USC FBE APPLIED ECONOMICS WORKSHOP
presented by Leon Zhu
FRIDAY, April 16, 2010
1:30 pm - 3:00 pm, **Room: HOH-506**

Submitted to *Management Science*
manuscript

Optimal Pre-order Discount and Information Release

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In this paper, we investigate the information release and pricing strategies for a seller who can take customer pre-orders before the release of a product. The pre-order option enables the seller to sell a product at an early date when consumers' valuations are relatively homogeneous. We find that the optimal pricing strategy is discontinuous with respect to the amount of information available at pre-order, and a small change in the amount of information may cause a dramatic change in the proportion of consumers who pre-order under the optimal pricing strategy. Furthermore, the seller's optimal information release strategy depends on a key measure, the normalized margin, which is the ratio between the expected margin and the standard deviation of consumer valuations. While the seller may want to release some (or zero) amount of information, depending on the normalized margin, the seller should never release all information. Finally, under the optimal information release strategy and pricing strategy, the benefit of pre-order is most pronounced when the seller can successfully position the product as a "mass-market" product by withholding information.

Key words: advance selling, pre-order, information release
History:

1. Introduction

From the end of the year 2006 through the year 2007, the world witnessed the phenomenal success of Wii. A game console produced by Nintendo Company Ltd., Wii was designed to appeal to a general public that was not normally interested in video games (Casey 2006), and it was sold at a lower price than its main competitors, Microsoft Corporation's Xbox and Sony Corporation's PlayStation. Riding on this initial success, Nintendo expanded its Wii Series by introducing a new game called Wii Fit, which guides players through various exercises with the aid of a new peripheral. While this highly anticipated game had been under development for over a year leading upto E3 2007 (the Electronic Entertainment Expo, widely regarded as the world's largest regular convention for video games), little was known at that time except its mere existence (announced in September 2006 when Wii was launched). At the E3 show in July 2007, enthusiasts were finally able to observe a demonstration of the game, but still with no details. Wii Fit was first launched in Japan on December 1st, 2007 and later released in the U.S. in May, 2008 at a manufacturer's suggested retail price (MSRP) of $89.99. After the initial release, Wii Fit was repeatedly "stocked out" nationwide (Schiesel 2008) and sold for over $120 on eBay and craigslist in the following months. Surprisingly, a customer could *pre-order* Wii Fit at $69.99 through Amazon back in September 2007. Such a deep discount seems warranted given that little information was provided at the time. However, could Nintendo do better had it provided more information and charged a higher pre-order price?

Pre-order is a common practice for books, CDs, videos, DVDs, video games, and software items. For instance, as reported by Zhao and Stecke (2009), *Harry Potter and Deathly Hallows* (Hardcover) could be pre-ordered (on Amazon) for $17.99, a 49% discount from the $34.99 list price. As of July 7th, 2009, two years after the release, the book was sold for $22.29 on Amazon. Recently, Microsoft Windows 7 Home Premium Upgrade could be pre-ordered at $49.99 (MSRP $119.99) until July 11th, 2009. The difference between the pre-order price and the MSRP is called the pre-order discount. For some products, discounts are offered at the pre-order date, and for others, various forms of incentives are given. For example, pre-order incentives for video games typically involve discounts, exclusive gifts, and/or early access to online game networks.

To induce consumers to pre-order, the discount must compensate for consumers' valuation uncertainty, which depends on how much information consumers have on the product. A manufacturer (or seller) can take a variety of ways to disseminate information about the product, such as running advertisement campaigns, demonstrating products in popular conventions, disclosing information on company websites, solicitating profesional reviews, and etc. These practices raise some intriguing questions. One such question is how much information should be released. Many product attributes are idiosyncratically valued and subject to consumers' personal preferences (which will be referred to as "horizontal attributes"), e.g., the aspect ratio (4:3 vs. 16:10) and screen finish (glossy vs. matte) of a notebook computer. A particular specification of such an attribute may appeal to some consumers but displease others. Should those attributes be disclosed (or highlighted if some sort of disclosure is unavoidable)?

Information management is an indispensable part of many companies' business strategies. A recent incident that caught much attention of the consumer technologist community is the launch of the tablet device, iPad, by Apple Inc. In a period of rampant speculation about the existence of the product, a Wall Street Journal article (Kane and Fowler 2010) confirmed the launch of the product and revealed some of its features, citing "people briefed by the company." A former senior marketing manager of Apple (Martellaro 2010) identified this article as an exercise of controlled information leak by Apple, which he claims the company has been doing for years.

In this paper, we investigate the optimal pricing and information release strategies for the manufacturer (or seller) as an integrated whole. We are particularly interested in the following questions: How much information should the seller release in the pre-order stage? How do the optimal pre-order and retail prices depend on the amount of information released? And when will pre-order be most effective for the seller?

To address these questions, we propose a two-period, continuous-valuation model and study the seller's optimal information release and pricing strategies. The continuous-valuation model is a non-trivial extension of the two-type valuation model predominant in the existing advance-selling literature, not only because it enables the control of product information at the pre-order date, but also because it offers new insights into the pre-order problem. For instance, we find that the optimal pricing strategy is discontinuous with respect to the amount of information available at pre-order, and a small change in the amount of information may cause a dramatic change in the proportion of consumers who pre-order under the optimal pricing strategy. For the optimal information strategy, we find that it depends on a key measure, namely normalized margin, as the ratio between the expected margin and the standard deviation of consumer valuations. We further identify the exact parameter regions in which the seller will release or withhold information. Interestingly, while the seller may want to release no information or part of it, he should never release all information. In addition, under the optimal information release and pricing strategies, the benefit of pre-order is most pronounced when the seller can successfully position the product as a "mass-market" product by withholding information.

The remainder of this paper is organized as follows. In Section 2, we review the literature related to pre-order. In Section 3, we introduce the model on the information structure. We study the seller's optimal pricing strategy in Section 4 and information release strategy in Section 5. Some extensions of the results are discussed in Section 6, and finally, we conclude in Section 7.

2. Literature Review

Though not precisely defined, pre-order is commonly regarded as a special form of advance selling and the two bodies of literature are intertwined. As defined by Xie and Shugan (2009), advance selling refers to the general practice that a seller induces buyers to commit to purchase a good before the time of consumption, which can take various forms. In contrast, pre-order usually refers to the practice that the seller allows the buyer to purchase a (typically new) product at a particular price till a specified time before its release. As an example of studies dedicated to pre-order, Hui, Eliashberg, and George (2008) propose a consumer behavior model based on an optimal stopping policy to explain the preorder and sales pattern for DVDs.[1] As another example, Zhao and Stecke (2009) focus on the pre-order of new products. Naturally, our paper is related to the literature on strategic pricing of new products and services, as discussed in Chaterjee (2009).

There are many plausible reasons for sellers to engage in advance selling (or pre-order). It may help sellers update demand forecasts and manage inventory, as pointed out by Moe and Fader (2002), Tang et al. (2004), and Li and Zhang (2010). It may also help the sellers effectively segment the market, especially under limited capacity. For example, efficiently allocating capacity is a key motive behind revenue management widely adopted in the travel industry, which capitalizes on the fact that late purchasers are often willing to pay higher prices. For reviews of the revenue management literature, we refer the reader to Boyd and Bilegan (2003) and Talluri and van Ryzin (2004). As illustrated by Xie and Shugan (2009), if capacity is limited and rationing is possible, the optimal price pattern under a dynamic information structure can take different forms — e.g., shutdowns and pre-order premiums instead of discounts. DeGraba (1995) shows that a seller may even benefit from purposely restricting its capacity and generating buying frenzies. Liu and Xiao (2008) study a seller's optimal return policy or inventory rationing policy when inventory is endogenously determined through a Newsvendor model. Prasad, Stecke, and Zhao (2009) and Li and Zhang (2010) both study the advance selling (pre-order) strategy of a Newsvendor retailer who must procure all units before the regular selling season starts. The structure of consumer demand is very different in these two papers: in the former, one group of consumers is informed about the pre-order opportunity yet the other group is not; in the latter, high-valuation consumers arrive in the pre-order period while low-valuation ones arrive in the regular season. Liu and van Ryzin (2008) and Su and Zhang (2008) show that capacity and inventory considerations may enhance the seller's credibility in charging a high retail price. In contrast to the above studies, we focus on situations such as the Windows 7 upgrade, Harry Potter book release, and motion picture DVD release, in which the seller has ample capacity.

If the seller can match the demand through continuous production and is not restricted by capacity or inventory, the problem can be framed as a sequential screening problem because consumers' valuations are private information at both the pre-order and release dates (with increasing accuracy). In this case, Courty and Li (2000) show that the seller's optimal pricing policy consists of a menu of refund contracts — i.e., customers pay different prices at the pre-order date, which are combined with different cancellation fees. Akan, Ata, and Dana (2007) show that a menu of refund contracts (with various expiration dates) is still optimal when each consumer learns his/her valuation at some privately-known future time. Xie and Shugan (2001) also note that a well-designed refund option can benefit the seller. The seller's optimal pricing policy can also take the form of a menu of options where consumers buy options at the pre-order date to acquire the product at the release date with various strike prices (Reiche 2008). However, the optimal refund contract or option contract is noticeably different from the simple contract prevalent in the pre-order practice, in which a single pre-order price and a single retail price are announced at the pre-order date. We restrict our attention to the latter form of pre-order mechanism in the paper.

All of the aforementioned papers assume exogenous information structures and focus on the seller's pricing decisions. In the real world, the seller can often influence consumers' knowledge about the product and can decide his information release strategy along with the pricing strategy. The more information the seller releases, the less uncertainty remains in consumers' valuations. Few papers in the literature have addressed the seller's information release strategy or commented

[1] In our paper, however, the precise time of monetary transfers is a minor consideration, because it is now commonplace for retailers to charge a customer when the item is shipped and for Hotels and car rental companies to charge after the service is provided.

on whether the seller prefers a more homogeneous consumer distribution or a heterogeneous one. Lewis and Sappington (1991) examine an extension of the standard screening model in which the seller can choose the probability with which the consumer receives perfect private information. They find that the seller will always set the probability at zero or one, corresponding to no release or full release of the information. A more detailed analysis of this model is provided in Lewis and Sappington (1994). Miravete (1996) compares the seller's profits generated by optimal two-part tariffs imposed either at an earlier date when consumer valuations are more homogeneous or at a later date when the valuations are more heterogeneous. Johnson and Myatt (2006) find that the seller's profit is typically a U-shaped function of the dispersion of consumer valuations, which again leads to extreme information release strategies. All these papers consider the static setting in which the seller interacts with consumers only once.

The operations management literature has recently seen papers studying whether to release all the information or withhold information as well. For instance, Shulman, Coughlan, and Savaskan (2009) study a problem that whether a seller should inform customers of the fit of the product with their preferences at the time of purchase if customers can return or exchange the product later (after paying a restocking fee). Guo (2009) studies a problem that whether a manufacturer should disclose quality information to consumers directly or through retailers. In this paper, we adopt a continuous model and investigate the optimal intermediate level of information release.

3. The Model

In this section, we introduce the model. We start at the lower level when product information is exogenously given and then proceed to the upper level when information provision is endogenously determined by the seller.

3.1. Exogenous Consumer Valuation

Consider the situation that a monopolist seller sells a product to a set of consumers (the seller will be referred to as "he" hereinafter, whereas a consumer will be referred to as "she"). Each consumer has a unit demand and has an idiosyncratic utility of consumption. The seller's marginal production cost is constant, denoted by $c > 0$. The seller releases the product at time 1 but can take pre-orders at time 0. The pre-order price p_0 and retail (or spot) price p_1 are announced before the pre-order takes place. All consumers are informed of these prices and can pre-order if they choose.[2] Throughout the paper, we assume that the seller can commit to the retail price at the pre-order date because he can earn a higher profit in this case.[3]

Consumers are risk-neutral and maximize their expected utilities. Each consumer learns her utility in two steps: at time 0, she learns an *initial valuation* θ_0; a random shock ε occurs between time 0 and time 1, and she learns her *final valuation* $\theta_1 = \theta_0 + \varepsilon$ at time 1. The final valuation is the consumer's true utility from consuming the product. Without loss of generality, we assume that ε has a zero mean. Thus, a consumer's initial valuation θ_0 can be viewed as an expectation of her true valuation θ_1. This dynamic valuation process describes a common phenomenon that consumers learn their valuations gradually over time. Both θ_0 and θ_1 are consumers' private information, and will be referred to as their *initial type* and *final type*, respectively. The distributions of θ_0 and ε (or $\theta_1|\theta_0$, more generally) are common knowledge, denoted by $F(\theta_0)$ and $G(\theta_1|\theta_0)$, respectively. The sequence of events are depicted in Figure 1.

[2] The model can be easily extended to accommodate the situation that only a subset of consumers are aware of or eligible for the pre-order opportunity. The optimality of adopting pre-order will continue to hold and the discontinuity in the optimal pricing and information release strategies will continue to exist.

[3] To establish credibility, the seller may (i) announce the pre-order discount and offer a refund to pre-order consumers if the retail price is reduced later (Amazon offers a similar type of pre-order price guarantee), (ii) build a reputation for a high spot price (Xie and Shugan 2009), or (iii) artificially set capacity and inventory limitations.

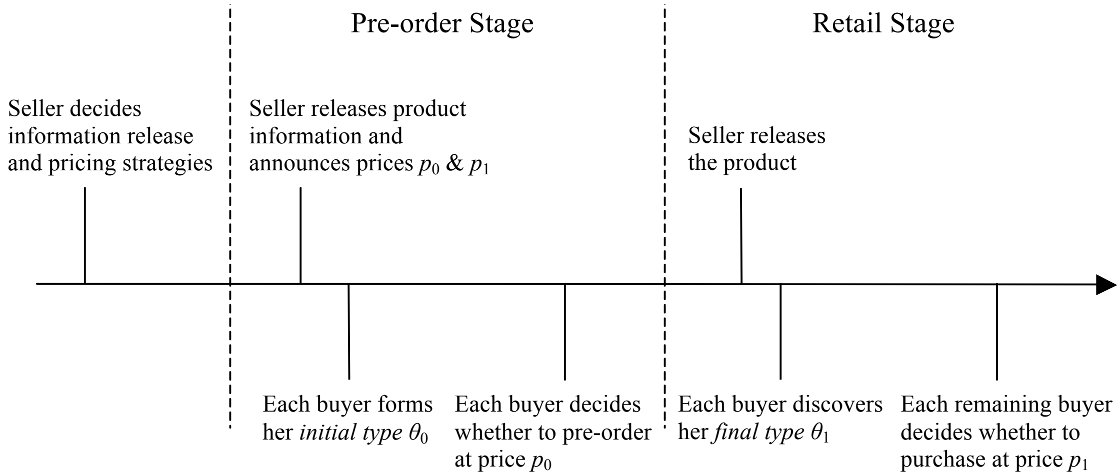


Figure 1 Sequence of Events.

3.2. Endogenous Consumer Valuation

As is common in reality, a consumer's initial valuation of a product, θ_0, before she actually owns (or even sees) the product can be influenced by many factors, one of which (arguably the most important one) is how much information the consumer has on the product. The seller can play a crucial role here. He can choose to release product information through advertisements (exhibitions, professional reviews, and etc.) or keep the information confidential; in the former case, he can also decide how much information to release. However, a consumer's intrinsic utility of a product, represented by the final valuation θ_1, can hardly be altered by the seller's information strategy, e.g., an advertising campaign. These important facts can be captured by the following assumptions:

(1) The unconditional distribution of a consumer's intrinsic valuation θ_1 follows a normal distribution $N(\mu, \kappa^2)$, with $\kappa > 0$. The variance κ^2 measures the *dispersion* of true consumer valuations. This distribution is determined by the attributes of the product and is independent from the product information available at the pre-order date.

(2) The seller can influence consumers' valuations of the product at the pre-order date through advertisements or other information provision mechanisms. Thus, each consumer observes an advertisement signal ψ at time 0. The distribution of ψ conditional on the true valuation θ_1 is $N(\theta_1, \xi^2)$, with $\xi \geq 0$.

(3) At time 0, each consumer forms her expectation of θ_1 based on the advertisement signal. That is, her initial type θ_0 is the conditional expectation of θ_1 given ψ, $\theta_0 = E(\theta_1|\psi)$.

