

Perjury versus Truth-Revelation: Quantity or Quality of Testimony

Winand Emons*

University of Bern and CEPR

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Abstract

In trials witnesses often slant their testimony to advance their interests. To obtain truthful testimony, courts rely on perjury rules. We show that truth-revelation is possible and that perjury rules are not truth-revealing. If the judge uses a truth-revealing mechanism, he will get little testimony because the defendant will not present a witness with unfavorable news; however, testimony is of high quality. Under perjury the court gets a different amount of testimony with lower informational content. A court striving for precision prefers truth-revelation to perjury. If the court is rational in the Bayesian sense, chances for the defendant to prevail are the same under perjury and truth-revelation from an *ex ante* point of view. Truth-revelation thus dominates perjury even when the different quantity of testimony is taken into account.

Keywords: litigation process, witness, truth-revelation, mechanism design, perjury rule.

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*Universität Bern, Volkswirtschaftliches Institut, Abteilung für Wirtschaftstheorie, Gesellschaftsstrasse 49, CH-3012 Bern, Switzerland, winand.emons@vwi.unibe.ch, www.vwi.unibe.ch/theory/emons_e.htm. I thank Bob Cooter, Dirk Engelmann, Claude Fluet, Thomas Liebi, Georg Nöldeke, and Gerd Weinrich for helpful comments.

1. Introduction

Witnesses often have a material interest in the court's judgment. For example, the plaintiff and defendant are interested in the stakes in the dispute, and an expert has an interest in future employment as a witness.¹ In deciding legal disputes, courts must rely on observers to report facts and experts to provide opinions. The interest of the witness in the case provides an incentive to distort testimony. To obtain undistorted testimony, witnesses must face legal sanctions for distortions that offset the gain.

The law relies on cross-examination under the threat of prosecution for perjury to deter distorted testimony. Cross-examination probes the quality of testimony by the witness, searching for internal inconsistencies or contradictions with testimony by other witnesses. In a criminal trial for perjury, the plaintiff must prove that the defendant lied or recklessly disregarded the truth.² If the law allows civil liability for false testimony, the plaintiff in a civil case usually must prove something similar to what the prosecutor must prove in a criminal case for perjury.

Establishing guilt or liability often requires more information than anyone can prove in court, so perjury trials or civil trials of false witnesses are rare. In practice, a skillful witness can slant testimony without fear of prosecution or liability. Moreover, as we show formally, even when all this information is available, perjury rules and rules of civil liability for false testimony are not perfectly truth-revealing.

We consider a model where the outcome of a case depends on the likelihood the court attaches to four states of nature; the states represent very good, good, bad, and very bad news for the defendant. To illustrate, a drug may have no, minor, major, and lethal side-effects. We assume that the defendant's (the pharmaceutical company's) chances to prevail are linear in the probabilities that the court attaches to these events after collecting all the facts and opinions. The defendant will present evidence in the courtroom so as to maximize the probability to prevail.

¹For the rapid growth of economists acting as expert witnesses see Posner (1999a), Thornton and Ward (1999), Mandel (1999), and Slottje (1999). This form of consulting is now designated "forensic economics." Several associations such as, e.g., the National Association of Forensic Economics (NAFE) as well as a couple of journals like, e.g., the Journal of Forensic Economics have emerged due to this boom in the demand for economists as experts.

²According to the Model Penal Code, perjury requires testimony in court under oath that is false and material. In addition, perjury requires knowledge that the assertion was false when made, or, possibly, that the defendant recklessly disregard for the truth. U.S. law closely resembles the Model Penal Code. False testimony in an American court cannot support a civil suit for damages, so a victim of slander or libel in court has no legal remedy. For details see Cooter and Emons (2004).

The defendant may find a witness who observes the true state of nature. The witness testifies in court on her observation. Later the court receives a noisy signal about the witness's observation. The court uses this signal to possibly sanction the witness.

We first derive a mechanism that is fully truth-revealing: given the witness testifies in the courtroom, she will tell the truth. This task is straightforward if we impose no further restrictions on the mechanism. Then we show that within our model perjury rules are not truth-revealing; they are at best partially truth-revealing, i.e., they elicit the truth for some, but not for all states of nature.³

The at first glance desirable property of truth-revelation creates, however, a problem if the defendant decides whether or not to present the witness. If the witness has unfavorable news for the defendant and the witness reports this truthfully in court under the truth-revealing mechanism, the defendant does better not to present the witness in the first place. Accordingly, under truth-revelation only witnesses with good news will be presented in the courtroom.

Under the perjury rule as a partially truth-revealing mechanism, witnesses will report good news in the courtroom even when the actual news is bad. Now two things may happen. The defendant may present a witness in the courtroom even when the actual news is bad because the witness lies under perjury. Then we observe a lot of low quality testimony under perjury. Interestingly, the defendant may also withhold a witness with favorable news: if the witness reports the good news also for many bad states, the court considers the bad states very likely and the defendant does better not to present the witness. If the witness has good news, the defendant curses that the witness would make the same report for bad states; the good report is inflated. Then perjury gives rise to very little testimony which is, however, of high quality; no testimony, in contrast, has very little informational content. Thus, if we take the defendant's decision to present a witness into account, we see that quality and quantity of testimony are not independent.

The purpose of this paper is to analyze this dependence. We assume that the court is not interested in who wins the case. The court wants to make the "right" decision; to do so, the court wants as much information as possible. The court thus strives for precision, which we measure by the entropy.

We distinguish between the fully truth-revealing mechanism and partially truth-revealing mechanisms, such as, e.g., perjury rules. For each mechanism we specify under which conditions the defendant will present the witness in court and what

³To allow for a fair comparison, in the Appendix we derive a truth-revealing mechanism working under the same set of restrictions as the perjury rule. These restrictions make the derivation of the truth-revealing mechanism somewhat complicated.

the witness will report. The rational court takes both decisions into account when he updates his beliefs about the likelihoods of the states of nature.

To illustrate, if under the truth-revealing mechanism the witness is presented in the courtroom and reports good news, the court knows that this message is true. If, however, the defendant presents no witness under the truth-revealing mechanism, the court knows that either the defendant found no witness or that he found a witness who observed bad news.

If, in contrast, the court uses a mechanism providing no incentives at all, the witness will always report good news for the defendant in the courtroom, irrespective of his actual observation. The defendant will thus always present the witness and the court will ignore her testimony altogether, i.e., the court will not update his prior beliefs.

For each possible report of the witness we compute the a posteriori probability distribution over the four states of nature and the corresponding value of the entropy, summarizing the precision obtained by the court. Then we compute the expected value of the entropy where we take the expectation over the four states of nature, reflecting the idea that the court has to choose a mechanism (a rule) from an ex ante point of view. In Proposition 1 we show that the court's ranking over the mechanisms is monotonic: the court prefers the fully truth-revealing mechanism over partially truth-revealing mechanisms over mechanisms giving no incentives at all.

