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How Should Property Rights Be Allocated?*

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ABSTRACT: We study the allocation of property rights to individuals who may differ in their valuations of those rights and in their abilities to pay. Agents' limited abilities to pay prevent them from achieving allocative efficiency through voluntary negotiation, so initial allocation of property rights matters. We show that non-market assignment of the rights, as simple as random rationing at below-market price, yields more efficient allocation than a competitive market if the rights are fully transferable. Further, need-based assignment schemes that favor the poor and merit-based schemes that favor those with high valuations are socially desirable. Conversely, if speculation is prevalent, it may be beneficial to render rights inalienable (i.e., prohibit transferability). Our result provides a potential rationale for government intervention of competitive markets and some implications for limitations of market oriented assignment rules, in the presence of financing costs.

KEYWORDS: Efficiency, Non-market assignment of resources, Transferability, Merit-Based Assignment Rules, Need-Based Assignment Rules.

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1 Introduction

Governments play an integral role in the allocation of resources. One way they play this role is by awarding individuals property rights — the right to extract minerals; the right to harvest forest or sea resources; the right to use airport landing slots and radio spectrum; the right (for foreign nationals) to immigrate legally; the right to be exempt from military or jury duty; and the right to access public health care, housing and education; to name just a few. In particular, government control of resource allocation has been a crucial element of economic development policies adopted by many countries. For instance, Korean economic development during its industrialization process was led by the government targeting industries and firms, and promoting them using subsidies, loans, export quotas and other privileges. During the period dubbed the "licence raj," the Indian government controlled large areas of economic activity through the licensing of various rights and "permissions." Whether such a role for government is justified, how a government should allocate rights, and what rights the owners should be allowed to enjoy are clearly issues of importance.

According to the Coase theorem, how property rights are assigned initially will not matter if property includes the right to transfer, and if transfers do not entail transaction costs. Then, individuals will arrive at a Pareto efficient allocation through voluntary negotiation.[1] Further, the outcome of voluntary exchange will be efficient in the utilitarian sense if the individuals' utilities are transferable. These conditions are rarely met in practice, however. In particular, utility would not be (fully) transferable if some individuals have limited wealth and limited access to capital markets. Limited wealth and capital market imperfections are important when individuals' valuations of the resource are potentially large and are not observable by investors, and/or the capital markets are not well developed. Entrepreneurial opportunities in developing economies, immigration visas, human organs, and housing provide some relevant examples. Financial constraints render individuals unable to pay as much as they are willing, so a socially desirable transfer may not occur through voluntary negotiation. Then, the initial assignment of property rights and their

[1]There is no formal statement or proof of such a theorem in Coase (1960). A standard version of the Coase theorem states: "Given traditional assumptions of substantial knowledge, perfect rationality and the absence of transactions costs and income effects, the assignment of legal entitlements ... will be neutral as to the goal of economic efficiency" (Coleman (1988)).

scope will affect the allocative efficiency of the outcome attained. How property should be assigned in such a case and what role governments can play are clearly important.[2] Unfortunately, economic theory offers very little in the way of guidance or frameworks for analyzing alternative methods of assigning property rights in such cases. We aim to fill this gap.

We develop a model in which a scarce object may be allocated to a mass of agents. This object could be a consumption good such as housing or health care, in which case agents' willingness to pay reflects their utility of consuming the good. Alternatively, the object could be a productive asset such as a license to operate a business, to exploit resources, or to export goods, in which case the willingness to pay reflects the monetary payoff that the asset will generate for the recipient. The agents differ not only in their willingness to pay for the object but also in their ability to pay, which is binding for some for the reasons described above. Using this model, our analysis will focus on several questions: *Can the government do better than a competitive market in allocating the object? How should the assignment of property treat agents with different attributes? Should the owner of property be allowed to resell?* These questions may require some motivation.

□ MARKET VS. NON-MARKET ASSIGNMENT OF "INITIAL" OWNERSHIP: This comparison serves a couple of purposes. First, for resources that are supplied by (possibly competitive) markets, this comparison will help investigate the rationale for government intervention in their provision. Housing, health care, and education are examples of private goods that are often publicly provided or are subject to government regulation. For instance, the prices of many housing units are held below market-clearing levels, and a lottery or priority scheme is then used to allocate them. A notable example comes from Singapore, where most citizens live in units sold by the government at prices that are well below the market.[3] Such a practice begs the question of whether a "competitive" market warrants government intervention. Human organs are assigned to patients in need of a transplant by a priority

[2] Although many scholars continue to focus on the case of zero transactions costs, Coase was interested in how rights should be allocated in a world in which transactions costs are *not* zero. (See the discussion in Cole and Grossman (2004, p. 78), for example.)

[3] Some 86 per cent of Singapore's citizens live in such units and 92 per cent of those residents own their units. (See "Building Homes, Shaping Communities," at http://www.mnd.gov.sg/, accessed on May 28, 2005.) The price cap is as low as half of the price on the resale market (Tu and Wong (2002)).

rule that depends on such factors as the recipient's age, the severity of the condition, and the distance between the donor and recipient. Likewise, access to public education is not assigned by the market, but rather by a priority rule. Second, for resources supplied by governments, this comparison helps us understand the efficiency of markets as a method for assigning initial ownership of resources.

Competitive markets in the form of auctions have been employed to assign government rights for timber harvesting, mineral extraction, and radio spectrum; and their use is expanding to areas where non-market methods such as hearings or lotteries had been used. Non-market assignment is common, though. Allocation of preferential treatment by the Korean government was clearly not based on markets. Immigration visas are allocated by eligibility criteria and lotteries. Jury duty (and thus exemptions from it) is assigned by a lottery. A military draft may select conscripts (and thus exemptions or deferments) by lottery as well. Various other methods have been used to allocate unclaimed property or government property. For instance, land was deeded on a first-come-first-served basis during the 1889 *Oklahoma Land Rush*, and by lottery in 1901.

Absent budget constraints, competitive markets are clearly an efficient way to assign property rights since a competitive price perfectly screens those who are more willing to pay. Budget constraints mitigate this screening role of prices, jamming the willingness-to-pay signal, so the allocation will be inefficient; some agents with low willingness but high ability to pay will be selected over agents with high willingness but low ability to pay. It is not clear that government can assign property better than competitive markets can, though, if it lacks information about agents' types. Hence, the comparison is not trivial.

□ MERIT-BASED VS. NEED-BASED ASSIGNMENT: Signals concerning agents' types are often available, so the initial assignment of property can be conditioned on these signals. For an object such as housing or a human organ, how much an agent values it may be learned, albeit imperfectly, from her existing living arrangements or a doctor's assessment of her medical condition. Likewise, the social value of assigning a productive asset to an agent can be inferred from his previous job performance or test scores. Similarly, an agent's ability to pay may be inferred from earnings and asset holdings. How an agent should be treated based on these signals in the assignment of initial ownership matters greatly. This question has a great deal of policy relevance as well, for many real world assignment rules treat individuals based on their merits (willingness to pay signal) and needs (ability to pay signal). Need-based assignment can be found in the preferences for "designated entities" (i.e., small businesses) in license auctions and in college financial aid and admissions decisions. While merit-based assignment (i.e., favoring those with the highest apparent valuation) would be clearly justifiable on allocative efficiency grounds, an efficiency justification for a need-based assignment (i.e., favoring the poor) is not so clear. We seek to establish one.

□ Transferability of Property: Whenever a non-market method is used to assign an object, some recipients may have an incentive to resell the good. Although voluntary trade is mutually beneficial, transferability of goods is often limited. For instance, a student cannot normally sell his spot in an oversubscribed class. This issue arose recently when a student at NYU Law School attempted to purchase a spot in two classes. Students there are permitted to trade spots in classes, but they may not offer cash or "cash substitutes."[4] Likewise, the subletting of rent-controlled apartments is often prohibited. In addition, most countries prohibit the sale of human organs. While there are moral or paternalistic reasons for these restrictions, the benefits from mutually beneficial trade make it appropriate to examine the efficiency implications of restricting alienation of property rights.

Our findings are as follows.

□ Efficiency of Non-market Assignment: We first consider the simplest possible non-market assignment rule — random assignment. Specifically, we suppose that a government imposes a binding price cap and assigns the object randomly among those who demand the object at the price cap. Such an assignment scheme involves minimal administration and can be used even if the government has no information about the agents' types. Hence, if such a scheme yields a more efficient allocation than market assignment does, the efficiency case for non-market assignment will be very strong. Indeed, we build such a case, but the argument depends crucially on whether resale of the randomly assigned object is permitted. We show that random assignment yields a more efficient allocation

[4] See Bitkower (2005). The initial allocation was determined by a priority scheme based on submitted preferences. There was an appreciation by administrators and students alike that allowing the sale of spots could induce misrepresentation of preferences by students. Those issues do not arise here because only a single good is allocated.

than a competitive market *if and only if resale is allowed.*

When resale is prohibited, random assignment at a below-market price has two effects relative to the competitive market: It shifts assignment from agents with higher ability to pay toward agents with lower ability to pay, and from agents with higher willingness to pay to agents with lower willingness to pay. The former shift has no effect on allocative efficiency if agents with different ability to pay have the same distribution of willingness to pay; the latter shift clearly reduces allocative efficiency.

Now suppose that resale is permitted. Random assignment has the same effect as above, initially.[5] But, low-valuation agents will resell the object to those with high valuations. More importantly, shifting the assignment from the wealthy to the poor (relative to the market assignment) improves efficiency, since the latter would never receive the good in the competitive market but now do so with some probability; they will keep the good if and only if they value the good more than the resale price. The resulting allocation is more efficient than the market.

This result suggests that government intervention can be justified, even with a competitive market, if wealth constraints are important and if resale is permitted. It also suggests that an auction may not be a socially desirable assignment method for various entitlements.

☐ Desirability of Need-based assignment:

The logic of the previous results suggests that shifting the initial assignment to the poor improves allocative efficiency if resale is permitted. While the wealthy can purchase the good on the resale market, the poor are unable to do so. Favoring the poor in the initial allocation means that some poor, high-valuation consumers get the good who otherwise would not. *Need-based* schemes that favor buyers with low wealth have no effect on efficiency if resale is not permitted, but they improve efficiency if resale is permitted. This observation provides an efficiency rationale for assignment schemes and social programs that favor the poor.

