

# Executive Stock Options as a Screening Mechanism <sup>\*</sup>

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# Executive Stock Options as a Screening Mechanism

## Abstract

We show that a possible explanation for the widespread use of options in compensation contracts might be that they provide a way to screen executives. In particular, we consider the problem of a risk-neutral firm that tries to hire a risk-averse executive. There are several types of executives, of varying quality. Executives know their own types, but the firm only knows the probability distribution of types. Executives affect stock price dynamics through the choice of volatility, and by applying costly effort. The firm offers executives a compensation contract consisting exclusively of stock or options. We show that the optimal contract with type uncertainty requires more leverage than contracts with perfect information. Even when the optimal contract with perfect information requires stock compensation, in the case of incomplete information it might be optimal to offer options compensation to filter out bad executives.

# 1 Introduction

Stock options are an important component of compensation packages. As reported by Murphy (1999), in 1996, 39% of the compensation packages of CEO's of companies in the S&P 500 consisted of options. This percentage goes up to 47% in 1999. Additionally, 94% of companies in the S&P 500 granted options to their CEO's. An obvious explanation for the widespread use of options as compensation has been the accounting advantage that options had not to be expensed the moment they were granted. This is about to change: FASB has decided mandatory expensing of options starting on June 15, 2005. However, it does not seem this will be the end of the use of options as compensation. It is still a relevant question whether options should be part of compensation packages.

Jensen and Murphy (1990) show that the part of the compensation of executives linked to company performance depends mostly on their holdings of stock and options rather than on bonuses (more on this topic can be found in Murphy 1999, Hall and Leibman 1998 and Carpenter 1998). A number of papers consider whether it is optimal to grant options as part of the compensation package. Among the first references in this area are Lambert, Lanen and Larcker (1989) who argue that executive stock options induce a reduction in dividend payments. Yermack (1995) reviews some possible reasons argued in the literature in favor of the use of options for compensation, but finds little empirical support for most of them. Lazear (2001) introduces sorting as a possible argument in favor of option compensation: options will be a cheaper way to compensate optimistic employees. Oyer and Schaefer (2004) find empirical support in favor of sorting. Assef and Santos (2004) argue that option compensation provides the right incentives in a moral hazard setting: in a calibration exercise they find that options can be an optimal contract for plausible risk-aversion parameter values. In a related paper, Palmon, Bar-Yosef, Chen and Venezia (2004) argue in favor of the optimality of granting in-the-money options in a moral hazard setting, versus the standard practice of granting at-the-money options, as examined by Hall and Murphy (2000). Stoughton and Wong (2004) argue against the use of options in industries where firms compete to hire because of the extra flexibility features of options through repricing and resetting policies. Kadan and Swinkels (2004) consider the possibility of bankruptcy: stock is more likely to be the optimal compensation when the probability of bankruptcy is high. One of the potential problems of stock options compensation (see, for instance, John and John, 1993, Johnson and Tian, 2000a, 2000b) is the incentive for the executive to increase volatility, since options values increase with volatility. Carpenter (2000) addresses this problem in a dynamic setting, and shows that this is not necessarily the case for a risk averse executive. Ross (2004) discusses the effects of different compensation schedules and shows that, in fact, convex fees might make a risk averse executive to behave more conservatively.

Ittner, Lambert and Larcker (2002) find that a major reason why companies use equity-based compensation is to attract new employees. In their empirical study

they do not differentiate between restricted stock and stock options compensation, although they argue that “option-based contracts are (...) more attractive to employees with higher skill levels who have greater ability to take actions that cause their options to finish in the money.” In this paper we present a theoretical model that studies the mechanics of that conjecture and shows that options might help to screen low-quality managers. We consider a risk-neutral firm whose objective is to maximize expected stock price minus cost of the compensation package. The firm needs to hire a risk-averse executive and chooses the number of options and strike price to offer. Executives can affect the dynamics of the stock price in two ways: through the choice of volatility (the manager chooses among a menu of projects) and through costly effort, that affects the expected stock price appreciation. The higher the volatility, the higher also the expected appreciation of the stock price. The risk-neutral firm would like the executive to choose a high level of volatility, and apply a high level of effort. There are different executive types, depending on the effect their effort level will have on the stock price. In a setting with perfect information about the executive type, the optimal contract is to grant options to high-type executives and stock to low-type executives. When the firm is uncertain about the type, the optimal contract involves a security with higher implicit leverage (higher strike price) than it would be optimal to the highest-type executive. Even if the executive types are such that the optimal contract with perfect information would consist of stock, it might be optimal to offer options. The reason is that low-type executives will not be willing to accept such a contract because the implicit risk will make their value lower than their reservation wage, even if this is lower than the reservation wage of the high-type executives.

The paper is structured as follows. In section 2 we describe verbally the problem we are considering in this paper. In section 3 we formally introduce our model and describe the dynamics of the stock price and the effects of the actions of the executive on those dynamics, as well as the objectives of the two players in our model: the executive and the firm. In section 4 we derive the optimal effort and volatility to be chosen by the executive, as well as the choice of the optimal number of securities and their strike price by the firm, both under complete and incomplete information. In section 5 we explain the algorithm we use to derive the solution, compute some numerical examples and derive the main economic conclusions of the paper.

## 2 The Economic Problem

In this paper we analyze the optimal strategy of a risk-neutral company that wants to hire a risk-averse executive but does not know the type of the executive, although it knows the probability distribution of types. Type here can represent the skill of the executive in applying effort to the business of the firm. Alternatively, we can think of executives of similar skills but whose dedication to the firm is different: for example, a particular executive might be willing to be fully devoted to the firm business, while another executive might plan to spend a lot of time on other things, but this is

unknown to the firm. We say that the former executive is high-type, while the latter is low-type. The firm knows the probability distribution of types and the reservation wage of each type.

However, when dealing with individual executives, the firm does not know their type. We assume that executives will sign a fixed-term contract and the company will reward them with a compensation package consisting of options or stock. Arguably, high-type executives will have a reservation wage higher than low-type executives. In this case, it might be optimal for a low-type executive to lie and declare a high-type. The firm has to offer a contract that clears the participation constraint of the executive according to the type declared by the executive. However, the same contract has different value for executives of different types. Therefore, it might be optimal for the firm to offer a contract such that it is optimal for executives to declare their true type, otherwise, they will receive a contract that has a value for them lower than their reservation wage. In those cases, the firm actually knows the type of the executive it is hiring (although the contract might be different from the one that it would offer in the case of perfect information about the types). Following the tradition of the literature on Principal-Agent problems, we call such an outcome *separating equilibrium*. However, in other cases, it might be optimal for the firm to offer a contract such that some executives have an incentive to lie and declare a different type. The firm does not know “ex ante” the type of executive it is hiring, and it knows there is a positive probability the executive is of a different type. We call this outcome a *pooling equilibrium*. The firm will eventually find out the type, but cannot impose any penalty if the executive lied. In this paper we are interested in the role that stock options compensation might have in a setting of incomplete information.

### 3 The Model

The framework in this paper is similar to that of Cadenillas, Cvitanić and Zapatero (2004), but in that paper the firm knows the type of the executive. In particular, in this paper we consider the problem of a risk-neutral firm that has to decide whether to grant stock or options as compensation to a risk-averse executive that can affect the dynamics of the stock by applying costly effort or choosing the level of volatility. The objective of the firm is to maximize the expected value of the stock price minus the cost of the compensation package. Our results depend on our use of a dynamic setting: as we will see, the optimal effort and volatility adapt dynamically to the resulting state.

### 3.1 Stock Dynamics

Our benchmark stock has a price that follows a geometric Brownian motion process,

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

with starting value  $S_0$ . The process  $W$  is a standard Brownian motion process and  $\mu$  and  $\sigma$  are exogenous constants. However, when the company is managed by the executive whose problem we address in the next subsection, the dynamics of the stock price  $S$  is given by

$$dS_t = \delta a_t dt + \alpha \sigma_t S_t dt + \sigma_t S_t dW_t, \quad (1)$$

(without loss of generality, we assume  $\mu = 0$ ), where  $a$  and  $\sigma$  are adapted stochastic processes chosen by the executive; in the case of perfect information  $\delta$  and  $\alpha$  are known constants,  $\delta \in [0, \infty)$  and  $\alpha \in (0, \infty)$ . We will later consider the case in which  $\delta$  is a random variable. We assume that  $E[\int_0^T |a_t|^2 dt] < \infty$  and  $E[\int_0^T |\sigma_t S_t|^2 dt] < \infty$ . The control  $a$  is the level of effort the executive puts in the management of the company. The higher the effort  $a$ , the higher the expected value of the stock. Effort produces disutility for the executive in a way we model later. The choice of  $\sigma$  is equivalent to the choice of the volatility of the stock: we assume that the executive can choose within a menu of different projects, such that riskier projects also offer higher expected return. We interpret the choice of  $\sigma$  as a choice of projects, and the parameter  $\alpha$  is a measure of the benefits of taking more risk, and it is a characteristic of the firm. Parameter  $\delta$  measures the effect the effort of the executive will have on the stock price appreciation. It can be interpreted as an indicator of the type (quality) of the executive, but other interpretations are possible, as we will discuss when we present the objective of the executive. Carpenter (2000) studies the optimal choice of  $\sigma$  for the case in which  $\delta = 0$ .