Standard calculation yields $\theta_0 = \frac{\mu\xi^2+\psi\kappa^2}{\xi^2+\kappa^2}$, and the distributions of θ_0 and $\theta_1|\theta_0$ (F and G) are given by $N(\mu, \frac{\kappa^4}{\xi^2+\kappa^2})$ and $N(\theta_0, \frac{\xi^2\kappa^2}{\xi^2+\kappa^2})$, respectively. By changing ξ, the seller can arbitrarily allocate the uncertainty in consumers' valuations between the two random variables θ_0 and $\theta_1|\theta_0$. The variance ξ^2 measures the noise in the advertisement signal. If ξ^2 is small, consumers are well informed at time 0, while if ξ^2 is large, little information is released. Note that although an advertisement provides the same information to all consumers, it can send distinct signals to different consumers, because consumers' reactions to the same piece of information can be rather different, depending on their intrinsic perferences. Real-world examples are ubiquitous, e.g., a notebook computer has a 13 inch screen (rather than 15 inch), or the screen has a glossy finish (as opposed to a matte one). On one extreme, a perfectly informative advertisement helps consumers learn their true valuations of the product, i.e. sending a perfect singal $\psi = \theta_1$ to a consumer with intrinsic valuation θ_1; and on the other extreme, a totally uninformative advertisement does not help at all, which sends a completely noisy singal $\psi \sim N(\theta_1, \infty)$ to the same consumer.

The above information model has an equivalent interpretation through the random walk perspective. That is, a consumer has a random valuation $\theta_0 \sim N(\mu, \frac{\kappa^4}{\xi^2+\kappa^2})$ at time 0, a random shock

$\varepsilon \sim N(0, \frac{\xi^2\kappa^2}{\xi^2+\kappa^2})$ occurs between time 0 and time 1, and the consumer's final valuation $\theta_1 = \theta_0 + \varepsilon$ is revealed at time 1. By changing ξ, the seller can arbitrarily allocate the uncertainty between θ_0 and $\theta_1|\theta_0$.

3.3. Micro Foundation of Information Structure: Product Attributes

We can model the seller's handling of information in more detail. Assume that a product can be described by a set of attributes and consumers form their valuations based on their preferences on these attributes. More specifically, we focus on "horizontal attributes," for which a particular specification may appeal to some consumers but displease others. Examples of such attributes include the enclosure material, screen size, and operating system of a notebook computer, the shape, color, keyboard type, and carrier of a cellular phone, as well as the user interface and platform of a video game.

Assume that a product has n horizontal attributes and a consumer's true valuation of the product is $V = \mu + \sum_{i=1}^{n} X_i$, where μ represents the aggregate value from the product's other (non-horizontal) attributes and each $X_i \in \{1, -1\}$ represents whether the consumer likes the specification of attribute i or not. If the consumer is informed about the specification of attribute i, she knows her actual X_i; otherwise, she only knows its expectation. Consider a stylized case that each horizontal attribute of the product polarizes the consumer population — half of them like a particular specification and half of them do not, i.e., $P(X_i = 1) = P(X_i = -1) = 0.5$ and $E[X_i] = 0$. At time 0, the seller can decide to release information on $n_0 \leq n$ attributes. Then, a consumer's valuation of the product at time 0 is given by $V_0 = \mu + \sum_{i=1}^{n_0} X_i$ and that at time 1, given V_0, is given by $V|V_0 = V_0 + \sum_{i=n_0+1}^{n} X_i$. (This model can be viewed as a special case of the additive Martingale Model of Forecast Evolution, developed by Graves et al. (1986, 1998) and Heath and Jackson (1994).) The valuations V_0 and $V|V_0$ follow the distributions $\mu + 2B(n_0, 0.5) - n_0$ and $V_0 + 2B(n - n_0, 0.5) - (n - n_0)$, respectively, where $B(n,p)$ denotes the binomial distribution with n trials and success probability p.

It is well known that when n is large enough and p is not near 0 or 1 the binomial distribution $B(n,p)$ can be closely approximated by the normal distribution $N(np, np(1-p))$ (Box, Hunter and Hunter 2005).[4] Therefore, when n_0 and $n - n_0$ are reasonably large, V_0 and $V|V_0$ can be approximated by the normally distributed θ_0 and $\theta_1|\theta_0$ introduced previously. When the number of horizontal attributes n is fixed, the distribution of a consumer's total valuation V (approximated by θ_1) is fixed, and the seller can allocate the valuation uncertainty between V_0 and $V|V_0$ by choosing the number of attributes n_0 to advertise. From this perspective, a small (or large) n_0 corresponds to a large (or small) variance ξ^2 of the advertisement signal ψ. The attributes model also justifies the continuous (normal) valuation model we have proposed; a simple two-type model as commonly seen in the literature would be too coarse to capture the seller's ability to control information.

4. Seller's Optimal Pricing Strategy

In this section, we study the seller's optimal pricing strategy when the distributions F and G are exogenously given. We first show that the seller's problem can be formulated in terms of threshold consumer types. We then streamline the model around two key parameters — information intensity (or availability) at pre-order and normalized margin of the product. Next, we discuss the features of the seller's profit function and show that it is optimal for the seller to exercise both pre-order and retail. Finally, we illustrate the possible scenarios that may arise in the optimal solution through numerical analysis. In Section 5, we will allow the seller to control the information available at the pre-order date, i.e., alter distributions F and G, and study his information release strategy.

[4] A rule of thumb for an excellent approximation is that both np and $n(1-p)$ should be greater than 5.

4.1. Seller's Pricing Problem and Consumer Valuation Thresholds

The seller tries to maximize his profit, while consumers try to maximize their expected utilities given the pre-order price p_0 and retail price p_1. If the distributions F and G are given, the seller solves the following pricing problem:[5]

$$\max_{p_0,p_1,x(\cdot),y(\cdot)} (p_0-c)\int_{\theta_0} x(\theta_0)dF(\theta_0)+(p_1-c)\int_{\theta_0}\left[\int_{\theta_1} y(\theta_1)dG(\theta_1|\theta_0)\right](1-x(\theta_0))dF(\theta_0) \tag{1}$$

$$\text{s.t. } y(\theta_1)=\begin{cases}1,\ \theta_1\geq p_1\\ 0,\ \theta_1<p_1\end{cases}, \tag{2}$$

$$x(\theta_0)=\begin{cases}1,\ \theta_0-p_0\geq \int_{\theta_1} y(\theta_1)(\theta_1-p_1)dG(\theta_1|\theta_0)\\ 0,\ \theta_0-p_0< \int_{\theta_1} y(\theta_1)(\theta_1-p_1)dG(\theta_1|\theta_0)\end{cases}. \tag{3}$$

Constraint (2) describes whether a type θ_1 consumer will purchase the product at time 1, and constraint (3) describes whether a type θ_0 consumer will make the purchase at time 0.

We first show that consumers' purchase decisions at both dates are threshold policies, which implies that pre-order enables the seller to capture high-valuation consumers at an early date.

THEOREM 1. *Given any pre-order price p_0 and retail price p_1, a consumer with initial type θ_0 pre-orders at time 0 if and only if $\theta_0\geq\tau_0$, and she purchases at time 1 after observing her final type θ_1 (if not pre-ordered already) if and only if $\theta_1\geq\tau_1$, where τ_0 and τ_1 are determined by:*

$$\tau_1=p_1, \tag{4}$$

$$p_0=\int_{-\infty}^{p_1}\theta_1 dG(\theta_1|\tau_0)+p_1\overline{G}(p_1|\tau_0)=p_1-\int_{-\infty}^{p_1}G(\theta_1|\tau_0)d\theta_1. \tag{5}$$

A sketch of the proof is presented here for completeness and more details are provided in Appendix A. Clearly, a consumer will purchase the product at time 1 if and only if $\theta_1-p_1\geq 0$. It can be shown that she should pre-order the product at time 0 if and only if $p_1-p_0\geq\int_{-\infty}^{p_1}(p_1-\theta_1)dG(\theta_1|\theta_0)$. That is, she pre-orders if and only if the pre-order discount p_1-p_0 exceeds her potential regret, $\int_{-\infty}^{p_1}(p_1-\theta_1)dG(\theta_1|\theta_0)$, or equivalently, $\int_{-\infty}^{p_1}G(\theta_1|\theta_0)d\theta_1$. Because $G(\theta_1|\theta_0)$ is decreasing in θ_0 for any θ_1, $\int_{-\infty}^{p_1}G(\theta_1|\theta_0)d\theta_1$ is decreasing in θ_0 and hence a consumer's optimal pre-order strategy is a threshold policy.

Because $\int_{-\infty}^{p_1}G(\theta_1|\tau_0)d\theta_1$ is monotone in τ_0, τ_0 is uniquely determined by p_0 and p_1 from expression (5). The set of equations (4) and (5) implies that the price pair (p_0,p_1) and threshold-type pair (τ_0,τ_1) have a one-to-one correspondence. Thus, the seller's pricing problem (1)-(3) can be reformulated based on the threshold types as follows:

$$\max_{\tau_0,\tau_1}\Pi(\tau_0,\tau_1)=(p_0(\tau_0,\tau_1)-c)\overline{F}(\tau_0)+(\tau_1-c)\overline{G}(\tau_1|\theta_0<\tau_0)F(\tau_0), \tag{6}$$

$$\text{where } p_0(\tau_0,\tau_1)=\tau_1-\int_{-\infty}^{\tau_1}G(\theta_1|\tau_0)d\theta_1=\tau_1\overline{G}(\tau_1|\tau_0)+\int_{-\infty}^{\tau_1}\theta_1 dG(\theta_1|\tau_0). \tag{7}$$

In the objective function, $\overline{G}(\tau_1|\theta_0<\tau_0)F(\tau_0)$ is a shorthand notation for $\int_{-\infty}^{\tau_0}\int_{\tau_1}^{+\infty}dG(\theta_1|\theta_0)dF(\theta_0)$. Notice that setting $\tau_0=+\infty$ (or $p_0\geq p_1$) is equivalent to a pure retailing strategy.

[5] To focus on the joint effect of pricing and information release instead of capacity/inventory planning, the seller is assumed to have ample capacity and can perfectly match the supply with demand. In addition, we assume without loss of generality that a consumer will purchase the product if she is indifferent between buying and not buying.

4.2. Simplification Through Normalized Margin and Pre-order Information Intensity

Given the valuation distributions $F(\theta_0) \sim N(\mu, \frac{\kappa^4}{\xi^2+\kappa^2})$ and $G(\theta_1|\theta_0) \sim N(\theta_0, \frac{\xi^2\kappa^2}{\xi^2+\kappa^2})$, the seller's problem (1)-(3) or (6)-(7) is fully determined by four parameters (μ, c, κ, ξ). We show that it is essentially determined by only two measures: the normalized margin, defined as $z = \frac{\mu - c}{\kappa}$, and the pre-order information intensity, defined as $\lambda = \frac{\kappa^2}{\xi^2+\kappa^2}$. The measure z captures the intrinsic profitability of the product, while the measure λ captures the amount of information available at the pre-order date and belongs to the closed interval $[0,1]$ (if $\frac{\kappa^2}{\infty}$ is interpreted as 0). Then, the distributions of θ_0 and $\theta_1|\theta_0$ can be rewritten as $N(\mu, \lambda\kappa^2)$ and $N(\theta_0, (1-\lambda)\kappa^2)$, respectively, i.e., $F(\theta_0) = \Phi\left(\frac{\theta_0 - \mu}{\sqrt{\lambda}\kappa}\right)$ and $G(\theta_1|\theta_0) = \Phi\left(\frac{\theta_1 - \theta_0}{\sqrt{1-\lambda}\kappa}\right)$ (where $\Phi(\cdot)$ and later $\phi(\cdot)$ denote the c.d.f. and p.d.f. of the standard normal distribution). It is also convenient to use the notation $\sigma_0 = \sqrt{\lambda}\kappa$ and $\sigma_1 = \sqrt{1-\lambda}\kappa$, and there is a one-to-one correspondence between (κ, λ) and (σ_0, σ_1).

As the next result shows, the seller's problem (6)-(7) can be boiled down to a two-parameter problem.

THEOREM 2. *Given the normalized margin $z = \frac{\mu - c}{\kappa}$ and pre-order information intensity λ, the seller solves the following normalized model without loss of generality:*

$$\max_{\tau_0, \tau_1} p_0 \overline{\Phi}\left(\frac{\tau_0 - z}{\sigma_0}\right) + \tau_1 \int_{-\infty}^{\tau_0} \overline{\Phi}\left(\frac{\tau_1 - \theta_0}{\sigma_1}\right) d\left(\Phi\left(\frac{\theta_0 - z}{\sigma_0}\right)\right), \tag{8}$$

where $p_0 = \tau_0 - \sigma_1 \phi\left(\frac{\tau_1 - \tau_0}{\sigma_1}\right) + (\tau_1 - \tau_0)\overline{\Phi}\left(\frac{\tau_1 - \tau_0}{\sigma_1}\right)$, $\sigma_0 = \sqrt{\lambda}$, and $\sigma_1 = \sqrt{1-\lambda}$.

The normalized model can be viewed as a special instance of the original model (6)-(7) with marginal cost $c = 0$, standard deviation $\kappa = 1$, and mean value $\mu = z$. From the solution of the normalized model, (τ_0, τ_1), the solution of the original model can be obtained by an affine transformation, $(\kappa\tau_0 + c, \kappa\tau_1 + c)$, and the optimal profit under the original model is κ times the normalized one. Because of this simple relationship, we will focus on the normalized model in the rest of this paper.

We like to point out that unlike the existing literature (e.g., Xie and Shugan 2001 and Liu and Xiao 2008) that study the impacts of the expected margin and consumer valuation dispersion separately, the normalized margin ties the two factors together and provides a single measure that fundamentally influences both the pricing decision and the information decision of the seller.

4.3. Seller's Profit Function and Optimality of Pre-order

The seller's total profit $\Pi(\tau_0, \tau_1)$ consists of two parts, the pre-order profit $\Pi_0(\tau_0, \tau_1)$ at time 0 and the retail profit $\Pi_1(\tau_0, \tau_1)$ at time 1. From the normalized model (8), we have:

$$\Pi_0(\tau_0, \tau_1) = p_0 \overline{\Phi}\left(\frac{\tau_0 - z}{\sigma_0}\right) = \left(\tau_0 - \sigma_1 \phi\left(\frac{\tau_1 - \tau_0}{\sigma_1}\right) + (\tau_1 - \tau_0)\overline{\Phi}\left(\frac{\tau_1 - \tau_0}{\sigma_1}\right)\right) \overline{\Phi}\left(\frac{\tau_0 - z}{\sigma_0}\right), \tag{9}$$

$$\Pi_1(\tau_0, \tau_1) = \tau_1 \int_{-\infty}^{\tau_0} \overline{\Phi}\left(\frac{\tau_1 - \theta_0}{\sigma_1}\right) d\left(\Phi\left(\frac{\theta_0 - z}{\sigma_0}\right)\right) = \frac{\tau_1}{\sigma_0} \int_{-\infty}^{\tau_0} \overline{\Phi}\left(\frac{\tau_1 - \theta_0}{\sigma_1}\right) \phi\left(\frac{\theta_0 - z}{\sigma_0}\right) d\theta_0. \tag{10}$$

The three profit functions, $\Pi_0(\tau_0, \tau_1)$, $\Pi_1(\tau_0, \tau_1)$, and $\Pi(\tau_0, \tau_1)$, are illustrated in Figure 2, from left to right, for the instance $(z, \lambda) = (1, 0.2)$.[6]

As shown in Figure 2, the graphs of $\Pi_0(\tau_0, \tau_1)$ and $\Pi_1(\tau_0, \tau_1)$ resemble two mountain ridges roughly parallel to the τ_1 axis and τ_0 axis, respectively, and the sum of the two, $\Pi(\tau_0, \tau_1)$, resembles

[6] The attentive reader may have noticed that the pre-order and total profits are negative at $(\tau_0, \tau_1) = (0, 0)$. This is because $\tau_0 = \tau_1 = 0$ corresponds to $p_0 < p_1 = 0$.

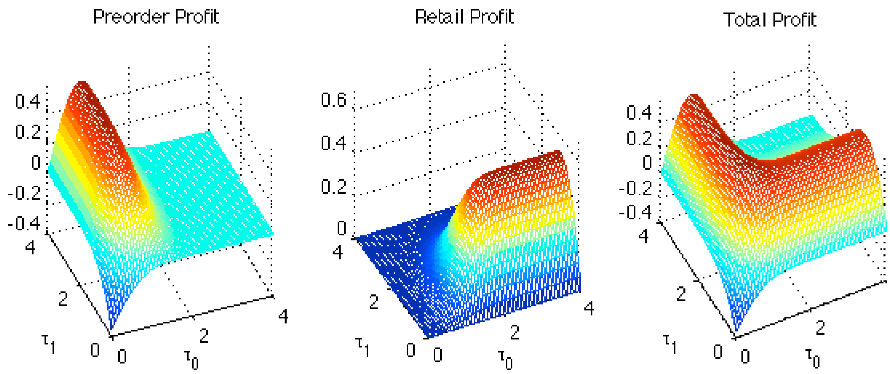


Figure 2 **Profit functions** $\Pi_0(\tau_0,\tau_1)$**,** $\Pi_1(\tau_0,\tau_1)$**, and** $\Pi(\tau_0,\tau_1)$**, from left to right, for** $(z,\lambda)=(1,0.2)$**.**

an L-shaped ridge. The non-concavity is an important feature of the seller's total-profit function. One implication, as will soon be seen, is the possible existence of multiple optimal solutions, which is rarely discussed in the existing pre-order (advance selling) literature.