☐ Desirability of Allowing for Resale:

Non-market assignment generates inefficiencies, thereby creating an incentive for resale. As seen above, resale is crucial for non-market assignment — and thus government

[5]The set of participants may differ, however.

intervention into a competitive market — to be justified on allocative efficiency grounds. Speculation tempers the benefits of resale, however. If there is a binding price cap, the equilibrium resale price will exceed it. This creates a motive for speculation, inviting those who would derive no consumption value from the object to compete initially for the assignment. Clearly, speculation dilutes the chances that poor agents with high valuations obtain the assignment. In fact, as the number of speculators grows without bound, the ultimate allocation approaches the competitive market allocation under general conditions. Hence, if the assignment technology is sufficiently effective at selecting high-valuation agents, and if there are sufficiently many speculators, then it may be beneficial to restrict transferability of the object, even though resale is beneficial ex post. Thus, there is an efficiency argument for prohibiting transferability.

The possible gains from transferability shed light on how existing schemes perform and how they can be improved. Consider U.S. immigration policy. The U.S. allocates 50,000 immigration visas per year by lottery. Becker (1987) proposed selling visas to a pool of qualified applicants. A simple alternative is to retain the current lottery system but permit recipients to resell their visas to other qualified applicants. Our results suggest that this change would yield greater efficiency than both the current mechanism and the Becker proposal.[6] Similar points hold with public school admissions decisions. The use of a lottery to allocate transferable vouchers may be preferable to a system of local attendance zones or to the use of a lottery to allocate nontransferable vouchers.

Transferability may also improve very sophisticated allocation schemes. Some recent studies have proposed ingenious algorithms for improving allocative efficiency—without using transfers—in settings where wealth constraints may be important. (See Abdulkadiroğlu and Sönmez (1999) and (2003), or Roth, Sönmez, and Ünver (2004).) The best algorithm may still not deliver full efficiency, however. The results here imply that introducing transfers in the form of resale could improve efficiency of these allocation schemes, so our approach is complementary.

Section 2 lays out the basic model and describes the efficient allocation and the competitive market allocation. Section 3 characterizes the outcome when the good is subject to a

[6] An implicit assumption is that immigrants create rents and those who value visas most highly create the most rents. Other factors may enter a policymaker's welfare function. For example, visa sales would generate revenue for the Treasury.

price cap and the available supply is allocated randomly. Section 4 analyzes general allocation schemes and derives conditions under which they outperform the market. Extensions are in Section 5, with concluding remarks in Section 6.

2 The Model

2.1 Primitives

A good is available in fixed supply, $S \in (0, 1)$. There exists a unit mass of buyers who each demand one unit. Each buyer is characterized by two attributes: her wealth, w, and her valuation, v, both distributed continuously over $[0, 1]^2$. We refer to (w, v) as the buyer's *type*. The two components are independent, with w having the cumulative distribution function (cdf) $G(w)$ and v having the cdf $F(v)$. The corresponding densities are non-zero for almost every v and w. Independence helps to isolate the role that each attribute plays and the effect of policy treatments that focus on a single one.

There also exist individuals who participate solely for speculative reasons. In particular, there is a mass, m, of *non-buyers*. These individuals all have a valuation $v = 0$, while their wealth has the cdf $G(w)$. In other words, they are like the buyers in terms of wealth, but they do not value consumption of the good.

Consumers are risk-neutral, with quasilinear utility. Specifically, a consumer with valuation v and money m gets utility of $v + m$ if she consumes the good and m if she does not. A buyer with $v > w$ is called *wealth-constrained* since she is not able to pay as much as she is willing to pay. This situation could arise from capital market imperfections, which limit a buyer's ability to borrow against future income, or simply from a low lifetime income. Whatever the source, wealth constraints inhibit the transfer of utility between individuals.

The good is indivisible, and it is supplied competitively at a constant marginal cost, $c > 0$, up to S. Marginal cost is sufficiently low that there is a shortage of supply. Inelastic supply and shortages are characteristic of many markets in which instruments other than price are employed.

We evaluate the welfare consequences of using different allocation mechanisms. Before describing the mechanisms, it is necessary to discuss our welfare criterion.

2.2 Welfare Criterion and Efficient Allocation

The welfare criterion we use is *total realized value*, which equals the sum of valuations for those who consume. Given quasilinear preferences, total realized value reflects the sum of utilities, which is the sensible criterion when utility is fully transferable. Total realized value is also a compelling criterion when utility is not transferable, since it reflects (ex ante) expected payoffs. Suppose that consumers and producers are drawn from a large pool, with each individual being equally likely to be selected for the consumer pool as the buyer pool. Then, a second drawing selects "active" participants and their types. If the agents were to select an allocation mechanism prior to realizing their types, they would vote for the one with the higher total realized value since they value consumer and producer surplus equally.[7] Thus, we employ total realized value as the welfare measure here. We refer to an allocation with greater total realized value as the more (*utilitarian*) *efficient* allocation.

The efficient allocation provides the good to the buyers with the highest valuations. Let $v^* > 0$ denote the critical valuation such that $1 - F(v^*) = S$. If all buyers with valuations of v^* and above acquire the good, the total realized value is

$$V^* := \int_0^1 \int_{v^*}^1 v dF(v) dG(w) = \int_{v^*}^1 v dF(v) = S\phi(v^*),$$

where

$$\phi(z) := \frac{\int_z^1 v dF(v)}{1 - F(z)}$$

is the expectation of a buyer's valuation, conditional on exceeding z. The aggregate quantity is S, and the average valuation among those who acquire the good is equal to $\phi(v^*)$, so the total realized value in the first-best allocation is $S\phi(v^*)$. That allocation and the associated realized value, V^*, will serve as benchmarks for the mechanisms that we consider.

2.3 Assignment Schemes

Throughout the paper we will compare the performance of three regimes: (1) *a competitive market*, (2) *an allocation scheme for a non-transferable good*, and (3) *an allocation scheme for a transferable good*. The latter two schemes entail a binding price cap, and they allocate

[7] Analogous arguments are provided in Vickrey (1945) and Harsanyi (1953). A formal justification of this utilitarian approach is in Harsanyi (1955).

the good according to some exogenous rule. The allocation scheme is described by the probability of allocation for those who participate. Formally, an *assignment rule* is a (measurable) function, $x : [0,1]^2 \mapsto [0,1]$, which maps each type, (w, v), to a probability of allocation. If the good is transferable, there will be a resale market subsequently in which those who obtained the good may resell it to those who did not.

The three regimes are seen prominently, as illustrated by the following examples:

- **Fugitive Property and Government-Owned Resources:** Fugitive property—an object or resource whose ownership is not yet established—is allocated in many different ways. It can be allocated to the individual who claims it first (*the rule of first possession*), or to the individual who owns property tied to that object (*tied ownership*).[8] Ownership can also be established by an auction. The first two methods correspond to allocation of a transferable good. Auctions correspond to the market. The 1889 Oklahoma Land Rush and the 1901 Oklahoma Land Opening are examples of allocation with transferability. In 1906, government land in Oklahoma was sold at auction. Government resources such as radio spectrum, timber rights, and oil rights have been allocated using a number of methods including hearings, lotteries, and auctions.

- **Education:** Public school enrollment is typically tied to one's residence, so the housing market serves indirectly as a market for school enrollment. Suppose that two public schools draw students from mutually exclusive attendance areas, and more than half of the students prefer school A. The valuation, v, now represents the premium that an individual is willing to pay for the right to attend A. Since the nominal price of attending the school is zero, the preference for A will be capitalized in housing prices. This scenario corresponds to the market regime. Now suppose that slots in school A are awarded by lottery. This is an allocation scheme with a non-transferable good. The final regime arises if a lottery awards *transferable* vouchers that confer the right to attend A.

- **Housing:** The three regimes arise naturally in housing markets. Consider a market that is subject to rent control. If leases are not transferable, the regime is that of

[8]For instance, a landowner has the subsurface right to natural gas deposits underneath the land.

allocation without transferability. If leases are transferable, we have allocation with transferability. If rent control is abolished, we have a competitive market.

- **Health care:** Health care is often provided via an allocation scheme without transferability. A specific example involves organ transplants. Patients are placed in a queue and cannot sell or swap their places in the queue.[9] In principle, a competitive market could allocate organs to those willing and able to pay the market price.[10] Finally, one could employ a general allocation scheme with transferability in which patients waiting for a transplant may sell their places in the queue.

- **Military recruitment:** A draft in which conscripts are called to duty based on the outcome of a lottery is effectively an allocation scheme for a non-transferable good — military deferments. A draft with tradable deferments represents an allocation scheme with a transferable good. An early example occurred in ancient Korean kingdoms when wealthy families were allowed to pay sharecroppers to enlist on their behalf. In the U.S. Civil War, conscripts could avoid service in the Union Army by hiring a substitute.

In the remainder of this section, we examine the competitive market.

2.4 A Competitive Market

A competitive market is characterized by the price at which the supply is exhausted by the effective demand. When the market price is $p > 0$, the measure of consumers willing and able to pay p^e is

$$D(p) := [1 - G(p)][1 - F(p)].$$

The competitive equilibrium, if it exists, is achieved at a price, p^e, such that

$$D(p^e) = [1 - G(p^e)][1 - F(p^e)] = S. \tag{1}$$

[9]In the U.S., waiting lists for organ transplants are maintained by the United Network for Organ Sharing, which matches donated organs to recipients. Additional issues arise with kidney transplants because of the possibility of live donors. See Roth, Sönmez, and Ünver (2004) for a discussion of kidney exchanges wherein patients with incompatible live donors essentially trade donor kidneys.

[10]Organs are available in certain countries at market prices.

Assuming that $p^e > c$, this is a competitive equilibrium.

A couple of remarks are in order. First, $1 - F(v^*) = S$, so $[1 - G(v^*)][1 - F(v^*)] < S$, which implies $p^e < v^*$. This means that the market equilibrium does not yield the efficient allocation. As can be seen in Figure 1, the first-best allocates the good to all buyers in the region $A + B$ while the market allocates to those in $B + C$. Relative to the first-best allocation, the market favors high-wealth-low-valuation buyers (region C) over low-wealth-high-valuation buyers (region A).

[PLACE FIGURE 1 ABOUT HERE.]

