_n'))\\ \geq\ &(1-\delta)(w + \theta b_s + (1-\theta)b_n - c(e)) + \delta(\theta u(\pi_s) + (1-\theta)u(\pi_n)),\end{aligned}$$

where the inequality follows from the concavity of u. In other words, we can (weakly) increase $u(\pi)$ by shortening the distance between π_s and π_n until $\mathrm{TT_N}$ binds, and this proves Part (i.).

For Part (ii.), suppose $u(\pi)$ is supported with w, b_s, b_n, and the associated effort and continuation payoffs. Replace w, b_s,and b_n with $w + b_s, 0$, and $b_n - b_s$ and keep the rest. All constraints remain satisfied with this change, and this proves Part (ii.). ■

LEMMA 6'. *The PPE frontier* $u(\pi)$ *satisfies the following. For all* $\pi \in [\underline{\pi}, \overline{\pi}]$, $\pi + u(\pi)$ *is equal to*

$$\max_{e,w,\pi_s,\pi_n} (1-\delta)(y(e) - c(e)) + \theta\delta(\pi_s + u(\pi_s)) + (1-\theta)\delta(\pi_n + u(\pi_n)) - (1-\delta)\theta\alpha w \qquad (4)$$

such that

$$\pi = (1-\delta)\, y(e) + \delta\pi_s - (1-\delta)(1+\theta\alpha)\, w, \tag{PK$_\mathrm{M}$}$$

$$\pi_s \geq \underline{\pi}, \tag{SE$_\mathrm{S}$}$$

$$\pi_n \leq \overline{\pi}, \quad \textit{and} \tag{SE$_\mathrm{N}$}$$

$$w \geq 0. \tag{NN$_\mathrm{W}$}$$

Proof: By Lemma 2', every point on the PPE frontier can be supported by pure actions other than the outside options. Lemma 3' and the definition of u then imply that $\pi + u(\pi)$ is given by (1) subject to the constraints discussed in the List of Constraints subsection. In particular b_s and b_n do not appear in the maximization program because by Lemma 4' and 5', we can use w, π_s and π_n to substitute out b_s and b_n.

Now to reduce the constraints in the List of Constraints subsection, we first consider the feasibility constraints. By Lemma 5', $\mathrm{NN_S}$ and $\mathrm{NN_N}$ no longer exists. From the proof in Lemma 2', we see that $\mathrm{NN_e}$ is always satisfied. In addition, Lemma 3' implies that $\mathrm{SE_S}$ and $\mathrm{SE_N}$ can be reduced to

$$\begin{aligned}
\underline{\pi} &\leq \pi_s \leq \overline{\pi}; \\
\underline{\pi} &\leq \pi_n \leq \overline{\pi}.
\end{aligned}$$

Since $b_n \geq b_s$ (by Lemma 4'), Part (i.) of Lemma 5' then implies $\pi_n \geq \pi_s$. This implies that feasibility constraints above can be reduced to $\underline{\pi} \leq \pi_s$ and $\pi_n \leq \overline{\pi}$. In other words, the remaining feasibility constraints are $\mathrm{SE_N}$ $\mathrm{SE_S}$, and $\mathrm{NN_W}$.

Next, we examine the No-Deviation constraints. $\mathrm{NR_S}$ follows immediately from Lemma 5' and $\mathrm{SE_S}$. $\mathrm{NR_N}$ follows from $\mathrm{SE_S}$, Lemma 4', and Part (i.) of Lemma 5'. $\mathrm{TT_N}$ and $\mathrm{TT_S}$ are satisfied by Lemma 4'. This eliminates all of the No-Deviation constraints.

Finally, $\mathrm{PK_M}$ in Lemma 6' is obtained by substituting b_s and b_n out of the original $\mathrm{PK_M}$ and $\mathrm{PK_W}$ is replaced by the restriction that $u(\pi)$ is equal to the value from the constrained maximization problem. ■

9.3 Dynamics

Define the Lagrangian as

$$\begin{aligned}
L &= (1-\delta)(y(e) - c(e)) + \theta\delta(\pi_s + u(\pi_s)) + (1-\theta)\,\delta(\pi_n + u(\pi_n)) - (1-\delta)\,\theta\alpha w \\
&\quad + \lambda_1(\pi - (1-\delta)\, y(e) - \delta\pi_s + (1-\delta)(1+\theta\alpha)\, w) \\
&\quad + \lambda_2(\delta\pi_s - \delta\underline{\pi}) + \lambda_3(1-\delta)w + \lambda_4(\delta\overline{\pi} - \delta\pi_n).
\end{aligned}$$

Note that this is a well-defined concave program. In particular, we have the following conditions. The FOCs with respect to π_s, w, and π_n are given by

$$\theta(1+u'(\pi_s)) - \lambda_1 + \lambda_2 = 0; \tag{FOC$_\text{S}$}$$

$$-\theta\alpha + \lambda_1(1+\theta\alpha) + \lambda_3 = 0; \tag{FOC$_\text{W}$}$$

$$(1-\theta)(1+u'(\pi_n)) - \lambda_4 = 0; \tag{FOC$_\text{N}$}$$

$$y'(e) - c'(e) - \lambda_1 y'(e) = 0. \tag{FOC$_\text{e}$}$$

The envelop condition is given by

$$1 + u'(\pi) = \lambda_1. \tag{envelop}$$

Now we can prove the missing part in Lemma 2' on the lower bound of $u'(\pi)$.

LEMMA 2". *For each* $\pi \in [\underline{\pi}, \overline{\pi}]$, $u'(\pi) > -1$.

Proof: To prove this, we first show that $u'(\pi_n) \leq u'(\pi)$. To see this, consider two cases. In Case 1, $\lambda_4 > 0$. In this case, we have $\pi_n = \overline{\pi}$, so $u'(\pi_n) \leq u'(\pi)$ by the concavity of u. In Case 2, $\lambda_4 = 0$. In this case, adding FOC_S and FOC_N, and replacing λ_1 with $1 + u'(\pi)$, we get

$$\theta u'(\pi_s) + (1-\theta)u'(\pi_n) + \lambda_2 = u'(\pi).$$

Note that $u'(\pi_s) \geq u'(\pi_n)$ by Lemma 4' and the concavity of u. The equality above then implies that

$$u'(\pi_n) \leq u'(\pi)$$

since $\lambda_2 \geq 0$. This finishes proving that $u'(\pi_n) \leq u'(\pi)$.

Next, we show that $u'(\pi) \geq -1$ for all π. Note that from FOC_N, we see that $u'(\pi_n) \geq -1$ since $\lambda_4 \geq 0$. Since $u'(\pi_n) \leq u'(\pi)$ by above, this gives that $u'(\pi) \geq -1$ for all π.

Now suppose to the contrary that $u'(\pi) = -1$. Then the following conditions must hold. First, we have $\lambda_1 = 0$ from the envelop condition, implying that $u'(\pi_s) = -1$ from FOC_S. Second, $u'(\pi_n) = -1$ since $u'(\pi_n) \leq u'(\pi)$. Third, $e = e^{FB}$ by FOC_e. Fourth, $w = 0$ because of FOC_W.

The four conditions above imply that there is a line segment on the PPE frontier with slope -1 such that a) $w = 0$ and $e = e^{FB}$ and b) the continuation payoffs (without public deviation) always stay on this line segment. This implies that the manager can maximize her pay by always announcing that the state is a shock state (since the worker always puts in e^{FB} and the manager always pays out 0 in this case). But this is a contradiction because in this case, the worker's payoff is $-c(e^{FB})$, which is smaller than his outside option. ■

Proof of Proposition 1: We first prove the results by neglecting the IC_W constraint. In other words, we first show that Conditions (a.)-(e.) are satisfied in each optimal relational contract in which the worker's effort were contractible. We then show that for such optimal relational contracts, the IC_W constraint is satisfied. This implies that the optimal relational contract with contractible effort coincides with the optimal relational contract in our model, and, thus, finishes the proof.

Part (e.) follows from Lemma 2" and FOC_N. For Part (d.), consider FOC_S, and there are two cases. In Case 1, $\lambda_2 > 0$, so $\pi_s = \underline{\pi}$, and trivially Part (d.) holds. In Case 2, $\lambda_2 = 0$, so

$$1 + u'(\pi_s) = \frac{1}{\theta}\lambda_1 = \frac{1}{\theta}(1 + u'(\pi)),$$

where the second equality comes from the envelop condition. Since $1 + u'(\pi) > 0$ by Lemma 2", we have $u'(\pi_s) > u'(\pi)$, and again (d.) holds by the concavity of u.

Part (c.) follows from TT_N. Part (a.) follows from combining FOC_e and the envelop condition. Note that since $c'(e)/y'(e)$ is strictly increasing, $e^*(\pi)$ is unique.