### 3.2 The Executive

In our model, the executive chooses  $a$  and  $\sigma$  so as to maximize expected utility. The executive is risk-averse and effort is costly in terms of utility. The objective of the executive is

$$e(K, n) := \max_{a, \sigma} E \left[ \log \{n(S_T - K)^+\} - \frac{1}{2} \int_0^T a_t^2 dt \right] \quad (2)$$

In (2),  $n$  is the number of call options or shares of stock the executive receives as a part of the compensation package.  $T$  is the horizon considered by the executive, that we make equal to the vesting period and the maturity of the option. We ignore the possibility of re-negotiation. As usual,  $K$  is the exercise price of the options. The case  $K = 0$  represents the choice of the firm (whose objective we will discuss below) to grant stock rather than options. The second term of the objective function of the executive represents the disutility from effort and we assume it is quadratic. We

point out that instead of characterizing types by the value of their parameter  $\delta$  we could have instead considered a cost parameter in front of the quadratic term of (2). As it will become clear when we present the solution, the two parameterizations are equivalent (there is a one to one mapping that yields the same solution). However, this allows us a more general interpretation of the type: the class of low-type executives will include executives that might be very skilled, but are not really committed to the firm (for personal reasons like family obligations or for professional reasons as responsibilities in other firms). The other control,  $\sigma$ , involves the choice of projects the firm will undertake and has no effect on the disutility of the executive, since it does not require any effort, but it affects the expected value of the compensation package: the executive has a menu of projects and decides the level of risk to undertake. The projects are, in principle, comparable in quality since the projects with higher risk also offer a higher expected return. Higher volatility has two conflicting effects on the executive utility. On one hand it increases the value of the payoff through higher stock appreciation. On the other hand, it makes the payoff more volatile and, therefore, less desirable from an utility point of view. Options magnify both effects through their implicit leverage properties.

Our choice of logarithmic utility is justified for tractability purposes. A simplifying consequence of our choice is the fact that the number of options  $n$  becomes irrelevant for incentive purposes, although it is important to determine the total compensation of the executive and whether the compensation satisfies the participation constraint (that we discuss later). Additionally, we assume that the total compensation package consists of only stock or options. As we will see later, however, the intuition of our results seems to be robust to more general types of utility (at least of the CRRA class) and more general compensation packages, at least as long as the equity-based part of the compensation is substantive. In the next section we discuss the solution to this problem.

### 3.3 The Firm With Complete Information

The firm will compensate the executive with stock, or call options on the stock, with maturity  $T$ . The firm will choose both the number of options  $n$  and the strike price  $K$ . A strike price of 0 indicates that the firm is giving stock to the executive. We assume that the firm is risk-neutral, and cares about the final value of the stock, as well as about the value of the compensation. On the other hand, the firm has to guarantee that the utility of the executive is at least as large as a reservation utility  $R$ . This can be interpreted as the utility that the executive would achieve in the best alternative offer from another firm. This restriction amounts to a *participation constraint*, standard in the Principle-Agent literature. Let us define

$$h(K, n) := \lambda E[S_T] - nE[(S_T - K)^+] \quad (3)$$

and

$$A(R) := \left\{ (K, n) \in [0, \infty)^2 : \max_{a, \sigma} E \left[ \log \{n(S_T - K)^+\} - \frac{1}{2} \int_0^T a_t^2 dt \right] \geq R \right\}. \quad (4)$$

The objective of the firm is, then,

$$\max_{(K, n) \in A(R)} h(K, n) \quad (5)$$

where  $\lambda$  is an exogenous constant that represents the relative importance of the expected value of the stock with respect to the compensation package, for the firm. For a given strike price  $K$ , the ratio  $n/\lambda$  would be an indicator of the “option overhang” (or the proportion of the firm granted in options). The time horizon of the firm matches the time horizon of the executive and the maturity of the options. The value  $R$ , as we said above, represents the minimum utility the executive has to be able to achieve through the optimal choice of effort and volatility, in order to work for the firm.

### 3.4 The Firm With Incomplete Information

The setting is as before, but we assume further that the firm does not know the type of the executive. That is, the company does not know the value of  $\delta$ . As we explained above, this is equivalent to the firm not knowing the cost of effort for the executive, which might be a more realistic interpretation in some cases. For example, the executive might have already a reputation (see, for example, Zwiebel 1995 for that type of consideration). However, the firm does not really know the level of commitment of the executive. More formally, we assume that the firm knows that the type of the executive is a discrete random variable  $Y$  that takes values in the set  $\{y_1, y_2, \dots, y_M\}$  and has probability distribution  $\{p_i, i \in \{1, 2, \dots, M\}\}$ . We will assume that the random variable  $Y$  is independent of the Brownian motion process  $W = \{W_t; t \in [0, \infty)\}$ . We will denote by  $\tilde{P}\{B\}$  the probability of an event  $B$  when we model the type of the executive as the random variable  $Y$ . Similarly, we will denote by  $\tilde{E}[X]$  the expected value of a random variable  $X$  when we model the type of the executive as the random variable  $Y$ . Thus,

$$p_i = \tilde{P}\{Y = y_i\}. \quad (6)$$

We define the firm’s objective function,

$$j(K, n) := \lambda \tilde{E}[S_T] - n \tilde{E}[(S_T - K)^+]. \quad (7)$$

The firm still has to satisfy the participation constraint of the executive, that is, the objective of the firm is,

$$\max_{(K, n) \in A(R)} j(K, n). \quad (8)$$



In summary, the firm only knows the probability distribution of the type of the executive. However, in choosing the parameters of the optimal contract (within the class of contracts we consider in this paper) the firm will have to make sure that the compensation satisfies the participation constraint of the executive, according to the terms reported by the executive. As we will show in section 5, depending on the characteristics of firm and executive, it might or might not be optimal for the executive to declare his/her true type.

## 4 Optimal Strategies

In this section we derive the solution to the problems of the executive and firm described in the previous section. As we will show, the solution to the problem of the executive is essentially dynamic, since optimal controls are state contingent (unlike in other papers that consider principal-agent models in a dynamic setting).

### 4.1 Optimal Effort and Volatility

This problem is similar to the problem considered in Cadenillas, Cvitanić and Zapatero (2004). We repeat the results for convenience. We denote the optimal effort  $\hat{a}$  and the optimal choice of volatility  $\hat{\sigma}$  of the executive. First we introduce the following auxiliary exponential martingale  $Z$ ,

$$Z_t = \exp \left\{ -\frac{1}{2}\alpha^2 t - \alpha W_t \right\}, \quad (9)$$

where  $\alpha$  is the parameter in (1) that represents the tradeoff between volatility and expected return of the projects the executive can choose among. Also, consider the following function of time  $\bar{T}$ ,

$$\bar{T}_t = \frac{e^{\alpha^2(T-t)} - 1}{\alpha^2}. \quad (10)$$

Using the previous notation and given the following quadratic equation in  $z$ ,

$$\delta^2 \bar{T}_0 z^2 + (S_0 - K)z - 1 = 0, \quad (11)$$

where  $\delta$  is the parameter that measures the type of the executive and  $K$  is the strike price of the options, we denote by  $\tilde{z}$  the positive solution of (11):

$$\tilde{z} = \frac{1}{2\delta^2 \bar{T}_0} \left( (K - S_0) + \sqrt{(K - S_0)^2 + 4\delta^2 \bar{T}_0} \right). \quad (12)$$

We now find the optimal controls of the executive:

**Proposition 1** Consider the problem of the executive described in Section 2.2. Consider also the exponential martingale of (9),  $\tilde{z}$ , the positive number given by (12), and  $\bar{T}$ , the time function of (10). Assume  $\delta > 0$ . The optimal effort  $\hat{a}$  of the executive is

$$\hat{a}_t = \delta \tilde{z} Z_t. \quad (13)$$

The optimal choice of volatility  $\hat{\sigma}$  is given by

$$\hat{\sigma}_t S_t = \frac{\alpha}{\tilde{z} Z_t} + \alpha \tilde{z} \delta^2 Z_t \bar{T}_t. \quad (14)$$

