

# Investment-Based Expected Stock Returns

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## Abstract

We derive and test implications of  $q$ -theory for the cross section of stock returns. Under constant returns to scale, stock returns equal levered investment returns, which are tied directly to firm characteristics. We use GMM to match moments of levered investment returns to those of observed stock returns. The model captures the average stock returns of portfolios sorted by earnings surprises, book-to-market equity, and capital investment. Stock return volatilities predicted from the model are largely comparable to those observed in the data. However, the model falls short in matching expected returns and volatilities simultaneously.

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# 1 Introduction

We use the  $q$ -theory of investment to derive and test predictions for the cross section of stock returns. Under constant returns to scale, stock returns equal levered investment returns, which are directly tied to firm characteristics via the conditions for optimal investment. We use Generalized Methods of Moments (GMM) to match means and variances of levered investment returns with those of stock returns. We conduct the GMM tests using data on portfolios sorted by earnings surprises, book-to-market equity, and capital investment—firm characteristics that are tied closely to cross sectional patterns in returns. We also compare the performance of the  $q$ -theory model with the performance of traditional asset pricing models such as the Capital Asset Pricing Model (CAPM), the Fama-French (1993) three-factor model, and the standard consumption CAPM with power utility.

The  $q$ -theory model outperforms traditional asset pricing models in matching expected returns. We estimate an average pricing error of 0.74% per annum for ten equal-weighted portfolios sorted by earnings surprises. This average error is much lower than those from the CAPM, 5.67%, the Fama-French model, 4.01%, and the standard consumption CAPM, 3.62%. The error for the return on the portfolio that is long on high earnings surprise stocks and short on low earnings surprise stocks (high-minus-low earnings surprise portfolio) is  $-0.40\%$  per annum. This error is negligible compared to the errors of 12.55% from the CAPM, 14.06% from the Fama-French model, and 13.38% from the standard consumption CAPM. Similarly, the  $q$ -theory model produces an error for the high-minus-low book-to-market portfolio of only 1.21%, which is substantially smaller than 18.56% from the CAPM, 7.30% from the Fama-French model, and 12.31% from the standard consumption CAPM. Finally, the high-minus-low investment portfolio has an error from the  $q$ -theory model of  $-0.49\%$ , which is much smaller than the error of  $-6.30\%$  from the CAPM,  $-6.34\%$  from the Fama-French model, and  $-8.38\%$  from the standard consumption CAPM.

When we use the  $q$ -theory model to match both the average returns and variances of the testing portfolios, the variances predicted by the model are largely comparable to stock return variances. The average stock return volatility across the earnings surprise portfolios is 21.06% per annum, which is close to the average levered investment return volatility of 20.43%. The average realized and predicted volatilities also are close for the book-to-market portfolios: 24.99% versus 23.55%, and for the investment portfolios: 24.84% versus 24.35%. However, the model falls short in two ways. First, while we find no discernible relation between volatilities and firm characteristics in the data, the model predicts a positive relation between volatilities and book-to-market. Second,

average return errors vary systematically with earnings surprises and investment, and are comparable in magnitude to those from the traditional models. However, the  $q$ -theory model still performs better in matching the average returns of the book-to-market portfolios.

Although  $q$ -theory originates in Brainard and Tobin (1968) and Tobin (1969), our work is built more directly on Cochrane (1991), who first uses  $q$ -theory to study stock market returns, as well as on Cochrane (1996), who uses aggregate investment returns to parameterize the stochastic discount factor in cross-sectional tests. Several more recent articles model cross-sectional returns based on firms' dynamic optimization problems (e.g., Berk, Green, and Naik (1999) and Zhang (2005)). We differ by doing structural estimation of closed-form Euler equations. Our work is also connected to the literature that estimates investment Euler equations using aggregate or firm level investment data (e.g., Shapiro (1986) and Whited (1992)). Our work differs because we use this framework to study the cross section of returns rather than investment dynamics or financing constraints. Most important, our  $q$ -theory approach to understanding cross-sectional returns represents a fundamental departure from the traditional consumption-based approach (e.g., Hansen and Singleton (1982) and Lettau and Ludvigson (2001)) in that we do not make any assumptions on preferences.

## 2 The Model of the Firms

Time is discrete and the horizon infinite. Firms use capital and costlessly adjustable inputs to produce a homogeneous output. These latter inputs are chosen each period to maximize operating profits, defined as revenues minus the expenditures on these inputs. Taking operating profits as given, firms choose optimal investment to maximize the market value of equity.

Let  $\Pi(K_{it}, X_{it})$  denote the maximized operating profits of firm  $i$  at time  $t$ . The profit function depends on capital,  $K_{it}$ , and a vector of exogenous aggregate and firm-specific shocks,  $X_{it}$ . The profit function exhibits constant returns to scale, that is,  $\Pi(K_{it}, X_{it}) = K_{it} \partial \Pi(K_{it}, X_{it}) / \partial K_{it}$ . If firm  $i$  has a Cobb-Douglas production function, the marginal product of capital is given by  $\partial \Pi(K_{it}, X_{it}) / \partial K_{it} = \alpha Y_{it} / K_{it}$ , in which  $\alpha > 0$  is capital's share and  $Y_{it}$  is sales. This parametrization assumes that shocks to operating profits,  $X_{it}$ , are reflected in sales.

End-of-period capital equals investment plus beginning-of-period capital net of depreciation:  $K_{it+1} = I_{it} + (1 - \delta_{it}) K_{it}$ , in which capital depreciates at an exogenous proportional rate of  $\delta_{it}$ , which is firm-specific and time-varying. Firms incur adjustment costs when investing. The adjustment cost function, denoted  $\Phi(I_{it}, K_{it})$ , is increasing and convex in  $I_{it}$ , decreasing in  $K_{it}$ , and exhibits con-

stant returns to scale in  $I_{it}$  and  $K_{it}$ , that is,  $\Phi(I_{it}, K_{it}) = I_{it}\partial\Phi(I_{it}, K_{it})/\partial I_{it} + K_{it}\partial\Phi(I_{it}, K_{it})/\partial K_{it}$ . We use a standard quadratic functional form:  $\Phi(I_{it}, K_{it}) = (a/2)(I_{it}/K_{it})^2 K_{it}$ .

Firms can finance investment with debt. We follow Hennessy and Whited (2007) and model only one-period debt. At the beginning of time  $t$ , firm  $i$  can issue an amount of debt, denoted  $B_{it+1}$ , which must be repaid at the beginning of period  $t+1$ . The gross corporate bond return on  $B_{it}$ , denoted  $r_{it}^B$ , can vary across firms and over time. Taxable corporate profits equal operating profits less capital depreciation, adjustment costs, and interest expenses:  $\Pi(K_{it}, X_{it}) - \delta_{it}K_{it} - \Phi(I_{it}, K_{it}) - (r_{it}^B - 1)B_{it}$ , in which adjustment costs are expensed, consistent with treating them as forgone operating profits. Let  $\tau_t$  denote the corporate tax rate at time  $t$ . The payout of firm  $i$  equals:

$$D_{it} \equiv (1 - \tau_t)[\Pi(K_{it}, X_{it}) - \Phi(I_{it}, K_{it})] - I_{it} + B_{it+1} - r_{it}^B B_{it} + \tau_t \delta_{it} K_{it} + \tau_t (r_{it}^B - 1) B_{it}, \quad (1)$$

in which  $\tau_t \delta_{it} K_{it}$  is the depreciation tax shield and  $\tau_t (r_{it}^B - 1) B_{it}$  is the interest tax shield.

Let  $M_{t+1}$  be the stochastic discount factor from  $t$  to  $t+1$ , which is correlated with the aggregate component of  $X_{it+1}$ . We can formulate the cum-dividend market value of equity as follows:

$$V_{it} \equiv \max_{\{I_{it+s}, K_{it+s+1}, B_{it+s+1}\}_{s=0}^{\infty}} E_t \left[ \sum_{s=0}^{\infty} M_{t+s} D_{it+s} \right], \quad (2)$$

subject to a transversality condition that prevents firms from borrowing an infinite amount to distribute to shareholders:  $\lim_{T \rightarrow \infty} E_t [M_{t+T} B_{it+T+1}] = 0$ .

**Proposition 1.** Firms' value-maximization implies that  $E_t [M_{t+1} r_{it+1}^I] = 1$ , in which  $r_{it+1}^I$  is the investment return, defined as:

$$r_{it+1}^I \equiv \frac{(1 - \tau_{t+1}) \left[ \alpha \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2} \left( \frac{I_{it+1}}{K_{it+1}} \right)^2 \right] + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) \left[ 1 + (1 - \tau_{t+1}) a \left( \frac{I_{it+1}}{K_{it+1}} \right) \right]}{1 + (1 - \tau_t) a \left( \frac{I_{it}}{K_{it}} \right)}. \quad (3)$$

Define the after-tax corporate bond return as  $r_{it+1}^{Ba} \equiv r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1}$ , then  $E_t [M_{t+1} r_{it+1}^{Ba}] = 1$ . Define  $P_{it} \equiv V_{it} - D_{it}$  as the ex-dividend equity value,  $r_{it+1}^S \equiv (P_{it+1} + D_{it+1})/P_{it}$  as the stock return, and  $w_{it} \equiv B_{it+1}/(P_{it} + B_{it+1})$  as the market leverage, then the investment return is the weighted average of the stock return and the after-tax corporate bond return:

$$r_{it+1}^I = w_{it} r_{it+1}^{Ba} + (1 - w_{it}) r_{it+1}^S. \quad (4)$$

**Proof.** See Appendix A. ■

The investment return in equation (3) is the ratio of the marginal benefit of investment at time  $t + 1$  divided by the marginal cost of investment at  $t$ . Optimality means that the marginal cost of investment in the denominator equals the marginal benefit (marginal  $q$ ), which is the expected present value of the marginal profits from investing in one additional unit of capital. In the numerator the term  $(1 - \tau_{t+1})\alpha Y_{it+1}/K_{it+1}$  is the marginal after-tax profit produced by an additional unit of capital, the term  $(1 - \tau_{t+1})(a/2)(I_{it+1}/K_{it+1})^2$  is the marginal after-tax reduction in adjustment costs, the term  $\tau_{t+1}\delta_{it+1}$  is the marginal depreciation tax shield, and the last term in the numerator is the marginal continuation value of an extra unit of capital net of depreciation. In addition, the first term in brackets in the numerator divided by the denominator is analogous to a dividend yield. The second term in brackets in the numerator divided by the denominator is analogous to a capital gain because this ratio is the growth rate of marginal  $q$ .

Equation (4) is exactly the weighted average cost of capital in corporate finance. Without leverage, this equation reduces to the equivalence between stock and investment returns, a relation first established by Cochrane (1991), and is an algebraic restatement of the equivalence between marginal  $q$  and average  $q$  established by Hayashi (1982). Solving for  $r_{it+1}^S$  from equation (4) gives:

$$r_{it+1}^S = r_{it+1}^{Iw} \equiv \frac{r_{it+1}^I - w_{it} r_{it+1}^{Ba}}{1 - w_{it}}, \quad (5)$$

in which  $r_{it+1}^{Iw}$  is the levered investment return.

### 3 Econometric Methodology

#### 3.1 Moments for GMM Estimation and Tests

To test whether cross-sectional variation in average stock returns matches cross-sectional variation in firm characteristics, we test the ex-ante restriction implied by equation (5): expected stock returns equal expected levered investment returns,

$$E [r_{it+1}^S - r_{it+1}^{Iw}] = 0. \quad (6)$$

A long-standing puzzle in financial economics is that stock returns are excessively volatile (e.g., Shiller (1981)). Cochrane (1991) reports that the annual aggregate investment return volatility is only about 60% of the annual value-weighted stock market volatility. To study whether the  $q$ -theory model can reproduce empirically plausible stock return volatilities, we also test whether

stock return variances equal levered investment return variances:

$$E \left[ (r_{it+1}^S - E[r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E[r_{it+1}^{Iw}])^2 \right] = 0. \quad (7)$$

As noted by Cochrane (1991), equation (5), taken literally, says that levered investment returns equal stock returns for every stock, every period, and every state of the world. Because no choice of parameters can satisfy these conditions, equation (5) is formally rejected at any level of significance. However, we can test the weaker conditions in equations (6) and (7). To do so, we must add statistical assumptions about the sources of error that might invalidate these two moment conditions (model errors). These errors arise because of either measurement or specification issues. For example, components of investment returns such as the capital stock are difficult to measure; adjustment costs might not be quadratic; and the marginal product of capital might not be exactly proportional to the sales-to-capital ratio.