For Part (b.), recall from Lemma 2' that $u'(\pi) \leq -1/(1+\alpha\theta)$. Now consider two cases. In Case 1, suppose $u'(\pi) < -1/(1+\alpha\theta)$ for all $\pi \in [\underline{\pi}, \overline{\pi}]$. In this case, we have $w \equiv 0$ by FOC_W and the envelop condition.

In Case 2, there exists a line segment with $u'(\pi) = -1/(1+\alpha\theta)$. Now recall that $\widehat{e}$ is the unique effort level with Figure $3(\widehat{e})/y'(\widehat{e}) = 1/(1+\alpha\theta)$. On this line segment, we have $e = \widehat{e}$ by Part (a.). In addition, we must have $\pi_s = \underline{\pi}$. To see this, note that when $u'(\pi) = -1/(1+\alpha\theta)$, we also have $u'(\pi_s) = -1/(1+\alpha\theta)$ by Part (d.) and Lemma 2'. Substituting out $u'(\pi_s)$ and λ_1 in FOC_S, we obtain that $\lambda_2 > 0$. The complementary slackness then implies that $\pi_s = \underline{\pi}$. With $\pi_s(\pi) = \underline{\pi}$ and $e(\pi) = \widehat{e}$ for π on the line segment, we have, by PK_M, that

$$w^*(\pi) = \frac{1}{(1-\delta)(1+\alpha\theta)}((1-\delta)y(\widehat{e}) + \delta\underline{\pi} - \pi).$$

Note that the right end of this line segment, $\widehat{\pi}$, satisfies $w^*(\pi) = 0$. This proves Part (b.).

Note that in all of the derivations above, $e^*(\pi), w^*(\pi)$, and $\pi_n^*(\pi)$ are unique. Then by PK_M, $\pi_s^*(\pi)$ is also unique. This implies that the implementation of the optimal relational contract is unique.

Finally, we check that IC_W is satisfied for each point on the PPE frontier, i.e.,

$$\delta\underline{u} + (1-\delta)w^*(\pi) \leq u(\pi).$$

The inequality is clearly satisfied when $w = 0$. From Part (b.), this then implies that the inequality is satisfied for all $\pi \geq \widehat{\pi}$.

For $\pi < \widehat{\pi}$, Part (b.) implies that

$$\frac{d(\delta \underline{u} + (1-\delta) w^*(\pi))}{d\pi} = -\frac{1}{1+\alpha\theta} = u'(\pi).$$

Therefore, for all $\pi < \widehat{\pi}$,

$$u(\pi) - (\delta \underline{u} + (1-\delta) w^*(\pi)) = u(\widehat{\pi}) - (\delta \underline{u} + (1-\delta) w^*(\widehat{\pi})) > 0.$$

This shows that $\mathrm{IC_W}$ is satisfied for all $\pi \in [\underline{\pi}, \overline{\pi}]$. ■

10 Appendix B: Allowing for Liquidity Constraints

In this appendix we prove the results in Section 6 that analyzes the model with liquidity constraints. Specifically, the firm is subject to the liquidity constraint that

$$\max\{w + b_s, w + b_n\} \leq (1 + m)y$$

for some $m > 0$. This constraint significantly complicates the analysis. In particular, the PPE frontier is no longer differentiable. To make the analysis more tractable, we assume that the manager cannot pay the worker in a shock state ($\alpha = \infty$). Consequently, this implies that $w \equiv 0$ and $b_s \equiv 0$.

Since the analysis with the liquidity constraints follows similar steps as the main model, we omit the proofs for results that are obtained from identical arguments. Below, we first describe the basic properties of the PPE frontier in the background subsection and then prove the main results in the dynamics subsection. The last subsection provides several sufficient conditions that allow for further characterization of the dynamics.

10.1 Background

Proceeding in the same way as the main model, we can show that for each payoff on the PPE frontier sustainable by pure actions (other than the outside options) must satisfy the following:

$$\pi + u(\pi) = \max_{e,\pi_s,\pi_n} (1-\delta)\left(y\left(e\right) - c\left(e\right)\right) + \theta\delta\left(\pi_s + u\left(\pi_s\right)\right) + \left(1-\theta\right)\delta\left(\pi_n + u\left(\pi_n\right)\right) \tag{5}$$

subject to

$$\pi = (1-\delta)\, y\left(e\right) + \delta\pi_s, \tag{LPK$_\mathrm{M}$}$$

$$\delta\pi_n \leq \pi + (1-\delta)my(e), \tag{LIQ$_\mathrm{F}$}$$

$$\pi_s \geq \underline{\pi}, \text{ and} \tag{LSE$_\mathrm{S}$}$$

$$\pi_n \leq \overline{\pi}. \tag{LSE$_\mathrm{N}$}$$

The functional equation above corresponds with that in Lemma 6 with proper modifications. In particular, $\mathrm{LIQ_F}$ is the extra liquidity constraint for the firm, and there is no $\mathrm{NN_W}$ because $\alpha = \infty$. Moreover, $\mathrm{LPK_M}$ is obtained with a simplified truth-telling condition of the manager that

$$\delta(\pi_n - \pi_s) = (1-\delta)b_n. \tag{LTT$_\mathrm{N}$}$$

In addition to these modifications, there are some additional differences between the main model and the liquidity constraint case. First, unlike the main model, it is no longer true that each point on the PPE frontier is sustainable by pure actions. Define π_0 as the smallest payoff of the manager at which $u(\pi_0)$ can be sustained by pure actions other than the outside options. The following lemma captures the difference.

LEMMA 7. *There exists a critical level of expected profits $\pi_0 \in [\underline{\pi}, \overline{\pi})$ such that for all $\pi \geq \pi_0$ the PPE frontier $u(\pi)$ is supported by pure actions and for all $\pi \in (\underline{\pi}, \pi_0)$ it is supported by randomization. Specifically, for any $\pi < \pi_0$ the manager and the worker randomize between terminating their relationship and playing the strategies that deliver expected payoffs π_0 and $u(\pi_0)$.*

Proof: The part that for all $\pi \geq \pi_0$ the PPE frontier $u(\pi)$ is supported by pure actions follows from identical proof as in Lemma 2'. It remains to show that for any $\pi \in (\underline{\pi}, \pi_0)$ the manager and the worker randomize between terminating their relationship and playing the strategies that deliver expected payoffs π_0 and $u(\pi_0)$. Now consider two cases. In Case 1, $\pi_0 = \underline{\pi}$, we return to Lemma 2', and the lemma is clearly satisfied.

In Case 2, $\pi_0 > \underline{\pi}$. In this case, since $(\underline{\pi}, u(\underline{\pi}))$ is an extremal point of the PPE payoff set, it must be supported by pure actions. Given that $\pi_0 > \underline{\pi}$, $(\underline{\pi}, u(\underline{\pi}))$ must be supported by the outside options. It then follows that $(\underline{\pi}, u(\underline{\pi})) = (\underline{\pi}, \underline{u})$.

Now for any $\pi \in (\underline{\pi}, \pi_0)$, $(\pi, u(\pi))$ is sustained by randomization by the definition of π_0. We may assume that $(\pi, u(\pi))$ is the weighted average of $(\pi', u(\pi'))$ and $(\pi'', u(\pi''))$ such that $\pi' < \pi < \pi''$ and that $u(\pi')$ and $u(\pi'')$ are both sustained by pure actions.

Since $\pi' < \pi < \pi_0$ and that $u(\pi')$ is supported by pure actions, we must have $\pi' = \underline{\pi}$ by the definition of π_0. Now by the concavity of u, it is clear that we can have $\pi'' = \pi_0$. In fact, we can show that π'' must equal to π_0. The argument follows the exact same logic as Step 2 and Step 3 in Lemma B3 below and is omitted. ■

Second, the PPE frontier u is no longer differentiable for all π for some parameter value m. However, since u is again concave (given the public randomization device), both the left and the right derivatives exist. This implies that the results written as equalities of derivatives can be replaced with a pair of corresponding inequalities involving left and right derivatives.

Third, while it remains true that $u'_{-}(\pi) > -1$ for all π, we no longer always have $\pi_n = \overline{\pi}$. When LIQ_F binds, $\pi_n < \overline{\pi}$. The lemma below gives the exact expression for π_n.

LEMMA B1. *The continuation payoff following a no-shock period satisfies the following:*

$$\pi_n = \min\{\overline{\pi}, \frac{1}{\delta}(\pi + (1-\delta)my)\}.$$

Proof: Immediate. ■

Fourth, unlike the main model in which there is flexibility in choosing between w and b_s, the maximizers $(e(\pi), \pi_s(\pi), \pi_n(\pi))$ are unique. Note that since the maximizers are upper-hemicontinuous, a direct consequence of the uniqueness result is that e, π_s, and π_n are continuous in π.

