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Informational Externalities in Market Games

Xavier Vives[*]

IESE Business School

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Abstract

This paper performs a welfare analysis of market games with private information in which public information is endogenously generated and agents can condition on noisy public statistics in the rational expectations tradition. We overturn the presumption that information externalities will lead economic actors to put too little weight on private information and consequently prices will contain too little information. Equilibrium is not (restricted) efficient even when feasible allocations share similar properties to the market context (e.g., linear in information). The reason is that the market in general does not balance optimally non-fundamental volatility and the dispersion of actions. Under strategic substitutability, agents will put too *much* weight on private information when the allocational role of prices prevails over its informational role, and too *little* in the opposite case. Under strategic complementarity, the latter case obtains. The welfare loss at the market solution may be increasing in the precision of private information. These results extend to the internal efficiency benchmark (accounting only for the collective welfare of the active players). Received results—on the relative weights placed by agents on private and public information, when the latter is exogenous—may be overturned.

Keywords: information externality, market games, complementarity and substitutability, asymmetric information, pecuniary externalities, excess volatility, team solution, rational expectations

1. <u>Introduction</u>

This paper overturns in the context of market games the presumption that information externalities will lead economic actors to put too little weight on private information and consequently prices will contain too little information. We show that agents may put too much weight on private information and prices may contain too much information for reasons other than the well-known Hirshleifer effect of destruction of insurance opportunities. This happens, in fact, in normal scenarios under strategic substitutes competition.

There has been a recent surge of interest in the welfare analysis of economies with private information and in particular on the role of public information in such economies (see, e.g., Morris and Shin 2002; Angeletos and Pavan 2007; Amador and Weill 2010). Agents may fail to place welfare-optimal weights on private and public information owing to payoff and information externalities. In this paper we examine the issue in a context where public information is endogenously generated and agents can condition on public statistics when making their choices. In the rational expectations tradition, agents learn from prices and from public statistics in general, which are themselves the aggregate outcome of individual decisions.

Endogenous public information is relevant for a broad array of markets and situations. In financial markets, prices are noisy statistics that arise from the decisions of traders. In goods markets, prices aggregate information on the preferences of consumers and the quality of the products. In the overall economy, the release of GDP data is a noisy public signal that is the outcome of actions taken by economic agents.[1] On the empirical front, initial evidence of herding of analysts forecasts (see Gallo et al. 2002 for GDP forecasts; Trueman 1994, Hong et al. 2000 for securities), and therefore of "insufficient" weight placed on private information, has been reversed by subsequent work. For example, both Bernardt et al. (2006) and Pierdzioch et al. (2010) find strong evidence of anti-herding behavior by, respectively, professional financial analysts and oil-price forecasters.[2] According to those authors forecasters issue predictions biased towards their private information. Effinger and Polborn (2001) and

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[1]   See, for example, Rodríguez-Mora and Schulstad (2007).

[2]   Other papers with anti-herding results by forecasters on asset prices and macro variables are Pierdzioch and Rülke (2012), Pierdzioch et al. (2013), and Frenkel et al. (2012).

Levy (2004) explain anti-herding behavior with reputational concerns. In the paper we will offer a novel explanation of why agents may put excessive weight on private information in the context of market games.

Any welfare analysis of rational expectations equilibria faces several difficulties. First of all, it must employ a model capable of dealing in a tractable way with the dual role of prices as conveyors of information and determinants of traders' budget constraints. Grossman and Stiglitz (1980) were pioneers in this respect with their CARA-normal model. Second, we require a welfare benchmark against which to test market equilibria in a world with asymmetric information. The appropriate benchmark for measuring inefficiency at the market equilibrium is the team solution in which agents internalize collective welfare but must still rely on private information when making their own decisions (Radner 1979; Vives 1988; Angeletos and Pavan 2007). This is in the spirit of Hayek (1945), where the private signals of agents cannot be communicated to a center. The team-efficient solution internalizes the payoff and information externalities associated with the actions of agents in the market. Collective welfare may refer to the surplus of all market participants, active or passive, or may be restricted to the internal welfare of the active agents. The third challenge for such welfare analysis is dealing with the interaction of payoff and informational externalities. If we take as a benchmark a pure prediction model with no payoff externalities, then agents will typically rely too much on public information. The reason is that agents do not take into account that their reaction to private information affects the informativeness of public statistics and general welfare. In other words, agents do not internalize an information externality. Pure information externalities will make agents insufficiently responsive to their private information (Vives 1993, 1997; Amador and Weill 2012) and, in the limit to disregard it (Banerjee (1992), Bikhchandani et al. (1992)).

The result of under-reliance on private information extends to some classes of economies with endogenous public information. Indeed, consider an economy in which equilibria are restricted efficient when public information is exogenous. If in this case the market is (restricted) efficient then increasing public information has to be good marginally, and under regularity conditions the result is global. This implies that more weight to private information is needed (Angeletos and Pavan 2009). This

logic breaks down in a market game where the price depends on a fundamental on which agents receive a private signal (say cost shock) and on a payoff relevant shock (say in demand). With this new ingredient when the price is more informative on cost it conveys less information on the demand shock (which matters for consumers). The result that we will show is that agents may put too much weight on private information in an economy with endogenous public information when with exogenous public information the market would be restricted efficient (and this is not a pathological case).

We consider a tractable linear-quadratic-Gaussian model that allows us to address the three challenges just described when public information is endogenously generated and influenced by the actions of agents. The context is a market game, where external effects go through the price and therefore externalities are pecuniary. There is uncertainty about a common valuation parameter about which agents have private information, and the endogenous public statistic or "price" is noisy. We use a model with a rational expectations flavor but in the context of a well-specified game, where a continuum of agents compete in schedules, and allow actions to be strategic substitutes or complements. We focus our attention on linear Bayesian equilibria. The model is flexible and admits several interpretations in terms of firms competing in a homogenous product market, investment complementarities, monopolistic competition, trading in a financial market, and asset auctions. (We will follow the first interpretation until the extensions section.)

Let us discuss the results in some more detail and start with the charcaterization of equilibrium. We show that in equilibrium agents correct the slope of their strategy according to what they learn from the public statistic and the character of competition. Under strategic substitutes competition the price's informational and allocational roles conflict. With strategic substitutes and private information, a high price is bad news (high cost) and the equilibrium schedule is steeper than with full information. In fact, in equilibrium schedules may slope the "wrong" way (e.g., downward for a supply schedule) when the informational role of prices dominates their allocational role. This will occur when there is little noise in the public statistic. With strategic complements there is no conflict: a high price is good news, and the equilibrium schedule is flatter than with full information. The impact on the slope of the equilibrium schedule of a

change in the exogenous (prior) precision of public information is opposite to the change in the precision of the noise in the endogenous public signal; consequently, market depth is increasing in the former and decreasing in the latter. The reason is that an increase in the *exogenous* precision of public information decreases the informational component of the public statistic whereas an increase in the *endogenous* precision increases it.

Consider an economy in which not only the full information equilibrium is efficient but also the equilibrium with private information when public information is exogenous (as in Vives 1988 or Section 5.3 in Angeletos and Pavan 2007). We show that market equilibria will not be team-efficient even when the allowed allocations have properties (e.g., being linear in information) similar to those of the market equilibrium. This is because the market in general does not internalize the informational externality that results from public statistics (e.g., prices) conveying information. Indeed, a competitive agent is an information taker while the precision of the public statistic is endogenous. The market equilibrium is characterized by the *privately* efficient use of private information. Team efficiency instead makes *socially* efficient use of private information. Market equilibria will be team-efficient only in exceptional circumstances (as when the information externality vanishes). This occurs, for example, when public information is exogenous. We find that, under strategic substitutability, equilibrium prices will tend to convey too little information when the informational role of prices prevails and too much information when its allocational role prevails. The normal case is the second one since in the first one the equilibrium schedules slope the "wrong" way. At the boundary of those situations there is a knife-edge case where parameters are such that agents use vertical schedules (as in a Cournot game), non contingent on the price (public statistic), and therefore the information externality disappears. In this particular case constrained efficiency is restored. Under strategic complementarity, prices always convey too little information.

The results can be explained as follows. Consider a homogenous product market with random demand and a continuum of firms competing in supply schedules with increasing and symmetric marginal costs with uncertain intercept. Each firm receives a private signal on the marginal cost intercept. In this situation there is both aggregate and productive inefficiency. Aggregate inefficiency refers to a distorted total output

and productive inefficiency refers to a distorted distribution of a given total output. The equilibrium in the complete information economy is efficient since it is competitive. In this equilibrium all firms produce the same amount since they all have full information on costs, which are symmetric. The team-efficient solution in an economy with asymmetric information optimally trades off the tension between the two sources of welfare loss, aggregate and productive inefficiency. Aggregate inefficiency is proportional to non-fundamental price volatility and productive inefficiency to the dispersion of individual actions. A higher response to private information makes prices more informative and reduces aggregate inefficiency (since the total quantity is closer to the full information first best), as well as non-fundamental price volatility, but at the same time the dispersion of quantities increases and with it productive inefficiency. The somewhat surprising possibility that prices are too informative arises then since at the market solution firms may respond excessively to private information generating too much productive inefficiency. In this case there is too little non-fundamental price volatility. This happens under strategic substitutability, when the dual role of prices conflict, in the normal case where the allocational role of prices dominates the information role and supply is upward sloping. When this does not happen and prices convey too little information, which is always the case with strategic complementarity, then there is excessive volatility at the market solution.

More precise information, be it public or private, reduces the welfare loss at the team-efficient solution. The reason is that the direct impact of the increased precisions is to decrease the welfare loss and this is the whole effect since at the team-efficient solution the responses to private and public information are already (socially) optimized (this is as in Angeletos and Pavan 2009). In contrast, at the market solution an increase in, say, the precision of private information will increase the response of an agent to his private signal and this will tend to increase the welfare loss when the market calls already for a too large response to private information. If this indirect effect is strong enough the welfare loss may be increasing with the precision of private information. In principle the same effect could happen with the precision of public information but we can show that the indirect effect of changes in both the exogenous public precision of information and the precision of the noise in the endogenous public signal are always dominated by the direct effect. The result is that

the welfare loss at the market solution is always decreasing with the precisions of public information.

The results can be extended to the internal team-efficient benchmark (where only the collective welfare of the players is taken into account, for example, ignoring passive consumers). Then the full information market does not achieve an efficient outcome. In this case also, endogenous public information may overturn conclusions reached using exogenous information models (e.g., Angeletos and Pavan 2007) when the informational role of the price is in conflict and dominates its allocational role.

The paper follows the tradition of the literature on the welfare analysis of private information economies (Palfrey 1985, Vives 1988, Angeletos and Pavan 2007), extending the analysis to endogenous public information. The results subvert the usual intuition of informational externality models (Vives 1997, Angeletos and Pavan 2009, Amador and Weill 2010, 2012) in a model of pecuniary externalities. Those externalities are the indirect welfare effects that arise through the price system out of the interaction of economic agents. It is worth noting that pecuniary externalities are associated to inefficiency even in competitive but incomplete markets and/or in the presence of private information since then the conditions of the first fundamental welfare theorem are not fulfilled. Competitive equilibria are not constrained efficient in those circumstances (Greenwald and Stiglitz 1986). For example, pecuniary externalities in market with financial frictions (borrowing or collateral constraints) can explain market failure (see, e.g., Caballero and Krishnamurthy 2001 and Jeanne and Korinek 2010). In our paper information frictions given rise to informational externalities through prices and we show that competitive noisy rational expectations equilibria, in which traders take into account information from prices, are not constrained efficient (as in Laffont 1985). If REE where to be fully revealing then there would be (ex-post) Pareto optimal (Grossman 1981) and in our case, since we have quasilinear utility, also ex ante Pareto optimal. In our quasilinear utility model there is no room for the Hirshleifer (1971) effect according to which fully revealing REE may destroy insurance opportunities by revealing too much information (and then REE need not be ex ante efficient). We provide therefore an instance of REE which may reveal too much information on a fundamental on which agents have private information which is independent of the Hirshleifer effect.