To construct a formal test of equation (5), we define the model errors from the empirical moments as follows:

$$e_i^q \equiv E_T [r_{it+1}^S - r_{it+1}^{Iw}], \quad (8)$$

$$e_i^{\sigma^2} \equiv E_T \left[ (r_{it+1}^S - E_T[r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E_T[r_{it+1}^{Iw}])^2 \right], \quad (9)$$

in which  $E_T[\cdot]$  is the sample mean of the series in brackets. We call  $e_i^q$  the mean error and  $e_i^{\sigma^2}$  the variance error, and assume that both errors have a mean of zero. While recognizing that specification and measurement errors, unlike forecast errors, do not necessarily have a zero mean, we note that this assumption is simple and that similar assumptions underlie most Euler equation tests.<sup>1</sup>

We estimate the parameters  $a$  and  $\alpha$  using GMM to minimize a weighted average of  $e_i^q$  or a weighted average of both  $e_i^q$  and  $e_i^{\sigma^2}$ . We use the identity weighting matrix in one-stage GMM to preserve the economic structure of the testing assets. After all, we choose testing assets precisely because the underlying characteristics are economically important in providing a wide spread in the cross section of average stock returns. The identity weighting matrix also gives potentially more robust, albeit less efficient, estimates. The estimates from second-stage GMM are similar to the

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<sup>1</sup>Cochrane (1991, p. 220) articulates this point as follows: “The consumption-based model suffers from the same problems: unobserved preference shocks, components of consumption that enter nonseparably in the utility function (for example, the service flow from durables), and measurement error all contribute to the error term, and there is no reason to expect these errors to obey the orthogonality restrictions that the forecast error obeys. Empirical work on consumption-based models focuses on the forecast error since it has so many useful properties, but the importance in practice of these other sources of error may be part of the reason for its empirical difficulties.”

first-stage estimates. To conduct inferences, we nevertheless need to calculate the optimal weighting matrix. We use a standard Bartlett kernel with a window length of five. The results are insensitive to the window length. To test whether all (or a subset of) model errors are jointly zero, we use a  $\chi^2$  test from Hansen (1982, Lemma 4.1). Appendix B provides additional econometric details.

We conduct the GMM estimation and tests at the portfolio level. The main reason is that the stylized facts in cross-sectional returns we wish to understand are most evident in portfolio level data. As such, the usage of portfolios befits our economic question. Further, the portfolio approach has the advantage that portfolio investment data are relatively smooth, whereas investment data are lumpy at the firm level, probably because of nonconvex adjustment costs (e.g., Whited (1998)). In addition, Thomas (2002) shows that aggregation substantially reduces the effect of lumpy investment in equilibrium business cycle models, and Hall (2004) shows that nonconvexities are not important for estimating investment Euler equations at the industry level. Conducting GMM tests using returns at the firm level with nonconvexities embedded into our framework is a potentially interesting extension, but is unnecessary for understanding cross-sectional return patterns.

## 3.2 Data

We construct annual levered investment returns to match annual stock returns. Our sample of firm-level data is from the Center for Research in Security Prices (CRSP) monthly stock file and the annual and quarterly 2005 Standard and Poor's Compustat industrial files. We select our sample by first deleting any firm-year observations with missing data or for which total assets, the gross capital stock, debt, or sales are either zero or negative. We include only firms with fiscal yearend in December. Firms with primary SIC classifications between 4900 and 4999 or between 6000 and 6999 are omitted because  $q$ -theory is unlikely to be applicable to regulated or financial firms.

### 3.2.1 Portfolio Definitions

We use 30 testing portfolios: ten Standardized Unexpected Earnings (SUE) portfolios as in Chan, Jegadeesh, and Lakonishok (1996), ten book-to-market (B/M) portfolios as in Fama and French (1993), and ten corporate investment (CI) portfolios as in Titman, Wei, and Xie (2004). SUE is a measure of earnings surprises or shocks to earnings, B/M is the ratio of accounting value of equity divided by the market value of equity, and CI is a measure of firm-level capital investment. The relations of stock returns with SUE and B/M represent what are arguably the two most important stylized facts in the cross section of returns (e.g., Fama (1998)). We use the CI portfolios because

our framework characterizes optimal investment behavior. We use equal-weighted returns for all testing portfolios because equal-weighted returns are harder for asset pricing models to capture than value-weighted returns. Our basic results are similar if we value-weight portfolio returns.

*Ten SUE Portfolios.* Following Chan, Jegadeesh, and Lakonishok (1996), we define SUE as the change in quarterly earnings (Compustat quarterly item 8) per share from its value four quarters ago divided by the standard deviation of the change in quarterly earnings over the prior eight quarters. We rank all stocks by their most recent SUEs at the beginning of each month  $t$  and assign all the stocks to one of ten portfolios using NYSE breakpoints. We calculate average monthly returns over the holding period from month  $t + 1$  to  $t + 6$ . The sample is from January 1972 to December 2005. The starting point is restricted by the availability of quarterly earnings data.

*Ten B/M Portfolios.* Following Fama and French (1993), we sort all stocks at the end of June of year  $t$  into ten groups based on NYSE breakpoints for B/M. The sorting variable for June of year  $t$  is book equity for the previous fiscal yearend in year  $t - 1$  divided by the market value of common equity for December of year  $t - 1$ . Book equity is common equity (Compustat annual item 60) plus balance sheet deferred tax (item 74). The market value of common equity is the closing price per share (item 199) times the number of common shares outstanding (item 25). We calculate equal-weighted annual returns from July of year  $t$  to June of year  $t + 1$  for the resulting portfolios, which are rebalanced at the end of each June. The sample is from January 1963 to December 2005.

*Ten CI Portfolios.* Following Titman, Wei, and Xie (2004), we define  $CI_{t-1}$ , the sorting variable in the portfolio formation year  $t$ , as  $CE_{t-1}/[(CE_{t-2}+CE_{t-3}+CE_{t-4})/3]$ , in which  $CE_{t-1}$  is capital expenditures (Compustat annual item 128) scaled by sales (item 12) in year  $t - 1$ . The prior three-year moving average of CE aims to measure the benchmark investment level. We sort all stocks on CI at the end of June of year  $t$  into ten portfolios using breakpoints based on NYSE, Amex, and Nasdaq stocks. Equal-weighted annual portfolio returns are calculated from July of year  $t$  to June of year  $t + 1$ . The sample runs from January 1963 to December 2005.

### 3.2.2 Variable Measurement

*Capital, Investment, Output, Debt, Leverage, and Depreciation.* The capital stock,  $K_{it}$ , is gross property, plant, and equipment (Compustat annual item 7), and investment,  $I_{it}$ , is capital expenditures minus sales of property, plant, and equipment (the difference between items 128 and 107). We set sales of property, plant, and equipment to be zero when item 107 is missing. Our basic



results are similar when we measure the capital stock as the net property, plant, and equipment (item 8) and investment as item 128. Output,  $Y_{it}$ , is sales (item 12), and total debt,  $B_{it}$ , is long-term debt (item 9) plus short term debt (item 34). Our basic results are similar when we use the Bernanke and Campbell (1988) algorithm to convert the book value of debt into the market value of debt. We measure market leverage as the ratio of total debt to the sum of total debt and the market value of equity. The depreciation rate,  $\delta_{it}$ , is the amount of depreciation (item 14) divided by capital stock.

Both stock and flow variables in Compustat are recorded at the end of year  $t$ . However, the model requires stock variables subscripted  $t$  to be measured at the beginning of year  $t$  and flow variables subscripted  $t$  to be measured over the course of year  $t$ . We take, for example, for the year 1993 any beginning-of-period stock variable (such as  $K_{i1993}$ ) from the 1992 balance sheet and any flow variable measured over the year (such as  $I_{i1993}$ ) from the 1993 income or cash flow statement.

We follow Fama and French (1995) in aggregating firm-specific characteristics to portfolio-level characteristics:  $Y_{it+1}/K_{it+1}$  is the sum of sales in year  $t+1$  for all the firms in portfolio  $i$  formed in June of year  $t$  divided by the sum of capital stocks at the beginning of  $t+1$  for the same firms;  $I_{it+1}/K_{it+1}$  in the numerator of  $r_{it+1}^I$  is the sum of investment in year  $t+1$  for all the firms in portfolio  $i$  formed in June of year  $t$  divided by the sum of capital stocks at the beginning of  $t+1$  for the same firms;  $I_{it}/K_{it}$  in the denominator of  $r_{it+1}^I$  is the sum of investment in year  $t$  for all the firms in portfolio  $i$  formed in June of year  $t$  divided by the sum of capital stocks at the beginning of year  $t$  for the same firms; and  $\delta_{it+1}$  is the total amount of depreciation for all the firms in portfolio  $i$  formed in June of year  $t$  divided by the sum of capital stocks at the beginning of  $t+1$  for the same firms.

*Corporate Bond Returns.* Firm-level corporate bond data are rather limited, and few or none of the firms in several portfolios have corporate bond ratings. To construct bond returns,  $r_{it+1}^B$ , for firms without bond ratings, we follow closely the approach in Blume, Lim, and MacKinlay (1998) for imputing bond ratings not available in Compustat. First, we estimate an ordered probit model that relates categories of credit ratings to observed explanatory variables. We estimate the model using all the firms that have data on credit ratings (Compustat annual item 280). Second, we use the fitted value to calculate the cutoff value for each rating. Third, for firms without credit ratings we estimate their credit scores using the coefficients estimated from the ordered probit model and impute bond ratings by applying the cutoff values for the different credit ratings. Finally, we assign the corporate bond returns for a given credit rating from Ibbotson Associates as the corporate bond returns to all the firms with the same credit rating.

The explanatory variables in the ordered probit model are interest coverage defined as the ratio of operating income after depreciation (item 178) plus interest expense (item 15) divided by interest expense, the operating margin as the ratio of operating income before depreciation (item 13) to sales (item 12), long-term leverage as the ratio of long-term debt (item 9) to assets (item 6), total leverage as the ratio of long-term debt plus debt in current liabilities (item 34) plus short-term borrowing (item 104) to assets, and the natural log of the market value of equity deflated to 1973 by the Consumer Price Index (item 24 times item 25). Following Blume, Lim, and MacKinlay (1998), we also include the market beta and residual volatility from the market model. For each calendar year we estimate the beta and residual volatility for each firm with at least 200 daily returns. Daily stock returns and value-weighted market returns are from CRSP. We adjust for nonsynchronous trading with one leading and one lagged value of the market return.

*The Corporate Tax Rate.* We measure  $\tau_t$  as the statutory corporate income tax rate. From 1963 to 2005, the tax rate is on average 42.3%. The statutory rate starts at around 50% in the beginning years of our sample, drops from 46% to 40% in 1987 and further to 34% in 1988, and stays at that level afterward. The source is the Commerce Clearing House, annual publications.