LEMMA B2. *For each $\pi \geq \pi_0$, there is a unique set of $e(\pi), \pi_s(\pi)$, and $\pi_n(\pi)$ that maximizes $u(\pi)$.*

Proof: Let $e_i, \pi_{s_i}, \pi_{n_i}$, $i = 1, 2$ be the associated effort and continuation payoffs as maximizers. Then for some $\rho \in (0, 1)$, let

$$\pi_s = \rho\pi_{s_1} + (1-\rho)\pi_{s_2};$$
$$\pi_n = \rho\pi_{n_1} + (1-\rho)\pi_{n_2};$$

Let e be the unique effort level such that

$$y(e) = \rho y(e_1) + (1-\rho)y(e_2).$$

The strict concavity of y then implies $e \leq \rho e_1 + (1-\rho)e_2$.

It is clear that e, π_s, π_n also supports π. Moreover, the strict concavity of y and the strict convexity of the cost function c implies that the value generated by this new set of choices is strictly larger than those from $e_i, \pi_{s_i}, \pi_{n_i}$, $i = 1, 2$ when $e_1 \neq e_2$. Therefore, we must $e_1 = e_2$. It then follows that $\pi_{s1} = \pi_{s2}$ from LPK_M. Finally, by Lemma B1, $\pi_n = \min\{\overline{\pi}, (\pi + (1-\delta)my)/\delta\}$, so π_n is also unique. ■

Fifth, the proof that $e > 0$ in the liquidity constraints case is different from that in the main model. We state this result as a separate lemma below.

LEMMA B3. *For each $\pi \geq \pi_0$, $e(\pi) > 0$.*

Proof: We first prove that $e(\pi) > 0$ for $\pi > \underline{\pi}$. Suppose to the contrary that there exists a $\pi' > \underline{\pi}$ with $e = 0$. In this case, the liquidity constraint implies that $b_n = 0$, and LTT_N then implies that $\pi_s(\pi') = \pi_n(\pi')$. Now from LPK_M, we have $\pi_s(\pi') = \pi_n(\pi') = \pi/\delta > \pi$. This allows us to derive a contradiction in three steps.

In Step 1, we show that $u'_-(\pi') = u'_-(\pi'/\delta)$, so u is a line segment in $[\pi', \pi'/\delta]$. To see this, suppose to the contrary that $u'_-(\pi') > u'_-(\pi'/\delta)$. Now for small enough $\varepsilon > 0$, we can change the continuation payoffs (both in the shock and no-shock state) to $(\pi'/\delta - \varepsilon, u(\pi'/\delta - \varepsilon))$ with the actions unchanged. This change generates a PPE payoff that gives the manager $\pi' - \delta\varepsilon$ and the worker $u(\pi') + \delta(u(\pi'/\delta - \varepsilon) - u(\pi'/\delta))$.

By the definition of u, we have

$$u(\pi' - \delta\varepsilon) \geq u(\pi') + \delta(u(\frac{\pi'}{\delta} - \varepsilon) - u(\frac{\pi'}{\delta})).$$

Sending ε to 0, the above implies that

$$u'_-(\pi') \leq u'_-(\pi'/\delta),$$

contradicting the assumption that $u'_-(\pi') > u'_-(\pi'/\delta)$.

In Step 2, we show that u is a line segment in $[\underline{\pi}, \pi'/\delta]$. To see this, for all $\pi < \pi'$, let $e(\pi) = 0$, $b_n(\pi) = 0$, and $\pi_n(\pi) = \pi_s(\pi) = \pi/\delta$. It can be checked that this choice generates a PPE payoff that lies on the left extension of the line segment between $(\pi', u(\pi'))$ and $(\pi'/\delta, u(\pi'/\delta))$. By the concavity of u, it follows that these payoffs are on the PPE frontier. It follows that u is a line segment in $[\underline{\pi}, \pi'/\delta]$.

By Step 2, the PPE frontier contains a line segment in the left. Let π_+ be the right end point of this segment. In Step 3, we derive a contradiction on the left derivative of u for payoffs near $\delta\pi_+$. To do so, first note that by the same construction as in Step 2, we must have $e(\delta\pi_+) = 0$, $\pi_s(\delta\pi_+) = \pi_s(\delta\pi_+) = \pi_+$ by the uniqueness of the maximizer. It then follows that for any $\pi'' \in (\delta\pi_+, \delta\pi_+ + \varepsilon)$ for small enough $\varepsilon > 0$, we must have $\pi_n(\pi'') > \pi_+$ by LIQ_F and $\pi_s(\pi'') > \underline{\pi}$ by the continuity of π_s. Now by the definition of π_+, we have

$$u'_-(\pi_n(\pi'')) < u'_-(\pi'').$$

In addition, $\pi_s(\pi'') > \underline{\pi}$, so $u'_-(\pi_s(\pi''))$ exists. Moreover,

$$u'_-(\pi_s(\pi'')) \leq u'_-(\pi'')$$

since u is a line segment in $[\underline{\pi}, \pi_+]$. The two inequalities above then imply that

$$(1-\theta)u'_-(\pi_n(\pi'')) + \theta u'_-(\pi_s(\pi'')) < u'_-(\pi'').$$

Now, starting at $(\pi'', u(\pi''))$, by changing the continuation payoffs to $(\pi_s(\pi'') - \varepsilon, u(\pi_s(\pi'') - \varepsilon))$ and $(\pi_n(\pi'') - \varepsilon, u(\pi_n(\pi'') - \varepsilon))$ and keeping the actions unchanged, the same argument as in Step 1 implies that

$$(1-\theta)u'_-(\pi_n(\pi'')) + \theta u'_-(\pi_s(\pi'')) \geq u'_-(\pi''),$$

which contradicts the earlier inequality. This proves that we cannot have $e(\pi) = 0$ for $\pi > \underline{\pi}$.

The argument above also helps prove $e(\underline{\pi}) > 0$. Suppose to the contrary that $e(\underline{\pi}) = 0$, then $\pi_s(\underline{\pi}) = \pi_n(\underline{\pi}) > \underline{\pi}$. The same argument as above can be used to show that u is a line segment in $[\underline{\pi}, \pi_n(\pi)]$, and we can derive a similar type of contradiction as above. ■

The next lemma shows that for $\pi \geq \pi_0$, the PPE frontier can be divided into (at most) three regions. In the right region, the liquidity constraints are slack. In the left region, the liquidity constraints are binding and $\pi_n < \overline{\pi}$. In the middle region, the liquidity constraints are binding and $\pi_n = \overline{\pi}$.

LEMMA B4. *There exists π_1 and π_2 with $\pi_0 \leq \pi_1 \leq \pi_2 < \overline{\pi}$ such that the following holds: (i.) If $\pi > \pi_2$, $\pi_n = \overline{\pi}$ and $\pi + (1-\delta)my > \delta\overline{\pi}$, (ii.) if $\pi \in [\pi_1, \pi_2]$, $\pi_n = \overline{\pi}$ and $\pi + (1-\delta)my = \delta\overline{\pi}$, and (iii.) If $\pi < \pi_1$, $\pi_n < \overline{\pi}$ and $\pi + (1-\delta)my = \delta\overline{\pi}$.*

Proof: To prove Part (i.), if suffices to show that for any $\pi' \geq \pi$, if $\pi + (1-\delta)my(e(\pi)) > \delta\overline{\pi}$, then we also have $\pi' + (1-\delta)my(e(\pi')) > \delta\overline{\pi}$. Now take a manager's payoff π with $\pi + (1-\delta)my(e(\pi)) > \delta\overline{\pi}$, the same argument as in Lemma 2' shows that u is differentiable at π with

$$u'(\pi) = -\frac{c'(e(\pi))}{y'(e(\pi))}.$$

In addition, for $\pi' > \pi$, again the same argument as in Lemma 2' shows that

$$\frac{c'(e(\pi'))}{y'(e(\pi'))} \geq -u'_+(\pi').$$

Note that $u'_+(\pi') \leq u'_+(\pi)$ since u is concave, it follows that

$$\frac{c'(e(\pi'))}{y'(e(\pi'))} > \frac{c'(e(\pi))}{y'(e(\pi))}.$$

This gives that $e(\pi') \geq e(\pi)$, and, thus, $\pi' + (1-\delta)my(e(\pi')) > \delta\overline{\pi}$. This proves Part (i.).