The optimal effort and volatility determine that the price of the stock be given by the equation

$$S_t = \frac{1}{\tilde{z} Z_t} + K - \tilde{z} \delta^2 Z_t \bar{T}_t. \quad (15)$$

The value for the executive is

$$e = \log(n/\tilde{z}) + \frac{\alpha^2}{2} T - \frac{1}{2} \delta^2 \tilde{z}^2 \bar{T}_0. \quad (16)$$

**Proof:** See the Appendix.  $\diamond$

We observe that the optimal effort and volatility can also be written as functions of the price of the stock. That is,

$$\hat{a}_t = \frac{1}{2\delta^2 \bar{T}_t} \left( (K - S_t) + \sqrt{(K - S_t)^2 + 4\delta^2 \bar{T}_t} \right).$$

and

$$\begin{aligned} \hat{\sigma}_t S_t &= \frac{\alpha \delta}{\hat{a}_t} + \alpha \delta \hat{a}_t \bar{T}_t \\ &= \frac{2\alpha \delta^2 \bar{T}_t}{(K - S_t) + \sqrt{(K - S_t)^2 + 4\delta^2 \bar{T}_t}} + \frac{\alpha}{2} \left[ (K - S_t) + \sqrt{(K - S_t)^2 + 4\delta^2 \bar{T}_t} \right]. \end{aligned}$$

With respect to the optimal effort,  $\hat{a}$  is increasing in the strike price  $K$ : as  $K$  goes to infinity, the effort goes to infinity as well. The reason for that result is that, for a given initial price of the stock, as we increase the strike price, the delta of the option decreases and the implicit leverage in the call increases; this, of course, provides an incentive to the executive to exercise more effort. Obviously, the nature of the utility function (logarithmic, but this property extends to all utilities with CRRA and risk-aversion equal or larger than logarithmic, since they have infinite slope at zero) is such that the executive, who only receives options (or stock) as compensation, is forced to push the stock so that the option will finish in-the-money. However, the relationship between  $\hat{a}$  and the strike price is independent of whether the option is in-the-money or not, and seems to be driven by incentives rather than by the type of utility function. Besides, we note that  $\tilde{z}$  is decreasing in  $T$ , the time to maturity of

the option ( $\bar{T}_0$  is increasing in  $T$  and  $\bar{z}$  is decreasing in  $\bar{T}_0$ ). Therefore, the larger the maturity of the option, the lower the effort of the executive. The intuition is clear: a larger  $T$  has a similar effect on the executive as a lower strike price.

The effect of  $\delta$  (the “type” of executive) depends on whether the option is in-, out-, or at-the-money. When the option is at-the-money, the optimal effort is independent of  $\delta$ , as we can see by substituting (12) in (13). We can also check that when the option is in-the-money the effort is increasing in  $\delta$ , and when the option is out-of-the-money the optimal effort decreases with  $\delta$ . The fact that the relationship changes at-the-money is due to the logarithmic utility function and the absence of cash compensation. However, the intuition of this result is useful to understand the main conclusion of the paper (which does not appear to be driven by this result). CRRA forces the executive to put the option in-the-money: when the option is out-of-the-money, low-type executives apply more effort in order to push the price of the stock upwards; high-type executives prefer to chose higher volatility, that guarantees a higher expected return; the problem with choosing higher volatility for low-type executives is that they will have to apply even higher effort later on, if the higher volatility leads to lower prices of the stock; since high-type executives are more efficient at affecting the price of the stock, they accept this possibility. When options are in-the-money, the main risk (finishing out-of-the-money) is greatly lowered, and then, the higher the marginal productivity of effort, the higher the effort exercised by the executive.

Since  $Z$  is a martingale, the expected value of the effort at any point in time is,

$$E[\hat{a}_t] = \delta \bar{z}. \quad (17)$$

With respect to the effect of  $\alpha$ , we note that  $\bar{T}_0$  is increasing in  $\alpha$  and, therefore,  $\bar{z}$  is decreasing in  $\alpha$ . Expected effort is, then, decreasing in  $\alpha$  (everything else constant): the better the menu of projects the executive can choose among, the lower the expected effort of the executive.

The analysis of the optimal volatility is more complicated. Since  $\bar{T}_T = 0$ , the second term of (14) decreases in expected value as we approach maturity, and will tend to be negligible relative to the first term. Therefore, for short maturities, optimal volatility will tend to decrease with higher strike price. For maturities long enough, the relation will tend to be the opposite. We also see that the volatility is increasing in the type of executive  $\delta$  ( $\bar{z}$  is decreasing in  $\delta$ , and  $\bar{z}\delta^2$  is increasing in  $\delta$ ). The economic intuition is the same we presented before: a high-type executive can afford more volatility because if the price of the stock drops fast, the high-type is more effective applying effort in order to counteract the drop in the value. It is straightforward to see that the expected value of the volatility at a future date  $t$  is

$$E[\hat{\sigma}_t S_t] = \frac{\alpha}{\bar{z}} e^{\alpha^2 t} + \alpha \bar{z} \delta^2 \bar{T}_t. \quad (18)$$

Since  $\bar{T}_t$  is increasing in  $\alpha$ , the expected volatility is increasing in  $\alpha$ . In other words, the higher the expected return-risk tradeoff, the higher the risk the executive will be willing to undertake, on average.

It is also interesting to study the correlation between optimal effort and optimal volatility. By Ito's lemma, and equation (14), the dynamics of the optimal volatility are

$$d(\hat{\sigma}_t S_t) = (\cdot)dt + \alpha^2 \left( \frac{1}{\tilde{z} Z_t} - \delta^2 \tilde{z} Z_t \bar{T}_t \right) dW_t. \quad (19)$$

Equation (13) yields,

$$d\hat{a}_t = (\cdot)dt - \alpha \hat{a}_t dW_t. \quad (20)$$

It is clear that their correlation can be either positive or negative. Considering only the instantaneous correlation and ignoring the drift terms, we see that for a short maturity of the option they tend to be negatively correlated, and increases in the optimal effort will be typically associated with decreases of the optimal level of volatility.

## 4.2 Optimal Strike Price and Number of Options with Complete Information

We assume that the firm has full information about the parameters that characterize the dynamics of the stock, as well as the preferences of the executive. The objective of the firm is given by (5).

From equation (15), we obtain

$$\begin{aligned} h(K, n) &= \lambda E[S_T] - nE[(S_T - K)^+] \\ &= \lambda \left( g(K)e^{\alpha^2 T} + K \right) - ng(K)e^{\alpha^2 T}, \end{aligned} \quad (21)$$

where

$$\begin{aligned} g(K) &= \frac{1}{\tilde{z}} \\ &= \frac{2\delta^2 \bar{T}_0}{(K - S_0) + \sqrt{(K - S_0)^2 + 4\delta^2 \bar{T}_0}}. \end{aligned} \quad (22)$$

We see that the optimal  $n$  is the smallest  $n$  that we can take, namely the smallest  $n$  such that the participation constraint of the executive is satisfied. As shown in the Appendix, the participation constraint (4) yields,

$$\frac{1}{2}\delta^2 \bar{T}_0 \tilde{z}^2 + \log \tilde{z} - \log n - \frac{\alpha^2}{2}T + R = 0. \quad (23)$$

From this it follows that the firm will take the value of  $n$  equal to

$$n(K) = f(K) \exp \left\{ R - \alpha^2 T / 2 + \delta^2 \bar{T}_0 f(K)^2 / 2 \right\}. \quad (24)$$

Here,

$$f(K) = \frac{1}{g(K)} = \frac{1}{2\delta^2 \bar{T}_0} \left( (K - S_0) + \sqrt{(K - S_0)^2 + 4\delta^2 \bar{T}_0} \right).$$

Substituting back in  $h$ , we get the objective function of the firm as the function of one argument only:

$$\psi(K) = \lambda g(K)e^{\alpha^2 T} + \lambda K - e^{\alpha^2 T/2} \exp \left\{ R + \delta^2 \bar{T}_0 f(K)^2 / 2 \right\}. \quad (25)$$

We now state the result about the optimal strike price and the number of options for the firm.