In a next step we analyze how the defendant ranks the different mechanisms. Here we also take the ex ante point of view and compute the expected probability to win the case, where we take the expectation at the time before the defendant has found a witness and, therefore, has the same priors as the court over the different states of nature. Our result is that the expected probability to prevail is the same for all mechanisms. The defendant is, therefore, indifferent as to which mechanism is used. This result holds because we model the probability to prevail as a linear function of the court's assessments and, moreover, the court updates his beliefs rationally.

The literature of applying mechanism design to courts is rather small. Sanchirico (2000, 2001) investigates the role of evidence production in the regulation of private behavior via judicial and administrative process. Bernardo, Talley, and Welch (2000) analyze how legal presumptions can mediate between costly litigation and ex ante incentives. Dewatripont and Tirole (1999) and Shin (1998) compare the adversarial with inquisitorial procedures in arbitration. Daughety and Reinganum (2000a) model the adversarial provision of evidence as a game in which two parties engage in strategic sequential search. Daughety and Reinganum (2000b) use axiomatic and Bayesian methods to model information and decisions in a hierarchical judicial system; axioms represent constraints that rules

of evidence impose at the trial. Miller (2001) shows that when the court has information when the witness testifies and information that surfaces thereafter, perjury rules should give greater weight to the latter. All of these papers are of different focus than ours.

Closest to this paper are Cooter and Emons (2003, 2004). There we look at the problem of inducing a witness who is presented in the courtroom to tell the truth. To justify that the witness is presented by the defendant in the first place, there we assume that any observation of the witness is good for the defendant, some observations are, however, better than the others. There we describe in great detail the class of truth-revealing mechanisms, whether they are individually rational, and we distinguish between interested and neutral witnesses. Here we look only at interested witnesses and, more importantly, we take bad observations into account. The three papers should be seen as complements.

In the next section we describe the basic model. In section 3 we derive a truth-revealing and the perjury mechanisms. In the subsequent section we describe the dependence between quantity and quality of testimony. Section 5 concludes.

2. The Model

A court has to decide a case. The decision depends on the probability the court attaches to the four states $X \in \{A, B, C, D\}$. The four states are mutually exclusive, meaning that only one of them will be realized.⁴ The state A means very good news for the defendant, B means good news, C bad news, and D means very bad news.⁵ To illustrate, in an antitrust case A might mean “the defendant’s market share is below 20%”, B “the market share is between 20% and 50%”, C “the market share is in the range of 50% and 80%”, and D “the market share is above 80%”; in a liability suit A might be “the defendant was certainly not negligent”, B “the defendant was likely not negligent”, C “the defendant was negligent and the plaintiff was probably contributorily negligent”, and D “only the defendant was negligent”. To keep matters simple, we assume that a priori the court considers the four events equally likely, i.e., $P(A) = P(B) = P(C) = P(D) = 1/4$.

The probability that the defendant prevails, $P(\text{win})$, depends on the probabilities the court attaches to the four states after collecting all the facts and opinions, $P(X|\cdot)$ with $\sum_{X \in \{A, B, C, D\}} P(X|\cdot) = 1$. As an interpretation think of a judge who

⁴Four is the minimum number of states such that perjury is not fully truth-revealing. See Section 3.

⁵In a trial, news that favors one party disfavors the other party. We will view testimony from the viewpoint of one party, we will take the defendant, and scale values accordingly.

draws at the end of the trial from an urn with $P(\text{win})$ “win-” and $(1 - P(\text{win}))$ “lose-” balls.⁶

Specifically, we assume

$$P(\text{win}) = \mathbf{a}P(A|\cdot) + \mathbf{b}P(B|\cdot) + \mathbf{c}P(C|\cdot) + \mathbf{d}P(D|\cdot)$$

with $1 \geq \mathbf{a} > \mathbf{b} > \mathbf{c} > \mathbf{d} \geq 0$. This mapping from $S^3 \mapsto [\mathbf{d}, \mathbf{a}]$ has the following properties. Suppose $\mathbf{a} = 1$ and $\mathbf{d} = 0$. Then the two states A and D alone can determine the outcome: if $P(A|\cdot) = 1$, the defendant wins for sure, whereas he loses for sure if $P(D|\cdot) = 1$. If $\mathbf{a} < 1$ and $\mathbf{d} > 0$, other factors also play a role in the court’s decision: even when $P(A|\cdot) = 1$, the defendant does not win for sure but only with probability $\mathbf{a} < 1$. The magnitude of the four parameters $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ reflects the influence of the four states in the decision finding process.

We take the influence of the other factors as given, i.e., we do not further explain the magnitude of $\mathbf{a}, \mathbf{b}, \mathbf{c}$, and \mathbf{d} . In any case it is good for the defendant to shift mass from the bad to the good states, or, to put it differently, it is bad for the defendant if the court takes weight from the good to the bad states. To have some more structure, let $\mathbf{c} - \mathbf{d} > \mathbf{a} - \mathbf{b}$. Then the defendant prefers that the court shifts mass from \mathbf{d} to \mathbf{c} rather than from \mathbf{b} to \mathbf{a} . We assume that the defendant tries to maximize the probability to prevail. He will present evidence in the courtroom so as to maximize $P(\text{win})$.

We have chosen the $P(\text{win})$ to be linear mainly for reasons of simplicity. Our idea is that this exogenously given function reflects the “law.” The judge cannot use this function strategically to reveal information. In a related context Daughety and Reinganum (2000b) derive liability assessment functions from a set of five axioms. One of the functions satisfying the axioms is a linear one. Although their set-up is not exactly ours, their result can be taken as a justification for our linear assessment function. Moreover, our analysis can be performed using more general assessment functions and we believe that our results will still hold qualitatively.

The court, in contrast, is not interested in who wins the case. The court wants to decide the case “correctly.” Suppose, for example, the court wants to deter wrongful acts. More accurate fact-finding increases deterrence, or to put it differently, greater accuracy in the determination of guilt increases the returns to being innocent.⁷ To achieve accuracy the court wants as much information as

⁶As an alternative interpretation suppose the plaintiff sued the defendant for damages L . $(1 - P(\text{win}))$ is the share of the the damages the defendant has to pay.

⁷See Posner (1999b) for a discussion whether the law of evidence has multiple goals rather than just the goal of accuracy in fact-finding. Note that we ignore the cost of fact-finding in our paper. Using Posner’s terminology, in our problem marginal cost of evidence production is below the marginal benefit.

possible as to which of the four states will materialize. The court thus strives for precision.