Recent literature has examined the circumstances under which more public information actually reduces welfare (as in Burguet and Vives 2000; Morris and Shin 2002; Angeletos and Pavan 2007; Amador and Weill 2010, 2011). In Burguet and Vives (2000) a higher (exogenous) public precision may discourage private information acquisition and lead to a higher welfare loss in a purely informational externality model. In Morris and Shin (2002) the result is driven by a socially excessive incentive to coordinate by agents. Angeletos and Pavan (2007) qualify this result and relate it to the payoff externalities present in a more general model. In Amador and Weill (2010) a public release of information reduces the informational efficiency of prices and this effect may dominate the direct information provision effect. Their model is purely driven by information externalities in the presence of strategic complementarities in terms of responses to private information.[3] In our model more public information is not damaging welfare but more private precision may be. This happens when at the market solution there is already too much dispersion of actions and an increase in private precision exacerbates the problem.

The plan of the paper is as follows. Section 2 presents the model and the leading interpretation of firms competing in a homogenous product market. Section 3 characterizes the equilibrium. Section 4 performs a welfare analysis, and Section 5 studies the internal team-efficient benchmark. Section 6 deals with the comparative statics properties of the equilibrium and the value of information. Section 7 presents alternative interpretations of the model and applications. Concluding remarks are given in Section 8. Proofs are gathered in the Appendix.

2. The market game

Consider a quadratic payoff market game with a continuum of players indexed within the interval $[0,1]$. Player i has the payoff function

$$\pi(x_i, \tilde{x}) = (p - \theta)x_i - \frac{\lambda}{2}x_i^2,$$

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[3]   Ganguli and Yang (2009) develop the implications of strategic complementarities for information acquisition in noisy rational expectations models.

where x_i is the individual action of the player, $p = \alpha + u - \beta \tilde{x}$ is a public statistic or "price", $\tilde{x} = \int_0^1 x_i \, di$ is the aggregate action, θ and u are parameters that, for the moment, are simply given, and α, λ are positive parameters. Then $\partial^2 \pi / (\partial x_i)^2 = -\lambda < 0$ and $\partial^2 \pi / \partial x_i \partial \tilde{x} = -\beta$, and the slope of the best reply of a player is $m \equiv \left(\partial^2 \pi / \partial x_i \partial \tilde{x} \right) / \left(-\partial^2 \pi / (\partial x_i)^2 \right) = -\beta / \lambda$. Thus we have strategic substitutability (complementarity) for $\beta > 0$ (for $\beta < 0$), and m can be understood as the degree of complementarity in the payoffs. (In the rest of this paper, when discussing strategic substitutability or complementarity we refer to this meaning). We assume that $m < 1/2$ or $2\beta + \lambda > 0$, limiting the extent of strategic complementarity. The condition $2\beta + \lambda > 0$ guarantees that $\pi(x,x)$ is strictly concave in x ($\partial^2 \pi / (\partial x)^2 = -(2\beta + \lambda) < 0$).

Consider now a game with uncertainty and in which θ and u are random. The parameter θ is uncertain; it has prior Gaussian distribution with mean $\bar{\theta}$ and variance σ_θ^2 (we write $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$ and, to ease notation, set $\bar{\theta} = 0$). Player i receives a signal $s_i = \theta + \varepsilon_i$ with $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$. Error terms are uncorrelated across players, and the random variables $\{\theta, \varepsilon_i, u\}$ are mutually independent. We establish the convention that error terms cancel in the aggregate: $\int_0^1 \varepsilon_i \, di = 0$ almost surely (a.s.). Then the aggregation of all individual signals will reveal the underlying uncertainty: $\int_0^1 s_i \, di = \theta + \int_0^1 \varepsilon_i = \theta$.[4]

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[4] That is, I assume that the strong law of large numbers (SLLN) holds for a continuum of independent random variables with uniformly bounded variances. Suppose that $\left(q_i \right)_{i \in [0,1]}$ is a process of independent random variables with means $E[q_i]$ and uniformly bounded variances $\mathrm{var}[q_i]$. Then we let $\int_0^1 q_i \, di = \int_0^1 E[q_i] \, di$ a.s. This convention will be used while taking as given the usual linearity property of integrals. Equality of random variables must be assumed to hold almost surely. It can be checked that the results obtained in the continuum economy are the limit of finite economies under the usual SLLN.

Players have access to the (endogenous) public statistic $p = \alpha + u - \beta\tilde{x}$, where $u \sim N\left(0, \sigma_u^2\right)$; this can be interpreted as the marginal benefit of taking action level x_i, which has cost $\theta x_i + \left(\lambda/2\right)x_i^2$.[5]

It is worth to remark that in this market game both payoff and informational externalities go through the market price or public statistic p, which has both an allocational and an informational role. Indeed, when $\beta = 0$, there are neither payoff nor informational externalities among players. The dual role of β as both a parameter in the payoff function and in the public statistic should be noted. This situation arises naturally in market games.

The timing of the game is as follows. At $t = 0$, the random variables θ and u are drawn but not observed. At $t = 1$, each player observes his own private signal s_i and submits a schedule $X_i\left(s_i, \cdot\right)$ with $x_i = X_i\left(s_i, p\right)$, where p is the public statistic. The strategy of a player is a map from the signal space to the space of schedules. Finally, the public statistic or price is formed (the "market clears") by finding a p that solves

$p = \alpha + u - \beta\left(\int_0^1 X_j\left(s_j, p\right)dj\right)$, and payoffs are collected at $t = 1$.

Let us assume that there is a unique public statistic $\hat{p}\left(\left(X_j\left(s_j, \cdot\right)\right)_{j\in[0,1]}\right)$ for any realization of the signals.[6] Then, for a given profile $\left(X_j\left(s_j, \cdot\right)\right)_{j\in[0,1]}$ of players' schedules and realization of the signals, the profits for player i are given by

$$\pi_i = \left(p - \theta\right)x_i - \frac{\lambda}{2}x_i^2,$$

where $x_i = X_i\left(s_i, p\right)$, $\tilde{x} = \int_0^1 X_j\left(s_j, p\right)dj$, and $p = \hat{p}\left(\left(X_j\left(s_j, \cdot\right)\right)_{j\in[0,1]}\right)$. This formulation has a rational expectations flavor but in the context of a well-specified

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[5] Normality of random variables means that prices and quantities can be negative with positive probability. The probability of this event can be controlled, if necessary, by an appropriate choice of means and variances. Furthermore, for this analysis the key property of Gaussian distributions is that conditional expectations are linear. Other prior-likelihood conjugate pairs (e.g., beta-binomial and gamma-Poisson) share this linearity property and can display bounded supports.

[6] We assign zero payoffs to the players if there is no p that solves the fixed point problem. If there are multiple solutions, then the one that maximizes volume is chosen.

schedule game. We will restrict our attention to linear Bayesian equilibria of the schedule game. The model admits several interpretations and we present below the leading one in terms of supply function competition (see Section 6 for the other interpretations).[7]

Firms competing in a homogenous product market with quadratic production costs.
In this case, $p = \alpha + u - \beta\tilde{x}$ is the inverse demand for the homogenous product, x_i is the output of firm i, u is a demand shock, and the cost function of firm i is given by $C(x_i) = \theta x_i + (\lambda/2) x_i^2$. Firms use supply functions as strategies, and markets clear: $p = \alpha + u - \beta\left(\int_0^1 X_i(s_i, p)\, di\right)$. Costs are random and firm i has a noisy estimate of the intercept of marginal cost $s_i = \theta + \varepsilon_i$ at the time of submitting the supply function. If $\beta > 0$, then demand is downward sloping and we have strategic substitutability in the usual partial equilibrium market. If $\beta < 0$, we have strategic complementarity and demand is upward sloping. The latter situation may arise in the case of a network good with compatibility.

As an example, the cost θ could be a unit ex post pollution damage that is assessed on firm i, say an electricity generator, and for which the firm has an estimate s_i before submitting its supply function.

We will maintain a supply interpretation of the model up to Section 7. We let $p \equiv \alpha + u - \beta\tilde{x}$ be the marginal benefit or "price" of taking an action and let $MC(x_i) \equiv \theta + \lambda x_i$ be the marginal cost.

3. Equilibrium

We are interested in a linear (Bayesian) equilibrium—equilibrium, for short—of the schedule game for which the public statistic functional is of type $P(\theta, u)$. Since the payoffs and the information structure are symmetric and since payoffs are strictly

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[7]   See Chapter 3 in Vives (2008) for an overview of the connection between supply function competition and rational expectations models, as well as examples.

concave, there is no loss of generality in restricting our attention to symmetric equilibria. Indeed, the solution to the problem of player i,

$$\max_{x_i} E\left[\left(p - \theta - \frac{\lambda}{2}x_i\right)x_i \mid s_i, p\right],$$

is both unique (given strict concavity of profits) and symmetric across players (since the cost function and signal structure are symmetric across firms):

$$X\left(s_i, p\right) = \lambda^{-1}\left(p - E\left[\theta \mid s_i, p\right]\right),$$

where $p = P\left(\theta, u\right)$. A strategy for player i may be written as

$$x_i = \hat{b} + \hat{c}p - as_i,$$

in which case the aggregate action is given by

$$\tilde{x} = \int_0^1 x_i \, di = \hat{b} + \hat{c}p - a\theta.$$

It then follows from $p = \alpha + u - \beta\tilde{x}$ that, provided $\hat{c} \neq -\beta^{-1}$,

$$p = P\left(\theta, u\right) = \left(1 + \beta\hat{c}\right)^{-1}\left(\alpha - \beta\hat{b} + z\right);$$

here the random variable $z = \beta a\theta + u$ is informationally equivalent to the "price" or public statistic p. Because u is random, z (and the public statistic) will typically generate a noisy signal of the unknown parameter θ. Let τ denote the precision of the price p or z in the estimation of θ, $\tau \equiv \left(\text{var}\left[\theta \mid z\right]\right)^{-1}$. From the properties of Gaussian random variables it is immediate that $\tau = \tau_\theta + \tau_u \beta^2 a^2$.

Market depth—that is, the inverse of how much the price moves to accommodate a unit increase in u—is given by $\left(\partial P / \partial u\right)^{-1} = 1 + \beta\hat{c}$.[8] Excess demand is given by

$$\Xi\left(p\right) \equiv \beta^{-1}\left(\alpha + u - p\right) - \hat{b} + a\theta - \hat{c}p.$$

The information available to player i is $\{s_i, p\}$ or, equivalently, $\{s_i, z\}$. Since $E\left[\theta \mid s_i, p\right] = E\left[\theta \mid s_i, z\right]$, we can posit strategies of the form

$$X\left(s_i, z\right) = b - as_i + cz$$

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8   See, for example, Kyle (1985).

and obtain that $p = \alpha - \beta b + (1 - \beta c)z$. If $1 + \beta \hat{c} > 0$ then $1 - \beta c > 0$ (since $\hat{c} = (c^{-1} - \beta)^{-1}$ and $1 + \beta \hat{c} = (1 - \beta c)^{-1}$) and so p and z will move together. The strategy of player i is then given by

$$X(s_i, z) = \lambda^{-1}(\alpha - \beta b + (1 - \beta c)z - E[\theta | s_i, z]).$$

We can solve for the linear equilibrium in the usual way: identifying coefficients with the candidate linear strategy $x_i = b - as_i + cz$ by calculating $E[\theta | s_i, z]$ and using the supply function of a player.

The following proposition characterizes the equilibrium.

<u>Proposition 1</u>. *Let* $\tau_\varepsilon \geq 0$ *and* $\tau_u \geq 0$. *Then there is a unique (and symmetric) equilibrium*

$$X(s_i, p) = \lambda^{-1}(p - E[\theta | s_i, p]) = \hat{b} - as_i + \hat{c}p,$$

where a is the unique (real) solution of the equation $a = \tau_\varepsilon \lambda^{-1}(\tau_\varepsilon + \tau)^{-1}$ *where*

$\tau = \tau_\theta + \tau_u \beta^2 a^2$, $\hat{c} = ((\beta + \lambda)(1 - \beta \lambda \tau_u a^2 \tau_\varepsilon^{-1})^{-1} - \beta)^{-1}$, *and* $\hat{b} = \alpha(1 - \lambda \hat{c})/(\beta + \lambda)$. *In equilibrium,* $a \in (0, \tau_\varepsilon \lambda^{-1}(\tau_\theta + \tau_\varepsilon)^{-1})$ *and* $1 + \beta \hat{c} > 0$.

<u>Remark 1.</u> We have examined linear equilibria of the schedule game for which the public statistic function is of type $P(\theta, u)$. In fact, these are the equilibria in strategies with bounded means and with uniformly (across players) bounded variances. (See Claim 1 in the Appendix.)