**Proposition 2** *Consider the firm whose objective is given by (5). The optimal strike price  $\hat{K}$  is the value of  $K$  that maximizes (25). Then, the optimal number of options is given by  $n(\hat{K})$ . The value for the firm is  $\psi(\hat{K})$ .*

**Proof:** See the Appendix.

◇

### 4.3 Optimal Strike Price and Number of Options with Partial Information

We assume that the firm has full information about all the parameters that characterize the dynamics of the stock, except for the parameter  $\delta$ . Instead, the firm only knows that the type of the executive is a discrete random variable  $Y$  that takes values in the set  $\{y_1, y_2, \dots, y_M\}$  and has probability distribution  $\{p_i, i \in \{1, 2, \dots, M\}\}$  given by

$$p_i = \tilde{P}\{Y = y_i\}. \quad (26)$$

We assume that the firm knows the utility function of the executive. The objective of the firm is given by (8). From equation (15), we obtain

$$\begin{aligned} j(K, n) &= \lambda \tilde{E}[S_T] - n \tilde{E}[(S_T - K)^+] \\ &= \lambda \left( \tilde{g}(K)e^{\alpha^2 T} + K \right) - n \tilde{g}(K)e^{\alpha^2 T}, \end{aligned} \quad (27)$$

where

$$\begin{aligned} \tilde{g}(K) &= \tilde{E} \left[ \frac{1}{\tilde{z}} \right] \\ &= \tilde{E} \left[ \frac{2Y^2 \bar{T}_0}{\left( (K - S_0) + \sqrt{(K - S_0)^2 + 4Y^2 \bar{T}_0} \right)} \right] \\ &= \sum_{i=1}^M \frac{2y_i^2 \bar{T}_0}{\left( (K - S_0) + \sqrt{(K - S_0)^2 + 4y_i^2 \bar{T}_0} \right)} p_i \end{aligned} \quad (28)$$

As in the case of complete information, the optimal  $n$  is the smallest  $n$  that we can take, namely the smallest  $n$  such that the participation constraint of the executive is satisfied. We show in the Appendix that the participation constraint (4) yields

$$\frac{1}{2} \delta^2 \bar{T}_0 \tilde{z}^2 + \log \tilde{z} - \log n - \frac{\alpha^2}{2} T + R = 0,$$

and this implies that the firm will take the value of  $n$  equal to

$$n(K) = f(K) \exp \left\{ R - \alpha^2 T / 2 + \delta^2 \bar{T}_0 f(K)^2 / 2 \right\}. \quad (29)$$

This coincides with (24). Substituting back in  $j$ , we get the objective function of the firm as the function of one argument only:

$$\tilde{\psi}(K) = \lambda \tilde{g}(K) e^{\alpha^2 T} + \lambda K - e^{\alpha^2 T / 2} \tilde{g}(K) f(K) \exp \left\{ R + \delta^2 \bar{T}_0 f(K)^2 / 2 \right\}. \quad (30)$$

We now state the result about the optimal strike price and the number of options for the firm.

**Proposition 3** *Consider the firm whose objective is given by (8). The optimal strike price  $\hat{K}$  is the value of  $K$  that maximizes (30). Then, the optimal number of options is given by  $n(\hat{K})$ . The value for the firm is  $\tilde{\psi}(\hat{K})$ .*

**Proof:** See the Appendix.

◇

In the next section we perform some numerical exercises and discuss the economic implications of our model.

## 5 Numerical Results and Analysis

### 5.1 Computation

In the previous section, we discussed the optimal effort and volatility strategy of the executive, as well as the optimal contract offered by the firm. In the case of incomplete information, we take as given the probability distribution of types. Here we are going to compute some numerical examples and study their economic implications. In order to have a tractable setting that will allow us to derive numerical examples, we make the following assumptions, in addition to the assumptions discussed in the previous sections.

First, we consider only two types of executives, that we call “high” and “low,” endowed with different values for  $\delta$ , that we denote by  $\delta_H$  and  $\delta_L$ , with  $\delta_H > \delta_L$ . They might also have different reservation utility values, that we denote, respectively, by  $R_H$  and  $R_L$ . The firm does not know if the parameter that characterizes the type of executive is  $\delta_H$  or  $\delta_L$ , but it knows what these two values are and their probabilities. We assume that the firm knows the reservation utility associated to each type: that is, the firm knows that an executive with a  $\delta_i, i = H, L$ , can command a minimum expected utility  $R_i, i = H, L$ . Finally, we assume that there is no penalty for lying: that is, executives will declare their types (and, therefore, the reservation utility the contract offered by the firm has to provide them); after the company offers a contract and the executive accepts, the executive will choose the optimal level of effort and

volatility for that contract, independently of the type initially declared; however, when executives are indifferent between declaring their true type and lying (that is, in both cases they achieve the same  $R$ ), they will declare the truth. This guarantees a unique equilibrium.

A specific case we are interested in is the following: suppose that high and low types have different reservation utilities, in particular  $R_H > R_L$ . It might be the case that an executive of a given type (more likely the low type) declares high type so as to receive a contract that might be worth more than the reservation utility corresponding to the true type. As described in section 2, we will distinguish between a pooling equilibrium as the setting in which the optimal contract offered by the firm is such that both types have utility incentives to declare the same type, and a separating equilibrium, that arises when the optimal contract offered by the firm is such that both types have utility incentives to declare their true type.

We now characterize each type of equilibrium from a technical point of view. This will allow us to perform some numerical exercises and derive conclusions about the use of options for screening purposes (that is, in the case of separating equilibrium). We extend the notation we have used in the previous two sections in the following way:  $(K(R_i), n(R_i))$  denotes a contract in which  $n(R_i)$  has been computed according to equation (24), so that the contract satisfies the participation constraint of an agent of type  $i$ ;  $(\hat{K}^I(R_i), \hat{n}^I(R_i))$  is the optimal contract offered by the firm with incomplete information to the executive who declares type  $i$ , that is, the contract that maximizes (30) for a reservation expected utility  $R_i$ ;  $(\hat{K}^C(R_i), \hat{n}^C(R_i))$  is the optimal contract offered by the firm with perfect information to an executive of type  $i$ , that is, the contract that maximizes (25) for a reservation expected utility  $R_i$ ;  $e^i(K, n)$  is the expected utility of the executive who receives a contract of  $n$  securities with strike price  $K$ ; we denote by  $\psi^C(K, n, R_i)$  the expected utility for the firm with perfect information that offers a contract  $(K, n)$  to an executive of type  $i$ ; finally, we denote by  $\psi^I(K, n, R_i)$  the expected utility for the firm with incomplete information that offers a contract  $(K, n)$  to an executive who has declared type  $i$ .

**Remark 1** *Consider the model described in this paper, specialized to the two types discussed in this section. From our definition of pooling and separating equilibria, we get the following characterization. A contract  $(\hat{K}^I(R_i), \hat{n}^I(R_i))$  for  $i = H$  or  $i = L$  is a pooling equilibrium if and only if it induces type  $j \neq i$  to lie, that is,  $e^j(\hat{K}^I(R_i), \hat{n}^I(R_i)) > R_j$ , and none of the following two types of contract exist:*

1. A contract  $(\hat{K}^I(R_j), \hat{n}^I(R_j))$  such that:
  - (a)  $e^i(\hat{K}^I(R_j), \hat{n}^I(R_j)) > R_i$ , that is, it induces type  $i$  to lie and,
  - (b)  $\psi^I(\hat{K}^I(R_j), \hat{n}^I(R_j)) > \psi^I(\hat{K}^I(R_i), \hat{n}^I(R_i))$ , that is, it is better for the firm.
2. A contract  $(K(R_l), n(R_l))$ ,  $l = H$  or  $l = L$ , such that:

(a)  $e^k(K(R_l), n(R_l)) \leq R_k, k \neq l$ , so that it induces both types to tell the truth, and

(b)  $\psi^C(K(R_l), n(R_l)) > \psi^I(\hat{K}^I(R_i), \hat{n}^I(R_i))$ , so that it is better for the firm.

A contract  $(K(R_i), n(R_i))$  for  $i = H$  or  $i = L$  is a separating equilibrium if and only if it induces the other type  $j \neq i$  to tell the truth, that is,  $e^j(K(R_i), n(R_i)) \leq R_j$ , and:

1. There is no other contract  $(K'(R_l), n'(R_l))$ , for  $l = H$  or  $l = L$  which would make both types to tell the truth and be better for the firm, that is, such that the following two conditions hold:

(a)  $e^k(K'(R_l), n'(R_l)) \leq R_k, k \neq l$

(b)  $\psi^C(K'(R_l), n'(R_l)) > \psi^C(K(R_i), n(R_i))$

2. There is no contract  $(\hat{K}^I(R_l), \hat{n}^I(R_l))$ , for  $l = H$  or  $l = L$  (so that  $i = l$  is possible), which induces type  $k \neq l$  to lie, and it is better for the firm, that is, such that the following two conditions hold:

(a)  $e^i(\hat{K}^I(R_l), \hat{n}^I(R_l)) > R_k$ ,

(b)  $\psi^I(\hat{K}^I(R_l), \hat{n}^I(R_l)) > \psi^C(K^C(R_i), n(R_i))$ .

