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"Personal Influence":
Social Context and Political Competition

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Abstract

This paper studies the effect of social learning on political outcomes in a model of informative campaign advertising. We find that communication networks among voters have important effects on parties' incentives to disclose political information, on voters' learning about candidates running for office, and on the polarization of the electoral outcome. In particular, in richer communication networks parties disclose less political information and voters are more likely to possess erroneous beliefs about the characteristics of the candidates running for office. In turn, a richer communication network among voters may lead to political polarization. These results are reinforced when interpersonal communication occurs more frequently among ideologically homogeneous individuals and parties can target political advertising.

JEL classification: D72, D83, Z13.

Keywords: Social Learning, Homophily, Polarization, Campaign Advertising.

"The mass do not now take their opinions from dignitaries in Church or State, from ostensible leaders, or from books. Their thinking is done for them by men much like themselves, addressing them or speaking in their name, on the spur of the moment." J. S. Mill (1869). On Liberty.

"Rational citizens will seek to obtain their free political information from other persons if they can. This expectations seems to be borne out by the existing evidence." A. Downs (1957). An Economic Theory of Democracy.

1   Introduction

In modern societies a large majority of individuals rely on others in order to obtain most of their political information. Empirical evidence of the importance of political information sharing in affecting individuals' voting behavior dates back to the early 1950s when, through a series of pioneering field experiments, Columbia sociologist Paul Lazarsfeld and coauthors documented the primacy of face-to-face interaction in spreading political information and showed that this information was more likely to reach undecided voters.[1]

In spite of the predominant role of the mass media in political advertising, recent empirical works show that word-of-mouth communication is still a fundamental input of the learning process of voters. For example, in an empirical study of the 1992 American presidential election campaign, Beck et al. [2] conclude that interpersonal discussions outweigh the media in affecting voting behavior. In a recent study on political disagreement within communication networks, Huckfeldt et al. [17] observe that: "Democratic electorates are composed of individually interdependent, politically interconnected decision makers. [...] they depend on one another for political information and guidance."[2]   Moreover, there

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[1]See, e.g., Lazarsfeld et al. [20], Berelson et al. [3], and Katz and Lazarsfeld [19]. The work of the Columbia sociologists is the "existing evidence" to which Downs refers in the quotation. See Downs [9] pages 222 and 229.

[2]Apparently, not much has changed since W. Lippmann's [22] treatise on public opinion where he states: "Inevitably our opinions cover a bigger space, a longer reach of time, a greater number of things, than we can directly observe. They have, therefore, to be pieced together out of what others have reported and what we can imagine". The recent use of internet in political campaign and,

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is evidence that interpersonal communication occurs more frequently among ideologically similar individuals, as documented by McPherson et al. [26].

In light of this evidence, understanding the relationship between interpersonal communication, individuals' voting behavior and political outcomes is of considerable interest. However, very little theoretical work has been done on this topic. This paper proposes a framework in which interpersonal communication between voters is embedded in a standard model of strategic electoral competition. We show that social learning has important effects on political outcomes. In particular, the structure of communication among voters is important in determining to what extent voters obtain information about candidates running for office and on the polarization of the electoral outcome.

We explore the implications of social learning within a citizen-candidate model where the policy space is unidimensional. There are three groups of citizens: leftists and rightists (the "partisans"), and independent voters. Citizens have distance preferences over policy, independents are decisive in the election, and the identity of the median independent voter is ex-ante uncertain. There are two policy-motivated parties, representative of the left and right partisans, and their objective is to maximize the expected utility of their median member.

In the political game, parties select candidates running for office and the level of informative campaign advertising. Advertising is costly, and the information disclosed by a party perfectly reveals its candidate's policy position to a fraction of voters.[3] Voters do not observe these decisions, so that *ex-ante* they do not know the ideological position of the two candidates. However, voters may learn this information by directly receiving informative advertising from parties (*direct exposure*) as well as by talking about politics with other voters (*contextual exposure*). Based on the information a voter receives, he updates his beliefs about the position of the candidates and casts his vote. The candidate that wins a

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in particular, of on-line social networking sites (e.g., MySpace, Flickr, YouTube, Facebook), suggests that interpersonal communication is becoming a crucial aspect of political advertising. See, for example, http://politicalinsider.com/2007/05/the_facebook_campaign.html.

[3]For evidence about the importance of political advertising in providing voters with information see, e.g., Lodge et al. [23], and Coleman and Manna [7]. See also Zaller [37] for evidence on the effect of media content on policy preferences.

simple majority of votes is elected and implements his most preferred policy.

The novelty of our framework rests in the introduction of social learning in a political game, which is reflected in the fact that voters may learn through contextual exposure. Clearly, different assumptions on the structure of communication among voters may have different effects on the final political outcome. Our aim is to describe the communication structure in a simple and parsimonious way. Yet, we intend to have a rich enough model which is able to incorporate different empirically relevant dimensions of political communication networks.

In the first part of the paper we consider a model where the communication network does not entail any form of correlation between voters' communication links–*who interact with whom*– and voters' ideologies. In particular, we assume that each independent voter randomly samples a finite number of other independents, who truthfully report the information they have obtained, if any, from parties' advertising.[4] Furthermore, we assume that parties advertise randomly. We believe that abstracting away from possible correlations between the communication network and the distribution of political ideologies, and from the possibility that parties strategically target their advertising based on such correlations represents a useful benchmark model. In fact, within this model, the structure of communication between voters can be captured by a single parameter, which is the level of contextual exposure (i.e., voters' sample size), allowing us to derive a simple yet powerful comparative statics result. Indeed, by comparing the political outcome for different levels of contextual exposure we can study how the richness of communication networks affects the extent to which voters obtain information about candidates running for office as well as the the likelihood that moderate policies are implemented.

In the benchmark model, our first result shows that when informative advertising is sufficiently expensive parties always select extremist candidates and do not disclose any

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[4]Our model of acquisition of political information formalizes the idea of "two-step flow communication", which played a central role in the analysis of the Columbia sociologists. They describe the two-step flow communication as a relay function of interpersonal relations, where political information flows directly from mass media to a subset of voters, the "opinion leaders" (which corresponds to direct exposure), and from them to other voters they are in contact with (which corresponds to contextual exposure). See Lazarsfeld et al. [20].

political information. Otherwise, parties select a moderate candidate with positive probability and disclose political information only when the candidate is a moderate. Focusing on the interesting case where parties select moderate candidates with positive probability, our second result shows that an increase in the level of contextual exposure decreases the political information that parties choose strategically to disclose. This equilibrium effect has striking implications on social learning: when the network of communication between voters becomes richer, it is more likely that a voter holds erroneous beliefs about the ideological position of the candidates running for office. In other words, it is more likely that a voter believes that a candidate is a moderate (extremist) when in fact he is an extremist (moderate). An immediate consequence of this is that in the presence of a richer communication structure it is more likely that an extremist candidate defeats a moderate candidate. Finally, we show that when the cost of advertising is sufficiently low, an increase in the level of interpersonal communication between voters also increases the probability that parties select extremist candidates. Overall, in the presence of richer communication networks the (ex-ante) expected probability that an extreme policy is implemented increases.

The second part of the paper extends our benchmark model to incorporate for the empirically relevant case in which *i)* parties can target political advertising to ideologically similar voters, and *ii)* interpersonal communication occurs more frequently among ideologically similar individuals.[5] The latter assumption is a very simple form of the so-called "value homophily," according to the original formulation of Lazarsfeld and Merton [21].[6] In

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[5]The fact that parties target their political advertising to specific subset of the population is well-known. As Ken Mehlman, Chairman of the Republican National Committee, recently argued in a radio interview: "[. . .] does this person have children, owns a gun [. . .] that can be predictive of vote behavior and issues that voters care about [. . .] it helps you figure out the best way to reach out to them. [. . .] one of the reasons we in Ohio doubled our support among African Americans in 2004 is because we were able to reach out in places where before you couldn't [. . .] before you would mail 10 people and get one [. . .] it was unaffordable [. . .] it becomes affordable if you can find out one family that's willing to consider voting Republican." See also Green and Gerber [13] for experimental results on the effectiveness of targeting.

[6]The word "homophily" literally means "love of the same". The presence of homophily in social relations is a robust observation which applies very broadly. See, e.g., McPherson et al. [26] for a survey of research on homophily, and Myatt [24] for evidence on the effect of homophily on voting decisions. See also Currarini et al. [8] for a simple model in which homophily emerges as an equilibrium outcome.

our framework, absence of homophily corresponds to a situation in which the frequency of interpersonal communication between voters does not depend on their ideological similarity. In contrast, pure homophily subsumes a society in which communication links are only active within ideologically-homogeneous groups. In this sense, the level of homophily can also be interpreted as the level of segregation between different ideologically-homogeneous groups.

We show that in the extended model there exists an equilibrium in which parties choose to target only their ideologically closer subset of independent voters, both parties select a moderate candidate with positive probability, and they disclose political information only when the candidate is a moderate. An increase in the level of homophily harms social learning (i.e, voters are more likely to hold erroneous beliefs about the candidates' positions) and parties are more likely to select extremist candidates. Overall, as the structure of communication among voters entails higher level of correlation between communication links and voters' ideologies the expected probability that an extremist candidate wins the election increases.

This paper builds on two different strands of theoretical literature. The first strand focuses on the effects of political advertising on electoral competition and voters' welfare, e.g., Coate [5], [6] and Prat [30], [31]. The second strand studies interpersonal communication and learning.[7] We model electoral competition and direct exposure to political information following Coate [6], while the model of interpersonal communication follows the approach of Ellison and Fudenberg [10], [11], and Galeotti and Goyal [12]. To the best of our knowledge, the present paper is the first to embed informal communication among voters in a political economy framework. Our results also relate to the existing empirical literature on polarization in US politics, e.g., Poole and Rosenthal [29] and McCarthy et al. [25]. This literature documents an increase in polarization of the Democratic and Republican parties in the last thirty years, an increase that was not accompanied by a corresponding polarization in the preference of the electorate. Our analysis shows that changes in social context – an increase in the level of interpersonal communication and in the frequency of communication between ideologically similar voters – may be important to understand

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[7]For a survey of the existing literature on interpersonal communication and local learning see Goyal [15].

these empirical findings.

The paper is organized as follows. Section 2 presents the model. Section 3 studies the effect of the level of interpersonal communication on political outcomes. Section 4 extends the model to the case in which interpersonal communication is more frequent between ideologically similar individuals and parties can target political advertising. In Section 5 we conclude and suggest some avenues for future research.

2 Model

Voters and Parties: Ideologies. There is a continuum of citizens of unit measure. The policy space is unidimensional, and citizens are exogenously divided into three groups: left partisans, right partisans, and independents. Partisans represent an equal fraction of the population, and their ideology is symmetrically distributed in $[0, m]$ and $[1 - m, 1]$, respectively. The ideology of independents is uniformly distributed in the interval $[\mu - \tau, \mu + \tau]$, where $\tau > 0$, and μ is drawn from a uniform distribution with support $[1/2 - m, 1/2 + m]$. Hence, the identity of the median independent is ex-ante uncertain. We assume that $m < 1/4 - \tau/2$ so that ideologies of independents are always between those of partisans.

There are two policy-motivated political parties: party L and party R. Party $L(R)$ consists of a representative subgroup of the left (right) partisans. A representative of each party is selected to be a candidate in an election. For simplicity, we restrict the candidates' type space to be $T = \{e, m\}$, where $e \equiv m/2$. Let $\mathbf{t} = (t_L, t_R) \in T \times T$ be a profile of types, where $t_L \in \{e, m\}$ denotes the ideology of party L's candidate, and $1 - t_R$ denotes the ideology of party R's candidate. Henceforth, a candidate is an extremist if his type is $t = e$, otherwise a candidate is a moderate. Figure 1 illustrates the ideologies of voters and parties.

