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Trading Know-How*

William Fuchs — UC Berkeley
Luis Garicano — LSE

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Abstract

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We study how improvements in communication technology allow for the development of anonymous, internet-mediated, problem-solving markets. These markets are plagued by informational asymmetries, since the difficulty of the question posed, the quality of the consultants offering help, and often, whether a real solution to the problem has been found are all unknown. We study efficient contracting in these markets in a general equilibrium setting, where heterogeneous agents may decide whether to generate productive opportunities or become consultants. We show that efficiency often requires that the market exclude some agents so that the hardest questions are not posed, and the worse agents do not solve problems for others. However, under asymmetric information it is impossible to prevent 'pretenders' from showing up in the market- so that if contingent contracts are impossible the market disappears and no advice is sought or given. Instead, if output (whether problems are solved) is verifiable, output contingent contracts are feasible. In this case, however, equilibrium may involve too much advice– as no agent can be excluded in equilibrium. The presence of 'pretenders' thus leads to inefficiencies involving either too much trade, or no trade at all. We show that improvements in communication technology increase efficiency in these markets and may allow these anonymous markets to attain first best.

I Introduction

Know-how is generally tacit, and thus cannot itself be transferred; instead, an agent can pay another agent to use his know-how to help him solve a particular problem. But such trades in know-how are plagued by informational asymmetries. First, the difficulty of the problem posed is often hard to assess. Second, the skill of the agent (a 'consultant') offering his services in the market is also unobservable. Finally, the output of the consultant (whether the problem is or not solved) is itself

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often uncertain or at least ambiguous in nature. As a result, these markets have tended, in the past, to either be replaced by inside the firm 'knowledge-hierarchies' which allow for the exchange of know-how within the firm (Garicano, 2000); or to rely strongly on reputations (like in the case of consulting firms).

The development of the internet has created an opportunity for spot, anonymous, fee-based markets.[1] Companies such as NineSigma and Innocentive offer a platform for problem originators and problem solvers to meet and trade problem solutions for cash.[2] In these marketplaces a problem originator (called a 'seeker' at Innocentive) poses a query for a solver to solve within a given time frame; problem solvers ('solvers' at Innocentive) may choose to pick up the query and work on finding an answer. In case they do solve the problem, a monetary prize can be obtained. The informational challenges that this 'open innovation' or 'know-how trading' approach involves are many. This paper aims to start posing some of them and showing how to deal with them contractually.

Absent information asymmetries, the first best, which we analyze in Section II, has the less knowledgeable agents being originators of opportunities and those with higher knowledge levels becoming consultants. Furthermore, there is positive sorting: more knowledgeable consultants tackle the harder problems, which are those that the most skilled originators could not resolve by themselves.[3] When communication costs are high and hence consultants can only apply their know-how to a few problems, highly skilled originators do not enter the market for know–how. They are not knowledgeable enough to offer to solve others problems, but pose questions which are in expectation very hard and it is not worth their time for any consultant to tackle them. The existence of these agents 'in the middle' (too smart to ask but not smart enough to answer), as we shall see, is a key obstacle for the development of a problem solving market.

The value of the market to agents is non-monotonic: the market benefits most the most skilled agents, who can leverage their know-how to solve a lot of problems, and the least skilled agents, who without the market could extract very little value from the opportunities they originate. The intermediate agents, on the other hand, are not much better off than in autarky. As originators they ask for advice on problems which are hard to solve and thus require very talented (and expensive) consultants; as problem solvers they cannot solve a large share of the problems for which their help was requested.

The contractual problems appear because generally agents' knowledge and hence the (expected) difficulty of the problems is unobservable. We first show, that if informational asymmetries are one-sided, the first best can still be attained. It is well known for bilateral relationships that if the

[1] As well as free knowledge-exchanges in which the only reward for those who participate is reputational. such as Yahoo Answers, Windows Live Q&A, or Wikipedia:Reference Desk. For example, Wikipedia Reference Desk description states that it "works like a library reference desk. Users leave questions on the reference desk and Wikipedia volunteers work to help you find the information you need."(http://en.wikipedia.org/wiki/Wikipedia:Reference_desk)

[2] Innocentive, was spun off by Ely Lilly; the information we refer to can be found at www.ninesigma.com and at http://www2.innocentive.com.

[3] Garicano and Rossi-Hansberg (2004) introduced and sketched a solution of this full information problem.

party with private information can be made the residual claimant (and if the agents are risk neutral) an efficient outcome can be attained. A similar logic extends to a two sided market. Markets can be set so that prices are based on the observable type- a referral market when the originator's type is observable or a consulting market when the consultant's type is observable. Equilibrium prices in turn induce the side of the market with private information to self-select into the efficient match.

Efficiency is harder to attain when asymmetric information is double sided, that is, neither the knowledge of agents nor the expected difficulty of the problems they pose can be observed. In this case, the market is characterized by double sided adverse selection. In particular, those seeking advice have incentives to pretend that their problems are easier than they are so that they can pay less for advice. On the other hand, consultants want to pretend they are smarter than they are, so that they can get a higher fee. In other words, consultants want to play smart, while originators want to play dumb. Moreover, and further complicating the problem, whether a problem is or not actually solved is often unverifiable. Consider, for example, a firm that needs advice on its future strategy– how can the quality of advice given be evaluated?

Not surprisingly, as we show in Section III, a matching market where neither the quality of sellers nor that of the buyers can be observed works inefficiently. In fact, if the the problem, or the good or opportunity associated with it, is non-transferable, and output is unverifiable, the market completely breaks down, and no advising actually takes place: the worst agent in the economy can always pass himself for a consultant, and thus advice is worthless as the 'pretender' cannot be identified either ex ante or ex post. [4]

If it is possible to verify whether the advice given solved the problem posed, conditional payments (a fee paid only if the problem posed is solved) are possible, and contingent contracts can be used. We characterize the equilibrium in this case and compare it to the first best, analyze the properties of the contingent contracts, and the impact of imperfect information on the distribution of income. Each originator type pays or receives a different fixed fee for the right to a varying conditional payment (or share) in case the problem is solved. This, we show, disciplines both originators and problem solvers and may allow for the first best to be attained, depending on the level of communication costs.

The first key result under double sided informational asymmetries is that no agent can be excluded from the market, so that even mediocre problem solvers and originators with too difficult problems want to trade. As a result, market efficiency decreases to the extentent that, if communication is sufficiently expensive, the market completely breaks down and a (separating) equilibrium with trade is impossible to attain. When communication costs decrease, so that each consultant can work on a larger number of problems, an equilibrium with too much trade exists- since keeping originators

[4] A certification mechanism able to set minimum standards for access into the role of advisor provides a partial solution in this environment, which we studied in Fuchs and Garicano (2010). We also provided there a characterization of the optimal certification level. In particular, we showed that entry regulation involves less experts than the first best, since those seeking advice are matched with the average advisor in the market, which makes advice less valuable than under optimal matching.

with hard problems out of the market is not possible, and the first best requires that some problems remain unsolved, the competitive equilibrium has problems that should not be solved getting solved. When communication costs are low enough, efficiency requires that all problems be solved; since all problems are indeed solved in the market, the first best is attained. Thus surprisingly, when communication costs are low enough, the first best allocation can be attained even in the presence of two sided adverse selection.

We find that the output contingent payments increase with the difficulty of problem to the point that the smartest consultants (who tackle the hardest problems) actually become full residual claimants to the output. On the other hand, the fixed fee paid by originators is non-monotonic. At first it is increasing and then it is decreasing to the point that it becomes negative i.e. the consultant starts buying a share of the venture. The non-monotonicity arises because of the asymmetric matching implications that a local deviation has for consultants versus originators in different segments of the market. At the low end of the market, since there is a large demand for consultants from low types, pretending to be a slightly smarter originator leads to being matched with a much better consultant. But for the consultants that are matched with these agents, a local deviation leads only to slightly different match. The originators would then have a stronger incentive to exaggerate their type, the fixed fee they have to pay needs to be increasing to dissuade such behavior. On the other hand, high originator types do not demand much consultant time, since they overcome most difficulties on their own. Hence a local deviation will only lead to a slightly better match for them. In contrast, for the top consultants, a local deviation will lead to a much easier problem to solve. Thus now the consultants have the strongest incentives to deviate and therefore the fixed fee needs to be decreasing to prevent these deviations. The non-monotonicity of the payments leads to two originators paying the same fixed fee but giving up different shares; separation is ensured because the matching is different- the originator that gives up a higher share is being compensated by being matched with a more skilled consultant.

Even when the market does not break down, the asymmetry of information has distributional consequences. Agents in the middle of the distribution are made better off by the absence of information, preventing them from imitating others requires that they capture some rents; as a consequence the agents on the extremes (that trade with them) are made worse off. These effects are augmented when there is entry of more consultants than in the first best: the best problem solvers now in addition receive harder problems and the low originators are in turn matched to worse problem solvers.

Finally, to gain a partial insight into the mechanisms used by the new internet sites, we study mechanisms that only inolve a contingent payment. We show that no separation of types exists here- this is a pooling equilibrium with random matching. As a result, these markets cannot achieve full efficiency, since an unskilled originator may unwittingly choose to work on a problem that is too hard. This pooling equilibrium, however, does achieve partial efficiency, as long as communication costs are not too high– someone does work on all problems. If communications costs are too high, on the other hand, then problem solving may only take place when originators give up their problems for free and problem solvers become the residual claimants to the entire problem.

No previous literature has, to our knowledge, examined the double sided adverse selection issue raised by the matching of consultants to problems under asymmetric information. The previous literature on consultant services has emphasized moral hazard issues involved in the provision of consultant services - consultants have little incentive to provide the right level of effort. Demski and Sappington (1987), examines the trade-off between productive effort and information gathering incentives by the consultant. In Wolinsky (1993) the issue is the incentives of consultants to recommend the right treatment- small treatment for small problem and big treatment for large problem. He shows that specialization is optimal in this case. Similarly, Pesendorfer and Wolinsky (2003) study the provision of adequate diagnosis effort by consultants. Taylor (1995) studies how insurance can solve informational asymmetries in a context where only the consultant can tell the treatment needed. Garicano and Santos (2004) does study the matching of opportunities and consultants take place, but in a context with moral hazard and one-sided adverse selection: the agent who knows about the exisstence of an opportunity must be compensated but this gives him incentives to retain it, but this creates moral hazard on the part of the agent receiving the referral. We depart from all of this literature in that we emphasize the double sided nature of the informational asymmetries- neither the agent knows the quality of the consultant, nor the consultant knows a priori the difficulty of the problem posed.