To facilitate the analysis, we examine the first-order conditions next.

PROPOSITION 1. *The first-order condition* $\frac{\partial\Pi(\tau_0,\tau_1)}{\partial\tau_0}=0$ *is equivalent to the following equation:*

$$\tau_0-\frac{\sigma_0\overline{\Phi}\left(\frac{\tau_0-z}{\sigma_0}\right)}{\phi\left(\frac{\tau_0-z}{\sigma_0}\right)}=\frac{\sigma_1\phi\left(\frac{\Delta\tau}{\sigma_1}\right)}{\Phi\left(\frac{\Delta\tau}{\sigma_1}\right)}, \tag{11}$$

where $\Delta\tau=\tau_1-\tau_0$*. For any* $\Delta\tau\in(-\infty,+\infty)$*, there is a unique* τ_0 *satisfying (11). Furthermore, an optimal* τ_0 *must be greater than the exclusive pre-order price (when retail is excluded).*

PROPOSITION 2. *The first-order condition* $\frac{\partial\Pi(\tau_0,\tau_1)}{\partial\tau_1}=0$ *is equivalent to the following equation:*

$$\overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right)\overline{\Phi}\left(\frac{\tau_0-z}{\sigma_0}\right)+\int_{-\infty}^{\tau_0}\Phi\left(\frac{\theta_0-\tau_0-\Delta\tau}{\sigma_1}\right)d\left(\Phi\left(\frac{\theta_0-z}{\sigma_0}\right)\right)=\tau_1\phi(\tau_1-z)\Phi\left(\frac{\sigma_1^2\tau_0-\sigma_0^2\Delta\tau-\sigma_1^2z}{\sigma_0\sigma_1}\right). \tag{12}$$

Furthermore,

$$\overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right)\overline{\Phi}\left(\frac{\tau_0-z}{\sigma_0}\right)<(\tau_0+\Delta\tau)\phi(\tau_0+\Delta\tau-z)\Phi\left(\frac{\sigma_1^2\tau_0-\sigma_0^2\Delta\tau-\sigma_1^2z}{\sigma_0\sigma_1}\right)<\overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right). \tag{13}$$

Equation (11) enables us to express τ_0 as a function of $\Delta\tau$ and the total profit function $\Pi(\tau_0,\tau_1)$ as $\Pi(\tau_0(\Delta\tau),\Delta\tau+\tau_0(\Delta\tau))$, which reduces the two-variable optimization problem into a one-variable problem and can be easily solved by standard computing software such as MATLAB. This will be the basis for the numerical analysis in the remainder of this paper.

With the aid of the first-order conditions, we show that it is always optimal for the seller to sell at both dates. This differs from the result of Shugan and Xie (2004), who adopt a discrete-type model and show that the optimality of pre-order depends on model parameters.

THEOREM 3. *Given normal distributions* F *and* G*, the optimal threshold pair* (τ_0,τ_1) *that maximizes the seller's expected profit must satisfy* $\tau_0<+\infty$ *and* $\tau_1<+\infty$*. That is, it is optimal for the seller to adopt both pre-order and retail.*

The proof builds on the monotone hazard-ratio properties of normal distributions: (1) $\frac{\overline{\Phi}(\theta)}{\phi(\theta)}$ is decreasing in θ and $\lim_{\theta\to+\infty}\frac{\overline{\Phi}(\theta)}{\phi(\theta)}=0$; and (2) $\frac{\Phi(\theta)}{\phi(\theta)}$ is increasing in θ and $\lim_{\theta\to-\infty}\frac{\Phi(\theta)}{\phi(\theta)}=0$. For a fixed τ_1 (or p_1), choosing $\tau_0<+\infty$ has two effects. On one hand, it leaves positive surplus to consumers

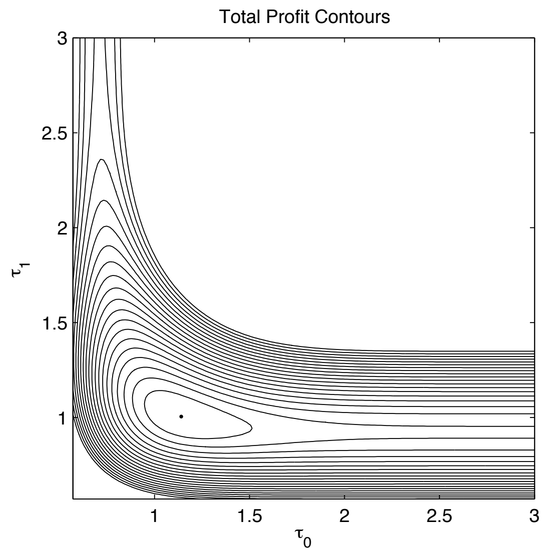


Figure 3 **Contour map of the seller's total profit** $\Pi(\tau_0, \tau_1)$**, given** $(z, \lambda) = (0.5, 0.5)$**.**

with high initial valuations. On the other hand, it increases social welfare, because given the pre-order opportunity some consumers will pre-order the product only to find their final valuations between p_1 and $c(< p_1)$. Those consumers would not have purchased the product without the pre-order option. The proof of the proposition suggests that the second effect dominates the first. Therefore, for a fixed p_1, both the seller and consumers benefit from the pre-order practice because pre-order increases market participation and social welfare and mitigates the adverse impact on the social welfare imposed by the threshold price p_1.

4.4. Optimal Solution Through Numerical Analysis

In this subsection, we explore the seller's optimal pricing strategy through representative examples. We choose the normalized margin $z = 0.5$ and consider different values of pre-order information intensity λ to illustrate the variations in the optimal solution of the seller's pricing problem. This discussion also prepares for the detailed investigation of the impact of λ in the next section.

Benchmark Case: No Pre-order ($\boldsymbol{\lambda}$= 1). We start with the case in which the seller uses retail exclusively, which is equivalent to the situation $\lambda = 1$ where consumers are fully informed at the pre-order date. In this situation, the seller should charge the optimal exclusive retail price 0.9220. At this price, 33.65% of the consumers purchase the product, and the seller's profit, consumer surplus, and social welfare are 0.3103, 0.2229, and 0.5332, respectively.

Case I: $\boldsymbol{\lambda}$= 0.5. The seller's profit function $\Pi(\tau_0, \tau_1)$ in this case is illustrated in Figure 3 in the form of a contour map. The map shows a unique local (and global) maximizer $(\tau_0, \tau_1) = (1.1412, 1.0047)$, with seller profit 0.3147. The contour interval (the profit difference between consecutive contour lines) is 0.0025, and for clarity of exhibition, the profit function is truncated after 20 contour levels.

At this optimal solution, the seller should charge a pre-order price $p_0 = 0.7856$ and a retail price $p_1 = 1.0047$, corresponding to consumer valuation thresholds $\tau_0 = 1.1412$ and $\tau_1 = 1.0047$. The seller captures high-valuation consumers early, through a deep pre-order discount, by setting the pre-order (retail) price lower (higher) than the exclusive retail price 0.9220. Consumer surplus at time 0 depends on whether pre-order is permitted or not. The difference between the two further depends on consumers' initial type θ_0. If the seller adopts pre-order and charges the optimal prices, consumers with low initial valuations will see their surplus decline (compared with the benchmark case of exclusive retail) due to the higher retail price, while those with high initial

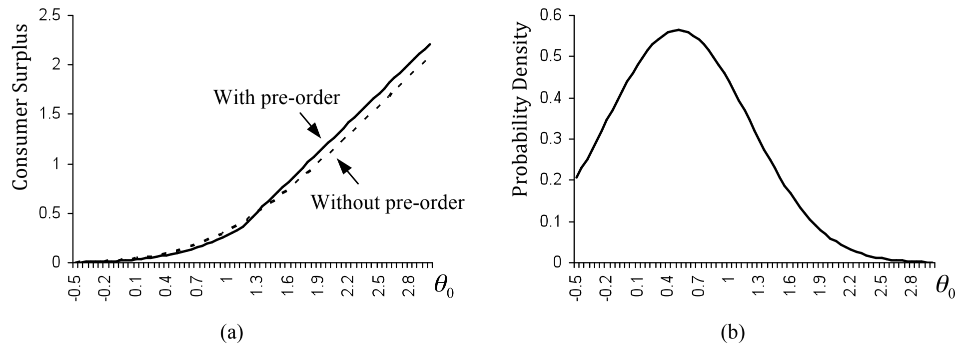


Figure 4 **Consumer surplus (a) and p.d.f. (b) of the initial type** θ_0**, when** $(z,\lambda)=(0.5,0.5)$.

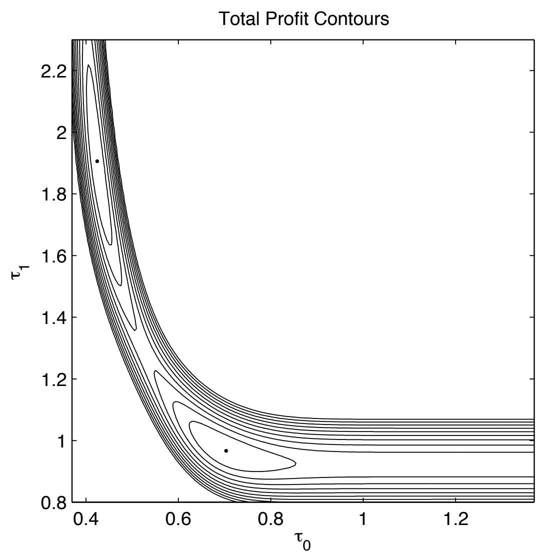


Figure 5 **Contour map of the seller's total profit** $\Pi(\tau_0,\tau_1)$**, given** $(z,\lambda)=(0.5,0.0193)$.

valuations will enjoy higher surplus due to the pre-order discount and relatively low risk of regret. We depict consumer surplus for various initial types in Figure 4(a). The solid line and dotted line represent consumer surplus with and without the pre-order option, respectively. Figure 4(b) shows the probability (normal) density function of the initial type θ_0 in the given range.

Because consumers are reasonably well informed of the product when $\lambda=0.5$, the market participation rate with pre-order is almost the same as that without it. The social welfare declines by less than 1%, the seller's profit increases by 1.4%, and the aggregate consumer surplus decreases by about 3%.

Case II: $\boldsymbol{\lambda}$**= 0.0193.** In this case, the contour map of the seller's total profit is illustrated in Figure 5. The map shows two optimal solutions $(\tau_0,\tau_1)=(0.7030,0.9670)$ and $(0.4242,1.9059)$, with the same seller profit 0.3112. It also reveals a saddle point at $(0.5276,1.2876)$. In the figure, the contour interval is 0.00075, and for clarity, the profit function is truncated after 10 contour levels.

The two optimal solutions represent two distinctive pricing strategies. (1) The valuation thresholds $(\tau_0,\tau_1)=(0.7030,0.9670)$ correspond to optimal pre-order and retail prices $(p_0,p_1)=(0.4260,0.9670)$. Even though the pre-order discount in this solution is larger than the one in Case I, the seller can only capture a smaller portion of high-valuation consumers through pre-order here because consumers are less certain about their valuations. At this solution, the market participation, social welfare, seller's profit, and consumer surplus are all similar to those in Case I.

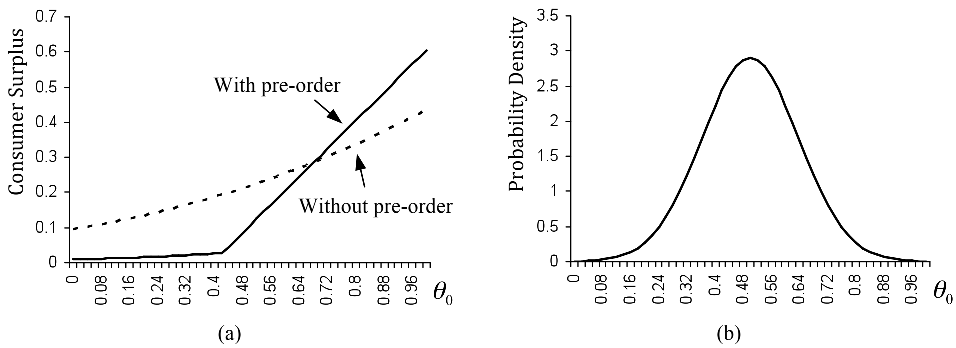


Figure 6 **Consumer surplus (a) and p.d.f. (b) of the initial type θ_0, when $(z,\lambda) = (0.5, 0.0193)$ and $(\tau_0, \tau_1) = (0.4242, 1.9059)$.**

(2) The valuation thresholds $(\tau_0, \tau_1) = (0.4242, 1.9059)$ correspond to optimal prices $(p_0, p_1) = (0.3949, 1.9059)$. Using this pricing strategy, instead of capturing only high-valuation consumers early, the seller charges a prohibitively high retail price to induce the majority of consumers to pre-order. Over 70% of consumers will buy the product, more than double of the participation rate in the benchmark case. Compared with the benchmark case, while the participation rate increases dramatically, the social welfare in fact decreases by 20% due to the inefficiency in allocating the good to ill-informed consumers who will likely regret at time 1 (the low normalized margin $z = 0.5$ makes this situation especially relevant). The difference in consumer surplus with and without pre-order also becomes larger, as can be seen in Figure 6(a). The aggregate consumer surplus declines by over 40% because consumers are almost homogeneous at the pre-order date and the seller can extract their surplus (information rent) relatively easily. This example is a vivid contrast to Xie and Shugan (2001), who show that advance selling benefits the seller through the increase of market participation, not the reduction of buyer surplus. With the combination of information rent reduction and social welfare loss, the seller merely improves his profit by less than half a percent.

However, the seller can improve his profit by releasing more information at the pre-order date (e.g. increasing λ to 0.5 as in Case I). If the seller releases information, social welfare can be improved because better allocation can be made, but consumers can acquire higher information rent as well. How should the seller make the trade-off between information rent extraction and social welfare improvement? This will be addressed in the next section as we study the seller's information release strategy.

Finally, it is worth noting that we can easily find examples in which market participation, social welfare, and seller's profit all increase significantly through pre-order when z becomes larger. We select a small $z = 0.5$ in this subsection in order to demonstrate an intriguing situation related to a later discussion in Subsection 5.3 (as will be seen, when $z = 0.5$, $\lambda = 0.0193$ represents a local minimizer of the optimal profit function over λ).

5. Seller's Optimal Information Release Strategy

In the above analysis of the seller's pricing strategy, we have treated the pre-order information intensity λ as exogenously given. In the real world, however, the seller can often control the amount of information available at the pre-order date. The more information the seller releases (i.e., the larger the λ), the better a consumer understands the (horizontal) attributes of the product and her own preferences at time 0.

The existing literature on information release (e.g. Lewis and Sappington 1991 and Johnson and Myatt 2006) finds that either no information or full information is optimal in the static environment in which the seller and consumers interact only once. This is not the case in our setting. When the seller releases all information, a consumer's initial valuation θ_0 equals her final valuation θ_1 and the maximum profit attainable by the seller is the same as that under pure (or exclusive) retail. By Theorem 3, given any F and G (with unbounded supports), pure retail is sub-optimal. Therefore, full information release is never optimal in our model, as summarized below.

COROLLARY 1. *If the seller can choose the pre-order information intensity λ in problem (8), the optimal λ must be strictly less than* 1*. That is, full information release is not optimal.*

The departure from the extent literature implies that the trade-off between consumer rents and social welfare is more complex in a dynamic environment such as ours. To find the seller's optimal information strategy, we start with the following benchmark case.

5.1. Benchmark Case: No Information at Pre-order ($\lambda = 0$)

We start with the extreme case in which consumers have no initial information about the product at time 0 and the seller decides not to release any information at all. In that situation, the pre-order information intensity is given by $\lambda = 0$ and consumers' initial valuations reduce to a single point, $\theta_0 = z$. Then, the seller should choose either pre-order or retail instead of both, because either all or no consumers will pre-order. (This is different from the situation studied in previous sections, in which θ_0 was assumed to have an infinite support and hence Theorem 3 applies. Nevertheless, as shown in Appendix B, the seller's profit function and pricing strategy are continuous at $\lambda = 0$.) We consider the seller's two extreme strategies, pure retail and pure pre-order, next.