Voters and Parties: Preferences. Citizens have distance preferences over ideology and, in particular, a citizen with ideology i derives utility $-|t - i|$ if a candidate of ideology t wins the election. The objective of each party is to maximize the expected utility of its median member.[8] Partisans always vote for their party, while each independent votes as if

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[8]Note that, when m is small, the assumption that $t \in \{e, m\}$ is without loss of generality. Indeed, a
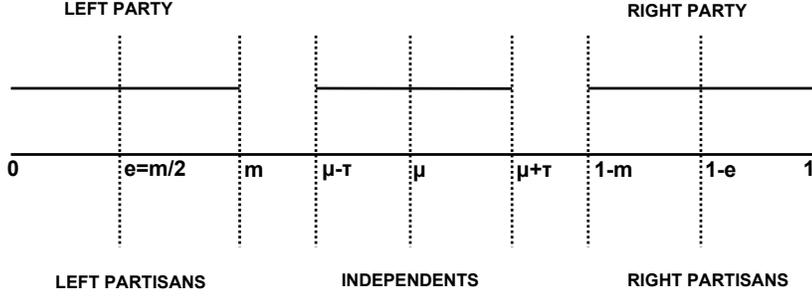
Figure 1: Voters' Ideologies and Parties' Ideologies.

he is pivotal.[9]

Sources of Learning. A crucial element of our model is that independents are *ex-ante* ignorant about candidates' types and they may learn this information from two sources: parties' informative advertising (direct exposure) and interaction with other voters (contextual exposure). Our model of social learning follows Ellison and Fudenberg [10], [11] and it captures the basic idea that the amount of learning does not only depend on the level of contextual exposure, but it also depends on the proportion of informed voters in the population, which is determined by parties' informative advertising. This natural interplay between contextual exposure and direct exposure is both analitically simple and rich enough, and it is the most important feature of our framework. We now specify the details of these two technologies.

Direct Exposure. Each party $j = \{L, R\}$, after having selected its candidate, chooses a level $x_j \in [0, 1]$ of campaign advertising. Advertising is truthful and fully informative. In particular, if a party chooses x_j, then a random fraction x_j of independents perfectly learn

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party maximizing the expected utility of its median member will never select a candidate that is more extreme than its median member e. Moreover, as the uncertainty about the median voter is sufficiently small, i.e., m is sufficiently small, it is possible to show that a party will never select a candidate with ideology lying in the interior of the interval $[e, m]$.

[9]For evidence about the fact that partisans tend to be little affected by campaigns, see, e.g., Zaller [36], and Huckfeldt et al. [17].

party j candidate's position. The cost of informing x_j voters is $C(x_j) = \alpha x_j$, where α is a positive constant measuring the efficiency of the advertising technology. Section 3.2 discusses the robustness of our results with respect to different specification of the cost function.

Contextual Exposure. In addition to direct exposure, independents may learn candidates' types by talking to other voters. In particular, each independent randomly samples a finite number $k > 0$ of other independents and each sampled independent reports truthfully the information obtained from parties' advertising.[10] We refer to the parameter k as to the level of contextual exposure and our main interest is to understand how political equilibrium outcomes are affected by different levels of k. As it is common in models of social learning, we take the informal communication structure, which in our case corresponds to the parameter k, as exogenously given.[11]

Timing of the Political Game. We study the following Bayesian Game. In the first stage, parties choose simultaneously their own candidate and, conditional on the candidate selected, a campaign advertising intensity. Independent voters do not observe these choices. In the second stage, independents may be exposed to political information either directly or via interpersonal communication. Based on the information received, independents update their beliefs about candidates' types and cast their vote. In the spirit of the citizen-candidate approach, the candidate that wins a simple majority of votes is elected and implements the policy which corresponds to his ideology.[12]

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[10] In our model independents only communicate with other independents. It is easy to extend the basic framework to the case where independents sample other voters from the entire population of citizens. Depending on what we assume about the report of a partisan (truthful or not), this would change the specification of the posterior beliefs of independents, but it would not affect qualitatively our results.

[11] To this end, Huckfeldt and Sprague [16] point out: "People often choose their associates and the content of their conversations, but each of these choices is, in turn, bounded by an environment that for many purposes must be taken as given rather than chosen." Also, Huckfeldt et al. [17] report that: "[...] individuals exercise limited discretion in the selection of informants. [...] the construction of a communication network occurs with pronounced constraints on supply."

[12] An alternative specification, more in the spirit of the Downsian tradition, would be to consider the case where parties choose policies instead of candidates and they can perfectly commit. The citizen-candidate

Parties' Strategies and Parties' Utilities. A strategy for party j is a probability distribution over candidates' types and an intensity of informative advertising for each candidate's type. Formally, let $\sigma_j : T \to [0,1]$, where $\sigma_j(t)$ denotes the probability that party j selects a candidate of type t, and $\sigma_j(e) + \sigma_j(m) = 1$. Analogously, let $x_j : T \to [0,1]$, where $x_j(t)$ denotes the intensity of informative advertising of party j when candidate t is selected. We denote a strategy of party j as $s_j = (\sigma_j, x_j)$, while $s = (s_L, s_R)$ denotes a strategy profile for parties.

Let $\pi_L(s|\mathbf{t})$ denote the expected probability that party L wins, given a pair of candidates $\mathbf{t}$ and a strategy profile s. The expected payoff to party L when its candidate is t_L can be written as follows,

$$U_L(s|t_L) = \sum_{t_R \in \{e,m\}} \sigma_R(t_R) \left[\pi_L(s|\mathbf{t}) (1 - t_R - t_L) - (1 - t_R - e) \right] - \alpha x_L.$$

Voting Behavior of Independent Voters. *Ex-post*, the information of an independent about party j's candidate can be summarized by $I_{k,j} \in T \cup \emptyset$, where $I_{k,j} = t$ means that the independent knows that party j's candidate is t, while $I_{k,j} = \emptyset$ indicates that the independent did not receive any information about party j's candidate.

Let $\rho_j(t|I_{k,j}, s, k)$ denote the belief of an independent that party j's candidate is t, given $I_{k,j}$ and s. Whenever possible, $\rho_j(t|I_{k,j}, s, k)$ is derived using Bayes' rule. Hence, $\rho_j(t|t, s, k) = 1$, $\rho_j(t|t', s, k) = 0$, for $t \neq t'$, and

$$\rho_j(t|\emptyset, s, k) = \frac{\sigma_j(t)(1 - x_j(t))^{k+1}}{\sum_{t' \in T} \sigma_j(t')(1 - x_j(t'))^{k+1}}, \tag{1}$$

for every $t \in T$ such that $\sigma_j(t) > 0$ and $x_j(t) > 0$. We also assume that equation (1) holds at zero probability events, i.e., when $\sigma_j(t') = 0$ and/or $x_j(t') = 0$.[13]

Since each independent votes as if he is pivotal, an independent with ideology i and information $(I_{k,L}, I_{k,R})$ votes for party L if and only if $i < i^*(I_{k,L}, I_{k,R})$, where $i^*(I_{k,L}, I_{k,R})$

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approach (see Besley and Coate [4], and Osborne and Slivinski [27]) provides a natural rationale for the commitment assumption.

[13]Note that this is a necessary condition for a Bayesian equilibrium to be a sequential equilibrium. See also Footnote 15 for a discussion of the role of this condition in the characterization of equilibria.

is the identity of the indifferent independent voter with information set $(I_{k,L}, I_{k,R})$. The expression for $i^*(I_{k,L}, I_{k,R})$ is given by:

$$i^*(I_{k,L}, I_{k,R}) = \frac{1}{2} + \frac{\sum_{t \in T} \rho_L(t|I_{k,L}, s, k)t - \sum_{t \in T} \rho_R(t|I_{k,R}, s, k)t}{2}.$$

Given (t_L, t_R) and s, party L's candidate gets at least half of the independents' votes if and only if $\mu < \mu_L^*(s|t_L, t_R)$, where:

$$\mu_L^*(s|t_L, t_R) = \sum_{(I_{k,L}, I_{k,R})} i^*(I_{k,L}, I_{k,R}) \Pr(I_{k,L}|s, t_L) \Pr(I_{k,R}|s, t_R).$$

Therefore,

$$\pi_L(s|\mathbf{t}) = \begin{cases} 0 & \text{if } \mu_L^*(s|\mathbf{t}) \leq \frac{1}{2} - m \\ \frac{\mu_L^*(s|\mathbf{t}) + m - \frac{1}{2}}{2m} & \text{if } \mu_L^*(s|\mathbf{t}) \in \left(\frac{1}{2} - m, \frac{1}{2} + m\right) \\ 1 & \text{if } \mu_L^*(s|\mathbf{t}) \geq \frac{1}{2} + m. \end{cases}$$

Political Equilibrium. A political equilibrium consists of (i) parties' strategies, $s^* = (s_L^*, s_R^*)$; (ii) voter belief functions $\rho_j^*(\cdot)$, $j = \{L, R\}$, and indifferent independent voters $i^*(\cdot)$ such that:

1. (s_L^*, s_R^*) are mutual best responses given subsequent voting behavior;

2. $\rho_j^*(\cdot)$ are consistent with s^* for all $j = \{L, R\}$, and $i^*(\cdot)$ are consistent with $\rho_j^*(\cdot)$ and s for all $j = \{L, R\}$.

3 Characterization of Political Equilibrium

Our first result shows that there exists a unique symmetric political equilibirum. When the advertising technology is sufficiently inefficient (i.e., for high α), parties only select extremist candidates and they do not advertise. Otherwise, parties randomize between selecting a moderate candidate and an extremist candidate, and they only advertise moderate candidates. The characteristics of the political equilibrium are pinned down by a simple measure of voters' *misperception* about the types of candidates running for office, which we define next.

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For a symmetric strategy profile s, the fraction of independents who learn (directly or indirectly) that the leftist candidate is of type t is:

$$y_L(x_L(t), k) = 1 - (1 - x_L(t))^{k+1}.$$

We define the misperception of an independent about the leftist candidate t as the probability that a (randomly selected) independent believes that the leftist candidate is of type $t' \neq t$. Formally,

$$Q[t'|t, s, k] = [1 - y_L(x_L(t), k)]\rho(t'|\emptyset, s, k),$$

where $1 - y_L(x_L(t))$ is the probability that an independent does not observe that the leftist candidate is of type t, and $\rho(t'|\emptyset, s, k)$ (defined in equation (1)) is the probability that an uninformed independent places on the event that the leftist candidate is of type t'. A measure of overall voters' misperception is then the *ex-ante* probability that a randomly selected independent misperceives the type of the candidate running for office. Formally,

$$\Psi(s, k) \equiv Q[e|m, s, k] + Q[m|e, s, k].$$

In every pure strategy equilibrium the level of misperception is zero, while a mixed-strategy equilibrium always entails some positive level of misperception. Next we characterize symmetric political equilibria.

Proposition 1 *A symmetric political equilibrium exists and it is unique. For every k, there exists a critical level $\alpha^*(k) > 0$ such that in equilibrium:*

(I.) If $\alpha \geq \alpha^*(k)$ parties always select an extremist candidate and they do not advertise, i.e., $\sigma^*(e) = 1$, and $x^*(e) = 0$;

(II.) If $\alpha < \alpha^*(k)$ parties randomize between selecting an extremist candidate and a moderate candidate, and they only advertise moderate candidates. In particular, $x^*(e) = 0$, and $x^*(m)$ and $\sigma^*(e)$ jointly solve:

$$(k+1)(1 - x^*(m))^k \rho(e|\phi, s^*, k)\frac{2 - 4m + \sigma^*(e)}{16} = \alpha \tag{2}$$

$$1 - \Psi(s^*, k) = \frac{4m + 16\alpha x^*(m)}{2 - 3m}. \tag{3}$$

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Furthermore, $\alpha^*(k)$ is increasing in k.

To see the intuition behind this result, first consider the case of symmetric pure-strategy equilibria.[14] Note that a profile in which parties always select a moderate candidate cannot be part of equilibrium. For suppose not, then in equilibrium voters would anticipate that the two candidates are moderates and therefore parties would not find it profitable to advertise. In this case, a party could increase its utility by selecting an extremist candidate. In fact, this would not change its expected probability of winning and, since each party prefers to implement the extreme policy, its utility would be higher.[15]

Next, consider the alternative scenario in which parties always select extremist candidates and they do not advertise. In this case a party could find it profitable to deviate from this strategy by selecting a moderate and informing some voters about its candidate. The benefit of such deviation is the increase in the probability of winning the election, which in turn is increasing in the proportion of voters that the party informs. Since disclosing information is costly, when the advertising technology is sufficiently inefficient ($\alpha > \alpha^*(k)$), this deviation cannot be profitable. Furthermore, the lower is the level of contextual exposure, the higher is the level of advertising needed in order to make such a deviation profitable. This implies that when voters have higher chances to learn the candidates' types from communicating with other voters, the pure strategy equilibrium exists for higher level of α, i.e., $\alpha^*(k)$ is increasing in k.

We now consider the case of symmetric mixed-strategy equilibria. Clearly, in any equilibrium, parties only advertise moderate candidates. Moreover, when choosing their political strategy, parties face the following trade-off. Since they are policy motivated and they never advertise extremists, conditional on winning the election, they derive higher

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[14]It is easy to see that pure-strategy asymmetric equilibria do not exist in this political game.