<u>Remark 2.</u> We can show that the equilibrium in the continuum economy is the limit of equilibria in replica economies that approach the limit economy. Take the homogenous market interpretation with a finite number of firms n and inverse demand $p_n = \alpha + u - \beta \tilde{x}_n$, where $\tilde{x}_n$ is the average output per firm, and with the same informational assumptions. In this case, given the results in Section 5.2 of Vives (2011), the supply function equilibrium of the finite n-replica market converges to the equilibrium in Proposition 1.

The public statistic or price serves a dual role as index of scarcity and conveyor of information. Indeed, a high price has the direct effect of increasing an agent's competitive supply, but it also conveys news about costs—namely, that costs are high (low) if $\beta > 0$ ($\beta < 0$). In equilibrium, the "price impact" (or inverse of the depth of the market) is always positive, $\partial P / \partial u = (1 + \beta \hat{c})^{-1} > 0$, and excess demand is downward or upward sloping depending on β : $\Xi' = -(\beta^{-1} + \hat{c})$ or $\operatorname{sgn}\{\Xi'\} = \operatorname{sgn}\{-\beta\}$. That is, the slope's direction depends on whether the competition is in strategic substitutes or in strategic complements.

In equilibrium, agents take public information z, with precision $\tau \equiv \left(\operatorname{var}\left[\theta | z \right] \right)^{-1}$, as given and use it to form probabilistic beliefs about the underlying uncertain parameter θ. We have that $E\left[\theta / s_i, z \right] = \gamma s_i + (1 - \gamma) E\left[\theta / z \right]$ with $\gamma = \tau_\varepsilon (\tau_\varepsilon + \tau)^{-1}$. Revised beliefs and optimization, in turn, determine the coefficients a and c for private and public information, respectively. In equilibrium, the informativeness of public information z depends on the sensitivity of strategies to private information $a : \tau = \tau_\theta + \tau_u \beta^2 a^2$. Agents behave as information takers and so, from the perspective of an individual agent, public information is exogenous. This fact is at the root of the equilibrium's *informational externality*. That is, agents fail to account for the impact of their own actions on public information and hence on other agents.

Consider as a benchmark the full information case with perfectly informative signals ($\tau_\varepsilon = \infty$). This puts us in a full information competitive equilibrium and we have $c = (\beta + \lambda)^{-1}$, $a = \hat{c} = \lambda^{-1}$, and $X(\theta, p) = \lambda^{-1}(p - \theta)$. In this case, agents have nothing to learn from the price. If signals become noisy ($\tau_\varepsilon < \infty$) then $a < \lambda^{-1}$ and $\hat{c} < \lambda^{-1}$ for $\beta > 0$, with supply functions becoming steeper (lower $\hat{c}$) as agents protect themselves from adverse selection. The opposite happens ($\hat{c} > \lambda^{-1}$ and flatter

supply functions) when $\beta < 0$, since then a high price is good news (entailing lower costs). [9] There is then "favorable" selection.

There are several cases in which $\hat{c} = \lambda^{-1}$ and there is no learning from the price: (i) When signals are uninformative about the common parameter θ $(\tau_\varepsilon = 0)$ or when there is no uncertainty ($\tau_\theta = \infty$ and $\theta = \overline{\theta}$ (a.s.)), the price has no information to convey; $a = 0$ and $X(s_i, p) = \lambda^{-1}(p - \overline{\theta})$; (ii) When the public statistic is extremely noisy ($\tau_u = 0$) or when $\beta = 0$ (in which case there is no payoff externality, either), then public information is pure noise, $a = \lambda^{-1}\tau_\varepsilon(\tau_\theta + \tau_\varepsilon)^{-1}$, with $X(s_i, p) = \lambda^{-1}(p - E[\theta | s_i])$. In all these cases, there is no information externality via the public statistic.

As τ_u tends to ∞, the precision of prices τ also tends to ∞, the weight given to private information a tends to 0, and the equilibrium collapses (with market depth $1 + \beta\hat{c} \to 0$). Indeed, the equilibrium becomes fully revealing and is not implementable. With $\beta > 0$, as τ_u increases from 0, $\hat{c}$ decreases from λ^{-1} (and the slope of supply increases) because of the price's increased informational component (a high price indicates higher costs). As τ_u increases more, $\hat{c}$ becomes zero at some point and then turns negative; as τ_u tends to ∞, $\hat{c}$ tends to $-\beta^{-1}$. At the point where the scarcity and informational effects balance, agents place zero weight ($\hat{c} = 0$) on the public statistic. In this case, agents do not condition on the price and the model reduces to a quantity-setting model à la Cournot (however, not reacting to the price is optimal). However, if τ_θ increases then the informational component of the price diminishes since the agents are now endowed with better prior information, and induces a higher $\hat{c}$ (and a more elastic supply).

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[9]   This follows because, with upward-sloping demand, we assume that $2\beta + \lambda > 0$ and therefore $\lambda > -\beta$.

When $\beta < 0$ then a high price conveys the good news that average quantity tends to be high and that costs therefore tend to be low. In this case, increasing τ_u, which reinforces the informational component of the price, increases $\hat{c}$ —the opposite of what happens when τ_θ increases. It follows that in either case ($\beta > 0$ or $\beta < 0$) market depth $(\partial P / \partial u)^{-1} = 1 + \beta \hat{c}$ is decreasing in τ_u and increasing in τ_θ. (See Proposition 5 for a complete statement of the comparative statics properties of the equilibrium.)

4. Welfare analysis

Consider the homogeneous product market with quadratic production costs. The inverse demand $p = \alpha + u - \beta \tilde{x}$ arises from a benefit or surplus function $\left(\alpha + u - (\beta/2) \tilde{x} \right) \tilde{x}$, and the welfare criterion is total surplus:

$$\text{TS} = \left(\alpha + u - \beta \frac{\tilde{x}}{2} \right) \tilde{x} - \int_0^1 \left(\theta x_i + \frac{\lambda}{2} x_i^2 \right) di .$$

Under our assumptions, $\beta + \lambda > 0$ and the TS function is strictly concave for symmetric solutions.

The equilibrium is partially revealing (with $0 < \tau_u < \infty$ and $0 < \tau_\varepsilon < \infty$), so expected total surplus should be strictly greater in the first-best allocation (full information) $x^o = (\lambda + \beta)^{-1} (\alpha + u - \theta)$, which is just the market solution with full information, than at the equilibrium. The reason is that suppliers produce under uncertainty and rely on imperfect idiosyncratic estimation of the common cost component; hence they end up producing different amounts even though costs are identical and strictly convex. However, since producers are competitive they produce in expected value the right amount at the equilibrium: $E[\tilde{x}] = E[x^o] = \alpha (\lambda + \beta)^{-1}$.

The welfare benchmark that we use is the team solution maximizing expected total surplus subject to employing linear decentralized strategies (as in Vives 1988; Angeletos and Pavan 2007). This team-efficient solution internalizes the information externalities of the actions of agents, and it is restricted to using the same type of strategies (decentralized and linear) that the market employs. Indeed, when reacting to

information, an agent in the market does not take into account the influence her own actions have on public statistics.

It is worth noting that in the economy considered if firms would not condition on prices, i.e. if each firm would set quantities conditioning only on its private information, then the market solution would be team-efficient (Vives 1988). This will not be the case in general when public information is endogenous because of information externalities. That is, in the economy considered the full information equilibrium is (first best) efficient and the private information equilibrium is team-efficient for given public information.

At the team-efficient solution, expected total surplus $E[\text{TS}]$ is maximized under the constraint that firms use decentralized linear production strategies. That is,

$$\max_{a,b,c} E[\text{TS}]$$

$$\text{subject to} \quad x_i = b - as_i + cz, \quad \tilde{x} = b - a\theta + cz, \quad \text{and} \quad z = u + \beta a\theta.$$

Equivalently, the team-efficient solution minimizes, over the restricted strategies, the expected welfare loss WL with respect to the full information first best. It is possible to show that

$$\text{WL} = \left((\beta + \lambda) E\left[(\tilde{x} - x^o)^2 \right] + \lambda E\left[(x_i - \tilde{x})^2 \right] \right) \Big/ 2,$$

where the first term in the sum corresponds to aggregate inefficiency (how distorted is the average quantity $\tilde{x}$ while producing in a cost-minimizing way), which is proportional to $E\left[(\tilde{x} - x^o)^2 \right]$, and the second term to productive inefficiency (how distorted is the distribution of production of a given average quantity $\tilde{x}$), which is proportional to the dispersion of outputs $E\left[(x_i - \tilde{x})^2 \right]$. Let p^o be the full information first best price. Note that the non-fundamental price volatility is given by $E\left[(p - p^o)^2 \right] = \beta^2 E\left[(\tilde{x} - x^o)^2 \right]$ and therefore it is proportional to aggregate inefficiency.

It is easily seen that the form of the optimal team strategy is $x_i = \lambda^{-1}\left[p - \left(\gamma s_i + (1-\gamma) E[\theta \mid z]\right)\right]$ where the weight to private information $\gamma = \lambda a$ may differ from the market weight. Note that both in the market and the team solutions we have that $\gamma = \lambda a$. It follows then that the welfare loss at any candidate team solution will depend only on the response to private information a since we have $E\left[(\tilde{x} - x^o)^2 \right] = (1 - \lambda a)^2 \big/ \left(\tau (\beta + \lambda)^2 \right)$, $\tau = \tau_\theta + \tau_u \beta^2 a^2$, and $E\left[(x_i - \tilde{x})^2 \right] = a^2 / \tau_\varepsilon$. This yields a strictly convex WL as a function of a. Changing a has opposite effects on both sources of the welfare loss since allocative inefficiency decreases with a, as price informativeness τ increases and the average quantity gets close to the full information allocation, but productive inefficiency increases with a as dispersion increases. A more informative price reduces allocative inefficiency and non-fundamental price volatility but increases productive inefficiency. The team solution optimally trades them off among decentralized strategies.

If there was no information externality a would not affect τ (which would be exogenous). In this case it is easy to see that the team and the market solution coincide. Otherwise there is an information externality and the market is inefficient.

In principle we may think that the usual intuition that makes agents put too little weight on private information due to the information externality (Vives (1997), Amador and Weill (2012)) will apply as well to our economy. The reason is that since the market equilibrium is (restricted) efficient for given public information we should be able to improve on the market by increasing the precision of public information and therefore by having agents put more weight to private information to increase the precision of the public statistic.[10] Indeed, if with exogenous public information the market is efficient then increasing public information has to be good marginally, and under our linear-normal framework the result should be global. This implies that more weight to private information is needed. However, this argument does not apply to our market game since the demand shock is payoff relevant (for the consumers) and the

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[10]   See Angeletos and Pavan (2009) for a more formal argument.

price reflects both the fundamental on which agents have private information (cost sock) and the demand shock, and when it is more informative on cost it conveys less information on demand (since the sufficient statistic for public information is $z = u + \beta a\theta$). The usual intuition would hold, and we would have the usual under-reliance on private information, if noise in the public statistic had no payoff relevance (a similar situation arises in Amador and Weill (2010) where only the posterior precision on fundamental matters and with no payoff externalities).

The sign of the information externality can be found easily by breaking down the impact of the sensitivity to private information a on $E[\text{TS}]$ between the market effect, where the public statistic z is taken as given, and the information externality effect (IE), where the impact on z is taken into account.

$$\frac{\partial E[\text{TS}]}{\partial a} = \underbrace{E\left[(p - \text{MC}(x_i))\left(\frac{\partial x_i}{\partial a}\right)_{z\ \text{ct.}}\right]}_{\text{Market}} + \underbrace{E\left[(p - \text{MC}(x_i))\left(\frac{\partial x_i}{\partial z}\frac{\partial z}{\partial a}\right)\right]}_{\text{IE}}.$$

The market term is null at the market solution (denoted *) and the IE term can be evaluated as follows:

$$\text{sgn}\left\{\left.\frac{\partial E[\text{TS}]}{\partial a}\right|_{a=a^*}\right\} = \text{sgn}\{\text{IE}\} = \text{sgn}\{-\beta c^*\}.$$

The sign of the informational externality depends on whether we have strategic substitutes or complements competition and on whether supply slopes upwards or downwards. If $\beta > 0$ and supply is upward sloping ($c^* > 0$) and, say, costs are high ($\theta - \bar{\theta} > 0$) then an increase in a will increase x_i ($\frac{\partial x_i}{\partial z}\frac{\partial z}{\partial a} = c\beta(\theta - \bar{\theta}) > 0$) while ($p - \text{MC}(x_i)$) will tend to be low (since at the market solution $E\left[(p - \text{MC}(x_i))(\theta - \bar{\theta})\right] < 0$). This means that IE < 0 and that a must be reduced. If supply is downward sloping ($c^* < 0$) in the same situation an increase in a will decrease x_i, which is welfare enhancing. The same will happen if $\beta < 0$ since then

$c* > 0$ and an increase in a will decrease x_i. In the last two situations $IE > 0$ and a must be increased.