Our paper also fits within a literature studying trade in markets with bilateral asymmetric information. Most of the literature develops from the original Myerson and Satherwaite's (1983) analysis of trade mechanisms under asymmetric information about buyer and seller valuation, but with multiple buyers and sellers of a commodity for which they have unknown valuations (e.g. Lu and Robert, 2001); they do not care about each other's types generally, but only about the value of the object at stake and thus matching is irrelevant. The only paper we are aware that studies equilibrium in matching markets with two-sided incomplete information is Gale (2001). There are several important differences between our models. First, Gale takes as exogenously given the side of the market agents are in, instead, in our model, agents get to choose if ex-post they will be buyers or sellers of consultant advice. This endogenous choice makes the analysis a bit harder on one hand but on the other, the indifference conditions of the cutoff types help pin down the equilibrium. Also, in Gale's paper the outside option of agents from not participating in the market is equal for all types. Instead, in out setting higher types have a higher outside option. This increase the adverse selection problems since when the market is not very attractive it is the best agents that exit first. Finally, trading partners in our model are determined in a centralized way instead, he has separate markets for each type of contract. Because our problem has more structure we are able to go substantially further in characterizing the market equilibrium.

II The Model and First Best

There is a continuum of income maximizing agents who are indexed by their level of know-how $z \in [0,1]$, loosely measuring what level of difficulty are the problems they can solve. Without loss of generality, we choose the index z so that z is measured in percentiles of the know-how distribution – the distribution of z is thus uniform. Agents must first decide if they become problem originators or specialized problem solvers ('consultants'). We will let O denote the set of agents that become originators and by S the set of agents that become solvers or consultants. If an agent is an originator then at the beginning of the period he draws a problem, with an associated difficulty level $q \in [0,1]$; q is unobserved and i.i.d. across problems and distributed according to $F(q)$, a continuous function with density $f(q)$. If $z > q$ then the agent can solve the problem by himself and gets a payoff of 1. If $z < q$ then he cannot solve the problem. But he can then seek a consultant who can potentially solve the problem for a fee. Since not all originators need to seek advice, we will denote by A the set of agents that do seek advice and by I the set of agents that remain independent.

The other option is for the agent to decide to become a consultant. Consultants can advise up to $1/h$ agents on their problems, where $h < 1$ is the helping cost (it costs a fraction h of time to help one other agent) in a given period. Consultants don't generate any problems of their own, that is, they are specialized in solving problems. Like originators, consultants with know-how z can solve problems q as long as they are not 'too hard' for them i.e. $z > q$. We will assume for simplicity that if neither the original agent nor the hired consultant can solve the problem then the problem goes unsolved.[5]

To represent the (potentially random) matching between advice seekers and solvers we will use the CDF $M(s,z)$. That is, $M(s,z)$ will determine with what probability agent $z \in A$ gets matched with $s \in S$. We will use $\mu(s,z)$ to denote its density. In some cases the matching will not have any random component and $M(s,z)$ will hence be degenerate. In these cases will simply use the matching function $m(z) : A \to S$

The Information Structure. Our objective is to characterize the optimal contracts and division of labor that will arise in this economy. This will depend critically on the assumptions we make about what is observable and what is verifiable. We start by assuming that there is perfect information, all agents types are observable. Then we look at the case in which only side of the market is observable and finally the case in which all types are unobservable. For this case we consider both verifiable and non-verifiable output.

[5] See Garicano (2000) and Garicano and Rossi-Hansberg (2006) for a setting (without asymmetric information) in which several layers of advice are available, with homogeneous and with heterogenous agents respectively. In those papers z is a choice of agents, whereas here it is given.

A Full Information Benchmark

Properties of the First Best Allocation We start by studying the first best. Suppose that a social planner could allocate optimally agents into those who seek advice or originators (O), those who neither seek nor give advice(I for indepenent), and consultants or solvers (S), and could choose which type of consultants help with which problems.

The planners objective can be written as:

$$\max_{O,S,M(s,z)} \int_{z\in A} (F(z) + (1-F(z)) \Pr(q < s|q > z, M(s,z)))\, dz + \int_{z\in I} F(z)\, dz$$

With full information, the only constraint that the planner faces is the resource constraint that the demand for advice be not larger than the supply. Formally, for any subset $D \in S$:

$$\int_{z\in D} (1-F(z))\, dz \le \int_{z\in D} \frac{\int_{s\in S} \mu(s,z)}{h} dz \tag{1}$$

Intuitively, it seems clear that, more skilled originators must ask questions to more skilled consultants, and that consultants should be those more skilled at problem solving. We show below that this is indeed the case. (The proofs are in the appendix)

Lemma 1 (Assortative Matching) *Let s be a solver or consultant who is solving problems posed by originator z and s' one who is solving problems posed by originator z'. If $z > z'$ optimality requires that $s > s'$.*

To illustrate why the planner would choose assortative matching consider the highest originator type. If he couldn't solve the problem then the problem is fairly hard in expectation. In contrast, the unsolved problem from the lowest originator is fairly easy in expectation. Assigning the smartest consultant to the easy problem is inefficient. Most likely a less able consultant could handle this problem and the smartest consultant's time would be better spend solving those problems that are hard in expectation and hence less able consultants have a lower probability of solving.

To show that some matching always takes place in equilibrium, consider a situation where all workers are unmatched, the least skilled agent $z = 0$ produces $F[0] = 0$, while the most skilled agent produces $F[1] = 1$. Now consider the value of the match between the best and worst workers. This value is $\frac{F[1]-F(0)}{h(1-F(0))} = \frac{1}{h} > 1$ as long as $h < 1$, and thus this match is welfare increasing.

Assortative matching together with the fact that it is never optimal to have an under utilized consultant implies that with full information $M(s,z)$ is degenerate and hence we can focus on characterizing the matching function $m(z)$. Before doing so it is useful to establish the following two lemmas:

Lemma 2 (Independents are Smart) *Suppose there are originators with ability z that do not seek advice. Then, there cannot be any originator $z' > z$ that seeks advice when he cannot solve a problem. If $z \in I$ then if $z' > z$ $z' \notin O$.*

Intuitively, if some problems are going to be passed on, they must be the easier ones– those have the highest likelihood of being solved by a problem solver. If a problem is too hard to be passed on and is dropped, then all the problems originated by smarter agents are even harder and hence are not worth being passed on to the consultants.

Lemma 3 (Experts are Smarter) *Agents who become consultants are smarter than those who become originators. If $z \in S$ then for all $z' \in O,\ z > z'$.*

It is easy to see that if an agent is a consultant, then someone smarter than him should not be an originator. If this were not the case, then the roles can be swapped and output increased, so that the originator solves the problems that were previously solved by the consultant, but with a higher probability. The gain is larger than the loss, since each consultant solves multiple problems.

The ordering implied by the two lemmas above together with the fact that $M(s, z)$ is degenerate allows us to write the resource constraint on advising time (eq. 1) as:

$$\text{for all } z: \int_0^z (1 - F(q))\, dq = \int_{m(0)}^{m(z)} \frac{1}{h} dt$$

Equivalently, the integral equation above can be written as:

$$m(z) = m(0) + h \int_0^z (1 - F(q))\, dq \tag{2}$$

and therefore:

$$m'(z) = h(1 - F(z))$$

We summarize our results in the following Proposition:

Proposition 1 *The first best allocation can be characterized by a matching function $m(z)$ and 2 cutoff types z_1 and z_2 where $z_1 \leq z_2$. Types $z \in [0, z_1] = O$ are originators who seek advice, types $z \in (z_1, z_2) = I$ originate problems but do not seek advice and $z \in [z_2, 1] = S$ are consultants. The matching function satisfies: $m(0) = z_2$, $m(z_1) = 1$ and $m'(z) = h(1 - F(z))$.*

This follows from the lemmas above and the need to guarantee the right proportion of consultants to originators so that demand for consultants and their available time are equal type by type.

$$0\underbrace{-------}_{O}z_1\underbrace{---------}_{I}z_2\underbrace{---------}_{S}1$$

Solving for the first best allocation. Given the characterization provided in the Proposition above, the only thing that remains to be determined are the optimal cutoff types z_1 and z_2. In fact, the boundary conditions $m(z_1) = 1$ and $m(0) = z_2$ imply that:

$$\int_0^{z_1} (1 - F(z))\, dz = \frac{1 - z_2}{h}$$

Hence, the planner can only choose z_1. This in turn determines the cutoff type

$$z_2 = Z(z_1) = 1 - h \int_0^{z_1} (1 - F(z))\, dz$$

and the matching function

$$m(z; z_1) = 1 - h \int_z^{z_1} (1 - F(q))\, dq$$

Given the previous results solving for the first best allocation reduces to solving the following:

$$\max_{z_1} \int_0^{z_1} F(m(z; z_1)) dz + \int_{z_1}^{Z(z_1)} F(z)\, dz$$

Taking the derivative with respect to z_1 and grouping the terms to facilitate the interpretation, the FOC can be written as:

$$\begin{aligned} & -\left(\underbrace{\int_0^{z_1} f(m(z; z_1)) \frac{\partial m(z; z_1)}{\partial z_1} dz}_{loss\ from\ worse\ matches} + \underbrace{F(Z(z_1)) \frac{\partial Z(z_1)}{\partial z_1}}_{loss\ from\ less\ originators} \right) \\ = & \underbrace{F(m(z_1; z_1)) - F(z_1)}_{Extra\ output} \end{aligned} \tag{3}$$

The condition can be readily interpreted. As z_1 increases, the marginal gain (the second line of expression 3) comes from more output being produced as more agents are able to seek advice. They now produce with probability $F(m(z_1))$ instead of working on their own and producing with probability $F(z_1)$. There are two sources of losses. First, as more agents are asking for advice, the quality of advice each agent receives is reduced. The reason for this is that with positive sorting, as more agents become consultants, the worst consultant (the one who advises the worst originator), is lower quality, and so on for all originators. The final loss is the output loss from those originators who were self employed and now, instead of generating productive opportunities on their own they provide advise to other agents.