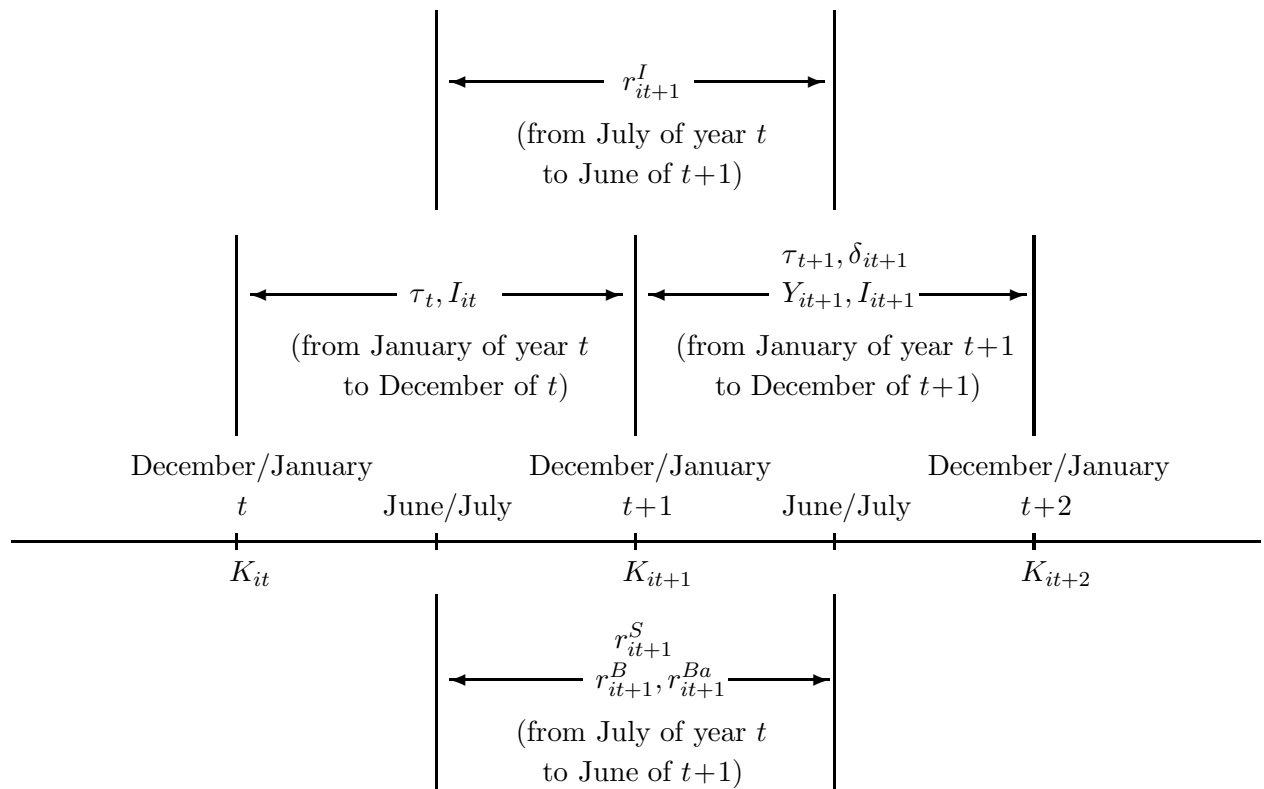
We have experimented with firm-specific tax rates using the trichotomous variable approach of Graham (1996). The trichotomous variable is equal to i) the statutory corporate income tax rate if the taxable income defined as pretax income (Compustat annual item 170) minus deferred taxes (item 50) divided by the statutory tax rate is positive and net operating loss carryforward (item 52) is nonpositive; ii) one-half of the statutory rate if one and only one condition in i) is violated; and iii) zero otherwise. The trichotomous variable does not vary much across our testing portfolios. The portfolio-level tax rate is on average 36.0% for the low SUE portfolio, 37.9% for the high SUE portfolio, 34.8% for the low CI portfolio, and 37.4% for the high CI portfolio. The spread across the B/M portfolios is only slightly larger: the tax rate is 40.2% in the low B/M portfolio and 35.1% in the high B/M portfolio. As such, we use time-varying but portfolio invariant tax rates for simplicity. The results are largely similar using portfolio-specific tax rates.

### 3.2.3 Timing Alignment

To match annual levered investment returns with annual stock returns, we need to align the timing across the two types of returns. Figure 1 illustrates the timing convention. Specifically, we use the Fama-French portfolio approach to form the B/M and CI portfolios by sorting stocks in June of year  $t$  based on characteristics at the end of fiscal year  $t-1$ . The characteristics used to sort portfolios in

year  $t$  are measured at the end of year  $t-1$  or, equivalently, the beginning of year  $t$ . Portfolio stock returns,  $r_{it+1}^S$ , are calculated from July of year  $t$  to June of year  $t+1$ , and the portfolios are rebalanced in June of year  $t+1$ . To construct the annual investment returns in equation (3),  $r_{it+1}^I$ , we use the tax rate and investment observed at the end of year  $t$  ( $\tau_t$  and  $I_{it}$ ) and other variables at the end of year  $t+1$  ( $\tau_{t+1}$ ,  $Y_{it+1}$ ,  $I_{it+1}$ , and  $\delta_{it+1}$ ). Because time  $t$  stock variables are measured at the beginning of year  $t$ , and because time  $t$  flow variables are realized over the course of year  $t$ , the investment return constructed using  $I_{it}$ ,  $K_{it}$ ,  $Y_{it+1}$ ,  $I_{it+1}$ ,  $\delta_{it+1}$ , and  $K_{it+1}$  in equation (3) goes roughly from the middle of year  $t$  to the middle of year  $t+1$ . The bottomline is that the investment return timing matches naturally with the stock return timing from the Fama-French portfolio approach.

**Figure 1: Timing Alignment between Annual Stock and Investment Returns**



The changes in stock composition in a given portfolio from portfolio rebalancing raise further subtleties. In the Fama-French portfolio approach, for the annually rebalanced B/M and CI portfolios, the set of firms in a given portfolio formed in year  $t$  is fixed when we aggregate returns from July of year  $t$  to June of  $t+1$ . The stock composition changes only at the end of June of year  $t+1$  when we rebalance. Correspondingly, we fix the set of firms in a given portfolio in the formation year  $t$  when

aggregating characteristics, dated both  $t$  and  $t + 1$ , across firms in the portfolio. In particular, to construct the numerator of  $r_{it+1}^I$ , we use  $I_{it+1}/K_{it+1}$  from the portfolio formation year  $t$ , which is different from the  $I_{it+1}/K_{it+1}$  from the formation year  $t+1$  used to construct the denominator of  $r_{it+2}^I$ .

The SUE portfolios are initially formed monthly. We time-aggregate monthly returns of the SUE portfolios from July of year  $t$  to June of  $t+1$  to obtain annual returns. Constructing the matching annual investment returns,  $r_{it+1}^I$ , requires care because the composition of the SUE portfolios changes from month to month. First, consider the 12 low SUE portfolios formed in each month from July of year  $t$  to June of  $t+1$ . For each month we calculate portfolio level characteristics by aggregating individual characteristics over the firms in the low SUE portfolio. We use the following specific characteristics:  $I_{it}$  and  $\tau_t$  observed at the end of year  $t$ ,  $K_{it}$  at the beginning of year  $t$ ,  $K_{it+1}$  at the beginning of  $t + 1$ , and  $\tau_{t+1}$ ,  $Y_{it+1}$ ,  $I_{it+1}$ , and  $\delta_{it+1}$  at the end of year  $t+1$ . Because the portfolio composition changes from month to month, these portfolio level characteristics also change from month to month. We therefore average these portfolio characteristics over the 12 monthly low SUE portfolios, and use these averages to construct  $r_{it+1}^I$ , which is in turn matched with the annual  $r_{it+1}^S$  from July of  $t$  to June of  $t+1$ . We then repeat this procedure for the remaining SUE portfolios.

The after-tax corporate bond return,  $r_{it+1}^{Ba}$ , depends on the tax rate and the pre-tax bond return,  $r_{it+1}^B$ , which we measure as the observed corporate bond returns in the data. The timing of  $r_{it+1}^B$  is the same as that of stock returns: after sorting stocks on characteristics measured at the end of fiscal year  $t-1$ , we measure  $r_{it+1}^B$  as the equal-weighted corporate bond return from July of year  $t$  to June of  $t+1$ . However, calculating  $r_{it+1}^{Ba} = r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1}$  is slightly complicated:  $\tau_{t+1}$  is applicable from January to December of year  $t+1$ , but  $r_{it+1}^B$  is applicable from July of year  $t$  to June of  $t+1$ . We deal with this timing-mismatch by replacing  $\tau_{t+1}$  in the calculation of  $r_{it+1}^{Ba}$  with the average of  $\tau_t$  and  $\tau_{t+1}$  in the data. This timing-mismatch matters little for our results because the tax rate exhibits little time series variation. In particular, we have experimented with time-invariant tax rates in calculating  $r_{it+1}^{Ba}$ , and the results are largely similar.

## 4 Empirical Results

Section 4.1 reports tests of the CAPM, the Fama-French model, and the standard consumption CAPM on our portfolios. Sections 4.2 and 4.3 report tests of the  $q$ -theory model in matching expected returns and in matching both expected returns and variances, respectively.

## 4.1 Testing Traditional Asset Pricing Models

To test the CAPM, we regress annual portfolio returns in excess of the risk-free rate on market excess returns. The risk-free rate, denoted  $r_{ft+1}$ , is the annualized return on the one-month Treasury bill from Ibbotson Associates. The regression intercept measures the model error from the CAPM. To test the Fama-French model, we regress annual portfolio excess returns on annual returns of the market factor, a size factor, and a book-to-market factor (the factor returns data are from Kenneth French's Web site). The intercept measures the error of the Fama-French model. We also estimate the standard consumption CAPM with the pricing kernel  $M_{t+1} = \beta(C_{t+1}/C_t)^{-\gamma}$ , in which  $\beta$  is time preference coefficient,  $\gamma$  is risk aversion, and  $C_t$  is annual per capita consumption of nondurables and services from the Bureau of Economic Analysis. We use one-stage GMM with the identity weighting matrix to estimate the moments  $E[M_{t+1}(r_{it+1}^S - r_{ft+1})] = 0$ . We also include  $E[M_{t+1}r_{ft+1}] = 1$  as an additional moment condition to identify  $\beta$ . The error of the standard consumption CAPM is calculated as  $E_T[M_{t+1}(r_{it+1}^S - r_{ft+1})]/E_T[M_{t+1}]$ .

The SUE, B/M, and CI effects cannot be captured by traditional asset pricing models. Panel A of Table 1 shows that from the low SUE to the high SUE portfolio the average return increases monotonically from 10.89% to 23.39% per annum. The portfolio volatilities are largely flat at around 22%. The CAPM error of the high-minus-low SUE portfolio is 12.55% per annum ( $t = 5.53$ ), and the average absolute value of the pricing errors, denoted a.a.p.e., is 5.67%. The Gibbons, Ross, and Shanken (1989, GRS) statistic, which tests the null hypothesis that all the individual intercepts are jointly zero, rejects the CAPM. (The intercepts do not add up to zero because we equal-weight the portfolio returns.) The performance of the Fama-French model is similar: the a.a.p.e. is 4.01% and the GRS test rejects the model. The error of the high-minus-low SUE portfolio from the Fama-French model is 14.06% per annum ( $t = 5.31$ ). The consumption CAPM error increases from  $-8.07\%$  for the low SUE portfolio to 5.13% per annum for the high SUE portfolio. Although the errors are not individually significant, probably because of large measurement errors in consumption data, the  $\chi^2$  test rejects the null hypothesis that the pricing errors are jointly zero at the 1% significance level. The parameter estimates also are unrealistic: the estimate of the time preference coefficient is 2.76, and estimate of the risk aversion parameter is 127.59.

Panel B of Table 1 shows that value stocks with high B/M ratios earn higher average stock returns than growth stocks with low B/M ratios, 25.78% versus 8.65% per annum. The difference of 17.13% is significant ( $t = 5.53$ ). There is no discernible relation between B/M and stock return

volatility: both the value and the growth portfolios have volatilities around 27%. The CAPM error increases monotonically from  $-4.91\%$  for growth stocks to  $13.65\%$  for value stocks. The average magnitude of the errors is  $6.34\%$  per annum and the GRS test strongly rejects the CAPM. Even the Fama-French model fails to capture the equal-weighted returns: the high-minus-low portfolio has an error of  $7.30\%$  ( $t = 3.25$ ). The consumption CAPM error increases from  $-5.43\%$  for growth stocks to  $6.88\%$  for value stocks with an average magnitude of  $2.36\%$ , and the model is rejected by the  $\chi^2$  test.