[15]This argument relies on the assumption that voters' beliefs are constant at zero probability events. There is only one equilibrium that does not satisfy this condition: parties always select a moderate, they set $x(m) = 1$ and, out-of-equilibrium, an uniformed voter believes that the candidate is an extremist with strictly positive probability. This equilibrium exists for a value of α sufficiently small and it is not robust to small imperfections of the advertising technology. For example, suppose that there is a positive probability that a voter remains uninformed even if parties set $x(m) = 1$. Given that parties always select moderate candidates, uniformed voters will always believe that the candidates are moderate and therefore setting $x(m) = 1$ would not be optimal.

utility when they select an extremist candidate. However, since independents are decisive, a moderate candidate has a higher chance of winning the election relatively to an extremist, and this advantage is higher the lower is voters' misperception. Indeed, in a symmetric mixed-strategy profile s, the difference between the expected probabilities of winning of, say, party L when choosing a moderate rather than an extremist candidate is

$$\hat{\pi}_L(s|t_L = m) - \hat{\pi}_L(s|t_L = e) = \frac{1}{2} - \hat{\pi}_L(s|t_L = e, t_R = m) = \frac{1 - \Psi(s, k)}{8}.$$

In a mixed-strategy equilibrium each party must be indifferent between selecting a moderate candidate and an extremist candidate, which requires that condition (3) holds. Furthermore, the equilibrium level of $x^*(m)$ must equate marginal returns of advertising a moderate with marginal costs. This is summarized by condition (2), where the right hand side represents the marginal cost α. The left hand side of (2) represents the marginal returns of advertising, which consist of the marginal decrease in the fraction of uninformed voters, weighted by their belief of facing an extremist candidate and by the increase in utility in case of electoral victory.

3.1 The Equilibrium Effect of Contextual Exposure

We now explore the equilibrium relation between the level of interpersonal communication, voters' misperception and policy polarization. In order to do so, we compare the political equilibrium when the level of contextual exposure is k with the political equilibrium when independents communicate with $k + 1$ other voters. We focus on the case $\alpha < \alpha^*(k)$ described in part (II) of Proposition 1 since this is the only non trivial situation.

As a measure of policy polarization we define the *ex-ante* expected probability that in equilibrium an extremist candidate is elected. This is denoted by $\Pi(s^*)$ and it is equal to:

$$\Pi(s^*) = \sigma^*(e)^2 + 2\sigma^*(e)(1 - \sigma^*(e))\pi(s^*|e, m), \tag{4}$$

where the first term of the right hand side of (4) is the probability that two extremist candidates compete in the election, while the last term is the probability that an extremist candidate wins against a moderate candidate.

The next proposition summarizes the results.

Proposition 2 *Suppose that $\alpha \in (0, \alpha^*(k))$.*

(I.) *If k increases then the equilibrium advertising of moderate candidates decreases, voters' misperception increases and the probability that an extreme candidate wins against a moderate candidate increase;*

(II.) *For every k, there exists $\hat{\alpha}(k) \in (0, \alpha^*(k))$ such that, for all $\alpha < \hat{\alpha}(k)$, if k increases then parties select extreme candidates with higher probability and polarization increases.*

The crucial step behind the result of part (I) in Proposition 2 is to show that the level of informative advertising is decreasing in the level of contextual exposure. This follows from a simple economic intuition, which hinges on the relation between contextual exposure and advertising. On the one hand, the level of contextual exposure and the intensity of advertising are complements. When voters are part of a richer communication network, an increase in advertising reaches an additional fraction of otherwise uniformed voters. Hence voters' misperception towards moderate candidates decreases and, as a consequence, the probability that a moderate candidate wins the election increases. This "network multiplier effect" increases the incentive for parties to advertise. On the other hand, the level of contextual exposure and the level of advertising are substitutes. In fact, when the voters' communication network becomes richer, the amount of "wasted" informative advertising reaching voters who would eventually become informed by communicating with others increases. In other words, when k increases, parties may reach the same total fraction of voters with a lower level of advertising. This decreases the incentive for parties to advertise.

Intuitively, the substitution effect dominates the complementarity effect when informative advertising is high to begin with. Recall that from the parties' viewpoint selecting a moderate candidate is worthwhile only if voters' misperception is sufficiently low, which requires a high enough level of advertising. That is, in a mixed-strategy equilibrium, the level of informative advertising must always be bounded below, and we can show that at this lower bound the substitution effect always dominates the complementarity effect. Therefore, an increase in k reduces the level of informative advertising.[16] Since parties

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[16]In our model total informative advertising and the level of interpersonal communication are substitutes.

advertise less their moderate candidates, the cost of selecting a moderate decreases, which implies that the benefit of selecting a moderate instead of an extremist candidate must also decrease. This occurs only if voters' misperception increases and, as a consequence, the difference between the probabilities of winning when the candidate is a moderate rather than an extremist decreases.

The first part of Proposition 2 is silent about how an increase in the level of contextual exposure affects the probability that parties select extremist candidates and, therefore, how it affects the level of polarization. Note that, even if a larger value of k may increase polarization because the probability that an extremist defeats a moderate increases, if parties react to the presence of richer networks of communication by selecting moderate candidates more often, overall the level of polarization may still decrease. The extent to which parties adjust their equilibrium selection strategy to a change in the level of contextual exposure crucially depends on the change in the total fraction of informed voters in response to an increase in k. As we illustrate now, this latter equilibrium response depends solely on whether the marginal cost of total exposure is decreasing or increasing in the level of contextual exposure.

We can write the total advertising cost of reaching a fraction $y(x(m), k)$ of independents as $C(y(x(m), k)) = \alpha y(x(m), k)$, where $y(x(m), k) = 1 - (1 - x(m))^{k+1}$ takes into account both direct and indirect exposure. Hence, the marginal cost of total exposure is

$$\frac{\partial C(y(x(m), k))}{\partial y} = \frac{\alpha}{(k+1)(1 - y(x(m), k))^{\frac{k}{k+1}}}.$$

While in recent years we observed an increase in both interpersonal communication and in total campaign spending, only a fraction of this spending is devoted to informative advertising, which is the form of political advertising we are focusing on. In fact, part of the increase in campaign advertising is accounted for by an increased use of negative political advertising, see, e.g., Sabato [33]. In his recent survey on political advertising, Prat [32] states: "often ads convey verifiable information on the political record of the candidate [...] it is also true that some commercials are extremely expensive but appear to contain little new information." See Gerber et al. [14] for mixed evidence on the effectiveness of media advertising, and in particular Ansolabehere and Iyengar [1] for experimental evidence supporting the claim that political advertising is effective even when it is not directly informative.

Using the expression above, we can rewrite the equilibrium condition (2) as

$$\rho(e|\phi, s^*, k)\frac{2 - 4m + \sigma^*(e)}{16} = \frac{\partial C(y(x^*(m), k))}{\partial y}.$$

When α is sufficiently low, parties advertise with relatively high intensity and therefore $y(x(m), k)$ is large. In this case, an increase in k increases the marginal cost of total exposure, *ceteris paribus*. Formally, we have that:

$$\frac{\partial^2 C(y(x(m), k))}{\partial y \partial k} = -\frac{\alpha\left[1 + \frac{1}{k+1}\ln(1 - y(x(m), k))\right]}{(k + 1)^2(1 - y(x(m), k))^{\frac{k}{k+1}}},$$

which is strictly positive for sufficiently high $y(x(m), k)$. Intuitively, when many voters are already informed, a marginal increase in informative advertising will only have a relatively small effect on the total fraction of informed voters, i.e., the marginal cost of total exposure is very high. This implies that, when α is sufficiently low, not only the levels of informative advertising and of contextual exposure are substitutes, but also total exposure and contextual exposure are substitutes. Hence, for low values of α, an increase in k decreases the equilibrium level of informative advertising insomuch that the total fraction of informed voters decreases as well. This unambiguously softens political competition and therefore parties select extremists more often. Overall, polarization increases.

We conclude this section with two observations. First, the second part of Proposition 2 does not hold for sufficiently inefficient advertising technology, i.e., $\alpha \in (\hat{\alpha}(k), \alpha^*(k)]$. In line with the intuition above, for sufficiently high α, an increase in the level of contextual exposure leads parties to decrease the level of informative advertising only slightly, so that the total fraction of informed voters increases. As a result, parties select moderate candidates more often and overall the level of polarization decreases.[17]

Second, the effect of an increase in the level of interpersonal communication on the endogenous variables is rather different from the effect of a decrease in the marginal costs

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[17]As an example, suppose that $m = 0.1$ and $\alpha = 0.1$. For $k = 1$, in equilibrium parties select extremist candidates with probability $\sigma^* = 0.8$, the level of voters' misperception is $\Psi^* = 0.3$, and the level of polarization is $\Pi^* = 0.76$. In contrast, when $k = 2$ in equilibrium parties select extremist candidates with probability $\sigma^* = 0.48$, the level of voters' misperception is $\Psi^* = 0.43$, and the level of polarization is $\Pi^* = 0.45$.

of advertising α (i.e., direct exposure becomes less expensive). The reason is that a decrease in the marginal costs of direct exposure unambiguously leads parties to increase the level of informative advertising. This is exactly the opposite effect of an increase in the level of contextual exposure. Since parties advertise more their moderate candidates, it immediately follows that voters' misperception decreases, the proportion of informed voters increases, moderate candidates are selected with higher probability, and therefore the level of polarization decreases.[18]

3.2   Discussion

We now discuss the robustness of the results presented in Proposition 2. We first focus on our assumption of linear advertising cost. Next, we provide a simple purification argument for the mixed equilibrium. This shows that our results in Proposition 2 are not driven by the mixed strategy nature of the equilibrium. Finally, we elaborate on different possible assumptions regarding the technology of interpersonal communication.

Cost of Advertising. We start by noticing that the results in part (I) of Proposition (2) do not depend on the specific functional form of the cost function. Consider a general cost function $C(\alpha, x)$, which is increasing and convex in α and x, respectively. The equilibrium condition (2) can be rewritten as:

$$(k+1)(1 - x^*(m))^k \rho(e|\phi, s^*, k)\frac{2 - 4m + \sigma^*(e)}{16} = \frac{\partial C(\alpha, x^*(m))}{\partial x^*(m)}.$$

Following the same line of reasoning developed in the previous section, we can conclude that the marginal return of advertising is decreasing in the level of contextual exposure. Hence, the level of informative advertising is also decreasing in k. For a general cost function $C(\alpha, x)$, the equilibrium condition 3 reads as follows:

$$1 - \Psi(s^*, k) = \frac{4m + 16C(\alpha, x^*(m))}{2 - 3m},$$

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[18]The formal proof of the comparative static result with respect to α is straightforward and therefore omitted.

and since $x^*(m)$ decreases in k, it follows that $C(\alpha, x^*(m))$ is also decreasing. Thus, the equilibrium level of voters' misperception is higher when voters belong to richer communication networks.

In contrast, the results in part (II) of Proposition 2 depend on the specific formulation of the cost function. To see why this is the case, recall that the level of polarization increases with the richness of voters' communication network whenever total exposure and contextual exposure are substitutes. This requires that, in equilibrium, the marginal cost of total exposure is increasing in k. This relation holds more generally than for the linear cost function case, which we have considered before. For example, it holds for the family of cost functions $C(\alpha, x) = \alpha x^{\beta}$, where $\beta \geq 1$. However, it does not hold for all increasing and convex cost functions. For example, it does not hold for the cost function $C(\alpha, x) = \alpha x/(1-x)$, which is the one used by Coate [6]. Indeed, in this case the marginal cost of total exposure is

$$\frac{\partial C(y(x(m), k))}{\partial y} = \frac{\alpha}{(k+1)(1 - y(x(m), k))^{\frac{k+2}{k+1}}},$$

which is decreasing in k. Hence, even if it is still true that an increase in the level of contextual exposure decreases the amount of information that parties strategically choose to disclose, the total fraction of informed voters always increases in k. Consequently, the higher is the level of contextual exposure, the stronger is political competition between parties and therefore parties select moderate candidates more often. Overall, the level of polarization decreases. Table 1 summarizes these observations by showing how political equilibrium outcomes change with the level of contextual exposure for three different formulations of the cost function.