The total realized value under the market is:

$$V^e := \int_{p^e}^1 \int_{p^e}^1 v dF(v) dG(w) = [1 - G(p^e)] \int_{p^e}^1 v dF(v) = S\phi(p^e) < S\phi(v^*) = V^*.$$

The second equality holds by integrating over w; the third holds since $[1-G(p^e)][1-F(p^e)] = S$, by (1); and the inequality holds since $p^e < v^*$ and ϕ is an increasing function. The inefficiency is entirely attributable to the binding wealth constraints. If no buyers were constrained, the effective demand would simply be $1 - F(p)$, so the market-clearing price would satisfy $p = v^*$, yielding an efficient allocation.

Second, even though the market allocation is inefficient, it would not trigger any resale. Those who have the good would only be willing to resell at prices exceeding p^e, but there are no additional buyers willing and able to pay that much.[11] The inefficiency resulting from the market will not be resolved by opening another market. With a price cap, by contrast, inefficiencies may be partially resolved through resale.

3 Analysis with Random Assignment

In this section, we consider a market with a binding price cap, $\overline{p} \in [c, p^e)$. The good is assigned completely randomly to participants. A random allocation is simple to implement as it does not require knowledge of buyers' preferences or budgets. While a government

[11] By the same token, if there were an active resale market, the resale market price would be the same as the price in the original equilibrium; otherwise, buyers would switch from one market to the other. Hence, the equilibrium price must be p^e, so the allocation is the same.

may try to screen buyers to favor one group, such screening may be imperfect because types are private information and screening may be subject to manipulation. (For instance, if the allocation scheme favors buyers with low wealth, buyers may hide or divest their wealth.) We will analyze general allocation schemes in Section 4, so details of the equilibrium analysis will be presented then.

3.1 Random Assignment with No Resale

Many of the goods we have discussed are not transferable because the supplier mandates it or because there are legal restrictions. We now analyze the case of a non-transferable good. Enforcement of non-transferability is easy in many cases. Health care providers and educational institutions require identification of the patient or student. Licenses to provide cellular telephone or cable television service can easily be made non-transferable since ownership or use can be monitored.

Buyers whose ability-to-pay and willingness-to-pay both exceed $\overline{p}$ will want to buy the good.[12] Each of these buyers receives the good with probability

$$\frac{S}{[1 - F(\overline{p})][1 - G(\overline{p})]}.$$

The expected valuation for such buyers is $\phi(\overline{p})$, and the aggregate quantity is S, so random allocation gives a total realized value of $S\phi(\overline{p})$. Since $\overline{p} < p^e$, we have $S\phi(\overline{p}) < S\phi(p^e)$, so the allocation is less efficient than under the market.

The reason that capping price and randomly allocating the good does worse than the market differs from the standard explanation, which is that capping price leads to a reduction in quantity, resulting in a deadweight loss. There is no quantity effect here; rather, the fixed quantity is simply allocated inefficiently.[13] When the cap binds, it expands the set of buyers who may get the good. The effect on efficiency is negative because some buyers with lower valuations crowd out others with higher valuations. Although some buyers with

[12] We can assume that a buyer who participates must take delivery and pay for the good if successful. If the buyer does not fulfill these obligations, she will not get the good and she must pay a small penalty. This will mean that buyers only participate if they are able to pay the price.

[13] The same point is made by Glaeser and Luttmer (2003), who note that the existing stock of rent-controlled apartments is allocated inefficiently.

lower wealth now acquire the good, this factor in isolation has no effect since v and w are independent.

3.2 Random Assignment with Resale

When a good is assigned randomly, some high-valuation buyers do not get the good whereas some low-valuation buyers do. This produces a motive for resale. We now assume that the good is transferable. Since it will be profitable to acquire multiple units and resell some, we assume that each individual may only attempt to purchase one unit of the good in the initial allocation.

There will be a competitive resale market. The equilibrium resale price, $r(\overline{p})$, must exceed $\overline{p}$. (If not, $r(\overline{p}) < p^e$, so there would have to be excess demand on the resale market.) A successful speculator can pocket a strictly positive gain, $r(\overline{p}) - \overline{p}$, by reselling. Thus, all buyers who can afford to pay $\overline{p}$ will participate in the initial allocation, and they each receive the good with probability

$$\rho(\overline{p}) := \frac{S}{1 - G(\overline{p})}.$$

Given a price cap $\overline{p}$, all $[1 + m][1 - G(\overline{p})]$ consumers with $w \geq \overline{p}$ participate, so each receives the good with probability

$$\rho(\overline{p}; m) := \frac{S}{[1 + m][1 - G(\overline{p})]}.$$

Resale demand at a price r comprises the buyers willing and able to pay r who did not get the good initially:

$$RD(r) := [1 - F(r)][1 - G(r)][1 - \rho(\overline{p}; m)].$$

Resale supply comprises the successful speculators who choose to resell the good. If a consumer with valuation v gets the good and keeps it, she will receive a net surplus of $v - \overline{p}$. Reselling nets $r - \overline{p}$, so the consumer will resell if $v < r$. Resale supply comprises the participants with $v < r$ (including the non-buyers) who get the good initially. Their measure is $\rho(\overline{p}; m)[F(r) + m][1 - G(\overline{p})]$, so

$$RS(r) := S\left(\frac{F(r) + m}{1 + m}\right).$$

Equating demand and supply, we have

$$[1-F(r)][1-G(r)][1-\rho(\overline{p};m)] = S\left(\frac{F(r)+m}{1+m}\right)$$
$$\Rightarrow D(r) = D(r)\rho(\overline{p};m) + S\left(\frac{F(r)+m}{1+m}\right)$$
$$\Rightarrow D(r) = S - \rho(\overline{p};m)[1-F(r)][G(r)-G(\overline{p})].$$

The last term is the measure of buyers who receive the good and retain it, but would be unable to purchase on the resale market. For any $r \leq (<)\ p^e$, $D(r) \geq (>)\ S$, so $RD(r) > RS(r)$. Hence, the resale equilibrium must have $r(\overline{p}) > p^e$. This can be seen intuitively through Figure 2.

[PLACE FIGURE 2 ABOUT HERE.]

Suppose that the resale price were $r = p^e$. The buyers who are both willing and able to pay $r = p^e$ (area B) all end up with the good, just as in the competitive market equilibrium. (They either acquire it initially and keep it, or they acquire it in the resale market.) As noted, some buyers with $(w, v) \in [\overline{p}, r] \times [r, 1]$ obtain the good and keep it. This group accounts for the area A. Since they keep the good, there must be excess demand on the resale market when $r = p^e$, so the resale price must exceed p^e.

The total realized value is now $S\phi(r(\overline{p})) > S\phi(p^e) = V^e$.[14] This tells us that capping price and randomly allocating a transferable good produces a strictly more efficient allocation than either randomly allocating a non-transferable good or operating a competitive market.[15] Despite the speculation that it engenders, permitting transferability is beneficial.

The welfare ranking rests on the following logic: The post-resale allocation becomes more efficient when the initial allocation shifts away from those with the wealth to acquire the good and toward those without. A random assignment scheme effectively reallocates the good from the wealthy to the poor; those with high valuations keep the good while those

[14] Independence of v and w implies that, for any given $w \geq \overline{p}$, the expected value of v, conditional on exceeding $r(\overline{p})$, is $\phi(r(\overline{p}))$. When buyers with wealth w and valuation $v \in [r(\overline{p}), 1]$ all get the good with probability ρ_w, say, the expected value for such bidders is $\phi(r(\overline{p}))$. Since this holds for all $w \geq \overline{p}$, and since quantity equals S, the total realized value is $S\phi(r(\overline{p}))$.

[15] High-valuation buyers with wealth $w < \overline{p}$ cannot get the good, so $r(\overline{p}) < v^*$. Since $S\phi(r(\overline{p})) < S\phi(v^*) = V^*$, there is not full efficiency.

with low valuations resell. The same logic that supports random allocation also applies to reductions in $\overline{p}$. As the cap drops, more of the good is assigned to the poor. The ultimate allocation becomes more efficient since $r(\overline{p})$ increases as $\overline{p}$ falls, so efficiency is highest when the cap is at the lowest level at which supply is available, c. The formal results are now given.

PROPOSITION 1. *Random allocation of a non-transferable good is less efficient than the competitive market, and it becomes progressively less so as the price cap falls. Random allocation of a transferable good is more efficient than the competitive market, however, and it becomes strictly more so as the price cap falls.*

A final point is that speculation reduces the benefits from capping price and allocating the good randomly. It does this by lowering the quantity assigned to buyers with high valuations and low wealth. To see this, let

$$\psi(m, r) := D(r) - S + \rho(\overline{p}; m)[1 - F(r)][G(r) - G(\overline{p})]$$

denote the excess demand on the resale market. Along a level set of ψ, the slope is $-\frac{\psi_m}{\psi_r} < 0$ since excess demand declines with r and the probability of receiving the good declines with m. A rise in m therefore lowers the total realized value, $S\phi(r(\overline{p}))$. As m increases without bound, $\psi(m, r)$ approaches $D(r) - S$ so the equilibrium resale price approaches p^e. Speculators acquire the entire supply in the limit so the resale market mimics the original competitive market. The next section shows the robustness of the results here.

4 Analysis with a General Assignment Scheme

The previous section assumed that the good was assigned randomly. In practice, the government or the suppliers may have information about consumers' wealths or valuations, allowing them to implement alternative schemes. For instance, when one patient is given priority for an organ transplant based on age and medical urgency, that is an implicit assessment that the valuation is higher for the one patient than the other.

Allocation schemes may also depend on the government's or suppliers' objectives. Some schemes are geared to help the poor and some are geared to select those deemed to have the greatest merit. For example, need-based scholarships target those with low wealth while merit-based scholarships can be seen as targeting those with high valuations. Even a first-come-first-served scheme will assign the good in a manner that depends on the correlation between cost of time and bidders' types. In this section we will consider a family of general allocation schemes. We do not model seller objectives or the information available when selecting the assignment rule; instead, we simply take the assignment rule as given.

An assignment rule, x, gives the probability that a buyer gets the good initially. The assignment rule is *feasible* if

$$\int_0^1 \int_0^1 x(w,v)dF(v)dG(w) + m\int_0^1 x(w,0)dG(w) = S.$$

A feasible allocation rule is *separable* if $x(w,v) = \alpha_x(w)\beta_x(v)$ for functions $\alpha_x : [0,1] \mapsto \Re_+$ and $\beta_x : [0,1] \mapsto \Re_+$. Let $\mathcal{X}$ denote the set of feasible, separable assignment rules. We henceforth consider rules in this family. Separable allocation schemes allow us to examine the effect of policies targeting a single attribute in the most transparent way.