From Panel C, high CI stocks earn lower average stock returns than low CI stocks:  $15.16\%$  versus  $22.12\%$  per annum, and the difference is more than four standard errors from zero. The volatilities of the low CI stocks are slightly higher than those of the high CI stocks:  $32.42\%$  versus  $26.73\%$ . The high-minus-low CI portfolio has an error of  $-6.30\%$  ( $t = -3.88$ ) from the CAPM and an error of  $-6.34\%$  ( $t = -3.99$ ) from the Fama-French model. Both models are rejected by the GRS test. The consumption CAPM error decreases from  $4.03\%$  for the low CI portfolio to  $-4.35\%$  for the high CI portfolio with an average magnitude of  $2.36\%$ , and the  $\chi^2$  test rejects the model.

## 4.2 The $q$ -theory Model: Matching Expected Returns

### 4.2.1 Point Estimates and Overall Model Performance

We estimate only two parameters in our parsimonious model: the adjustment cost parameter,  $a$ , and capital's share,  $\alpha$ . Panel A of Table 2 provides estimates of  $\alpha$  that range from 0.21 to 0.50, and are often significant. These estimates are reasonably close to the approximate 0.30 figure for capital's share used in Rotemberg and Woodford (1992). The estimates of  $a$  are not as stable across the different sets of testing portfolios. We find significant estimates of 7.68 and 0.97 for the SUE and CI portfolios, respectively. The estimate is 22.34 for the B/M portfolios but with a high standard error of 25.47. Although some of these estimates of  $a$  are large, they fall within the wide range of estimates from studies using quantity data. Finally, the evidence implies that firm's optimization problem has an interior solution because the positive estimates of  $a$  mean that the adjustment cost function is increasing and convex in  $I_{it}$ .

Panel A of Table 2 also reports two measures of overall model performance: the average absolute pricing error, a.a.p.e., and the  $\chi^2$  test. The model performs quite well in accounting for the average returns of the ten SUE portfolios. The a.a.p.e. is  $0.74\%$  per annum, which is substantially lower than those from the CAPM,  $5.67\%$ , the Fama-French model,  $4.01\%$ , and the standard consumption CAPM,  $3.62\%$ . Unlike the traditional models that are rejected using the SUE portfolios, the  $q$ -theory model is not rejected by the  $\chi^2$  test. The overall performance of the model is more modest

in capturing the average B/M portfolio returns. Although the model is not formally rejected by the  $\chi^2$  test, the a.a.p.e. is 2.32% per annum, which is comparable to that from the Fama-French model, 2.79%, and that from the standard consumption CAPM, 2.36%, but is lower than that from the CAPM, 6.34%. The model does better in pricing the ten CI portfolios. The a.a.p.e. is 1.51% per annum, which is lower than those from the CAPM, 5.71%, the Fama-French model, 2.24%, and the standard consumption CAPM, 2.36%. The  $q$ -theory model is again not rejected by the  $\chi^2$  test.

#### 4.2.2 Euler Equation Errors

The average absolute pricing errors and  $\chi^2$  tests only indicate overall model performance. To provide a more complete picture, we report each individual portfolio error,  $e_i^q$ , defined in equation (8), in which levered investment returns are constructed using the estimates from Panel A of Table 2. We also report the  $t$ -statistic, described in Appendix B, testing that an individual error equals zero.

Panel A of Table 3 reports that the magnitude of the individual mean errors varies from 0.05% to 1.72% per annum across ten SUE portfolios. None of the mean errors are significant. In particular, the high-minus-low SUE portfolio has a mean error of  $-0.40\%$  per annum ( $t = -0.34$ ), which is negligible compared to the economically large errors from the traditional models: 12.55% for the CAPM, 13.38% for the Fama-French model, and 14.06% for the standard consumption CAPM.

Figure 2 provides a visual presentation of the fit. Panel A plots the average levered investment returns of the ten SUE portfolios against their average stock returns. If the model performs perfectly, all the observations should lie precisely on the 45-degree line. Panel A shows that the scatter plot from the  $q$ -theory model is closely aligned with the 45-degree line. The remaining panels contain analogous plots for the CAPM, the Fama-French model, and the standard consumption CAPM. In all three cases the scatter plot is largely horizontal, meaning that the traditional models almost completely fail to predict the average returns across the SUE portfolios.

Moving to the B/M portfolios, we observe relatively large mean errors in Panel A of Table 3. Three out of ten portfolios have mean errors with magnitudes higher than 3% per annum and six out of ten have mean errors with magnitudes higher than 2.5%. The growth portfolio has a mean error of  $-3.94\%$ . However, the mean errors in the  $q$ -theory model do not vary systematically with B/M. The high-minus-low B/M portfolio only has a mean error of 1.21% per annum, which is substantially smaller than 18.56% in the CAPM, 7.30% in the Fama-French model, and 12.31% in the standard consumption CAPM. The scatter plots in Figure 3 show that, although the errors from the  $q$ -theory

model are largely similar in magnitude to those from the Fama-French model and the standard consumption CAPM, the average return spread between the low and the high B/M portfolios predicted by the  $q$ -theory model is much larger than the spreads from the traditional models.

From Panel A of Table 3, the mean errors from the CI portfolios are somewhat larger than those from the SUE portfolios but are much smaller than those from the B/M portfolios. Only three out of the ten CI portfolios have mean errors larger than 2.5% per annum. The high-minus-low CI portfolio has a small mean error of  $-0.49\%$  ( $t = -0.43$ ), meaning that the  $q$ -theory model generates a realistic average return spread across the two extreme CI portfolios. The scatter plot in Panel A of Figure 4 confirms this observation. In contrast, none of the traditional models are able to reproduce the average return spread, as shown in the rest of Figure 4.

### 4.2.3 Drivers of Expected Stock Returns

The intuition behind our estimation results comes from the investment return equation (3) and the levered investment return equation (5). The equations suggest several economic forces driving the cross section of average stock returns. The first driver operates through the marginal benefit of investment, whose first component is the marginal product of capital at  $t+1$  in the numerator of the investment return. The second driver is roughly proportional to the growth rate of investment, which corresponds to the “capital gain” component of the investment return: investment-to-capital is an increasing function of marginal  $q$ , denoted  $q_{it}$ , which is related to firm  $i$ 's stock price.

The third driver works through  $I_{it}/K_{it}$  in the denominator of the investment return. Because investment today increases with the net present value of one additional unit of capital, and because the net present value decreases with the cost of capital, a low cost of capital means high net present value and high investment. As such, investment today and average stock returns are negatively correlated. Relatedly, because investment is an increasing function of marginal  $q$ , and because marginal  $q$  is in turn inversely related to book-to-market equity, expected stock returns and book-to-market equity are positively correlated.

The fourth driver is the rate of depreciation,  $\delta_{it+1}$ . Collecting terms involving  $\delta_{it+1}$  in the numerator of equation (3) yields  $-(1 - \tau_{t+1})[1 + a(I_{it+1}/K_{it+1})]\delta_{it+1}$ , meaning that high rates of depreciation tomorrow imply lower average returns. The fifth driver is market leverage: taking the first-order derivative of equation (5) with respect to  $w_{it}$  shows that expected stock returns should increase with market leverage today. In sum, all else equal, firms with high investment-to-capital



today, low expected investment growth, low sales-to-capital tomorrow, high rates of depreciation tomorrow, and low market leverage today should earn lower average stock returns.

#### 4.2.4 Expected Returns Accounting

To understand our estimation results, Table 4 presents averages of the drivers underlying levered investment returns across testing portfolios. From Panel A, the average  $I_{it}/K_{it}$ ,  $\delta_{it+1}$ , and the bond returns,  $r_{it+1}^B$ , are largely flat across ten SUE portfolios. The average  $(I_{it+1}/K_{it+1}) / (I_{it}/K_{it})$  (future investment growth) and  $Y_{it+1}/K_{it+1}$  both increase from the low SUE portfolio to the high SUE portfolio, going in the right direction to capture average stock returns. However, going in the wrong direction, market leverage decreases from the low SUE portfolio to the high SUE portfolio.

For ten B/M portfolios,  $I_{it}/K_{it}$  decreases from 18% to 8% per annum from the low to the high B/M portfolio. The low B/M firms also have higher rates of depreciation (10% versus 7%) and lower market leverage (8% versus 53%) than the high B/M firms. All three characteristics go in the right direction to match average stock returns. However, going in the wrong direction, the low B/M firms have higher average  $Y_{it+1}/K_{it+1}$  (1.95 versus 1.38) than the high B/M firms. Average corporate bond returns and investment growth are roughly flat across the B/M portfolios. Not surprisingly, sorting on CI produces a large spread in  $I_{it}/K_{it}$  of 7.3%. This characteristic increases monotonically from 9% for the low to 16% for the high CI portfolio. Compared to the high CI firms, the low CI firms also have much higher future investment growth (1.25 versus 0.81) and higher market leverage (35% versus 28%). All three patterns go in the right direction to match average stock returns. The remaining characteristics are largely flat across the CI portfolios.

The observed patterns in characteristics help explain the differences in the estimates of the adjustment cost parameter,  $a$ , across the different sets of portfolios. For the B/M portfolios, for example, the characteristic  $Y_{it+1}/K_{it+1}$  goes strongly in the wrong direction to match average stock returns. Equation (3) suggests that this backward cross-sectional movement in the numerator of the investment return must be countered with a relatively strong movement in the denominator if average levered investment returns are to match average stock returns. A high estimate of  $a$ , 22.3, accomplishes this goal by magnifying the movement in the denominator. For the CI portfolios both future investment growth and  $I_{it}/K_{it}$  go strongly in the right direction. As such, the magnifying effect of  $a$ , 0.97, does not need to be large. The cross-sectional movement in  $I_{it}/K_{it}$  for the SUE portfolios is negligible, meaning that the parameter  $a$  only operates via its effect on future investment growth. The estimate of 7.68 falls between the extreme estimates for the other testing portfolios.

To quantify the role of each driver in matching expected stock returns, we conduct the following accounting exercises. We start with the parameters reported in Panel A of Table 2. We set a given driver equal to its cross-sectional average in each year. We then use the estimated parameters to reconstruct levered investment returns, while keeping all the other characteristics unchanged. In the case of future investment growth we hold constant the capital gain component of the investment return,  $[1 + (1 - \tau_{t+1})a(I_{it+1}/K_{it+1})]/[1 + (1 - \tau_t)a(I_{it}/K_{it})] = q_{it+1}/q_{it}$ , while allowing all other components to vary. We focus on the resulting change in the magnitude of the mean errors: a large change would suggest that the driver in question is quantitatively important.

Panel B of Table 4 reports several insights. First, the most important driver for the SUE portfolio returns is  $q_{it+1}/q_{it}$ : eliminating its cross-sectional variation makes the  $q$ -theory model underpredict the average stock return of the high-minus-low SUE portfolio by a mean error of 8.85% per annum. In contrast, this error is only  $-0.40\%$  in Table 3.  $Y_{it+1}/K_{it+1}$  also is important: without its cross-sectional variation, the mean error of the high-minus-low SUE portfolio becomes 4.31%.

Second, investment and leverage are both important for the B/M portfolios. Fixing  $I_{it}/K_{it}$  to its cross-sectional average produces a mean error of 90.23% per annum for the high-minus-low B/M portfolio. This huge error mostly reflects the large estimate of the parameter  $a$  for this set of testing portfolios. Setting  $w_{it}$  to its cross-sectional average produces a mean error of 11.58% for the high-minus-low B/M portfolio.  $Y_{it+1}/K_{it+1}$  and  $q_{it+1}/q_{it}$  also play a role, but they are quantitatively less important than  $w_{it}$ . Third, the dominating force in driving the average stock returns across the CI portfolios is  $I_{it}/K_{it}$ . Eliminating its cross-sectional variation gives rise to a mean error of  $-8.53\%$  per annum for the high-minus-low CI portfolio. Fixing  $q_{it+1}/q_{it}$  produces a substantial mean error of 4.60%, and  $w_{it}$  contributes 2.71% per annum. The effect of  $Y_{it+1}/K_{it+1}$  is negligible.