Given Part (i.), we prove Parts (ii.) and (iii.) simultaneously by showing that if $\pi_n(\pi) = \overline{\pi}$, then for all $\pi' > \pi$, $\pi_n(\pi') = \overline{\pi}$. Suppose the contrary. Then there exists a pair of $\pi' > \pi$ such that

$$\pi'_n = \pi' + (1-\delta)my(e(\pi')) < \pi + (1-\delta)my(e(\pi)) = \overline{\pi}.$$

Now at π', increase $e(\pi')$ to $e(\pi') + \varepsilon$, keep $\pi_s(\pi')$ the same, and increase $\pi_n(\pi')$ correspondingly so that the liquidity constraint remains to bind. This change is feasible, so it creates a payoff that falls weakly below the PPE frontier. Sending ε to zero, we get

$$\frac{c'(e(\pi'))}{y'(e(\pi'))} \geq (m+1)(1-\theta)(1 + u'_+(\pi_n(\pi')) - u'_+(\pi').$$

Similarly, at π, decrease $e(\pi)$ to $e(\pi) - \varepsilon$, keep $\pi_s(\pi)$ the same, and decrease $\pi_n(\pi)$ correspondingly, we get

$$\frac{c'(e(\pi))}{y'(e(\pi))} \leq (m+1)(1-\theta)(1 + u'_-(\overline{\pi})) - u'_-(\pi).$$

Since u is concave, we have $u'_+(\pi_n(\pi') > u'_-(\overline{\pi})$ and $u'_+(\pi') \leq u'_-(\pi)$. The two inequalities above then imply that

$$\frac{c'\left(e\left(\pi'\right)\right)}{y'\left(e\left(\pi'\right)\right)} \geq \frac{c'\left(e\left(\pi\right)\right)}{y'\left(e\left(\pi\right)\right)},$$

so $e(\pi') \geq e(\pi)$. But this contradicts

$$\pi' + (1-\delta)my(e\left(\pi'\right)) < \pi + (1-\delta)my(e\left(\pi\right)). \quad \blacksquare$$

Note that the right region always exist (so that $\pi_2 < \overline{\pi}$) because for all $\pi > \delta\overline{\pi}$, $\pi + (1-\delta)my(e\left(\pi\right)) > \delta\overline{\pi}$. In contrast, the middle region or the left region does not always exist. This can occur, for example, when m is large and when $\underline{\pi}$ is large. In this case, the firm's liquidity constraint is always slack, and we return to the main model. At the end of this section, we give sufficient conditions for the existence of the left and the middle region.

10.2 PPE Frontier

In this subsection, we describe the dynamics of the optimal relational contract.

LEMMA B5. *The set of effort and continuation payoffs* $(e(\pi), \pi_s(\pi)$, *and* $\pi_n(\pi))$ *satisfies the following.*

(*a.*) *For* $\pi > \pi_2$, *the PPE frontier is differentiable with*

$$\begin{aligned} \frac{c'\left(e\right)}{y'\left(e\right)} &= -u'(\pi) \text{ and} \\ -\left(1-\theta\right) + \theta u'_+\left(\pi_s\right) &\leq u'\left(\pi\right) \leq -\left(1-\theta\right) + \theta u'_-\left(\pi_s\right). \end{aligned}$$

In this region, both e *and* π_s *weakly increase with* π.

(*b.*) *For* $\pi \in [\pi_1, \pi_2]$, *if* $m \neq 0$, *then*

$$y = \frac{\delta\overline{\pi} - \pi}{(1-\delta)m} \text{ and } \pi_s = \frac{(m+1)\pi + \delta\overline{\pi}}{m}.$$

In this region, e *strictly decreases with* π *and* π_s *strictly increases with* π.

If $m = 0$, *then* $\pi_1 = \pi_2 = \delta\overline{\pi}$. u *is not differentiable at* $\delta\overline{\pi}$, *and* e *and* π_s *satisfy*

$$\begin{aligned} -u'_+(\delta\overline{\pi}) &\leq \frac{c'\left(e\right)}{y'\left(e\right)} \leq -u'_-(\delta\overline{\pi}) \text{ and} \\ -\left(1-\theta\right) + \theta u'_+\left(\pi_s\right) &\leq u'\left(\pi\right) \leq -\left(1-\theta\right) + \theta u'_-\left(\pi_s\right). \end{aligned}$$

(*c.*) *For* $\pi \in [\pi_0, \pi_1]$, e, π_s, *and* π_n *satisfy*

$$\begin{aligned}(1+m)(1-\theta)(1+u'_+(\pi_n)) - \frac{c'(e)}{y'(e)} &\leq u'_+(\pi) \leq u'_-(\pi) \\ &\leq (1+m)(1-\theta)(1+u'_-(\pi_n)) - \frac{c'(e)}{y'(e)}.\end{aligned} \tag{L-e-n}$$

When $\pi_s > \underline{\pi}$,

$$\begin{aligned}(1+m)(-(1-\theta)+\theta u'_+(\pi_s)) + \frac{c'(e)}{y'(e)} &\leq mu'_+(\pi) \leq mu'_-(\pi) \\ &\leq (1+m)(-(1-\theta)+\theta u'_-(\pi_s)) + \frac{c'(e)}{y'(e)},\end{aligned} \tag{L-e-s}$$

and

$$\theta u'_+(\pi_s) + (1-\theta)\, u'_+(\pi_n) \leq u'_+(\pi) \leq u'_-(\pi) \leq \theta u'_-(\pi_s) + (1-\theta)\, u'_-(\pi_n). \tag{L-s-n}$$

In this region, π_s *weakly increases in* π.

Proof: The inequalities in this lemma are all equalities if u is differentiable. In this case, the equalities can be obtained directly from the Kuhn-Tucker conditions of Lagrangian associated with the constrained maximization problem (5). The formal proof of the inequalities is standard and is omitted here. Below, we show that u is not differentiable at $\delta\overline{\pi}$ and that π_s is weakly increasing in $[\pi_0, \pi_1]$.

First, to see that u is not differentiable at $\delta\overline{\pi}$ when $m = 0$, note that $\pi = \delta\overline{\pi}$ is the only point in the middle region. Moving from the right, we have by Part (a.) that

$$u'_+(\delta\overline{\pi}) = -\frac{c'(e)}{y'(e)}.$$

Moving from the left, we have by L-e-n that

$$u'_-(\delta\overline{\pi}) \geq -\frac{c'(e)}{y'(e)} + (1+m)(1-\theta)(1+u'_+(\pi_n)).$$

Since $u'_+(\pi_n) > -1$, we have

$$u'_-(\delta\overline{\pi}) > u'_+(\delta\overline{\pi}).$$

This shows that u is not differentiable at $\delta\overline{\pi}$.

Second, to see that π_s is weakly increasing for $\pi \in [\pi_0, \pi_1]$, we assume that u is differentiable at π and π_s to ease exposition. The argument can be adapted to the non-differentiable case. When u is differentiable at π and π_s, L-e-s becomes

$$-(1+m)(1-\theta) + (1+m)\theta u'(\pi_s) + \frac{c'(e)}{y'(e)} = mu'(\pi).$$

Now as π increases, the right hand side weakly decreases. Now suppose to the contrary that π_s decreases. In this case $u'(\pi_s)$ weakly increases. Moreover, when π increases and π_s decreases, e increases by the LPK_M. Consequently, $c'(e)/y'(e)$ strictly increases. Therefore, if π_s decreases, the left hand side strictly increases. This contradicts that the right hand side decreases. ■

Since $\pi_s(\pi)$ is weakly increasing in π in all three regions, the continuity of $\pi_s(\pi)$ then implies that $\pi_s(\pi)$ is weakly increasing for all $\pi \in [\pi_0, \overline{\pi}]$. In contrast, $e(\pi)$ is decreasing in the middle region. In other words, the worker's effort level increases as the manager's payoff decreases. Next, we characterize the dynamics of the optimal relational contract.

LEMMA B6. *For all $\pi \in [\pi_0, \overline{\pi}]$, $\pi_n(\pi) > \pi$.*

Proof: It is clear that $\pi_n(\pi) > \pi$ for π in the middle and the right region. In the left region, suppose the contrary. Then by the continuity of $\pi_n(\pi)$, there exists a largest $\pi < \overline{\pi}$ with $\pi_n(\pi) = \pi$. At this point, note that $u'_+(\pi_s) \geq u'_+(\pi_n) = u'_+(\pi)$. Then the first inequality in L-s-n implies that $u'_+(\pi_s) = u'_+(\pi_n)$. In other words, u is a line segment between π_s and π_n. Let π_+ be the right end point of this segment. Now consider two cases.

In Case 1, $\pi_+ = \overline{\pi}$. This is not possible because it implies that, for all $\pi' > \pi$, we have $u'_+(\pi_s(\pi')) = u'_+(\pi_n(\pi')) = u'_+(\pi')$. This violates the last inequality in Part (a.) of Lemma B5 since $u'_-(\pi_s) > -1$.