A Simple Purification Argument. We can purify the symmetric mixed-strategy equilibrium described in part (II) of Proposition 1 in the following way. Suppose that each party can be either a "high-cost" type, with probability σ, or a "low-cost" type with probability $1 - \sigma$. A "high-cost" party has a high marginal cost of advertising, say α_H, while the marginal cost of a "low-cost" party is $\alpha \ll \alpha_H$. Each party observes its own type, but it does not observe the type of its opponent. If the difference between α_H and α is sufficiently large, in equilibrium the high cost party selects an extremist candidate and

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$m = 0.1, \alpha = 0.01$	$C(\alpha, x) = \alpha x$	$C(\alpha, x) = \alpha x^2/2$	$C(\alpha, x) = \alpha x/(1 - x)$
$k = 1$	$\sigma^*(e) = 0.011$	$\sigma^*(e) = 0.008$	$\sigma^*(e) = 0.1542$
	$x^*(m) = 0.846$	$x^*(m) = 0.847$	$x^*(m) = 0.517$
	$\Psi^* = 0.685$	$\Psi^* = 0.731$	$\Psi^* = 0.664$
	$\Pi^* = 0.010$	$\Pi^* = 0.008$	$\Pi^* = 0.141$
$k = 2$	$\sigma^*(e) = 0.015$	$\sigma^*(e) = 0.009$	$\sigma^*(e) = 0.088$
	$x^*(m) = 0.672$	$x^*(m) = 0.703$	$x^*(m) = 0.465$
	$\Psi^* = 0.701$	$\Psi^* = 0.741$	$\Psi^* = 0.686$
	$\Pi^* = 0.014$	$\Pi^* = 0.009$	$\Pi^* = 0.082$

Table 1: The effect of an increase in the level of contextual exposure for different cost functions.

does not advertise, while the low cost party always selects a moderate and advertises with intensity x^*, which is the solution to equation (2).

Using the same intuition developed in the previous section, it can be shown that for a sufficiently low level of α, both the level of informative advertising as well as the total fraction of informed voters are decreasing in the level of contextual exposure. As before, this implies that when voters belong to richer communication networks the level of voters' misperception is higher. Furthermore, since now the probability that a party selects a moderate *versus* an extremist candidate is exogenously given, it immediately follows that polarization increases with the level of contextual exposure.

The Technology of Social Learning. The social learning technology can be enriched in many ways without loosing tractability and without changing the qualitatively insights of our results. First, we could allow for the realistic possibility that voters are heterogeneous with respect to their exposure to social learning. A simple way of capturing this feature is to consider that social ties are described by a distribution $P : [0, ..., \bar{k}] \to [0, 1]$, where $P(k)$ indicates the fraction of voters who sample k other voters. In this case, the effect of an increase in the level of contextual exposure on political equilibrium outcomes can be studied by taking first order stochastic shifts in the distribution P. While in this paper we presented the analysis for the case of homogeneous voters, all our results are robust to the introduction of heterogeneity with respect to social-learning exposure.

Second, we assumed that interpersonal communication only travels one step in the un-

derlying social structure: if voter i samples voter j, then i obtains the information that j obtained from the parties, but not the information that j may have obtained by communicating with other voters. In principle, communication may also be indirect, thereby allowing for more complex patterns of social learning. This can be formalized by assuming that information travels $r \geq 1$ steps in the underline communication networks. Allowing for indirect social learning will increase the complexity of the analysis, without adding new insights. In particular, it is immediate to see that the effect of an increase in the radius of information on political equilibria is analogous to the effect of an increase in the level of contextual exposure in our basic formulation.

4 Homophily and Targeted Political Advertising

So far, we assumed that voters learn from sampling randomly other voters. However, a well-known documented fact is that interpersonal communication occurs more frequently among similar individuals.[19] One possible way to think about homophily in a political context is that individuals with similar political ideology will have higher chances to interact and learn from each other. In our model, this phenomenon would entail a higher probability of interpersonal communication between voters with closer political ideologies. Note that if parties cannot target political advertising, our results will be unaffected by any form of such correlation. However, if parties are able to target advertising to ideologically similar voters, homophily will affect electoral competition and political outcomes. In this section we modify our benchmark model in order to capture these additional features in a simple and parsimonious way.

First, we define group l as the group of independents in the interval $[\mu-\tau, \mu]$; analogously we call group r the remaining group of independents. Each independent samples k other voters. For each draw, a group-l member samples a voter in his own group with probability β, while with the remaining probability he samples a group-r voter. We assume that $\beta \in [1/2, 1)$, and we interpret this parameter as the level of homophily in the society. Absence of homophily corresponds to $\beta = 1/2$. When instead $\beta = 1$, voters only communicate with

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[19]See, McPherson et al. [26] and references in Footnote 6.

members of their own group, which exemplifies a society in which ideology-based groups are totally segregated. Since the focus of this section is to study the effect of homophily on political outcomes, we set $k = 1$, hereafter.[20]

Second, we assume that a party can choose either to advertise its candidate at a cost $\alpha > 0$ or not to advertise, and that advertising is targeted to the ideologically closer group of independents so that if the left party chooses to advertise its candidate, then all members of group l learn perfectly the type of the leftist candidate.[21] This is a simplfied version of a more general model where a party chooses whether to target advertising to one of the two groups (at a cost α), or to disclose political information to both groups (for example at a cost 2α). In Appendix B (Proposition 5) we show that the equilibria we characterize by assuming that advertising is targeted to the closer group of independents (see Proposition 3 below) are indeed equilibria in the more general model where parties can choose where to target advertising.

In this context, we can define voters' misperception as follows. Consider a strategy profile in which party L selects an extremist candidate with probability $\sigma(e)$ and advertises only a moderate candidate. Clearly, in equilibirum, all group-l voters are informed about the position of the leftist candidate. However, the probability that a voter in group r believes that the leftist candidate is an extremist (resp. moderate) when in fact he is a moderate (resp. extremist) is $\beta\sigma(e)$ (resp. $\beta[1 - \sigma(e)]$). Hence the level of voters' misperception is simply $\Psi(s, \beta) = \beta$. The next proposition characterizes the equilibrium.

Proposition 3 *A symmetric political equilibrium always exists and it is unique. For every* β, *there exists* $\underline{\alpha}(\beta) < \bar{\alpha}$ *such that in equilibrium:*

(I.) For all $\alpha > \bar{\alpha}$ *parties select extremist candidates and they do not advertise;*

(II.) For all $\alpha \in (0, \max[0, \underline{\alpha}(\beta)])$, *parties select moderates and they advertise;*

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[20]We assume that $k = 1$ merely for expositional reasons; all the results hold for arbitrary finite k.

[21]Here, we assume that advertising is discrete, i.e., a party can decide whether or not to advertise but not how much to advertise. This assumption is only needed for tractability. The fact that when a party advertises its candidate then *all* idelogically close independents observe perfectly the candidate's ideology is not crucial for our results. Moreover, note that, abusing notation, we are denoting by α the total cost of advertising, while in the benchmark model α was the marginal cost of advertising.
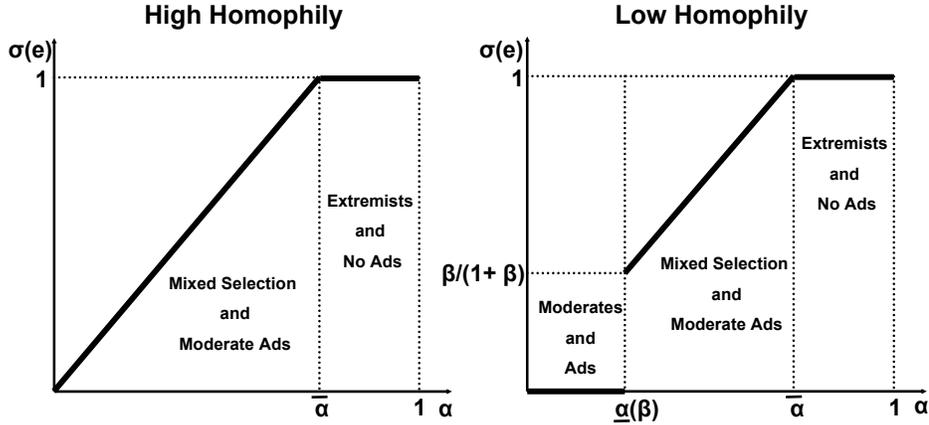
Figure 2: Political Equilibrium under Homophily.

(III.) *For all $\alpha \in (\max[0, \underline{\alpha}(\beta)], \bar{\alpha})$ parties select extremist candidates with probability*

$$\sigma^*(e) = 1 - \frac{2 - 7m - 16\alpha}{\beta(2 - 3m)}, \tag{5}$$

and they advertise when they select a moderate.

Furthermore, $\underline{\alpha}(\beta)$ is decreasing in β and $\underline{\alpha}(\beta) > 0$ if and only if $\beta < \frac{2-7m}{4m}$.

Figure 2 provides a graphical illustration of the equilibrium in the $(\alpha, \sigma(e))$ parameter space. The intuitions behind part (I) and part (III) of Proposition 3 are analogous to the intuitions behind Proposition 1. When the level of homophily is sufficiently high these are the only equilibria. However, when the level of homophily and the cost of advertising are low, a party cannot be indifferent between selecting the two candidates. Indeed, a low value of β implies that information often travels across groups and therefore voters' misperception is low. In this case, a party has much higher chances of winning the election when selecting a moderate rather than an extremist. Similarly, when α is low, the cost of selecting a moderate candidate is also low. Hence, in equilibrium parties always select moderate candidates and they advertise.[22] Our final result shows how the level of homophily affects political outcomes.

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[22]This equilibrium requires specific out-of-the-equilibrium beliefs: when a voter in group $l(r)$ is uninformed about party's $L(R)$ candidate, then he believes that party $L(R)$ selected an extremist.
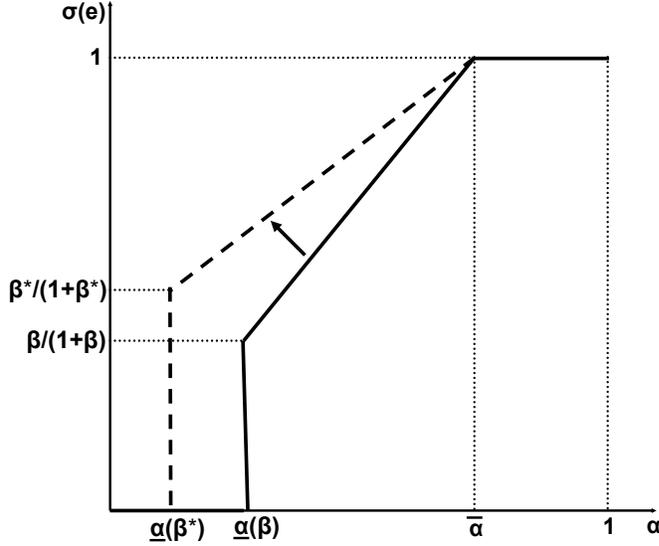
Figure 3: Comparative statics under Homophily, i.e., $\beta^* > \beta$.

Proposition 4 *Suppose $\alpha \in (\max[0, \underline{\alpha}(\beta)], \bar{\alpha})$. If β increases then parties select extremist candidates with (strictly) higher probability, voters' misperception increases and the policy outcome is more polarized.*

Figure 3 illustrates the comparative statics with respect to β. The intuition behind Proposition 4 is simple. Since parties target political information to distinct groups, an increase in homophily leads to a lower probability that a voter will possess information about both candidates. Hence, the level of voters' misperception increases, which softens political competition between parties. As a consequence, policy-motivated parties exploit an increase in voters' misperception by selecting extremist candidates more often. Overall, the level of polarization increases with the level of homophily.[23]

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[23]In the context of the mixed-strategy equilibrium described in Proposition 3, we can compute the *ex-ante* equilibrium probability that a randomly selected group-$l(r)$ independent votes for the Left(Right) party. This probability is increasing in β, meaning that the higher is the level of homophily, the higher is the correlation between vote choice and ideology. This result is consistent with empirical evidence suggesting that receiving information about the ideologically closer candidate reduces the probability that voters switch their votes from their initial disposition. See Velazquez Nunez [35] and Pattie and Johnston [28].

5 Conclusion

The importance of interpersonal communication in affecting voters' choices has been empirically documented in economics, political science, and sociology. To the best of our knowledge there is no theoretical model that examines how communication networks among voters affect social learning and its consequences on political equilibrium outcomes. This paper embeds social learning in a strategic model of informative political advertising and it provides novel insights on the equilibrium relation between relevant aspects of interpersonal communication and electoral outcomes.

In our model interpersonal communication is truthful. A first step toward relaxing this assumption is to consider the case in which voters can strategically choose to omit information. In this case, a voter who learns that the right (left) candidate is a moderate and who strictly prefers to vote for the left (right) candidate, will choose to omit information about the right (left) candidate. Preliminary work suggests that this extension does not affect qualitatively our results. In fact, if any, strategic communication creates endogenously homophily in the sense that "good" information will only be passed along to voters with a similar ideology, while "bad" information only to voters with a different ideology. In principle, this might decreases political competition between parties thereby leading towards more polarized outcomes.