### 4.3 The $q$ -theory Model: Matching Both Expected Returns and Variances

#### 4.3.1 Point Estimates and Overall Model Performance

Panel B of Table 2 reports the point estimates and overall model performance when we use the  $q$ -theory model to match both the expected returns and variances of the testing portfolios. Capital's share,  $\alpha$ , is estimated from 0.35 to 0.61, and all estimates are significant. The estimates of the adjustment cost parameter,  $a$ , are on average higher than those reported in Panel A. The estimates are 11.48 and 16.23 for the B/M and CI portfolios, and both are significant. The estimate of  $a$  for the SUE portfolios is 28.88, but with a large standard error of 16.25.

As explained in Erickson and Whited (2000), it can be misleading to interpret the parameter  $a$  in terms of adjustment costs or speeds. We therefore follow their suggestion of gauging the economic magnitude of this parameter in terms of the elasticity of investment with respect to marginal  $q$ . Evaluated at the sample mean, this elasticity is given by  $1/a$  times the ratio of the mean of  $q_{it}$  to the mean of  $I_{it}/K_{it}$ . The estimates in Panel B imply elasticities that range from 0.35 to 0.65. A similar inelastic response of 0.11 is implied by the estimate of  $a$  for the B/M portfolios in Panel A. However, the implied elasticity for the SUE portfolios is greater than one, and that for the CI portfolios is over ten. Although this last estimate seems unreasonable, the others fall in a reasonable range between zero and 1.3, and the general inference given by these estimates is that investment responds to  $q$  inelastically.

Panel B of Table 2 reports three tests of overall model performance.  $\chi^2_{(2)}$  is the  $\chi^2$  test that all the variance errors are jointly zero,  $\chi^2_{(1)}$  is the  $\chi^2$  test that all the mean errors are jointly zero, and the statistic labeled  $\chi^2$  tests that all the mean and variance errors are jointly zero. The  $\chi^2_{(2)}$  tests do not reject the model. More important, the average variance errors, denoted a.a.p.e.(2), are relatively small. To better interpret their economic magnitude, we use the parameter estimates from Panel B of Table 2 to calculate the average levered investment return volatility (instead of variance). At 20.43%, this average predicted volatility is close to the average realized volatility, 21.06%, across the ten SUE portfolios. For the ten B/M portfolios, the average stock return volatility is 24.99%, and the average levered investment return volatility is 23.55%. Finally, for the ten CI portfolios the average stock return volatility is 24.84%, and their average levered investment return volatility is 24.35%.

These volatility results complement Cochrane's (1991) in several ways. First, we account for leverage, while Cochrane does not. Second, we use portfolios as testing assets, in which firm-specific shocks are unlikely to be completely diversified away, while Cochrane studies the stock market portfolio. Third, and most important, we formally choose parameters to match volatilities, while Cochrane calibrates his parameters to match means exactly but allows volatilities to vary freely.

Although the  $\chi^2_{(1)}$  tests on the mean errors do not reject the model, the magnitudes of the mean errors, denoted a.a.p.e.(1), are relatively large. The a.a.p.e.(1) for the SUE portfolios is 3.45% per annum, up from 0.74% when matching expected returns only. The a.a.p.e.(1) for the B/M portfolios increases slightly from 2.32% to 2.58%, while that for the CI portfolios goes up from 1.51% to 2.22%. This increase is to be expected because we are asking more of the model by matching more moments.

### 4.3.2 Euler Equation Errors

Panel B of Table 3 reports individual variance errors, defined as in equation (9), and mean errors, defined as in equation (8), in which levered investment returns,  $r_{it+1}^{Iw}$ , are constructed using the estimates from Panel B of Table 2. The  $t$ -statistics of the errors, described in Appendix B, are calculated using the variance-covariance matrix from one-stage GMM.

Panel B of Table 3 shows that the magnitude of the variance errors is generally small relative to stock return variances. Most variance errors are insignificant. The left panels in Figure 5 plot levered investment return volatilities against stock return volatilities for the testing portfolios. To facilitate interpretation, we plot volatilities instead of variances. The points in the scatter plot are generally aligned with the 45-degree line. However, while there is no discernible relation between stock return volatilities and the characteristics in the data, the model predicts a negative relation between levered investment return volatilities and SUE (Panel A) and a positive relation between the predicted volatilities and B/M (Panel C). Panel B of Table 3 also shows that the variance errors increase with SUE and decrease with B/M. The difference in the variance errors is 0.08 ( $t = 1.83$ ) between the high and low SUE portfolios and is  $-0.20$  ( $t = -2.39$ ) between the high and low B/M portfolios.

Panel B of Table 3 shows that the mean error varies systematically with SUE: it increases from  $-6.99\%$  per annum for the low SUE portfolio to  $5.38\%$  for the high SUE portfolio. The difference of  $12.37\%$  ( $t = 2.51$ ) is similar in magnitude to those from the traditional models. Panel B of Figure 5 plots the average levered investment returns against the average stock returns. The pattern is largely horizontal, similar to those from the traditional models.

The mean errors for the B/M portfolios in Panel B also are larger than those in Panel A from matching only expected returns. However, the model still predicts an average return spread of  $11.26\%$  per annum between the extreme B/M portfolios. The mean error for the high-minus-low B/M portfolio is  $5.89\%$  per annum in the  $q$ -theory model, which is lower than  $7.30\%$  from the Fama-French model. (The CAPM and the standard consumption CAPM produce even higher mean errors,  $18.56\%$  and  $12.31\%$ , respectively.) The model's performance in reproducing the average returns of the CI portfolios deteriorates to the same level as in the traditional models. The mean error difference between the high and low CI portfolios is  $-6.60\%$ , which is similar to those from the CAPM and the Fama-French model. From Panel F of Figure 5, the scatter plots of average returns from the  $q$ -theory model are largely horizontal, similar to the patterns in Figure 4 for the traditional models.

### 4.3.3 A Correlation Puzzle

As noted, equation (5), taken literally, predicts that stock returns should equal levered investment returns at every data point. We have so far examined the first and second moments of returns that are the focus of much work in financial economics. We can explore yet another (even stronger) prediction of the model: stock returns should be perfectly correlated with levered investment returns.

Table 5 reports that the contemporaneous time series correlations between stock and levered investment returns are weakly negative, while those between one-period-lagged stock returns and levered investment returns are positive. When we pool all the observations in the SUE portfolios together, the contemporaneous correlation is  $-0.11$ , which is significant at the 5% level. However, the correlation between one-period-lagged stock returns and levered investment returns is  $0.19$ , which is significant at the 1% level. Replacing levered investment returns with investment growth yields similar results, meaning that the correlations are insensitive to the investment return specifications.

Lamont (2000) shows that investment lags (lags between the decision to invest and the actual investment expenditure) can temporally shift the correlations between investment growth and stock returns. Lags prevent firms from adjusting investment immediately in response to discount rate changes. Consider a one-year lag. A discount rate fall in year  $t$  increases investment only in year  $t+1$ . When stock returns rise in year  $t$  (due to the discount rate fall), investment growth rises in year  $t+1$ : lagged stock returns should be positively correlated with investment growth. The discount rate fall in year  $t$  also means low average stock returns in year  $t+1$ , coinciding with high investment growth in year  $t+1$ . As such, the contemporaneous correlation between stock returns and investment growth should be negative. These lead-lag correlations are consistent with the evidence in Table 5.

However, incorporating investment lags into the model is beyond our scope. After all, these lags appear less important for the first and potentially even the second moments of stock returns that we focus on. Incorporating lags will likely only improve the fit along these two crucial dimensions.

## 5 Summary and Future Work

We use GMM to estimate a structural model of cross-sectional stock returns derived from the  $q$ -theory of investment. We construct empirical first and second moment conditions based on the  $q$ -theory prediction that stock returns equal levered investment returns, the latter of which can be constructed from firm characteristics. Our parsimonious model (with only two parameters) goes a long way toward capturing the average returns of stock portfolios sorted by earnings surprises,

book-to-market equity, and capital investment. The volatilities from the model also are empirically plausible. However, the model falls short in matching expected returns and volatilities simultaneously and in reproducing the correlation structure between stock returns and investment growth. In sum, we interpret our results as saying that on average portfolios of firms do a good job of aligning investment policies with their costs of capital, and that this alignment drives many stylized facts in cross-sectional stock returns. In particular, because we avoid the parametrization of the stochastic discount factor, our work is silent about why average return spreads across characteristics-sorted portfolios are not matched with spreads in covariances.

We view our contribution as mainly providing a simple structure that links the cross section of returns to characteristics in an economically interpretable way. To preserve the transparency of the economic mechanisms that drive our results, we have not searched deliberately for alternative specifications to maximize the model fit. This simplicity leaves open many paths for future work.

One can introduce capital heterogeneity, labor adjustment costs, costly reversibility, flow fixed costs of production (or investment), financing constraints, investment-specific technological shocks, and non-quadratic adjustment costs, while preserving the closed-form Euler equation tests. Decreasing returns to scale, investment lags, and pure fixed costs can be incorporated as well. However, the analytical link between stock and investment returns breaks, and the resulting models can only be estimated via simulation-based methods. The asset pricing literature on corporate bonds has traditionally built on the contingent-claims framework with exogenous investment and cash flows. Taking stock returns as given, one can turn equation (4) around as a theory for bond returns. A dynamic trade-off theory of optimal leverage can be embedded into the  $q$ -theory framework to provide a more complete description of the relations between leverage and expected stock and bond returns.

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## A Proof of Proposition 1

Let  $q_{it}$  be the Lagrangian multiplier associated with  $K_{it+1} = I_{it} + (1 - \delta_{it})K_{it}$ , and  $q_{it}$  is therefore the expected present value of the marginal benefits of an additional unit of capital. The optimality conditions with respect to  $I_{it}$ ,  $K_{it+1}$ , and  $B_{it+1}$  from maximizing equation (2) are, respectively,

$$q_{it} = 1 + (1 - \tau_t) \frac{\partial \Phi(I_{it}, K_{it})}{\partial I_{it}} \quad (\text{A1})$$

$$q_{it} = E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left[ \frac{\partial \Pi(K_{it+1}, X_{it+1})}{\partial K_{it+1}} - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial K_{it+1}} \right] + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) q_{it+1} \right] \right] \quad (\text{A2})$$

$$1 = E_t \left[ M_{t+1} \left[ r_{it+1}^B - (r_{it+1}^B - 1) \tau_{t+1} \right] \right]. \quad (\text{A3})$$

Equation (A1) equates the marginal purchase and adjustment costs of investing to the marginal benefit,  $q_{it}$  (marginal  $q$ ). Equation (A2) is the investment Euler condition, which describes the evolution of  $q_{it}$ . The term  $(1 - \tau_{t+1}) \partial \Pi(K_{it+1}, X_{it+1}) / \partial K_{it+1}$  captures the marginal after-tax profit generated by an additional unit of capital at  $t + 1$ , the term  $-(1 - \tau_{t+1}) \partial \Phi(I_{it+1}, K_{it+1}) / \partial K_{it+1}$  captures the marginal after-tax reduction in adjustment costs, the term  $\tau_{t+1} \delta_{it+1}$  is the marginal depreciation tax shield, and the term  $(1 - \delta_{it+1}) q_{it+1}$  is the marginal continuation value of an extra unit of capital net of depreciation. Discounting these marginal profits of investment dated  $t + 1$  back to  $t$  using the stochastic discount factor yields  $q_{it}$ .