In Case 2, $\pi_+ < \overline{\pi}$. By the definition of π, we have $\pi_n(\pi') > \pi'$ for all $\pi' > \pi$. The continuity of π_n then implies that there exists a $\pi'' \in (\pi, \pi_+)$ with $\pi_n(\pi'') > \pi_+$. In addition, we have $\pi_s(\pi'') \in (\pi_s(\pi), \pi_+)$ since π_s increases with π. At π'', we have $u'_-(\pi_s(\pi'')) = u'_-(\pi'') > u'_-(\pi_n(\pi''))$, violating the last inequality in L-s-n. ■

LEMMA B7. *For all $\pi \in [\pi_0, \overline{\pi}]$, $\pi_s(\pi) < \pi$.*

Proof: It is clear that $\pi_s(\pi) < \pi$ for π in the middle and the right region. In the left region, suppose to the contrary there exists a manager's payoff π with $\pi_s(\pi) \geq \pi$. By L-s-n, u must be a line segment between π and $\pi_n(\pi)$. Let π_+ be the right end point of this line segment. Just as in the proof in Lemma B6, we must have $\pi_+ < \overline{\pi}$. Now by Lemma B6 and the monotonicity of π_s, there exists a $\pi' \in (\pi, \pi_+)$ such that $\pi_n(\pi') > \pi_+$ and $\pi_s(\pi') \in (\pi, \pi_+)$. This implies that $u'_-(\pi_s(\pi')) = u'_-(\pi') > u'_-(\pi_n(\pi'))$, violating L-s-n. ■

The previous two lemma describe the dynamics of the relationship for $\pi \geq \pi_0$. The next lemma provides further information on the dynamics by characterizing π_0.

LEMMA B8. *Either $\pi_0 = \underline{\pi}$ or $\pi_s(\pi_0) = \underline{\pi}$.*

Proof: Suppose $\pi_0 > \underline{\pi}$ and to the contrary $\pi_s(\pi_0) > \underline{\pi}$. L-s-n implies that $u'(\pi_0) = u'(\pi_s) = u'(\pi_n)$. Using the same argument as in Lemma B6, we can show that there exists $\pi_+ \geq \pi_0$ such that u is a line segment in $[\underline{\pi}, \pi_+]$ and derive a contradiction. ■

Proof of Proposition 2: Part (a.) is direct consequence of Lemma B5. Part (b.) follows from LTT_N. Part (c.) follows from Lemma B6. Part (d.) follows from Lemma B7 and B8. ■

10.3 Sufficient Conditions

In this subsection, we first provide a condition for the left region to exist, implying that the liquidity constraint is relevant. A sufficient condition for $\pi_0 > \underline{\pi}$ is given next. Finally, we provide a sufficient condition for the existence of the middle region.

Define $\overline{\pi}^u$ as the maximal equilibrium payoff of the manager in the main model.

LEMMA B9. *The PPE frontier contains more than the right region, i.e., ($\pi_2 > \underline{\pi}$) if and only if the following Condition L holds:*

$$\delta\overline{\pi}^u > (1+m)\underline{\pi}. \tag{L}$$

Proof: It suffices to look for the condition on whether the liquidity constraint is violated at $\underline{\pi}$ for the problem in the main model. Under the unconstrained problem, $\pi_n(\underline{\pi}) = \overline{\pi}^u$ and $\pi_s(\underline{\pi}) = \underline{\pi}$. In addition, NR_S states that

$$\underline{\pi} = \delta\pi_s(\underline{\pi}) + (1-\delta)y(e(\underline{\pi})).$$

This implies that $y(e(\underline{\pi})) = (1-\delta)\underline{\pi}$. Therefore, the liquidity constraint that $\delta\pi_n \leq \pi + (1-\delta)my(e)$ is equivalent to

$$\delta\overline{\pi}^u \leq (1+m)\underline{\pi}. \quad ■$$

Next, we describe a sufficient condition for $\pi_0 > \underline{\pi}$.

LEMMA B10. *Suppose Condition L holds. $\pi_0 > \underline{\pi}$ if $m < \frac{\theta}{1-\theta}$ and*

$$\underline{u} > \frac{(1-\theta)(1+m)}{\theta - m(1-\theta)} \frac{y'(\underline{e}) - c'(\underline{e})}{y'(\underline{e})}\underline{\pi} - c(\underline{e}),$$

where $\underline{e}$ is the unique effort level satisfying $y(\underline{e}) = \underline{\pi}$.

Proof: To prove that $\pi_0 > \underline{\pi}$ if the conditions above hold, we proceed as if u were differentiable to simplify the exposition. The argument can be adapted for the non-differentiable case by replacing the equalities involving derivatives with inequalities involving left and right derivatives. Now suppose to the contrary $\pi_0 = \underline{\pi}$, define the Lagrangian as

$$\begin{aligned}\pi + u(\pi) &= L = (1-\delta)\left(y\left(e\right) - c\left(e\right)\right) + \theta\delta\left(\pi_s + u\left(\pi_s\right)\right) + (1-\theta)\,\delta\left(\pi_n + u\left(\pi_n\right)\right) \\ &\quad + \lambda_1(\pi - (1-\delta)\,y\left(e\right) - \delta\pi_s) \\ &\quad + \lambda_2(\pi + (1-\delta)my(e) - \delta\pi_n) \\ &\quad + \lambda_3(\delta\pi_s - \delta\underline{\pi}) + \lambda_4\delta(\overline{\pi} - \pi_n).\end{aligned}$$

The FOCs are given by

$$\theta(1 + u'\left(\pi_s\right)) - \lambda_1 + \lambda_3 = 0. \qquad (\mathrm{FOC_S})$$

$$(1-\theta)\left(1 + u'\left(\pi_n\right)\right) - \lambda_2 - \lambda_4 = 0. \qquad (\mathrm{FOC_N})$$

$$(1 - \lambda_1 + m\lambda_2)y'\left(e\right) - c'\left(e\right) = 0. \qquad (\mathrm{FOC_e})$$

The envelop condition is given by

$$1 + u'(\pi) = \lambda_1 + \lambda_2. \qquad \text{(Envelop Condition)}$$

We now proceed in two steps. In Step 1, we provide an upper bound to $u'(\underline{\pi})$. To do this, note that by Condition L, $\pi_n(\underline{\pi}) < \overline{\pi}$. $\mathrm{FOC_N}$ then implies that at $\pi = \underline{\pi}$,

$$(1-\theta)(1 + u'\left(\pi_n(\underline{\pi})\right)) = \lambda_2.$$

By the envelop condition,

$$1 + u'(\underline{\pi}) = \lambda_1 + \lambda_2.$$

Since u is concave, $u'(\underline{\pi}) \geq u'\left(\pi_n\right)$. This then implies that $\lambda_2 \leq (1-\theta)\lambda_1/\theta$, and it follows that

$$1 + u'(\underline{\pi}) \leq \lambda_1 + \frac{1-\theta}{\theta}\lambda_1 = \frac{\lambda_1}{\theta}.$$

Now to give an upper bound to λ_1, note that at $\underline{\pi}$, we have $\pi_s(\underline{\pi}) = \underline{\pi}$ by Proposition 2, and consequently, by $\mathrm{LTT_N}$, $y(e(\underline{\pi})) = \underline{\pi}$. In other words, $e(\underline{\pi}) = \underline{e}$. $\mathrm{FOC_e}$ then implies that

$$\frac{y'\left(\underline{e}\right) - c'\left(\underline{e}\right)}{y'(\underline{e})} = \lambda_1 - m\lambda_2 \geq \frac{\theta - m(1-\theta)}{\theta}\lambda_1,$$

where the inequality follows because $\lambda_2 \leq (1-\theta)\lambda_1/\theta$ as shown above.

Combining the two inequalities above and noting $\theta - m(1-\theta) > 0$, we then obtain

$$1 + u'(\underline{\pi}) \leq \frac{\lambda_1}{\theta} \leq \frac{1}{\theta - m(1-\theta)}\frac{y'\left(\underline{e}\right) - c'\left(\underline{e}\right)}{y'(\underline{e})},$$

which concludes Step 1.

In Step 2, we derive a contradiction on the joint payoff at $\underline{\pi}$ using the upper bound in Step 1. Since the liquidity constraint binds at $\underline{\pi}$,

$$\delta\pi_n(\underline{\pi}) = \underline{\pi} + (1-\delta)my(\underline{e}) = (1 + (1-\delta)m)\underline{\pi}.$$

It follows that

$$\delta(\pi_n(\underline{\pi}) - \pi_s(\underline{\pi})) = (1-\delta)(1+m)\underline{\pi}.$$

The concavity of u then implies that

$$\begin{aligned} & u\left(\pi_n(\underline{\pi})\right) - u\left(\pi_s(\underline{\pi})\right) \\ \leq\ & u'\left(\pi_s(\underline{\pi})\right))(\pi_n(\underline{\pi}) - \pi_s(\underline{\pi})) \\ =\ & u'\left(\underline{\pi}\right)\left(\frac{(1-\delta)(1+m)}{\delta}\right)\underline{\pi}. \end{aligned}$$

Therefore,

$$\begin{aligned} & \underline{\pi} + u(\underline{\pi}) \\ =\ & (1-\delta)\left(y(\underline{e}) - c\left(\underline{e}\right)\right) + \delta\left(\pi_s(\underline{\pi}) + u\left(\pi_s(\underline{\pi})\right)\right) \\ & + (1-\theta)\,\delta((\pi_n(\underline{\pi}) + u\left(\pi_n(\underline{\pi})\right) - ((\pi_s(\underline{\pi}) + u\left(\pi_s(\underline{\pi})\right)) \\ \leq\ & (1-\delta)\left(\underline{\pi} - \underline{c}\right) + \delta\left(\underline{\pi} + u(\underline{\pi})\right) + (1-\theta)\left(1-\delta\right)(1+m)(1+u'\left(\underline{\pi}\right))\underline{\pi}. \end{aligned}$$

Rearranging the above and substituting the inequality at the end of Step 1, we get

$$\underline{\pi} + u(\underline{\pi}) \leq \underline{\pi} - \underline{c} + \frac{(1-\theta)\,(1+m)}{\theta - m(1-\theta)} \frac{y'\left(\underline{e}\right) - c'\left(\underline{e}\right)}{y'(\underline{e})}\underline{\pi}.$$

This contradicts the condition in the lemma. ■

Finally, we provide a sufficient condition for the middle region to exist.