It would thus be very bad for the judge if he believed a lie. Fortunately, this does not happen in our setup. Our judge can work out the witness's incentives so that he knows when the witness will possibly lie. This means that if the witness makes a report which could both be true or not, the court updates rationally. For example, if the witness always makes the same report irrespective of the true state of nature, the judge completely ignores this report and sticks to his priors. Our judge is never misled and rationally extracts whatever information there is in the testimony. Consequently, the judge's posterior $P(X|\cdot)$, $X \in \{A; B; C; D\}$, correctly reflects all available information and the judge wishes to maximize the accuracy of this distribution. Accordingly, in our set-up the worst scenario for the court is that after collecting all the evidence, the four states are equally likely.

We need, therefore, a good measure of the amount of uncertainty contained in the probability distribution $P(X|\cdot)$, $X \in \{A; B; C; D\}$. One measure of uncertainty which has the desired properties is the entropy

$$H = - \sum_{X \in \{A, B, C, D\}} P(X|\cdot) \ln P(X|\cdot)$$

where we put $0 \ln 0 = 0$ to ensure continuity of the function $-x \ln x$ at the origin.⁸

This function has the following properties. H is non-negative and continuous for any distribution over the four states. If the probability that one state materializes equals one, i.e., if $P(X|\cdot) = 1$ for any of the four states, then $H = 0$. If uncertainty is maximal in the sense that the four states are equally likely, H is maximal, i.e., $H(1/4, 1/4, 1/4, 1/4) = -\ln 1/4$. The uniform distribution maximizes the entropy. This is just Laplace's Principle of Insufficient Reason according to which if there is no reason to discriminate between several events, the best strategy is to consider them as equally likely. The court wishes to minimize the entropy; more precisely, the judge minimizes the expected value of H where we take the expectation at the time when the defendant starts looking for a witness.⁹

⁸We use the entropy because the outcomes of the random variable "states of the world" are of qualitative nature. We cannot use, e.g., the variance which requires quantitative outcomes.

⁹See Guiasu and Shenitzer (1985) for more on why the entropy is a good measure of the amount of uncertainty contained in a probability distribution. Rather than working with the entropy, we could also use the normalized version of Simpson's D , $D = (4/3)[1 - \sum_{X \in \{A, B, C, D\}} P(X|\cdot)^2]$.

With probability $P(W) \in (0, 1)$ the defendant finds a witness. The witness observes the state X .¹⁰ To illustrate, the defendant may find an industrial organization expert who knows the defendant's market share; in the liability case the defendant may find an expert who can determine whether or not the defendant and/or the plaintiff were negligent.

If the defendant presents the witness in court, the witness reports $x \in \{a, b, c, d\}$ where a means that the witness has observed A , etc.¹¹ If $x = X$, the witness tells the truth; otherwise, the witness lies.¹² We thus confine our attention to a direct revelation problem.

Depending on her reported values, the witness receives a remuneration (wage) $w(x)$ from the defendant. Taking future consequences into account, remuneration is higher when the testimony is more favorable to the defendant. We look at the case where the witness is interested, i.e., we assume $w(a) > w(b) > w(c) > w(d)$, where $w(c)$ and $w(d)$ may be negative, reflecting the fact that c and d are bad news for the defendant.¹³ In the case of an expert the contract between the defendant and the witness may stipulate a fixed payment. If the witness reports a , future business from defendants in similar situations is more likely than if the witness reports say c . $w(x)$ then reflects the expected future income. If the witness is directly related to the defendant, $w(x)$ reflect her stakes in the dispute.

After the case has been decided, the court receives a signal $\chi \in \{\alpha, \gamma\}$ about the true state; we call α the good and γ the bad signal. Think of the signal simply as the opinion of a second expert who is, e.g., less able than the original witness. Denote the a priori probability of signal χ by $P(\chi)$.¹⁴

¹⁰As in Shin (1999) we treat the information collection process as exogenous in order to focus on the incentives to disclose the evidence. In Dewatripont and Tirole (1999) information gathering is costly; their focus is on the incentive to gather information.

¹¹Obviously, the actual and the reported values are in the same set and using a, b, c, d is an abuse of notation. The formally correct notation $x \in \{A, B, C, D\}$ might, however, lead to confusion in what follows.

¹²In our set-up the witness can lie, i.e., report false information. There is a related literature comparing the adversarial (partisan) procedure of the Anglo-Saxon law in which partisan advocates present their cases to an impartial jury with the inquisitorial procedures of Roman-Germanic countries in which judges take an active role in investigating a case (Dewatripont and Tirole (1999) and Shin (1999)). In these papers a party can conceal information but cannot report false information.

¹³If $w(a) = w(b) = w(c) = w(d)$ the witness is neutral and has no incentive to distort testimony; see Cooter and Emons (2003, 2004). We do by no means claim that all witnesses are interested. We look at interested witnesses because only for them truth-revelation is a problem.

¹⁴We assume that the process generating the evidence confirming or disconfirming the testimony is exogenous. We do not model how this evidence comes into existence and how it is brought to the attention of the court. In the inquisitorial system the court or the party against which the witness has testified may create the new evidence; in the adversarial system only the

If the true state of the world is a good one, the good signal α is more likely than the bad signal γ and vice versa if the true state of the world is a bad one. Formally, we have $P(\alpha|X) \in (1/2, 1)$, $X \in \{A, B\}$ and $P(\gamma|X) \in (1/2, 1)$, $X \in \{C, D\}$. Note that $P(\alpha|X) = 1 - P(\gamma|X)$, $\forall X$.

Conditional on the relationship between the testimony x and the court's signal χ , the witness can be rewarded or sanctioned. Formally, we denote a sanction/reward by $S(\chi, x)$ where $S > 0$ is a sanction and $S < 0$ a reward. We will say that testimony is confirmed if the court receives the good signal α after reports a, b , and the bad signal γ for reports c, d ; otherwise, testimony is not confirmed. The witness's expected payoff equals her wage minus the expected sanction. Formally, the payoff is given as $w(x) - E(S(\tilde{\chi}, x)|X)$ where $E(S(\tilde{\chi}, x)|X)$ stands for the expected sanction given her reported testimony x and the true information X . The witness chooses her reported testimony x so as to maximize her expected payoff. If the witness is indifferent between the truth and another report, she reports truthfully.

3. A Truth-revealing Mechanism and the Perjury Rule

Let us now derive a system of sanctions that induces the witness to be honest. We call such a mechanism truth-revealing. This means that reporting the true signal must generate at least as much payoff as announcing any other signal. Formally, the *truth-revealing* requirement means

$$w(X) - E(S(\tilde{\chi}, X)|X) \geq w(x) - E(S(\tilde{\chi}, x)|X) \\ \forall x \in \{a, b, c, d\}, \forall X \in \{A, B, C, D\}.$$

The revelation principle implies that we can always find a direct mechanism under which the witness reports the truth.¹⁵ For example, the mechanism $S(\chi, x) = w(x) + k$, $k \in \mathbb{R}$ is truth-revealing. This mechanism simply charges the witness for every report x the wage $w(x)$ she receives from the defendant; in addition the witness gets some constant k which is independent of her report. Under this mechanism the witness's payoff is k for every report. Being completely indifferent, the witness will tell the truth.