Dividing both sides of equation (A2) by  $q_{it}$  and substituting equation (A1), we obtain  $E_t[M_{t+1} r_{it+1}^I] = 1$ , in which  $r_{it+1}^I$  is the investment return, defined as:

$$r_{it+1}^I \equiv \frac{(1 - \tau_{t+1}) \left[ \frac{\partial \Pi(K_{it+1}, X_{it+1})}{\partial K_{it+1}} - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial K_{it+1}} \right] + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) \left[ 1 + (1 - \tau_{t+1}) \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial I_{it+1}} \right]}{1 + (1 - \tau_t) \frac{\partial \Phi(I_{it}, K_{it})}{\partial I_{it}}}. \quad (\text{A4})$$

The investment return is the ratio of the marginal benefit of investment at time  $t + 1$  divided by the marginal cost of investment at  $t$ . Substituting  $\partial \Pi(K_{it+1}, X_{it+1}) / \partial K_{it+1} = \alpha Y_{it+1} / K_{it+1}$  and  $\Phi(I_{it}, K_{it}) = (a/2)(I_{it}/K_{it})^2 K_{it}$  into equation (A4) yields the investment return equation (3).

Equation (A3) says that  $E_t[M_{t+1} r_{it+1}^B] = 1 + E_t[M_{t+1} (r_{it+1}^B - 1) \tau_{t+1}]$ . Intuitively, because of the tax benefit of debt, the unit price of the pre-tax bond return,  $E_t[M_{t+1} r_{it+1}^B]$ , is higher than unity. The difference is precisely the present value of the tax benefit. Because we define the after-tax corporate bond return,  $r_{it+1}^{Ba} \equiv r_{it+1}^B - (r_{it+1}^B - 1) \tau_{t+1}$ , equation (A3) says that the unit price of the after-tax corporate bond return is one:  $E_t[M_{t+1} r_{it+1}^{Ba}] = 1$ .

To prove equation (4), we first show that  $q_{it} K_{it+1} = P_{it} + B_{it+1}$  under constant returns to scale.

We start with  $P_{it} + D_{it} = V_{it}$  and expand  $V_{it}$  using equations (1) and (2):

$$\begin{aligned}
& P_{it} + (1 - \tau_t)[\Pi(K_{it}, X_{it}) - \Phi(I_{it}, K_{it}) - r_{it}^B B_{it}] - \tau_t B_{it} - I_{it} + B_{it+1} + \tau_t \delta_{it} K_{it} = \\
& (1 - \tau_t) \left[ \Pi(K_{it}, X_{it}) - \frac{\partial \Phi(I_{it}, K_{it})}{\partial I_{it}} I_{it} - \frac{\partial \Phi(I_{it}, K_{it})}{\partial K_{it}} K_{it} - r_{it}^B B_{it} \right] - \tau_t B_{it} - I_{it} + B_{it+1} + \tau_t \delta_{it} K_{it} \\
& - q_{it}(K_{it+1} - (1 - \delta_{it})K_{it} - I_{it}) + E_t[M_{t+1}((1 - \tau_t) \left[ \Pi(K_{it+1}, X_{it+1}) - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial I_{it+1}} I_{it+1} \right. \\
& \left. - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial K_{it+1}} K_{it+1} - r_{it+1}^B B_{it+1} \right] - \tau_{t+1} B_{it+1} - I_{it+1} + B_{it+2} \\
& \left. + \tau_{t+1} \delta_{it+1} K_{it+1} - q_{it+1}(K_{it+2} - (1 - \delta_{it+1})K_{it+1} - I_{it+1}) + \dots \right] \tag{A5}
\end{aligned}$$

Recursively substituting equations (A1), (A2), and (A3), and simplifying, we obtain:

$$\begin{aligned}
& P_{it} + (1 - \tau_t)[\Pi(K_{it}, X_{it}) - \Phi(I_{it}, K_{it}) - r_{it}^B B_{it}] - \tau_t B_{it} - I_{it} + B_{it+1} + \tau_t \delta_{it} K_{it} = \\
& (1 - \tau_t) \left[ \Pi(K_{it}, X_{it}) - \frac{\partial \Phi(I_{it}, K_{it})}{\partial K_{it}} K_{it} - r_{it}^B B_{it} \right] - \tau_t B_{it} + q_{it}(1 - \delta_{it})K_{it} + \tau_t \delta_{it} K_{it} \tag{A6}
\end{aligned}$$

Simplifying further and using the linear homogeneity of  $\Phi(I_{it}, K_{it})$  yield:

$$P_{it} + B_{it+1} = (1 - \tau_t) \frac{\partial \Phi(I_{it}, K_{it})}{\partial I_{it}} I_{it} + I_{it} + q_{it}(1 - \delta_{it})K_{it} = q_{it} K_{it+1} \tag{A7}$$

Finally, we are ready to prove equation (4):

$$\begin{aligned}
w_{it} r_{it+1}^{Ba} + (1 - w_{it}) r_{it+1}^S &= \frac{\left[ \begin{aligned} & (1 - \tau_{t+1}) r_{it+1}^B B_{it+1} + \tau_{t+1} B_{it+1} + P_{it+1} \\ & + (1 - \tau_{t+1}) [\Pi(K_{it+1}, X_{it+1}) - \Phi(I_{it+1}, K_{it+1}) - r_{it+1}^B B_{it+1}] \\ & - \tau_{t+1} B_{it+1} - I_{it+1} + B_{it+2} + \tau_{t+1} \delta_{it+1} K_{it+1} \end{aligned} \right]}{P_{it} + B_{it+1}} \\
&= \frac{1}{q_{it} K_{it+1}} \left[ \begin{aligned} & q_{it+1}(I_{it+1} + (1 - \delta_{it+1})K_{it+1}) + (1 - \tau_{t+1}) [\Pi(K_{it+1}, X_{it+1}) \\ & - \Phi(I_{it+1}, K_{it+1})] - I_{it+1} + \tau_{t+1} \delta_{it+1} K_{it+1} \end{aligned} \right] \\
&= \frac{q_{it+1}(1 - \delta_{it+1}) + (1 - \tau_{t+1}) \left[ \frac{\partial \Pi(K_{it+1}, X_{it+1})}{\partial K_{it+1}} - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial K_{it+1}} \right] + \tau_{t+1} \delta_{it+1}}{q_{it}} = r_{it+1}^I. \tag{A8}
\end{aligned}$$

## B Estimation Details

Following the standard GMM procedure (e.g., Hansen and Singleton (1982)), we estimate the parameters,  $\mathbf{b} \equiv (a, \alpha)$ , to minimize a weighted combination of the sample moments (8) or (8) and (9). Specifically, let  $\mathbf{g}_T$  be the sample moments. The GMM objective function is a weighted sum of squares of the model errors across assets,  $\mathbf{g}'_T \mathbf{W} \mathbf{g}_T$ , in which we use  $\mathbf{W} = \mathbf{I}$ , the identity matrix. Let  $\mathbf{D} = \partial \mathbf{g}_T / \partial \mathbf{b}$  and  $\mathbf{S}$  a consistent estimate of the variance-covariance matrix of the sample errors  $\mathbf{g}_T$ . We estimate  $\mathbf{S}$  using a standard Bartlett kernel with a window length of five.

The estimate of  $\mathbf{b}$ , denoted  $\hat{\mathbf{b}}$ , is asymptotically normal with variance-covariance matrix:

$$\text{var}(\hat{\mathbf{b}}) = \frac{1}{T} (\mathbf{D}'\mathbf{W}\mathbf{D})^{-1} \mathbf{D}'\mathbf{W}\mathbf{S}\mathbf{W}\mathbf{D} (\mathbf{D}'\mathbf{W}\mathbf{D})^{-1} \quad (\text{B1})$$

To construct standard errors for the model errors on individual portfolios or groups of model errors, we use the variance-covariance matrix for the model errors,  $\mathbf{g}_T$ :

$$\text{var}(\mathbf{g}_T) = \frac{1}{T} [\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}] \mathbf{S} [\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}]' \quad (\text{B2})$$

In particular, the  $\chi^2$  test whether all model errors are jointly zero is given by:

$$\mathbf{g}'_T [\text{var}(\mathbf{g}_T)]^+ \mathbf{g}_T \sim \chi^2(\# \text{ moments} - \# \text{ parameters}) \quad (\text{B3})$$

The superscript  $+$  denotes pseudo-inversion.

**Table 1 : Descriptive Statistics of Testing Portfolio Returns**

For testing portfolio  $i$ , we report in annualized percent the average stock return,  $\bar{r}_i^S$ , the stock return volatility,  $\sigma_i^S$ , the intercept from the CAPM regression,  $e_i$ , the intercept from the Fama-French three-factor regression,  $e_i^{FF}$ , and the model error from the standard consumption CAPM,  $e_i^C$ . In each panel we only report results for three (Low, 5, and High) out of ten portfolios to save space. The H–L portfolio is long in the high portfolio and short in the low portfolio. The heteroscedasticity-and-autocorrelation-consistent  $t$ -statistics for the model errors are reported in brackets beneath the corresponding errors. a.a.p.e. is the average of the absolute values of the errors for a given set of ten testing portfolios. For the CAPM and the Fama-French model, the  $p$ -values in brackets in the last column are for the Gibbons, Ross, and Shanken (1989) tests of the null hypothesis that the intercepts for a given set of ten portfolios are jointly zero. For the standard consumption CAPM the  $p$ -values are for the  $\chi^2$  test from one-stage GMM that the moment restrictions for all ten portfolios are jointly zero. In Panel A for the standard consumption CAPM the estimate of the time preference coefficient is  $\beta = 2.76$  with a standard error (ste) of 1.05 and the estimate of risk aversion is  $\gamma = 127.59$  (ste = 59.07). In Panel B  $\beta = 3.31$  (ste = 1.38) and  $\gamma = 142.08$  (ste = 63.73). In Panel C  $\beta = 3.30$  (ste = 1.39) and  $\gamma = 143.28$  (ste = 62.71).

	Low	5	High	H–L	a.a.p.e.	[ $p$ ]	Low	5	High	H–L	a.a.p.e.	[ $p$ ]	Low	5	High	H–L	a.a.p.e.	[ $p$ ]
	Panel A: Ten SUE portfolios						Panel B: Ten B/M portfolios						Panel C: Ten CI portfolios					
$\bar{r}_i^S$	10.89	18.95	23.39	12.50			8.65	17.93	25.78	17.13			22.12	18.10	15.16	–6.96		
$\sigma_i^S$	22.35	22.51	21.13	8.46			27.93	24.92	26.97	20.54			32.42	22.26	26.73	11.37		
$e_i$	–1.69	6.56	10.86	12.55	5.67	[0.00]	–4.91	5.19	13.65	18.56	6.34	[0.00]	8.21	5.89	1.91	–6.30	5.71	[0.01]
[ $t$ ]	[–0.84]	[2.83]	[5.74]	[5.53]			[–2.11]	[2.59]	[4.66]	[2.51]			[2.91]	[3.10]	[0.87]	[–3.88]		
$e_i^{FF}$	–4.59	1.96	9.47	14.06	4.01	[0.00]	–0.54	1.80	6.76	7.30	2.79	[0.00]	6.45	1.54	0.11	–6.34	2.24	[0.01]
[ $t$ ]	[–2.27]	[0.89]	[5.20]	[5.31]			[–0.24]	[1.08]	[3.10]	[3.25]			[2.81]	[1.12]	[0.06]	[–3.99]		
$e_i^C$	–8.07	–0.04	5.31	13.38	3.62	[0.00]	–5.43	0.27	6.88	12.31	2.36	[0.00]	4.03	0.46	–4.35	–8.38	2.36	[0.00]
[ $t$ ]	[–1.19]	[0.01]	[1.36]	[1.35]			[–0.66]	[0.06]	[1.96]	[0.26]			[0.75]	[0.11]	[–0.71]	[–1.35]		