The following proposition characterizes the response to private information at the team solution (superscript T) and compares it with the equilibrium solution (superscript *).

Proposition 2. Let $\tau_\varepsilon > 0$. Then the team problem has a unique solution with $\lambda^{-1} > a^T > 0$, and $\mathrm{sgn}\{a* - a^T\} = \mathrm{sgn}\{\beta c*\}$.

Corollary (market quality). At the market solution:
- In relation to the team optimum, when $\beta c* > 0$ price informativeness τ and dispersion $E\left[(x_i - \tilde{x})^2\right]$ are too high, and market depth $1 + \beta \hat{c}$ and non-fundamental volatility $E\left[(\tilde{x} - x^o)^2\right]$ too low. The opposite is true when $\beta c* < 0$.
- In relation to the first best (where $E\left[(\tilde{x} - x^o)^2\right] = E\left[(x_i - \tilde{x})^2\right] = 0$), price informativeness and market depth are too low, and non-fundamental volatility and dispersion are too high.

If $\beta = 0$ then there no informational externality, and the team and market solutions coincide. For $\beta \neq 0$, $\tau_\varepsilon > 0$, and $\tau_u > 0$, the solutions coincide only if $c* = 0$. This occurs only at the equilibrium when $a = \tau_\varepsilon / \left(\lambda(\tau_\varepsilon + \tau_\theta) + \beta \tau_\varepsilon\right)$ (with $\beta > 0$). When firms do not respond to the price $(c = 0)$, the model reduces to a quantity-setting model with private information. This is consistent with Vives (1988), where it is shown that a Cournot market with private information and a continuum of suppliers solves a team problem whose objective function is expected total surplus. If $c* < 0$ then a should be increased, and the contrary holds for $c* > 0$. Indeed, in the usual case with strategic substitutability, $\beta > 0$, and upward sloping supply functions, $c* > 0$, the price is too informative about θ , and there is too much dispersion and productive inefficiency. (In the electricity example, the price of electricity would be too informative about the pollution damage and not enough about the shock to

demand.) With downward sloping supply functions, $c* < 0$, the price is too little informative about θ, and there is too much aggregate inefficiency. It follows that since $c*$ is decreasing in τ_u, there is too much (not enough) weight given to private information whenever τ_u is small (large) and supply functions are increasing (decreasing). With strategic complementarity ($\beta < 0$) we have that both $c* > 0$ and $\text{sgn}\{\partial E[\text{TS}]/\partial a\} = \text{sgn}\{-\beta c*\} > 0$ always, agents give insufficient weight to private information and the market displays too much aggregate inefficiency.

The conclusion is that, with strategic substitutability, team efficiency requires a decrease (increase) in c when $c*$ is negative (positive). When $c* < 0$, the informational role of the price dominates and the price reveals too little information. In this case, more weight should be given to private signals so that public information becomes more revealing to reduce allocative inefficiency. Conversely, when the price has mainly an allocational role, $c* > 0$, it reveals too much information and a should be decreased to reduce excessive dispersion. Only in the knife-edge (Cournot) case, where $c* = 0$, is the equilibrium team-efficient. With strategic complementarity, agents place too little weight on private information. When $\beta < 0$, the informational externality is aligned with the price scarcity effect; in this case, it is always preferable to induce agents to rely more on their private information to reduce allocative inefficiency.

Remark 3. There is no information externality when signals are uninformative ($\tau_\varepsilon = 0$). Then the team and the market solution coincide since $a = 0$. Note that when the price contains no information ($\tau_u = 0$) both for the team and the market solutions $c = 1/(\beta + \lambda)$ but then $E[\text{TS}]$ is infinite.

Remark 4. If the signals of agents can be communicated to a center, then questions arise concerning the incentives to reveal information and how welfare allocations may be modified. This issue is analyzed in a related model by Messner and Vives (2006), who use a mechanism design approach along the lines of Laffont (1985).

The question arises as of how the welfare loss WL at the market solution depends on information precisions τ_ε, τ_u and τ_θ. We know that WL at a linear allocation as a function of a is given by the strictly convex function

$$\mathrm{WL}(a) = \frac{1}{2}\left(\frac{(1-\lambda a)^2}{(\tau_\theta + \tau_u \beta^2 a^2)(\beta + \lambda)} + \frac{\lambda a^2}{\tau_\varepsilon} \right).$$

It is immediate then that at the team-efficient solution $\mathrm{WL}(a^{\mathrm{T}})$ is decreasing in τ_ε, τ_u and τ_θ. This is so since WL is decreasing in τ_ε, τ_u and τ_θ for a given a and $\mathrm{WL'}(a^{\mathrm{T}}) = 0$. Things are potentially different at the market solution $a*$ since then $\mathrm{WL'}(a*) > 0$ or $\mathrm{WL'}(a*) < 0$ depending on whether $a* > a^{\mathrm{T}}$ or $a* < a^{\mathrm{T}}$. Since $a*$ is decreasing in τ_u and τ_θ, and increasing in τ_ε we have thus that $\mathrm{WL}(a*)$ is decreasing in τ_u and τ_θ when $a* > a^{\mathrm{T}}$ and in τ_ε when $a* < a^{\mathrm{T}}$. It is possible in principle that increasing precisions of public information τ_u and τ_θ increases the welfare loss when $a* < a^{\mathrm{T}}$ when the direct effect of the increase of τ_u or τ_θ is dominated by the indirect effect via the induced decrease in $a*$ (and similarly for an increase in τ_ε when $a* > a^{\mathrm{T}}$). We can check, however, that $\mathrm{WL}(a*)$ is always decreasing in τ_θ and τ_u because the direct effect always dominates the indirect effect. This need not be the case when changing τ_ε. In any case, as the information precisions τ_θ, τ_u, and τ_ε tend to infinity $\mathrm{WL}(a*)$ tends to 0.[11] The following proposition summarizes the results.

<u>Proposition 3</u>. The welfare loss at the team-efficient solution is decreasing in τ_ε, τ_u and τ_θ. The welfare loss at the market solution is also decreasing in τ_θ and τ_u, and it may be decreasing or increasing in τ_ε (it will be increasing for $\beta > \lambda$ and $\tau_\varepsilon/\tau_\theta$ small enough). As any of the information precisions τ_θ, τ_u, and τ_ε tend to infinity welfare losses tend to zero.

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[11]   This follows since as $\tau_\varepsilon \to \infty$, $a* \to \lambda^{-1}$; and as τ_θ or $\tau_u \to \infty$, $a* \to 0$ and $\tau \to \infty$.

In summary, more precise public or private information reduces the welfare loss at the team-efficient solution. This is in accordance with the results in Angeletos and Pavan (2007) where more information can not hurt when it is used efficiently. The welfare loss at the market solution is also always decreasing with the precision of public information. However, the welfare loss at the market solution may be increasing with the precision of private information when the market calls already for a too large response to private information. The reason is that an increase in the precision of private information will increase the response of an agent to his private signal and this indirect effect may dominate.

The welfare result of the market solution is in contrast with received results in the literature where more *public* information may be damaging to welfare (Burguet and Vives 2000; Morris and Shin 2002; Angeletos and Pavan 2007; Amador and Weill 2010, 2011). In those papers more public information discourages the use and/or acquisition of private information. In the present paper this also happens but the direct effect of public information provision prevails.

5. Internal welfare benchmark

A different benchmark is provided by the collective welfare of the players, the producers in our case. At the internal team–efficient solution, expected average profit $E[\tilde{\pi}]$ (where $\tilde{\pi} = \int_0^1 \pi_i \, di$ and $\pi_i = (\alpha + u - \beta \tilde{x} - \theta) x_i - (\lambda/2) x_i^2$) is maximized under the constraint that agents use decentralized linear strategies. Since the solution is symmetric we have that $E[\tilde{\pi}] = E[\pi_i]$. This is the cooperative solution from the firms' perspective. That is,

$$\max_{a,b,c} E[\pi_i]$$

subject to $\quad x_i = b - as_i + cz$, $\quad \tilde{x} = b - a\theta + cz$, and $\quad z = u + \beta a\theta$.

It should be clear that the market solution, not even with complete information, will attain the full information cooperative outcome (denoted M for monopoly, for which $x^M = (\lambda + 2\beta)^{-1}(\alpha + u - \theta)$) where joint profits are maximized under full information. This is so since the market solution does not internalize the payoff externalities and therefore if $\beta \neq 0$ it will produce an expected output

$E[\tilde{x}*] = \alpha(\beta + \lambda)^{-1}$ which is too high (low) with strategic substitutes (complements) in relation to the optimal $E[x^M] = \alpha(2\beta + \lambda)^{-1}$. Furthermore, the market solution does not internalize the information externalities. At the *internal team* (IT) benchmark, joint profits are maximized and information externalities internalized with decentralized strategies.[12] The question is whether the market solution allocates the correct weights (from the players' collective welfare viewpoint) to private and public information. We show that the answer to this question is qualitatively similar to the one derived when analyzing the *total surplus* team benchmark but in this case with a larger bias towards the market displaying too much weight on private information.

As before, it can be seen that the internal team-efficient solution minimizes, over the restricted strategies, the expected loss L with respect to the full information cooperative outcome x^M, and that

$$L = \left((2\beta + \lambda) E\left[(\tilde{x} - x^M)^2 \right] + \lambda E\left[(x_i - \tilde{x})^2 \right] \right) / 2.$$

The first term in the sum corresponds to aggregate inefficiency in the average quantity, which is proportional to $E\left[(\tilde{x} - x^M)^2 \right]$, and the second term to productive inefficiency, which is proportional to $E\left[(x_i - \tilde{x})^2 \right]$.

It can checked that the form of the internal optimal team strategy is $x_i = (\lambda + \beta)^{-1} \left(p - (\gamma s_i + (1 - \gamma) E[\theta | z]) \right)$ where $\gamma = (\lambda + \beta)a$ (while at the market solution we have that $\gamma = \lambda a$). The loss at any candidate internal team solution (which internalizes the payoff externality and for which $E[\tilde{x}] = \alpha(2\beta + \lambda)^{-1}$) will depend only on the response to private information a since at this candidate solution we have $E\left[(\tilde{x} - x^M)^2 \right] = (1 - (\lambda + \beta)a)^2 / (\tau(2\beta + \lambda)^2)$ and $E\left[(x_i - \tilde{x})^2 \right] = a^2 / \tau_\varepsilon$.

This yields a strictly convex L as a function of a. As before, changing a has opposite effects on both sources of the loss. Now the internal team solution optimally trades off

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12 Indeed, when $\beta = 0$ there are no externalities (payoff or informational) and the internal team and market solutions coincide.

the sources of the loss with respect to the responsiveness to private information among decentralized strategies which internalize payoff externalities.