Pure Retail and Pure Pre-Order. When the seller chooses pure retail, consumers can learn their true valuations at time 1 and decide whether to purchase or not.[7] Consumers order if and only if $\theta_1 \geq p_1$. Because the distribution of θ_1 is given by $\Phi(\theta_1 - z)$, the seller's problem reduces to a static pricing problem, $\Pi^R(z) = \max_p p\Phi(z-p)$, and the optimal price satisfies the first-order condition $\Phi(z - p^R) - p^R\phi(z - p^R) = 0$, or $p^R = \frac{\Phi(z-p^R)}{\phi(z-p^R)}$ (the superscript "R" stands for pure "retail"). It is straightforward to show that both $p^R(z)$ and $\Pi^R(z)$ are increasing in z, and that $(\Pi^R)'(z) = \Phi(z - p^R) \in (0,1)$, which approaches 1 as z approaches infinity.

If the seller wants all consumers to pre-order, he can set the pre-order price $p_0 = z$ and the retail price $p_1 = +\infty$. The seller's pure pre-order profit is then $\Pi^P(z) = z$ ("P" stands for pure "pre-order"). Because $(\Pi^P)'(z) = 1 > (\Pi^R)'(z)$, there exists a single threshold $z^\dagger$ such that $\Pi^P(z) \geq \Pi^R(z)$ if and only if $z \geq z^\dagger$. This threshold can be found to be 0.2267 through straightforward computation. Therefore, we arrive at the following result:

PROPOSITION 3. *When $\lambda = 0$, the seller should choose either pre-order or retail, and the former is better than the latter if and only if $z > z^\dagger = 0.2267$.*

Rotation of the Demand Curve. If the seller only sells at a single date, his choice between pure pre-order and pure retail is essentially the choice between homogeneous and heterogeneous consumers. Proposition 3 implies that the right choice depends on the normalized margin z. This can be explained through the rotation of the demand curve.[8]

If all consumers have the same valuation z, the demand curve is flat; when consumers become more and more heterogeneous, the demand curve rotates clockwise. When z is large enough, the optimal static price p should be no more than z; a clockwise rotation of the demand curve would result in less sales at the price p and hence the seller would prefer more homogeneous consumers. This scenario is illustrated in Figure 7(a) for $p' < z$. Conversely, when z is small enough or negative, the optimal static price p should be larger than z; a clockwise rotation of the demand curve would

[7] This scenario is equivalent to $\lambda = 1$, where consumers learn their true valuations at time 0 and the pre-order stage and retail stage essentially collapse into one.

[8] With a unit of consumers, the demand curve can be obtained from the c.d.f. of the valuation θ through a 90° counter-clockwise rotation and proper re-labeling of the axes, as can be seen in Figure 7.

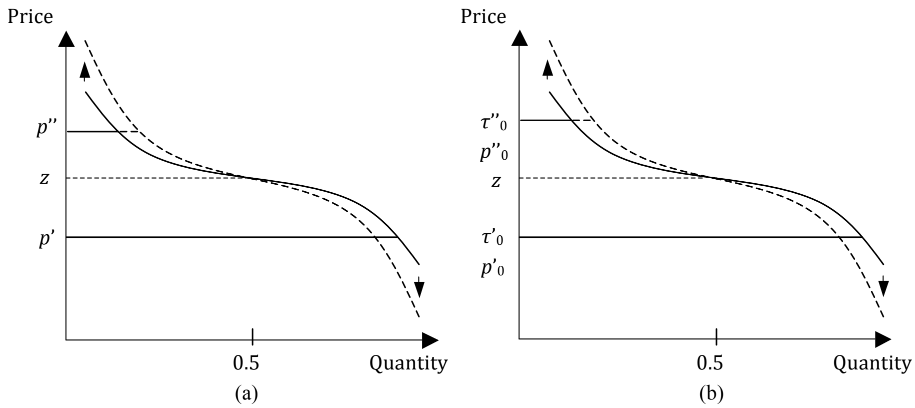


Figure 7 Effects of demand curve rotation in (a) the one-period setting and (b) the first period of a two-period setting.

bring in more sales and be preferred by the seller, as illustrated in Figure 7(a) for $p'' > z$. Following Johnson and Myatt's (2006) terminology, we say that the seller faces a "mass" market or has a mass product when z is large and faces a "niche" market or has a niche product when z is small. In a similar spirit as that of Proposition 3, Johnson and Myatt (2006) also argue that to raise profit the seller should reduce the variance of consumer valuations for a mass product while increase the variance for a niche product.

When the seller adopts both pre-order and retail, Figure 7(b) demonstrates the rotation effects of the pre-order demand curve, where the initial valuation threshold τ_0 is brought into the picture, as will be discussed soon.

5.2. Information Release: Impact of λ on Seller's Profit

We have shown above that when $\lambda = 0$, the seller only sells at a single date and the optimal price is governed by the normalized margin z. In this subsection, we examine the impact of a small change of λ on the seller's profit. In Section 4, the seller's profit function was expressed as $\Pi(\tau_0, \tau_1)$, in terms of the thresholds (τ_0, τ_1). It can also be written in terms of the prices (p_0, p_1), which turns out to be more convenient in explaining the impact of λ. By the envelope theorem, if $(p_0(\lambda), p_1(\lambda))$ are the optimal prices given λ, the derivative of the seller's profit with respect to λ is $\frac{d\Pi(\lambda, p_0(\lambda), p_1(\lambda))}{d\lambda} = \frac{\partial\Pi(\lambda, p_0(\lambda), p_1(\lambda))}{\partial\lambda}$ and thus it suffices to investigate the impact of λ for fixed p_0 and p_1 (both at their optimal values).

If the seller only sells at time 0 (and hence only p_0 is relevant), whether an increase in λ results in more or less sales depends on whether p_0 is greater or less than z, as shown in Figure 7(a). If the seller sells at both dates, the purchasing threshold at time 0 is given by τ_0, instead of p_0, and the proportion of consumers who pre-order depends on two effects simultaneously: (1) the demand rotation effect, as demonstrated in Figure 7(b), that may increase or decrease the pre-order demand depending on τ_0 and z, and (2) a regret (or really, diminishing regret) effect, $\frac{\partial\tau_0}{\partial\lambda} < 0$, as shown below, that always increases the pre-order demand. When the prices p_0 and p_1 are fixed, as λ increases, consumers become more heterogeneous at the pre-order date and hence the pre-order demand curve rotates clockwise. In the meantime, consumers become more assertive about their true valuations at the pre-order date and, as a result, are more willing to purchase early to secure the pre-order discount, which pushes τ_0 downward.

LEMMA 1. *The partial derivative of the pre-order threshold τ_0 with respect to the pre-order information intensity λ is given by*

$$\frac{\partial\tau_0(\lambda, p_0, p_1)}{\partial\lambda} = -\frac{1}{2\sigma_1}\frac{\phi\left(\frac{p_1-\tau_0}{\sigma_1}\right)}{\Phi\left(\frac{p_1-\tau_0}{\sigma_1}\right)} < 0. \tag{14}$$

That is, τ_0 is decreasing in λ for fixed p_0 and p_1.

Define $u \equiv \frac{\tau_0 - z}{\sigma_0}$. Then, the proportion of consumers who pre-order, under the joint influence of the above two forces, is given by $\overline{F}(\tau_0) = \overline{\Phi}(u)$. When $z < 0$ (which implies $z < 0 < p_0 < \tau_0$), as more information is available at pre-order, both the demand rotation effect (illustrated by the scenario $z < p_0'' < \tau_0''$ in Figure 7(b)) and the diminishing regret effect propels the pre-order demand, so the aggregate pre-order amount (or proportion) goes up, as summarized below.

Proposition 4. *When $z < 0$, for fixed p_0 and p_1, the percentage of consumers who pre-order, $\overline{\Phi}(u(\lambda, p_0, p_1))$, is increasing in λ, or equivalently, $\frac{\partial u(\lambda, p_0, p_1)}{\partial \lambda} < 0$.*

On the other direction, we can show that as z approaches infinity, the proportion of consumers who pre-order approaches 1 and the demand rotation effect suppresses the pre-order demand:

Proposition 5. *For any $\alpha < 1$, there exists a z_0 such that for any $z > z_0$, $\overline{\Phi}(u) > \alpha$ under the optimal pricing policy.*

Because $\overline{\Phi}(u) > 0.5$ is equivalent to $z > \tau_0 (> p_0)$, the above proposition implies that the demand rotation effect reduces the pre-order demand when z is large enough, as demonstrated in Figure 7(b). Thus, in this case, the demand rotation effect and diminishing regret effect are opposite to each other. Through numerical analysis, we find that when z is large the demand rotation effect dominates the diminishing regret effect and that $\overline{\Phi}(u)$ becomes decreasing in λ (or equivalently, $\frac{\partial u}{\partial \lambda} > 0$), under the optimal pricing policy.

The change of λ also impacts the seller's retail profit. For fixed p_0 and p_1, the proportion of consumers who purchase through retail also depends on two effects: (1) a demand substitution effect that decreases or increases consumer participation at the release date (depending on the sign of $\frac{\partial \overline{\Phi}(u)}{\partial \lambda}$), and (2) the diminishing regret effect that always dampens the retail demand.

The next theorem aggregates the profit changes at the pre-order date and release date. (In the theorem, the prices (p_0, p_1) are always fixed at their optimal levels $(p_0(\lambda), p_1(\lambda))$, λ is dropped from the right hand side of the expression for simplicity, and the functions $u(\lambda, p_0(\lambda), p_1(\lambda))$ and $\Delta\tau(\lambda, p_0(\lambda), p_1(\lambda))$ are simply denoted by u and $\Delta\tau$.)

Theorem 4. *The derivative of the seller's total profit with respect to λ is given by*

$$\frac{d\Pi(\lambda, p_0(\lambda), p_1(\lambda))}{d\lambda} = -\sigma_0 \overline{\Phi}(u) \frac{\partial u}{\partial \lambda} - \frac{p_1}{2\sigma_0 \sigma_1} \phi(u) \phi\left(\frac{\Delta\tau}{\sigma_1}\right), \tag{15}$$

where $\sigma_0 = \sqrt{\lambda}$, and $\sigma_1 = \sqrt{1-\lambda}$.

The second term on the right-hand side of expression (15) measures the diminishing regret effect on the retail profit. It is negative because with more accurate initial valuations, the lower $\Phi(u)$ quartile of consumers are less likely to buy at the release date (by definition, they do not buy at the pre-order date). The first term captures the demand substitution effect on the total profit and has the same sign as $\frac{\partial}{\partial \lambda} \overline{\Phi}(u)$.

The theorem implies that if more information results in lower pre-order demand (i.e., $\frac{\partial u}{\partial \lambda} > 0$), the seller should withhold information at the pre-order date. As argued above, this should happen when the normalized margin z is large enough.

5.3. Optimal Information Strategy

Equation (15) facilitates the investigation of information release at the pre-order date. We find that the seller's optimal strategy depends crucially on the normalized margin z. We also extend our model so that consumers can possess some initial information at time 0, represented by the initial information intensity $\lambda_0 \in [0, 1)$. In reality, the seller may have to disclose some product information anyway and consumers may also have some initial knowledge about the product beyond the seller's control. The total amount of information that can be released by the seller at pre-order is therefore limited to $\lambda \in [\lambda_0, 1]$. Clearly, the seller's optimal information strategy depends on λ_0 as well. Thus, we draw a strategy map for the seller on how much additional information to release for any given normalized margin z and initial information intensity λ_0, as shown in Figure 8. The map can be best explained in three sections, according to the value of z.

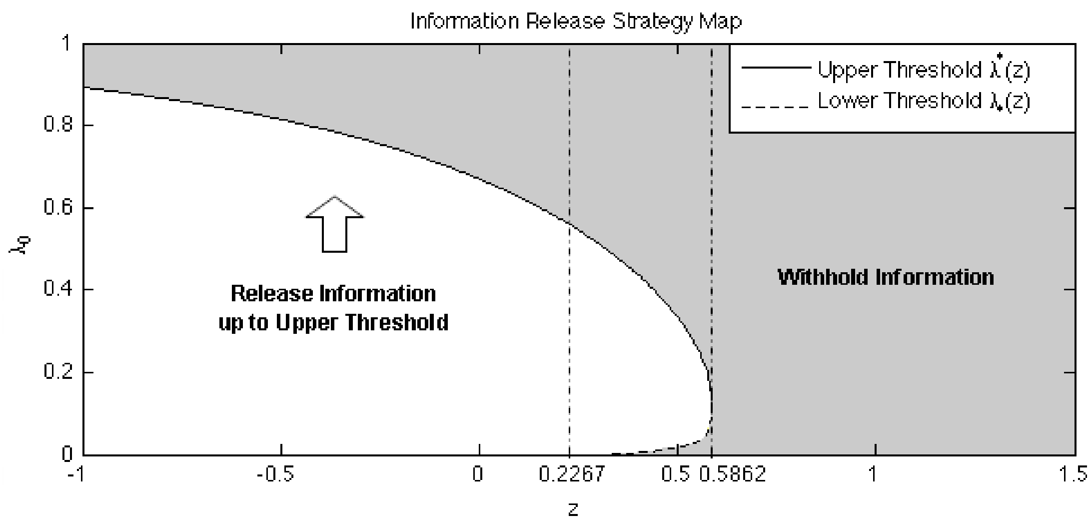


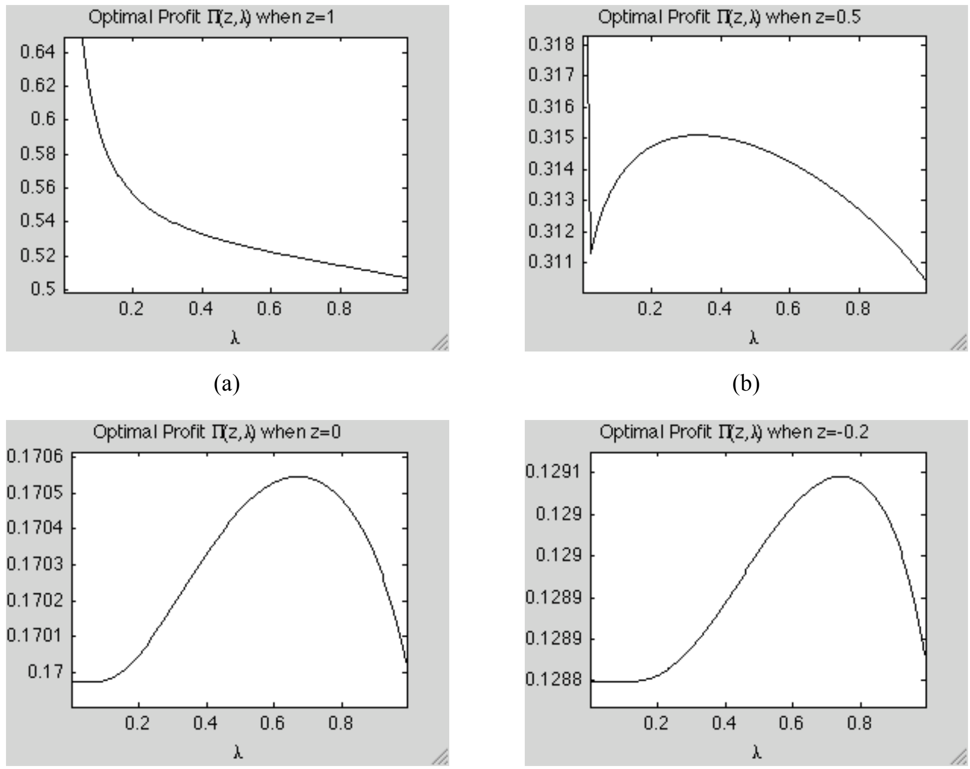
Figure 8 Optimal information release strategy.

I. High-Margin Section. When z is relatively large ($z > 0.5862$), the seller's optimal profit $\Pi(z, \lambda)$ is decreasing in λ in the entire interval $[0, 1]$. In this region of the strategy map, the seller should not disclose any information in addition to the initial information already acquired by consumers. Because the expected margin is large enough, the seller should advertise the product as a mass product and use pre-order effectively to capture average consumers. As a result, he should withhold information.