4.1 Characterization of Assignment Rules

Let

$$a_x(w) := \frac{\alpha_x(w)}{\int_0^1 \alpha_x(\tilde{w})dG(\tilde{w})}$$

and

$$b_x(v) := \frac{\beta_x(v)}{\int_0^1 \beta_x(\tilde{v})dF(\tilde{v}) + m\beta_x(0)}.$$

The probability of allocation then satisfies

$$x(w,v) = S \cdot a_x(w)b_x(v),$$

since feasibility implies

$$\int_0^1 \int_0^1 \alpha_x(\tilde{w})\beta_x(\tilde{v})dF(\tilde{v})dG(\tilde{w}) + m\int_0^1 \alpha_x(\tilde{w})\beta_x(0)dG(\tilde{w})$$
$$= \left(\int_0^1 \alpha_x(\tilde{w})dG(\tilde{w})\right)\left(\int_0^1 \beta_x(\tilde{v})dF(\tilde{v}) + m\beta_x(0)\right) = S.$$

Then,

$$A_x(w) := \int_0^w a_x(\tilde{w})dG(\tilde{w})$$

and

$$B_x(v) := \int_0^v b_x(\tilde{v})dF(\tilde{v}) + \frac{m\beta_x(0)}{\int_0^1 \beta_x(\tilde{v})dF(\tilde{v}) + m\beta_x(0)}$$

can be seen as cumulative distribution functions for *quantity* across different wealths and valuations, respectively. It will prove useful for efficiency comparisons to characterize assignment rules in terms of these functions.

We begin with rules that favor those with higher valuations. We say that the assignment rule $x \in \mathcal{X}$ *merit-dominates* $y \in \mathcal{X}$ if B_x first-order stochastically dominates (FOSD) B_y: $\forall v$, $B_x(v) \leq B_y(v)$.[16] In words, x assigns a higher probability to high-valuation consumers than y does. Rules x and y are *merit-equivalent* if $b_x(\cdot) = b_y(\cdot)$. (Note that this implies $B_x(0) = B_y(0)$.)

The assignment rule $x \in \mathcal{X}$ is *merit-blind* if $b_x(v)$ is constant for all $v \geq \overline{p}$. Random allocation is an obvious example of a merit-blind rule as it awards the good with the same probability to all participants whose valuations exceed the price cap.

There are analogous conditions for cases in which *lower* wealth is favored. We say that $x \in \mathcal{X}$ *need-dominates* $y \in \mathcal{X}$ if A_y FOSD A_x: $\forall w$, $A_x(w) \geq A_y(w)$. In words, x is more likely to assign the good to low-wealth buyers than y is. Rules x and y are said to be *need-equivalent* if $a_x(\cdot) = a_y(\cdot)$. Finally, $x \in \mathcal{X}$ is *need-blind* if $a_x(w)$ is constant for all $w \geq \overline{p}$. Then, the probability of receipt does not depend on wealth.

The outcomes with and without transferability can now be characterized.

4.2 Assigning a Non-transferable Good

Given a price cap, $\overline{p} < p^e$, only buyers with $(w, v) \geq (\overline{p}, \overline{p})$ will participate if the good is not transferable. The allocation rule will then have $x(w, v) = 0$ if $w < \overline{p}$ or $v < \overline{p}$. The total realized value is

$$V_x \ := \ \int_{\overline{p}}^1 \int_{\overline{p}}^1 vx(w, v)dF(v)dG(w).$$

The following proposition provides a ranking based on merit-dominance.

Proposition 2. *If $x \in \mathcal{X}$ [strictly] merit-dominates $y \in \mathcal{X}$, then $V_x \geq [>] V_y$.*

[16]The merit-dominance is *strict* if the inequality is strict for a positive measure of valuations. The analogous condition makes subsequent dominance definitions strict as well.

PROOF: Rewrite the total realized value as

$$\begin{aligned} V_x &= S\int_0^1\int_0^1 va_x(w)b_x(v)dF(v)dG(w) \\ &= S\int_0^1 vb_x(v)dF(v) \\ &= S\int_0^1 vdB_x(v). \end{aligned}$$

If $x \in \mathcal{X}$ [strictly] merit-dominates $y \in \mathcal{X}$, then B_x [strictly] FOSD B_y, so the result follows. ∎

This result says that an assignment rule that puts relatively more weight on high valuations yields greater efficiency. A couple of implications can be drawn immediately. We first note that it does not matter how an assignment rule treats consumers with different wealth levels here: All that matters for efficiency is how it screens based on valuations.

COROLLARY 1. (IRRELEVANCE OF NEED-BASED SCREENING) *If $x \in \mathcal{X}$ and $y \in \mathcal{X}$ are merit-equivalent, then $V_x = V_y$.*

Any assignment rule that is merit-dominated by a merit-blind rule is less efficient than the merit-blind rule, which is welfare-equivalent to random allocation, by Corollary 1. The previous section established that random allocation is strictly less efficient than the competitive market, so the following result is also immediate.

COROLLARY 2. (DRAWBACK OF ALLOCATION WITHOUT TRANSFERABILITY) *Any assignment rule in $\mathcal{X}$ that is merit-dominated by the random assignment rule (given the same binding price cap) yields a strictly less efficient allocation than the market does.*

It follows that a merit-blind assignment rule (which is merit-equivalent to random allocation) is strictly less efficient than the competitive market. That is, any assignment rule associated with purely need-based screening cannot be justified from an efficiency perspective, *when the good is not transferable.*

The *strict* dominance by the market over all merit-blind rules implies that any rule that is modestly merit-superior to random allocation cannot be justified from an efficiency perspective either. In order for the allocation of a non-transferable good to improve efficiency, substantial merit-based screening must be feasible. In the extreme case in which buyers' valuations are observable, the first-best allocation can be achieved. In practice, however, such precise information will rarely be available.

4.3 Assigning a Transferable Good

We now consider general allocation schemes when the good is transferable. Suppose that there is a price cap, $\overline{p} < p^e$, with a resale market price $r > \overline{p}$. A consumer would participate if and only if $(w, v) \geq (\overline{p}, 0)$. Hence, the assignment rule must have $x(w, v) = 0$ for any $w < \overline{p}$.

A buyer who fails to get the good initially will demand a unit on the resale market if she is willing and able to pay r. Such buyers represent the resale demand:

$$RD(r) := \int_r^1 \int_r^1 [1 - x(w, v)] dF(v) dG(w).$$

Buyers who get the good initially will keep it if $v \geq r$. The quantity supplied on the resale market is therefore

$$RS(r) := S - \int_{\overline{p}}^1 \int_r^1 x(w, v) dF(v) dG(w).$$

Observe that $RD(\cdot)$ is nonincreasing and $RS(\cdot)$ is nondecreasing, and both are continuous functions. Further, $RD(1) = 0 < S = RS(1)$, and $RD(0) = 1 - S > 0 = RS(0)$. Hence, there exists a resale price, r_x, that clears the market: $RD(r_x) = RS(r_x)$.

Rewrite the market-clearing condition as:

$$RD(r) - RS(r) = D(r) - S + K_x(r) = 0, \tag{2}$$

where

$$K_x(r) := \int_{\overline{p}}^r \int_r^1 x(w, v) dF(v) dG(w) = S \cdot A_x(r)[1 - B_x(r)]$$

is the measure of buyers with wealth $w \in [\overline{p}, r]$ who get the good and keep it. Note that if $K_x(p^e) > 0$, then $RD(p^e) - RS(p^e) = K_x(p^e) > 0$. It follows that $RD(r) > RS(r)$ for any $r \leq p^e$, implying that $r_x > p^e$ (which confirms that the resale price exceeds $\overline{p}$).

The total realized value is now

$$\hat{V}_x := \int_{\overline{p}}^{r_x} \int_{r_x}^1 v x(w, v) dF(v) dG(w) + \int_{r_x}^1 \int_{r_x}^1 v dF(v) dG(w).$$

The first term pertains to high-valuation-low-wealth buyers who keep the good, while the second covers those with high valuations and high wealth; some of them get the good initially, while the others purchase it on the resale market.

The subsequent characterization refers to a new property. We say that the assignment rule x *relatively merit-dominates* the rule y if $B_x(0) \leq B_y(0)$ and, for every $v' \in (v, 1)$,

$$\frac{b_x(v')}{b_x(v)} \geq \frac{b_y(v')}{b_y(v)}.$$

This notion implies merit-dominance as it requires the latter to hold for all subsets of valuations. We say that x is *meritorious* if it relatively merit-dominates a merit-blind rule; i.e., $b_x(v') \geq b_x(v)$ for all $v' > v$. Likewise, x is *demeritorious* if $b_x(v'') \leq b_x(v)$ for all $v'' < v$. Roughly speaking, meritorious (demeritorious) allocation schemes dominate (are dominated by) merit-blind rules in a stronger sense than merit-dominance.

Let $\mathcal{X}^+$ and $\mathcal{X}^-$ denote, respectively, the set of all meritorious and demeritorious allocation schemes in $\mathcal{X}$. Note that both sets include merit-blind rules.

PROPOSITION 3. *If a meritorious rule, $x \in \mathcal{X}^+$, relatively merit-dominates $y \in \mathcal{X}$, and if $r_x \geq [>] r_y$, then $\hat{V}_x \geq [>] \hat{V}_y$.*

PROOF: See the Appendix.

As was the case with random allocation, Proposition 3 shows that a higher resale price indicates a more efficient allocation. Several important implications can then be drawn. The first is that we can provide a sufficient condition for allocation schemes to improve upon the competitive market equilibrium.

COROLLARY 3. (SUPERIORITY OF MERITORIOUS ALLOCATION SCHEMES FOR A TRANSFERABLE GOOD) *Any meritorious assignment rule, $x \in \mathcal{X}^+$, with $\overline{p} < p^e$ and $A_x(p^e) > 0$, produces a strictly more efficient allocation than the competitive market.*

PROOF: Fix $x \in \mathcal{X}^+$ with $\overline{p}_x < p^e$. The total realized value in the competitive market is equal to $\hat{V}_y$ for any $y \in \mathcal{X}$ satisfying $\overline{p}_y = r_y = p^e$. (Then, only consumers with $w \geq p^e$ participate.) In particular, we can choose y to be relatively merit-dominated by x. Since x is meritorious, we have $B_x(p^e) \leq p^e < 1$; together with $A_x(p^e) > 0$, this means $K_x(p^e) > 0$.