Proposition 2 *The number of independent agents is increasing in h. If $h > h'$ then $I' \sqsubseteq I$.*

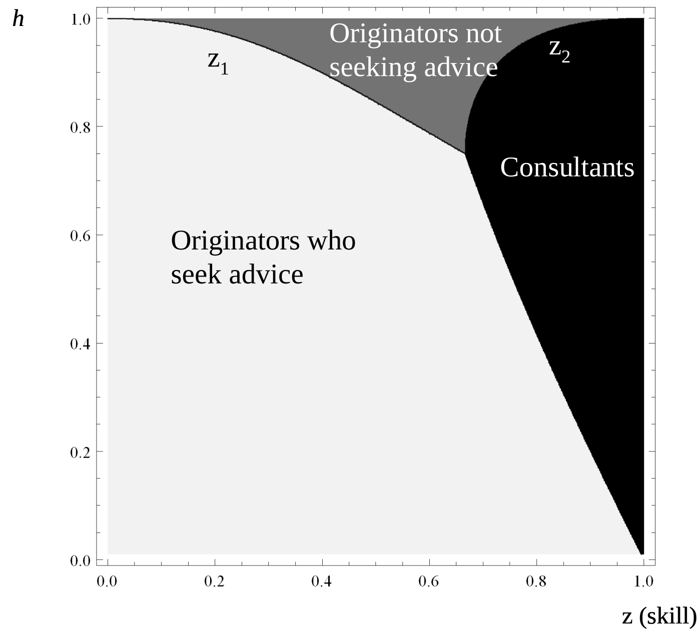

Figure 1: First best allocation of agents as a function of communication costs h

Figure 1 illustrates the solution for the uniform case for each value of the parameter h.[6]

As can be seen from the figure above, the number of consultants is non-monotonic in the number of problems a consultant can address. When the consultants cannot leverage their consultantise with many problems (h close to 1) then it is not worth having knowledge hierarchies and most agents are independents. As h decreases it is more efficient to have consultants. The number of independents decreases monotonically. For *the* uniform case, for $h \leq 0.75$ it is efficient not to have any independents at all. From this point, if we continue to increase the number of problems a consultant can address (decreasing h) the number of consultants starts falling. This is simply because fewer consultants are needed to address all the unsolved problems.

B Competitive Equilibrium and One-Sided Informational Asymmetries

With perfect information, the first best can be attained in a decentralized way as a competitive equilibrium. In fact, there are many different decentralizations that can implement the first best; they are all equivalent in the allocation they support, which is unique. In particular, we will consider

[6] In the uniform case:

$$\begin{aligned} m(z; z_1) &= 1 - h\left(z_1 - z - \frac{z_1^2 - z^2}{2}\right) \\ Z(z_1) &= 1 - h\left(z_1 - \frac{z_1^2}{2}\right) \end{aligned}$$

specifically two decentralizations, which are readily interpretable and will be useful later on. They differ in the agent who obtains the residual income from solving the problem. As a result, they deal differently with asymmetric information.

Letting the consultant of an agent z be $m(z)$, the joint output that a matched pair produces is given by:

$$F(z) + (1 - F(z))\frac{F(m(z)) - F(z)}{(1 - F(z))} = F(m(z))$$

That is, with probability $F(z)$ the originator produces on his own, and with probability $(1-F(z))$ he needs help. Thus the (ex post) output of the match (conditionally on advice being needed) is given by $\frac{F(m(z))-F(z)}{(1-F(z))}$ per worker or, given that a problem solver may have $1/h$ originators, $y = \frac{F(m(z))-F(z)}{h(1-F(z))}$. This function displays increasing differences $\partial^2 y(z; z_s)/\partial z \partial z_s > 0$, so the competitive equilibrium must be characterized by positive sorting, $m'(z) > 0$. The competitive equilibrium must result in occupational choices for all agents among originating or advising, in an earnings stream for originators and solvers, and in an allocation of originators to consultants (a matching function). The two decentralizations differ in who claims the residual income from the problem potentially being solved. We define them next.

Definition 1 *In a* ***consulting market*** *originators pay a fixed price for advice* $w(z)$ *and claim the residual income from the problem solution.*

Earnings of originators z who hire consultants z_s are:

$$W_o^c(z; z_s) = F(z) + (1 - F(z))\left(\frac{F(z_s) - F(z)}{(1 - F(z))} - w(z_s)\right),$$

while earnings of consultants of skill z are:

$$W_s^c(z) = \frac{w(z)}{h}.$$

Definition 2 *A* ***referral market*** *has consultants claiming the residual income from the problem solution; they pay a fixed price* $r(z)$ *in exchange of the problem.*

Earnings of originators z are then

$$W_o^r(z) = F(z) + (1 - F(z))r(z);$$

those of consultants who buy problems from originators of skill z_o are

$$W_s^r(z; z_o) = \frac{1}{h}\left(\frac{F(z) - F(z_o))}{1 - F(z_o))} - r(z_o)\right).$$

We proceed now to characterize the equilibrium in each of these markets. We will show that the allocations and earnings are identical, and identical to the first best.

A Consulting Services Market

In a consulting services market originators hire consultants of skill z_s for a fixed fee $w(z_s)$. Originators remain the residual claimants to output. Earnings of consultants do not depend on who they match with; their earnings are simply determined by the equilibrium consulting fee (they make no choices): $W(z) = \frac{w(z)}{h}$; on the other hand, originators earns the residual, so they cares directly about the choice of partner:

$$\begin{aligned} W_o^c(z; z_s) &= \max_{z_s} F(z) + (1 - F(z)) \left(\frac{F(z_s) - F(z)}{(1 - F(z))} - w(z_s) \right) \\ &= \max_{z_s} F(z_s) - (1 - F(z))\, w(z_s) \end{aligned}$$

With first order condition for the optimal choice of consultant:

$$f(z_s) - (1 - F(z)) w^{'}(z_s) = 0 \tag{4}$$

Before characterizing the competitive equilibrium, note that since $w'(z_s)$ must be increasing in equilibrium, $\partial^2 W_o^c(z; z_s)/\partial z \partial z_s > 0$, and the matching function $z_s = m(z)$ must be increasing, $m'(z) > 0$. The competitive equilibrium in this case can be characterized as follows:

Definition 3 *A competitive equilibrium in a consulting service market consist of:*
(i) a fee schedule $w(z)$ paid for by problem originators to consultants,
(ii) a matching function $m(z) : A -> S$ allocating consultants to originators;
(iii) a pair of cutoffs $\{z_1, z_2\}$, such that $A = [0, z_1]$ is the set of originators who seek advice, $I = [z_1, z_2]$ is the set of originators who do not seek advice (independent) and $S = [z_2, 1]$ is the set of problem solvers;
Such that:
(1) Supply equals demand point by point;
(2) The matching is such that no originator can do better by choosing a different consultant;
(3) No agent can be made better off by an occupational (from originator to consultant) change or by deciding to seek or forgo advice.

To construct the equilibrium, start from the supply and demand conditions. Supply equals demand pointwise implies: $m'(z) = (1 - F(z))h$. With $m(0) = z_2$, we can write the matching function as $m(z; z_2)$. Then for a given z_2, the matching function evaluated at the the highest originator is:

$$m(z_1; z_2) = 1$$

implies that the match is entirely pinned down up to one constant z_1. Trivially, $m(z_1; z_2) = 1$ implies a function $z_2^{sd}(z_1)$ with $z_2^{sd'} < 0$ (intuitively, if the supply of problems requiring advise increases $-z_1$ goes up– you need more problem solvers – z_2 must decrease).

Notice also that the first order condition (4) must hold for all z. Thus given some matching $m(z, z_2))$, the first order condition determines a wage function for each z_2.

$$w^{'}(z; z_2) = \frac{f(z)}{(1 - F(m^{-1}(m(z; z_2)))} \tag{5}$$

This differential equation can be solved simply by integration, as there is no $w(.)$ on the right hand side, and generates a wage function $w(z, z_2)$. To solve for the constant of integration, use $\frac{1}{h}w(z_2; z_2) = F(z_2)$. Finally, optimal occupational choices also requires that the top originator be indifferent between seeking or not advice: $W_o^c(z_1; 1) = 1 - (1 - F(z_1))\, w(1; z_2) = F(z_1)$, which implies $w(1; z_2) = 1$. This allows us to solve for z_2. The following proposition summarizes this analysis (see the Appendix for a detailed proof).

Proposition 3 *The competitive equilibrium in a consulting services market, is unique and achieves the first best.*

We show that the competitive equilibrium achieves the first best in the appendix. Note that nothing in the argument above requires that we observe the ability of the originators. The consultants do not make any choice, so they do not need to observe anything. Thus suppose that in fact, the originator skill is unobservable, but the skill of consultants is not. This could be the case, for example, if consultants have developed a reputation that allows agents to know who is knowledgeable and who is not, while problem originators are unknown, and so are their types. In this case, the consulting market we have just described would work exactly in the same way we suggested. We state this in the following corollary.

Corollary 1 *Under one sided asymmetric information, where only the consultant skill can be observed but not the originator skill, the consulting service market still attains the first best.*

A Referral Market

In a referral market originators transfer the whole residual ownership of the problem to consultants, in exchange for a fixed referral price $r(z)$. The earnings of originators, for a given per problem price, are given– originators now do not need to choose anything:

$$W_o^r(z) = F(z) + (1 - F(z))r(z) \tag{6}$$

While consultants earnings are a function of whom they choose to buy problems from:

$$W_s^r(z; z_o) = \max_{z_o} \frac{1}{h}\left(\frac{F(z) - F(z_o)}{1 - F(z_o)} - r(z_o)\right) \tag{7}$$

The optimal choice of z_o by a consultant problem solver with skill z requires:

$$-\frac{f(z_o)(1-F(z))}{(1-F(z_o))^2}=r'(z_o) \tag{8}$$

Again, note that as in the first best, and in the consulting services market, the competitive equilibrium must be characterized by assortative marching since $\frac{\partial^2 w(z_s)}{\partial z_s \partial z_0}>0$. We can define the competitive equilibrium analogously to the consulting case.

Definition 4 *A competitive equilibrium in problem referrals consists of:*
(i) a price schedule $r(z)$ paid by consultants in exchange for an unsolved opportunity from type z,
(ii)a matching function $m(z): A->S$ allocating opportunities to consultants;
(iii) a pair of cutoffs $\{z_1,z_2\}$, such that $A=[0,z_1]$ is the set of originators who sell their unsolved opportunities, $I=[z_1,z_2]$ is the set of originators who do not sell their opportunities and $S=[z_2,1]$ is the set of problem solvers;
Such that:
(1) Supply equals demand point by point;
(2) No consultant can do better by choosing to buy problems from a different originator;
(3) No agent can be made better off by an occupational change or by deciding to seek or forgo buying/selling unsolved opportunities.

The first part of the equilibrium construction, using the supply equal demand condition, leads to the same function $m(z;z_2)$ and the supply and demand condition result in a downward sloping marginal manager function $z_2^{sd}(z_1)$. Substituting again in the first order condition, we have

$$-\frac{f(z)(1-F(m(z;z_2))}{(1-F(z))^2}=r'(z) \tag{9}$$

Which we can integrate for each z_2 to obtain a function $r(z;z_2)$ and a constant. Again we can solve for the constant by using $W_s^r(z_2)=\frac{1}{h}\left(F(z_2)-r(z;z_2)\right)=F(z_2)$ so that $r(z,z_2)=F(z_2)(1-h)$. And we can finally, as previously, obtain a second condition for z_1 and z_2 by using the indifference condition of the top originator between seeking or not advice: $W_o^c(z_1;1)=F(z_1)+\left(1-F\left(z_1\right)\right)r(z_1)=F(z_1)$ thus $r(z_1;z_2)=0$. This generates a condition $z_2^r(z_1)$ with $z_2^{r'}(z_1)>0$, as we show in the appendix. Moreover, occupational choice is optimal. The following proposition summarizes this analysis (see the Appendix for a detailed proof).