Note that this simple truth-revealing mechanism operates under fewer restrictions than the perjury rule does which we will describe next. In particular, it sanctions the witness independently of whether the testimony is confirmed or not. If we make additional assumptions on the signal structure, we can also derive a

latter will have an incentive to search for new evidence. Note, however, that the perjury rule also needs new evidence to be triggered. Comparing truth-revealing mechanisms with perjury given that new evidence pops up thus seems to be fair.

¹⁵For more on the revelation principle see, e.g., Myerson (1985).

truth-revealing mechanism working under the same set of restrictions as perjury law. We describe such a mechanism in the Appendix.

Let us now turn to the perjury rule. First note that the perjury rule does not reward the witness, i.e., $S(\chi, x) \geq 0 \forall (\chi, x)$. Moreover, the perjury rule depends on whether testimony is confirmed or not. It does not sanction the witness if testimony is confirmed meaning $S(\alpha, a) = S(\alpha, b) = S(\gamma, c) = S(\gamma, d) = 0$. With this restriction, the incentive constraints have the following structure. Consider, for example, the case in which the true state is C . Here one of our tasks is to guarantee that announcing $x = c$ is at least as good as reporting $x = b$. Formally, this means $w(c) - P(\alpha|C)S(\alpha, c) \geq w(b) - P(\gamma|C)S(\gamma, b)$. If the witness tells the truth, she receives the wage $w(c)$. With probability $P(\alpha|C)$ the signal α materializes and the witness has to pay the sanction $S(\alpha, c)$. If, in contrast, she reports b , she receives the higher wage $w(b)$. Now the sanction is $S(\gamma, b)$, triggered by the signal γ which occurs with the probability $P(\gamma|C)$. Recall that the signals are informative, i.e., $P(\gamma|C) > P(\alpha|C)$. Accordingly, the probability of being sanctioned is higher when the witness lies.

Analogous incentive constraints hold for the other 3 signals so that overall we end up with 12 incentive constraints. After some algebraic manipulation and rearranging we have the following 6 chains of weak inequalities.

- (1) $P(\gamma|B)S(\gamma, a) - P(\gamma|B)S(\gamma, b) \geq w(a) - w(b) \geq P(\gamma|A)S(\gamma, a) - P(\gamma|A)S(\gamma, b),$
- (2) $P(\alpha|D)S(\alpha, c) - P(\alpha|D)S(\alpha, d) \geq w(c) - w(d) \geq P(\alpha|C)S(\alpha, c) - P(\alpha|C)S(\alpha, d),$
- (3) $P(\alpha|B)S(\gamma, b) - P(\gamma|B)S(\alpha, d) \geq w(b) - w(d) \geq P(\alpha|D)S(\gamma, b) - P(\gamma|D)S(\alpha, d),$
- (4) $P(\gamma|C)S(\gamma, a) - P(\alpha|C)S(\alpha, c) \geq w(a) - w(c) \geq P(\alpha|C)S(\gamma, a) - P(\gamma|C)S(\alpha, c),$
- (5) $P(\gamma|D)S(\gamma, a) - P(\alpha|D)S(\alpha, d) \geq w(a) - w(d) \geq P(\gamma|A)S(\gamma, a) - P(\alpha|A)S(\alpha, d),$
- (6) $P(\gamma|C)S(\gamma, b) - P(\alpha|C)S(\alpha, c) \geq w(b) - w(c) \geq P(\gamma|B)S(\gamma, b) - P(\alpha|B)S(\alpha, c);$

call the first inequality in such a chain (a) and the second one (b).

Before proceeding with the description of the perjury rule, we can already state two preliminary results. First, (1a) and (2a) imply that truth-revealing requires that the sanctions increase with the strength of the testimony. Formally, truth-revealing sanctions for interested witnesses satisfy $S(\gamma, b) < S(\gamma, a)$ and $S(\alpha, d) < S(\alpha, c)$. Second, (3a) implies that $S(\gamma, b)$ and $S(\alpha, d)$ cannot both be zero. These two observations taken together imply that incentive compatible sanctions have to take on at least three different values.

Let us now proceed with the description of the perjury rule.¹⁶ If under the perjury rule testimony is not confirmed, the court uses this information to compute the probability ϕ that the witness did not tell the truth. If this probability exceeds a legal standard $\bar{\phi}$, the court imposes a sanction $s > 0$; if the probability is below the legal standard, the sanction is zero.¹⁷ Formally,

$$S_P(\chi, x) = \begin{cases} s, & \text{if } \phi(\chi, x) \geq \bar{\phi}; \\ 0, & \text{otherwise.} \end{cases}$$

If the witness has reported, say a , and nature chooses γ , the probability of not having told the truth is

$$\phi(a, \gamma) = P(\neg A|\gamma) = 1 - P(A|\gamma) = 1 - (P(\gamma|A)P(A))/P(\gamma).$$

The probability that A was not the true state given γ equals the sum of the probabilities that the witness has observed B, C , or D , which in turn equals 1 minus the probability that A was the true state given γ . Analogously, we compute

$$\begin{aligned} \phi(b, \gamma) &= P(\neg B|\gamma) = 1 - (P(\gamma|B)P(B))/P(\gamma), \\ \phi(c, \alpha) &= P(\neg C|\alpha) = 1 - (P(\alpha|C)P(C))/P(\alpha), \text{ and} \\ \phi(d, \alpha) &= P(\neg D|\alpha) = 1 - (P(\alpha|D)P(D))/P(\alpha). \end{aligned}$$

Notice that however we set $\bar{\phi}$, the perjury rule can take on only two values, 0 and s . If we set, e.g., $\bar{\phi}$ very low, then $S(\gamma, a) = S(\gamma, b) = S(\alpha, c) = S(\alpha, d) = s$; if we set $\bar{\phi}$ very high, then $S(\gamma, a) = S(\gamma, b) = S(\alpha, c) = S(\alpha, d) = 0$; if we set $\bar{\phi}$ to some intermediate value, then $S(\gamma, a) = S(\alpha, c) = s$ and $S(\gamma, b) = S(\alpha, d) = 0$.

Now recall that truth-revelation requires that sanctions have to take on at least three different values. The perjury rule takes on at most two values. Consequently, the perjury rule is not fully truth-revealing. The perjury rule is at most partially truth-revealing. We will describe possible reporting strategies under the perjury rule in the following section.

4. Quantity versus Quality of Testimony

In what follows we will call a mechanism (*fully*) *truth-revealing* if, given the witness testifies in court, she always reports the true state. We will call a mechanism *partially truth-revealing*, if, given the witness testifies in court, for some states of

¹⁶We model a Bayesian court's decision process. There are also indications that a trial court process of fact finding and aggregation is not purely Bayesian but is constrained by rules of evidence and procedure; see, e.g., Posner (1999b). Therefore, Daughety and Reinganum (2000a,b) use axiomatic methods to model information and decisions in court.