**Table 2 : Parameter Estimates and Tests of Overidentification**

Estimates and tests are from one-stage GMM estimation with the identity weighting matrix. In Panel A the moment conditions are  $E[r_{it+1}^S - r_{it+1}^{Iw}] = 0$ .  $a$  is the adjustment cost parameter and  $\alpha$  is capital's share. Their standard errors, denoted ste, are reported in brackets beneath the estimates.  $\chi^2$  is the statistic from one-stage GMM that the moment conditions are jointly zero. d.f. is the degrees of freedom, and  $p$  is the  $p$ -value associated with the test. a.a.p.e. is the average absolute value of the model errors,  $E_T[r_{it+1}^S - r_{it+1}^{Iw}]$ , in which  $E_T[\cdot]$  is the sample mean of the series in brackets, in annual percent across a given set of testing portfolios. In Panel B the moment conditions are  $E[r_{it+1}^S - r_{it+1}^{Iw}] = 0$  and  $E[(r_{it+1}^S - E[r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E[r_{it+1}^{Iw}])^2] = 0$ .  $\chi_{(2)}^2$ , d.f.(2), and  $p(2)$  are the statistic, degrees of freedom, and  $p$ -value for the  $\chi^2$  test that the variance errors, defined as  $E_T[(r_{it+1}^S - E_T[r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E_T[r_{it+1}^{Iw}])^2]$ , are jointly zero. a.a.p.e.(2) is the average magnitude of the variance errors in annual decimals.  $\chi_{(1)}^2$ , d.f.(1), and  $p(1)$  are the statistic, degrees of freedom, and  $p$ -value for the  $\chi^2$  test that the mean errors, defined in the same way as in Panel A, are jointly zero. a.a.p.e.(1) is the average magnitude of the mean errors in annual percent.  $\chi^2$ , d.f., and  $p$  are the statistic, degrees of freedom, and  $p$ -value of the test that both the mean and variance errors are jointly zero.

Panel A: Matching expected returns				Panel B: Matching expected returns and variances			
	SUE	B/M	CI		SUE	B/M	CI
$a$	7.68	22.34	0.97	$a$	28.88	11.48	16.23
[ste]	[1.72]	[25.47]	[0.29]	[ste]	[16.25]	[4.75]	[5.53]
$\alpha$	0.32	0.50	0.21	$\alpha$	0.61	0.35	0.36
[ste]	[0.03]	[0.31]	[0.02]	[ste]	[0.27]	[0.07]	[0.08]
$\chi^2$	4.37	5.99	6.52	$\chi_{(2)}^2$	5.14	6.18	6.05
d.f.	8	8	8	d.f.(2)	8	8	8
$p$	0.82	0.65	0.59	$p(2)$	0.74	0.63	0.64
a.a.p.e.	0.74	2.32	1.51	a.a.p.e.(2)	0.03	0.04	0.02
				$\chi_{(1)}^2$	5.22	4.38	4.81
				d.f.(1)	8	8	8
				$p(1)$	0.73	0.82	0.78
				a.a.p.e.(1)	3.45	2.58	2.22
				$\chi^2$	5.45	6.17	6.62
				d.f.	18	18	18
				$p$	1.00	1.00	0.99

**Table 3 : Euler Equation Errors**

Euler equation errors and  $t$ -statistics are from one-stage GMM estimation with an identity weighting matrix. In Panel A the moment conditions are  $E[r_{it+1}^S - r_{it+1}^{Iw}] = 0$ . The mean errors are defined as  $e_i^q \equiv E_T[r_{it+1}^S - r_{it+1}^{Iw}]$ , in which  $E_T[\cdot]$  is the sample mean of the series in brackets. In Panel B the moment conditions are  $E[r_{it+1}^S - r_{it+1}^{Iw}] = 0$  and  $E[(r_{it+1}^S - E[r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E[r_{it+1}^{Iw}])^2] = 0$ . The variance errors are defined as  $e_i^{\sigma^2} \equiv E_T[(r_{it+1}^S - E_T[r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E_T[r_{it+1}^{Iw}])^2]$ . The mean errors are defined as in Panel A. In the last column we report the difference in the mean errors and the difference in the variance errors between the high and low portfolios, as well as their  $t$ -statistics. Mean errors are in annual percent, and variance errors are in annual decimals. In each set of ten portfolios, we only report results for three (Low, 5, and High) out of the ten portfolios to save space.

	Low	5	High	H-L	Low	5	High	H-L	Low	5	High	H-L
Panel A: Euler equation errors from matching expected returns												
	Ten SUE portfolios				Ten B/M portfolios				Ten CI portfolios			
$e_i^q$	0.26	1.66	-0.15	-0.40	-3.94	2.35	-2.73	1.21	-0.97	2.72	-1.45	-0.49
$[t]$	[0.61]	[1.70]	[-0.14]	[-0.41]	[-1.76]	[1.37]	[-1.37]	[0.79]	[-0.51]	[1.74]	[-1.24]	[-0.41]
Panel B: Euler equation errors from matching expected returns and variances												
	Ten SUE portfolios				Ten B/M portfolios				Ten CI portfolios			
$e_i^{\sigma^2}$	-0.04	0.02	0.03	0.08	0.10	0.01	-0.10	-0.20	0.01	0.03	-0.06	-0.07
$[t]$	[-1.93]	[0.95]	[1.47]	[1.83]	[2.35]	[0.50]	[-1.99]	[-2.39]	[0.34]	[1.33]	[-1.77]	[-1.36]
$e_i^q$	-6.99	2.60	5.38	12.37	-6.46	1.71	-0.58	5.89	1.29	3.47	-5.32	-6.60
$[t]$	[-2.24]	[1.91]	[2.01]	[2.51]	[-1.89]	[0.94]	[-0.15]	[1.08]	[0.49]	[1.97]	[-1.97]	[-2.04]

**Table 4 : Expected Returns Accounting**

Panel A reports the averages of investment-to-capital,  $I_{it}/K_{it}$ , future investment growth,  $(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$ , sales-to-capital,  $Y_{it+1}/K_{it+1}$ , the depreciation rate,  $\delta_{it+1}$ , market leverage,  $w_{it}$ , and corporate bond returns in annual percent,  $r_{it+1}^B$ . In each set of ten portfolios we only report results for three (Low, 5, and High) out of the ten portfolios to save space. The column H-L reports the average differences between high and low portfolios and the column  $[t_{H-L}]$  reports the heteroscedasticity-and-autocorrelation-consistent  $t$ -statistics for the test that the differences equal zero. Panel B performs four comparative static experiments denoted  $\overline{I_{it}/K_{it}}$ ,  $\overline{q_{it+1}/q_{it}}$ ,  $\overline{Y_{it+1}/K_{it+1}}$ , and  $\overline{w_{it}}$ , in which  $q_{it+1}/q_{it} = [1 + (1 - \tau_{t+1})a(I_{it+1}/K_{it+1})]/[1 + (1 - \tau_t)a(I_{it}/K_{it})]$ . In the experiment denoted  $\overline{Y_{it+1}/K_{it+1}}$ , we set  $Y_{it+1}/K_{it+1}$  for a given set of ten portfolios, indexed by  $i$ , to be its cross-sectional average in  $t+1$ . We then use the parameters reported in Panel A of Table 2 to reconstruct the levered investment returns, while keeping all the other characteristics unchanged. The other three experiments are designed analogously. We report the mean errors defined as  $e_i^q \equiv E_T [r_{it+1}^S - r_{it+1}^w]$  for the testing portfolios, the high-minus-low portfolios, and the average absolute value of  $e_i^q$  (a.a.p.e.) across a given set of ten testing portfolios.

Panel A: Characteristics in levered investment returns															
	Low	5	High	H-L	$[t_{H-L}]$	Low	5	High	H-L	$[t_{H-L}]$	Low	5	High	H-L	$[t_{H-L}]$
	Ten SUE portfolios					Ten B/M portfolios					Ten CI portfolios				
$I_{it}/K_{it}$	0.12	0.11	0.12	0.00	[0.70]	0.18	0.11	0.08	-0.10	[-7.95]	0.09	0.11	0.16	0.07	[11.06]
$(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$	0.89	1.00	1.06	0.17	[4.06]	0.98	1.00	1.02	0.04	[0.68]	1.25	1.04	0.81	-0.44	[-7.23]
$Y_{it+1}/K_{it+1}$	1.52	1.50	1.83	0.31	[5.16]	1.95	1.45	1.38	-0.57	[-6.77]	1.84	1.58	1.89	0.05	[0.38]
$\delta_{it+1}$	0.08	0.08	0.08	0.00	[0.63]	0.10	0.07	0.07	-0.03	[-5.01]	0.08	0.07	0.08	0.00	[-0.46]
$w_{it}$	0.30	0.28	0.21	-0.10	[-5.83]	0.08	0.27	0.53	0.44	[12.44]	0.35	0.25	0.28	-0.07	[-2.59]
$r_{it+1}^B$	9.44	9.76	9.38	-0.06	[-0.27]	8.17	8.09	8.52	0.35	[1.05]	8.47	8.27	8.44	-0.03	[-0.15]
Panel B: Mean errors from comparative static experiments															
	Low	5	High	H-L	a.a.p.e.	Low	5	High	H-L	a.a.p.e.	Low	5	High	H-L	a.a.p.e.
	Ten SUE portfolios					Ten B/M portfolios					Ten CI portfolios				
$\overline{I_{it}/K_{it}}$	-2.48	4.45	-4.26	-1.78	2.35	-42.06	4.69	48.17	90.23	21.25	2.86	3.50	-5.67	-8.53	2.26
$\overline{q_{it+1}/q_{it}}$	-5.23	1.76	3.62	8.85	2.62	-1.92	2.11	-4.06	-2.14	1.87	0.73	2.97	-3.87	-4.60	1.71
$\overline{Y_{it+1}/K_{it+1}}$	-0.78	0.39	3.53	4.31	1.34	0.16	0.92	-6.33	-6.49	1.94	0.57	-0.44	0.09	-0.48	0.40
$\overline{w_{it}}$	0.13	1.89	-1.46	-1.58	0.88	-6.00	2.19	5.58	11.58	3.57	1.80	2.61	-0.91	-2.71	1.35