LEMMA B11. *Suppose Condition L holds. The middle region exists, i.e.,* $(\pi_2 > \pi_1)$ *if* $m < \theta/(1-\theta)$ *and* $(1+m)^2(1-\theta)\theta/m < 1$.

Proof: Condition L implies that the PPE frontier contains more than the right region. Now suppose to the contrary that the middle region does not exist. Let π_d be the payoff that divides the left and the right region. The same argument as in Proposition 2 shows that u is not differentiable at π_d with

$$\begin{aligned} u'_+\left(\pi_d\right) &= -\frac{c'\left(e\right)}{y'\left(e\right)}, \text{ and} \\ u'_-\left(\pi_d\right) &= (1+m)\left(1-\theta\right)\left(1+u'\left(\overline{\pi}\right)\right) - \frac{c'\left(e\right)}{y'\left(e\right)}. \end{aligned}$$

Let $\Delta u'(\pi_d) = u'_+(\pi_d) - u'_-(\pi_d) > 0$. Then L-e-s implies that

$$\Delta u'(\pi_d) \leq \frac{(1+m)\theta}{m} \Delta u'(\pi_s(\pi_d)).$$

This implies that u is not differentiable at $\pi_s(\pi_d)$.

Note that by L-e-n, we have

$$\Delta u'(\pi_s(\pi_d)) \leq (1+m)(1-\theta)\Delta u'(\pi_n(\pi_s(\pi_d))).$$

This implies that u is not differentiable at $\pi_n(\pi_s(\pi_d))$.

Since u is differentiable for all $\pi \in (\pi_d, \overline{\pi}]$, the above implies that either $\pi_n(\pi_s(\pi_d)) = \pi_d$ or $\pi_n(\pi_s(\pi_d)) \in (\pi_s(\pi_d), \pi_d)$. In the later case, we can show, using the same argument as above, that either $\pi_n^2(\pi_s(\pi_d)) = \pi_d$ or $\pi_n^2(\pi_s(\pi_d)) \in (\pi_n(\pi_s(\pi_d)), \pi_d)$, where the superscript denotes that applying π_n twice. Since $\pi_n > \pi$, the sequence of π_n^k is monotone in k. It follows that there exists some K such that

$$\pi_d = \pi_n^K(\pi_s(\pi_d)).$$

Note that for all $k \leq K$, we have by above

$$\Delta u'(\pi_n^k(\pi_s(\pi_d))) \leq (1+m)(1-\theta)\Delta u'(\pi_n^{k+1}(\pi_s(\pi_d)))$$

Linking this chain of inequalities, we get

$$\Delta u'(\pi_d) \leq \frac{(1+m)\theta}{m}(1+m)^K(1-\theta)^K \Delta u'(\pi_d).$$

This is a contradiction because

$$\frac{(1+m)\theta}{m}(1+m)^K(1-\theta)^K < 1$$

by assumption. ■

11 Appendix C: Discussion

In this appendix we prove the results that we discussed in Section 7.

11.1 Failure to Achieve First Best

First, we show that the Folk Theorem holds in our setting. In particular, we show that as $\delta \to 1$, the interior of the feasible payoff set is included in the set of PPE payoff set. To state the proposition, define $\pi^{FB} = y(e^{FB}) - c(e^{FB}) - \underline{u}$.

PROPOSITION C1. *For all* $\pi \in [\underline{\pi}, \pi^{FB})$ *and for any* $\varepsilon > 0$, *there exists a* $\delta(\varepsilon)$ *sufficiently large such that for all* $\delta \geq \delta(\varepsilon)$,

$$u(\pi) > y(e^{FB}) - c(e^{FB}) - \pi - \varepsilon.$$

Proof: It is equivalent to show that for any π, the expected surplus destruction $(y(e^{FB}) - c(e^{FB}) - \pi - u(\pi))$ goes to 0 as $\delta \to 1$. To show this, first consider the sequence of contracts constructed in the long-term contracting section, which is independent of this section. As $\delta \to 1$, we see that this sequence of contracts becomes self-enforcing for arbitrarily large deadline T (with the off-equilibrium path specifies that the parties take the outside options forever). Since the surplus destruction goes to 0 as $T \to \infty$ and that the worker always receives $\underline{u}$ in this sequence of PPEs, this implies that, for arbitrarily small $\varepsilon > 0$, $(\pi^{FB} - \varepsilon, \underline{u})$ can be sustained as a PPE payoff.

Second, consider the surplus destruction at $\pi = \underline{\pi}$ as follows. In particular, let d be the expected surplus destruction in the (first) stage game under the optimal relational contract at $\pi = \underline{\pi}$. From Proposition 1, we have

$$d = \begin{cases} y(e^{FB}) - c(e^{FB}) - (y(\widehat{e}) - c(\widehat{e}) - \frac{\theta}{(1+\alpha\theta)}(y(\widehat{e}) - \underline{\pi})) & y(\widehat{e}) > \underline{\pi} \\ y(e^{FB}) - c(e^{FB}) - (y(\underline{e}) - c(\underline{e})) & \textit{otherwise} \end{cases},$$

where this expression is obtained from comparing the first best joint payoff with the expected joint payoff in period 1 under the optimal relational contract.

Now define $\varepsilon(\delta) = \pi^{FB} - \overline{\pi}(\delta)$ as the surplus destruction at $\pi = \overline{\pi}$. Denote D as the normalized destruction of surplus at $\pi = \underline{\pi}$, then

$$D(\delta) = (1-\delta)d + \delta((1-\theta)\varepsilon(\delta) + \theta D(\delta)).$$

Solving for D, we have

$$D(\delta) = \frac{(1-\delta)d + \delta(1-\theta)\varepsilon(\delta)}{1-\delta\theta}.$$

As $\delta \to 1$, we have

$$\lim_{\delta \to 1} D(\delta) = \lim_{\delta \to 1} \varepsilon(\delta) = 0.$$

This shows that at $\pi = \underline{\pi}$, the PPE payoff approaches the set of feasible payoffs.

Finally, since u is concave, for each δ, the expected surplus is decreasing in π. Given that the normalized destruction goes to 0 at $\pi = \underline{\pi}$, the destruction of surplus must also goes to 0 for all $\pi > \underline{\pi}$. ■

Next, suppose that whenever the firm is not hit by a shock, with some probability $p \in [0, 1)$ it becomes publicly known that the firm's opportunity costs are low. And whenever the firm is hit by a shock, with some probability $q \in [0, 1)$ it becomes publicly known that the firm's opportunity costs are high. The next proposition shows that first best can be achieved for sufficiently high discount factors if and only if $p > 0$.

PROPOSITION C2. *If $p = 0$, there does not exist a PPE in which the joint payoff of the manager and worker is equal to $y(e^{FB}) - c(e^{FB})$. Otherwise, when*

$$\delta \geq \frac{c(e^{FB}) + \underline{u}}{c(e^{FB}) + \underline{u} + (1-\theta)p(y(e^{FB}) - c(e^{FB}) - \underline{u} - \underline{\pi})},$$

there exists a PPE such that the joint payoff of the manager and worker is equal to $y(e^{FB}) - c(e^{FB})$.

Proof: Suppose $p = 0$. In this case, the manager's action includes w, b_n, b_{sk}, b_{su}, where b_{sk} stands for the bonus payment when it is publicly known that the firm is hit by a shock and b_{su} stands for the bonus payment when the shock is unknown to the worker.

Suppose to the contrary that the first best can be achieved. Let π_f be the smallest PPE payoff of the manager in which first best is achieved. Note that at π_f, we have $w = b_{sk} = b_{su} = 0$, the worker puts the first best level of effort, and the continuation payoffs of the manager fall weakly to the right of π_f. Therefore, the promise-keeping constraint of the manager implies that

$$\pi_f = (1-\delta)(y(e^{FB}) - (1-\theta)b_n) + \delta(\theta q \pi_{sk} + \theta(1-q)\pi_{su} + (1-\theta)\pi_n),$$

where π_{sk}, π_{su}, and π_n are the respective continuation payoffs.