Proposition 4 *The competitive equilibrium allocation in a problem referrals market is unique and achieves the first best. Moreover, the equilibrium allocation and earnings in the referral market are the same as in the consulting market. .*

Similarly to the consulting market, nothing about the equilibrium in this market requires observing the skill of consultants. This means that a referral market can achieve the first best in a

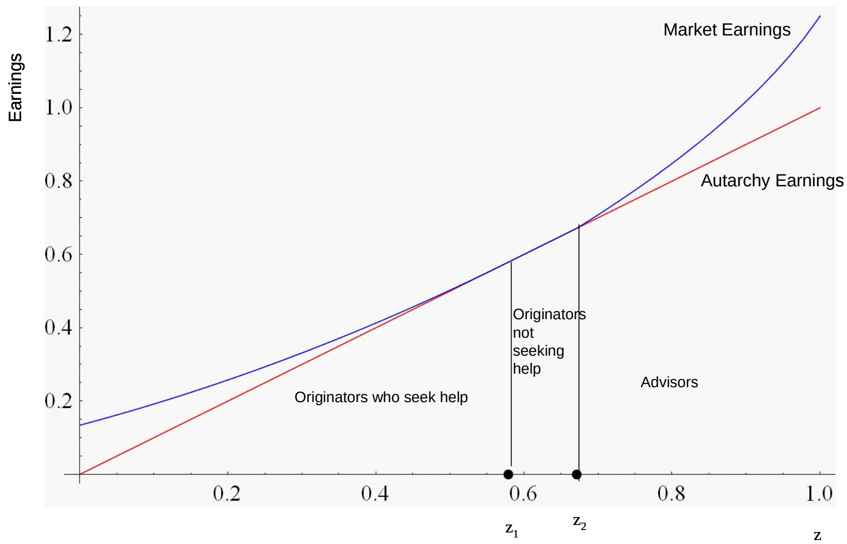

Figure 2: Market Allocation (the curved line) versus Autarchy (the straight, 45^oline)

situation in which the consultant skill is unobservable. For example, suppose all agents can see the skill of agents less skilled than themselves. Then one sided asymmetric information follows. In this case, having the informed side, the consultants, be the buying side, results in the first best.

Corollary 2 *Under one sided asymmetric information, where only the skill of originators can be observed (for example, all agents can observe the skill of those less skilled than themselves) the referrals market still attains the first best.*

Thus straightforward institutional arrangements can achieve efficiency if the informational problems are only one sided. In general, in bilateral relationships making the party with private information the residual claimant allows for efficiency. We have shown that a similar logic extends to this two sided market. As long as the market is set up so that prices are based on the observable type, -a referral market when the originator's type is observable or a fee based market for advice when the consultant's type is observable-, equilibrium prices will induce the side of the market with private information to self-select the efficient match.

Who gains most from the market for advice?

Inspection of figure 2 above leads to an important intuition on the value of skill in this market. The agents who gain most from being able to give and ask for advice are those in the extremes: the best problems solvers and the worst originators. In a way, the better the originator the worse the quality of the goods he sells, in the sense that he is seeking advice on harder problems. Thus 'quality' is decreasing in skill on the advice seeking side of the market. In the margin, originator z_1 is indifferent between seeking advice or not- to him, advice has no value– his earnings are the same with or without it. On the other side of the market, quality increases in skill- the better the agent, the better advice he can give; thus he benefits most from the market for skill.

III Two-Sided Asymmetric Information

We now turn to the case in which the agents types are their private information. This becomes a trading problem with two-sided adverse selection. Consultants might want to pretend they are smarter than they truly are and originators might want to pretend that their unresolved problems are simpler than they really are.

We first analyze the case in which output is unverifiable and ownership is non transferable. The only type of market which could be set up in this case is one with uncontingent wages in exchange for consultant services. This market breaks down because at wages high enough to motivate high types to become consultants low types would want to enter the consultant sector.

Then we look at the case in which either output is verifiable or an ownership stake can be sold and hence fully contingent contracts can be written. In this case, for low values of h the planner can achieve the first best allocation. This is possible when h is such that the first best calls for all originators seeking advice on their unsolved problems. Keeping originators from bringing their problems to the market is not possible and that leads to too much trade relative to first best when the first best would have some originators not seeking help with their unsolved problems.

A Unverifiable output and non-transferable ownership

When output contingent contracts cannot be written and ownership cannot be transferred, the originators are full residual claimants and consultants can only be paid an uncontingent fee. Since their payoff is uncontingent all consultants must receive the same payment. Under these circumstances, the market breaks down completely. No trade can take place, as the lowest skilled agents in the economy can pretend to be smarter and become a seller of consulting services. Any fixed fee that is high enough to entice a highly skilled agent to become a consultant will induce the least skilled agents to misrepresent their knowledge and offer their "services" for this fee.

Proposition 5 *When output contingent contracts cannot be written and ownership cannot be transferred there cannot be a competitive equilibrium with trade of consultant services.*

The intuition for this result is that the expected earnings of becoming an originator depend on the agent's type but the expected earnings of becoming a consultant (or pretending to be one) are independent of type. Hence, if there is some type that prefers to become a consultant then all types below want to follow the same path.

Note that in contrast to the classic lemons problem like in Akerlof (1970) the main reason for the market to break-down is not coming from an unravelling from high types exiting but rather from the excessive entry of low types. To illustrate, consider the market for brain surgeons. The fact that top brain surgeons are not differentially compensated from good brain surgeons is second order. The first order problem arises from the average Joe putting a white robe and offering to crack your head open. Hence, to be able to get the market operating, we must find a way to prevent the low types from becoming false consultants. In Fuchs and Garicano (2010) we show how a certification process achieves this and partially restores efficiency. As we show in the next section, if output is contractible full efficiency can be attained (for low values of h) without resorting to certification.

B Verifiable Output: Contingent Contracts

Suppose now that it is possible to contract on output. Agents can pay conditional on the solution being found. Without loss of generality, contracts can be characterized by two parameters w, α. w being the uncontingent payment to the problem solver and α the additional payment to the problem solver is he succeeds in solving the problem. Note that $w < 0$ would correspond to the consultant paying for giving advice- purchasing, in a way, the problem, in exchange for a share of the output.

We will characterize incentive feasible allocations in which each type of originator offers a different contract and each type of solver works for a different originator, that is, the matching function is strictly monotonic. Let $\omega_z = \{w_z, \alpha_z\}$ denote the contract offered by type z and $m(z; z_1)$ denote the solver type that attempts to solve a problem originated by type z.

Definition 5 *A set of contracts ω_z, a matching function $m(z; z_1)$ and a pair of cutoffs for occupational choice $\{z_1; z_2\}$ are a separating incentive feasible allocation if: (i) demand for advice equals supply of consultant services; (ii) only those agents $z > z_2$ choose to become consultants; (iii) only agents with $z < z_1$ choose to seek help with their unsolved problems; (iv) both types of agents truthfully reveal their types.*

We first show that the equilibrium must exhibit positive assortative matching.

Lemma 4 *Any separating incentive feasible allocation must exhibit positive assortative matching, $m'(z; z_1) > 0$.*

Since $m(z; z_1)$ is strictly increasing market clearing type by type essentially pins down $m'(z; z_1)$. The only degree of freedom left comes from z_1, how many agents become originators. In principle, there will be two possibilities either, types $z \in [0, z_1]$ will be originators, $[z_1, z_2]$ independents and types $z \in [z_2, 1]$ will be solvers; or $z_1 = z_2 = z^*$, where then $z \in [0, z^*]$ are originators and $z \in [z^*, 1]$ are solvers– no agents are independent. In next Lemma we show that there does not exist a separating incentive feasible allocation where some agents do not seek advice, and hence we focus on the case where all originators seek advice.

Lemma 5 *In any separating equilibrium all originators must seek advice in equilibrium.*

Proof. For there to be an equilibrium where some agents do not seek advice there must exist a z_1 and $z_2 > z_1$ such that type z_2 is ex-ante indifferent between being a consultant or an originator who leaves his problem unsolved. If he remains an originator but he does not seek advice, he earns $F(z_2)$. If he becomes a consultant, he gets paired with the worst worker type (since he is the marginal consultant), he then earns$\frac{1}{h}\left(w(0)+\alpha(0)\frac{F(z_2)-F(0)}{1-F(0)}\right)$, where the last term is the conditional probability that he can solve a problem that worker 0 could not solve. Indifference requires:

$$F(z_2) = \frac{1}{h}\left(w_0 + \alpha_0 F(z_2)\right)$$

or

$$F(z_2)\left(1-\frac{\alpha_0}{h}\right) = \frac{w_0}{h}. \tag{10}$$

Furthermore, for an interval of types that prefer to be originators to exist, there must exist a type z_1 that must strictly prefer to be an originator than to obtain the contract offered to the worst consultant:

$$F(z_1) \geq \frac{1}{h}\left(w_0 + \alpha_0 F(z_1)\right)$$

or

$$F(z_1)\left(1-\frac{\alpha_0}{h}\right) \geq \frac{w_0}{h} \tag{11}$$

Now, note that $w_0 \geq 0$ is a necessary condition in an equilibrium where some problems remain unsolved. Thus the two conditions above only can hold if $z_1 \geq z_2$ which cannot be the case. ■

Intuitively, a separating incentive feasible allocation with some problems unsolved must keep some agents out of the market. This restricts the prices to be positive $w \geq 0$ (since otherwise anyone not seeking advice could always receive a payment by entering the market place) which limits excessively the space of available contracts. In particular, the contract that is sufficient to keep the worst problem solver in the market makes it too attractive for the best originator to remain out- he also wants to be a problem solver.

Given the two Lemmas above, we can conclude that there can be at most one separating equilibrium allocation.

Proposition 6 *There is at most one separating equilibrium allocation.*

The qualifier "at most" is used in the proposition above because for high values of h there is indeed no separating equilibrium. Before showing that, we first characterize the equilibrium.

At this point, we specialize the density of problems to uniform $F(z) = z$; this considerably simplifies the analysis as it allows us to find closed form solutions for the fee and share schedules.

Originators and Problem Solvers Problem.