It follows that $r_x > p^e$. In sum, $x \in \mathcal{X}^+$ relatively merit-dominates y, and $r_x > p^e = r_y$. Proposition 3 then implies $\hat{V}_x > \hat{V}_y$. ∎

The condition $A_x(p^e) > 0$ simply means that x awards the good to some buyers who are willing but unable to pay the competitive market price. These buyers would not resell the good if $r = p^e$, so the resale price must exceed the competitive market price. Except for this condition, the result does not require much in terms of how the allocation depends on wealth levels. In other words, a weakly meritorious assignment rule does strictly better than the competitive market, largely independent of how it treats the poor relative to the wealthy. Also worth noting is that even a merit-blind assignment rule strictly dominates the market. This means that some demeritorious allocation schemes could do better than the market when the good is transferable.

We next examine the effect of a change in the price cap. This analysis involves a delicate issue: When the cap changes, it alters the set of buyers who participate, so the assignment rule itself changes. One must therefore specify how the rule changes when the price cap changes. We make the following assumptions.

Condition (RC): Suppose that x and x' are assignment rules induced by a given allocation technology, with price caps $\overline{p} < p^e$ and $\overline{p}' < \overline{p}$, respectively. Then, (i) x and x' are merit-equivalent (i.e., $b_{x'}(\cdot) = b_x(\cdot)$), and (ii) $a_{x'}(w) < a_x(w)$ for $w \in [\overline{p}, 1]$.

Property (i) is appealing since a change in the price cap affects the set of participants along the wealth dimension only. It is then reasonable that the merit aspect of the assignment does not change. Property (ii) reflects the equally plausible assertion that adding buyers with lower wealth to the pool reduces the allocation probability for all of the original participants. A special case of Condition (RC) arises if the relative probabilities among the original participants are unchanged; i.e., $x'(w, v) = \lambda x(w, v)$ for $w \in [\overline{p}, 1]$, for some $\lambda < 1$.

Corollary 4. (Benefit of lowering price caps) *Lowering the price cap increases efficiency, given a meritorious allocation technology satisfying Condition (RC).*

Proof: Let x and x' be meritorious allocation rules induced by a given allocation technology, with price caps $\overline{p} < p^e$ and $\overline{p}' < \overline{p}$, respectively. Then, Condition (RC) means that x and x' are also merit-equivalent. Hence, x' relatively merit-dominates x, and $B_{x'}(\cdot) =$

$B_x(\cdot)$.

We next prove that $A_{x'}(w) > A_x(w)$ for all $w \in [\overline{p}, 1)$. Observe that, $\forall w \geq \overline{p}$, we have

$$A_x(w) = \int_{\overline{p}}^{w} a_x(\tilde{w}) dG(\tilde{w}),$$

and

$$A_{x'}(w) = \int_{\overline{p}'}^{w} a_{x'}(\tilde{w}) dG(\tilde{w}).$$

Hence, for $w \geq \overline{p}$,

$$\frac{dA_{x'}(w)}{dw} = a_{x'}(w)g(w) < a_x(w)g(w) = \frac{dA_x(w)}{dw},$$

where the inequality follows from Condition (RC). This, together with $A_{x'}(1) = A_x(1)$, implies that $A_{x'}(w) > A_x(w)$ for all $w \in [\overline{p}, 1)$.

Combining these facts, we have

$$K_{x'}(r) = A_{x'}(r)[1 - B_{x'}(r)] > A_x(r)[1 - B_x(r)] = K_x(r)$$

for any $r > \overline{p}$, from which it follows that $r_{x'} > r_x$. Consequently, Proposition 2 implies that $\hat{V}_{x'} > \hat{V}_x$. ∎

This result shows that the lowest feasible cap is optimal. The proposition also allows us to investigate which allocation schemes promote efficiency.

Corollary 5. (Benefit of need-based allocation schemes) *If $x \in \mathcal{X}$ relatively merit-dominates and need-dominates $y \in \mathcal{X}$, then $\hat{V}_x \geq \hat{V}_y$. If either dominance is strict, then $\hat{V}_x > \hat{V}_y$.*

Proof: Given Proposition 3, it suffices to show that $r_x \geq r_y$, with a strict inequality for a strict ranking. Relative merit-dominance by x over y implies $1 - B_x(r) \geq 1 - B_y(r)$ for all r, whereas need-dominance implies $A_x(r) \geq A_y(r)$. Hence, $K_x(r) \geq K_y(r)$, for all r, which implies $r_x \geq r_y$. If either dominance is strict, then $K_x(r) > K_y(r)$ when $r = r_y$, so $r_x > r_y$. ∎

Need-based allocation schemes can yield greater efficiency than the market does. While it is not surprising that merit-based rules can improve efficiency, it is striking that need-based rules can have the same effect. If x and y are merit-equivalent, but the former need-dominates the latter, then x produces a more efficient allocation. Whereas wealthy buyers can buy the good from a reseller if they do not get it initially, the poor lack the means to do so. As a consequence, an initial allocation that gives the good disproportionately to the poor allows the resale market to produce a more efficient allocation.

Corollary 5 also provides a strong result if the allocation rule favors low-wealth consumers completely. Consider a merit-blind rule, $x^* \in \mathcal{X}$, with $x^*(w, v) = 1$ if $w \leq w^*$, and $x^*(w, v) = 0$ otherwise, where $(1 + m)G(w^*) = S$. (It is implicit that $\overline{p} = 0$ here.) That is, x^* assigns the good to consumers with wealth w^* or below (the region $A + B$ in Figure 3). This rule will achieve full efficiency if $v^* \leq w^*$, where $1 - F(v^*) = S$.[17] To see this, suppose that $v^* \leq w^*$ and that the resale price is r. Then, the resale supply will be $RS(r) = S\frac{F(r)+m}{1+m}$. Meanwhile, those not assigned the good are willing to buy if $v \geq r$, so resale demand is $RD(r) = [1 - F(r)][1 - G(\max\{r, w^*\})]$. Since $RD(r) \gtreqless RS(r)$ if $r \lesseqgtr v^*$, the unique resale equilibrium price is v^*. Consequently, the buyers with $v \geq v^*$ will end up with the good, which is fully efficient. This group comprises the region $A + C$ in Figure 3.

[PLACE FIGURE 3 ABOUT HERE.]

When the supply is sufficiently large, the infimum wealth among buyers who do not get the good initially is high. These buyers are then able to purchase on the resale market. An important observation is that this benefit from need-based screening does not depend on independence of v and w.

A final point concerns the impact of speculation by non-buyers. Consider a more general (not necessarily separable) allocation technology now. As m rises, the assignment rule itself must change to remain feasible. We note this dependence by writing the allocation probability as $x(w, v; m)$. Let the set of participants be Ω^* and let $\mu(\cdot)$ denote the measure of a set. Then, an assignment rule is called *non-concentrating* if, for all sets of buyers, $\Omega \subset \Omega^*$, we have

$$\int_{\Omega} x(w, v; m)dF(v)dG(w) \leq \left(\frac{N\mu(\Omega)}{\mu(\Omega^*)}\right) S$$

for some fixed $N > 1$. This condition says that any set of types involved in the allocation process gets a quantity that is not too large relative to the measure of the set. This leads

[17] This condition is satisfied if S is sufficiently large. Letting $t(z) := [1 - F(z)] - (1 + m)G(z)$, we just need to find a root of the equation $t(z) = 0$. Since $t(0) = 1$, $t(1) = -(1 + m)$, and $t(\cdot)$ is continuous and strictly decreasing, there is a unique root, z^*. Full efficiency requires $S \geq (1 + m)G(z^*) = [1 - F(z^*)]$.

to an asymptotic version of the Coase Theorem, which says that the initial allocation does not matter much when there is substantial speculation by non-buyers.

PROPOSITION 4. *Given a non-concentrating allocation rule, as m rises without bound the equilibrium resale price converges to the competitive market price.*

PROOF: The current analog to the market-clearing condition in (2) is:

$$RD(r) - RS(r) = D(r) - S + \tilde{K}_x(r) = 0, \tag{3}$$

where

$$\tilde{K}_x(r) := \int_{\overline{p}}^{r} \int_{r}^{1} x(w, v; m) dF(v) dG(w)$$

is the measure of buyers with wealth $w \in [\overline{p}, r]$ who get the good initially and keep it. Since the assignment rule is non-concentrating, we have

$$\int_{\overline{p}}^{r} \int_{r}^{1} x(w, v; m) dF(v) dG(w) \leq \left(\frac{N \int_{\overline{p}}^{r} \int_{r}^{1} dF(v) dG(w)}{[1 + m][1 - G(\overline{p})]} \right) S \leq \frac{NS}{1 + m}.$$

Thus, $\tilde{K}_x(p^e)$ must converge to zero as m rises without bound, so $RD(p^e) - RS(p^e)$ also converges to zero, meaning that r_x converges to p^e. ∎

It follows that prohibiting transferability may be welfare-enhancing when speculation is a major concern. With transferability, the allocation approaches the original competitive market allocation as m rises without bound. Without transferability, if the corresponding assignment rule is sufficiently meritorious, the ultimate outcome will be more efficient than the market outcome (and the outcome of the allocation scheme with transferability).

COROLLARY 6. *Consider a non-concentrating allocation technology that merit-dominates the competitive market allocation when the good is not transferable. Then, prohibiting transferability raises total realized value if $m > M$, for some fixed $M > 0$.*

When the allocation technology allows sufficient merit-based screening, it is preferable to rely on that technology instead of the resale market since speculation blunts the benefits of resale. At the same time, one can often screen out speculators, making this result less relevant. For instance, it is possible to verify whether people seeking an organ transplant actually need the operation, whether people seeking a military deferment are eligible to be drafted, or whether people seeking transferable education vouchers have children who are eligible to attend the school.[18]

5 Discussion

In this section we explore the impact of several natural extensions. We will see that the basic qualitative results continue to hold in a range of circumstances. For ease of exposition, we consider the case of $m = 0$ here so there are no non-buyers.