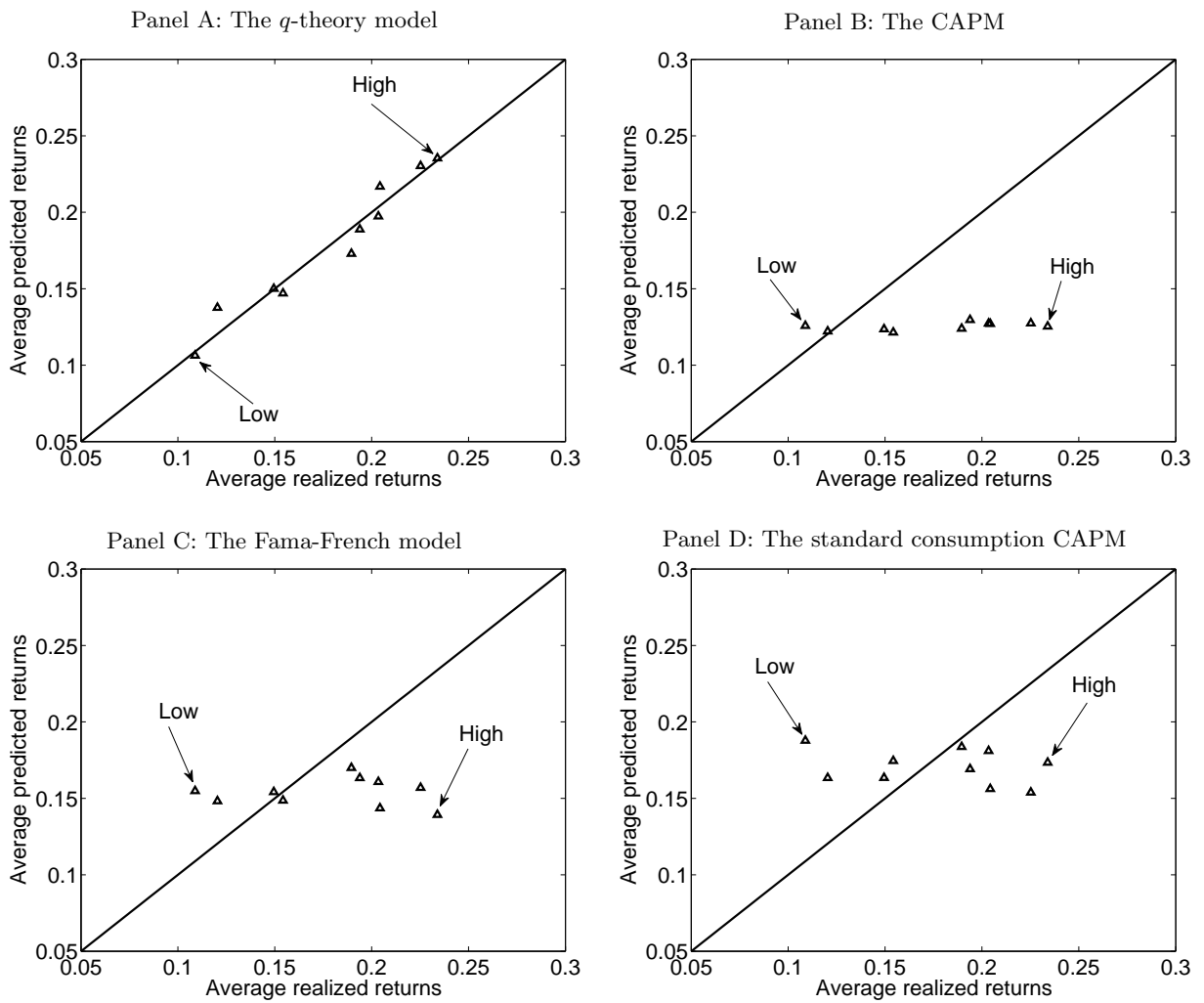
**Table 5 : Correlations**

We report time series correlations of stock returns (contemporaneous,  $r_{it+1}^S$ , and one-period-lagged,  $r_{it}^S$ ) with levered investment returns,  $r_{it+1}^{Iw}$ , and with investment growth,  $I_{it+1}/I_{it}$ . In each panel we only report results for three (Low, 5, and High) out of ten portfolios to save space.  $\rho(\cdot, \cdot)$  denotes the correlation between the two series in the parentheses. We report the significance of a given correlation with a star system: 10%, 5%, and 1% significance levels are indicated by one, two, and three stars, respectively. In the last column, All, we report the correlations and their significance by pooling all the observations for a given set of ten testing portfolios (SUE, B/M, or CI). The levered investment returns are constructed using the parameters in Panel A of Table 2.

	Low	5	High	All	Low	5	High	All	Low	5	High	All
	Panel A: Ten SUE portfolios				Panel B: Ten B/M portfolios				Panel C: Ten CI portfolios			
$\rho(r_{it+1}^S, r_{it+1}^{Iw})$	-0.28	-0.21	-0.26	-0.11**	-0.23	-0.17	-0.05	-0.12**	0.22	-0.34**	-0.30*	-0.06
$\rho(r_{it}^S, r_{it+1}^{Iw})$	0.22	0.01	0.14	0.19***	0.06	0.23	0.33***	0.22***	0.44***	0.16	0.30*	0.21***
$\rho(r_{it+1}^S, I_{it+1}/I_{it})$	-0.29	-0.24	-0.23	-0.08	-0.14	-0.06	-0.13	-0.15***	0.28*	-0.33**	-0.12	-0.04
$\rho(r_{it}^S, I_{it+1}/I_{it})$	0.18	0.01	-0.00	0.14**	0.12	0.17	0.29*	0.14***	0.20	0.10	0.26	0.16***



**Figure 2 : Average Predicted Stock Returns versus Average Realized Stock Returns, Ten SUE Portfolios**



**Figure 3 : Average Predicted Stock Returns versus Average Realized Stock Returns, Ten B/M Portfolios**

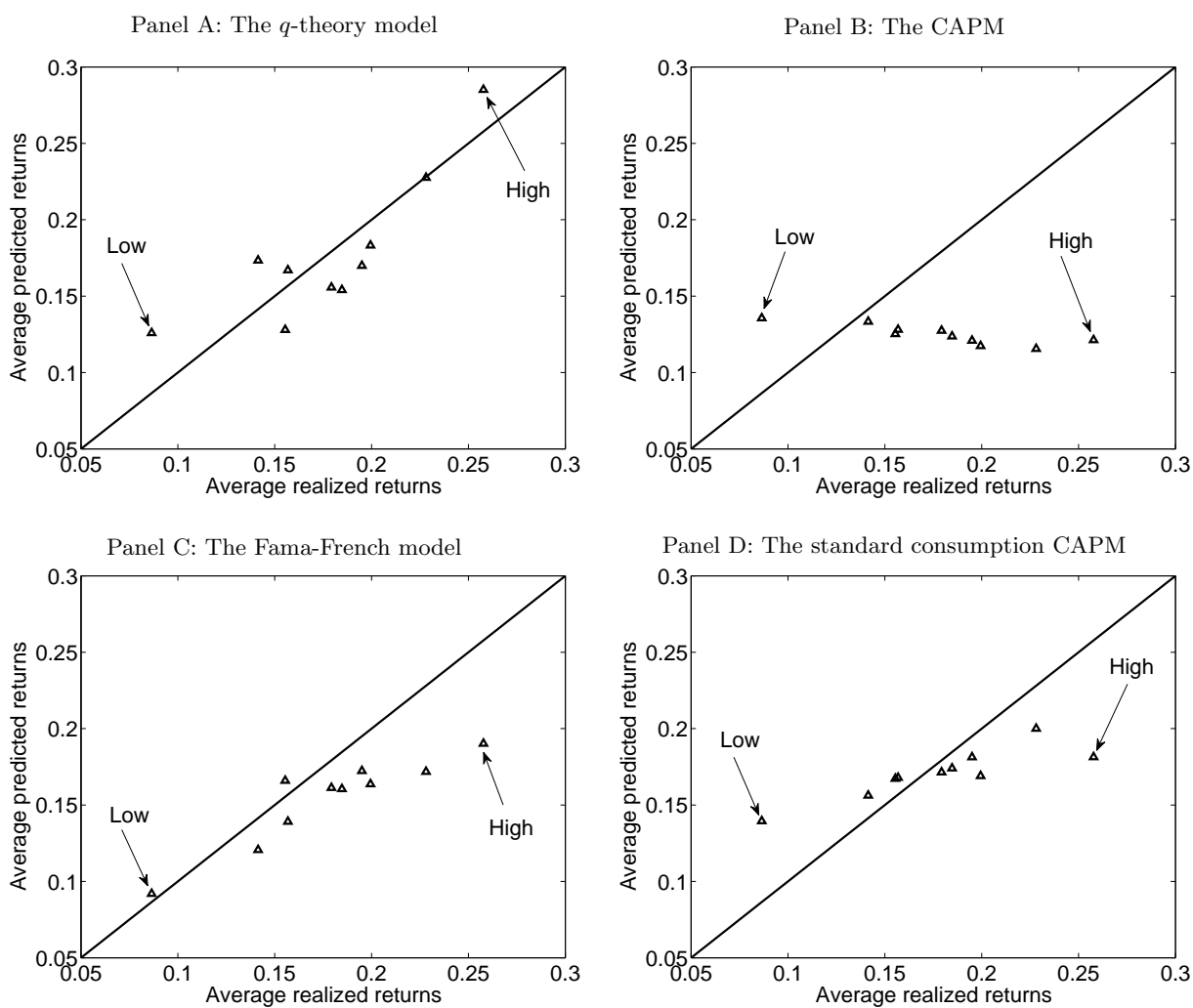
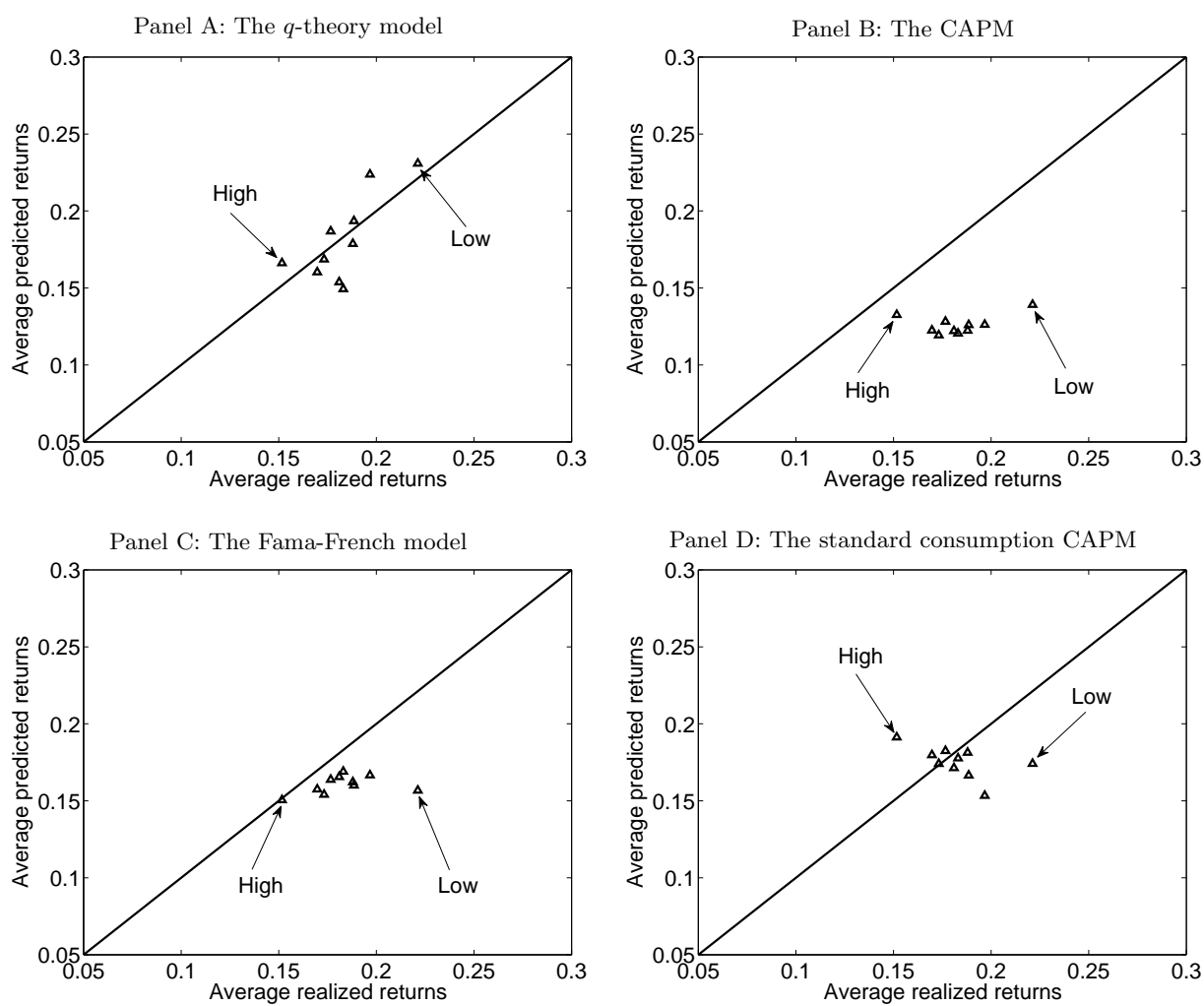


Figure 4 : Average Predicted Stock Returns versus Average Realized Stock Returns, Ten CI Portfolios



**Figure 5 : Predicted Stock Return Volatilities versus Realized Stock Return Volatilities, Average Predicted Stock Returns versus Average Realized Stock Returns, The  $q$ -theory Model, Matching Both Expected Returns and Variances**