Let $V(z, \tilde{z})$ denote the value (ex-post) of an originator of type z who failed to solve his problem from pretending to be type $\tilde{z}$:

$$\begin{aligned} V(z,\tilde{z}) &\equiv -w_{\tilde{z}} + (1-\alpha_{\tilde{z}}) \Pr\left(q < m(\tilde{z};z_1)\, | q > z\right) \\ &\equiv -w_{\tilde{z}} + (1-\alpha_{\tilde{z}}) \frac{m(\tilde{z};z_1) - z}{1-z}. \end{aligned}$$

Hence, we can define the ex-ante expected value of becoming an originator and pretending to be type $\tilde{z}$ if trading with a consultant:

$$R(z,\tilde{z}) \equiv z + (1-z)\max\{V(z,\tilde{z}), 0\}$$

Let $S(z,\tilde{z})$ denote the value of a problem solver of type z who pretends to be $\tilde{z}$ and thus buys problems from type $m^{-1}(\tilde{z})$:

$$\begin{aligned} S(z,\tilde{z}) &\equiv \frac{1}{h}\left(w_{m^{-1}(\tilde{z};z_1)} + \alpha_{m^{-1}(\tilde{z})} \Pr\left(q < z | q > m^{-1}(\tilde{z};z_1)\right)\right) \\ &\equiv \frac{1}{h}\left(w_{m^{-1}(\tilde{z};z_1)} + \alpha_{m^{-1}(\tilde{z})} \frac{z - m^{-1}(\tilde{z};z_1)}{1 - m^{-1}(\tilde{z};z_1)}\right) \end{aligned}$$

Equilibrium Contracts and Matching

Given that $z_1 = z_2 = z^*$ the conditions for equilibrium imply:

(i) **Supply and Demand.** Given that there is assortative matching, the matching function is like in the first best, that is it is given by (2) up to the parameter z^*. In particular, given the uniform problem density,

$$z^* = \left(1 - \frac{1}{h}(\sqrt{h^2+1} - 1)\right)$$

and the matching function is:

$$m = hz - \frac{1}{2}hz^2 + \left(1 - \frac{1}{h}(\sqrt{h^2+1} - 1)\right). \tag{12}$$

(ii) **Occupational Choice.** We need that each type chooses the right occupation.[7]

For consultants ($z > z^*$) not to prefer to originate their own problems,

$$S(z,z) \geq R(z,\tilde{z}) \quad \textit{for all } \tilde{z} < z^*;$$

and for originators ($z < z^*$) not to pretend to be consultants:

$$S(z,\tilde{z}) \leq R(z,z), \quad \textit{for all } \tilde{z} > z^*.$$

[7] Since we require thruthtelling to be optimal conditional on choosing the right occupation we assume without loss that agents would be truthful if they choose the right occupation.

With equality for the boundary type:

$$S(z^*, z^*) = R(z^*, z^*)$$

(iii) **Ex post advice seeking.** Third, advice seeking must be ex post optimal when prescribed by the equilibrium. This requires that those with $z < z^*$ must strictly prefer to seek advice, that is, $V(z, z) > 0$, for all $z < z^*$.

(iv) **Truthelling.** For originators and solvers to be willing to report truthfully their type we require that:

$$\begin{aligned} V(z,z) &= \max_{\tilde{z}} V(z,\tilde{z}) \\ S(z,z) &= \max_{\tilde{z}} S(z,\tilde{z}) \end{aligned}$$

Equilibrium Construction

To construct the equilibrium, we will first construct the marginal conditions for truthtelling. We start by considering the problem of the originator who draws problem z. For him to report truthfully his type we need that:

$$V(z,z) = \max_{\tilde{z}} V(z,\tilde{z}),$$

that is, $\forall z \in [0, z^*]$:

$$\begin{aligned} \left(\frac{\partial V(z,\tilde{z})}{\partial \tilde{z}} | \tilde{z} = z\right) &= 0 \\ -w'_{\tilde{z}} - \alpha'_{\tilde{z}} \frac{m(\tilde{z}) - z}{1 - z} + (1 - \alpha_{\tilde{z}}) \frac{m'(\tilde{z})}{1 - z} &= 0 \end{aligned}$$

thus, in equilibrium ($\tilde{z} = z$)

$$-w'_z - \alpha'_z \frac{m(z) - z}{1 - z} + (1 - \alpha_z) \frac{m'(z)}{1 - z} = 0$$

or, since $m'(z) = h(1 - z)$:[8]

$$-w'_z - \alpha'_z \frac{m(z) - z}{1 - z} + (1 - \alpha_z) h = 0 \tag{13}$$

While for a problem solver to report his type:

$$S(z,z) = \max_{\tilde{z}} S(z,\tilde{z})$$

[8]The second order condition is:
$-w''_{\tilde{z}} - \alpha''_{\tilde{z}} \frac{m(\tilde{z}) - z}{1-z} - 2\alpha'_{\tilde{z}} \frac{m'(\tilde{z})}{1-z}$
$+ (1 - \alpha_{\tilde{z}}) \frac{m''(\tilde{z})}{(1-z)} \leq 0.$

that is, $\forall z \in [z_2, 1]$:

$$\left(\frac{\partial S(z,\tilde{z})}{\partial \tilde{z}}|\tilde{z}=z\right) = 0$$

$$\frac{1}{h}\left(\begin{array}{c} w'_{m^{-1}(\tilde{z})} + \alpha'_{m^{-1}(\tilde{z})}\frac{z-m^{-1}(\tilde{z})}{1-m^{-1}(\tilde{z})} \\ +\alpha_{m^{-1}(\tilde{z})}\frac{-1+m^{-1}(\tilde{z})+z_s-m^{-1}(\tilde{z})}{(1-m^{-1}(\tilde{z}))^2}\end{array}\right)\frac{dm^{-1}(\tilde{z})}{d\tilde{z}} = 0$$

thus, in equilibrium ($\tilde{z} = z$)

$$w'_{m^{-1}(z)} + \alpha'_{m^{-1}(z)}\frac{z-m^{-1}(z)}{1-m^{-1}(z)} + \alpha_{m^{-1}(z)}\frac{-(1-z)}{(1-m^{-1}(z))^2} = 0$$

or equivalently, in terms of worker skill, rather than manager skill we have:

$$w'_z + \alpha'_z\frac{m(z)-z}{1-z} + \alpha_z\frac{-(1-m(z))}{(1-z)^2} = 0 \tag{14}$$

By adding equations (14) and (13) we can solve for α_z:

$$\alpha_z = \frac{h(1-z)^2}{h(1-z)^2 + (1-m(z))} \tag{15}$$

This is a closed form solution for the α_z function– everything is known. Specifically, $\alpha_0 > 0$, and (since the denominator of α_z is larger than the numerator and it grows slower –recall that $m'(z) > 0$) $\alpha'_z > 0$. Moreover, α_{z^*} is:

$$\alpha_{z^*} = \frac{h(1-z^*)^2}{h(1-z^*)^2 + (1-m(z^*))} = 1$$

thus the best consultant is the full residual claimant of the output, and $V(z^*, z^*) = -w_{z^*}$. Note that occupational choice (ii) requires that $w_{z^*} \leq 0$: the best originator is not getting any share, $1 - \alpha_{z^*} = 0$, and he is passing the problem up, so he cannot be paying for advice.

The fact that $\alpha'_z > 0$ is intuitive since higher originators want to keep a lower share of the output and higher consultants are willing to accept a larger fraction of variable compensation. Although one might be tempted to think that the fixed payments must be strictly decreasing to make up for the increasing variable share this is not the case.

Substitute α_z in (13) to solve for w'_z :

$$w'_z = (1-\alpha_z)\,h - \alpha'_z\frac{m(z)-z}{1-z} \tag{16}$$

Since $\alpha'_z > 0$, and $\alpha_{z^*} = 1$, indeed $w'_{z^*} < 0$ but for $z = 0$ we have $w'_0 = h - \alpha'_z z^* > 0$. The non-monotonicity of the fixed fees arises from the fact that the matching function introduces an asymmetry in the local incentives to deviate. This asymmetry shows up in the difference in the last terms of 13 and 14.[9] In particular, consider the incentives for a type $z = 0$ from pretending to be a slightly higher type. Because the slope of the matching function is high for z close to zero, pretending to be slightly higher leads to a much better match for the originator. This incentive to exaggerate is partly offset by having $\alpha' > 0$ but note that the value of α' cannot be chosen arbitrarily since it must simultaneously provide incentives for z^* consultants not to want to pretend to be of type $z^* + \varepsilon$. Since the cost of exaggerating its type for a consultant comes from receiving a harder match and the matching function is steep at zero, the α' needed to satisfy the consultants IC is lower than that necessary to satisfy the originators incentives. It is therefore necessary to have $w_0' > 0$ to be able to satisfy both IC simultaneously.

From the fundamental theorem of calculus, given w_0 we can obtain the whole w_z schedule,

$$w_z(w_0) = w_0 + \int_0^z \left((1 - \alpha_t)\, h - \alpha_t' \frac{m(t) - t}{1 - t} \right) dt \quad (17)$$

$$w_z(w_0) = w_0 + A(z) \quad (18)$$

The integral is involved and cannot be obtained in closed form. Yet, we can fully characterize this function for a given w_0. Since there are no independents for a given h supply and demand uniquely determine z^* (from $Z_2(z^*) = z^*$). Given z^*, the matching function is uniquely pinned down (see equation 12) and so is α_z. The only object left to solve for is w_0 in (17). The condition that the marginal originator z^* is indifferent between being an originator and becoming a consultant pins down w_0:

$$\begin{aligned} S(z^*, z^*) &= R(z^*, z^*) \\ \frac{1}{h}(w_0 + \alpha_0 z^*) &= z^* - (1 - z^*) w_{z^*}(w_0, z^*)\,, \text{ that is:} \\ w_0 &= \frac{z^*(1 - \frac{1}{h}\alpha_0) - A(z^*, z^*)(1 - z^*)}{\frac{1}{h} + 1 - z^*} \end{aligned}$$

This completes the equilibrium construction. To verify that this is in equilibrium, note first that conditions (i), and (iv) are met by construction. Thus we need to verify that ex post advise seeking (condition iii) is optimal, that is $V(z, z) > 0$ for all $z < z^*$. We show in the Appendix (see Lemma (11)) that this condition is indeed satisfied. Finally, we need to verify that occupational choice is optimal, that is condition (ii) holds. One can verify numerically that this is indeed the case for $h < h^* = .76$; while for $h > h^*$there exist some consultants that prefer to exit the market.

Proposition 7 *For $h < 0.76$ there exists a separating equilibrium with the properties that:*

1. *The share of the opportunity transferred $\alpha_z = \frac{h(1-z)^2}{h(1-z)^2 + (1-m(z))}$, which is increasing in z and $\alpha_{z^*} = 1$.*

[9] This also explains why the contingent schedule on its own is not sufficient to provide incentives to both type of agents.

2. *The fixed payment w_z non monotonic, increasing at* 0 *and decreasing at z^*, z and $w_{z^*} \leq 0$.*

3. *An increase in communication costs h increases the fixed price of advice (shifts w_z up) and reduces the share of the solution transferred to the consultants (α_z goes down).*

4. *The equilibrium is efficient for $h < .75$*

Intuitively, truthtelling is attained through both the fixed and variable portions of the contract and the quality of the match. In fact, since w_z is non-monotonic, there exist $z < z'$ such that $w_z = w_{z'}$ and yet truthtelling is attained even though $\alpha_z < \alpha_{z'}$. The high type z' would seem to prefer the contract for z, since it costs the same fixed payment to get advice and a lower share must be offered (and thus a higher kept). However, at that price and share the advice received is worse, since $m(z) < m(z')$; this ensures truth-telling. Conversely, the low type z does not prefer the better advise since that requires offering a higher share, and the problem is sufficiently easy that it can be solved by the worse consultant with relatively high probability.