¹⁷Being a crime, one element for perjury is the intention to do wrong (*mens rea*, guilty mind). Here we may argue that a personal gain from lying is a necessary condition for intent. A neutral witness gains nothing from lying. Accordingly, a neutral witness should not be prosecuted for perjury. Only when the witness is interested, as we assume, the perjury rule is triggered.

the world she will not report the truth; we will specify the witness's reporting incentives under partial truth-revelation in the context.¹⁸

Let us now take the defendant's decision whether or not to present the witness in the courtroom into account. The sequencing of events is as follows. The judge announces a mechanism. Then defendant searches for a witness. If he does not find one, he cannot present a witness. If he finds a witness, she reports her observation to the defendant. The defendant works out the witness's report under the mechanism in place. Then the defendant decides whether or not to present the witness. The court then updates his beliefs. The equilibrium concept is perfect Bayesian. In an event with zero ex ante probability the court sticks to his uniform priors; the judge thus has passive beliefs.

To illustrate that the defendant's decision is of some interest, suppose first the court employs a truth-revealing mechanism and the witness has observed D . If the witness is presented in the courtroom, she will honestly report d , which is bad for the defendant. In this case it seems better for the defendant not to present the witness and claim that he hasn't found a witness.¹⁹

Suppose next the court uses the perjury rule with $\bar{\phi}$ high enough (so that the witness always reports a) and the witness has observed D : Here it seems a good idea to present the witness because she will report a in the courtroom which is rather good news for the defendant after all. In what follows we will make these ideas precise.

Recall that the defendant finds a witness with probability $P(W)$ and that he finds no witness with $P(NW) = 1 - P(W)$. The two random variables "state of the world" and "finding a witness" are independent, so that $P(X \cap W) = (1/4)P(W)$ and $P(X \cap NW) = (1/4)P(NW)$, $X \in \{A, B, C, D\}$. If the defendant finds no witness, under any mechanism he cannot present a witness in the courtroom.

We will solve the game for different mechanisms; we will then determine which of the mechanisms minimizes uncertainty. To distinguish the different mechanisms, we will from now on identify them by the number of lies by the witness

¹⁸Fully truth-revealing mechanisms are thus high-powered and partially truth-revealing ones low-powered incentive schemes.

¹⁹Suppose the defendant has approached a potential expert witness to find out her opinion about the case. If the witness indicates that the true state might be C or D , the defendant simply doesn't pursue the matter of hiring the witness any further. If the defendant approaches the witness cleverly enough (as any good lawyer can), he can always claim that he didn't find a witness. In this case asking the defendant under a truth-revealing mechanism whether or not he found a witness wouldn't help either. Accordingly, the defendant cannot concoct false evidence but may withhold information unfavorable to his cause. This assumption is a standard feature of disclosure games; see, e.g., Milgrom and Roberts (1986).

they give rise to.²⁰ Let us start with the truth-revealing mechanism. Since under this mechanism the witness tells the truth whatever the state of world, we will call this the *no lie mechanism*. If the defendant finds a witness who observes A or B , clearly he will present her in court and the witness will report the truth. Rather than writing $W \cap a$ and $W \cap b$, we will denote these events as a shortcut simply by a and b . If, however, the witness observes C and D , under truth-revelation the defendant will claim that he found no witness which we denote by NW . Taking the defendant's presentation policy and the witness's reporting strategy into account, the court's updated probabilities of the states of the world are:

$$\begin{aligned}
P(A|a, \text{no lie}) &= 1, & P(B|b, \text{no lie}) &= 1, \\
P(A|NW, \text{no lie}) &= P(A \cap NW)/(P(NW) + P(C \cap W) + P(D \cap W)) \\
&= (1/4)P(NW)/(P(NW) + (1/2)P(W)), \\
P(B|NW, \text{no lie}) &= (1/4)P(NW)/(P(NW) + (1/2)P(W)), \\
P(C|NW, \text{no lie}) &= \frac{P(C \cap NW) + P(C \cap W)}{P(NW) + P(C \cap W) + P(D \cap W)} \\
&= (1/4)/(P(NW) + (1/2)P(W)), \text{ and} \\
P(D|NW, \text{no lie}) &= (1/4)/(P(NW) + (1/2)P(W)).
\end{aligned}$$

Let us now compute the entropy. Here we have:

$$\begin{aligned}
H(P(X|a, \text{no lie})) &= 0, & H(P(X|b, \text{no lie})) &= 0, \quad \text{and} \\
H(P(X|NW, \text{no lie})) &= \\
&= \frac{-1/2}{P(NW) + (1/2)P(W)} \left[P(NW) \ln \frac{(1/4)P(NW)}{P(NW) + (1/2)P(W)} + \right. \\
&\quad \left. \ln \frac{1/4}{P(NW) + (1/2)P(W)} \right].
\end{aligned}$$

Let us now look at the expected entropy. We take the expectation at the time when the defendant looks for a witness. From an ex ante point of view the expected entropy is given as

$$\begin{aligned}
EH(\text{no lie}) &= P(A \cap W)H(P(X|a, \text{no lie})) + P(B \cap W)H(P(X|b, \text{no lie})) \\
&\quad + (P(NW) + P(C \cap W) + P(D \cap W))H(P(X|NW, \text{no lie})) = \\
&= (-1/2)[(1 + P(NW))[\ln(1/4) - \ln(P(NW) + (1/2)P(W)) \\
&\quad + P(NW) \ln P(NW)].
\end{aligned}$$

²⁰Note that we only count the lies of the witness to distinguish the mechanisms. We do not count the lies of the defendant.

Let us now look at the other extreme and consider a mechanism providing no incentives at all. Here the witness reports a for states A, B, C , and D . Accordingly, we will call it the *3 lies mechanism*. Take, for example, the perjury rule with the legal standard $\bar{\phi}$ so high (low) that the witness is never (always) sanctioned.

Here matters are straightforward. Whenever the defendant finds a witness, he will present her in court and she will report a , whatever the true state of nature. We have thus a high quantity of low quality testimony. Anticipating this, the court will not update his beliefs and

$$P(X|a, 3 \text{ lies}) = 1/4, \quad X \in \{A, B, C, D\}, \quad \text{so that } H(P(X|a, 3 \text{ lies})) = -\ln 1/4.$$

Similarly,

$$P(X|NW, 3 \text{ lies}) = 1/4, \quad X \in \{A, B, C, D\}, \quad \text{and } H(P(X|NW, 3 \text{ lies})) = -\ln 1/4.$$

The expected entropy is given as

$$EH(3 \text{ lies}) = P(W)H(P(X|a, 3 \text{ lies})) + P(NW)H(P(X|NW, 3 \text{ lies})) = -\ln 1/4.$$

Now consider partially truth-revealing mechanisms. Let us start with the perjury rule where $\bar{\phi}$ and s are such that the witness tells the truth whenever she observes A and B and where the witness reports b for C and D . We will call this the *2 lies mechanism*. Here the defendant has two potentially attractive presentation policies.