We now sketch the algorithm for the derivation of the equilibrium contract. The idea is the following: we compute the values of the objective function for both potential equilibria with incomplete information (for  $R_H$  and  $R_L$ ). We verify if they are actually pooling (executives have incentives to lie). Then we choose the highest pooling (potential) equilibrium and look for all possible separating equilibria with a value of the objective function at least as high as that of the candidate pooling equilibrium. A preliminary step that might save time is to check if the best possible equilibria with perfect information is separating. We present and analyze several examples in the next subsection. We consider the same assumptions, and use the same notation as in Remark 1. Additionally, we denote by  $\mathcal{A}(R_i, M)$  the set of contracts that would make type  $j \neq i$  declare the true type, and would have a value for the firm with complete information no smaller than  $M$ , that is,

$$\mathcal{A}(R_i, M) = \{(K(R_i), n(R_i)), e^j(K(R_i), n(R_i)) \leq R_j, \psi^C(K(R_i), n(R_i)) \geq M\}.$$

We denote  $\mathcal{A}(R_{HL}, M) = \mathcal{A}(R_H, M) \cup \mathcal{A}(R_L, M)$ . The algorithm is the following:

1. We compute  $(\hat{K}^C(R_i), \hat{n}^C(R_i))$  and  $\psi^C(\hat{K}^C(R_i), \hat{n}^C(R_i))$  for both  $i = H$  and  $i = L$ .
2. We compare  $\psi^C(\hat{K}^C(R_H), \hat{n}^C(R_H))$  and  $\psi^C(\hat{K}^C(R_L), \hat{n}^C(R_L))$  and take the higher of the two, say  $\psi^C(\hat{K}^C(R_k), \hat{n}^C(R_k))$ .



3. If  $e^l(\hat{K}^C(R_k), \hat{n}^C(R_k)) \leq R_l, l \neq k$ , we have that  $(\hat{K}^C(R_k), \hat{n}^C(R_k))$  is a separating equilibrium, and we stop there.
4. Otherwise, we compute  $e^k(\hat{K}^C(R_l), \hat{n}^C(R_l))$  and compare with  $R_k$ .
5. If  $e^k(\hat{K}^C(R_l), \hat{n}^C(R_l)) \leq R_k$ , this is a candidate for a separating equilibria; in this case, let us denote  $\psi^C(\hat{K}^C(R_l), \hat{n}^C(R_l)) = \bar{\psi}^C$ , and we say that “we got a  $\bar{\psi}^C$ .”
6. Next, regardless of whether  $e^k(\hat{K}^C(R_l), \hat{n}^C(R_l)) \leq R_k$ , that is, regardless of whether we got a  $\bar{\psi}^C$  or not, we compute  $(\hat{K}^I(R_k), \hat{n}^I(R_k)), \psi^I(\hat{K}^I(R_k), \hat{n}^I(R_k))$  and  $e^l(\hat{K}^I(R_k), \hat{n}^I(R_k))$  (since  $e^l(\hat{K}^C(R_k), \hat{n}^C(R_k)) > R_l$  from step 3, we now want to check if type  $l$  has an incentive to declare type  $k$  under the optimal contract with incomplete information). If  $e^l(\hat{K}^I(R_k), \hat{n}^I(R_k)) \geq R_l$  we say that “we got a  $\bar{\psi}_k^I$ .” We note that if we didn’t get a  $\bar{\psi}_k^I$ , we have that  $(\hat{K}^I(R_k), \hat{n}^I(R_k)) \in \mathcal{A}(R_k, 0)$ .
7. If we did not get a  $\bar{\psi}^C$ , that is,  $e^k(\hat{K}^C(R_l), \hat{n}^C(R_l)) > R_k$  from step 5, we also compute  $(\hat{K}^I(R_l), \hat{n}^I(R_l)), \psi^I(\hat{K}^I(R_l), \hat{n}^I(R_l))$  and  $e^l(\hat{K}^I(R_l), \hat{n}^I(R_l))$  (we want to check whether type  $k$  has an incentive to declare type  $l$  when the firm optimizes under incomplete information.) If  $e^k(\hat{K}^I(R_l), \hat{n}^I(R_l)) \geq R_k$  we say that “we got a  $\bar{\psi}_l^I$ .” We note that if we didn’t get a  $\bar{\psi}_l^I$ , we have that  $(\hat{K}^I(R_l), \hat{n}^I(R_l)) \in \mathcal{A}(R_l, 0)$ .
8. We now take  $\max\{\bar{\psi}^C, \bar{\psi}_k^I, \bar{\psi}_l^I\} = \bar{\psi}$ . If we did not get  $\bar{\psi}^C, \bar{\psi}_k^I$  and  $\bar{\psi}_l^I$ , we make  $\bar{\psi} = 0$ .
9. Next we find  $\mathcal{A}(R_H, \bar{\psi})$  and  $\mathcal{A}(R_L, \bar{\psi})$ ; if these sets are both empty, our equilibrium is the contract that yields  $\bar{\psi}$ , which can be pooling or separating. Additionally, it is clear from the comment at the end of steps 6 and 7 that, if  $\bar{\psi} = 0$ , the sets  $\mathcal{A}(R_H, \bar{\psi})$  and  $\mathcal{A}(R_L, \bar{\psi})$  are not empty.
10. If the sets  $\mathcal{A}(R_H, \bar{\psi})$  and  $\mathcal{A}(R_L, \bar{\psi})$  are not empty, they are possibly non-finite, so we characterize them approximately by plotting the value function of the firm; the fact that there are several local maxima makes this task easier than it would otherwise appear. Figure 1 is an example of a plot of the objective function of the firm, and illustrates this point.
11. Next, we compute

$$\max_{\{(\bar{K}, \bar{n}) \in \mathcal{A}(R_{HL}, \bar{\psi})\}} \psi^C(\bar{K}, \bar{n}) = \bar{\psi}'.$$

This is a separating equilibrium.

## 5.2 Numerical Examples

In this section we want to study the economic properties of the optimal strategy of the firm (strike price and number of securities) when the type of the executive is unknown. Our solutions are quasi-analytic, so we need to perform some numerical exercises for our analysis.

First, to gain some intuition, we compute optimal parameters for the case of perfect information. We report the results in Table 1. We point out that, due to the curvature properties of the value function, the optimal strike price is not continuous in the values of the model parameters. For example, the optimal strike price decreases as  $\alpha$  increases and, for some threshold value of  $\alpha$  (that will depend on the other parameter values) it jumps to zero, so that the optimal contract consists of stock for that and higher value of  $\alpha$ . In summary, we observe that the strike price (and therefore the delta or implicit leverage of the option) increases with the type of the executive  $\delta$  and with  $\lambda$  (that we interpret as the size of the firm), and it decreases with the trade-off between risk and volatility,  $\alpha$ . The intuition is the following: the firm is risk-neutral and would like the executive to be aggressive and choose a high level of volatility, because it offers a high expected return. The higher the  $\alpha$ , the better a high choice of volatility for the firm. Options grants provide executives with a positive incentive (both through higher expected return and through the convexity effect pointed by Ross, 1973, 2004) and a negative incentive through the increase in risk due to the implicit leverage in options. For executives with high  $\delta$  (high type) the positive effect outweighs the risk. The reason is that in bad states executives will have to put more effort to drive up the price of the stock, but for executives with high  $\delta$  the cost of that potential extra effort is acceptable. However, for bad executives, the cost of that potential extra effort is too high, and the optimal contract will be options with lower strike price or stock.

In the remaining tables we compute the optimal contract for the case in which the firm does not know the type ( $\delta$ ) of the executive. We focus on the two-type case discussed in the previous section,  $(\delta_H, \delta_L)$  with  $\delta_H > \delta_L$ , and compute the resulting equilibrium. We assume that both types have the same probability,  $p^H = p^L = 0.5$ .

In order to find the optimal contract (within the set of contracts considered in our model), we have to find numerically the strike price  $K$  that maximizes the right-hand side of equation (30). The number of options needed to satisfy the participation constraint of the executive (that is, to clear the reservation wage  $R$ ) is given by equation (29). We search for the resulting equilibrium according to Remark 1.