As to the last point, since $h < 75$ is the condition for the equilibrium to not have independents in the first, best, the equilibrium with double sided asymmetric information is efficient in that case. This is important, as we have a market that can achieve, when communication costs h are low enough, first best efficiency even though both seller and buyer types are unobservable. The key inefficiency in this market is that it is impossible to exclude those in the middle from it; when communication costs are sufficiently low, there is no efficiency reason to exclude them, and the equilibrium is efficient.

Finally, we can compare this equilibrium with the first best. It is clear that, as long as the equilibrium exists, trade is weakly larger than in the first best. Moreover, matching is worse. The asymmetric information helps the 'worse' agents, who in this economy are the intermediate ones- those are the least skilled consultants and the most skilled (thus less value to add) originators of problems.

Lemma 6 *The agents around z^* benefit from the asymmetry of information at the expense of types close to the extremes* $\{0, 1\}$

Proof. Clearly, those who were independent in the first best (middle types) are now in the market, thus they must be better off (they always could originate their own problems and keep them). To see that the highest type is worse off when the independents are around, note that in the first best he is solving all problems but paying 0 for them (to the last originator, who is indifferent between seeking and not seeking advice); but in the asymmetric information equilibrium there are no independents, and thus he must share some rents– pay a positive price. Similarly, the lowest problem solver is pinned down at the outside value, and all his rents are captured by the lowest origininator- the worst agent in the economy, who earns : $w_0 = \frac{1}{h}(z_2 - 0)/(1 - 0) - z_2 = z_2(1 - h)/h$. Without independents, there are two effects: first, z_2 decreases; second, some rents go to the consultant. Thus clearly the best and worst agents are worse off than in the first best, and by continuity there is an interval of the best and worst agents some other agents are worse off as well. ■

Summarizing, the equilibrium with double sided asymmetric information has the following properties:

1. When the first best allocation has no independents (low values of h) efficiency can be attained even in the presence of two-sided asymmetric information. There are still redistributive effects.

2.For intermediate values of h the equilibrium exists but there is too much trade relative to first best. Originators are getting weakly worse advice than in the first best. consultants are getting weakly harder problems. The agents around z^* benefit from the asymmetry of information at the expense of types close to the extremes $\{0, 1\}$

3. For high values of h there is no separating equilibrium.

C Contracts without fixed transfers: Pooling equilibria

The internet sites that have appeared to this date involve only a contingent payment, and no fixed transfer or payment for participate. Clearly, no separation of types can exist, as can be easily verified by setting $w(z) = 0$ in 13 and 14 and trying to solve the two differential equations in one function $\alpha(z)$.

Corollary 3 *No separation of types can exist using only contingent payments, $w(z) = 0$.*

Pooling equilibria do exist, however, in this case. The simplest case, which we examine briefly here, involves announcing a single contingent fee and having all problem solvers match with all problem originators randomly.[10] No problem remains unsolved, since it is free for an originator to offer a contingent fee to a problem solver to help him solve his problem. Thus the only equilibrium object in this case are the contingent fee (α) (since no type separation exists, the price cannot depend on the difficulty of the problem or on the ability of the originator, but only on whether output was or was not produced) and the cutoff z^*.

Market clearing conditions imply a unique cutoff type z^* which must satisfy:

$$\underbrace{\frac{1-z^*}{h}}_{Demand} = \underbrace{\int_0^{z^*} (1 - F(q))\, dq}_{Supply} \tag{19}$$

Note that the demand is decreasing in h so z^* must be decreasing in h as well.[11]

Given z^* the contingent price α must satisfy that type z^* be indifferent between becoming an originator or a consultant.

[10] Of course as we showed above, if the fee is non-contingent no equilibrium exists.

[11] Note that demand is decreasing in z^*, and supply is increasing in z^*. Moreover, if $z^* = 0$, there is excess demand, while if $z^* = 1$ there is excess supply.

The expected earnings for the marginal originator z^* are given in this pooling equilibrium p by:

$$R^p(z,\alpha) = F(z_0) + (1 - F(z_0)) \left(\frac{EF[z_s|z > z^*] - F(z_0)}{1 - F(z_0)} \right) (1 - \alpha) = \quad (20)$$

$$= z^*\alpha + (1 - \alpha)\frac{1 + z^*}{2} \quad (21)$$

The expected earnings of problem solvers are given by:

$$S^p(z^*, \alpha) = \frac{\alpha}{h} \left(E_x \Pr\left[q < z^* | q > x \right] \right)$$

Since,

$$E_x \Pr\left[q < z^* | q > x \right] = \frac{\int_0^{z^*} \left(\int_x^{z^*} f(q)\, dq \right) dx}{\int_0^{z^*} \left(\int_x^1 f(q)\, dq \right) dx} = \frac{\int_0^{z^*} \left(F(z^*) - F(x) \right) dx}{\int_0^{z^*} \left(1 - F(x) \right) dx} = $$

$$= \frac{z^*}{2 - z^*}$$

$$S^p(z^*, \alpha) = \frac{\alpha}{h} \frac{z^*}{2 - z^*}$$

Since $R^p(z, \alpha)$ is strictly decreasing in α and $S^p(z^*, \alpha)$ is strictly increasing in α we can find the unique α^* that guarantees that $R^p(z, \alpha) = S^p(z^*, \alpha)$

$$z^*\alpha + (1 - \alpha)\frac{1 + z^*}{2} = \frac{\alpha}{h} \frac{z^*}{2 - z^*}$$

$$\alpha = \frac{2 + z^*(1 - z)}{\frac{2}{h} z^* + 2 - (3 - z^*) z^*} \quad (22)$$

In the expression above, α is strictly increasing in h and for $h = \frac{3}{4}$, $\alpha = 1$. Therefore, for the uniform case, an equilibrium with$\alpha \leq 1$ only exists for $h \leq \frac{3}{4}$. Note that an equilibrium with $\alpha > 1$ is not incentive compatible, as originators would have to pay for asking for help with their problem.

With $\alpha = 1$ there could still be equilibria in which some agents decide to become consultants. In fact, there is a continuum of such equilibria. In all of them, there is an excess supply of problems. Essentially, in all of these equilibria an originator abandons his claim to the problem, and transfers the problem wholly to the problem solver. We can find an upper and lower bound to the entry into the consultanty market by assuming that the problems from the dumbest (smartest) originators are passed on to the consultants. Letting the interval of consultants be $[z_2, 1]$, the easiest case corresponds to the case where the problems are in the interval $[0, \bar{z}]$ with $\bar{z} < z_2$; the hardest case when the problems are in $[\underline{z}, z_2]$.

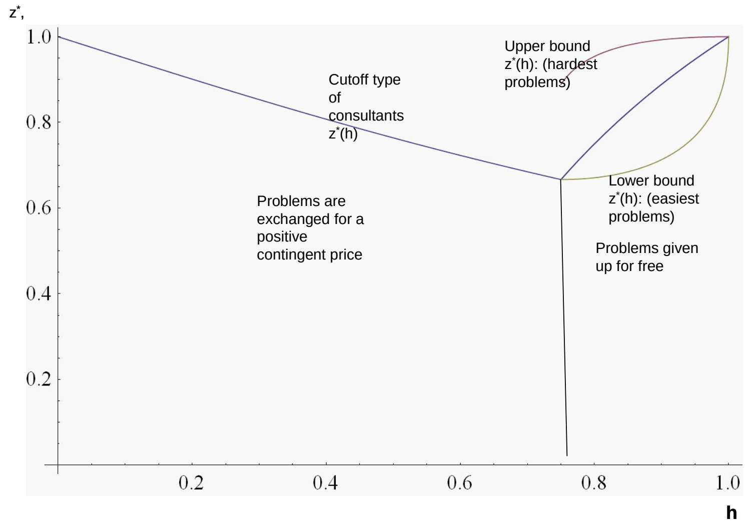

Figure 3: Pooling Equilibrium, Uniform case. For $h > .75$ the contingent fee is $\alpha = 1$:the entire claim is transferred. In this case there always exists an excess supply of problems, and the decision to enter into consulting depends on the selection of problems that do get transferred. Maximum entry (lower bound) occurs when the easiest problems get transferred; minimum entry (upper bound) when only the hardest problems get transferred.

For example, for the uniform case and in the intermediate case in which problems are drawn randomly from the pool of unsolved problems:

$$\begin{aligned} S^p(z^*, 1) &= R^p(z^*, 1) \\ &\Updownarrow \\ z^* &= 2 - \frac{1}{h}; \quad h > \frac{3}{4} \end{aligned}$$

Figure 3 illustrates the properties of the equilibria for the uniform case, with the bound on the worst consultant $(z^*(h))$ for each h depending on whether the problems selected are the worst problems, a random selection of problems, or the best problems drawn by originators.

The results above where formulated for the particular case of a uniform distribution, but the following proposition summarizes them for the general case, which has the same properties.

Proposition 8 (Pooling) *When only a contingent payment can be offered there exists an* $h^* \in (0, 1)$ *such that*
i) For any $h \leq h^*$ *a unique pooling equilibrium with a single contingent fee exists, in which agents,* $z \in [0, z^*(h)]$ *are problem originators and agents* $z \in [z^*(h), 1]$ *are problem solvers, and there is a contingent fee* $\alpha^*(h)$ *per problem referred. The cutoff* $z^*(h)$ *is strictly decreasing in* h *and is*

larger than the first level cutoff z_1.

ii) For any $h > h^*$ *there exist a continuum of equilibria with* $\alpha = 1$. *The equilibria differ in the value of* h *and the distribution of problems that are passed on to consultants. Both the lower bound and upper bound of equilibrium values for* z *are increasing in* h *and as* $h \to 1$ *there is a unique equilibrium with* $z = 1$.

Thus, while the pooling market with a single contingent payment does not break down, it suffers from three types of efficiency losses with respect to the full information problem; first, problems that are too hard to be referred are in the pool of problems passed on; second, there is too much entry into consulting; and third, there is inefficient matching- conditional on a problem being passed, the probability that it is solved is much lower, as the matching is now random instead of assortative.