Either he will always present the witness who reports a when the true state is A , and b when the true states are B, C , and D . Under this policy #1 the court's updated probabilities are

$$\begin{aligned} P(A|a, 2 \text{ lies}, \#1) &= 1, \\ P(A|b, 2 \text{ lies}, \#1) &= 0, \quad P(X|b, 2 \text{ lies}, \#1) = 1/3, \quad X \in \{B, C, D\}, \quad \text{and} \\ P(X|NW, 2 \text{ lies}, \#1) &= 1/4, \quad X \in \{A, B, C, D\}. \end{aligned}$$

Or the defendant presents the witness only when she has observed A and reports a ; otherwise, he claims to have found no witness. Under this policy #2 the court's updated probabilities are

$$\begin{aligned} P(A|a, 2 \text{ lies}, \#2) &= 1, \\ P(A|NW, 2 \text{ lies}, \#2) &= P(NW)/(3 + P(NW)), \\ P(X|NW, 2 \text{ lies}, \#2) &= 1/(3 + P(NW)), \quad X \in \{B, C, D\}. \end{aligned}$$

It turns out that given our assumptions on $P(\text{win})$, namely that $\mathbf{a} > \mathbf{b} > \mathbf{c} > \mathbf{d}$, the probability to win is higher under the presentation policy #2 than under policy #1. The reason is that under policy #1 the report b is rather bad news for the defendant because the court puts weight of $2/3$ on the bad outcomes C

and D . The defendant does better to claim NW ; the court then shifts some mass from the bad outcomes to the best state A . Here perjury leads the defendant to withhold favorable news because it is inflated. Accordingly, the defendant follows policy #2 and

$$\begin{aligned} H(P(X|a, 2 \text{ lies})) &= 0, \text{ and} \\ H(P(X|NW, 2 \text{ lies})) &= \\ &= \frac{-1}{3 + P(NW)} \left[P(NW) \ln \frac{P(NW)}{3 + P(NW)} + 3 \ln \frac{1}{3 + P(NW)} \right], \end{aligned}$$

and

$$\begin{aligned} EH(2 \text{ lies}) &= (1/4)P(W)H(P(X|a, 2 \text{ lies})) \\ &+ (3/4 + (1/4)P(NW))H(P(X|NW, 2 \text{ lies})) \\ &= \frac{-1}{4} \left[P(NW) \ln \frac{P(NW)}{3 + P(NW)} + 3 \ln \frac{1}{3 + P(NW)} \right]. \end{aligned}$$

To complete the picture we look at the following *1 lie mechanism*: the witness tells the truth for A , B and D , but reports b if the true state is C . We haven't derived a mechanism giving rise to this reporting strategy but using the techniques given in the Appendix, this is a fairly straightforward task.²¹ Here the defendant will present the witness in states A , B , and C , but not in D . If the true state is A , the witness reports a , for B and C the witness reports B .²² Accordingly,

$$\begin{aligned} P(A|a, 1 \text{ lie}) &= 1, \\ P(B|b, 1 \text{ lie}) &= 1/2, \quad P(C|b, 1 \text{ lie}) = 1/2, \\ P(A|NW, 1 \text{ lie}) &= P(A \cap NW)/(P(NW) + P(D \cap W)) = \\ &= 1/4P(NW)/(P(NW) + (1/4)P(W)), \\ P(B|NW, 1 \text{ lie}) &= 1/4P(NW)/(P(NW) + (1/4)P(W)), \\ P(C|NW, 1 \text{ lie}) &= 1/4P(NW)/(P(NW) + (1/4)P(W)), \text{ and} \\ P(D|NW, 1 \text{ lie}) &= (P(D \cap NW) + P(D \cap W))/(P(NW) + P(D \cap W)) = \\ &= (1/4)/(P(NW) + (1/4)P(W)). \end{aligned}$$

²¹One can think of other examples of 1 lie mechanisms. The basic structure of the following arguments remains, however, the same.

²²Presenting the witness only when she has observed A and otherwise claiming to have found no witness is unattractive given our assumptions on $P(\text{win})$.

Therefore,

$$\begin{aligned}
H(P(X|a, 1 \text{ lie})) &= 0, & H(P(X|b, 1 \text{ lie})) &= -\ln 1/2 \quad \text{and} \\
H(P(X|NW, 1 \text{ lie})) &= \\
&\frac{-1/4}{P(NW) + (1/4)P(W)} \left[3P(NW) \ln \frac{(1/4)P(NW)}{P(NW) + (1/4)P(W)} + \right. \\
&\left. \ln \frac{1/4}{P(NW) + (1/4)P(W)} \right],
\end{aligned}$$

and

$$\begin{aligned}
EH(1 \text{ lie}) &= (1/4)P(W)H(P(X|a, 1 \text{ lie})) + (1/2)P(W)H(P(X|b, 1 \text{ lie})) + \\
&\quad (P(NW) + (1/4)P(W))H(P(X|NW, 1 \text{ lie})) \\
&= (1/2)P(W)(-\ln 1/2) - 1/4 \left[3P(NW) \ln \frac{(1/4)P(NW)}{P(NW) + (1/4)P(W)} \right. \\
&\quad \left. + \ln \frac{1/4}{P(NW) + (1/4)P(W)} \right].
\end{aligned}$$

We are now in the position to state that the court prefers mechanisms giving rise to as little lies as possible. When the court has the choice between, e.g., the truth-revelation and perjury, clearly he opts for truth-revelation.

Proposition 1: *For the expected entropy we have $EH(\text{no lie}) < EH(1 \text{ lie}) < EH(2 \text{ lies}) < EH(3 \text{ lies})$ for $0 < P(W) < 1$.*

The proof is relegated to the Appendix. The intuition for this result is straightforward. If the mechanism provides no incentives, the court gets a lot of testimony, which is, however, worthless. If the mechanism provides incentives to tell the truth, the court gets fewer testimony but this testimony is of higher quality. Moreover, even the message that the defendant found no witness is now informative: the court can infer that the bad states are more likely than the good ones. If the mechanism gives rise to a lie-structure such that the defendant withholds even good news, the report “no witness” contains less information than under truth revelation. Accordingly, using high powered incentives to elicit the truth is more informative for courts even when the quantity of testimony is accounted for. In our game the judge will thus pick a truth-revealing mechanism.

Given that the court prefers mechanisms giving rise to as little lies by the witness as possible, the next question to ask is: How does the defendant rank the different mechanisms? Let us look at the expected probability to win the case. Here we take the expectation as in the case of the expected entropy at the time when the defendant looks for a witness. The somewhat surprising result is that

with a rational (Bayesian) court, the expected probability to win the case is the same for all mechanisms.