Note that

$$\begin{aligned} & (1-\delta)(y(e^{FB}) - (1-\theta)b_n) + \delta(\theta q \pi_{sk} + \theta(1-q)\pi_{su} + (1-\theta)\pi_n) \\ \geq \; & (1-\delta)y(e^{FB}) + \delta(\theta q \pi_{sk} + \theta(1-q)\pi_{su} + (1-\theta)\pi_{su}) \\ \geq \; & (1-\delta)y(e^{FB}) + \delta \pi_f, \end{aligned}$$

where the first inequality follows from the manager's truth-telling constraint in the no-shock state ($\delta(\pi_n - \pi_{su}) \geq (1-\delta)(b_n - b_{su}) = (1-\delta)b_n$) and the second inequality follows from the definition of π_f.

Therefore, the inequality above implies that $\pi_f \geq y(e^{FB})$, but this is a contradiction because the maximal feasible payoff of the manager that gives the worker a payoff of least $\underline{u}$ is $y(e^{FB}) - c(e^{FB}) - \underline{u} < y(e^{FB})$.

Next, we construct a PPE that reaches first best when $p > 0$. Consider the following strategy in which the bonus payment satisfies $b_{sk} = b_{su} \equiv b_s$.

Along the equilibrium path: (i.) the worker puts in the first best level of effort e^{FB}, (ii.) the manager offers $w = 0$, $b_s = 0$, and $b_n = \frac{c(e^{FB})+\underline{u}}{(1-\theta)p}$, and (iii.) the manager pays out a bonus $b_n = (c(e^{FB}) + \underline{u})/((1-\theta)p)$, when it is publicly revealed that the opportunity cost is low. Note that this happens with probability $(1-\theta)p$.

Off the equilibrium path: (i.) the parties take their outside options, (ii.) the worker puts in $e = 0$, (iii.) the manager offers a base wage $w = 0$, $b_s = 0$, and $b_n = 0$, and (iv.) the manager does not pay out the bonus in any state.

For the strategy above to be an equilibrium, we need to check that the worker is willing to participate and to put in effort, that is

$$-c(e^{FB}) + (1-\theta)pb_n \geq \underline{u} \text{ and } c(e^{FB}) \leq (1-\theta)pb_n.$$

Given $b_n = (c(e^{FB}) + \underline{u})/((1-\theta)p)$, it is immediate that both inequalities above are satisfied.

In addition, we need to check that the manager is willing to pay to bonus:

$$(1-\delta)b_n \leq \delta(y(e^{FB}) - c(e^{FB}) - \underline{u} - \underline{\pi}).$$

Note that once this constraint is satisfied, it implies that the manager's participation constraint is also satisfied. Since $b_n = (c(e^{FB}) + \underline{u})/((1-\theta)p)$, the inequality above is equivalent to

$$\delta \geq \frac{c(e^{FB}) + \underline{u}}{c(e^{FB}) + \underline{u} + (1-\theta)p(y(e^{FB}) - c(e^{FB}) - \underline{u} - \underline{\pi})}.$$

This is exactly the condition in the proposition, so the strategy is a PPE that reaches first best. ■

11.2 Benchmarks: Public Information and Long-term Contracts

In this subsection, we analyze the dynamics of the relationship when the state of the world is public information. We characterize the PPE frontier, and for each payoff pair on the frontier, we state the associated effort, base wage, bonuses, and the continuation payoffs. This essentially specifies the dynamics of the relationship. Since the analysis is similar to and simpler than that in the private information case, we only state and prove the main results.

LEMMA C1. *With public information, the PPE frontier satisfies the following. For each PPE payoff of the manager* π,

$$\pi + u(\pi) = \max_{e,w,\pi_s,b_n} (1-\delta)(y(e) - c(e)) + \theta\delta(\pi_s + u(\pi_s)) + (1-\theta)\delta(\overline{\pi} + u(\overline{\pi})) - (1-\delta)\theta\alpha w \tag{Public Program}$$

subject to

$$\pi = (1-\delta)(y(e) - (1+\theta\alpha)w - (1-\theta)b_n) + \delta(\theta\pi_s + (1-\theta)\overline{\pi}), \tag{PK$_\mathrm{M}$}$$

$$w \geq 0, \text{ and} \tag{NN$_\mathrm{W}$}$$

$$(1-\delta)b_n \leq \delta(\overline{\pi} - \underline{\pi}). \tag{NR$_\mathrm{S}$}$$

Lemma C1 directly corresponds to Lemma 6 in the main model. As in the main model, $b_s \equiv 0$ and $\pi_n \equiv \overline{\pi}$, so they do not appear as choice variables. A key difference, however, is that the maximization problem does not contain the truth-telling constraint in the no-shock state ($(1-\delta)(b_n - b_s) = \delta(\pi_n - \pi_s)$) since information is public. This implies that b_n is now a choice variable included in the program. It follows that we need to include the non-reneging constraint in the no-shock state. Since the proof of Lemma C1 parallel with that of Lemma 6, we omit it here. The next proposition is the main result of this subsection.

PROPOSITION C3. *The followings hold:*

(*a.*) *For all* $\pi \geq \underline{\pi}$,

$$u'(\pi) \leq -\frac{1}{1+\theta\alpha}.$$

(*b.*) *For all* $\pi \geq \underline{\pi}$, *the associated effort level and the continuation payoff following the no-shock state satisfies*

$$\begin{aligned} \frac{c'(e)}{y'(e)} &= -u'(\pi) \text{ and} \\ \pi_n &= \overline{\pi}. \end{aligned}$$

(*c.*) *(The middle region) If* $u'(\pi) \in (-1, -1/(1+\theta\alpha))$, *then*

$$\begin{aligned} w &= 0, \\ \pi_s &= \pi, \text{ and} \\ b_n &= \frac{\delta}{1-\delta}(\overline{\pi} - \underline{\pi}). \end{aligned}$$

In this region, $u'(\pi)$ is strictly decreasing.

(*d.*) *(The right region) If $u'(\pi) = -1$, then*

$$\begin{aligned} u(\pi) &= y(e^{FB}) - c(e^{FB}) - \pi \text{ and} \\ w &= 0. \end{aligned}$$

There can be multiple choices of π_s and b_n. One such choice is

$$\begin{aligned} \pi_s &= \pi \text{ and} \\ b_n &= \frac{(1-\delta)y(e) - (1+\delta\theta)\pi - \delta\,(1-\theta)\,\overline{\pi}}{(1-\delta)(1-\theta)}, \end{aligned}$$

where $\overline{\pi} = y(e^{FB}) - c(e^{FB}) - \underline{u}$.

The right region exists if and only if

$$(1-\delta)\frac{c(e^{FB}) + \underline{u}}{1-\theta} \leq \delta(y(e^{FB}) - c(e^{FB}) - \underline{u} - \underline{\pi}),$$

and its left boundary is given by $((1-\delta)y(e^{FB}) + (1-\theta)\delta\underline{\pi})/(1-\delta\theta)$.

(*e.*) *(The left region) When $u'(\pi) = -1/(1+\theta\alpha)$, then*

$$b_n = \frac{\delta}{1-\delta}(\overline{\pi} - \underline{\pi}).$$

There can be multiple choices of π_s and b_n. One such choice is

$$\begin{aligned} \pi_s &= \pi \text{ and} \\ w &= \frac{1}{(1-\delta)(1+\alpha\theta)}((1-\delta)y(\widehat{e}) + \delta(1-\theta)\underline{\pi} - (1-\delta\theta)\pi). \end{aligned}$$

The left region exists if $y(\widehat{e}) > \underline{\pi}$, where recall $\widehat{e}$ is the unique effort level satisfying $c'(e)/y'(e) = 1/(1+\alpha\theta)$. The right boundary of this region is given by $((1-\delta)y(\widehat{e}) + \delta(1-\theta)\underline{\pi})/(1-\delta\theta)$.