IV Empirical Implications and Conclusions

A One-sided asymmetric Information in Problem Solving Markets: Consulting markets

As we show in Section II.B, when information about the quality of those giving advice can be easily obtained (maybe through reputation or through well functioning certification mechanisms) contracts should take the form of consulting contracts: consultantise is provided in exchange for a fee; those buying the advice can easily internalize the difficulty of their own problem, and the consulting fee internalizes properly the match between the expected difficulty of the problem and the skill of the consultant required. This is consistent with the use of consulting by firms in many contexts, where essentially the consultant names a price and a quality pair and the client sorts among firms.[12]

On the other hand, when opportunities are transferable and the quality of consultants who would be appropriate for a given opportunity is more difficult to observe, we expect referral contracts to be preferred. In this context, originators post their opportunities in exchange for a fee and consultants bid for them. The market price for these opportunities will be such that, again, consultants will sort themselves so that the best consultant will end up with the a priori more difficult opportunities. Such markets are observed in biotech, for example, where firms which have discovered molecules and want to take them to market try to find the right company to do this by selling their IP, the profitable opportunity they generated; they post the opportunity and idea, and the pharma companies sort themselves among opportunities.

B Markets for advice with two-sided asymmetric information

In Section III we studied the design of consultant markets when asymmetric information is relevant on both sides – the originating and the problem solving side. We argue, first, that whether a solution

[12] Of course, there is an element of risk sharing in the hourly fee structure, but the total price of the project is actually basically known in advance with a high degree of certainty in this market.

exists depends on the extent to which output is verifiable; if it is not, absent certification the market will disappear. If it is, then we find that (1) the optimal contract involves a fixed payment and an equity stake for both problem solvers and originators; that (2) the equity share increases with the quality of the problem solving required; (3) that income distribution in the market is tighter than in the first best, as rents are capture by those agents in the middle of the distribution, either the worst originators (the most skilled ones) or as worst problem solvers (the least skilled ones among them); and that (4) there is too much problem solving in equilibrium, as all problems are solved. We discuss next three instances of problem solving markets under asymmetric information.

Online consultant Markets A range of companies have emerged to help companies get problem solving advice. The pioneer is in this industry is Innocentive, other rivals are Innovation Exchange, Fellowforce, NineSigma, yet2.com, and YourEncore.[13] The market has two sides, those who post problems for which no solution is yet known, called the 'seekers' by Innocentive and those who attempt to provide a solution who are called 'solvers' in the site. Seekers post 'challenges' which are unsolved problems. Like in our case, there is asymmetric information both about how difficult the challenge will eventually prove and about the skill of those attempting a solution.

The leading site at the moment is Innocentive.com. The site had, as of May 2008, 145000 solvers registered, who had submitted to date 7,011 solutions; 620 challenges have been posted, with a total award of $16m, of which 215 have been solved with $3m paid out.[14]

This generation of sites operate along the lines of a tournament model; a prize is posted, and it is awarded to the agent who solved the challenge. The system has an important inefficiency– the effort of those who do not win the challenge is wasted. Moreover, this inefficiency is compounded strategically, as participants try to figure out which challenges will attract just the right number of solvers to ensure an adequate probability of winning[15] Of course, it is hard to know a priori how hard and how attractive a challenge will be, but the system as set up has the seed of its own destruction. If it becomes too popular, the probability of being the chosen solution collapses, and those with a substantially higher opportunity cost of time –presumably the best solvers– drop out of it.

Our analysis suggest that the system should be replaced by one in which a restricted number of solvers, potentially grouped into a team, are given an opportunity to solve the problem in exchange for a fee and a share in the output that may result, where the share should be higher the harder the problem. This will, unlike a tournament, attract the right level of talent. Absent an arrangement

[13] See innocentive.com, fellowforce.com, ninesigma.com, yet2.com yourencore.com.

[14] This information is from the InnoCentive.com (A) HBS case 9-6098-170, by Karim R. Lakharni, dated June 10, 2008.

[15] A top solver in Innocentive, David H. Tracy declared "I'm good enough mathematician (barely) to know better than to play the lottery. ... If I thought that a given Challenge would attract say 100 strong solutions –solutions likely to be roughly as wonderful as mine- then I might choose not to invest the time needed to create and submit a solution with just 1% probability of winning."

along these lines, these sites are likely to remain small and out of the mainstream, attracting only the non-mainstream solvers.

Rent sharing and Referrals in the Law Like in our model, lawyers generally pass on clients to one another in exchange for a referral fee. This is particularly the case in litigation, where these payments take the form, as in the contracts we describe, of referral shares. While such compensation arrangements involve clearly team production and moral hazard issues as well (see Garicano and Santos, 2004), there is also a sorting element along the lines of our analysis. Thus, we expect better lawyers to receive larger shares of output, and to be matched with harder problems. Empirically we should find referral shares increasing with the quality of the claim or of the lawyer.[16]

Venture Capital: Sorting and Contracting Venture capital markets have very similar features to the ones in our second best contracts. Those who originated a business idea must find consultants that help them take them to market. Venture capital contracts generally will involve cash transfers and equity stakes– that is a share in the profits if the idea is successful.

Our model has clear implications for this type of markets. First, as pointed out above, there should be positive sorting between the quality of the deal and the quality of the venture capitalist; only a good venture capitalist is able to add sufficient value to a good entrepreneur. A direct test of this is Sorensen (2007), who finds that more experienced venture capitalists make more successful investments and invest in 'better' companies– late stage and biotechnology companies. Second, this positive sorting should covary with increasing revenue shares for the venture capitalist– the higher the unobserved quality of the entrepreneur (and of the venture capitalist) the higher should be the revenue share accruing to the venture capitalist, as the better entrepreneur signals good quality by offering a large residual share. Kaplan and Stronberg (2002) come closest to being able to test this, as they have contracting data on VC contracts; however, their regressions do not test for these sorting and share effects.

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[16] A test along these lines was conducted in small sample by Stephen Spurr (1988). However, rather than presenting the regression of share on either claim value or quality of lawyer he includes both in the only specification he studies and finds them insignificant.

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V Appendix A: Ommited Proofs

Lemma (1) **Assortative Matching.** We show that $s < s'$ for $z > z'$ cannot be optimal. Conditional on being consulted on one problem each, the expected number of solved problems is:

$$\frac{F(s)}{1-F(z)} + \frac{F(s')}{1-F(z')}$$

If instead we reverted the matching so that type $m(z)$ is tries to solve type z' problems and vice-versa the number of solved problems would be:

$$\frac{F(s')}{1-F(z)} + \frac{F(s)}{1-F(z')}$$

We show that the second arrangement is more productive if $s < s'$ for $z > z'$:

$$\begin{aligned}
s' &> s \\
F\left(s'\right) &> F(s) \\
F\left(s'\right)\left(F\left(z'\right) - F(z)\right) &< F(s)\left(F\left(z'\right) - F(z)\right) \\
F\left(s'\right)F\left(z'\right) - F(z)F\left(s'\right) &< F(s)F\left(z'\right) - F(z)F(s) \\
F\left(s'\right)F\left(z'\right) + F(z)F(s) &< F(s)F\left(z'\right) + F(z)F\left(s'\right) \\
F\left(s'\right)\left(1 - F\left(z'\right)\right) + (1-F(z))F(s) &> F(s)\left(1 - F\left(z'\right)\right) + (1-F(z))F\left(s'\right) \\
\frac{F(s')}{1-F(z)} + \frac{F(s)}{1-F(z')} &> \frac{F(s)}{1-F(z)} + \frac{F(s')}{1-F(z')}
\end{aligned}$$

■

Lemma (**2**) **Independents are Smart.** If $z' > z$ could not solve a problem it means that the problem is harder to solve than the unsolved problem by type z. Hence, it is more likely that type

$m(z')$ will solve problem z than problem z' therefore the planner would be better off by leaving z' unmatched and matching z. This implies that no originator can be smarter than an independent. ■

Lemma (3)**Experts are Smarter.** Consider two agents and independent $z_I \in I$ and a consultant with type $m(z)$. Assume in search of a contradiction that $z_I > m(z)$ is optimal. The joint output of these two types is:

$$F(z_I) + \frac{F(m(z))}{hF(z)}$$

Since $\frac{1}{h} > 1$, if $z_I > m(z)$ then:

$$F(m(z)) + \frac{F(z_I)}{hF(z)} > F(z_I) + \frac{F(m(z))}{hF(z)}$$

Therefore the planner could improve by having z_I and $m(z)$ switch their roles. Hence consultants must always be smarter than independents. ■

Proof of Proposition 2. Follows from taking the derivative of the FOC with respect to h. ■

Rather than providing a separate proof for Propositions (3) and (4) separately we will prove them jointly with the Proposition below.

Proposition 9 *The competitive equilibrium allocation exists and is unique. It may be implemented equivalently through a referral or a consulting market. It attains the first best.*

We shall show that the competitive equilibrium is unique and attains the first best. To do this we abstract first from the actual implementation for now proceed and proceed through a series of three lemmas which follow below. We then show that the earnings and allocation in the referral and consulting formulations match the ones in the general derivation we follow.

Lemma 7 *The competitive equilibrium must display positive assortative matching.*

Proof. To see this consider the production function of a firm that hires solvers and originators of skill z_s and z_o. This firm's production function will be:

$$\pi(z_s, z_o) = F(z_s)n - w_o n - w_s$$

Subject to the time constraint of the problem solver, $h(1 - F(z_o)n = 1$. That is the profit function of this firm is:

$$\pi(z_s, z_o) = \frac{F(z_s) - w_o}{h(1 - F(z_o)} - w_s$$

It is clear this production function displays increasing differences (since $\frac{\partial^2 y}{\partial z_o z_s} > 0$ where $y = F(z_s)n$) and thus positive sorting must hold in equilibrium. ■

Lemma 8 (Market Clearing) *Equality of supply and demand means that the competitive equilibrium is pinned down up to the two cutoffs* z_1 *and* z_2*. Moreover, for each* z_1 *there exists a unique* z_2, $z_2^{sd}(z_1)$ *such that supply equals demand. Finally,* $z_2^{sd'} < 0$*.*

Proof. Suppose first that some agents are unmatched– there are independent originators. Supply equals demand pointwise implies: $m'(z) = (1 - F(z))h$. With $m(0) = z_2$, we can write the matching function as $m(z; z_2)$. Then for a given z_2 and

$$m(z_1; z_2) = 1$$

implies that the match is entirely pinned down up to one constant z_1. Trivially, $m(z_1; , z_2) = 1$ implies a function $z_2^{sd}(z_1)$ with $z_2^{sd'} < 0$ (intuitively, if there are more supply of problems, you need a larger supply of problem solvers). ■

Lemma 9 (Uniqueness) *There always exists a competitive equilibrium for* $h < 1$*. This equilibrium is unique.*

Proof. The proof is by construction. We move along the $z_2^{sd}(z_1)$ curve until either $z_2 = z_1$ or $z_2 = w_c(z_2)$.