As an example, we illustrate the optimal profit $\Pi(z, \lambda)$ for the case $z = 1$ in Figure 9(a). The two extreme scenarios are: $\Pi(z, 0) = \Pi^P(z) = 1$ (via pure pre-order, at price 1) and $\Pi(z, 1) = \Pi^R(z) = 0.5066$ (via pure retail, at price $p^R(z) = 1.1317$). The profit function is strictly decreasing in $\lambda \in [0, 1]$. Thus, the seller should not release any additional information regardless of λ_0.

II. Low-Margin Section. When z is small ($z < z^\dagger = 0.2267$), the seller's optimal profit $\Pi(z, \lambda)$ is quasi-concave in λ and has a unique internal maximizer $\lambda^*(z)$ (> 0.5600). In this region of the strategy map, the seller should release information up to $\lambda^*(z)$ if consumers have less information to begin with — i.e. $\lambda_0 < \lambda^*(z)$ — but should withhold information otherwise. Because the expected margin is low, consumers are skeptical at time 0. The seller should advertise the product as a niche product and release a great deal of information to stimulate demand from high-valuation consumers at the pre-order date; in the meantime, he should also withhold some information so as to benefit from both pre-order and retail (Corollary 1).

The case $z = 0$ is illustrated in Figure 9(c), with the following extreme scenarios: $\Pi(z, 0) = \Pi(z, 1) = \Pi^R(z) = 0.1700$ (via pure retail, at price $p^R(z) = 0.7518$). The case $z = -0.2$ is illustrated in Figure 9(d), with the following extreme scenarios: $\Pi(z, 0) = \Pi(z, 1) = \Pi^R(z) = 0.1288$ (via pure retail, at price $p^R = 0.6940$). We observe that both profit functions have a unique peak and that the optimal $\lambda^*(z)$ increases as z decreases because more information is needed to stir up the pre-order demand when z is smaller.



(a) (b) (c) (d)

Figure 9 Optimal profit $\Pi(z,\lambda)$ as a function of λ, when (a) $z=1$, (b) $z=0.5$, (c) $z=0$, and (d) $z=-0.2$.

III. Medium-Margin Section. When z is in the middle $(0.2267 < z < 0.5862)$, the optimal strategy is more intriguing because the profit function $\Pi(z,\lambda)$, for $\lambda \in [0,1]$, has two local maxima, corresponding to two selling strategies: mass market strategy and niche market strategy. By choosing a tiny λ $(\geq \lambda_0)$, the seller may effectively market (and pre-sell) the product as a mass product at the pre-order date because consumers are almost homogeneous at the pre-order date; by choosing a larger λ and inducing more heterogenous consumer preferences, the seller can market the product as a niche product at the pre-order date, which may be particularly beneficial when z is small. Which strategy is better for the seller is affected by how much information consumers initially possess.

We find that when $z < 0.2288$, the niche-market optimum dominates the mass-market one, and the seller should continue to adopt the niche-market strategy as he does in the low-margin situation, by releasing a large amount of information up to the threshold $\lambda^*(z)$. When $z > 0.2288$ and λ_0 is smaller than some threshold $\lambda_*(z)$, the mass-market optimum overtakes the niche-market optimum, and the seller should adopt the mass-market strategy and withhold information as he does in the high-margin situation. However, if $z > 0.2288$ but $\lambda_0 \in (\lambda_*(z), \lambda^*(z))$, the seller should adopt the niche-market strategy and release information up to $\lambda^*(z)$. Intuitively, when the information about a somewhat niche product is leaked (by just a tiny amount perhaps) to the extent that the product can no longer be perceived as a mass product, it would be wise for the seller not to position the product as a mass product. Finally, we note that the upper threshold function $\lambda^*(z)$ is decreasing in z while the lower threshold $\lambda_*(z)$ is increasing in z. At $z = 0.5862$, the two thresholds meet at $\lambda_*(z) = \lambda^*(z) = 0.1320$, and the profit function $\Pi(z,\lambda)$ becomes monotone in λ when $z > 0.5862$.

The case $z = 0.5$ is illustrated in Figure 9(b). The two extreme scenarios are: $\Pi(z,0) = \Pi^P(z) = 0.5$ (via pure pre-order, at price 0.5) and $\Pi(z,1) = \Pi^R(z) = 0.3103$ (via pure retail, at price $p^R(z) = 0.9220$). The seller's profit function $\Pi(z,\lambda)$ has two local optima, at $\lambda = 0$ and 0.3330, one internal

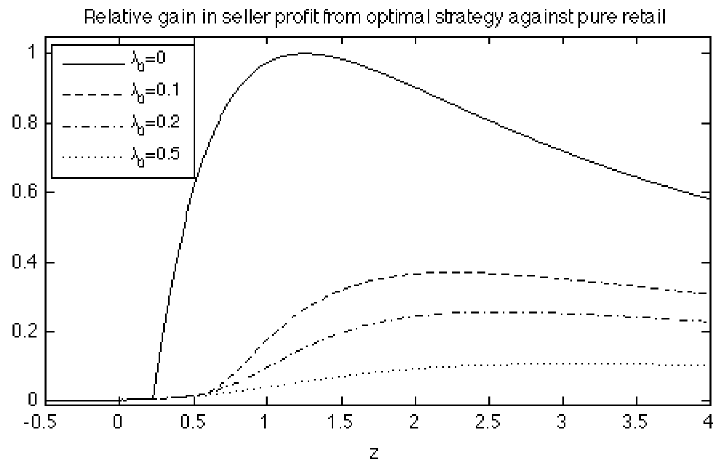


Figure 10 **Relative gain in the seller's profit from the optimal information and pricing strategies against pure retail.**

local minimum, at $\lambda = 0.0193$, and the global optimum is located at $\lambda = 0$. The derivative of $\Pi(z, \lambda)$ with respect to λ is discontinuous at $\lambda = 0.0193$ because the optimal pricing strategy jumps from one local optimum to the other at this λ, as illustrated in Figure 5 in Subsection 4.4. At this λ, the seller is indifferent between two pricing strategies: one attempts to capture a small portion of high-valuation consumers at the pre-order date, and the other aims to appeal to the majority of the consumers at the pre-order date. The seller decides the information release strategy before the pricing strategy. According to the information strategy map (Figure 8), for $z = 0.5$, if $\lambda_0 < \lambda_*(z) = 0.0171$, the seller should choose the mass-market strategy and withhold information; if $\lambda_*(z) < \lambda_0 < \lambda^*(z) = 0.3330$, the seller should adopt the niche-market strategy and release information up to $\lambda^*(z)$; if $\lambda_0 > \lambda^*(z)$, the seller should withhold further information to benefit the most from both pre-order and retail (Corollary 1).

6. Further Discussions

6.1. Relative Gain from Pre-order

Now, we discuss when pre-order is most beneficial to the seller. We compute his relative gain from the optimal information and pricing strategies when pre-order is adopted against the benchmark case of pure retail without pre-order — i.e., the ratio $\frac{\Pi(z,\lambda)-\Pi^R(z)}{\Pi^R(z)}$, where λ is optimally chosen for each z. The result is summarized in Figure 10.

Let us start with the line $\lambda_0 = 0$, which describes the situation in which consumers have no initial information. The pre-order option has only a minor impact on the seller's profit when z is small, because the seller needs to release a great amount of information to stir up demand for a niche product, which at the same time blurs the distinction between pre-order and retail and limits the gain from pre-order. Nevertheless, the gain becomes significant, and the slope of the curve $\lambda_0 = 0$ has a jump at $z = 0.2288$ when the seller switches from the niche-market strategy to the mass-market strategy at the pre-order date, as discussed in Subsection 5.3. When z becomes too large, the relative gain from adopting pre-order decreases, because pure retail is a very attractive option for the seller — by charging a retail price slightly lower than the normalized margin, the seller can capture almost all of the market without sacrificing much unit profit in relative terms, so the room for further improvements through pre-order shrinks relatively.

Whether or not the seller can implement a highly profitable mass-market strategy at the pre-order date depends on consumers' initial knowledge of the product. To account for the more realistic situation with nonzero λ_0, additional curves for $\lambda_0 = 0.1$, 0.2, and 0.5 are drawn in Figure 10. As λ_0 increases, the gain from the pre-order practice decreases because the seller can only operate in a smaller strategy domain. Nevertheless, the gain is still substantial for large z's when the seller can successfully position the product as a mass-market one at the pre-order date (e.g., over 10% at $z = 3$ when $\lambda_0 = 0.5$).

6.2. Risk-averse Consumers

We have focused on risk-neutral consumers throughout the paper. In this discussion, we show that it is still optimal for the seller to adopt pre-order if consumers are risk-averse and, interestingly, his profit is even higher in this case.

For simplicity, we assume that consumers obtain total utility $u(-p) + g$ if the good is purchased at price p or $u(0)$ if no purchase is made, where $u' > 0$ and $u'' < 0$. At time 1, a consumer knows exactly her utility derived from the good, g (which may be different across consumers). To facilitate the comparison between the risk-neutral case and risk-averse case, we define the *final type* θ_1 as the maximum money the consumer is willing to pay for the good, i.e., satisfying $u(-\theta_1) + g = u(0)$. At time 0, the consumer knows her *initial type* θ_0 and the conditional distribution of $\theta_1|\theta_0$, while the seller only knows the distributions of θ_0 and $\theta_1|\theta_0$. We assume that $\theta_1 = \theta_0 + \epsilon$ and both θ_0 and ϵ follow independent normal distributions as in Section 3.[9] We have the following result:

THEOREM 5. *Given any pre-order and retail prices, consumers' purchase decisions at both dates are threshold policies.*

The proof is analogous to the proof of Theorem 1 and is omitted. Given this threshold result, we can show that:

THEOREM 6. *It is optimal for the seller to practice pre-order when consumers are risk-averse.*

The proof is based on the observation that risk-averse consumers are willing to pay more at the pre-order date than risk-neutral consumers. A consumer is willing to pay a risk-premium because if the good turns out to be of high value, she will pay the higher retail price (comparing to the pre-order price). Such a monetary risk is undesirable by a risk-averse consumer. As a result, she is more willing to participate in pre-order and secure consumption at the lower price.

7. Conclusion

In this paper, we identified the seller's optimal pre-order information and pricing strategies under a continuous-valuation model. The seller's optimal strategies generally involve selling at both the pre-order and release dates with a pre-order discount and the optimal pricing strategy highly depends on the information strategy. We find that the seller can make a strategic choice at the pre-order date, whether to position the product as a "mass-market" one or a "niche-market" one, through information provision. We also find that while pre-order benefits the seller across the board, the effect is most pronounced when the seller can effectively position the product for the mass market at the pre-order date.

In the future, we would like to investigate other factors that influence the pre-order practice, including return policies, inventory and capacity considerations, heterogeneity in the information noise level, logistic costs at different dates, and etc. Given the existence of alternative theories on the pre-order practice, empirical studies offer another promising research direction.

[9] In Section 3, the initial type θ_0 coincides with the expected value of the good for a consumer and has a natural explanation. When consumers are risk-averse, θ_0 is not the maximum money the consumer is willing to pay at the pre-order date given the available information. Nevertheless, similar analysis still carries through even when θ_0 is no longer the expected value of the good. The essence is that the initial type θ_0 provides a consumer advantageous information about her final (true) type θ_1.

Appendix A: Proofs

Proof of Theorem 1. If a consumer waits until time 1, she will buy if and only if $\theta_1 - p_1 \ge 0$, which is a threshold policy with the threshold type $\tau_1 = p_1$. At time 0, the consumer will buy the product if and only if the expected net utility from buying is no less than that from waiting — i.e.,

$$\theta_0 - p_0 \ge \int_{p_1}^{+\infty} (\theta_1 - p_1) dG(\theta_1|\theta_0). \tag{16}$$

Recall that $\theta_0 = E[\theta_1|\theta_0]$. The above expression can be simplified to $p_0 \le \int_{-\infty}^{p_1} \theta_1 dG(\theta_1|\theta_0) + p_1 \overline{G}(p_1|\theta_0)$, where $\overline{G}(p_1|\theta_0) = 1 - G(p_1|\theta_0)$. Because $E[\theta_1|\theta_0]$ is well defined, we have $\int_{-\infty}^{p_1} \theta_1 dG(\theta_1|\theta_0) = p_1 G(p_1|\theta_0) - \int_{-\infty}^{p_1} G(\theta_1|\theta_0) d\theta_1$, by integration by parts. Therefore, the consumer pre-orders the product if and only if $p_1 - p_0 \ge \int_{-\infty}^{p_1} G(\theta_1|\theta_0) d\theta_1$.

Because $G(\theta_1|\theta_0) = G(\theta_1 - \theta_0|0)$ is decreasing in θ_0, the consumer's optimal pre-order strategy is also a threshold policy with the threshold type τ_0 determined by expression (5). □

Proof of Theorem 2. By expression (7) and the fact that $\phi'(x) = -x\phi(x)$, we have

$$\begin{aligned}
p_0 &= \int_{-\infty}^{\tau_1} \theta_1 dG(\theta_1|\tau_0) + \tau_1 \overline{G}(\tau_1|\tau_0) \\
&= \int_{-\infty}^{\tau_1} \theta_1 d\Phi\left(\frac{\theta_1 - \tau_0}{\sigma_1}\right) + \tau_1 \overline{\Phi}\left(\frac{\tau_1 - \tau_0}{\sigma_1}\right) \\
&= \int_{-\infty}^{\tau_1} \frac{\theta_1 - \tau_0}{\sigma_1} \phi\left(\frac{\theta_1 - \tau_0}{\sigma_1}\right) d\theta_1 + \int_{-\infty}^{\tau_1} \frac{\tau_0}{\sigma_1} \phi\left(\frac{\theta_1 - \tau_0}{\sigma_1}\right) d\theta_1 + \tau_1 \overline{\Phi}\left(\frac{\tau_1 - \tau_0}{\sigma_1}\right) \\
&= \sigma_1 \int_{-\infty}^{\frac{\tau_1 - \tau_0}{\sigma_1}} x_1 \phi(x_1) dx_1 + \tau_0 \int_{-\infty}^{\frac{\tau_1 - \tau_0}{\sigma_1}} \phi(x_1) dx_1 + \tau_1 \overline{\Phi}\left(\frac{\tau_1 - \tau_0}{\sigma_1}\right) \\
&= -\sigma_1 \phi\left(\frac{\tau_1 - \tau_0}{\sigma_1}\right) + \tau_0 \Phi\left(\frac{\tau_1 - \tau_0}{\sigma_1}\right) + \tau_1 \overline{\Phi}\left(\frac{\tau_1 - \tau_0}{\sigma_1}\right).
\end{aligned}$$

Therefore, given any $\tau_0, \tau_1 \in \mathbb{R}$ and $\sigma_1 > 0$, (1) for any $h \in \mathbb{R}$,

$$p_0(\tau_0 + h, \tau_1 + h, \sigma_1) = p_0(\tau_0, \tau_1, \sigma_1) + h, \tag{17}$$

and (2) for any $h > 0$,

$$p_0(h\tau_0, h\tau_1, h\sigma_1) = h p_0(\tau_0, \tau_1, \sigma_1). \tag{18}$$

The above expressions indicate a linear relationship between the pre-order price p_0 and the threshold types τ_0 and τ_1: (1) shifting the threshold types by h (keeping σ_1 constant) shifts the pre-order price by h as well, and (2) scaling the threshold types (along with the standard deviation σ_1) by a factor of h scales the pre-order price by h as well. This relationship provides the basis for normalizing the model parameters below.