In tables 2 and 3 we perform the following exercise. We assume that the executives have different reservation wages: the low type has a lower reservation wage  $R^L$  than the high type,  $R^H > R^L$ . Furthermore, and for comparison purposes, we assume that the reservation wages are such that, if the firm had full information, it would be indifferent between hiring the high type and the low type. In other words, the value function of the firm (5) is the same for  $(\delta^H, R^H)$  and for  $(\delta^L, R^L)$ . We further assume that the firm knows the reservation wage associated to each type of executive. The

question we address is whether it is optimal for executives to lie about their type or not. In general, it appears that it will not be optimal for the high type to declare low type since the reservation wage is lower; the subjective utility might be higher than the one declared, but it is unlikely (we verify that such is the case for the examples in the tables). However, for the case of the low type the answer is less clear. On one hand the low type will prefer the higher reservation wage of the high type. On the other hand, high types receive contracts that involve more implicit leverage. The difference between tables 2 and 3 is the “size” of the firm, as measured by the parameter  $\lambda$ . It is clear that when the equilibrium is separating, the value function of the firm is the same as in the case of complete information. However, it is lower in the case of pooling equilibrium. We point out that the optimal contract offered by the firm in the case of separating equilibrium always involves choosing the higher strike price (or options over stock, if those are the contracts in the case of perfect information). Options/leverage discourage one of the types.

In tables 4 and 5 we consider the case of a firm that faces uncertainty about the executive type, but would prefer the high-type with perfect information. We record the optimal contracts in case of perfect information and the optimal contract with unknown type: we choose all the examples so that the optimal contract with perfect information would be stock with both types of agents. As in tables 2 and 3, in some cases, the equilibrium is pooling, in which case the firm will have to grant the same number of shares as it would have granted to the high-type with perfect information, which lowers the expected value for the firm, since there is a 50% probability that a low-type executive will accept the contract. However, in many cases, the equilibrium is separating and it can take either of the two following forms: offer a small number of shares so as to attract the low-type for sure (but at the right price) or offer options, which will discourage the low-type and will attract the high-type. We observe that the separating equilibrium takes place when the types of the agents are relatively far apart with respect to the difference in reservation wage.

## 6 Conclusions

We study the use of stock options in compensation packages for screening purposes: we show that a possible role for options is to discourage low-type executives (even if low-type executives are less expensive). Our analysis is based on a standard Principal-Agent setting in which the Principal (the firm who cares about the stock price) is risk-neutral and the Agent (the executive who cares about the compensation package and costly effort to devote to the firm) is risk-averse. Furthermore, the firm does not know the type of the executive (how effective is the executive in affecting the price of the stock). To our knowledge, this is the first time this argument has been suggested as a possible explanation to explain the widespread use of options in compensation packages.

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# A Appendix: Proofs

## A.1 Proof of Proposition 1.

We consider the more general case in which the executive maximizes

$$\max_{a, \sigma} E \left[ F(S_T) - \int_0^T G(a_s) ds \right],$$

where

$$F(s) = \frac{1}{\gamma} [n(s - K)^+]^\gamma, \quad G(a) = \frac{a^2}{2}$$

and  $\gamma < 1$  is the risk-aversion parameter. The log-utility case  $F(x) = \log(x)$  corresponds to  $\gamma = 0$ . We approach this problem by familiar duality/martingale techniques, as introduced by Cox and Huang (1989), Karatzas, Lehoczky and Shreve (1987). Consider the dual function

$$\tilde{F}(z) = \max_{s \geq 0} [F(s) - sz].$$

The maximum is attained at the points of the form

$$\hat{s} = \hat{s}(z, b) = \left( \left( \frac{z}{n^\gamma} \right)^{\frac{1}{\gamma-1}} + K \right) \mathbf{1}_{\left\{ \left( \frac{z}{n^\gamma} \right)^{\frac{1}{\gamma-1}} > \frac{K\gamma}{1-\gamma} \right\}} + b \mathbf{1}_{\left\{ \left( \frac{z}{n^\gamma} \right)^{\frac{1}{\gamma-1}} = \frac{K\gamma}{1-\gamma} \right\}},$$

where  $b$  is either 0 or  $\left( \frac{z}{n^\gamma} \right)^{\frac{1}{\gamma-1}} + K$ . Consider also the dual function

$$\tilde{G}(z) = \max_a [-G(a) + \delta az],$$

where the maximum is attained at

$$\hat{a} = \hat{a}(z) = \delta z.$$

Define the stochastic process

$$M_t = Z_t S_t - \delta \int_0^t Z_s a_s ds. \tag{31}$$

where  $Z$  is the exponential martingale defined in (9). Applying Ito's rule, we get

$$dM_t = (\sigma_t - \alpha) S_t Z_t dW_t \quad \text{and} \quad M_0 = S_0. \tag{32}$$

Obviously,  $M$  is a local martingale, but we would like to prove that  $M$  is also a martingale. For that purpose, it is good enough to verify the condition

$$E \left[ \sup_{0 \leq t \leq T} |M_t| \right] < \infty.$$

According to the Burkholder-Davis-Gundy inequality (see, for instance, Theorem 3.3.28 of Karatzas and Shreve (1991)), it is enough to check that

$$E \left[ \left( \int_0^T (\sigma_t - \alpha)^2 S_t^2 Z_t^2 dt \right)^{1/2} \right] < \infty.$$

We observe that, according to Theorem 6.1.6 of Yong and Zhou (1999),  $E \left[ \sup_{0 \leq t \leq T} Z_t^2 \right] < \infty$ . Since  $E \left[ \int_0^T |\sigma_t S_t|^2 dt \right] < \infty$ , that theorem applied to equation (1) gives  $E \left[ \sup_{0 \leq t \leq T} S_t^2 \right] < \infty$ . Applying Hölder's inequality (see, for instance, Theorem 4.2 of Chow and Teicher (1988)) and again the condition  $E \left[ \int_0^T |\sigma_t S_t|^2 dt \right] < \infty$ , we note that

$$\begin{aligned} E \left[ \left( \int_0^T (\sigma_t S_t Z_t)^2 dt \right)^{1/2} \right] &\leq E \left[ \left( \sup_{0 \leq t \leq T} Z_t^2 \int_0^T (\sigma_t S_t)^2 dt \right)^{1/2} \right] \\ &= E \left[ \left( \sup_{0 \leq t \leq T} Z_t^2 \right)^{1/2} \left( \int_0^T (\sigma_t S_t)^2 dt \right)^{1/2} \right] \\ &\leq \left( E \left[ \sup_{0 \leq t \leq T} Z_t^2 \right] \right)^{1/2} \left( E \left[ \int_0^T (\sigma_t S_t)^2 dt \right] \right)^{1/2} \\ &< \infty. \end{aligned}$$

This implies that

$$E \left[ \left( \int_0^T ((\sigma_t - \alpha) S_t Z_t)^2 dt \right)^{1/2} \right] < \infty,$$

and therefore that  $M$  is a martingale. Thus,

$$E[M_T] = S_0.$$

By definitions, we get

$$E \left[ F(S_T) - \int_0^T G(a_s) ds \right] \leq E \left[ \tilde{F}(z Z_T) + \int_0^T \tilde{G}(z Z_s) ds \right] + z E[M_T],$$

where we can replace  $E[M_T]$  by  $S_0$ . Therefore, the above inequality gives an upper bound for our maximization problem. The upper bound will be attained if the maximums are attained, and if  $E[M_T] = S(0)$ . In other words, the optimal terminal stock price and the optimal effort  $\hat{a}$  are given by

$$S_T = \hat{s}(\tilde{z} Z_T, B) \quad \text{and} \quad \hat{a}_t = \delta \tilde{z} Z_t, \quad (33)$$

where  $B$  and  $\tilde{z}$  are chosen so that  $B$  is any  $\mathcal{F}_T$  measurable random variable taking only two possible values, 0 and  $(\frac{\tilde{z} Z_T}{n\gamma})^{\frac{1}{\gamma-1}} + K$ , and so that  $E[M_T] = S(0)$ .