In our model we abstract from the effect that social learning may have on electoral turnout. However, we believe this is an important issue and one possible way to analyze the effect of interpersonal communication on turnout is to consider an expressive theory of voting. That is, each voter votes as if he were pivotal, provided that the increase in his expected utility when is most preferred candidate gets elected is above his private cost of voting. To see how interpersonal communication can affect the decision of turning out to vote, consider the case of a voter informed only about party L's candidate. Conditionally on remaining uninformed, an increase in the level of interpersonal communication increases the voter's posterior beliefs that the right candidate is an extremist. This reduces the voter expected utility were this candidate being elected, which makes it more likely that this voter would choose to turn out and vote. Hence, the overall effect on turnout will depend on the extent to which voters learn about electoral candidates, which in turn depends on

the structure of the communication network. These and other extensions are the object of ongoing research.

APPENDIX

Appendix A. In this appendix we provide the proofs of Proposition 1 and Proposition 2.

Proof Proposition 1.

First Step: Symmetric Pure Strategy Equilibria. We first consider pure-strategy equilibria. Let $s^* = (s_L^*, s_R^*)$ be part of a symmetric pure-strategy equilibrium. Then s_j^* prescribes to select candidate $t^* \in T$ with probability 1, and to advertise with intensity $x^*(t^*)$, for all $j \in \{L, R\}$. Note that in any pure-strategy equilibrium, it has to be that $x^*(t^*) = 0$. Hence, $U_L(s^*|t^*) = -(1-m)/2$. There are two possibilities, which we now analyze.

Consider the case of $t^* = m$, and let party L deviate by selecting $t_L = e$; in this case the best advertising strategy is $x_L(e) = 0$, and denoting this strategy by $\tilde{s}_L$, we have that $U_L(\tilde{s}_L, s_R^*|e) = -(2-3m)/4 > U_L(s^*|m)$, which contradicts our hypothesis that s^* is an equilibrium.

Consider now the case of $t^* = e$, and let party L deviate by selecting $t_L = m$ and $x_L(m)$; denote this strategy by $\tilde{s}_L$. Observe that such deviation is profitable only if $x_L(m) \neq 0$. Thus, assume that $x_L(m) > 0$. We now derive the optimal advertising level given s_R^*, which we denote by $x_L^*(m)$. To do this, we start by observing that:

$$\mu_L^*(\tilde{s}_L, s_R^*|m, e) = \frac{1}{2} + \frac{m}{4} - \frac{m}{4}(1 - x_L(m))^{k+1},$$

and it is readily seen that $\mu_L^*(\tilde{s}_L, s_R^*|m, e) \in (1/2, 1/2 + m/4)$, for all $x_L(m) \in (0, 1)$, which implies that $\pi_L(\tilde{s}_L, s_R^*|m, e) \in (0, 1)$, for all $x_L(m) \in (0, 1)$. Next, since $\pi_L(\tilde{s}_L, s_R^*|m, e) \in (0, 1)$, it follows that party L's expected utility of playing $\tilde{s}_L$ against s_R^* is

$$U_L(\tilde{s}_L, s_R^*|m) = \left(\frac{5 - (1 - x_L(m))^{k+1}}{8}\right)\left(1 - \frac{3}{2}m\right) - (1-m) - \alpha x_L(m),$$

which is concave in $x_L(m)$. Hence, the optimal $x_L^*(m) \in (0, 1)$ solves

$$(k+1)(1 - x_L^*(m))^k = \frac{16\alpha}{2 - 3m}. \tag{6}$$

Note that $x_L^*(m)$ is decreasing in α, and $x_L^*(m) \geq 0$ if and only if $\alpha \leq (2-3m)(k+1)/16$, and $x_L^*(m) = 1$ if and only if $\alpha = 0$. Thus, if $\alpha \geq (2-3m)(k+1)/16$, then $x_L^*(m) = 0$ and a pure-strategy equilibrium exists.

If instead $\alpha < (2 - 3m)(k + 1)/16$, party L will not deviate from s_L^* if and only if $U_L(s^*|e) \geq U_L(\tilde{s}_L, s_R^*|m)$, where abusing notation $\tilde{s}_L$ prescribes to advertise a moderate candidate with intensity $x_L^*(m)$. The latter inequality is satisfied if and only if

$$(1 - x_L^*(m))^{k+1} \geq \frac{2 - 7m - 16\alpha x_L^*(m)}{2 - 3m}. \tag{7}$$

Defining $p = 1 - x$ and substituting (6) into (7), we have that $U_L(s^*|e) \geq U_L(\tilde{s}_L, s_R^*|m)$ if and only if

$$\alpha \geq \alpha^*(k) \equiv \frac{(2 - 7m)(k + 1)}{16(k + 1 - k\underline{p})}, \tag{8}$$

where $\underline{p}$ is the unique solution to

$$(k + 1)\underline{p}^k = \frac{16\alpha}{2 - 3m},$$

and $\alpha^*(k) < (2 - 3m)(k + 1)/16$ for all $p \in (0, 1)$. Putting together these observations it follows that a pure-strategy equilibrium exists if and only if $\alpha > \alpha^*(k)$, and in a pure-strategy equilibrium parties always select extreme candidates and they never advertise.

Second Step: Symmetric Mixed Strategy Equilibria. We now consider symmetric mixed-strategy equilibria. For convenience we use the notation $\sigma \equiv \sigma(e)$. Assume that a symmetric mixed-strategy equilibrium exists, and let $s_j^* = (\sigma^*, x^*(t))$, $j \in \{L, R\}$, denote the equilibrium strategy profile. Note that in equilibrium $x^*(e) = 0$. Consider a profile $s = (s_L, s_R)$ with $x_j(e) = 0$, $j \in \{L, R\}$; under this profile we have that:

$$U_L(s|e) = \sigma_R(1 - m)(\pi_L(s|e, e) - 1) + (1 - \sigma_R)\left(1 - \frac{3m}{2}\right)(\pi(s|e, m) - 1),$$

and

$$U_L(s|m) = \sigma_R\left(\pi_L(s|m, e)\left(1 - \frac{3m}{2}\right) - (1 - m)\right) +$$
$$+ (1 - \sigma_R)\left(\pi_L(s|m, m)(1 - 2m) - \left(1 - \frac{3m}{2}\right)\right) - \alpha x_L(m).$$

Moreover,

$$\frac{\partial \pi_L(s|m, e)}{\partial x_L(m)} = \frac{\partial \pi_L(s|m, m)}{\partial x_L(m)} =$$
$$= \frac{1}{8}(k + 1)(1 - x_L(m))^k \rho_L(e|\emptyset, s, k),$$

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and $U_L(s|m)$ is concave in $x_L(m)$. Hence, in a symmetric equilibrium, $\frac{\partial U_L(s|m)}{\partial x_L(m)}|_{s^*} = 0$ if and only if:

$$(k+1)(1-x^*(m))^k \rho(e|\emptyset, s^*, k) = \frac{16\alpha}{2-4m+\sigma m}, \tag{9}$$

where

$$\rho(e|\emptyset, s^*, k) = \frac{\sigma}{\sigma + (1-\sigma)(1-x^*(m))^{k+1}}.$$

Next, in a symmetric mixed-strategy political equilibrium each party is indifferent between selecting a moderate candidate and selecting an extremist candidate, i.e., $U_L(s^*|e) = U_L(s^*|m)$. Since in a symmetric equilibrium we have that

$$\pi(s^*|e, e) = \pi(s^*|m, m) = \frac{1}{2}$$

$$\pi(s^*|e, m) = \frac{1}{2} - \frac{1}{8}\rho(e|\emptyset, s^*, k)(1-(1-x^*(m))^{k+1}) = 1 - \pi(s^*|m, e),$$

it follows that $U_L(s^*|e) = U_L(s^*|m)$ if and only if:

$$\rho(e|\emptyset, s^*, k)(1-(1-x^*(m))^{k+1}) = \frac{4m + 16\alpha x^*(m)}{2-3m}. \tag{10}$$

Note that condition (10) is equivalent to condition (3) stated in Proposition 1.

We now show existence of a symmetric mixed-strategy equilibrium. We first investigate the equilibrium condition (9). Define

$$f(\sigma, p) = \frac{(k+1)p^k \sigma(2 - 4m + \sigma m)}{\sigma + (1-\sigma)p^{k+1}},$$

and note that the equilibrium condition (9) holds if and only if (σ, p) are such that $f(\sigma, p) = 16\alpha$. The following properties of $f(\cdot, \cdot)$ will prove useful for the proof.

Property 1: $f(0, p) = 0$ for all $p \in (0, 1)$;

Property 2: $f(1, p) = (k+1)p^k(2 - 3m)$, which is increasing in p;

Property 3:

$$\frac{\partial f(\sigma, p)}{\partial \sigma} = \frac{(k+1)p^k}{(\sigma + (1-\sigma)p^{k+1})^2}\left(p^{k+1}(2 - 4m + m\sigma) + m\sigma(\sigma + (1-\sigma)p^{k+1})\right) > 0.$$

Properties 1, 2, and 3 imply that $\widetilde{\sigma}(p) : f(\widetilde{\sigma}, p) = 16\alpha$ is a well defined function of p for all $p \in [\underline{p}, 1]$, where $\underline{p}$ solves $f(1, \underline{p}) = 16\alpha$, i.e.,

$$(k+1)\underline{p}^k = \frac{16\alpha}{2-3m}. \tag{11}$$

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Note that $\underline{p} \in (0,1)$ if and only if $\alpha < (2-3m)(k+1)/16$.

We now study how $\widetilde{\sigma}(p)$ behaves in $p \in [\underline{p}, 1]$. The following properties of $\widetilde{\sigma}(\cdot)$ are useful:

Property 4: $\widetilde{\sigma}(\underline{p}) = 1$;

Property 5: $\widetilde{\sigma}(1) \in (0,1)$ solves

$$\widetilde{\sigma}(1)(2 - 4m + \widetilde{\sigma}(1)m) = \frac{16\alpha}{k+1};$$

Property 6: $\partial f(p,\sigma)/\partial p$ may change sign only once, and

$$\frac{\partial f(\sigma,p)}{\partial p}\Big|_{\underline{p},\widetilde{\sigma}(\underline{p})} > 0.$$

Note that Property 6 follows from Property 4 and inspection of

$$\frac{\partial f(\sigma,p)}{\partial p} = \frac{(k+1)p^{k-1}\sigma(2 - 4m + \sigma m)(k\sigma - p^{k+1}(1-\sigma))}{[\sigma + (1-\sigma)p^{k+1}]^2}.$$

Using the implicit function theorem and invoking properties 1-6, we can summarize the results in the following claim:

Claim 1. For all $\alpha < (2-3m)(k+1)/16$ the following holds: $\widetilde{\sigma}(\underline{p}) = 1$, $\widetilde{\sigma}(p) \in (0,1)$ for all $p \in (\underline{p}, 1)$ and there exists a $v \in (\underline{p}, 1]$ such that $\frac{\partial \widetilde{\sigma}(p)}{\partial p} < 0$ for all $p \in [\underline{p}, v]$, while $\frac{\partial \widetilde{\sigma}(p)}{\partial p} > 0$ for all $p \in (v, 1]$, where $\underline{p}$ is the solution to (11). If $\alpha > (2-3m)(k+1)/16$ an equilibrium in mixed strategies does not exist.

We now consider the indifference equilibrium condition. Here, note that the equilibrium condition (10) can be rewritten as follows:

$$\bar{\sigma}(p) = \frac{p^{k+1}(4m + 16\alpha(1-p))}{(1 - p^{k+1})(2 - 7m - 16\alpha(1-p))}. \tag{12}$$

Furthermore,

$$\frac{\partial \bar{\sigma}(p)}{\partial p} = \frac{p^k \left(\frac{(k+1)(4m+16\alpha(1-p))}{1 - p^{k+1}} - \frac{16\alpha(2-3m)p}{2 - 7m - 16\alpha(1-p)} \right)}{(1 - p^{k+1})(2 - 7m - 16\alpha(1-p))}. \tag{13}$$

Case I. Assume that $2 - 7m - 16\alpha > 0$, which is equivalent to $\alpha < (2-7m)/16$.