Consider a profit maximizing firm that hires teams of originators z_0 and problem solvers z_s. Given that each problem solver can solve $1/h$ problems per unit of time, and that an originator only needs help with probability $(1 - F(z_o))$, the firm will need a measure $n = 1/(h(1 - F(z_o))$ of originators per problem solver, so that earnings are given by:

$$\pi(z_s, z_o) = F(z_s)n - w_o n - w_s$$

by the 0 profit condition these can be through of equivalently as the measure of consultants hires the originators:

$$w_s(z_s, z_o) = F(z_s)n - w_o n = \frac{F(z_s) - w_o}{h(1 - F(z_o)}$$

For the choice of originators of quality z_o to be an optimum, it must be the case that the wages are such that the choice of z_o is optimum:

$$w_s(z_s, z_o) = \max_{z_o} \frac{F(z_s) - w_o(z_o)}{h(1 - F(z_o)}$$

From here, using the first order condition and then the envelope we can obtain the slope of the earnings curve along the equilibrium:

$$\frac{\delta w_o(z)}{\delta z} = \frac{f(z)}{(1-F(z))}(F(m(z)) - w_o(z)) < f(z)\frac{(F(m(z)) - F(z)}{(1-F(z))} < f(z)$$
$$\frac{\delta w_s(z)}{\delta z} = \frac{f(z)}{h(1-F(m^{-1}(z))} > f(z)$$

Where we are using the matching schedule definition $z_s = m(z)$. The inequality in the first line uses the fact that, for originators who actually choose to be originators , earnings as originators are higher than earnings as self employed.

Now we move along $z_1^{sd}(z_2)$. Since the top consultant matches with originator z_1 and $w(z_1) = F(z_1)$ for optimal occupational choice, top consultants earnings are fixed at $\frac{1-F(\varepsilon)}{h(1-F(\varepsilon))} = \frac{1}{h} > 1$ as long as $h < 1$, and as long as the equilibrium is interior (there are originators). The earnings schedule of consultants thus starts at $w_c(1) = 1/h$ and decreases with slope $\frac{f(z)}{1-F(m^{-1}(z;z_2))}$. Specifically, since increasing z_1 raises the value of the match of every problem solver, this means that the rate of decrease of earnings as we reduce z is larger the higher z_1 : $\frac{d}{dz_1}\left(\frac{f(z)}{1-F(m^{-1}(z;z_2))}\right) > 0$. Start from $z_2 = 1$, $z_1 = 0$ We know this is a market clearing pair (that is $z_1^{sd}(1) = 0$), since there is no supply or demand of problems. The worst workers earn $F[0] = 0$ and the best ones earn $F[1] = 1$. Now consider a deviation along the market clearing condition so that $z_1 = \varepsilon$ and $1 = z_2^{sd}(z_1)$. Now the value of the match is $\frac{1-F(\varepsilon)}{h(1-F(\varepsilon))} = \frac{1}{h} > 1$ as long as $h < 1$. Managers clearly will chose to hire workers ε, pay them $z_1 = \varepsilon$ and earn themselves $1/h > 1$. However, this is not an equilibrium, as the agents at $z_2 = 1$ strictly prefer being problem solvers than independents(the earnings function is discontinuous at $z_2 = 1$). Raise now z_1 to $z_1^* = 2\varepsilon$. Now earnings of top solvers $z = 1$ are still $1/h$. Construct the earnings function of consultants by using $\frac{f(1)}{1-F(2\varepsilon))}$.The earnings of $z_2^* = z_2(2\varepsilon)$ are either still $w(z_2^*) > z_2$ or $w(z_2^*) = z_2$. In the second case, we have a competitive equilibrium and stop. In the first case, we go back and increase z_1 again by ε. Now the slope of the earnings function at 1 is stepper at every point, $\frac{f(1)}{1-F(3\varepsilon))} > \frac{f(1)}{1-F(2\varepsilon))}$ etc. Since $\frac{f(z)}{1-F(m^{-1}(z_1;z_2))} > f(z)$, and $z_2^{**} < z_2^*$ the distance $w(z_2) - z_2$ is unambiguously reduced with each step. We can continue taking these steps till $z_1 = z_2$. If at any point $w(z_2) = z_2$, we have an equilibrium, since $w(z_1) = z_1$, market clears, and matches are optimal (agents cannot gain by deviating since, by construction, the slope is always equal to the marginal contribution. Moreover, since $\frac{\delta w_o(z)}{\delta z} < f(z)$, if worker z_1 is indifferent between being a worker or an originator, all workers with $z < z_1$ strictly prefer to be workers. If instead at this point it is still the case that $w_c(z_2) > z_2$, then we have no independents, and we can obtain the cutoff simply from the market clearing condition, $z_1 = z_2 = z^*$, where $m(z^*; z^*) = 1$, ■

Lemma 10 *The unique competitive equilibrium is Pareto Optimal.*

Proof. All we need to show is that the cutoff types coincide. We do this by showing the that the

FOC of the planner's problems is satisfied with the CE cutoff z_1^{CE}

$$
\begin{aligned}
&-\left(\underbrace{\int_0^{z_1^{CE}} f(m\left(z;z_1^{CE}\right))\frac{\partial m\left(z;z_1^{CE}\right)}{\partial z_1}dz}_{loss\ from\ worse\ matches}+\underbrace{F\left(Z\left(z_1^{CE}\right)\right)\frac{\partial Z\left(z_1^{CE}\right)}{\partial z_1}}_{loss\ from\ less\ originators}\right) \\
&= \underbrace{F\left(m\left(z_1;z_1^{CE}\right)\right)-F\left(z_1^{CE}\right)}_{Extra\ output}
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
&-\left(\int_0^{z_1^{CE}}\left(f\left(m\left(z;z_1^{CE}\right)\right)\left(-h\left(1-F\left(z_1^{CE}\right)\right)\right)\right)dz+F\left(z_2\right)\left(-h\left(1-F\left(z_1^{CE}\right)\right)\right)\right) \\
&= F\left(1\right)-F\left(z_1^{CE}\right)
\end{aligned}
\tag{24}
$$

$$
\left(\int_0^{z_1^{CE}} f\left(m\left(z;z_1^{CE}\right)\right)dz+F\left(z_2\right)\right)=\underbrace{\frac{F\left(1\right)-F\left(z_1^{CE}\right)}{h\left(1-F\left(z_1^{CE}\right)\right)}}_{Earnings\ of\ z=1}
$$

Making a change of variables in the integral, the LHS can be written as:

$$
\left(\int_{z_2^{CE}}^1 \underbrace{\frac{f\left(z\right)}{h\left(1-F\left(z\right)\right)}}_{\frac{\partial w_s(z)}{\partial z}}dz+\underbrace{F\left(z_2\right)}_{Earnings\ of\ z=z_2}\right)=Earnings\ of\ z=1
$$

■

Proof of Proposition (5)**.** If consultants are paid a fixed fee ϕ and a given type z chooses to become a consultant then all types $z' < z$ will choose to become consultants as well. This follows from noting that for type z :

$$
\frac{\phi}{h}\geq F\left(z\right)+\left(1-F\left(z\right)\right)\left(\max\left\{0,\Pr\left(sol|z\right)-\phi\right\}\right)
$$

where $\Pr\left(sol|z\right)$ is the probability that the problem gets solved conditional on hiring a consultant and the difficulty of the problem being above z.

Furthermore, since type z could choose to not solve a problem of difficulty $q < z$ it must also follow that he can pretend his ability level is $\tilde{z} < z$ and therefore:

$$
\frac{\phi}{h}>F\left(\tilde{z}\right)+\left(1-F\left(\tilde{z}\right)\right)\left(\max\left\{0,\Pr\left(sol|\tilde{z}\right)-\phi\right\}\right)\quad\forall\tilde{z}<z
$$

but the RHS of the equation is exactly what any type $\tilde{z} < z$ would get. Note that these agents can also get $\frac{\phi}{h}$ since their type is not verifiable. Hence, they would all choose to become consultants. As a result, there would be nobody interested in hiring a consultant because consultants would not be able to solve the problem. ■

Proof of Lemma 4. In search of a contradiction, suppose that $m'(z)$ is negative. Suppose first that there are no independents. This would imply that the lowest problem solver z^* is meant to solve problems for the best originator, z^*. Clearly, no problem posed is solved, $\frac{z^*-z^*}{1-z^*}=0$ and hence there cannot be any trade between them. Second, suppose there are independents. Then type z_1, the highest originator must turn to z_2, the lowest problem solver, for help. The gains to z_1 from hiring z_2 must be 0, since he must be indifferent between getting help or not:

$$0 = -w_{z_1} + (1-\alpha_{z_1})\frac{z_2 - z_1}{1-z_1}$$

while z_2 must be indifferent between becoming a problem solver or an independent:

$$z_2 = \frac{1}{h}\left(w_{z_1} + \alpha_{z_1}\frac{z_2 - z_1}{1-z_1}\right)$$

This $z_2(z_1)$ is generically different that the one required to satisfy market clearing. ■

Lemma 11 *The candidate equilibrium satisfies ex-post advise seeking (condition iii). That is* $V(z,z) > 0$ *for all* $z < z^*$

Proof.

$$\begin{aligned} V(z,z) &= -\left(w_0 + A\left(z,z^*\right)\right) + \\ &\quad \left(\frac{\left(1-\left(hz-\frac{1}{2}hz^2+z^*\right)\left(hz-\frac{1}{2}hz^2+z^*-z\right)\right.}{\left(h(1-z)^2+\left(1-\left(hz-\frac{1}{2}hz^2+z^*\right)\right)(1-z)\right)}\right) \end{aligned}$$

$$V(z,z) = -w_z + (1-\alpha_z)\,\frac{m(z;z^*)-z}{1-z}$$

$$\frac{\partial V(z,z)}{\partial z} = -w_z' - \alpha_z'\frac{m(z;z^*)-z}{1-z} + (1-\alpha_z)\left(\frac{(1-z)\left(m'(z;z^*)-1\right)-\left(m(z;z^*)-z\right)}{(1-z)^2}\right)$$

and using the first order condition for truthtelling:

$$\begin{aligned} \frac{\partial V(z,z)}{\partial z} &= -\left(1-\alpha_z\right)h + \left(1-\alpha_z\right)\left(\frac{(1-z)\left(h(1-z)-1\right)-\left(m(z;z^*)-z\right)}{(1-z)^2}\right) \\ &= \frac{\left(1-\alpha_z\right)}{(1-z)^2}\left((1-z)\left(h(1-z)-1\right)-(m-z)-(1-z)^2 h\right) \end{aligned}$$

which is negative if $h - m + 4z - 2hz - z^2 + hz^2 - 2 < 0$. Replacing m in and simplifying:

$$h + 4z - 3hz - z^2 + \frac{3}{2}hz^2 + \frac{1}{h}\left(\sqrt{h^2+1}-1\right) - 3 < 0$$

which is easy to verify to be true. Now we need to show that it is indeed always positive. Since $\alpha_{z^*} = 1$, we require that $w_{z^*} < 0$. One can verify this is indeed the case. ■