5.1 A Dual System

We have so far assumed that the entire supply was distributed according to a single allocation scheme. Now suppose that part of the supply is allocated according to one allocation scheme, while the remainder is sold at a higher price on a black market.[19] There were black markets for beef, chicken, gasoline, and tires in the U.S. during World War II. For instance, some farmers sold part of their crop on the market at the price cap, and the remainder at the farm gate at higher prices. (For a general discussion see Rockoff (1998) and references therein.) Underground markets for a multitude of goods thrived for decades in Soviet-style planned economies. More recently, news reports from Baghdad indicated that gasoline was selling for 2,000 dinars on the black market whereas the price cap was only 80 dinars.[20] Finally, despite restrictions on the sale of human organs, there have been sales on black markets.[21]

Dual market systems may also arise by design. Consider school choice. A familiar scheme guarantees admission to students living in the vicinity of a public school and uses a

[18]It may be difficult to discern whether applicants for immigration visas or subsidized housing are speculators, however.

[19]There are other means of circumventing price caps. One is a tied sale in which the purchase of a good subject to a binding price cap is conditioned on the purchase of another good that is not.

[20]See "Iraqis' Dismay Surges as Lights Flicker and Gas Lines Grow," *The Washington Post*, Dec. 24, 2004, A1.

[21]A recent phenomenon is the advent of online services that match live donors and recipients. In jurisdictions where it is legal for recipients to reimburse donors for costs, this opens one avenue for compensation through inflated costs.

lottery to allocate the remaining slots (Abdulkadiroğlu and Sönmez (2003)). Since one can acquire a slot by purchasing a house in the immediate vicinity of the school, there is a high price to get a slot with probability one, and a low (zero) price to get it with probability below one through the lottery.

An analogous situation occurs when a high school senior applies to a college or university under the "early decision" program. All else equal, the probability of being accepted is higher when one applies for an early decision (Avery, Fairbanks, and Zeckhauser (2003)). The price is also higher since financial aid offers may be significantly lower when one applies for an early decision.[22] In other words, students who are accepted under the early decision program tend to pay more, all else equal.

Our model can be altered to accommodate dual systems. Suppose that a fraction $\gamma \in (0,1)$ of the supply is sold in a competitive market, while the remaining $1-\gamma$ is subject to a price cap, $\overline{p} < p^e$, and random allocation. Assume that the good is not transferable.

In equilibrium, buyers with $(w,v) \geq (\overline{p},\overline{p})$ participate in the allocation scheme. There is also a market price, $p^* > p^e$, such that unsuccessful buyers with $(w,v) \geq (p^*,p^*)$ will purchase on the market.[23] Let $L(z) := \{(w,v)|(w,v) \geq (\overline{p},z), \min\{w,v\} < p^*\}$ be the L-shaped set containing types with valuations exceeding z who participate in the allocation scheme but who will not purchase at p^*; and let $\ell(z) := \Pr\{(w,v) \in L(z)\}$ and $\xi(z) := \mathbb{E}[v|(w,v) \in L(z)]$. The total realized value from the dual system is

$$V_D := \sigma\ell(\overline{p})\xi(\overline{p}) + D(p^*)\phi(p^*),$$

where σ is the probability of allocation in the lottery, which satisfies the market-clearing condition, $\sigma\ell(\overline{p}) + D(p^*) = S$.

The efficiency comparison between the dual system and the unregulated market is actually ambiguous.[24] At the same time, incorporating random allocation of a transferable good would enhance the dual system. Suppose that the quantity $\hat{\gamma}S$ is allocated via transferable

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[22]See "The Benefits and Drawbacks of Early Decision and Early Action Plans," at http://www.collegeboard.com/prof/counselors/apply/5.html, accessed on June, 8, 2005.

[23]The organizer may be able to monitor participation in the dual system so that one individual may get access to either the allocation scheme or the market. The result is qualitatively the same.

[24]When $\gamma = 1$, this is the same as a competitive market. Lowering γ slightly raises p^*. Moreover, buyers in $L(\overline{p})$ now get the good with some small probability. These changes may raise or lower the total realized value, depending on the density in this region.

vouchers and the remaining $(1-\hat{\gamma})S$ is allocated via non-transferable vouchers, both at the price of $\overline{p}$. Consistent with the earlier approach, assume that each individual may only enter the pool for a single type of voucher. Those who are unsuccessful can purchase a transferable voucher in the resale market, which is again assumed to be competitive.

We look for an equilibrium with two properties: (1) There exists $\hat{v} \in [0, p^*]$ such that buyers with $v \geq \hat{v}$ enter the pool for non-transferable vouchers, and those with $v < \hat{v}$ enter the pool for transferable ones; and (2) the resale price is p^*. Sufficient conditions for such an equilibrium to exist are derived as follows. First, the critical type, $\hat{v}$, must be indifferent between the two types of vouchers:

$$\frac{(1-\hat{\gamma})S}{[1-F(\hat{v})][1-G(\overline{p})]}[\hat{v}-\overline{p}] = \frac{\hat{\gamma}S}{F(\hat{v})[1-G(\overline{p})]}[p^*-\overline{p}]. \tag{4}$$

Given (4), any buyer with $v > \hat{v}$ [resp., $v < \hat{v}$] will strictly prefer the non-transferable [resp., transferable] vouchers. Next, the resale market will clear at p^* if

$$\left(1 - \frac{(1-\hat{\gamma})S}{[1-F(\hat{v})][1-G(\overline{p})]}\right) D(p^*) = \hat{\gamma}S. \tag{5}$$

If there exists a pair, $(\hat{v}, \hat{\gamma})$, satisfying (4) and (5), then such an equilibrium exists. The aggregate value realized in that equilibrium is

$$\hat{V}_D = \hat{\sigma}\ell(\hat{v})\xi(\hat{v}) + D(p^*)\phi(p^*),$$

where $\hat{\sigma}\ell(\hat{v}) + D(p^*) = S$. Since $\hat{v} > \overline{p}$, we conclude that $\hat{V}_D > V_D$.

PROPOSITION 5. *For any dual system, there exists an allocation scheme with vouchers (some transferable) that is more efficient.*

PROOF: The proof is in the Appendix.

5.2 Regulation of Resale

When a good is transferable, speculation by low-valuation buyers reduces the probability that any given buyer gets the good initially, which ultimately lowers efficiency. One possible response is to tax resales. Suppose that the government caps the price of the good at $\overline{p} < p^e$, assigns it randomly, and imposes a lump-sum tax of $\tau \geq 0$ on resellers. All buyers with

$w \geq \overline{p}$ will seek to acquire the good if the resale price, $\tilde{r}(\tau)$, satisfies $\tilde{r}(\tau) \geq \overline{p} + \tau$, which we will assume. This condition means that it is profitable to engage in resale.[25] The equilibrium resale price increases with the tax, and the net proceeds from resale fall, in this region.

To see the effect of the higher tax, it is useful to distinguish the two groups that get the good ultimately. First, some buyers with $(w, v) \geq (\overline{p}, \tilde{r}(\tau) - \tau)$ get the good initially and find it optimal not to resell. Second, some buyers with $(w, v) \geq (\tilde{r}(\tau), \tilde{r}(\tau))$ purchase on the resale market. When the tax rate rises from τ to τ', the former group expands and the latter group contracts. In particular, some buyers with $v \geq \tilde{r}(\tau)$ now get the good with a probability below one. Conversely, buyers with $w \geq \overline{p}$ and $\tilde{r}(\tau') - \tau' \leq v \leq \tilde{r}(\tau) - \tau$ now get the good with strictly positive probability, rather than zero. Since the probability of receipt is unchanged in the other regions, the total realized value is lower with the higher tax, τ'. It follows that a tax on resales is counterproductive.[26]

5.3 Pre-payment Resale

Since resale has been shown to be beneficial, we now ask whether one should encourage even more resale. This can be done by allowing a buyer to resell the good *before* taking delivery. We consider such cases by allowing a buyer who acquires the good to resell it after paying a deposit $\delta \leq \overline{p}$.[27] This allows the buyer to engage in speculation even if his wealth is insufficient to pay the price.

Assume that the good is randomly assigned again. Fixing $\overline{p}$, speculation declines as the required deposit rises, so more high-valuation buyers with lower wealth get the good initially. The resale price must therefore rise, so a higher deposit means improved allocative efficiency. Setting the deposit equal to the price cap eliminates speculation by buyers with $w < \overline{p}$. These buyers would not be able to keep the good anyway, so there is no efficiency loss from excluding them. We conclude that a deposit equal to the cap is optimal, meaning

[25] If the inequality does not hold, speculation is unprofitable. Only those with $(w, v) \geq (\overline{p}, \overline{p})$ will participate in the initial allocation, and the resale market will be inactive.

[26] One can also interpret τ as a measure of resale transaction costs, unrelated to government regulation. Then, τ is a social cost as well.

[27] Requiring a deposit $\delta > \overline{p}$ has essentially the same effect as raising the price cap to δ, so there is no loss of generality in assuming that $\delta \leq \overline{p}$.

that there is no benefit from allowing pre-payment resale.

5.4 Direct Subsidy vs. Random Allocation

Another option for improving the allocation is to alleviate wealth constraints directly through a subsidy, and then let the market operate. Direct subsidies have two shortcomings, however. First, the financing of the subsidy may itself be distortionary. Second, if buyers' valuations and wealth are not readily observable, it is difficult to target the subsidy to the buyers who are wealth constrained. Random allocation of a transferable good subsidizes the poor while being budget-balanced as it does not entail any outlay from the government. The scheme is also targeted in the sense that not all of the wealthy are subsidized. (In fact, the wealthy may be worse-off, on average, since there is probability $(1 - S)$ that they pay more with the price cap and resale than without the cap.) The only unintended beneficiaries of random allocation of a transferable good are the speculators, who have low valuations. Enriching this group appears to be necessary to subsidize buyers in a budget-balanced fashion.

We now provide a comparison of two schemes. Consider a direct subsidy that is financed in a budget-balanced way, i.e., through the sale of the good itself. We will show that such a scheme produces a less efficient outcome than the random allocation scheme if $G(\cdot)$ is (weakly) concave. (This condition means, roughly speaking, that the poor are relatively more numerous than the wealthy.) We suppose that neither v nor w is observable so that only a uniform subsidy is feasible.