Given this, we have that $\bar{\sigma}(0) = 0$, $\bar{\sigma}(1) = \infty$. Moreover, $\frac{\partial \bar{\sigma}(p)}{\partial p} > 0$ for all $p \in (0,1)$. To see this note that $\frac{\partial \bar{\sigma}(p)}{\partial p} > 0$ if and only if

$$\frac{(k+1)(4m + 16\alpha(1-p))}{1 - p^{k+1}} - \frac{16\alpha(2-3m)p}{2 - 7m - 16\alpha(1-p)} > 0.$$

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Since the LHS is increasing in k, it is sufficient to show that:

$$\frac{(4m + 16\alpha(1 - p))}{1 - p} - \frac{16\alpha(2 - 3m)p}{2 - 7m - 16\alpha(1 - p)} > 0,$$

that is

$$4m(2 - 7m - 16\alpha(1 - p)) + 16\alpha(1 - p)(2 - 7m - 16\alpha(1 - p) - p(2 - 3m)) > 0,$$

which is equivalent to

$$B(p) = 4m(2 - 7m) + 16\alpha(1 - p)((1 - p)(2 - 3m - 16\alpha) - 8m) > 0.$$

Note that:

$$\frac{\partial B(p)}{\partial p} = -32\alpha((1 - p)(2 - 3m - 16\alpha) - 4m),$$

and $\frac{\partial B(p)}{\partial p}\big|_{p=0} = -32\alpha(2 - 7m - 16\alpha) < 0$ where the inequality follows because we are assuming that $2 - 7m - 16\alpha > 0$, while $\frac{\partial B(p)}{\partial p}\big|_{p=1} > 0$. Given these two observations and the fact that $B(p)$ is convex in p, it follows that $B(p)$ is minimized whenever p is such that $(1 - p)(2 - 3m - 16\alpha) = 4m$. In this case $B(p)$ equals $4m(2 - 7m - 16\alpha(1 - p)) > 0$, where the inequality follows from $2 - 7m - 16\alpha > 0$. This proves that $\frac{\partial \bar{\sigma}(p)}{\partial p} > 0$ for all $p \in (0, 1)$.

Since $\bar{\sigma}(0) = 0$, $\bar{\sigma}(1) = \infty$ and $\bar{\sigma}(p)$ is increasing in p, there exists a unique $\bar{p} \in (0, 1)$ such that $\bar{\sigma}(\bar{p}) = 1$, which is given by

$$\bar{p}^{k+1} = \frac{2 - 7m - 16\alpha(1 - \bar{p})}{2 - 3m}. \tag{14}$$

We summarize these observations in the following claim:

Claim 2. For all $\alpha < (2 - 7m)/16$ the following holds: $\bar{\sigma}(0) = 0$, $\bar{\sigma}(\bar{p}) = 1$, $\bar{\sigma}(p)$ is increasing for all $p \in [0, 1]$, and $\bar{p}$ is the solution to (14).

Combining Claim 1 and Claim 2 it follows that for all $\alpha < (2 - 7m)/16$ a mixed-strategy equilibrium exists if and only if $\bar{p} > \underline{p}$, which holds if and only if $\alpha < \alpha^*(k)$, where $\alpha^*(k)$ is defined as in (8). Since $\alpha^*(k) > (2 - 7m)/16$ and by assumption $\alpha < (2 - 7m)/16$, we conclude that for all $\alpha < (2 - 7m)/16$ a symmetric mixed-strategy equilibrium exists.

Case II. Assume that $2 - 7m - 16\alpha < 0$, which is equivalent to $\alpha \geq (2 - 7m)/16$.

Define $\hat{p}$ such that $2 - 7m - 16\alpha(1 - \hat{p}) = 0$, that is $\hat{p} = 1 - (2 - 7m)/(16\alpha) \in (0, 1)$. Note that for all $p \in (0, \hat{p})$, (12) implies that $\bar{\sigma}(p) < 0$. Hence, a necessary condition for an equilibrium is that $p \in [\hat{p}, 1)$.

Let $p \in [\hat{p}, 1)$; note that for all $p \in [\hat{p}, 1)$, the following holds $\bar{\sigma}(p) > 0$, $\bar{\sigma}(\hat{p}) = \infty$ and $\bar{\sigma}(1) = \infty$. Furthermore, there exists a $p = z$ such that $\bar{\sigma}(z) = 1$ if and only if

$$\psi(z, \alpha) = z^{k+1} - \frac{2 - 7m - 16\alpha(1 - z)}{2 - 3m} = 0.$$

Moreover, $\psi(\hat{p}, \alpha) = \hat{p}^{k+1} > 0$, $\psi(1, \alpha) > 0$, and

$$\frac{\partial \psi(z, \alpha)}{\partial z} = (k + 1)z^k - \frac{16\alpha}{2 - 3m},$$

where $\psi(z, \alpha)$ is convex in z. Recall from equation (11) that $\underline{p}$ is such that $\underline{p}^k = 16\alpha/((2 - 3m)(k+1))$. Hence, if $\hat{p} > \underline{p}$, then $\frac{\partial \psi(z, \alpha)}{\partial z} > 0$ for all $z \in (\hat{p}, 1)$, which implies that $\bar{\sigma}(p) > 1$ for all $p \in (\hat{p}, 1)$. Hence, for existence of an equilibrium in mixed strategies it must be the case that $\hat{p} < \underline{p}$.

Suppose that $\hat{p} < \underline{p}$. Then, $\psi(z, \alpha)$ is minimized when $z = \underline{p}$, and a necessary condition for existence of a mixed-strategy equilibrium is that $\bar{\sigma}(p) < 1$ for some p, which requires that

$$\psi(\underline{p}, \alpha) = \frac{16\alpha \underline{p}}{(2 - 3m)(k + 1)} - \frac{2 - 7m - 16\alpha(1 - \underline{p})}{2 - 3m} < 0,$$

which is true if and only if

$$\alpha < \frac{(2 - 7m)(k + 1)}{16(k + 1 - k\underline{p})} = \alpha^*(k).$$

Moreover, $\hat{p} < \underline{p}$ if and only if $1 - \frac{2 - 7m}{16\alpha} < \underline{p}$, which is always true when $\alpha \leq \frac{(2 - 7m)(k + 1)}{16(k + 1 - k\underline{p})} = \alpha^*(k)$.

We summarize these observations in the following claim:

Claim 3. Suppose that $\alpha \in \left((2 - 7m)/16, \frac{(2 - 7m)(k + 1)}{16(k + 1 - k\underline{p})} \right)$ then the following holds: there exists $(p_1, \bar{p})$ such that (i) $\hat{p} < p_1 < \underline{p} < \bar{p} < 1$ and (ii) $\bar{\sigma}(p_1) = \bar{\sigma}(\bar{p}) = 1$ and $\bar{\sigma}(p) \in (0, 1)$ for all $p \in [p_1, \bar{p}]$.

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Combining Claim 1 and Claim 3 it follows that if $\alpha \in ((2 - 7m)/16, \alpha^*(k))$ a mixed-strategy equilibrium exists. Finally, combining Case 1 together with Case 2 it follows that a mixed-strategy equilibrium exists if and only if $\alpha < \alpha^*(k)$.

Final Step: Uniqueness. If $\alpha > \alpha^*(k)$ we know that the equilibrium is in pure strategies and it is unique. So, let $\alpha < \alpha^*(k)$. In this case the equilibrium is in mixed strategies and recall that at the equilibrium $s^* = (\sigma^*, p^*)$, conditions (9) and (10) are mutually satisfied. To prove that there exists a unique equilibrium it is sufficient to prove that at p^* the following is true: (i) $\frac{\partial \bar{\sigma}(p)}{\partial p}\big|_{p^*} > 0$ and (ii) $\frac{\partial \tilde{\sigma}(p)}{\partial p}\big|_{p^*} < 0$. We now show that these two conditions hold.

We first show that $\frac{\partial \bar{\sigma}(p)}{\partial p}\big|_{p^*} > 0$. Note that, using expression (13), it follows that $\frac{\partial \bar{\sigma}(p)}{\partial p}\big|_{p^*} > 0$ if and only if

$$\frac{(k+1)(4m + 16\alpha(1 - p^*))}{1 - p^{*k+1}} - \frac{16\alpha(2 - 3m)p^*}{2 - 7m - 16\alpha(1 - p^*)} > 0. \tag{15}$$

At the equilibrium s^* condition (10) holds, which implies that

$$\rho(e|\emptyset, s^*, k) = \frac{4m + 16\alpha(1 - p^*)}{(1 - p^{*k+1})(2 - 3m)}, \tag{16}$$

and substituting this expression in the equilibrium condition (9), we obtain that in equilibrium

$$\frac{(k+1)(4m + 16\alpha(1 - p^*))}{1 - p^{*k+1}} = \frac{16\alpha(2 - 3m)}{(2 - 4m + \sigma^* m)p^{*k}}.$$

Thus, we can rewrite inequality (15) as follows:

$$16\alpha(2 - 3m)\left(\frac{1}{p^{*k}(2 - 4m + \sigma^* m)} - \frac{p^*}{2 - 7m - 16\alpha(1 - p^*)}\right) > 0,$$

which is satisfied if and only if

$$C(p^*) = 2 - 7m - 16\alpha(1 - p^*) - p^{*k+1}(2 - 4m + \sigma^* m) > 0.$$

To show that the last inequality holds, we observe that

$$\begin{aligned}
\frac{\partial C(p^*)}{\partial p^*} &= 16\alpha - (k+1)p^{*k}(2 - 4m + \sigma^* m) \\
&> 16\alpha - (k+1)\underline{p}(2 - 4m + \sigma^* m) \\
&> 16\alpha - (k+1)\underline{p}(2 - 3m) = 0,
\end{aligned}$$

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where the first inequality follows from $p^* > \underline{p}$, the second inequality follows from $\sigma^* \in (0,1)$, and the last equality follows from the definition of $\underline{p}$. Hence, it is sufficient to show that $C(\underline{p}) > 0$. Using the expression for $\underline{p}$ we obtain

$$C(\underline{p}) = 2 - 7m - 16\alpha(1 - \underline{p}) - \frac{16\alpha\underline{p}(2 - 4m + \sigma^*m)}{(k+1)(2-3m)} > 0$$

whenever

$$(k+1)(2-3m)(2-7m-16\alpha(1-\underline{p})) - 16\alpha\underline{p}(2-4m+\sigma^*m) > 0,$$

which holds because

$$
\begin{aligned}
& (k+1)(2-3m)(2-7m-16\alpha(1-\underline{p})) - 16\alpha\underline{p}(2-4m+\sigma^*m) \\
> \; & (k+1)(2-3m)(2-7m-16\alpha(1-\underline{p})) - 16\alpha\underline{p}(2-3m) \\
= \; & (2-3m)(k+1)(2-7m) - 16\alpha(2-3m)((k+1-k\underline{p})) > 0,
\end{aligned}
$$

where the first inequality follows from $\sigma^* < 1$, and the last inequality follows from $\alpha < (k+1)(2-7m)/(16(k+1-k\underline{p}))$. Hence, $\frac{\partial \tilde{\sigma}(p)}{\partial p}\big|_{p^*} > 0$.

We now show that $\frac{\partial \tilde{\sigma}(p)}{\partial p}\big|_{p^*} < 0$. To see this note that, using expression (16), the equilibrium condition (9) can be rewritten as follows:

$$(k+1)p^{*k}\frac{4m+16\alpha(1-p^*)}{(2-3m)(1-p^{*k+1})} = \frac{16\alpha}{2-4m+\sigma^*m}, \tag{17}$$

and to establish that $\frac{\partial \tilde{\sigma}(p)}{\partial p}\big|_{p^*} < 0$ it is sufficient to show that the LHS is increasing in p^*. Since $(k+1)p^{*k}$ is obviously increasing in p^*, we need to show that $\frac{4m+16\alpha(1-p^*)}{(2-3m)(1-p^{*k+1})}$ is increasing in p^*, which, taking the derivative with respect to p, holds if and only if

$$\frac{(k+1)p^{*k}(4m+16\alpha(1-p^*))}{1-p^{*k+1}} - 16\alpha > 0.$$

Using (17) we have that

$$
\begin{aligned}
& \frac{(k+1)p^{*k}(4m+16\alpha(1-p^*))}{1-p^{*k+1}} - 16\alpha \\
= \; & 16\alpha\left(\frac{2-3m}{2-4m+\sigma^*m} - 1\right) \\
= \; & \frac{16\alpha m(1-\sigma^*)}{2-4m+\sigma^*m} > 0.
\end{aligned}
$$

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Combining these observations, it follows that there exists a unique equilibrium.