Proof: From Lemma C1, we form the Lagrangian

$$\begin{aligned} L &= (1-\delta)\,(y\,(e) - c\,(e)) + \theta\delta\,(\pi_s + u\,(\pi_s)) + (1-\theta)\,\delta\,(\overline{\pi} + u\,(\overline{\pi})) - (1-\delta)\,\theta\alpha w \\ &\quad + \lambda_1(\pi - (1-\delta)\,(y\,(e) - (1+\theta\alpha)\,w - (1-\theta)\,b_n) - \delta(\theta\pi_s + (1-\theta)\,\overline{\pi})) \\ &\quad + \lambda_2(1-\delta)w + \lambda_3(\delta(\overline{\pi} - \underline{\pi}) - (1-\delta)b_n). \end{aligned}$$

The FOCs and the envelop condition are given by

$$1 + u'\,(\pi_s) = \lambda_1. \tag{FOC$_\text{S}$}$$

$$-\theta\alpha + \lambda_1\,(1+\theta\alpha) + \lambda_2 = 0. \tag{FOC$_\text{W}$}$$

$$y'(e) - c'(e) = \lambda_1 y'(e). \quad (\text{FOC}_{\text{e}})$$

$$\lambda_1 (1-\theta) = \lambda_3. \quad (\text{FOC}_{\text{N}})$$

$$1 + u'(\pi) = \lambda_1. \quad (\text{envelop})$$

Note that FOC_{W} implies $\lambda_1 \leq \theta\alpha/(1+\theta\alpha)$. From the envelop condition, we then obtain $u'(\pi) \leq -1/(1+\theta\alpha)$. This proves Part (a.).

By FOC_{e}, $\lambda_1 = (y'(e) - c'(e))/y'(e)$. Substituting this into the envelop condition, we get Part (b.).

To prove Part (c.), note that if $u'(\pi) \in (-1, -1/(1+\theta\alpha))$, $\lambda_1 > 0$ by the envelop condition. This then implies that $\lambda_3 > 0$ by FOC_{N}. From the complementarity slackness condition on the non-reneging constraint, we then get $b_n = \delta(\overline{\pi} - \underline{\pi})/(1-\delta)$. In addition, when $u'(\pi) < -1/(1+\theta\alpha)$, we have $\lambda_2 > 0$ from FOC_{W} and the envelop condition, and, thus, $w = 0$. Substituting the expression of b_n and w into PK_{M}, we get

$$\pi = (1-\delta)\, y(e) + \delta(\theta\pi_s + (1-\theta)\, \underline{\pi}).$$

Now from the FOC_{S}, we have $u'(\pi_s) = u'(\pi)$. Therefore, we must have $\pi_s = \pi$ unless there is an interval in which $u'(\pi)$ is constant. But if this were the case, $y(e)$ is a constant in the interval by Part (b.), and the expression above then implies that for all π in the interval

$$\frac{d\pi_s}{d\pi} = \frac{1}{\delta\theta} > 1.$$

But the above expression clearly cannot hold for the entire interval. Therefore, we must have $\pi_s = \pi$. This proves Part (c.).

For Part (d.), note that we have $e = e^{FB}$ in this region by Part (b.). FOC_{W} implies that $w = 0$ in this region. In addition, by the FOC_{S}, $u'(\pi_s) = u'(\pi) = -1$, so this region is self-generating and reaches first best.

In this region, there is some flexibility in choosing π_s. By having $\pi_s = \pi$ (to be more consistent with the middle region), the expression of b_n follows from the PK_{m}.

For this region to exist, a necessary and sufficient condition is that, at $\pi = \overline{\pi}$, the non-reneging constraint holds. PK_{M} at $\overline{\pi}$ implies that

$$b_n(\overline{\pi}) = \frac{y\left(e^{FB}\right) - \overline{\pi}}{1-\theta} = \frac{c\left(e^{FB}\right) + \underline{u}}{1-\theta},$$

where the second inequality uses that the first best is obtained at $\overline{\pi}$. Substituting this into the non-reneging constraint, we obtain the necessary and sufficient condition in Part (d.).

Now at the left boundary of the region, the non-reneging constraint must bind. This implies that $(1-\delta)b_n = \delta(\overline{\pi} - \underline{\pi})$. Substituting this into PK_{M}, we have

$$\pi = (1-\delta)\, y\left(e^{FB}\right) + \delta(\theta\pi + (1-\theta)\, \underline{\pi}).$$

This proves Part (d.).

For Part (e.), note that in this region we have $e = \widehat{e}$. In addition, we have $(1-\delta)b_n = \delta(\overline{\pi} - \underline{\pi})$ since $\lambda_3 > 0$ in this region. Substituting these into PK_{M}, we get

$$\pi = ((1-\delta)\, y\,(\widehat{e}) - (1+\alpha\theta)w) + \delta(\theta\pi_s + (1-\theta)\, \underline{\pi}).$$

There is again some flexibility is choosing π_s here except for at the right boundary of this region. To be consistent with the middle region, we have $\pi_s = \pi$ and this gives the expression for w.

For this region to exist, a necessary and sufficient condition is that at $\pi = \underline{\pi}$, we have $w > 0$, and this is equivalent to $\underline{\pi} < y\,(\widehat{e})$. Finally, at the right boundary of this region, we have $w = 0$ and $\pi_s = \pi$. Substituting these into the PK_{M} proves Part (e.). ■

Now we proceed to the long-term contracts. Recall that $h^t = \{y_1, ...y_t\}$ is the history of past outputs. Let $m^t = \{m_1, ..m_t\}$ be the history of past announcements, and t_n be the last time the manager announces no-shock (and $t_n = 0$ if the manager has never announced no-shock.) Let $b_t(h^t, m^t)$ be the manager's payment to the worker in period t.

PROPOSITION C4. *As T approaches ∞, the following sequence of contracts approaches first best.*

$$b_t(h^t, m^t) = \begin{cases} 0 & \textit{if } h^t \neq \{y^{FB}, ..., y^{FB}\} \\ 0 & \textit{if } m_t = n \textit{ and } t < t_n + T \\ (c(e^{FB}) + \underline{u})(1 + \delta^{-1} + \delta^{-2} + ... + \delta^{-(t-1-t_n)}) & \textit{otherwise.} \end{cases}$$

Proof: To simplify the exposition, we normalize $\underline{u}$ to be zero. We first show that this sequence of contracts are incentive compatible for sufficiently large T. Under the construction above, the worker's payoff is 0 by putting e^{FB} in each period. Any other effort choice (except $e = 0$) gives the worker a negative payoff. Therefore, choosing $e_t = e^{FB}$ along the equilibrium path is a best response for the worker.

It remains to check that it is incentive compatible for the manager to be truth-telling. Note that when the worker puts in first best level of effort, his expected payoff is always zero for any strategy taken by the manager. Therefore, the manager's payoff is equal to the value of the relationship. This implies that the manager's payoff is maximized when she minimizes the surplus destruction, i.e., when she minimizes the expected payment shock states. This makes it clear that it is incentive compatible for the manager to be truth-telling in a no-shock state.

It remains to check the manager's incentive compatibility in shock states. Note that once the manager announces a no-shock state, the contract immediately restarts in the next period. Given this renewal feature and that the manager is always truth-telling in no-shock states, it suffices to consider the following sequence of strategies in which the manager makes truthful announcements until period n after which she always announces that it is a no-shock state. For truth-telling to be optimal, it is equivalent that the surplus destruction from such strategies is minimized at $n = T$.

Now let V_i^n be the associated expected surplus destruction if the manager has announced shock states in the past i^{th} periods. Denote $k = \alpha c(e^{FB})$, then V_i^n satisfies the following.

$$\begin{aligned}
V_0^n &= (1-\theta)(0+\delta V_0^n) + \theta\delta V_1^n, \\
V_1^n &= (1-\theta)(0+\delta V_0^n) + \theta\delta V_2^n, \\
&\ldots \\
V_{n-1}^n &= (1-\theta)(0+\delta V_0^n) + \theta\delta V_n^n, \quad \text{and}
\end{aligned}$$

$$V_n^n = \theta k(1+\delta^{-1}+\delta^{-2}+\ldots+\delta^{-(n-1)}) + \delta V_0^n.$$

Solving for the n equations, we obtain that

$$V_0^n = \delta\theta k \frac{1-\theta\delta}{(1-\delta)^2} \frac{\theta^n(1-\delta^n)}{(1-(\theta\delta)^{n+1})}.$$

Note that

$$\begin{aligned}
& \frac{\theta^{n-1}(1-\delta^{n-1})}{1-(\theta\delta)^n} - \frac{\theta^n(1-\delta^n)}{1-(\theta\delta)^{n+1}} \\
> \; & \frac{\theta^{n-1}}{(1-(\theta\delta)^n)(1-(\theta\delta)^{n+1})}\left(1-\delta^{n-1}-\theta\right).
\end{aligned}$$

Therefore, for $n > 1 + \log(1-\theta)/\log\delta$,

$$\frac{\theta^{n-1}(1-\delta^{n-1})}{1-(\theta\delta)^n} > \frac{\theta^n(1-\delta^n)}{1-(\theta\delta)^{n+1}},$$

so V_0^n is decreasing in n.

Moreover, $\theta^n(1-\delta^n)/(1-(\theta\delta)^{n+1})$ goes to zero as n goes to infinity. It follows that for T sufficiently large, V_0^n is minimized at $n = T$. This proves the incentive compatibility. Since V_0^T goes to zero as T goes to infinity, this sequence of contracts approximates first best. ■

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