In this case at the market solution there is both an information (IE) and a payoff (PE) externality, even with full information the market solution is not efficient (i.e. cooperative). The impact of the externalities on the response to private information can be assessed similarly as before. The market takes the public statistic z or p as given while the internal team solution takes into account both the impact on public informativeness (IE) and on payoffs (PE):

$$\frac{\partial E[\pi_i]}{\partial a} = \underbrace{E\left[(p - MC(x_i))\left(\frac{\partial x_i}{\partial a}\right)_{z\ ct.}\right]}_{\text{Market}} +$$

$$\underbrace{E\left[(p - MC(x_i))\left(\frac{\partial x_i}{\partial z}\frac{\partial z}{\partial a}\right)\right]}_{\text{IE}} +$$

$$\underbrace{E\left[x_i\left(\frac{\partial p}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial a}\right)\right]}_{\text{PE}}.$$

The market term is null at the market solution and the sum of the IE and PE terms can be evaluated as follows:

$$\left.\frac{\partial E[\pi_i]}{\partial a}\right|_{a=a^*} = -\beta a^*\left(c^*\lambda\sigma_\varepsilon^2 + (c^*\beta - 1)^2 \sigma_\theta^2\right).$$

It is worth noting that while, as before, $\text{sgn}\{\text{IE}\} = \text{sgn}\{-\beta c^*\}$ we have that $\text{sgn}\{\text{PE}\} = \text{sgn}\{-\beta\}$ since $(c^*\beta - 1)^2 \sigma_\theta^2 > 0$, and therefore the PE term will call for a lower (higher) response to private information with strategic substitutes (complements) than the market solution. If $\beta > 0$ a high price indicates high costs. If, say, costs are high ($\theta - \bar{\theta} > 0$) then an increase in a will increase p ($\frac{\partial p}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial a} = -\beta(c\beta - 1)(\theta - \bar{\theta}) > 0$ since at the market solution $c\beta - 1 < 1$) while x_i will tend to be low (since at the market solution $E[(\theta - \bar{\theta})x_i] = a\sigma_\theta^2(c\beta - 1) < 0$). This means that if $\beta > 0$, $\text{PE} < 0$ and a must be reduced. Similarly, we have that $\text{PE} > 0$ if $\beta < 0$. The results on PE are in line with the results obtained by Angeletos and Pavan (Section 6.5, 2007) with exogenous public signals (and therefore

no information externality).[13] We will see how the effect of the informational externality term may overturn this result when $c < 0$.

The next proposition characterizes the response to private information.

Proposition 4. *Let $\tau_\varepsilon > 0$. Then the internal team problem has a unique solution with*

$$\left(\lambda + \beta\right)^{-1} > a^{\mathrm{IT}} > 0, \text{ and } \mathrm{sgn}\left\{a^* - a^{\mathrm{IT}}\right\} = \mathrm{sgn}\left\{\beta\left(c^* \lambda \sigma_\varepsilon^2 + \left(c^* \beta - 1\right)^2 \sigma_\theta^2\right)\right\}.$$

If $c^* \geq 0$ then $\mathrm{sgn}\left\{a^* - a^{\mathrm{IT}}\right\} = \mathrm{sgn}\left\{\beta\right\}$. Therefore, as before, under strategic complements ($\beta < 0$), there is too little response to private information, $a^* < a^{\mathrm{IT}}$. Indeed, the characterization yields the same qualitative result as in the previous section if $c^* > 0$: too much or too little response to private information in the presence of (respectively) strategic substitutability or strategic complementarity. In this case, however, if agents use Cournot strategies (i.e., if $c^* = 0$) then the market is not internal team–efficient. This should not be surprising when one considers that, when $c^* = 0$, there is no information externality yet the payoff externality is not internalized, as agents set a quantity that is too large (small) under strategic substitutability (complementarity). If $\beta > 0$ and $c^* < 0$, then $c^* \lambda \sigma_\varepsilon^2 + \left(c^* \beta - 1\right)^2 \sigma_\theta^2 > 0$ for c^* close to zero or sufficiently negative (τ_u large). Only for intermediate values of c^* we have $c^* \lambda \sigma_\varepsilon^2 + \left(c^* \beta - 1\right)^2 \sigma_\theta^2 < 0$ and $a^{\mathrm{IT}} > a^{\mathrm{LE}}$. With strategic substitutes the market will bias the solution more towards putting too high a weight on private information since we may have $c^* \lambda \sigma_\varepsilon^2 + \left(c^* \beta - 1\right)^2 \sigma_\theta^2 > 0$ even if $c^* < 0$.

This is the same qualitative result concerning the response to private information as derived previously using the total surplus team benchmark—with the following proviso: when $c^* < 0$, it need not be the case that there is too little response to private information.

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[13] Note also that in our model there are two random shocks to the welfare function while there is only one (the fundamental) in Angeletos and Pavan (2007).

<u>Remark 4</u>. The weights to private information in the internal team and market solutions are, respectively, $\gamma^{\text{IT}} = (\lambda + \beta) a^{\text{IT}}$ and $\gamma^* = \lambda a^*$. It is easy to see that for τ_u small enough (and $\tau_\theta > 0$) we have that $\gamma^{\text{IT}} > \gamma^*$. The same result applies when $\beta > 0$ and $c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2 < 0$ in which case $a^{\text{IT}} > a^*$ and therefore $\lambda \gamma^{\text{IT}} > (\lambda + \beta) \gamma^* > \lambda \gamma^*$.

6. Comparative statics and the value of information

This section studies the comparative statics properties of the equilibrium and how the weights and the responses to public and private information vary with underlying parameters. The following proposition presents a first set of results. The effects of changes in the degree of complementarity are dealt with afterwards.

<u>Proposition 5</u>. *Let* $\tau_\varepsilon > 0$ *and* $\tau_u > 0$. *In equilibrium, the following statements hold.*

(i) *Responsiveness to private information* a *decreases from* $\lambda^{-1} \tau_\varepsilon (\tau_\theta + \tau_\varepsilon)^{-1}$ *to* 0 *as* τ_u *ranges from* 0 *to* ∞, *decreases with* τ_θ, $|\beta|$ *and* λ, *and increases with* τ_ε.

(ii) *Responsiveness to the public statistic* $\hat{c}$ *goes from* λ^{-1} *to* $-\beta^{-1}$ *as* τ_u *ranges from* 0 *to* ∞. *Furthermore*, $\operatorname{sgn}\{\partial \hat{c}/\partial \tau_u\} = \operatorname{sgn}\{-\partial \hat{c}/\partial \tau_\theta\} = \operatorname{sgn}\{-\beta\}$ *and* $\operatorname{sgn}\{\partial \hat{c}/\partial \tau_\varepsilon\} = \operatorname{sgn}\{\beta (\beta^2 \tau_u \tau_\varepsilon^2 + 4\lambda^2 \tau_\theta^2 (\tau_\varepsilon - \tau_\theta))\}$. *Market depth* $1 + \beta \hat{c}$ *is decreasing in* τ_u *and increasing in* τ_θ.

(iii) *Price informativeness* τ *is increasing in* $|\beta|$, τ_u, τ_θ *and* τ_ε, *and decreasing in* λ.

(iv) *Dispersion* $E\left[(x_i - \tilde{x})^2\right]$ *decreases with* τ_u, τ_θ, $|\beta|$ *and* λ.

How the equilibrium weights to private and public information vary with the deep parameters of the model help to explain the results. We have that $E[\theta / s_i, z] = \gamma s_i + h z$ where $h = \beta a \tau_u (\tau_\varepsilon + \tau)^{-1}$. Identify the informational component of the price with the weight $|h|$ on public information z, with

$\mathrm{sgn}\{h\} = \mathrm{sgn}\{\beta\}$. When $\beta > 0$ there is adverse selection (a high price is bad news about costs) and $h > 0$ while when $\beta < 0$, $h < 0$ and there is favorable selection (a high price is good news). We have that $\mathrm{sgn}\{\partial|h|/\partial\beta\} = \mathrm{sgn}\{\beta\}$. As β is decreased from $\beta > 0$ adverse selection is lessened, and when $\beta < 0$ we have favorable selection with $h < 0$ and $\partial|h|/\partial\beta < 0$. The result is that an increase in $|\beta|$ increases the public precision[14] τ and decreases the response to private information. We have also that increasing the precision of the prior decreases the informational component of the price, $\partial|h|/\partial\tau_\theta < 0$, while that increasing the precision of the noise in the price increases it, $\partial|h|/\partial\tau_u > 0$. (See Claim 2 in the Appendix.) The effect of τ_ε is ambiguous.

Consider firts the case $\beta > 0$. As τ_u increases from 0, $\hat{c}$ decreases from λ^{-1} (and the slope of supply increases) because of the price's increased informational component $h > 0$. Agents are more cautious when seeing a high price because it may mean higher costs. As τ_u increases more, $\hat{c}$ becomes zero at some point and then turns negative; as τ_u tends to ∞, $\hat{c}$ tends to $-\beta^{-1}$.[15] At the point where the scarcity and informational effects balance, agents place zero weight ($\hat{c} = 0$) on the public statistic. In this case, agents do not condition on the price and the model reduces to a quantity-setting model à la Cournot (however, not reacting to the price is optimal). If τ_θ increases then the informational component of the price diminishes since the agents are now endowed with better prior information, and induces a higher $\hat{c}$ (and a more elastic supply). An increase in the precision of private information τ_ε always increases responsiveness to the private signal but has an ambiguous effect on the slope of supply. The parameter $\hat{c}$ is U-shaped with respect to τ_ε. Observe that $\hat{c} = \lambda^{-1}$ not only when $\tau_\varepsilon = \infty$ but also when $\tau_\varepsilon = 0$ and that $\hat{c} < \lambda^{-1}$ for $\tau_\varepsilon \in (0, \infty)$. If τ_ε is high, then a further increase in τ_ε (less noise in the signals) lowers adverse

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[14]   An increase in $|\beta|$ has a direct positive effect on τ and an indirect negative effect via the induced change in a. The direct effect prevails. Note that changing β modifies not only the public statistic p but also the degree of complementarity in the payoff.

[15]   See Wilson (1979) for a model in which adverse selection makes demand schedules upward sloping.

selection (and h) and increases $\hat{c}$. If τ_ε is low then the price is relatively uninformative, and an increase in τ_ε increases adverse selection (and h) while lowering $\hat{c}$.

If $\beta < 0$ then a high price conveys goods news in terms of both scarcity effects and informational effects, so supply is always upward sloping in this case. Indeed, when $\beta < 0$ we have $\hat{c} > \lambda^{-1}$. A high price conveys the good news that average quantity tends to be high and that costs therefore tend to be low ($h < 0$). In this case, increasing τ_u, which reinforces the informational component of the price, increases $\hat{c}$—the opposite of what happens when τ_θ increases. An increase in the precision of private information τ_ε increases responsiveness to the private signal but, as before, has an ambiguous effect on the slope of supply. Now the parameter $\hat{c}$ is hump-shaped with respect to τ_ε because $\hat{c} > \lambda^{-1}$ for $\tau_\varepsilon \in (0,\infty)$ and $\hat{c} = \lambda^{-1}$ in the extremes of the interval $(0,\infty)$.

In either case ($\beta > 0$ or $\beta < 0$) market depth $(\partial P/\partial u)^{-1} = 1 + \beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ.

Table 1 summarizes the comparative statics results on the equilibrium strategy.

<u>Table 1:</u> Comparative Statics on the Equilibrium Strategy

sgn	$\partial \tau_u$	$\partial \tau_\theta$	$\partial \tau_\varepsilon$
∂a	$-$	$-$	$+$
$\partial \hat{c}$	$-\beta$	β	$\beta\left(\beta^2 \tau_u \tau_\varepsilon^2 + 4\lambda^2 \tau_\theta (\tau_\varepsilon - \tau_\theta)\right)$

The degree of complementarity $m \equiv -\beta/\lambda$ depends on λ for a fixed β (it makes sense to keep β fixed since β also affects the public statistic $p = \alpha + u - \beta\tilde{x}$). For fixed β we have that $\text{sgn}\{\partial m/\partial\lambda\} = \text{sgn}\{\beta\}$. From Proposition 5 we have then that

$\operatorname{sgn}\left\{\partial a/\partial m\right\}=\operatorname{sgn}\left\{-\beta\right\}$, $\operatorname{sgn}\left\{\partial\gamma/\partial m\right\}=\operatorname{sgn}\left\{\beta\right\}$, and

$\operatorname{sgn}\left\{\partial\tau/\partial m\right\}=\operatorname{sgn}\left\{\partial E\left[\left(x_i-\tilde{x}\right)^2\right]\big/\partial m\right\}=\operatorname{sgn}\left\{-\beta\right\}$. The results are summarized in

Table 2.

Table 2: Effects of a Change in the Degree of Complementarity (β fixed)

sgn	∂a	$\partial\gamma$	$\partial\tau$	$E\left[\left(x_i-\tilde{x}\right)^2\right]$
∂m	$-\beta$	β	$-\beta$	$-\beta$

Increased reliance on public information as complementarity increases is a general theme in the work of Morris and Shin (2002) and Angeletos and Pavan (2007) when public signals are exogenous. In stylized environments more complementarity increases the value of public information in forecasting aggregate behavior and decreases the dispersion of actions (e.g., Cor. 1 in Angeletos and Pavan 2007). In our model this happens in the strategic substitutes case ($\beta>0$). With strategic complements ($\beta<0$) an increase in m (a lower λ) makes agents rely less on private information (γ decreases) but respond more to private information (a increases), and increases dispersion as well as increases the precision of public information. (See Table 2.)