Define $z = \frac{\mu - c}{\kappa}$, $\tau_i' = \frac{\tau_i - c}{\kappa}$, and $\sigma_i' = \frac{\sigma_i}{\kappa}$, for $i = 0, 1$. Then expressions (17) and (18) imply $p_0(\tau_0, \tau_1, \sigma_1) - c = \kappa p_0\left(\frac{\tau_0 - c}{\kappa}, \frac{\tau_1 - c}{\kappa}, \frac{\sigma_1}{\kappa}\right) = \kappa p_0(\tau_0', \tau_1', \sigma_1')$, and the seller's objective function (6) can be rewritten as

$$\Pi(\tau_0, \tau_1) = \kappa p_0(\tau_0', \tau_1', \sigma_1') \overline{\Phi}\left(\frac{\tau_0' - z}{\sigma_0'}\right) + \kappa \tau_1' \int_{-\infty}^{\tau_0'} \overline{\Phi}\left(\frac{\tau_1' - \theta_0'}{\sigma_1'}\right) d\left(\Phi\left(\frac{\theta_0' - z}{\sigma_0'}\right)\right).$$

Notice that $\sigma_0' = \sqrt{\lambda}$ and $\sigma_1' = \sqrt{1 - \lambda}$, which depend only on λ, and the scaled profit function $\Pi(\tau_0, \tau_1)/\kappa$ depends on μ, c, and κ only through $z = \frac{\mu - c}{\kappa}$. Therefore, the seller's problem can be simplified by using parameters (z, λ) and decision variables (τ_0', τ_1'), which is the normalized model. To simplify expressions, we use the same notation as in the original model — i.e., τ_0, τ_1, σ_0, σ_1, and etc. □

Proof of Proposition 1. It is useful to examine the first-order derivatives of $\Pi(\tau_0, \tau_1)$. From expression (7), the partial derivatives of p_0 with respect to τ_0 and τ_1, respectively, are given by:

$$\frac{\partial p_0(\tau_0, \tau_1)}{\partial \tau_0} = \int_{-\infty}^{\tau_1} \frac{\partial \overline{G}(\theta_1|\tau_0)}{\partial \tau_0} d\theta_1, \tag{19}$$

$$\frac{\partial p_0(\tau_0, \tau_1)}{\partial \tau_1} = \overline{G}(\tau_1|\tau_0). \tag{20}$$

From expression (19), it is straightforward to show that

$$\frac{\partial p_0(\tau_0,\tau_1)}{\partial \tau_0} = \Phi(\frac{\Delta\tau}{\sigma_1}). \tag{21}$$

(Note that the notation $\Delta\tau$ is used only to simplify the expression. When taking the partial derivative with respect to τ_0, we hold τ_1 constant, not $\Delta\tau$. The same is true for the expressions below.) From (9), (10), and (21), we obtain

$$\begin{aligned}\frac{\partial \Pi(\tau_0,\tau_1)}{\partial \tau_0} =&\Phi\left(\frac{\Delta\tau}{\sigma_1}\right)\overline{\Phi}\left(\frac{\tau_0-z}{\sigma_0}\right)-\frac{1}{\sigma_0}\left(\tau_0-\sigma_1\phi\left(\frac{\Delta\tau}{\sigma_1}\right)+\Delta\tau\overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right)\right)\phi\left(\frac{\tau_0-z}{\sigma_0}\right)\\ &+\frac{\tau_1}{\sigma_0}\overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right)\phi\left(\frac{\tau_0-z}{\sigma_0}\right)\\ =&\Phi\left(\frac{\Delta\tau}{\sigma_1}\right)\overline{\Phi}\left(\frac{\tau_0-z}{\sigma_0}\right)-\frac{1}{\sigma_0}\left(\tau_0\Phi\left(\frac{\Delta\tau}{\sigma_1}\right)-\sigma_1\phi\left(\frac{\Delta\tau}{\sigma_1}\right)\right)\phi\left(\frac{\tau_0-z}{\sigma_0}\right).\end{aligned}$$

Therefore, the first-order condition $\frac{\partial \Pi(\tau_0,\tau_1)}{\partial \tau_0}=0$ is equivalent to equation (11).

It can be verified that the right-hand side of (11) is strictly decreasing in $\Delta\tau$ and that the left-hand side is strictly increasing in τ_0 because of the monotone hazard-ratio properties of the normal distribution. As $\Delta\tau$ varies from $-\infty$ to $+\infty$, the right-hand side varies from $+\infty$ to 0; as τ_0 varies from $-\infty$ to $+\infty$, the left-hand side varies from $-\infty$ to $+\infty$ as well. Therefore, for any $\Delta\tau \in (-\infty,+\infty)$, there is a unique τ_0 satisfying equation (11). Furthermore, the monopoly price under exclusive pre-order solves the equation when the right-hand side is zero, which must be smaller than any τ_0 that solves the equation with a positive right-hand side. □

Proof of Proposition 2. By (20), it is straightforward to show that $\frac{\partial p_0(\tau_0,\tau_1)}{\partial \tau_1} = \overline{\Phi}(\frac{\Delta\tau}{\sigma_1})$, where $\Delta\tau=\tau_1-\tau_0$.

$$\begin{aligned}\frac{\partial \Pi_0(\tau_0,\tau_1)}{\partial \tau_1} &= \overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right)\overline{F}(\tau_0)=\overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right)\overline{\Phi}\left(\frac{\tau_0-z}{\sigma_0}\right).\\ \frac{\partial \Pi_1(\tau_0,\tau_1)}{\partial \tau_1} &= \int_{-\infty}^{\tau_0}\Phi\left(\frac{\theta_0-\tau_1}{\sigma_1}\right)\phi\left(\frac{\theta_0-z}{\sigma_0}\right)d\left(\frac{\theta_0-z}{\sigma_0}\right)-\frac{\tau_1}{\sigma_1}\int_{-\infty}^{\tau_0}\phi\left(\frac{\theta_0-\tau_1}{\sigma_1}\right)\phi\left(\frac{\theta_0-z}{\sigma_0}\right)d\left(\frac{\theta_0-z}{\sigma_0}\right).\end{aligned}$$

The last term can be simplified as follows:

$$\begin{aligned}&\int_{-\infty}^{\tau_0}\phi\left(\frac{\theta_0-\tau_1}{\sigma_1}\right)\phi\left(\frac{\theta_0-z}{\sigma_0}\right)d\left(\frac{\theta_0-z}{\sigma_0}\right)\\ =&\int_{-\infty}^{\tau_0}\frac{1}{\sqrt{2\pi}}e^{-\frac{(\theta_0-\tau_1)^2}{2\sigma_1^2}}\frac{1}{\sqrt{2\pi}}e^{-\frac{(\theta_0-z)^2}{2\sigma_0^2}}d\left(\frac{\theta_0-z}{\sigma_0}\right)=\int_{-\infty}^{\tau_0}\frac{1}{\sqrt{2\pi}}e^{-\frac{(\theta_0-\sigma_0^2\tau_1-\sigma_1^2 z)^2}{2\sigma_0^2\sigma_1^2}}\frac{1}{\sqrt{2\pi}}e^{-\frac{(\tau_1-z)^2}{2}}d\left(\frac{\theta_0-z}{\sigma_0}\right)\\ =&\sigma_1\phi(\tau_1-z)\int_{-\infty}^{\tau_0}\frac{1}{\sqrt{2\pi}}e^{-\frac{(\theta_0-\sigma_0^2\tau_1-\sigma_1^2 z)^2}{2\sigma_0^2\sigma_1^2}}d\left(\frac{\theta_0-\sigma_0^2\tau_1-\sigma_1^2 z}{\sigma_0\sigma_1}\right)=\sigma_1\phi(\tau_1-z)\Phi\left(\frac{\tau_0-\sigma_0^2\tau_1-\sigma_1^2 z}{\sigma_0\sigma_1}\right).\end{aligned} \tag{22}$$

Therefore,

$$\begin{aligned}\frac{\partial \Pi(\tau_0,\tau_1)}{\partial \tau_1} &= \overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right)\overline{\Phi}\left(\frac{\tau_0-z}{\sigma_0}\right)+\int_{-\infty}^{\tau_0}\Phi\left(\frac{\theta_0-\tau_1}{\sigma_1}\right)d\left(\Phi\left(\frac{\theta_0-z}{\sigma_0}\right)\right)-\tau_1\phi(\tau_1-z)\Phi\left(\frac{\tau_0-\sigma_0^2\tau_1-\sigma_1^2 z}{\sigma_0\sigma_1}\right)\\ &= \overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right)\overline{\Phi}\left(\frac{\tau_0-z}{\sigma_0}\right)+\int_{-\infty}^{\tau_0}\Phi\left(\frac{\theta_0-\tau_0-\Delta\tau}{\sigma_1}\right)d\left(\Phi\left(\frac{\theta_0-z}{\sigma_0}\right)\right)\\ &\quad-(\tau_0+\Delta\tau)\phi(\tau_0+\Delta\tau-z)\Phi\left(\frac{\sigma_1^2\tau_0-\sigma_0^2\Delta\tau-\sigma_1^2 z}{\sigma_0\sigma_1}\right),\end{aligned}$$

and at optimality, $\Delta\tau$ satisfies the following equation:

$$\begin{aligned}&\overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right)\overline{\Phi}\left(\frac{\tau_0-z}{\sigma_0}\right)+\int_{-\infty}^{\tau_0}\Phi\left(\frac{\theta_0-\tau_0-\Delta\tau}{\sigma_1}\right)d\left(\Phi\left(\frac{\theta_0-z}{\sigma_0}\right)\right)\\ =&(\tau_0+\Delta\tau)\phi(\tau_0+\Delta\tau-z)\Phi\left(\frac{\sigma_1^2\tau_0-\sigma_0^2\Delta\tau-\sigma_1^2 z}{\sigma_0\sigma_1}\right).\end{aligned}$$

Furthermore, $0 < \int_{-\infty}^{\tau_0} \Phi\left(\frac{\theta_0-\tau_0-\Delta\tau}{\sigma_1}\right) d\left(\Phi\left(\frac{\theta_0-z}{\sigma_0}\right)\right) < \overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right)\Phi\left(\frac{\tau_0-z}{\sigma_0}\right)$ because $0 < \Phi\left(\frac{\theta_0-\tau_0-\Delta\tau}{\sigma_1}\right) < \Phi\left(\frac{-\Delta\tau}{\sigma_1}\right)$ for $\theta_0 < \tau_0$. Therefore,

$$\overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right)\overline{\Phi}\left(\frac{\tau_0-z}{\sigma_0}\right) < (\tau_0+\Delta\tau)\phi(\tau_0+\Delta\tau-z)\Phi\left(\frac{\sigma_1^2\tau_0-\sigma_0^2\Delta\tau-\sigma_1^2 z}{\sigma_0\sigma_1}\right) < \overline{\Phi}\left(\frac{\Delta\tau}{\sigma_1}\right).$$

□

We will build the following lemma to prove Theorem 3. The proof does not rely on the specific function form of the normal distribution nor the normalization of the parameters, rather it relies on the hazard ratio properties.

LEMMA 2. *Given any $\tau_1 < +\infty$, the ratio $\frac{\int_{-\infty}^{\tau_1} G(\theta_1|\tau_0)d\theta_1}{G(\tau_1|\tau_0)}$ is decreasing in τ_0 and approaches 0 as τ_0 approaches infinity.*

Proof. We see that $\frac{\partial}{\partial\tau_0}\left(\frac{\int_{-\infty}^{\tau_1} G(\theta_1|\tau_0)d\theta_1}{G(\tau_1|\tau_0)}\right)$ has the same sign as $G(\tau_1|\tau_0)\int_{-\infty}^{\tau_1}\frac{\partial}{\partial\tau_0}G(\theta_1|\tau_0)d\theta_1 - \frac{\partial}{\partial\tau_0}G(\tau_1|\tau_0)\int_{-\infty}^{\tau_1} G(\theta_1|\tau_0)d\theta_1$. Because $\frac{\partial}{\partial\theta_0}G(\theta_1|\theta_0) = -\frac{\partial}{\partial\theta_1}G(\theta_1|\theta_0)$, the above expression equals $-G(\tau_1|\tau_0)\int_{-\infty}^{\tau_1} g(\theta_1|\tau_0)d\theta_1 + g(\tau_1|\tau_0)\int_{-\infty}^{\tau_1} G(\theta_1|\tau_0)d\theta_1$ (note that $g(\theta_1|\theta_0) = \frac{\partial}{\partial\theta_1}G(\theta_1|\theta_0)$). Because $\frac{G(\theta_1|\tau_0)}{g(\theta_1|\tau_0)} \le \frac{G(\tau_1|\tau_0)}{g(\tau_1|\tau_0)}$ for any $\theta_1 < \tau_1$, we have $-G(\tau_1|\tau_0)\int_{-\infty}^{\tau_1} g(\theta_1|\tau_0)d\theta_1 + g(\tau_1|\tau_0)\int_{-\infty}^{\tau_1} G(\theta_1|\tau_0)d\theta_1 \le 0$. Therefore, $\frac{\int_{-\infty}^{\tau_1} G(\theta_1|\tau_0)d\theta_1}{G(\tau_1|\tau_0)}$ is decreasing in τ_0.

$\frac{G(\tau_1|\tau_0)}{g(\tau_1|\tau_0)}$ approaches 0 as τ_0 approaches infinity, and hence $\frac{G(\tau_1|\tau_0)}{g(\tau_1|\tau_0)} \ge \frac{\int_{-\infty}^{\tau_1} G(\theta_1|\tau_0)d\theta_1}{\int_{-\infty}^{\tau_1} g(\theta_1|\tau_0)d\theta_1} = \frac{\int_{-\infty}^{\tau_1} G(\theta_1|\tau_0)d\theta_1}{G(\tau_1|\tau_0)} \ge 0$. The term in the middle approaches 0 as τ_0 approaches infinity by the sandwich theorem. □

Proof of Theorem 3. (1) We show the first part, $\tau_0 < +\infty$. If the seller excludes the pre-order option, he must charge $\tau_1 > c$ at time 1 to obtain a positive payoff. It suffices to show that for any $\tau_1 > c$, the partial derivative of $\Pi(\tau_0,\tau_1)$ with respect to τ_0 is negative as $\tau_0 \to +\infty$. Expressions (19) and (7) imply

$$\frac{\partial\Pi(\tau_0,\tau_1)}{\partial\tau_0} = \left(-\int_{-\infty}^{\tau_1}\frac{\partial G(\theta_1|\tau_0)}{\partial\tau_0}d\theta_1\right)\overline{F}(\tau_0) + \left(\int_{-\infty}^{\tau_1} G(\theta_1|\tau_0)d\theta_1\right)f(\tau_0) + (c-\tau_1)G(\tau_1|\tau_0)f(\tau_0).$$

Because $\left(-\int_{-\infty}^{\tau_1}\frac{\partial G(\theta_1|\tau_0)}{\partial\tau_0}d\theta_1\right)\overline{F}(\tau_0)$ and $\left(\int_{-\infty}^{\tau_1} G(\theta_1|\tau_0)d\theta_1\right)f(\tau_0)$ are positive while $(c-\tau_1)G(\tau_1|\tau_0)f(\tau_0)$ is negative, it suffices to show that the ratios between the positive terms and the negative term approach 0 as τ_0 approaches infinity.

Notice that $\left(-\int_{-\infty}^{\tau_1}\frac{\partial G(\theta_1|\tau_0)}{\partial\tau_0}d\theta_1\right)\overline{F}(\tau_0) = \left(\int_{-\infty}^{\tau_1} g(\theta_1|\tau_0)d\theta_1\right)\overline{F}(\tau_0) = G(\tau_1|\tau_0)\overline{F}(\tau_0)$. The ratio between $G(\tau_1|\tau_0)\overline{F}(\tau_0)$ and $(c-\tau_1)G(\tau_1|\tau_0)f(\tau_0)$ is $\frac{1}{(c-\tau_1)}\frac{\overline{F}(\tau_0)}{f(\tau_0)}$, which approaches 0 as τ_0 approaches infinity. The ratio between $\left(\int_{-\infty}^{\tau_1} G(\theta_1|\tau_0)d\theta_1\right)f(\tau_0)$ and $(c-\tau_1)G(\tau_1|\tau_0)f(\tau_0)$ is $\frac{1}{(c-\tau_1)}\frac{\int_{-\infty}^{\tau_1} G(\theta_1|\tau_0)d\theta_1}{G(\tau_1|\tau_0)}$, which approaches 0 as τ_0 approaches infinity, by Lemma 2 (see above). Therefore, given any τ_1, it is optimal for the seller to choose $\tau_0 < +\infty$ and exercise pre-order.

(2) We show the second part that $\tau_1 < +\infty$. It suffices to show that for any τ_0, the partial derivative with respect to τ_1 is negative as $\tau_1 \to +\infty$. The first order condition with respect to τ_1 implies:

$$\frac{\partial\Pi(\tau_0,\tau_1)}{\partial\tau_1} = \frac{\partial p_0(\tau_0,\tau_1)}{\partial\tau_1}\overline{F}(\tau_0) + \overline{G}(\tau_1|\theta_0<\tau_0)F(\tau_0) + (c-\tau_1)g(\tau_1|\theta_0<\tau_0)F(\tau_0).$$

Because $\frac{\partial p_0(\tau_0,\tau_1)}{\partial\tau_1}\overline{F}(\tau_0)$ and $\overline{G}(\tau_1|\theta_0<\tau_0)F(\tau_0)$ are positive while $(c-\tau_1)g(\tau_1|\theta_0<\tau_0)F(\tau_0)$ is negative for large τ_1, it suffices to show that the ratios between the positive terms and the negative term approaches 0 as τ_1 approaches infinity.