To complete the proof of Proposition 1 we show that $\alpha^*(k)$ is increasing in k. Note that

$$\frac{d\alpha^*(k)}{dk} = \frac{\partial\alpha^*(k)}{\partial k} + \frac{\partial\alpha^*(k)}{\partial \underline{p}}\frac{d\underline{p}}{dk},$$

where

$$\frac{d\underline{p}}{dk} = -\frac{1}{k\underline{p}^{k-1}}\left[\underline{p}^k\ln(\underline{p}) + \frac{16\alpha}{(2-3m)(k+1)^2}\right]$$

Using this last expression, and developing $\frac{\partial\alpha^*(k)}{\partial k}$ and $\frac{\partial\alpha^*(k)}{\partial \underline{p}}$, we have that

$$\frac{d\alpha^*(k)}{dk} = \frac{(2-7m)\underline{p}}{16(k+1-k\underline{p})^2}\left[1-\underline{p}^{k-1}-(k+1)\ln(\underline{p})\right] > 0.$$

This completes the proof of Proposition 1.■

Proof of Proposition 2. Let $A(p,\alpha) = 4m+16\alpha(1-p)$, $B(p,\alpha) = 2-7m-16\alpha(1-p)$ and $C(\sigma(p)) = 2-4m+\sigma(p)m$. With some abuse of notation we shall write A instead of $A(p,\alpha)$, and similarly for B and C. Recall that in equilibrium:

$$f(\sigma(p),p) - 16\alpha = \frac{(k+1)p^k AC}{(2-3m)(1-p^{k+1})} = 0$$

$$\sigma(p) = \frac{Ap^{k+1}}{B(1-p^{k+1})}.$$

We first prove the first part of the proposition, i.e., $x^*_{k+1}(m) < x^*_k(m)$, $\Psi(s^*_{k+1}, k+1) > \Psi(s^*_k, k)$ and $\pi(s^*_{k+1}|e,m) > \pi(s^*_k|e,m)$. To see this note that:

$$\frac{\partial f(\sigma(p),p)}{\partial k} = \frac{Ap^k\left(C(1-p^{k+1}+(k+1)\ln(p)) + (k+1)m(1-p^{k+1})\frac{\partial\sigma(p)}{\partial k}\right)}{(2-3m)(1-p^{k+1})^2}$$

$$\frac{\partial\sigma(p)}{\partial k} = \frac{Ap^{k+1}}{B(1-p^{k+1})^2}\ln(p).$$

Clearly, $\frac{\partial\sigma(p)}{\partial k} < 0$, and therefore to show that $\frac{\partial f(\sigma(p),p)}{\partial k} < 0$ it is sufficient to note that $(1-p^{k+1}+(k+1)\ln(p)) < 0$ for all $p \in (0,1)$. Recall that, in equilibrium, $\frac{\partial f(\sigma(p),p)}{\partial p} > 0$. Combining these two observations it follows that for all $\alpha < \alpha^*(k)$, an increase in k

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increases p, i.e., $x_{k+1}^* < x_k^*$. By investigation of equilibrium condition 3, it is immediate to see that if $x_{k+1}^* < x_k^*$, then $1 - \Psi(s_{k+1}^*, k+1) < 1 - \Psi(s_k^*, k)$, which implies that $\Psi(s_{k+1}^*, k+1) > \Psi(s_k^*, k)$. Moreover, since $\pi(s_k^*|e, m) = 1/2 - (1 - \Psi(s_k^*, k))/8$, it follows that $\pi(s_{k+1}^*|e, m) > \pi(s_k^*|e, m)$.

We now prove the second part of the proposition, i.e., for sufficiently low α, if k increases then σ increases and polarization increases. First, note that:

$$\frac{d\sigma(p)}{dk} = \frac{\partial f(\sigma(p), p)}{\partial p} \frac{\partial \sigma(p)}{\partial k} - \frac{\partial f(\sigma(p), p)}{\partial k} \frac{\partial \sigma(p)}{\partial p},$$

where

$$\frac{\partial f(\sigma(p), p)}{\partial p} = \frac{(k+1)p^{k-1}\left(AC(k + p^{k+1}) + p(1 - p^{k+1})Am\frac{\partial \sigma(p)}{\partial p} - 16C\alpha p(1 - p^{k+1})\right)}{(2 - 3m)(1 - p^{k+1})^2}$$

$$\frac{\partial \sigma(p)}{\partial p} = \frac{p^k}{B(1 - p^{k+1})}\left(\frac{A(k+1)}{1 - p^{k+1}} - \frac{16\alpha p(2 - 3m)}{B}\right).$$

Using these expressions and noting that as $\alpha \to 0$ then $p \to 0$, and A, B, and C are bounded, it follows that:

$$\lim_{\alpha \to 0} \frac{d\sigma(p)}{dk} = \lim_{\alpha \to 0}\left(\frac{A^2 C(k+1)}{B(2 - 3m)}\right)\lim_{\alpha \to 0}\left(-p^{2k}\ln(p)\right) = 0^+.$$

Hence, there exists a $\hat{\alpha}(k) \in (0, \alpha^*(k))$ such that for all $\alpha \in (0, \hat{\alpha}(k))$ an increase in k leads to an increase in σ.

We finally show that for sufficiently small α,

$$\Pi(s^*, k) = \sigma^{*2} + 2\sigma^*(1 - \sigma^*)\pi(s^*|e, m),$$

is increasing in k. Note that for small α, $\sigma^* < 1/2$ and therefore $\Pi(s^*, k)$ is increasing in σ, keeping constant $\pi(s^*|e, m)$. Since an increase in k leads to an increase in $\pi(s^*|e, m)$, the result follows. This concludes the proof of the proposition. ∎

Appendix B. This appendix contains the proofs of Proposition 3 and Proposition 4. Furthermore, we provide an additional result in which we show that the equilibria described

in Proposition 3 are also equilibria of a more general model in which parties can target political advertising to different groups of voters. This result is contained in Proposition 5 below.

In what follows, with some abuse of notation, we shall denote by $i_{j,in}$ the indifferent group-j voter that has sampled only voters belonging to group j, $j = l, r$; analogously, $i_{j,out}$ is the indifferent group-j voter that has sampled at least one voter in group j', where $j, j' = l, r$ and $j \neq j'$.

Proof of Proposition 3. We start by considering symmetric pure strategy equilibria. Note that in equilibrium a party never advertises an extremist candidate. Also note that a (pure) strategy profile in which each party selects a moderate and does not advertise cannot be part of equilibrium; for otherwise, a party, by switching to an extremist candidate, would win the election with the same probability and would obtain a higher expected payoff, which contradicts optimality. We are then left with two possible candidates: (1.) parties select extremist candidates and do not advertise and (2.) parties select moderates and do advertise. We now analyze these two possibilities.

(1.) Consider a strategy profile s^* such that $\sigma = 1$ and $x = 0$. In this case the utility of party L is $U_L^*(s^*) = -(1-m)/2$. It is easy to check that the best deviation of party L is to select a moderate and to advertise. Let s_L be such strategy profile. The utility of party L is then

$$U_L(s_L, s_R^*) = \pi_L(s_L, s_R^*|m, e)\left(1 - \frac{3m}{2}\right) - (1-m) - \alpha.$$

We now derive $\pi_L(s_L, s_R^*|m, e)$. First, all group-l voters observe that $t_L = m$, and, regardless of the realization of the sampling, they believe that $t_R = e$. Therefore, $i_{l,in} = i_{l,out} = 1/2 + m/4$. Second, all group-$r$ voters believe that $t_R = e$, a fraction β always samples group-r voters so that they believe that $t_L = e$, while the remaining group-r voters have sampled at least one voter in group l and therefore they know that $t_L = m$. Thus, $i_{r,in} = 1/2$, while $i_{r,out} = 1/2 + m/4 = i_{l,in}$. Putting together these facts it follows that for all $\mu > i_{l,in} = i_{r,out}$ party L never wins, and that for all $\mu \leq i_{l,in}$ party L wins with probability 1. Hence, $\pi_L(s_L, s_R^*|m, e) = \Pr[\mu \leq i_{l,in}] = 5/8$. Therefore, s^* is equilibrium if and only if

$$U_L(s_L, s_R^*) = \frac{5}{8}\left(1 - \frac{3m}{2}\right) - (1-m) - \alpha \leq U_L^*(s^*) = -(1-m)/2,$$

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which is satisfied if and only if

$$\alpha \geq \frac{2 - 7m}{16} = \bar{\alpha}. \tag{18}$$

(2.) Next, consider a strategy profile s^* such that $\sigma = 0$ and $x = 1$. In this case $U_L^*(s^*) = -(1-m)/2 - \alpha$. Suppose that if a group-l voter does not observe the ads from party L he believes that $t_L = e$; analogously for group-r voters. Clearly, the best deviation of party L is to select an extremist candidate and do not advertise. Let s_L be such strategy. The utility of party L is

$$U_L(s_L, s_R^*) = \left(1 - \frac{3m}{2}\right)[\pi_L(s_L, s_R^*|e, m) - 1].$$

We now derive $\pi_L(s_L, s_R^*|e, m)$. First, all group-l voters observe that party L does not advertise and therefore they believe that $t_L = e$; also, regardless of the sampling, all group-l voters believe that $t_R = m$. Hence, $i_{l,in} = i_{l,out} = 1/2 - m/4$. Second, all group-$r$ voters believe that $t_R = m$, a fraction β of group-r voters believe that $t_L = m$, while the remaining voters believe that $t_L = e$. Hence, $i_{r,in} = 1/2$, while $i_{r,out} = 1/2 - m/4 = i_{l,in}$. Using these considerations, three facts follow. One, for all $\mu \geq i_{r,in}$ party L never wins; two, for all $\mu \leq i_{r,out}$ party L wins with probability 1. Three, for all $\mu \in [i_{r,out}, i_{r,in}]$, total votes for party L are

$$TV_L = \frac{1}{2} + \frac{1+\beta}{2\tau}\left(\frac{1}{2} - \mu\right) - \frac{1}{\tau}\frac{m}{8},$$

and $TV_L > 1/2$ if and only if

$$\mu \leq \frac{1}{2} - \frac{m}{4}\frac{1}{1+\beta} = \mu^*,$$

where it is easy to check that $\mu^* \in [i_{r,out}, i_{r,in}]$. Combining these three facts we have that

$$\pi_L(s_L, s_R^*|e, m) = \Pr\left[\mu \leq \frac{1}{2} - \frac{m}{4}\frac{1}{1+\beta}\right] = \frac{3 + 4\beta}{8(1+\beta)}.$$

Hence, s^* is an equilibrium if and only if

$$U_L(s_L, s_R^*) = \left(1 - \frac{3m}{2}\right)\left(\frac{3 + 4\beta}{8(1+\beta)} - 1\right) \leq -\frac{1-m}{2} - \alpha,$$

which is satisfied if and only if

$$\alpha \leq \frac{2 - 7m - 4m\beta}{16(1+\beta)} = \underline{\alpha}(\beta). \tag{19}$$

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It is easy to verify that $\underline{\alpha}(\beta) < \bar{\alpha}$ and that $\underline{\alpha}(\beta) \geq 0$ if and only if $\beta \leq (2 - 7m)/(4m)$.

We now turn to symmetric mixed-strategy equilibria. Clearly, the only candidate is a strategy profile s^* in which each party selects an extremist candidate with probability $\sigma \in (0, 1)$ and advertises only moderates. Randomization implies that a party is indifferent between selecting an extremist and a moderate, which holds if and only if:

$$\pi_L(s^*|m, e) = \frac{1 - m + 2\alpha}{2 - 3m}. \tag{20}$$

We now derive the expression for $\pi_L(s^*|m, e)$. Suppose that $t_L = m$ and $t_R = e$. First, a fraction β of group-l voters believe that $t_L = m$ and, with probability σ, that $t_R = e$. The remaining fraction $1 - \beta$ believe that $t_L = m$ and that $t_R = e$. Hence, $i_{l,in} = 1/2 + m\sigma/4$, while $i_{l,out} = 1/2 + m/4$. Second, all group-r voters believe that $t_R = e$, a fraction β believe that $t_L = e$ with probability σ, and a fraction $1 - \beta$ believe that $t_L = m$. Hence, $i_{r,in} = 1/2 + m(1 - \sigma)/4$, while $i_{r,out} = 1/2 + m/4 = i_{l,out}$.

Given these observations there are two relevant cases to be considered: (1.) $\sigma \leq \frac{1}{2}$ and (2.) $\sigma \geq \frac{1}{2}$.