Let the government offer each consumer a lump-sum subsidy, $\sigma \geq 0$, and charge a price, p, such that the budget balances and the market clears. Budget-balance means that $(p-c)S = \sigma$. Given p and σ, a buyer with (w, v) will demand the good if and only if $v \geq p$ and $w + \sigma \geq p$, so aggregate demand is $(1 - F(p))(1 - G(p - \sigma))$. Using the budget-balance condition, market-clearing requires

$$ED^s(p) := (1 - F(p))(1 - G(p - (p - c)S)) - S = 0.$$

It is straightforward to check that a unique equilibrium price, p^*, exists. As in Section 3, the value realized is $S\phi(p^s)$.

PROPOSITION 6. *Suppose that $G(\cdot)$ is (weakly) concave. Then, setting a price cap $\overline{p} = c$ and randomly allocating a transferable good yields higher total realized value than the budget-balanced uniform subsidy.*

PROOF: It suffices to show that the equilibrium resale price under the random allocation scheme, $r(c)$, exceeds the market-clearing price under the subsidy regime. Let $ED^r(p) := RD(p) - RS(p)$ denote the excess demand under the former regime, where $RD(p) - RS(p)$ follows from (**??**) with $\overline{p} = c$. The difference between the excess demands is:

$$\Delta(p) := ED^r(p) - ED^s(p) = \left(\frac{1 - F(p)}{1 - G(c)}\right) \delta(p, S),$$

where

$$\delta(p, S) := (G(p - (p - c)S) - G(c))(1 - G(c)) - (G(p) - G(c))(1 - G(c) - S).$$

To prove $r(c) > p^s$, it suffices to show that $\delta(p, S) > 0$ for all p and $S \in (0, 1 - G(c))$. Note first that $\delta(p, 0) = 0$. Next, we show that $\partial\delta(p, S)/\partial S > 0$ whenever $\delta(p, S) = 0$:

$$\begin{aligned}
& \frac{\partial\delta(p,S)}{\partial S}\Big|_{\delta(p,S)=0} \\
= \ & \{-g(p - (p - c)S)(p - c)(1 - G(c)) + G(p) - G(c)\}_{\delta(p,S)=0} \\
= \ & \left(\frac{1 - G(c)}{1 - G(c) - S}\right) [G(p - (p - c)S) - G(c) - g(p - (p - c)S)(p - c)(1 - G(c) - S)] \\
> \ & \left(\frac{1 - G(c)}{1 - G(c) - S}\right) [G(p - (p - c)S) - G(c) - g(p - (p - c)S)(p - c)(1 - S)] \\
\geq \ & 0.
\end{aligned}$$

The last inequality holds by concavity of G: Letting $z := p - (p - c)S$, the terms in square brackets equal $G(z) - G(c) - (z - c)g(z) \leq 0$.

Since $\delta(p, 0) = 0$, $\partial\delta(p, S)/\partial S > 0$ whenever $\delta(p, S) = 0$, and $\delta(p, \cdot)$ is continuous, we conclude that $\delta(p, S) > 0$ for all $p \in (c, 1)$ and $S \in (0, 1 - G(c))$. The result then follows. ▌

This analysis presumed that the government sold the good and used the revenue to finance the subsidy. If the sellers are private firms, a direct subsidy may require financing through a distortionary tax, another source of inefficiency. In addition, any attempt to target the subsidy based on wealth may cause individuals to conceal their wealth. Targeting a subsidy based on individuals' choices is not foolproof either. Suppose that the government offers a subsidy only to those who purchase the good. This will raise each buyer's willingness-to-pay by exactly the amount of the subsidy, so wealth constraints are not alleviated at all. The same point would hold with uniform "bidding credits."

5.5 Social Cost of Speculation

Speculation produces an indirect welfare cost here by reducing the probability of allocation for those with low wealth but a high valuation. Speculation may also entail a direct welfare cost if it involves real transaction costs or opportunity costs. Indeed, the price caps on new housing in Korea have been criticized for encouraging speculation and diverting resources away from other productive investment activities. This cost need not overturn the desirability of the aforementioned schemes for allocating a transferable good, however.

Suppose that there exists a project requiring a capital investment of $k \in [0, 1)$ that yields a net return of $R \in (0, 1)$. An individual who participates in the allocation process must forego that project. (Assume that $k + c > 1$ so no individual can both purchase the good and pursue the project.) Clearly, it is socially desirable for individuals with $w \geq k$ and $v < R + c$ to invest in the project. This will occur in the competitive equilibrium since $p^e \geq c$, which implies $v - p^e < R$ if $v < R + c$.

Now suppose that the price is capped at $\overline{p} < p^e$ such that

$$\rho(\overline{p})[r(\overline{p}) - \overline{p}] \geq R,$$

where

$$\rho(\overline{p}) := \frac{S(\overline{p})}{1 - G(\overline{p})}$$

is the allocation probability for all participants. Speculation then offers a higher expected profit than the project return, so no agent will invest in the project. In this case, our scheme for allocating a transferable good has a direct welfare cost — the unrealized project return for agents with $w \geq k$ and $v < R + c$. This cost need not overturn the desirability of the scheme, although moderation in capping the price may be warranted. The improvement in efficiency that the cap brings may still outweigh the cost. But if not, there exists $\overline{p}$ sufficiently close to p^e such that $r(\overline{p}) - \overline{p} < R$. It follows that such a cap would not create the kind of speculation that entails inefficient project choices.[28]

[28]Essentially the same conclusion would arise with a continuum of projects with returns ranging from 0

5.6 Elastic Supply

Our analysis has assumed that supply is perfectly inelastic, which may be a reasonable assumption for health care, school choice, and even housing. Supply may be responsive to a price cap in situations where capacity is not an issue, however. Moreover, even if capacity is limited in the short run, the ability to invest means that supply may be elastic in the long run.[29] It is therefore important to establish the robustness of our results to the presence of elastic supply.

Suppose that the good is supplied competitively according to a twice-differentiable, increasing, aggregate cost function, $c(\cdot)$. The supply at price p is then given by $S(p) \in \arg\max_{q \geq 0} pq - c(q)$, or implicitly by $c'(S(p)) = p$; $S(\cdot)$ is increasing and differentiable. The competitive equilibrium is characterized by a price, p^e, satisfying $D(p^e) = S(p^e)$.

When the price is capped at $\overline{p} < p^e$, supply is $S(\overline{p}) < S(p^e)$. Since $D(\overline{p}) > D(p^e)$, there is excess demand, which will be resolved by allocating the good randomly. Assume that the good is transferable, so all individuals with $(w, v) \geq (\overline{p}, 0)$ will participate. A resale equilibrium exists and is characterized by the price $r(\overline{p})$ satisfying

$$D(r(\overline{p})) = S(\overline{p}) - \rho(\overline{p})[1 - F(r(\overline{p}))][G(r(\overline{p})) - G(\overline{p})]. \tag{6}$$

The total realized value is now

$$\hat{V}(\overline{p}) = S(\overline{p})\phi(r(\overline{p})) - c(S(\overline{p})). \tag{7}$$

Capping the price and randomly allocating the good has a tradeoff. On the positive side, it again reallocates the good from some low-valuation-high-wealth buyers to some with high valuations and low wealth. On the negative side, supply falls, so some current buyers lose access to the good. Moreover, this latter effect is nonnegligible, even if the cap is just below the market price, since the buyers losing access may have high valuations.

to $\overline{R} > 0$. In this case, random allocation of a transferable good will crowd out some low-return projects. Yet, lowering the cap slightly below p^e will entail only a negligible welfare cost, since the projects that are foregone in pursuit of speculation would have generated only small social returns, whereas the gain from improved efficiency would be nonnegligible.

[29]For example, new apartments can be built in the long run. Conversely, apartments can be converted to offices while rent control is in place.

(Some current buyers are wealth constrained, with wealths just above p^e but valuations well above p^e.) The net effect is positive if supply is not too elastic.

PROPOSITION 7. *If* $\frac{S'(p^e)}{S(p^e)} < \frac{f(p^e)}{F(p^e)}$, *there exists a price cap,* $\overline{p} < p^e$, *such that random allocation followed by transferability yields higher total realized value than the competitive market does.*

PROOF: The proof is in the Appendix.

The applicability of Proposition 7 is clear in the two polar cases. If supply is almost perfectly inelastic, the proposition applies. The previous results continue to hold and random allocation outperforms the competitive market. Conversely, if supply is almost perfectly elastic, the proposition does not apply. The drop in quantity is so great that this effect swamps the improved mix of buyers.

It is also worth noting that if the speculation problem could be addressed directly, the random allocation scheme would be desirable regardless of the supply elasticity. That is, if $x(w, v) = 0$ for all $v < \overline{p}$ (instead of random allocation), the result of Proposition 7 always holds. In addition, the same result would hold if the buyers' price could be lowered without altering the price that the sellers command, although such a policy would violate the budget-balancing property of the allocation scheme.

6 Related Literature

The literature offers additional rationales for allocation schemes that employ instruments besides price. Wijkander (1988) showed that capping price and allocating the good randomly may favor certain income groups. (Direct redistribution of endowments was assumed infeasible in that paper.) Given a social welfare function that puts different weights on different consumers' utility, capping price may raise welfare.

Weitzman (1977) justified a pro rata allocation scheme on similar redistributive grounds. He considered the allocation of divisible goods. In his model, consumers differed in their marginal utility of wealth and in a preference parameter. Weitzman asked what the allocation would be if all consumers had the mean income. This allocation was taken as the benchmark. He then showed that allocating the good evenly may produce an allocation closer to the benchmark than the market does.[30]

Guesnerie and Roberts (1984) offered an efficiency-based justification for quantity controls. They considered a market in which the equilibrium allocation was not Pareto optimal because of distortionary excise taxes, for example. In that context, imposing quantity controls (maximum or minimum purchases) may improve efficiency.[31] In particular, they show that placing an upper bound on consumption of a particular good may produce a Pareto improvement.

7 Concluding Remarks

This paper has asked how property rights should be assigned in the presence of buyers' limited wealth. We have shown that a range of allocation schemes may perform better than the unregulated market when wealth constraints are important. Schemes that place goods in the hands of high-valuation buyers obviously work well. It is striking, however, that placing goods in the hands of low-wealth buyers is also beneficial when the good is transferable. Even more striking is the possibility that full efficiency may be realized by allocating the entire stock to the poor.