For  $\gamma = 0$ , we can choose  $B \equiv 0$ , and we see that

$$S_T = \frac{1}{\tilde{z}Z_T} + K.$$

Using this and the martingale property of  $M$ , we get

$$Z_t S_t = E \left[ \frac{1}{\tilde{z}} + K Z_T - \tilde{z} \delta^2 \int_t^T Z_s^2 ds \mid \mathcal{F}_t \right] = \frac{1}{\tilde{z}} + K Z_t - \tilde{z} \delta^2 Z_t^2 \bar{T}_t, \quad (34)$$

in the notation of (10). In other words,

$$S_t = \frac{1}{\tilde{z}Z_t} + K - \tilde{z} \delta^2 Z_t \bar{T}_t. \quad (35)$$

Using Ito's rule we see that the diffusion term of  $S$  is given by

$$\hat{\sigma}_t S_t = \frac{\alpha}{\tilde{z}Z_t} + \alpha \tilde{z} \delta^2 Z_t \bar{T}_t,$$

as claimed in (14). We observe that the  $\hat{a}$  and  $\hat{\sigma}$  defined above are adapted stochastic processes with  $E[\int_0^T |\hat{a}_t|^2 dt] < \infty$  and  $E[\int_0^T |\hat{\sigma}_t S_t|^2 dt] < \infty$ . Finally, the requirement  $E[M_T] = S_0$ , obtained by setting  $t = 0$  in (35), gives

$$S_0 = \frac{1}{\tilde{z}} + K - \tilde{z} \delta^2 \bar{T}_0. \quad (36)$$

This is equivalent to (11), and we are done.

◇

## A.2 Proof of Proposition 2

Our first objective is to compute the objective function of the firm

$$h(K, n) := \lambda E[S_T] - n E[(S_T - K)^+], \quad (37)$$

and

$$e = e(n, K) := \max_{a, \sigma} E \left[ \log \{n(S_T - K)^+\} - \frac{1}{2} \int_0^T a_t^2 dt \right].$$

It is easily seen that

$$E[Z^2(t)] = e^{\alpha^2 t}, \quad E[Z(t)] = 1, \quad E[Z^{-1}(t)] = e^{\alpha^2 t}. \quad (38)$$

Thus, according to equation (15),

$$E[S_t] = \frac{1}{\tilde{z}} e^{\alpha^2 t} + K - \tilde{z} \delta^2 \bar{T}_t.$$



We also see that  $E[(S_T - K)^+] = e^{\alpha^2 T} / \check{z}$ , and, using (12), we verify that the value  $h(K, n)$  of (37) is equal to the value  $h(K, n)$  of (21).

Finally, using  $\hat{a} = \delta \check{z} Z$  and (38), we can compute

$$e = \log(n/z) + \frac{\alpha^2}{2} T - \frac{1}{2} \delta^2 \check{z}^2 \bar{T}_0 \quad ,$$

in terms of  $z$  and  $n$ . Here,  $\check{z}$  is given in equation (12). We can check that  $e(n(K), K) = R$ , with  $n$  given in (24).

Now, the firm wishes to maximize the function  $h$  as a function of  $K$ , so that the strike price is non-negative and the executive's rationality constraint is satisfied.

◇

### A.3 Proof of Proposition 3

Our first objective is to compute the objective function of the firm

$$j(K, n) := \lambda \tilde{E}[S_T] - n \tilde{E}[(S_T - K)^+] \quad , \quad (39)$$

and

$$e = e(n, K) := \max_{a, \sigma} E \left[ \log \{ n(S_T - K)^+ \} - \frac{1}{2} \int_0^T a_t^2 dt \right] \quad .$$

It is easily seen that

$$\tilde{E}[Z^2(t)] = E[Z^2(t)] = e^{\alpha^2 t}, \quad \tilde{E}[Z(t)] = 1, \quad \tilde{E}[Z^{-1}(t)] = E[Z^{-1}(t)] = e^{\alpha^2 t} \quad . \quad (40)$$

Thus, according to equation (15),

$$\tilde{E}[S_t] = \tilde{E} \left[ \frac{1}{\xi(Y)} \right] e^{\alpha^2 t} + K - \tilde{E} [\xi(Y) Y^2] \bar{T}_t \quad ,$$

where

$$\xi(Y) := \frac{1}{2Y^2 \bar{T}_0} \left( (K - S_0) + \sqrt{(K - S_0)^2 + 4Y^2 \bar{T}_0} \right) \quad .$$

We also see that  $\tilde{E}[(S_T - K)^+] = e^{\alpha^2 T} \tilde{E} \left[ \frac{1}{\xi(Y)} \right]$ , and, using (28), we verify that the value  $j(K, n)$  of (39) is equal to the value  $j(K, n)$  of (27). Finally, using  $\hat{a} = \delta \check{z} Z$  and (12), we can compute

$$e = \log(n/\check{z}) + \frac{\alpha^2}{2} T - \frac{1}{2} \delta^2 \check{z}^2 \bar{T}_0 \quad ,$$

in terms of  $\check{z}$  and  $n$ . Here,  $\check{z}$  is given in equation (12). We can check that  $e(n(K), K) = R$ , with  $n$  given in (29).

Now, the firm wishes to maximize the function  $j$  as a function of  $K$ , so that the strike price is non-negative and the executive's rationality constraint is satisfied.

◇

**Table 1**  
**Optimal strike price with perfect information**

The column  $\hat{K}$  measures the optimal strike price at the initial time for a fixed initial stock price of  $S_0 = 100$  and a fixed horizon  $T = 5$ .  $\hat{K} = 0$  means that the optimal contract consists of stock. The column  $\hat{n}$  represents the optimal number of call options or shares of stock to offer to the manager as compensation, at the initial time, and for the same initial stock price and time horizon. In this table,  $\lambda$  represents the parameter that measures the relative importance of the expected price of the stock with respect to the value of the compensation package,  $\alpha$  is the parameter that measures the additional expected return resulting from an additional unit of volatility,  $\delta$  is the type of the manager, and  $R$  is the reservation compensation of the manager.

	$\alpha = 0.1$				$\alpha = 0.2$			
	$\delta$	$R$	$\hat{K}$	$\hat{n}$	$\delta$	$R$	$\hat{K}$	$\hat{n}$
$\lambda = 100$	1	1	0	0.0265	1	1	0	0.0246
	1	2	0	0.072	1	2	0	0.0668
	1	5	0	1.447	1	5	0	1.3425
	2	1	111.279	94.519	2	1	0	0.0246
	2	2	109.584	94.351	2	2	0	0.0668
	2	5	0	1.446	2	5	0	1.3414
	5	1	131.682	94.622	5	1	132.712	79.99
	5	2	127.862	94.508	5	2	128.702	79.557
	5	5	112.676	93.447	5	5	0	75.301
$\lambda = 1000$	1	1	106.758	946.793	1	1	0	0.0246
	1	2	106.825	945.912	1	2	0	0.0668
	1	5	0	1.447	1	5	0	1.3425
	2	1	114.571	947.255	2	1	0	0.0246
	2	2	113.224	946.554	2	2	0	0.0668
	2	5	108.259	941.724	2	5	0	1.3414
	5	1	139.294	947.739	5	1	140.673	805.674
	5	2	136.155	947.204	5	2	137.393	803.652
	5	5	124.934	943.952	5	5	125.619	791.23

**Table 2**

**Optimal contract with unknown type for a “small” firm, that would be indifferent between types with perfect information**

The column “Type” indicates whether the executive is of known type “low” (L), known type “high” (H) or unknown type (U), which means the firm thinks it is type L with probability 50% and type H with probability 50%. Columns  $\delta^i$ , which measures the impact of the effort on the expected return, and  $R^i$ , which represents the reservation wage of each executive, characterize the particular type. The column  $V^i$  records the value of the objective of the firm for that case and optimal contract. For type H,  $R^i$  is calibrated so that  $V^H = V^L$ . The column labeled “equilibrium” denotes whether the resulting equilibrium is “separating” (S) or “pooling” (P). We assume the initial stock price to be  $S_0 = 100$  and a fixed horizon  $T = 5$ .  $K = 0$  means that the optimal contract consists of stock. We also assume that  $\lambda$  (the parameter that measures the relative importance of the expected price of the stock with respect to the value of the compensation package) has a value of 100.  $\alpha$  is the parameter that measures the additional expected return resulting from an additional unit of volatility.