7. Other interpretations of the model and applications.
In this section we extend the interpretation of the model to other applications.

7.1 *Investment complementarities*. In this case, $\beta<0$ and we have strategic complementarity among investment decisions of the agents. The marginal benefit of investing is $p=\alpha+u-\beta\tilde{x}$, and the cost is $C(x_i)=\theta x_i+\left(\lambda/2\right)x_i^2$. The shock to the marginal benefit (u) can be understood as a shock to demand, while the shock to costs (θ) can be viewed as a productivity shock. Agents condition their decisions on the marginal benefit of investment p, derived, for example, from the public signals on macroeconomic data released by the government (which in turn depend on the

aggregate activity level). This description need not be taken literally and is simply meant to capture the reduced form of a dynamic process. For example, consider competitive firms deciding about investment in the presence of macroeconomic uncertainty as represented by the random variable θ, which affects profitability. In predicting θ, each firm has access to a private signal as well as to public information, consisting of aggregate past investment figures compiled by a government agency. Data on aggregate investment incorporates measurement error and, at each period, a noisy measure of the previous period's aggregate investment is made public. [16] Propositions 2 and 4 indicate then that at the market solution agents respond too little to private information. This result is in line with the case of exogenous public information (Angeletos and Pavan, section 6.2, 2007). This should be not surprising since the informational and the alloacational role of the public statistic are aligned in this case.

7.2 *Monopolistic competition*. The model applies also to a monopolistically competitive market with quantity-setting firms; in this case, either $\beta > 0$ (goods are substitutes) or $\beta < 0$ (goods are complements). Firm i faces the inverse demand for its product, $p_i = \alpha + u - \beta \tilde{x} - (\lambda/2) x_i$, and has costs θx_i. Each firm uses a supply function that is contingent on its own price: $X(s_i, p_i)$ for firm i. It follows then that observing the price p_i is informationally equivalent (for firm i) to observing $p \equiv \alpha + u - \beta \tilde{x}$.

Under monopolistic competition, the total surplus function (consistent with the differentiated demand system) is slightly different:

$$\text{TS} = (\alpha + u - \theta)\tilde{x} - \left(\beta \tilde{x}^2 + (\lambda/2)\int_0^1 x_i^2 \, di\right)\Big/2.$$

Here the market is not efficient under complete information because price is not equal to marginal cost. Each firm has some residual market power. The results of Section 4 do not apply but those of Section 5 apply when firms collude. It is interesting to note then that, if agents cannot use contingent strategies and there is no information

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[16] For example, quarterly data on national accounts are subject to measurement error. Rodríguez-Mora and Schulstad (2007) show how government announcements regarding GNP growth affect growth via aggregate investment.

externality issue (as in, e.g., cases of Cournot or Bertrand competition), Angeletos and Pavan (Section 6.5, 2007) argue that the strategic substitutability case would exhibit always excessive response to private information in contrast with the case with endogenous public information, where either excessive or insufficient response to private information is possible.

7.3 *Demand schedule competition*. Let a buyer of a homogenous good with unknown ex post value θ face an inverse supply $p = \alpha + u + \beta \tilde{y}$, where $\tilde{y} = \int_0^1 y_i \, di$ and y_i is the demand of buyer i. The suppliers face a cost of supply of $\left(\alpha + u + \beta \frac{\tilde{y}}{2} \right) \tilde{y}$. The buyer's net benefit is given by $\pi_i = (\theta - p) y_i - (\lambda/2) y_i^2$, where λy_i^2 is a transaction or opportunity cost (or an adjustment for risk aversion). The model fits this setup if we let $y_i \equiv -x_i$. Some examples follow.

Firms purchasing labor. A firm purchases labor whose productivity θ is unknown— say, because of technological uncertainty—and faces an inverse linear labor supply (with $\beta > 0$) and quadratic adjustment costs in the labor stock. The firm has a private assessment of the productivity of labor, and inverse supply is subject to a shock. In particular, the welfare analysis of Section 4 applies letting $y_i \equiv -x_i$.

Traders in a financial market. Traders compete in demand schedules for a risky asset with liquidation value θ and face a quadratic adjustment cost in their position (alternatively, the parameter λ proxies for risk aversion). Each trader receives a private signal about the liquidation value of the asset. There are also liquidity suppliers who trade according to the elastic aggregate demand $(\alpha + u - p)/\beta$, where u is random. We can interpret $1/|\beta|$ as the mass of liquidity suppliers. When $\beta > 0$, liquidity suppliers buy (sell) when the price is low (high). When $\beta < 0$, liquidity suppliers buy (sell) when the price is high (low). In this case liquidity suppliers may be program traders following a portfolio insurance strategy.[17] Our inverse supply

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[17] As in Gennotte and Leland (1990). Hendershott and Seasholes (2009) find that program trading accounts for almost 14% of the average daily market volume at the NYSE in 1999-2005 and that program traders lose money on average.

follows from the market-clearing equation. It is worth noting that the normal case with $\beta > 0$ induces strategic substitutability in the actions of informed traders, while when $\beta < 0$ we have strategic complementarity in the actions of informed traders and the slope of excess demand $\Xi' = -\left(\beta^{-1} + \hat{c}\right)$ is positive.[18]

Increasing the mass of liquidity suppliers (i.e. decreasing $|\beta|$) increases the weight given to the private information by informed traders but decreases the informativeness of prices: $\tau = \tau_\theta + \beta^2 a^2 \tau_u$ (and $\beta^2 a^2$ is increasing in $|\beta|$, see Proposition 5). The direct effect on τ prevails over the indirect effect.

In the normal case with $\beta > 0$ and downward-sloping demand schedules for informed traders, prices will contain too much information (both from the perspectives of total surplus and the collective viewpoint of informed traders) about the value of the asset, and too little about the shock to liquidity suppliers. This will tend to happen when the volume of liquidity trading is high (i.e. when $\beta^2 \tau_u$ is low). In this case there will be insufficient price volatility with respect to the second best team benchmark (although still excessive from a first best perspective). In the region where demand schedules for the informed are upward sloping, prices will contain too little information about θ (with the total surplus benchmark). The same applies in the case $\beta < 0$.

Asset auctions. Consider the auction of a financial asset for which (inverse) supply is price elastic: $p = \alpha + \beta \hat{y}$ with $\beta > 0$, where $\hat{y}$ is the total quantity bid. The liquidation value θ of the asset may be its value in the secondary market (say, for a central bank liquidity or Treasury auction). The marginal valuation of a bidder is decreasing in the amount bid.[19] Each bidder receives a private signal about θ, and there are noncompetitive bidders who bid according to u/β. This setup yields $\hat{y} = \tilde{y} + u/\beta$, where $\tilde{y}$ is the aggregate of competitive (informed) bids, and an effective inverse supply for the competitive bidders: $p = \alpha + u + \beta \tilde{y}$.

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[18] See Shin (2010) for an explanation of upward sloping asset demand based on risk management considerations.

[19] A justification for the case of liquidity auctions is given in Ewerhart, Cassola, and Valla (2009).

From the viewpoint of general welfare or of competitive bidders prices contain too much information in the usual case of downward-sloping demand schedules, which obtain when the volume of *noncompetitive bidding* is large (low τ_u). When the volume generated by noncompetitive bids is small (high τ_u), demand schedules for the competitive bidders are upward sloping and prices will (may for intermediate values of τ_u within its high-value region) contain too little information from the perspective of total surplus (of collective competitive bidders).

7. <u>Concluding remarks</u>.

 In a market game where the price depends on a fundamental on which agents receive a private signal and on a payoff relevant shock we reverse the usual intuition leading to underweighting private information due to an information externality. In an economy which is restricted (team) efficient with exogenous public signals rational expectations equilibria are not team-efficient even when the allowed allocations share similar properties as the market equilibrium since the market does not internalize the informational externality when prices convey information. The market does not trade off optimally non-fundamental price volatility with the dispersion of individual actions. Only in exceptional circumstances (i.e., when the information externality vanishes) does the market get it right and strikes the optimal trade off between volatility and dispersion. Under strategic substitutability, prices will convey too much information in the normal case where the allocational role of prices prevails over their informational role and too little in the opposite situation. Under strategic complementarity prices always convey too little information. The inefficiency of the market solution opens the door to the possibility that more precise public or private information will lead to an increased welfare loss. This is the case when the market already calls for a too large response to private information, then more precise private information exacerbates the problem (but not more precise public information).

The results extend to an economy which is not restricted efficient with exogenous public signals, because of a payoff externality. Then the bias towards putting too much weight on private information is increased. It follows that received results on the optimal relative weights to be placed on private and public information (when the

latter is exogenous) may be overturned when the informational role of the price conflicts with its allocational role and the former is important enough.

Several extensions are worth considering. Examples include exploring tax-subsidy schemes to implement team-efficient solutions along the lines of Angeletos and Pavan (2009), Lorenzoni (2010), and Angeletos and Lao (2012); and studying incentives to acquire information (as in Vives 1988; Burguet and Vives 2000; Hellwig and Veldkamp 2009; Myatt and Wallace 2012; Llosa and Venkateswaran 2012; Colombo et al. 2012).

Appendix

Proof of Proposition 1: From the posited strategy $X(s_i, z) = b - as_i + cz$, where $z = u + \beta a\theta$ and $1 - \beta c \neq 0$, we obtain that $p = \alpha - \beta b + (1 - \beta c)z$. From the first-order condition for player i we have

$$X(s_i, z) = \lambda^{-1}(\alpha - \beta b + (1 - \beta c)z - E[\theta|s_i, z]).$$

Here $E[\theta/s_i, z] = \gamma s_i + (1 - \gamma)E[\theta/z]$ with $\gamma = \tau_\varepsilon(\tau_\varepsilon + \tau)^{-1}$, $E[\theta/z] = \beta\tau_u a\tau^{-1}z$ (recall that we have normalized $\bar{\theta} = 0$), and $\tau = \tau_\theta + \beta^2 a^2 \tau_u$ from the projection theorem for Gaussian random variables. Note that $E[\theta/s_i, z] = \gamma s_i + hz$ where $h = \beta a\tau_u(\tau_\varepsilon + \tau)^{-1}$. Identifying coefficients with $X(s_i, z) = b - as_i + cz$, we can immediately obtain

$$a = \frac{\gamma}{\lambda} = \frac{\tau_\varepsilon}{\lambda(\tau_\varepsilon + \tau)}, \quad c = \frac{1 - h}{\beta + \lambda} = \frac{1}{\beta + \lambda} - \frac{\beta a\tau_u}{(\beta + \lambda)(\tau_\varepsilon + \tau)}, \quad \text{and} \quad b = \frac{\alpha}{\beta + \lambda}.$$

It follows that the equilibrium parameter a is determined as the unique (real), of the following cubic equations, that is positive and lies in the interval $a \in \left(0, \tau_\varepsilon \lambda^{-1}(\tau_\theta + \tau_\varepsilon)^{-1}\right)$:

$$a = \frac{\tau_\varepsilon}{\lambda(\tau_\varepsilon + \tau_\theta + \beta^2 a^2 \tau_u)} \quad \text{or} \quad \beta^2 \tau_u a^3 + (\tau_\varepsilon + \tau_\theta)a - \lambda^{-1}\tau_\varepsilon = 0$$

and

$$c = \frac{1}{(\beta + \lambda)} - \frac{\beta\lambda\tau_u a^2}{(\beta + \lambda)\tau_\varepsilon}.$$

It is immediate from the preceding equality for c that $c < (\beta + \lambda)^{-1}$ (since $a \geq 0$) and that $1 - \beta c > 0$ (since $\beta + \lambda > 0$); therefore,

$$\beta c = \frac{\beta}{\beta + \lambda} - \frac{\beta^2 a\tau_u}{(\beta + \lambda)(\tau_\varepsilon + \tau)} < 1.$$

It follows that

$$X(s_i, p) = \hat{b} - as_i + \hat{c}p,$$

where $\hat{b} = b(1 - \lambda\hat{c})$, $b = \alpha/(\beta + \lambda)$, and $\hat{c} = c/(1 - \beta c)$ with $1 + \beta\hat{c} > 0$. From the equilibrium expression for $c = (\beta + \lambda)^{-1}(1 - \beta\lambda\tau_u a^2\tau_\varepsilon^{-1})$ we obtain the expression for

$$\hat{c} = (c^{-1} - \beta)^{-1}. \blacklozenge$$

Claim 1. *Linear equilibria in strategies with bounded means and with uniformly (across players) bounded variances yield linear equilibria of the schedule game for which the public statistic function is of type $P(\theta, u)$.*