Proposition 2: *The expected probability to prevail satisfies $EP(\text{win}, y \text{ lies}) = (1/4)(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d})$, $y \in \{0, 1, 2, 3\}$.*

This result may be explained as follows. Suppose the court uses the truth-revealing mechanism. If the witness observes good news, this is favorable for the defendant because the court believes the testimony. If, however, the news is bad and the witness is not presented in court, this is unfavorable for the defendant because the court puts now more weight on C and D which lowers $P(\text{win})$. Since the probability to prevail $P(\text{win})$ is linear in the court's assessment, the favorable and the unfavorable effect just cancel.²³ To put it differently: The defendant maximizes the probability of winning which is linear in the court's assessments. The court's assessment is a posterior. Regardless of the truth-revealing mechanism, by the law of iterative expectations, the expected posterior is always equal to the prior.²⁴

If we measure welfare by the expected entropy and the expected probability to prevail, we may say that the truth-revealing mechanism is better than any other mechanism in the weak Pareto sense. The court strictly prefers truth-revelation and the defendant does not care. Nevertheless, if we take the defendant's cost of hiring the witness $w(\cdot)$ into account, this welfare statement needs to be qualified. Suppose, for example, the defendant can commit ex ante not to call a witness at all in the trial. Then the defendant clearly prefers this alternative. The expected probability to win is the same as under every other mechanism and he saves the expenses of paying the witness.

6. Conclusions

In a simple framework we have analyzed the connection between quantity and quality of testimony. If the court uses a mechanism providing incentives to tell the truth, he obtains little testimony which is, however, of high quality. This also allows the court to make more precise inferences when he gets no testimony. If the court switches from a mechanism giving no incentives to a truth-revealing one, the defendant gains in the good states and loses in the bad states; the gains and the losses just cancel in our set-up. Overall then, we may argue that more incentives to tell the truth are better.

²³For the court who does not care who wins the case, any reduction in uncertainty is favorable; see Proposition 1.

²⁴By the law of iterative expectations we refer to the property $E(y) = E_x[E(y|x)]$.

A few qualifications are in order. First, we did not analyze the witness's effort to gather information. The more effort a witness provides, the more precise her signal, say. If effort were observable, the court could also use this information to infer the quality of the testimony. Second, we assume that the process generating the evidence confirming or disconfirming the testimony is exogenous.²⁵ Third, we do not ask the question what level of disconfirming evidence provides the best trigger for the sanction.²⁶ To illustrate, suppose the witness has made a statement, say b , about the defendant's market share. Disconfirming evidence γ is the observation that $z\%$ of a group of randomly chosen disinterested industrial economists disagree with the witness's statement. The optimal level of z is not addressed in our model. Fourth, in our set-up the court and the defendant have common priors about the states of nature. An analysis where the defendant is better informed than the court could be of some interest.

²⁵We ignore, e.g., the incentives of the other party to call a witness. For example, in adversarial systems competition between advocates who cannot prove every true statement can fully inform the fact-finder; see Lipman and Seppi (1995).

²⁶A related problem is analyzed by Bernardo, Talley, and Welch (2000).

Appendix

The Truth-revealing Mechanism under the Perjury Restrictions:

To derive a truth-revealing mechanism working under the same set of restrictions as the perjury rule, we need additional structure. We assume that the good signal is more likely for A than for B and the bad signal is more likely for C than for D . Formally, we have $1 > P(\alpha|A) > P(\alpha|B) > 1/2$ and $1 > P(\gamma|C) > P(\gamma|D) > 1/2$. Finally, to keep matters simple, let $P(\alpha|A) = P(\gamma|C)$ and $P(\alpha|B) = P(\gamma|D)$.²⁷

We focus on mechanisms working with minimal sanctions and no rewards. Formally, within the class of truth-revealing mechanisms we look for the mechanism S^* satisfying $0 \leq S^*(\chi, x) \leq S(\chi, x) \forall (\chi, x)$. The main reason we do not use rewards is that we want to compare our truth-revealing mechanism with the perjury rule which doesn't use rewards either. We make the sanctions as low as possible in order to minimize the monetary strain on the witness.²⁸

The truth-revealing mechanism using minimal sanctions and no rewards is given by

$$S^*(\chi, x) = \begin{cases} (w(a) - w(b))/P(\gamma|B) + \\ (w(b) - w(d))/P(\alpha|B), & \text{if } \chi = \gamma, x = a; \\ (w(b) - w(d))/P(\alpha|B), & \text{if } \chi = \gamma, x = b; \\ (w(c) - w(d))/P(\alpha|D), & \text{if } \chi = \alpha, x = c; \\ 0, & \text{otherwise.} \end{cases}$$

Proof: We use the first inequalities (1a)-(6a) to determine the smallest incentive compatible sanctions. Our preliminary result implies that we can set $S^*(\alpha, d) = 0$. (2a) then implies $S^*(\alpha, c) = (w(c) - w(d))/P(\alpha|D)$. (3a) then defines $S^1(\gamma, b) = (w(b) - w(d))/P(\alpha|B)$ while (6a) defines

$$S^2(\gamma, b) = \frac{1}{P(\gamma|C)} \left[w(b) - w(c) + \frac{P(\alpha|C)}{P(\gamma|B)} (w(c) - w(d)) \right]$$

Since $S^2(\gamma, b) =$

$$\frac{1}{P(\alpha|B)} \left[\frac{P(\alpha|B)}{P(\gamma|C)} w(b) + w(c) \left(\frac{P(\alpha|B) P(\alpha|C)}{P(\gamma|C) P(\gamma|B)} - \frac{P(\alpha|B)}{P(\gamma|C)} \right) - \frac{P(\alpha|B) P(\alpha|C)}{P(\gamma|C) P(\gamma|B)} w(d) \right] \leq \\ \frac{1}{P(\alpha|B)} \left[\frac{P(\alpha|B)}{P(\gamma|C)} (w(b) - w(d)) \right] \leq \frac{w(b) - w(d)}{P(\alpha|B)} = S^1(\gamma, b),$$

we have $S^*(\gamma, b) = S^1(\gamma, b)$. Given this, (1a) then defines

$$S^1(\gamma, a) = \frac{w(a) - w(b)}{P(\gamma|B)} + \frac{w(b) - w(d)}{P(\alpha|B)},$$

²⁷In Cooter and Emons (2003) we derive a truth-revealing mechanism for the case $P(\alpha|A) = P(\alpha|B) > P(\gamma|C) = P(\gamma|D)$. There we also show that under the restrictions of perjury truth-revelation is not possible if $P(\alpha|B) < P(\alpha|A)$ and/or $P(\gamma|C) < P(\gamma|D)$.

²⁸For a further discussion of why we restrict attention to mechanisms using minimal sanctions (viz individual rationality) and no rewards (viz frivolous witnesses), see also Emons and Sobel (1991) and Emons (1994). If the witness's remunerations are all non-negative, then the mechanism S^* is individually rational; see Cooter and Emons (2003).