	Type	$\delta^i$	$R^i$	$\hat{K}^i$	$\hat{n}^i$	$V^i$	Equilibrium
$\alpha = 0.1$	L	1	2	0	0.0720	10510.5	
	H	2	4.1403	104.918	93.3967	10510.5	
	U	-	-	104.918	93.3967	10510.5	S
	L	1.5	1	108.109	94.4785	10818.1	
	H	2	2.798	108.055	94.1402	10818.1	
	U	-	-	108.109	94.4785	10818.1	S
$\alpha = 0.2$	L	1.5	1	0	0.024581	12226.2	
	H	2	2.595	0	0.121084	12226.2	
	U	-	-	0	0.121084	12220.3	P
	L	4	1	125.482	79.9078	12624.3	
	H	5	2.8195	125.05	79.0309	12624.3	
	U	-	-	126.02	97.5789	12613	P

**Table 3**

**Optimal contract with unknown type for a “large” firm, that would be indifferent between types with perfect information**

The column “Type” indicates whether the executive is of known type “low” (L), known type “high” (H) or unknown type (U), which means the firm thinks it is type L with probability 50% and type H with probability 50%. Columns  $\delta^i$ , which measures the impact of the effort on the expected return, and  $R^i$ , which represents the reservation wage of each executive, characterize the particular type. The column  $V^i$  records the value of the objective of the firm for that case and optimal contract. For type H,  $R^i$  is calibrated so that  $V^H = V^L$ . The column labeled “equilibrium” denotes whether the resulting equilibrium is “separating” (S) or “pooling” (P). We assume the initial stock price to be  $S_0 = 100$  and a fixed horizon  $T = 5$ .  $K = 0$  means that the optimal contract consists of stock. We also assume that  $\lambda$  (the parameter that measures the relative importance of the expected price of the stock with respect to the value of the compensation package) has a value of 1000.  $\alpha$  is the parameter that measures the additional expected return resulting from an additional unit of volatility.

	Type	$\delta^i$	$R^i$	$\hat{K}^i$	$\hat{n}^i$	$V^i$	Equilibrium
$\alpha = 0.1$	L	1	3.5	0	0.3229	105147	
	H	2	6.427	104.961	934.111	105147	
	U	-	-	104.961	934.111	105147	S
	L	2	6.427	104.961	934.111	105147	
	H	2.5	7.022	104.873	930.095	105147	
	U	-	-	104.961	934.111	105147	S
$\alpha = 0.15$	L	1	2.5	0	0.1151	111954	
	H	2.5	5.716	0	2.8660	111954	
	U	-	-	0	2.8660	111800	P
	L	2	1	114.776	884.899	114945	
	H	3	4.6965	114.565	876.696	114945	
	U	-	-	114.776	884.899	114945	S

**Table 4**

**Optimal contract with unknown type for a “small” firm, that would prefer the high-type with perfect information**

The column “Type” indicates whether the executive is of known type “low” (L), known type “high” (H) or unknown type (U), which means the firm thinks it is type L with probability 50% and type H with probability 50%. Columns  $\delta^i$ , which measures the impact of the effort on the expected return, and  $R^i$ , which represents the reservation wage of each executive, characterize the particular type. The column  $V^i$  records the value of the objective of the firm for that case and optimal contract. The column labeled “equilibrium” denotes whether the resulting equilibrium is “separating” (S) or “pooling” (P). We assume the initial stock price to be  $S_0 = 100$  and a fixed horizon  $T = 5$ .  $K = 0$  means that the optimal contract consists of stock. We also assume that  $\lambda$  (the parameter that measures the relative importance of the expected price of the stock with respect to the value of the compensation package) has a value of 100.  $\alpha$  is the parameter that measures the additional expected return resulting from an additional unit of volatility.

	Type	$\delta^i$	$R^i$	$\hat{K}^i$	$\hat{n}^i$	$V^i$	Equilibrium
$\alpha = 0.15$	L	1	3	0	0.18982	11175.4	
	H	3	4	0	0.51490	11185.9	
	U	-	-	0	0.18982	11175.4	S
	L	2	3.5	0	0.31271	11179.3	
	H	3.4	4	0	0.51455	11200.8	
	U	-	-	111.261	86.567	11186.3	S
$\alpha = 0.25$	L	5	2	0	0.06275	13857.4	
	H	7	3	0	0.16941	14027	
	U	-	-	136.826	69.072	13962.4	S
	L	4	1.5	0	0.03816	13790.3	
	H	7	4	0	0.46052	13986.1	
	U	-	-	0	0.46052	13859.1	P

**Table 5**

**Optimal contract with unknown type for a “large” firm, that would prefer the high-type with perfect information**

The column “Type” indicates whether the executive is of known type “low” (L), known type “high” (H) or unknown type (U), which means the firm thinks it is type L with probability 50% and type H with probability 50%. Columns  $\delta^i$ , which measures the impact of the effort on the expected return, and  $R^i$ , which represents the reservation wage of each executive, characterize the particular type. The column  $V^i$  records the value of the objective of the firm for that case and optimal contract. The column labeled “equilibrium” denotes whether the resulting equilibrium is “separating” (S) or “pooling” (P). We assume the initial stock price to be  $S_0 = 100$  and a fixed horizon  $T = 5$ .  $K = 0$  means that the optimal contract consists of stock. We also assume that  $\lambda$  (the parameter that measures the relative importance of the expected price of the stock with respect to the value of the compensation package) has a value of 1000.  $\alpha$  is the parameter that measures the additional expected return resulting from an additional unit of volatility.

	Type	$\delta^i$	$R^i$	$\hat{K}^i$	$\hat{n}^i$	$V^i$	Equilibrium
$\alpha = 0.15$	L	2	4	0	0.51557	112086	
	H	2.5	5	0	1.40064	112119	
	U	-	-	0	0.51557	112086	S
	L	1.5	4	0	0.51581	111983	
	H	2.51	4.7	0	1.03761	112163	
	U	-	-	111.76	875.807	112088	S
$\alpha = 0.25$	L	4	1.5	0	0.03816	137950	
	H	4.5	2	0	0.06283	138281	
	U	-	-	0	0.06283	138114	P
	L	3.5	1.5	0	0.03820	137654	
	H	5.05	2.5	0	0.10344	138686	
	U	-	-	136.973	707.31	138446	S

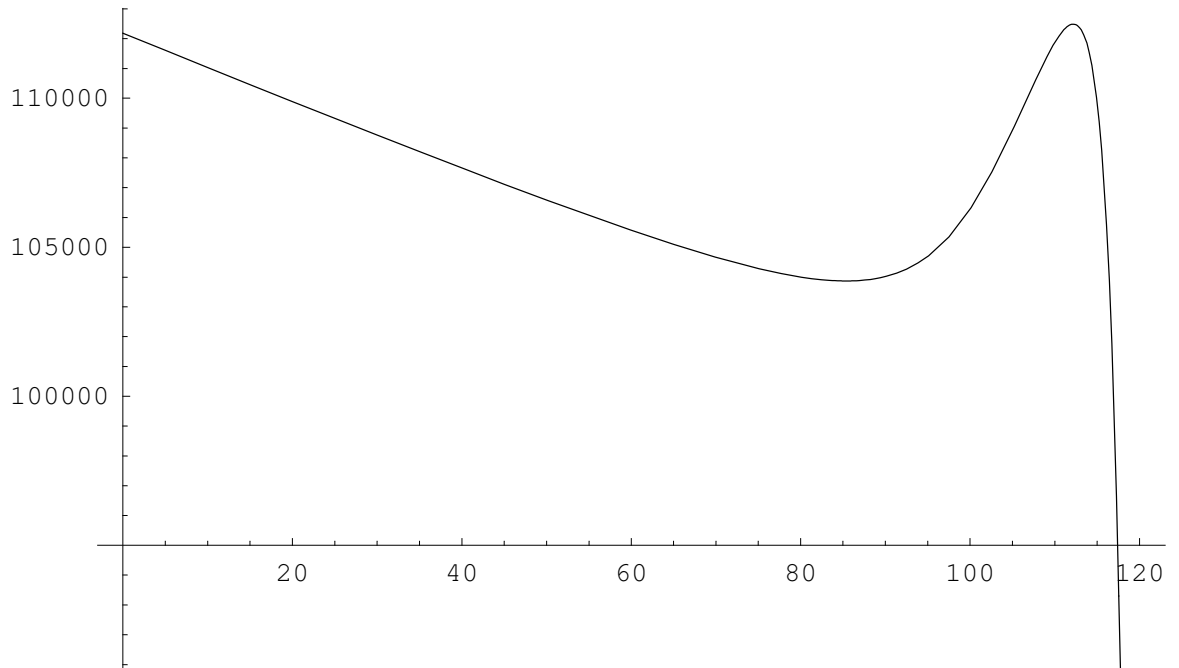


Figure 1: A plot of the value of the objective of the firm as a function of the strike price, for a number of options that satisfies the participation constraint of the executive. Parameter values are  $\alpha = 0.15$ ,  $T = 5$ ,  $\lambda = 1000$ ,  $\delta = 2.5$ , and  $R = 4.5$ .