Proof: If for player i we posit the strategy

$$x_i = \hat{b}_i + \hat{c}_i p - a_i s_i$$

then the aggregate action is given by

$$\tilde{x} = \int_0^1 x_i \, di = \hat{b} + \hat{c}p - a\theta - \int_0^1 a_i \varepsilon_i \, di = \hat{b} + \hat{c}p - a\theta,$$

where $\hat{b} = \int_0^1 \hat{b}_i \, di$, $\hat{c} = \int_0^1 \hat{c}_i \, di$, and $a = \int_0^1 a_i \, di$ (assuming that all terms are well-defined). Observe that, according to our convention on the average error terms of the signals, $\int_0^1 a_i\varepsilon_i \, di = 0$ a.s. provided that $\mathrm{var}[a_i \varepsilon_i]$ is uniformly bounded across agents (since $\mathrm{var}[\varepsilon_i] = \sigma_\varepsilon^2$, it is enough that a_i be uniformly bounded). In equilibrium, this will be the case. Therefore, if we restrict attention to candidate linear equilibria with parameters a_i uniformly bounded in i and with well-defined average parameters $\hat{b}$ and $\hat{c}$, then $\tilde{x} = \hat{b} + \hat{c}p - a\theta$ and the public statistic function is of the type $P(\theta, u)$. $\blacklozenge$

Proof of Proposition 2: Note first that it can be checked that $\partial^2 E[\mathrm{TS}]/\partial^2 b < 0$ and $\partial^2 E[\mathrm{TS}]/\partial^2 c < 0$ whenever $\beta + \lambda > 0$. Given that $\partial x_i/\partial b = 1$, and $\partial x_i/\partial c = z$, we can optimize with respect to b and c to obtain

$$\frac{\partial E[\mathrm{TS}]}{\partial b} = E[(p - \mathrm{MC}(x_i))] = 0,$$

$$\frac{\partial E[\mathrm{TS}]}{\partial c} = E[(p - \mathrm{MC}(x_i))z] = 0,$$

where $p = \alpha + u - \beta \tilde{x}$ and $\mathrm{MC}(x_i) = \theta + \lambda x_i$. The constraint $E\left[p - \mathrm{MC}(x_i)\right] = 0$ is equivalent to $b = \alpha/(\beta + \lambda)$ and $E\left[(p - \mathrm{MC}(x_i))z\right] = 0$ is equivalent to

$$c = c(a) \equiv \frac{1}{\beta + \lambda} - \frac{\beta a \tau_u (1 - \lambda a)}{\tau(\beta + \lambda)}.$$ Those constraints are also fulfilled by the market solution since the first-order condition (FOC) for player i is $E\left[p - \mathrm{MC}(x_i)|s_i, z\right] = 0$, from which it follows, according to the properties of Gaussian distributions, that

$$E\left[p - \mathrm{MC}(x_i)\right] = 0 \quad , \quad \text{and} \quad E\left[(p - \mathrm{MC}(x_i))z\right] = 0 \quad \text{(as} \quad \text{well} \quad \text{as}$$

$E\left[(p - \mathrm{MC}(x_i))s_i\right] = 0$).

It follows that the form of the team optimal strategy is $x_i = \lambda^{-1}\left[p - (\gamma s_i + (1 - \gamma)E[\theta \mid z])\right]$ where $\gamma = \lambda a$. We have that $\tilde{x} = \lambda^{-1}\left[p - (\gamma \theta + (1 - \gamma)E[\theta \mid z])\right]$ and that $\tilde{x} - x^o = (1 - \gamma)(\theta - E[\theta \mid z])/(\beta + \lambda)$ and, since $\tau = (\mathrm{var}[\theta | z])^{-1}$ we obtain $E\left[(\tilde{x} - x^o)^2\right] = (1 - \lambda a)^2/(\tau(\beta + \lambda)^2)$. We know that $E\left[(x_i - \tilde{x})^2\right] = a^2/\tau_\varepsilon$.

Let $\mathrm{WL} = E\left[\mathrm{TS}^o\right] - E\left[\mathrm{TS}\right]$. Similarly as in the proof of Proposition 3 in Vives (2011) we can obtain, using an exact Taylor expansion of total surplus around the full information first best allocation x^o, that

$$\mathrm{WL} = \left((\beta + \lambda)E\left[(\tilde{x} - x^o)^2\right] + \lambda E\left[(x_i - \tilde{x})^2\right]\right)\Big/ 2 \quad . \quad \text{It} \quad \text{follows} \quad \text{that}$$

$$\mathrm{WL}(a) = \frac{1}{2}\left(\frac{(1 - \lambda a)^2}{(\tau_\theta + \tau_u \beta^2 a^2)(\beta + \lambda)} + \frac{\lambda a^2}{\tau_\varepsilon}\right), \text{ which is easily seen strictly convex in } a$$

and with a unique solution $\lambda^{-1} > a^T > 0$. (Note that $\lambda^{-1} < a$ is dominated by $a = \lambda^{-1}$ and that $a < 0$ is dominated by $-a > 0$. Furthermore, it is immediate that $\mathrm{WL}'(0) < 0$ and therefore $a > 0$ at the solution.)

The impact of a on $E[\mathrm{TS}]$ is easily characterized (noting that $\partial E[\mathrm{TS}]/\partial c = 0$ and therefore disregarding the indirect impact of a on $E[\mathrm{TS}]$ via a change in c):

$$\frac{\partial E[\text{TS}]}{\partial a} = E\left[\left(p - \text{MC}(x_i)\right)\left(\frac{\partial x_i}{\partial a}\right)_{z\ \text{ct.}}\right] + E\left[\left(p - \text{MC}(x_i)\right)\left(\frac{\partial x_i}{\partial z}\frac{\partial z}{\partial a}\right)\right]$$

$$= E\left[\left(p - \text{MC}(x_i)\right)\left(-s_i + c\beta\theta\right)\right]$$

given that $\left(\partial x_i / \partial a\right)_{z\ \text{ct.}} = -s_i$, $\partial x_i / \partial z = c$ and $\partial z / \partial a = \beta\theta$.

Evaluating $\partial E[\text{TS}] / \partial a$ at the LE, where $E\left[\left(p - \text{MC}(x_i)\right)s_i\right] = 0$, we obtain that

$\partial E[\text{TS}] / \partial a = c\beta E\left[\left(p - \text{MC}(x_i)\right)\theta\right]$. Now, because

$$E\left[\left(p - \text{MC}(x_i)\right)s_i\right] = E\left[\left(p - \text{MC}(x_i)\right)\theta\right] + E\left[\left(p - \text{MC}(x_i)\right)\varepsilon_i\right] = 0,$$

it follows that

$$E\left[\left(p - \text{MC}(x_i)\right)\theta\right] = -E\left[\left(p - \text{MC}(x_i)\right)\varepsilon_i\right]$$

$$= E\left[\text{MC}(x_i)\varepsilon_i\right] = E\left[\left(\theta + \lambda x_i\right)\varepsilon_i\right] = -\lambda a^{\text{LE}}\sigma_\varepsilon^2 < 0$$

since ε_i is independent of all the model's other random variables and since $a^{\text{LE}} > 0$

when $\tau_\varepsilon > 0$. Hence

$$\text{sgn}\left\{\left.\frac{\partial E[\text{TS}]}{\partial a}\right|_{a=a^*}\right\} = \text{sgn}\{\text{IE}\} = \text{sgn}\{-\beta c^*\},$$

and this equals $\text{sgn}\{a^T - a^*\}$ because $E[\text{TS}]$ is single-peaked for $a > 0$ with a

maximum at a^T. $\blacklozenge$

Proof of Proposition 3. The welfare loss at the team-efficient solution is given by

$\text{WL}\left(a^{\text{T}}\right)$, which is decreasing in τ_ε, τ_u and τ_θ since WL is decreasing in τ_ε, τ_u and

τ_θ for a given a and $\text{WL}'\left(a^{\text{T}}\right) = 0$. With respect to the market solution we have that

$$\frac{d\text{WL}}{d\tau_\theta}(a^*) = \frac{\partial \text{WL}}{\partial a}\frac{\partial a^*}{\partial \tau_\theta} + \frac{\partial \text{WL}}{\partial \tau_\theta},$$

where $\dfrac{\partial a^*}{\partial \tau_\theta} = -\dfrac{a}{\tau_\theta + \tau_\varepsilon + 3a^2\beta^2\tau_u}$ and a^* solves $\beta^2\tau_u a^3 + \left(\tau_\varepsilon + \tau_\theta\right)a - \lambda^{-1}\tau_\varepsilon = 0$.

Given that

$$\text{WL} = \frac{1}{2}\left(\frac{\left(1 - \lambda a\right)^2}{\left(\tau_\theta + \tau_u\beta^2 a^2\right)\left(\beta + \lambda\right)} + \frac{\lambda a^2}{\tau_\varepsilon}\right),$$

it is possible to show that

$$\frac{d\mathrm{WL}}{d\tau_\theta}(a^*) < 0 \text{ if and only if } \frac{\tau_\theta + \tau_u \beta^2 a^2}{\tau_\varepsilon} > -\frac{2\beta + \lambda}{\lambda},$$

which is always true since $2\beta + \lambda > 0$. Exactly the same condition holds for $d\mathrm{WL}(a^*)/d\tau_u < 0$. Furthermore, we can show that $d\mathrm{WL}(a^*)/d\tau_\varepsilon \le 0$ if and only if $\beta - \lambda \le \frac{\tau_u \beta^2 a^*}{\tau_\theta}(a^*(\beta + \lambda) + 2) + (\beta + \lambda)\frac{\tau_\varepsilon}{\tau_\theta}$. It follows that WL will be increasing in τ_ε for $\beta > \lambda$ and $\tau_\varepsilon/\tau_\theta$ small enough (since a^* is increasing in $\tau_\varepsilon/\tau_\theta$). $\blacklozenge$

Proof of Proposition 4: It proceeds in a parallel way to the proof of Proposition 2. Again, it can be checked first that $\partial^2 E[\pi_i]/\partial^2 b < 0$ and $\partial^2 E[\pi_i]/\partial^2 c < 0$ whenever $2\beta + \lambda > 0$. Given that $\pi_i = px_i - \mathrm{C}(x_i)$, $p = \alpha + u - \beta\tilde{x}$, $\partial x_i/\partial b = 1$, and $\partial x_i/\partial c = \partial \tilde{x}/\partial c = z$ and $\partial p/\partial \tilde{x} = -\beta$ we can optimize with respect to b and c to obtain

$$\frac{\partial E[\pi_i]}{\partial b} = E\big[(p - \mathrm{MC}(x_i)) - \beta x_i\big] = 0,$$

$$\frac{\partial E[\pi_i]}{\partial c} = E\big[(p - \mathrm{MC}(x_i))z - \beta x_i z\big] = 0.$$

where $\mathrm{MC}(x_i) = \theta + \lambda x_i$. The constraint $E\big[(p - \mathrm{MC}(x_i)) - \beta x_i\big] = 0$ is equivalent to $b = \alpha/(2\beta + \lambda)$; we can also check that $E\big[(p - \mathrm{MC}(x_i))z - \beta x_i z\big] = 0$ is equivalent to $c = c^{\mathrm{IT}}(a)$, where

$$c^{\mathrm{IT}}(a) = \frac{1}{2\beta + \lambda} - \frac{\beta a \tau_u (1 - (\lambda + \beta)a)}{\tau(2\beta + \lambda)} \quad \text{and} \quad \tau = \tau_\theta + \beta^2 \tau_u a^2.$$

Note that due to payoff externalities ($\partial p/\partial \tilde{x} = -\beta$) the expressions for b and for c are different than in the market solution. It follows that the form of the internal team optimal strategy is $x_i = (\lambda + \beta)^{-1}\big[p - (\gamma s_i + (1 - \gamma)E[\theta \,|\, z])\big]$ where $\gamma = (\lambda + \beta)a$.

We have that $\tilde{x} = (\lambda + \beta)^{-1}\big[p - (\gamma\theta + (1 - \gamma)E[\theta \,|\, z])\big]$ and that

$\tilde{x} - x^{\mathrm{M}} = (1 - \gamma)(\theta - E[\theta \,|\, z])/(2\beta + \lambda)$ and, since $\tau = (\mathrm{var}[\theta|z])^{-1}$ we obtain

$E\big[(\tilde{x} - x^{\mathrm{M}})^2\big] = (1 - (\lambda + \beta)a)^2/(\tau(2\beta + \lambda)^2)$. We have that $E\big[(x_i - \tilde{x})^2\big] = a^2/\tau_\varepsilon$.