Notice that $\overline{G}(\tau_1|\theta_0<\tau_0) = \int_{-\infty}^{\tau_0}\overline{G}(\tau_1|\theta_0)d\left(\frac{F(\theta_0)}{F(\tau_0)}\right)$ and $g(\tau_1|\theta_0<\tau_0) = \int_{-\infty}^{\tau_0} g(\tau_1|\theta_0)d\left(\frac{F(\theta_0)}{F(\tau_0)}\right)$. $\overline{G}(\tau_1|\theta_0)/g(\tau_1|\theta_0)$ is increasing in θ_0. Therefore, the ratio between $\overline{G}(\tau_1|\theta_0<\tau_0)$ and $g(\tau_1|\theta_0<\tau_0)$ is bounded by $\overline{G}(\tau_1|\tau_0)/g(\tau_1|\tau_0)$, which approaches 0 as τ_1 approaches infinity. Thus, the ratio between the second term and the third term approaches 0 as τ_1 approaches infinity.

Because $\frac{\partial p_0(\tau_0,\tau_1)}{\partial\tau_1} = \overline{G}(\tau_1|\tau_0)$ by (20), we can rewrite the first term as $\overline{G}(\tau_1|\tau_0)\overline{F}(\tau_0)$.

Notice that $g(\tau_1|\tau_0)/\overline{G}(\tau_1|\tau_0)$ is monotone decreasing in τ_0. If we pick a small $\Delta > 0$, we also have $\frac{g(\theta_1|\tau_0)}{\overline{G}(\theta_1|\tau_0)} < \frac{g(\theta_1|\tau_0)}{\overline{G}(\tau_1|\tau_0)}$ for $\theta_1 \in (\tau_1-\Delta,\tau_1]$. Therefore, $\frac{g(\theta_1|\tau_0)}{\overline{G}(\tau_1|\tau_0)} \ge \frac{g(\tau_1-\Delta|\tau_0)}{\overline{G}(\tau_1-\Delta|\tau_0)}$ for $\theta_1 \in [\tau_1-\Delta,\tau_1]$. Integrating $\frac{g(\theta_1|\tau_0)}{\overline{G}(\tau_1|\tau_0)}$ with respect to θ_1 over $[\tau_1-\Delta,\tau_1]$ yields $\frac{\overline{G}(\tau_1-\Delta|\tau_0)-\overline{G}(\tau_1|\tau_0)}{\overline{G}(\tau_1|\tau_0)} \geq \Delta \frac{g(\tau_1-\Delta|\tau_0)}{\overline{G}(\tau_1-\Delta|\tau_0)}$. The right-hand side goes to infinity as τ_1 goes to infinity. Therefore, the ratio between $\overline{G}(\tau_1-\Delta|\tau_0)$ and $\overline{G}(\tau_1|\tau_0)$ goes to infinity as τ_1 goes to infinity. It is equivalent to say that the ratio between $\overline{G}(\tau_1+\Delta|\tau_0)$ and $\overline{G}(\tau_1|\tau_0)$ goes to 0 as τ_1 goes to infinity.

If we pick a small $\Delta>0$ and let $k = min\left\{\frac{f(\theta_0)}{F(\tau_0)}|\tau_0-\Delta\leq\theta_0\leq\tau_0\right\}$. $k>0$ as $f(\theta_0)$ is continuous, and $f(\theta_0)>0$ for all θ. $g(\tau_1|\theta_0<\tau_0)=\int_{-\infty}^{\tau_0} g(\tau_1|\theta_0)d\left(\frac{F(\theta_0)}{F(\tau_0)}\right) > k\int_{\tau_0-\Delta}^{\tau_0} g(\tau_1|\theta_0)d\theta_0 = k(\overline{G}(\tau_1|\tau_0)-\overline{G}(\tau_1+\Delta|\tau_0))$. The ratio between $\overline{G}(\tau_1|\tau_0)$ and $g(\tau_1|\theta_0<\tau_0)$ is no greater than $\frac{1}{k}$ as τ_1 approaches infinity.

As a result, the ratio between $\overline{G}(\tau_1|\tau_0)\overline{F}(\tau_0)$ and $(c-\tau_1)g(\tau_1|\theta_0<\tau_0)F(\tau_0)$ approaches 0 as τ_1 approaches infinity because $\frac{1}{c-\tau_1}$ approaches 0 and the ratio between $\overline{G}(\tau_1|\tau_0)\overline{F}(\tau_0)$ and $g(\tau_1|\theta_0<\tau_0)F(\tau_0)$ is bounded.

Therefore, it is optimal to adopt a $\tau_1<+\infty$ and sell at both dates. □

Proof of Lemma 1. Because $\tau_1=p_1$, we can rewrite p_0 as:

$$p_0=\tau_0-\sigma_1\phi\left(\frac{p_1-\tau_0}{\sigma_1}\right)+(p_1-\tau_0)\overline{\Phi}\left(\frac{p_1-\tau_0}{\sigma_1}\right), \tag{23}$$

where $\sigma_1=\sqrt{1-\lambda}$. Thus, τ_0 can be expressed as a function of λ, p_0, and p_1.

By equation (23) and the fact that $\phi'(x)=-x\phi(x)$, we have

$$\begin{aligned}0=&\frac{\partial\tau_0}{\partial\lambda}-\frac{d\sigma_1}{d\lambda}\phi\left(\frac{p_1-\tau_0}{\sigma_1}\right)+(p_1-\tau_0)\phi\left(\frac{p_1-\tau_0}{\sigma_1}\right)\frac{\partial}{\partial\lambda}\left(\frac{p_1-\tau_0}{\sigma_1}\right)\\&-\frac{\partial\tau_0}{\partial\lambda}\overline{\Phi}\left(\frac{p_1-\tau_0}{\sigma_1}\right)-(p_1-\tau_0)\phi\left(\frac{p_1-\tau_0}{\sigma_1}\right)\frac{\partial}{\partial\lambda}\left(\frac{p_1-\tau_0}{\sigma_1}\right)\\=&\frac{\partial\tau_0}{\partial\lambda}\Phi\left(\frac{p_1-\tau_0}{\sigma_1}\right)-\frac{d\sigma_1}{d\lambda}\phi\left(\frac{p_1-\tau_0}{\sigma_1}\right).\end{aligned}$$

Therefore,

$$\frac{\partial\tau_0}{\partial\lambda}=\frac{d\sigma_1}{d\lambda}\frac{\phi\left(\frac{p_1-\tau_0}{\sigma_1}\right)}{\Phi\left(\frac{p_1-\tau_0}{\sigma_1}\right)}=-\frac{1}{2\sigma_1}\frac{\phi\left(\frac{p_1-\tau_0}{\sigma_1}\right)}{\Phi\left(\frac{p_1-\tau_0}{\sigma_1}\right)}<0. \tag{24}$$

□

Proof of Proposition 4. The partial derivative $\frac{\partial}{\partial\lambda}\overline{\Phi}(u)=-\phi(u)\frac{\partial u}{\partial\lambda}=-\frac{1}{\sigma_0^2}\phi(u)\left(\sigma_0\frac{\partial\tau_0}{\partial\lambda}-(\tau_0-z)\frac{d\sigma_0}{d\lambda}\right)$. Because $z<0$ and $\tau_0>0$, the term $(\tau_0-z)\frac{d\sigma_0}{d\lambda}>0$, and hence $\sigma_0\frac{\partial\tau_0}{\partial\lambda}-(\tau_0-z)\frac{d\sigma_0}{d\lambda}<0$ by Lemma 1. Therefore, $\frac{\partial}{\partial\lambda}\overline{\Phi}(u)>0$, or $\frac{\partial u}{\partial\lambda}<0$. □

The proof of Proposition 5 will use the following bounds of the standard normal distribution function.

Lemma 3. *For any $x>0$, $\frac{x}{1+x^2}\phi(x)<1-\Phi(x)<\frac{1}{x}\phi(x)$. Therefore, $\epsilon_a(x)\equiv\frac{\phi(x)}{\overline{\Phi}(x)}-x>0$, $\epsilon_b(x)\equiv1-x\frac{\overline{\Phi}(x)}{\phi(x)}>0$, and both approach zero as x approaches infinity.*

Proof. Assume $x>0$. The upper bound is derived from

$$1-\Phi(x)=\int_x^\infty\phi(u)du<\int_x^\infty\frac{u}{x}\phi(u)du=\frac{1}{x}\int_x^\infty\frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}}d\left(\frac{u^2}{2}\right)=-\left.\frac{e^{-\frac{u^2}{2}}}{x\sqrt{2\pi}}\right|_x^\infty=\frac{\phi(x)}{x}.$$

The lower bound follows from the fact $\phi'(u)=-u\phi(u)$ and

$$\left(1+\frac{1}{x^2}\right)(1-\Phi(x))=\int_x^\infty\left(1+\frac{1}{x^2}\right)\phi(u)du>\int_x^\infty\left(1+\frac{1}{u^2}\right)\phi(u)du=-\left.\frac{\phi(u)}{u}\right|_x^\infty=\frac{\phi(x)}{x}.$$

Thus, $\epsilon_a(x)\in(0,\frac{1}{x})$, $\epsilon_b(x)\in(0,\frac{1}{1+x^2})$, and both approach 0 as x approaches infinity. □

Proof of Proposition 5. We first show that when $z>3.22$, $u<0$ under the optimal pricing policy.

If this is not true, there must exist $z>3.22$ and $u\geq0$ under an optimal pricing policy. By expression (11) and the monotonicity of $\left(\frac{\bar{\Phi}(u)}{\phi(u)}\right)$, we have

$$\frac{\sigma_1\phi(\frac{\Delta\tau}{\sigma_1})}{\Phi(\frac{\Delta\tau}{\sigma_1})}=\tau_0-\frac{\sigma_0\overline{\Phi}(\frac{\tau_0-z}{\sigma_0})}{\phi(\frac{\tau_0-z}{\sigma_0})}=z+\sigma_0u-\frac{\sigma_0\overline{\Phi}(u)}{\phi(u)}\geq z-\frac{\sigma_0\overline{\Phi}(0)}{\phi(0)}.$$

Because the right-hand side is at least 1.966 (as $z > 3.22$ and $\sigma_0 \le 1$) and the left-hand side is decreasing in $\frac{\Delta\tau}{\sigma_1}$ with $\frac{\phi(-1.532)}{\Phi(-1.532)} \approx 1.966$, we must have $\frac{\Delta\tau}{\sigma_1} < -1.532$. On the other hand, because $\Phi(x) = 1 - \Phi(-x) > \frac{-x}{1+x^2}\phi(x)$ for any $x < 0$ (by Lemma 3, see above), we have $\tau_0 - \frac{\sigma_0 \overline{\Phi}(\frac{\tau_0 - z}{\sigma_0})}{\phi(\frac{\tau_0 - z}{\sigma_0})} = \frac{\sigma_1 \phi(\frac{\Delta\tau}{\sigma_1})}{\Phi(\frac{\Delta\tau}{\sigma_1})} < \tau_0 - \tau_1 + \frac{\sigma_1^2}{\tau_0 - \tau_1}$ and hence

$$\tau_1 < \frac{\sigma_1^2}{\tau_0 - \tau_1} + \frac{\sigma_0 \overline{\Phi}(u)}{\phi(u)} < \frac{1}{1.532} + \frac{\overline{\Phi}(0)}{\phi(0)} < 1.91.$$

Under an optimal pricing policy, the seller's profit cannot exceed τ_1 because $p_0 < p_1 = \tau_1$. Thus, 1.91 is an upper bound for the seller's profit. However, by exclusive retailing at the price $p_1 = 2.5$, the seller can obtain a profit more than 1.91 (note that $P(\theta_1 > 2.5) > 0.764$ even when $z = 3.22$). Therefore, the above pricing policy is not optimal, which is a contradiction.

Now, we show that $u \to -\infty$ (or, $\overline{\Phi}(u) \to 1$) uniformly as $z \to \infty$ under the optimal pricing policy.

If this is not true, there must exist $\bar{u} < 0$ such that we can find a $u > \bar{u}$ and an arbitrarily large z under an optimal pricing policy. From expression (11), we have

$$\frac{\phi(\frac{\Delta\tau}{\sigma_1})}{\Phi(\frac{\Delta\tau}{\sigma_1})} \ge \frac{\sigma_1 \phi(\frac{\Delta\tau}{\sigma_1})}{\Phi(\frac{\Delta\tau}{\sigma_1})} = \tau_0 - \frac{\sigma_0 \overline{\Phi}(\frac{\tau_0 - z}{\sigma_0})}{\phi(\frac{\tau_0 - z}{\sigma_0})} = z + \sigma_0 \left(u - \frac{\overline{\Phi}(u)}{\phi(u)} \right) > z + \sigma_0 \left(\bar{u} - \frac{\overline{\Phi}(\bar{u})}{\phi(\bar{u})} \right).$$

The last inequality holds because $\left(u - \frac{\overline{\Phi}(u)}{\phi(u)} \right)' = 2 - \frac{u \overline{\Phi}(u)}{\phi(u)} > 0$ (as shown above, $u < 0$ when z is large enough). Thus, due to the monotonicity of $\left(\frac{\phi(x)}{\Phi(x)} \right)$, as z approaches infinity, $\frac{\Delta\tau}{\sigma_1}$ must approach negative infinity uniformly. By expression (11) and the definition of $\epsilon_a(\cdot)$ in Lemma 3, we obtain

$$\tau_1 = \sigma_1 \epsilon_a \left(-\frac{\Delta\tau}{\sigma_1} \right) + \frac{\sigma_0 \overline{\Phi}(u)}{\phi(u)} < \epsilon_a \left(-\frac{\Delta\tau}{\sigma_1} \right) + \frac{\overline{\Phi}(\bar{u})}{\phi(\bar{u})}.$$

By Lemma 3, $\epsilon_a(-\frac{\Delta\tau}{\sigma_1})$ is small (and positive) when $-\frac{\Delta\tau}{\sigma_1}$ is large, and hence for a given $\bar{u}$, we have $z > 2\tau_1$ when z is large enough. This is a contradiction because the seller's profit is bounded by τ_1 under an optimal pricing policy while he can easily secure a profit $z/2$ by exclusive retailing at price z. Therefore, we must have $u \to -\infty$ (or $\overline{\Phi}(u) \to 1$) uniformly as $z \to \infty$ under an optimal pricing policy. □

Proof of Theorem 4. In the following analysis, the prices (p_0, p_1) will always be fixed at the optimal levels $(p_0(\lambda), p_1(\lambda))$, and hence λ will be dropped from the expressions when it is clear from the context. The envelope theorem implies that $\frac{d}{d\lambda}\Pi(\lambda, p_0, p_1) = \frac{\partial}{\partial\lambda}\Pi(\lambda, p_0, p_1)$. Consequently, it suffices to evaluate $\frac{\partial}{\partial\lambda}\Pi(\lambda, p_0, p_1)$.

Recall that $\Pi(\lambda, p_0, p_1) = \Pi_0(\lambda, p_0, p_1) + \Pi_1(\lambda, p_0, p_1)$, where $\Pi_0(\lambda, p_0, p_1) = p_0 \overline{\Phi}\left(\frac{\tau_0 - z}{\sigma_0}\right)$ and $\Pi_1(\lambda, p_0, p_1) = p_1 \int_{-\infty}^{\tau_0} \Phi\left(\frac{\theta_0 - p_1}{\sigma_1}\right) d\left(\Phi\left(\frac{\theta_0 - z}{\sigma_0}\right)\right)$. Thus, we have $\frac{\partial \Pi_0}{\partial\lambda} = -p_0 \phi(u) \frac{\partial u}{\partial\lambda}$. The term $-p_0\phi(u)$ represents the loss of pre-order profit from the marginal consumers.

Define $t = \frac{\theta_0 - z}{\sigma_0}$ (i.e., $\theta_0 = z + t\sigma_0$) and $v(\lambda, p_1, t) = \frac{z + \sigma_0 t - p_1}{\sigma_1}$. By a change of variables, Π_1 can be rewritten as

$$\Pi_1 = p_1 \int_{-\infty}^{\tau_0} \Phi\left(\frac{\theta_0 - p_1}{\sigma_1}\right) d\left(\Phi\left(\frac{\theta_0 - z}{\sigma_0}\right)\right) = p_1 \int_{-\infty}^{u} \Phi\left(v(\lambda, p_1, t)\right) \phi(t)\, dt,$$

and thus,

$$\frac{\partial \Pi_1}{\partial\lambda} = p_1 \Phi\left(v(\lambda, p_1, u)\right) \phi(u) \frac{\partial u}{\partial\lambda} + p_1 \int_{-\infty}^{u} \phi\left(v(\lambda, p_1, t)\right) \frac{\partial v(\lambda, p_1, t)}{\partial\lambda} \phi(t)\, dt.$$

The first term on the right-hand side captures the gain of retail profit from the marginal consumers who switch from pre-order to retail purchase: given fixed p_0 and p_1 (at the optimal levels), a small increment $\partial\lambda$ translates into a loss of $\phi(u)\frac{\partial u}{\partial\lambda}\partial\lambda$ portion of consumers at the pre-order date; $\Phi\left(v(\lambda, p_1, u)\right)$ portion of them make purchase at time 1, at the price p_1. The second term captures the change of retail profit from the lower $\Phi(u)$ portion of consumers with relatively low initial valuations.