(5a) implies

$$S^2(\gamma, a) = \frac{w(a) - w(d)}{P(\gamma|D)},$$

and (4a) defines

$$S^3(\gamma, a) = \frac{w(a) - w(c)}{P(\gamma|C)} + \frac{P(\alpha|C)}{P(\gamma|C)} \frac{w(c) - w(d)}{P(\alpha|D)}.$$

While it is straightforward to see that $S^1(\gamma, a) \geq S^2(\gamma, a)$, proving the second inequality is more tricky. Here we have $S^1(\gamma, a) \geq S^3(\gamma, a) \Leftrightarrow$

$$\begin{aligned} & w(a) \left[\frac{P(\gamma|C)}{P(\gamma|B)} - 1 \right] + w(c) \left[1 - \frac{P(\alpha|C)}{P(\gamma|B)} \right] \geq \\ & w(b) \left[\frac{P(\gamma|C)}{P(\alpha|D)} - \frac{P(\gamma|C)}{P(\alpha|B)} \right] + w(d) \left[\frac{P(\gamma|C)}{P(\alpha|B)} - \frac{P(\alpha|C)}{P(\alpha|D)} \right] = \\ & w(b) \left[\frac{P(\gamma|C)}{P(\gamma|B)} - 1 \right] + w(d) \left[1 - \frac{P(\alpha|C)}{P(\gamma|B)} \right] + \left[\frac{P(\gamma|C)}{P(\alpha|B)} - 1 \right] (w(d) - w(b)) \end{aligned}$$

which holds given $P(\gamma|C) > P(\alpha|B)$ and our assumptions on $w(\cdot)$. Consequently,

$$S^*(\gamma, a) = \frac{w(a) - w(b)}{P(\gamma|B)} + \frac{w(b) - w(d)}{P(\alpha|B)}.$$

It remains to be shown that (1b)-(6b) also hold. (1b), (2b), and (3b) are obvious. Subtracting (2b) from (3b) yields $w(b) - w(c) \geq P(\alpha|D)S(\gamma, b) - P(\alpha|C)S(\alpha, c) \geq P(\alpha|D)S(\gamma, b) - P(\gamma|D)S(\alpha, c)$, implying (6b) since $P(\gamma|D) = P(\alpha|B)$. Adding (1b) to (3b) yields $w(a) - w(d) \geq P(\gamma|A)S(\gamma, a) + (P(\alpha|A) - P(\alpha|B))S(\gamma, b) > P(\gamma|A)S(\gamma, a) - P(\alpha|A)S(\gamma, b)$, meaning (5b) is satisfied. Last but not least, adding (1b) to (6b) yields $w(a) - w(c) \geq P(\gamma|A)S(\gamma, a) + (P(\alpha|A) - P(\alpha|B))S(\gamma, b) - P(\alpha|B)S(\alpha, c) \geq P(\alpha|C)S(\gamma, a) - P(\gamma|C)S(\alpha, c)$ which is (4b).

Q.E.D.

Proof of Proposition 1:

a)

$$\begin{aligned} & EH(\text{no lie}) = \\ & (-1/2)[(1 + P(NW))[\ln(1/4) - \ln(P(NW) + (1/2)P(W)) + P(NW) \ln P(NW)] < \\ & -\frac{1}{2}P(W) \ln \frac{1}{2} - \frac{1}{4} \left[3P(NW) \ln \frac{(1/4)P(NW)}{P(NW) + (1/4)P(W)} \right. \\ & \left. + \ln \frac{(1/4)}{P(NW) + (1/4)P(W)} \right] = EH(1 \text{ lie}) \quad \Leftrightarrow \\ & 2P(W) \ln 1/2 + (P(NW) - 1) \ln 1/4 + P(NW) \ln P(NW) - (1 + 3P(NW)) \cdot \\ & \ln(P(NW) + (1/4)P(W)) < -2(1 + P(NW)) \ln(P(NW) + (1/2)P(W)) \quad \Leftrightarrow \\ & P(NW) \ln P(NW) - (1 + 3P(NW)) \ln(1/4 + (3/4)P(NW)) < \\ & -2(1 + P(NW)) \ln(1/2 + (1/2)P(NW)) \end{aligned}$$

which holds for $P(NW) \in (0, 1)$.

b)

$$EH(1 \text{ lie}) < EH(2 \text{ lies}) = -\frac{1}{4} \left[P(NW) \ln \frac{P(NW)}{3 + P(NW)} + 3 \ln \frac{1}{3 + P(NW)} \right] \Leftrightarrow$$

$$2P(W) \ln(1/2) + (1 + 3P(NW))(\ln(1/4) - \ln(P(NW) + (1/4)P(NW))) +$$

$$2P(NW) \ln P(NW) + (3 + P(NW)) \ln(3 + P(NW)) > 0$$

which holds for $P(NW) \in (0, 1)$.

c) $EH(2 \text{ lies}) < EH(3 \text{ lies}) = -\ln 1/4$ which holds for $P(NW) \in (0, 1)$.

Q.E.D.

Proof of Proposition 2:

Under the 3 lies mechanism whenever the defendant finds a witness, he will present her in court and she will report a . Accordingly, the court will not update his beliefs and

$$EP(\text{win}, 3 \text{ lies}) = (1/4)(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}).$$

Under the 2 lies mechanism

$$EP(\text{win}, 2 \text{ lies}) = P(W)(1/4)\mathbf{a} +$$

$$[1 - (1/4)P(W)] [(P(NW)/(3 + P(NW)))\mathbf{a} + 1/(3 + P(NW))(\mathbf{b} + \mathbf{c} + \mathbf{d})]$$

$$= (1/4)(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}).$$

Under the 1 lie mechanism

$$EP(\text{win}, 1 \text{ lie}) = P(W)(1/4)\mathbf{a} + P(W)(1/4)(\mathbf{b} + \mathbf{c}) +$$

$$(P(NW) + \frac{1}{4}P(W)) \left[\frac{(1/4)P(NW)}{P(NW) + (1/4)P(W)}(\mathbf{a} + \mathbf{b} + \mathbf{c}) +$$

$$\frac{1/4}{P(NW) + (1/4)P(W)}\mathbf{d} \right] = (1/4)(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}).$$

Under the no lie mechanism

$$EP(\text{win}, \text{no lie}) = P(W)(1/4)\mathbf{a} + P(W)(1/4)\mathbf{b} +$$

$$(P(NW) + (1/2)P(W)) \left[\frac{(1/4)P(NW)}{P(NW) + (1/2)P(W)}\mathbf{a} + \frac{(1/4)P(NW)}{P(NW) + (1/2)P(W)}\mathbf{b} +$$

$$\frac{1/4}{P(NW) + (1/2)P(W)}(\mathbf{c} + \mathbf{d}) \right] = (1/4)(\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}).$$

Q.E.D.

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