The results here suggest a paradigm shift in how particular goods are assigned. Certain goods tend always to be assigned by a pure market mechanism whereas others are assigned using instruments other than price. The current paper suggests a blurring of this line when wealth constraints are important; specifically, one should consider removing prohibitions on transferability in markets with price caps, and one should consider non-price allocation schemes in certain unregulated markets.

There are numerous possible applications of the results. For instance, they suggest that using a lottery to assign transferable educational vouchers may be preferable to a system of local attendance zones or to the use of a lottery to assign non-transferable vouchers. The analysis also suggests that keeping the price of health care below competitive market levels and allowing patients to sell their spot in the queue could improve efficiency. Finally, the

[30]Closeness to the benchmark was determined using a mean-square-error criterion. This criterion does not give an exact measure of the welfare cost of misallocation, however.

[31]The authors also consider in-kind redistribution and taxation.

analysis shows that it may be beneficial to employ a draft with tradable deferments for military recruitment. In each of these cases there may be other objectives or institutional details that loom large, but the results here argue for consideration of different allocation schemes and transferability.

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Appendix: Proofs

PROOF OF PROPOSITION 3: Let $\psi_x(v) := \frac{\int_v^1 \tilde{v} dB_x(\tilde{v})}{1-B_x(v)}$ denote the expected value conditional on exceeding $v > 0$, given the ultimate distribution. The total realized value can be expressed as:

$$\begin{aligned}
\hat{V}_x &= \int_{\overline{p}}^{r_x}\int_{r_x}^{1} vx(w,v)dF(v)dG(w) + \int_{r_x}^{1}\int_{r_x}^{1} vdF(v)dG(w) \\
&= S\cdot\int_{\overline{p}}^{r_x}\int_{r_x}^{1} va_x(w)b_x(v)dF(v)dG(w) + (1-G(r_x))\int_{r_x}^{1} vdF(v) \\
&= S\cdot A_x(r_x)[1-B_x(r_x)]\left(\frac{\int_{r_x}^{1} vdB_x(v)}{1-B_x(r_x)}\right) + D(r_x)\left(\frac{\int_{r_x}^{1} vdF(v)}{1-F(r_x)}\right) \\
&= K_x(r_x)\psi_x(r_x) + D(r_x)\phi(r_x) \\
&= S\left[\left(1-\frac{D(r_x)}{S}\right)\psi_x(r_x) + \left(\frac{D(r_x)}{S}\right)\phi(r_x)\right];
\end{aligned}$$

where the first equality follows by definition; the second and third follow by substituting for $x(w,v)$ and integrating; and the last one follows from (2). If x is meritorious, then $\psi_x(r_x) \geq \phi(r_x)$.

Suppose, further, that x merit-dominates $y \in \mathcal{X}$ and $r_x \geq [>] r_y$. Then,

$$\begin{aligned}
\hat{V}_x &= S\left[\left(1-\frac{D(r_x)}{S}\right)\psi_x(r_x) + \left(\frac{D(r_x)}{S}\right)\phi(r_x)\right] \\
&\geq S\left[\left(1-\frac{D(r_y)}{S}\right)\psi_x(r_x) + \left(\frac{D(r_y)}{S}\right)\phi(r_x)\right] \\
&\geq S\left[\left(1-\frac{D(r_y)}{S}\right)\psi_y(r_x) + \left(\frac{D(r_y)}{S}\right)\phi(r_x)\right] \\
&\geq [>]\ S\left[\left(1-\frac{D(r_y)}{S}\right)\psi_y(r_y) + \left(\frac{D(r_y)}{S}\right)\phi(r_y)\right] \\
&= \hat{V}_y;
\end{aligned}$$

where the first inequality follows from $r_x \geq r_y$ (which implies $D(r_x) \leq D(r_y)$) and from $\psi_x(r_x) \geq \phi(r_x)$; the second follows from the relative merit-dominance of x over y; and the third one follows from the fact that the conditional expectations, $\psi_y(\cdot)$ and $\phi(\cdot)$, are strictly increasing in the relevant region. ∎

Proof of Proposition 5: It suffices to show that there exists a pair $(\hat{v},\hat{\gamma})$ satisfying the system of nonlinear equations (4) and (5). Observe first that for each $\hat{v} \in [\overline{p}, p^*]$, there exists $\hat{\gamma} = t_1(\hat{v})$ satisfying (4). Further, $t_1(\cdot)$ is increasing and satisfies $t_1(\overline{p}) = 0$. Next, let $\hat{p} > p^e$ be such that $[1-F(\hat{p})][1-G(\overline{p})] = S$. Then, for each $\hat{v} \in [\overline{p}, \hat{p}]$, there exists $\hat{\gamma} = t_2(\hat{v})$ satisfying (5). Further, $t_2(\cdot)$ is decreasing and satisfies $t_2(\overline{p}) > 0$ and

$t_2(\hat{p}) = 0$. Since both $t_1(\hat{v})$ and $t_2(\hat{v})$ are continuous, there exists a unique $\hat{v}^* \in (\overline{p}, \hat{p})$ such that $t_1(\hat{v}^*) = t_2(\hat{v}^*) =: \hat{\gamma}^*$. Obviously, $(\hat{v}^*, \hat{\gamma}^*)$ satisfies (4) and (5). Since $\hat{v}^* > \overline{p}$ and $\xi(\cdot)$ is increasing, we have $\hat{V}_D > V_D$. ∎

Proof of Proposition 7: It suffices to show that, given the condition, $\hat{V}'(p^e) < 0$, which would mean that lowering the cap increases the total realized value. To that end, for $\overline{p} \leq p^e$, rewrite

$$\hat{V}(\overline{p}) = \{\rho(\overline{p})[G(r(\overline{p})) - G(\overline{p})] + [1 - G(r(\overline{p}))]\} \int_{r(\overline{p})}^{1} v dF(v) - c(S(\overline{p})).$$

Since $r(p^e) = p^e$, we have

$$\begin{aligned} \hat{V}'(p^e) &= -(1 - G(p^e))f(p^e)r'(p^e)p^e - (1 - \rho(p^e))g(p^e)r'(p^e) \int_{p^e}^{1} v dF(v) \\ &\quad -\rho(p^e)g(p^e) \int_{p^e}^{1} v dF(v) - c'(S(p^e))S'(p^e) \\ &= -(1 - G(p^e))f(p^e)r'(p^e)p^e - F(p^e)[1 - F(p^e)]g(p^e)r'(p^e)\phi(p^e) \\ &\quad -[1 - F(p^e)]^2 g(p^e)\phi(p^e) - S'(p^e)p^e, \end{aligned} \tag{8}$$

where the second equality holds since $\rho(p^e) = S(p^e)/[1 - G(p^e)] = D(p^e)/[1 - G(p^e)] = 1 - F(p^e)$, $\phi(z) = \int_z^1 v dF(v)/[1 - F(z)]$, and $c'(S(p^e)) = p^e$.

Meanwhile, totally differentiating both sides of (6) and using $\rho(p^e) = 1 - F(p^e)$ yields

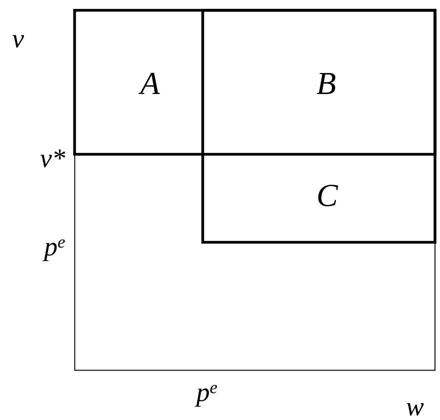
$$r'(p^e) = -\frac{S'(p^e) + g(p^e)[1 - F(p^e))]^2}{g(p^e)(1 - F(p^e))F(p^e) + f(p^e)(1 - G(p^e))}.$$

Substituting this into (8) and collecting terms, we get

$$\begin{aligned} \hat{V}'(p^e) &= -\frac{[\phi(p^e) - p^e](1 - F(p^e))g(p^e)[D(p^e)f(p^e) - S'(p^e)F(p^e)]}{g(p^e)(1 - F(p^e))F(p^e) + f(p^e)(1 - G(p^e))} \\ &= -\frac{[\phi(p^e) - p^e](1 - F(p^e))g(p^e)[S(p^e)f(p^e) - S'(p^e)F(p^e)]}{g(p^e)(1 - F(p^e))F(p^e) + f(p^e)(1 - G(p^e))}. \end{aligned}$$

Hence, $\hat{V}'(p^e) < 0$ if and only if $\frac{S'(p^e)}{S(p^e)} < \frac{f(p^e)}{F(p^e)}$. ∎

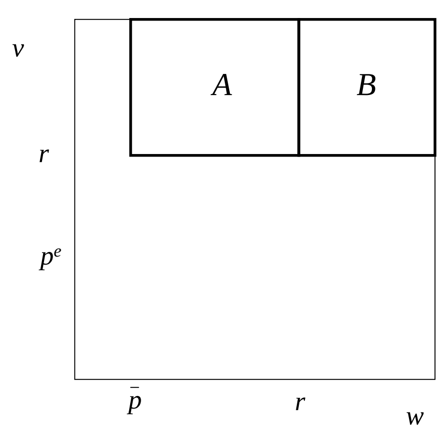
Figure 1: Benchmark allocations



B+*C*: Market

A+*B*: Efficient allocation

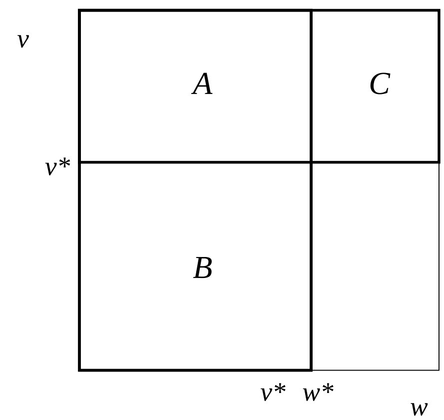
Figure 2: Rationing with resale



A: Probability = $\rho < 1$

B: Probability = 1

Figure 3: Full efficiency of need-based screening



A+*B*: Need-based allocation
A+*C*: Efficient allocation