We first examine the marginal consumers who change their purchase date. By substitution, we have $p_1 \Phi\left(v(\lambda, p_1, u)\right) \phi(u) = p_1 \Phi\left(\frac{-\Delta\tau}{\sigma_1}\right) \phi(u)$. Thus, the overall impact of the marginal consumers on the total profit (pre-order plus retail) is given by $-p_0\phi(u) + p_1 \Phi(\frac{-\Delta\tau}{\sigma_1})\phi(u) = (\sigma_1 \phi(\frac{\Delta\tau}{\sigma_1}) - \tau_0 \Phi(\frac{\Delta\tau}{\sigma_1}))\phi(u) = -\sigma_0 \overline{\Phi}(u)$, following equations (23) and (11).

Next, we examine the demand change at time 1 for the lower $\Phi(u)$ portion of consumers who do not pre-order. Because $\frac{d\sigma_0}{d\lambda} = \frac{1}{2}\frac{1}{\sqrt{\lambda}} = \frac{1}{2\sigma_0}$ and $\frac{d\sigma_1}{d\lambda} = \frac{1}{2}\frac{-1}{\sqrt{1-\lambda}} = -\frac{1}{2\sigma_1}$, we have $\frac{\partial v(\lambda,p_1,t)}{\partial\lambda} = \left((\frac{t}{2\sigma_0})\sigma_1 + (z+\sigma_0 t - p_1)\frac{1}{2\sigma_1}\right)\Big/\sigma_1^2 = \frac{t+\sigma_0(z-p_1)}{2\sigma_0\sigma_1^3}$. Furthermore, because $v(\lambda,p_1,t)^2 + t^2 = \frac{(z-p_1)^2}{\sigma_1^2} + \frac{2\sigma_0(z-p_1)t}{\sigma_1^2} + \frac{t^2}{\sigma_1^2} = (z-p_1)^2 + \frac{(t+\sigma_0(z-p_1))^2}{\sigma_1^2}$, we obtain

$$\begin{aligned}
&\int_{-\infty}^{u}\phi\left(v(\lambda,p_1,t)\right)\frac{t+\sigma_0(z-p_1)}{2\sigma_0\sigma_1^3}\phi(t)\,dt\\
&=\frac{1}{4\pi\sigma_0\sigma_1^3}\int_{-\infty}^{u}(t+\sigma_0(z-p_1))e^{-v(\lambda,p_1,t)^2/2}e^{-t^2/2}dt\\
&=\frac{1}{4\pi\sigma_0\sigma_1^3}\int_{-\infty}^{u}(t+\sigma_0(z-p_1))e^{-(z-p_1)^2/2}e^{-(t+\sigma_0(z-p_1))^2/(2\sigma_1^2)}dt\\
&=\frac{1}{4\pi\sigma_0\sigma_1}e^{-(z-p_1)^2/2}\int_{-\infty}^{u}\left(\frac{t+\sigma_0(z-p_1)}{\sigma_1}\right)e^{-(t+\sigma_0(z-p_1))^2/(2\sigma_1^2)}d\left(\frac{t+\sigma_0(z-p_1)}{\sigma_1}\right)\\
&=\frac{1}{4\pi\sigma_0\sigma_1}e^{-(z-p_1)^2/2}\int_{-\infty}^{u}d\left(-e^{-(t+\sigma_0(z-p_1))^2/(2\sigma_1^2)}\right)=-\frac{1}{4\pi\sigma_0\sigma_1}e^{-(z-p_1)^2/2}e^{-(u+\sigma_0(z-p_1))^2/(2\sigma_1^2)}\\
&=-\frac{1}{4\pi\sigma_0\sigma_1}e^{-u^2/2}e^{-(v(\lambda,p_1,u))^2/2}=-\frac{1}{2\sigma_0\sigma_1}\phi(u)\phi\left(\frac{\Delta\tau}{\sigma_1}\right)<0.
\end{aligned}$$

This result says that for those consumers who do not pre-order, the proportion of them who buy at the release date is decreasing in λ. Intuitively, the larger the λ, the smaller the difference between θ_0 and θ_1, and the less likely consumers will buy at a later date.

Therefore, with all combined, we arrive at expression (15):

$$\frac{\partial\Pi}{\partial\lambda} = -\sigma_0\overline{\Phi}(u)\frac{\partial u}{\partial\lambda} - \frac{p_1}{2\sigma_0\sigma_1}\phi(u)\phi\left(\frac{\Delta\tau}{\sigma_1}\right). \tag{25}$$

Further, by the definition of u, expression (14), and the facts that $\frac{d\sigma_0}{d\lambda} = \frac{1}{2\sigma_0}$ and $\frac{d\sigma_1}{d\lambda} = -\frac{1}{2\sigma_1}$, we have $\frac{\partial u}{\partial\lambda} = \left(\sigma_0\frac{\partial\tau_0}{\partial\lambda} - \frac{1}{2\sigma_0}(\tau_0 - z)\right)\Big/\sigma_0^2 = \left(-\frac{\sigma_0}{2\sigma_1}\frac{\phi\left(\frac{\Delta\tau}{\sigma_1}\right)}{\Phi\left(\frac{\Delta\tau}{\sigma_1}\right)} - \frac{1}{2\sigma_0}(\tau_0 - z)\right)\Big/\sigma_0^2 = -\left(u + \frac{\sigma_0}{\sigma_1}\frac{\phi\left(\frac{\Delta\tau}{\sigma_1}\right)}{\Phi\left(\frac{\Delta\tau}{\sigma_1}\right)}\right)\Big/2\sigma_0^2$. □

The proof of Theorem 6 needs the following lemma:

Lemma 4. *The seller prefers risk-averse consumers than risk-neutral ones when practicing pre-order.*

Proof. Let τ_0 and τ_1 be the optimal valuation thresholds when consumers are risk-neutral. Let p_0^n and $p_1^n = \tau_1$ denote the corresponding pre-order and retail prices (the superscript "n" means risk-"neutral"). Because of Theorem 5, risk-averse consumers' optimal purchasing strategies are also threshold policies. The same thresholds (τ_0, τ_1) can be implemented when consumers are risk-averse by charging $p_1^a = \tau_1$ and a proper $p_0^a \in (-\infty, p_1^a)$ such that the boundary initial type τ_0 is indifferent between pre-ordering and purchasing later (superscript "a" means risk-"averse").

Consider the seller's profit in the risk-neutral world and risk-averse world. At time 1, the seller would obtain the same profit because the same percentage of population, $\overline{G}(\tau_1|\theta_0 < \tau_0)F(\tau_0)$, purchases the good at the same price $p_1^n = p_1^a = \tau_1$. At time 0, the same percentage of population, $\overline{F}(\tau_0)$, pre-orders. To show that the seller makes a higher profit in the risk-averse world, it suffices to show that $p_0^a > p_0^n$.

Notice that by definition, $E_g[u(-p_0^a) + g|\theta_0 = \tau_0] = E_g[\max\{u(0), u(-\tau_1) + g\}|\theta_0 = \tau_0]$ and recall that $u(-\theta_1) + g = u(0)$, we have $p_0^a = -u^{-1}(u(-\tau_1)\overline{G}(\tau_1|\tau_0) + \int_{-\infty}^{\tau_1} u(-\theta_1)dG(\theta_1|\tau_0)) > \tau_1\overline{G}(\tau_1|\tau_0) + \int_{-\infty}^{\tau_1}\theta_1 dG(\theta_1|\tau_0) = p_0^n$. The inequality follows from the concavity of the utility function and the fact that G has unbounded support, and the last equality follows from the definition of p_0^n. Thus, the seller can charge a higher pre-order price and will be better off in the risk-averse world. □

Proof of Theorem 6. By Theorem 3, it is optimal for the seller to adopt pre-order when consumers are risk-neutral. That is, the seller can secure a higher profit through pre-order and retail than from pure retail. By Lemma 4 (see above), the seller can obtain a higher profit when consumers are risk-averse. Therefore, abandoning pre-order and obtaining the pure-retail profit is not optimal for the seller facing risk-averse consumers. □

Appendix B: Continuity of the Profit Function $\Pi(z,\lambda)$ at $\lambda=0$

We prove that the seller's profit Π is continuous at $\lambda=0$. When $\lambda=0$, $\Pi(\lambda)=\max\{\Pi^P(z),\Pi^R(z)\}$, where $\Pi^P(z)=z$ is the profit when only pre-order is allowed and $\Pi^R(z)=p^R\left(\Phi\left(z-p^R\right)\right)$ is the profit when only retail is allowed.

Denote $\hat{\Pi}(\lambda)=\max_p p\left(\Phi\left(\frac{z-p}{\sqrt{\lambda}}\right)\right)$ for $\lambda\in(0,1]$ and $\hat{\Pi}(0)=\max\{0,z\}$. Then, $\hat{\Pi}$ is the maximum profit when only pre-order is allowed. Let $p^*(\lambda)$ denote the optimal p for $\hat{\Pi}(\lambda)$. We first show that $\hat{\Pi}(\lambda)$ is a continuous function at $\lambda=0$.

Lemma 5. *If $p_i\uparrow+\infty$ and $\lambda_i\downarrow 0$ ($\lambda_i\in(0,1)$), $\lim_{i\to+\infty}p_i\Phi\left(\frac{z-p_i}{\sqrt{\lambda_i}}\right)=0$.*

Proof. $\lim_{i\to+\infty}p_i\Phi(z-p_i)=-\lim_{i\to+\infty}(z-p_i)\Phi(z-p_i)=\lim_{x\to+\infty}(x\Phi(-x))=\lim_{x\to+\infty}\frac{\Phi(-x)}{1/x}=\lim_{x\to+\infty}\frac{\phi(-x)}{1/x^2}=\lim_{x\to+\infty}\frac{1}{\sqrt{2\pi}}x^2e^{-x^2/2}=0$.

When i is sufficiently large, $p_i>z$, $0<\Phi\left(\frac{z-p_i}{\sqrt{\lambda_i}}\right)<\Phi(z-p_i)$ because $\lambda_i\in(0,1)$. Therefore, $0\le\lim_{i\to+\infty}p_i\Phi\left(\frac{z-p_i}{\sqrt{\lambda_i}}\right)\le\lim_{i\to+\infty}p_i\Phi(z-p_i)=0$. That is, $\lim_{i\to+\infty}p_i\Phi\left(\frac{z-p_i}{\sqrt{\lambda_i}}\right)=0$. □

Now, we show that:

Proposition 6. *$\Pi(\lambda)$ is continuous at $\lambda=0$.*

Proof. We first show that for any $\epsilon>0$, there exists δ such that $\Pi(\lambda)>\Pi(0)-\epsilon$ for $\lambda\in(0,\delta)$.

If $\Pi(0)=\Pi^R(z)=p^R\left(\Phi\left(z-p^R\right)\right)$, we can pick any $\delta\in(0,1)$ because the seller can forego the pre-order opportunity and $\Pi(\lambda)\ge\Pi^R(z)$. If $\Pi(0)=\Pi^P(z)=z$, we can pick a small δ so that $\Phi\left(\frac{\epsilon/2}{\sqrt{\delta}}\right)>1-\frac{\epsilon}{2z}$, then $\hat{\Pi}(\lambda)>\left(z-\frac{\epsilon}{2}\right)\left(\Phi\left(\frac{\epsilon/2}{\sqrt{\lambda}}\right)\right)>\left(z-\frac{\epsilon}{2}\right)\left(1-\frac{\epsilon}{2z}\right)>z-\epsilon$ for $\lambda\in(0,\delta)$; therefore, $\Pi(\lambda)>\hat{\Pi}(\lambda)>\Pi(0)-\epsilon$ for $\lambda\in(0,\delta)$.

To prove the proposition, it suffices to show that for any sequence of $\lambda_i\downarrow 0$ such that $\Pi(\lambda_i)$ converges, it converges to some point no more than $\Pi(0)$. Recall that the seller's problem is:

$$\max_{\tau_0,\tau_1} p_0\Phi\left(\frac{z-\tau_0}{\sigma_0}\right)+\tau_1\int_{-\infty}^{\tau_0}\overline{\Phi}\left(\frac{\tau_1-\theta_0}{\sigma_1}\right)d\left(\Phi\left(\frac{\theta_0-z}{\sigma_0}\right)\right), \tag{26}$$

$$\text{where } p_0(\tau_0,\tau_1)=\tau_0-\sigma_1\phi\left(\frac{\tau_1-\tau_0}{\sigma_1}\right)+(\tau_1-\tau_0)\overline{\Phi}\left(\frac{\tau_1-\tau_0}{\sigma_1}\right), \tag{27}$$

where $\sigma_0=\sqrt{\lambda}$ and $\sigma_1=\sqrt{1-\lambda}$.

It is straightforward to show that $\tau_1\int_{-\infty}^{\tau_0}\overline{\Phi}\left(\frac{\tau_1-\theta_0}{\sigma_1}\right)d\left(\Phi\left(\frac{\theta_0-z}{\sigma_0}\right)\right)\le\tau_1\Phi(z-\tau_1)\overline{\Phi}\left(\frac{z-\tau_0}{\sigma_0}\right)\le\hat{\Pi}(1)\overline{\Phi}\left(\frac{z-\tau_0}{\sigma_0}\right)=\hat{\Pi}(1)\overline{\Phi}\left(\frac{z-\tau_0}{\sqrt{\lambda}}\right)$ because pre-order captures the high-valuation consumers who would have a higher valuation at time 1 and only the $\overline{\Phi}\left(\frac{z-\tau_0}{\sqrt{\lambda}}\right)$ portion of the consumers are left at time 1.

For any such sequence λ_i where $\Pi(\lambda_i)$ converges, we can find a subsequence λ_{i_j} such that $\Phi\left(\frac{z-\tau_0(\lambda_{i_j})}{\sqrt{\lambda_{i_j}}}\right)$ converges to some point in $[0,1]$.

If $\Phi\left(\frac{z-\tau_0(\lambda_{i_j})}{\sqrt{\lambda_{i_j}}}\right)$ converges to $t\in(0,1)$, $p_0(\lambda_{i_j})<\tau_0(\lambda_{i_j})\to z$, $\lim_{i\to+\infty}\Pi(\lambda_i)=\lim_{j\to+\infty}\Pi(\lambda_{i_j})\le zt+\hat{\Pi}(1)(1-t)=\Pi^P(z)t+\Pi^R(z)(1-t)\le\Pi(0)$.

If $\Phi\left(\frac{z-\tau_0(\lambda_{i_j})}{\sqrt{\lambda_{i_j}}}\right)$ converges to $t=1$, $p_0(\lambda_{i_j})<\tau_0(\lambda_{i_j})<z$ for sufficiently large j, $\lim_{i\to+\infty}\Pi(\lambda_i)=\lim_{j\to+\infty}\Pi(\lambda_{i_j})\le z=\Pi^P(z)\le\Pi(0)$.

If $\Phi\left(\frac{z-\tau_0(\lambda_{i_j})}{\sqrt{\lambda_{i_j}}}\right)$ converges to $t=0$, $\lim_{j\to}p_0(\lambda_{i_j})\Phi\left(\frac{z-\tau_0(\lambda_{i_j})}{\sqrt{\lambda_{i_j}}}\right)=0$ because of Lemma 5, if $p_0(\lambda_{i_j})$ is unbounded, or $\Phi\left(\frac{z-\tau_0(\lambda_{i_j})}{\sqrt{\lambda_{i_j}}}\right)\to 0$, if $p_0(\lambda_{i_j})$ is bounded. Therefore, $\lim_{i\to+\infty}\Pi(\lambda_i)=\lim_{j\to+\infty}\Pi(\lambda_{i_j})\le\hat{\Pi}(1)=\Pi^R(z)\le\Pi(0)$.

In any case, $\Pi(\lambda)<\Pi(0)+\epsilon$ when λ is sufficiently small. □

Acknowledgments

The authors are grateful to the anonymous AE and three reviewers, whose comments and suggestions have helped improve the paper significantly. The authors also thank David Sappington, conference participants at the OCSAMSE conference, seminar participants at UC Riverside and UC Davis for their valuable feedback.

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