Case 1. Suppose $\sigma \leq 1/2$; then $i_{l,in} \leq i_{r,in} < i_{l,out} = i_{r,out}$. It follows that for all $\mu \leq i_{l,in}$ party L wins with probability 1. For all $\mu \in [i_{l,in}, i_{r,in}]$, total votes of party L are

$$TV_L = \frac{1}{2} + \frac{1 + \beta}{2\tau}\left(\frac{1}{2} - \mu\right) + \frac{1}{2\tau}\frac{m}{4},$$

and $TV_L \geq 1/2$ if and only if

$$\mu \leq \frac{1}{2} + \frac{m}{4}\frac{1}{1 + \beta} = \mu_1^*,$$

where $\mu_1^* \geq i_{l,in}$, and $\mu_1^* \leq i_{r,in}$ if and only if $\sigma \leq \beta/(1+\beta)$. Therefore, for all $\mu \in [i_{l,in}, i_{r,in}]$, if $\sigma \leq \beta/(1 + \beta)$ party L wins with probability $\Pr(\mu \leq \mu_1^*) = (5 + 4\beta)/(8(1 + \beta))$, while if $\sigma \geq \beta/(1 + \beta)$ then party L wins with probability 1. For all $\mu \in [i_{r,in}, i_{l,out}]$, total votes of party L are

$$TV_L = \frac{1}{2} + \frac{1}{2\tau}\left(\frac{1}{2} - \mu\right) + \frac{1}{2\tau}\frac{m}{4}(1 - \beta(1 - \sigma)),$$

and $TV_L \geq 1/2$ if and only if

$$\mu \leq \frac{1}{2} + \frac{m}{4}(1 - \beta(1 - \sigma)) = \mu_2^*.$$

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Clearly, $\mu_2^* \leq i_{l,out}$, and $\mu_2^* \geq i_{r,in}$ if and only if $\sigma \geq \beta/(1+\beta)$. Hence, for all $\mu \in [i_{r,in}, i_{l,out}]$, if $\sigma \leq \beta/(1+\beta)$, party L never wins, while if $\sigma \geq \beta/(1+\beta)$, party L wins with probability $\Pr(\mu \leq \mu_2^*) = (5 - \beta(1-\sigma))/8$. Finally, for all $\mu \geq i_{l,out}$ party L never wins. Summarizing, we have that: if $\sigma \leq \beta/(1+\beta)$ then $\pi_L(m,e|s^*) = (5+4\beta)/(8(1+\beta))$, while if $\sigma \in [\beta/(1+\beta), 1/2]$ then $\pi_L(m,e|s^*) = (5 - \beta(1-\sigma))/8$.

Case 2. Suppose $\sigma \geq 1/2$; then $i_{r,in} \leq i_{l,in} < i_{l,out} = i_{r,out}$. Here note that for all $\mu \leq i_{l,in}$ party L wins with probability 1. In contrast, for all $\mu \geq i_{l,out}$ party L never wins. Finally, for all $\mu \in [i_{l,in}, i_{l,out}]$, total votes of party L are

$$TV_L = \frac{1}{2} + \frac{1}{2\tau}\left(\frac{1}{2} - \mu\right) + \frac{1}{2\tau}\frac{m}{4}\left(1 - \beta(1-\sigma)\right),$$

and $TV_L \geq 1/2$ if and only if

$$\mu \leq \mu_2^*,$$

where it is easy to check that $\mu_2^* \in [i_{l,in}, i_{l,out}]$. Hence, for all $\mu \in [i_{l,in}, i_{l,out}]$, party L wins with probability $\Pr(\mu \leq \mu_2^*) = (5 - \beta(1-\sigma))/8$. Combining these observations it follows that if $\sigma \in [1/2, 1)$ then $\pi_L(s^*|m,e) = (5 - \beta(1-\sigma))/8$.

By combining case 1 and case 2, it follows that if $\sigma \in (0, \beta/(1+\beta)]$ then $\pi_L(s^*|m,e) = (5+4\beta)/(8(1+\beta))$, and if $\sigma \in [\beta/(1+\beta), 1)$ then $\pi_L(s^*|m,e) = (5 - \beta(1-\sigma))/8$.

Next, note that a mixed-strategy equilibrium exists only if $\sigma \in [\beta/(1+\beta), 1)$. Indeed, if $\sigma < \beta/(1+\beta)$, then $\pi_L(s^*|m,e)$ does not depend on σ. Therefore, the equilibrium condition (20) cannot be satisfied generically. Therefore, it must be that $\sigma \in [\beta/(1+\beta), 1)$, and the equilibrium condition (20) holds if and only if

$$\sigma^* = 1 - \frac{2 - 7m - 16\alpha}{\beta(2 - 3m)}. \tag{21}$$

Note that σ^* is increasing in α and, when $\alpha = \bar{\alpha}$ then $\sigma^* = 1$. Also if $\underline{\alpha}(\beta) \geq 0$, we have that at $\alpha = \underline{\alpha}(\beta)$, $\sigma^* = \beta/(1+\beta)$. If instead $\underline{\alpha}(\beta) \leq 0$ then as α approaches 0, σ^* converges to $1 - (2 - 7m)/(\beta(2 - 3m)) \geq \beta/(1+\beta)$, where the last inequality follows from the fact that $\underline{\alpha}(\beta) \leq 0$ (i.e., $\beta < (2 - 7m)/(4m)$). This concludes the proof of Proposition 3. ∎

Proof of Proposition 4. It is immediate to see that an increase in β, increases the probability that a party selects an extremist candidate. We now show that an increase in β

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it increases the ex-ante expected probability that an extremist candidate wins the election. To see this note that:

$$\Pi(s^*) = [\sigma^*]^2 + 2\sigma^*(1 - \sigma^*)\pi(s^*|e, m),$$

and therefore

$$\frac{d\Pi(s^*)}{d\beta} = 2\left((\sigma^* + \pi(s^*|e, m)(1 - 2\sigma^*))\frac{d\sigma^*}{d\beta} + \sigma^*(1 - \sigma^*)\frac{d\pi(s^*|e, m)}{d\beta}\right).$$

Since in equilibrium $\pi_L(s^*|m, e) = (1 - m + 2\alpha)/(2 - 3m)$, and $\pi(s^*|e, m) = 1 - \pi(s^*|m, e)$, it must be that $d\pi(s^*|e, m)/d\beta = 0$. Hence, it follows that:

$$\frac{d\Pi(s^*)}{d\beta} = 2\frac{d\sigma^*}{d\beta}\left(\sigma^* + \pi(s^*|e, m)(1 - 2\sigma^*)\right)$$

$$= 2\frac{d\sigma^*}{d\beta}\left(\sigma^*(1 - \pi(s^*|e, m)) + \pi(s^*|e, m)(1 - \sigma^*)\right) > 0.$$

This concludes the proof of Proposition 4. ∎

We now consider a model where parties choose whether to advertise to group l, to group r, to both groups, or not to advertise. Abusing notation, assume that advertising to one group costs α, while advertising to both groups costs 2α. The following proposition shows that the equilibria described in Proposition 3 are also equilibria of this model.

Proposition 5 *Suppose parties can target advertisement to group l, or group r, or to both groups. The following holds: (I.) If $\alpha > \bar{\alpha}$ there exists an equilibrium where parties select extremist candidates with probability one and they never advertise. (II.) If $\alpha \in (0, \max[0, \underline{\alpha}(\beta)])$ there exists an equilibrium in which parties select moderates and they only advertise to their closer group of independents. (III.) For every β, there exists $\alpha^* \in (\max[0, \underline{\alpha}(\beta)], \bar{\alpha})$, such that if $\alpha \in (\alpha^*, \bar{\alpha})$, in equilibrium parties select extremist candidates with probability σ^* and they advertise a moderate only to their closer group of independents, where σ^* is given by equation (5).*

Proof of Proposition 5. Consider the strategy s^* in which parties select extremist candidates with probability one and they never advertise. From Proposition 3 we know that for all $\alpha > \bar{\alpha}$ the following deviation is not profitable: a party selects a moderate

and advertises only to its own group. It is then sufficient to show that this is indeed the best deviation for a party. To see this, first note that if party L selects a moderate and advertises only to its own group, the probability of winning is $\Pr(\mu \leq 1/2 + m/4)$. Second, suppose that party L deviates by selecting a moderate and by advertising to group r only. It is immediate to check that in this case the probability of winning of party L will be lower than $\Pr(\mu \leq 1/2 + m/4)$. Therefore, the latter deviation is at least as good as the deviation considered above. The other possible deviation is one in which party L selects a moderate and advertises to both groups. Again, it is a matter of simple algebra to check that the probability of winning under this deviation cannot be higher than $\Pr(\mu \leq 1/2 + m/4)$. However, this deviation involves higher costs of advertising.

Consider now the strategy s^* in which parties select moderates and advertise to their own group. If $\underline{\alpha}(\beta) \leq 0$ then Proposition 3 implies that this is not an equilibrium. If instead $\underline{\alpha}(\beta) > 0$, Proposition 3 implies that for all $\alpha \leq \underline{\alpha}(\beta)$ the following deviation is not profitable: a party selects an extremist candidate and does not advertise. Moreover, if a party deviates from s^* by advertising to both groups, then simple algebra delivers that the party will face the same probability of winning as in s^*, and it will face an higher cost. Hence, we only need to check the following deviation s^d: party L selects a moderate and advertises only to group-r voters. However, note that s^* and s^d are cost equivalent, but the probability of winning of party L under s^d is lower than under s^*.

We now consider the mixed-strategy equilibrium defined in Proposition 3. We already know that for all $\alpha \in (\max[0, \underline{\alpha}(\beta)], \bar{\alpha})$ there exists a σ, defined by equation (5), such that parties are indifferent between selecting an extremist and selecting a moderate which they advertise only to their closer group. Suppose party R follows this strategy s_R^*. We consider the possible deviations of party L.

Deviation 1: consider the following strategy s_L: party L selects a moderate and advertises to group r only. In order to derive $\pi_L(s_L, s_R^* | m, e)$, note that a fraction β of group-l voters believe that $t_L = e$ and, with probability σ, that $t_R = e$, while the remaining fraction of group-l voters believe that $t_L = m$ and $t_R = e$. Hence, $i_{l,in} = 1/2 - m(1 - \sigma)/4$ and $i_{l,out} = 1/2 + m/4$. All group-r voters believe that $t_L = m$ and $t_R = e$, so that $i_{r,in} = i_{r,out} = 1/2 + m/4$. It follows that $\pi_L(s_L, s_R^* | m, e) = \Pr(\mu \leq 1/2 + (m/4)(1 - \beta(1 - \sigma))/(1 + \beta)) < \pi_L(s^* | m, e) = \Pr(\mu \leq 1/2 + (m/4)(1 - \beta(1 - \sigma)))$.

Next, we derive $\pi_L(s_L, s_R^*|m, m)$. In this case, a fraction β of group-l voters believe that $t_L = e$ and, with probability σ, that $t_R = e$. The remaining fraction of group-l voters believe that $t_L = t_R = m$. Hence $i_{l,in} = 1/2 - m(1 - \sigma)/4$ and $i_{l,out} = 1/2$. All group-r voters believe that $t_L = t_R = m$ and therefore $i_{r,in} = i_{r,out} = 1/2$. It is now easy to check that $\pi_L(s_L, s_R^*|m, m) \leq 1/2 = \pi_L(s^*|m, m)$. Putting together these two facts, it follows that deviation 1 is not profitable.

Deviation 2: consider the following strategy s_L: party L selects a moderate and advertises to both groups. We first derive $\pi_L(s_L, s_R^*|m, e)$. A fraction β of group-l voters believe that $t_L = m$ and, with probability σ, that $t_R = e$. The remaining fraction believe that $t_L = m$ and $t_R = e$. Hence, $i_{l,in} = 1/2 + m\sigma/4$ and $i_{l,out} = 1/2 + m/4$. All group-r voters believe that $t_L = m$ and $t_R = e$; therefore $i_{r,in} = i_{r,out} = 1/2 + m/4$. It is now easy to check that $\pi_L(s_L, s_R^*|m, e) = \Pr(\mu \leq 1/2 + (m/4)(1 + \beta\sigma)/(1 + \beta))$, which is now bigger than $\pi_L(s^*|m, e)$. We now derive $\pi_L(s_L, s_R^*|m, m)$. A fraction β of group-l voters believe that $t_L = m$ and, with probability σ, that $t_R = e$. The remaining fraction believe that $t_L = t_R = m$. Hence, $i_{l,in} = 1/2 + m\sigma/4$ and $i_{l,out} = 1/2$. All group-r voters believe that $t_L = t_R = m$ and therefore $i_{r,in} = i_{r,out} = 1/2$. It follows that $\pi_L(s_L, s_R^*|m, m) = 1/2 = \pi_L(s^*|m, m)$. Using these two facts we have that deviation 2 is not profitable if and only if

$$\alpha \geq \sigma^* \left(1 - \frac{3m}{2}\right) [\pi_L(s_L, s_R^*|m, e) - \pi_L(s^*|m, e)].$$

Given that the right hand side of the above inequality, which is a function of α, vanishes as α approaches $\bar{\alpha}$, there exists $\alpha^* \in (\max[0, \underline{\alpha}(\beta)], \bar{\alpha})$ such that the inequality holds if $\alpha > \alpha^*$. ∎

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