Let $\quad L = E\left[\pi_i^M\right] - E\left[\pi_i\right]\quad$. Similarly as before we can obtain that

$L = \left(\left(2\beta + \lambda\right)E\left[\left(\tilde{x} - x^M\right)^2\right] + \lambda E\left[\left(x_i - \tilde{x}\right)^2\right]\right)\Big/2$. It follows that

$$L = \frac{1}{2}\left(\frac{\left(1 - \left(\lambda + \beta\right)a\right)^2}{\left(\tau_\theta + \tau_u\beta^2a^2\right)\left(2\beta + \lambda\right)} + \frac{\lambda a^2}{\tau_\varepsilon}\right),$$

which is easily seen strictly convex in a and with a unique solution $\left(\lambda + \beta\right)^{-1} > a^{IT} > 0$. (Note that $\left(\lambda + \beta\right)^{-1} < a$ is dominated by $a = \left(\lambda + \beta\right)^{-1}$ and that $a < 0$ is dominated by $-a > 0$. Furthermore, it is immediate that $L'(0) < 0$ and therefore $a > 0$ at the solution.)

The impact of a on $E\left[\pi_i\right]$ is easily characterized (noting that $\partial E\left[\pi_i\right]/\partial c = 0$ and therefore disregarding the indirect impact of a on $E\left[\pi_i\right]$ via a change in c):

$$\frac{\partial E\left[\pi_i\right]}{\partial a} = \underbrace{E\left[\left(p - MC\left(x_i\right)\right)\left(\frac{\partial x_i}{\partial a}\right)_{z\,ct.}\right]}_{M} +$$
$$\underbrace{E\left[\left(p - MC\left(x_i\right)\right)\left(\frac{\partial x_i}{\partial z}\frac{\partial z}{\partial a}\right)\right]}_{IE} +$$
$$\underbrace{E\left[x_i\left(\frac{\partial p}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial a}\right)\right]}_{PE}$$
$$= E\left[\left(p - MC\left(x_i\right)\right)\left(-s_i + c\beta\theta\right) - \beta\left(c\beta - 1\right)\theta x_i\right]$$

given that $\left(\partial x_i/\partial a\right)_{z\,ct.} = -s_i$, $\partial x_i/\partial z = c$, $\partial z/\partial a = \beta\theta$, $\partial p/\partial \tilde{x} = -\beta$ and $\partial \tilde{x}/\partial a = \left(c\beta - 1\right)\theta$. Evaluating $\partial E\left[\pi_i\right]/\partial a$ at the equilibrium, where $E\left[\left(p - MC\left(x_i\right)\right)s_i\right] = 0$, we obtain

$$\frac{\partial E\left[\pi_i\right]}{\partial a} = \beta E\left[c\left(p - MC\left(x_i\right)\right)\theta - \left(c\beta - 1\right)\theta x_i\right].$$

As in the last section, we have $E\left[\left(p - \mathrm{MC}\left(x_i\right)\right)\theta\right] = -\lambda a \sigma_\varepsilon^2 < 0$ and, recalling that $\bar{\theta} = 0$, it is easily checked that $E\left[\theta x_i\right] = a\sigma_\theta^2\left(c\beta - 1\right)$. At the equilibrium we have therefore[20]

$$\frac{\partial E\left[\pi_i\right]}{\partial a} = -\beta a * \left(c * \lambda \sigma_\varepsilon^2 + \left(c * \beta - 1\right)^2 \sigma_\theta^2\right).$$

Since $E\left[\pi_i\right]$ is single-peaked for $a > 0$ and has a unique maximum at $a^{\mathrm{IT}} > 0$ and $a* > 0$, it follows that

$$\mathrm{sgn}\left\{a^{\mathrm{IT}} - a*\right\} = \mathrm{sgn}\left\{\left.\frac{\partial E\left(\pi_i\right)}{\partial a}\right|_{a=a*}\right\} = \mathrm{sgn}\left\{-\beta\left(c * \lambda \sigma_\varepsilon^2 + \left(c * \beta - 1\right)^2 \sigma_\theta^2\right)\right\}. \blacklozenge$$

Proof of Proposition 5: (i) From the equation determining the responsiveness to private information a, $\beta^2 \tau_u a^3 + \left(\tau_\varepsilon + \tau_\theta\right)a - \lambda^{-1}\tau_\varepsilon = 0$, it is immediate that a decreases with τ_u, τ_θ, β^2 and λ, that a increases with τ_ε. Note that $\mathrm{sgn}\left\{\partial a/\partial \beta\right\} = \mathrm{sgn}\left\{-\beta\right\}$. As τ_u ranges from 0 to ∞, a decreases from $\lambda^{-1}\tau_\varepsilon\left(\tau_\theta + \tau_\varepsilon\right)^{-1}$ to 0.

(ii) As τ_u ranges from 0 to ∞, the responsiveness to public information c goes from $\left(\beta + \lambda\right)^{-1}$ to $-\infty$ (resp. $+\infty$) if $\beta > 0$ (resp. $\beta < 0$). The result follows since, in equilibrium,

$$c = \frac{1}{\beta + \lambda} - \frac{\beta \lambda \tau_u a^2}{\left(\beta + \lambda\right)\tau_\varepsilon} = \frac{1}{\beta + \lambda} - \frac{1}{\left(\beta + \lambda\right)\beta}\left(\frac{1}{a} - \lambda\left(1 + \frac{\tau_\theta}{\tau_\varepsilon}\right)\right)$$

and $a \to 0$ as $\tau_u \to \infty$. It follows that $\mathrm{sgn}\left\{\partial c/\partial \tau_u\right\} = \mathrm{sgn}\left\{-\beta\right\}$ because $\partial a/\partial \tau_u < 0$. Similarly, from the first part of the expression for c we have $\mathrm{sgn}\left\{\partial c/\partial \tau_\theta\right\} = \mathrm{sgn}\left\{\beta\right\}$ since $\partial a/\partial \tau_\theta < 0$. Furthermore, with some work it is possible to show that, in equilibrium,

$$\frac{\partial c}{\partial \tau_\varepsilon} = \left(\beta + \lambda\right)^{-1}\lambda \beta \tau_u \tau_\varepsilon^{-1} a\left(2\frac{a\lambda - 1}{\lambda\left(\tau_\theta + \tau_\varepsilon + 3a^2\beta^2\tau_u\right)} + a\tau_\varepsilon^{-1}\right) \quad \text{and}$$

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[20]   Note also that at the equilibrium $c\beta - 1 < 0$.

$$\text{sgn}\left\{2\frac{a\lambda-1}{\lambda\left(\tau_\theta+\tau_\varepsilon+3a^2\beta^2\tau_u\right)}+a\tau_\varepsilon^{-1}\right\}=\text{sgn}\left\{a\lambda\tau_\theta-2\tau_\varepsilon+3a\lambda\tau_\varepsilon+3a^3\beta^2\lambda\tau_u\right\}$$

$$=\text{sgn}\left\{-2a\lambda\tau_\theta+\tau_\varepsilon\right\}$$

$$=\text{sgn}\left\{\beta^2\tau_u\tau_\varepsilon^2+4\lambda^2\tau_\theta^2\left(\tau_\varepsilon-\tau_\theta\right)\right\}.$$

Hence we conclude that $\text{sgn}\left\{\partial c/\partial\tau_\varepsilon\right\}=\text{sgn}\left\{\beta\left(\beta^2\tau_u\tau_\varepsilon^2+4\lambda^2\tau_\theta^2\left(\tau_\varepsilon-\tau_\theta\right)\right)\right\}$. Since

$\hat{c}=\left(c^{-1}-\beta\right)^{-1}$, it follows that $\hat{c}$ goes from λ^{-1} to $-\beta^{-1}$ as τ_u ranges from 0 to ∞,[21]

$\text{sgn}\left\{\partial\hat{c}/\partial\tau_u\right\}=\text{sgn}\left\{-\partial\hat{c}/\partial\tau_\theta\right\}=\text{sgn}\left\{-\beta\right\}$, and $\text{sgn}\left\{\partial\hat{c}/\partial\tau_\varepsilon\right\}=\text{sgn}\left\{\partial c/\partial\tau_\varepsilon\right\}$. It is then

immediate that $1+\beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ.

(iii) Price informativeness $\tau=\tau_\theta+\beta^2a^2\tau_u$ is increasing in τ_ε (since a increases with

τ_ε) and also in τ_u (since $a=\lambda^{-1}\tau_\varepsilon\left(\tau_\varepsilon+\tau\right)^{-1}$ and a decreases with τ_u). Using the

expression for $\partial a/\partial\tau_\theta$ we have that

$$\frac{\partial\tau}{\partial\tau_\theta}=1+2\beta^2\tau_ua\frac{\partial a}{\partial\tau_\theta}=1-\frac{2\beta^2a^2\tau_u}{\tau_\theta+\tau_\varepsilon+3a^2\beta^2\tau_u}=\frac{\tau_\theta+\tau_\varepsilon+a^2\beta^2\tau_u}{\tau_\theta+\tau_\varepsilon+3a^2\beta^2\tau_u}>0.$$

Furthermore,

$$\frac{\partial\tau}{\partial\beta}=\tau_u\left(2\beta a^2+2\beta^2a\frac{\partial a}{\partial\beta}\right)=2\beta a\tau_u\left(a-\frac{2a^4\tau_u\lambda\beta^2}{1+2a^3\tau_u\lambda\beta^2}\right)=\frac{2\beta a^2\tau_u}{1+2a^3\tau_u\lambda\beta^2},$$

and therefore $\text{sgn}\left\{\partial\tau/\partial\beta\right\}=\text{sgn}\left\{\beta\right\}$.

(iv) From $x_i=\lambda^{-1}\left[p-E\left[\theta|s_i,z\right]\right]$ and $E\left[\theta/s_i,z\right]=\gamma s_i+\left(1-\gamma\right)E\left[\theta/z\right]$ we obtain

$x_i-\tilde{x}=\lambda^{-1}\gamma\left(s_i-\theta\right)=\lambda^{-1}\gamma\varepsilon_i$ and, noting that $\gamma=\lambda a$ we conclude that

$E\left[\left(x_i-\tilde{x}\right)^2\right]=a^2\sigma_\varepsilon^2$. The results then follow from the comparative statics results for

a in (i). ◆

Claim 2. $E\left[\theta/s_i,z\right]=\gamma s_i+hz$ with $h=\lambda\beta\tau_\varepsilon^{-1}\tau_ua^2$, $\partial|h|/\partial\tau_\theta<0$, $\partial|h|/\partial\tau_u>0$ and

$\text{sgn}\left\{\partial|h|/\partial\beta\right\}=\text{sgn}\left\{\beta\right\}$.

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[21] Note that if $\beta<0$ and $\beta+\lambda>0$ then $\lambda^{-1}<-\beta^{-1}$.

<u>Proof</u>: From $h = \beta a \tau_u \left(\tau_\varepsilon + \tau \right)^{-1}$ in the proof of Proposition 1 it is immediate that

$h = \lambda \beta \tau_\varepsilon^{-1} \tau_u a^2$. We have that $\partial |h| / \partial \tau_\theta < 0$ since $\partial a / \partial \tau_\theta < 0$; $\partial |h| / \partial \tau_u > 0$ since

$\partial \tau / \partial \tau_u > 0$ and therefore $\partial \left(\tau_u a^2 \right) / \partial \tau_u > 0$. Finally, we have that in equilibrium

$$\frac{\partial c}{\partial \beta} = -\frac{1}{(\lambda + \beta) \tau_\varepsilon} \left(\frac{\tau_\varepsilon + \lambda^2 \tau_u a^2}{\lambda + \beta} + \frac{4 a^5 \lambda^3 \tau_u^2 \beta^2}{1 + 2 a^3 \tau_u \lambda \beta^2} \right) < 0 ,$$

and from $c = (1 - h)(\beta + \lambda)^{-1}$ we can obtain $\partial h / \partial \beta > 0$, and therefore,

$\mathrm{sgn} \left\{ \partial |h| / \partial \beta \right\} = \mathrm{sgn} \left\{ \beta \right\}$. ♦

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