USC FBE FINANCE SEMINAR
presented by Chris Parsons
THURSDAY, Mar. 10, 2011
10:15 am – 11:45 am, **Room: BRI-8**

The cost and timing of financial distress [*]

Redouane Elkamhi

Henry B. Tippie School of Business at the University of Iowa

Jan Ericsson

Desautels Faculty of Management at McGill University

Christopher A. Parsons

Kenan-Flagler School of Business at the University of North Carolina at Chapel Hill

March 9, 2011

Abstract

Assessments of the trade-off theory have typically compared the present value of tax benefits to the present value of *bankruptcy costs*. We show that this comparison overwhelmingly favors tax benefits, suggesting that firms are under-leveraged. However, when we allow firms to experience even modest financial distress costs *prior to bankruptcy* (e.g., investment distortions, damaged stakeholder relationships), the cumulative present value of such costs can easily offset the tax benefits.

1 Introduction

Trade-off theory is the canonical model of capital structure. It prescribes that at the margin, the costs of debt should offset its benefits, so that levered firm value is maximized at this optimum. Yet despite decades of research, there exists little consensus on the theory's empirical relevance. This lack of agreement is largely driven by differences in how researchers have estimated the present value of financial distress costs.[1]

This point can be appreciated by comparing two influential studies of financial distress costs, Andrade and Kaplan (1998) and Almeida and Philippon (2007). Andrade and Kaplan examine a sample of 31 LBO firms during the late 1980s that subsequently became financially distressed. They estimate that upon declaring bankruptcy, these firms lost an additional 10-23% of their value due to both direct and indirect costs.[2] That is, if such a firm's market value were $1 billion when it declared bankruptcy, it would expect to lose another $100-$230 million due to lost customers, restructuring fees, etc. The authors characterize their findings as "high relative to existing estimates (p. 1488)," but also note that they "seem low from an ex ante perspective that trades off the *expected* cost of financial distress against the tax and incentive benefits of debt (emphasis added)."

However, using precisely these estimates, Almeida and Philippon (2007) arrive at the opposite conclusion - namely that ex ante tax benefits and financial distress costs balance one another. To calculate the probability of distress, they extract risk-adjusted default probabilities from credit spreads, which is appropriate given that Andrade and Kaplan also condition on default/bankruptcy. However, they apply Andrade and Kaplan's 10-23% range of proportional losses to the firm's *current value*, rather than to *value near default* from which it was originally estimated. To the extent that a firm is not close to defaulting, this method will overstate the present value of its expected bankruptcy costs.

This paper has two primary goals. The first is to revisit the calculation performed by Almeida and Philippon (2007), but to perform the conditioning required to translate Andrade and Kaplan's (1998) ex post fractions to the ex ante fractions required to estimate the ex ante costs of financial distress. As we will see, this conditioning step reduces expected, risk-adjusted bankruptcy costs to the point where – in the typical case – they are incapable of offsetting the tax benefits of debt. The second goal is to then ask the question: "If

---

[1] Almost exclusively, tax benefits are characterized using estimates from Graham (2000), made available on John Graham's website.

[2] Andrade and Kaplan (1998) calculate distress costs as a percentage of the firm's value roughly six months before it enters bankruptcy. Specifically, they benchmark against the firm's value as of the end of the most recent fiscal year immediately prior to formally declaring bankruptcy, or attempting to restructure debt payments.

losing up to a quarter of a firm's value near default is not sufficient to offset the tax benefits, what is?" To answer this question, we characterize the ex ante costs when firms experience financial distress cost *prior* to declaring bankruptcy, such as investment distortions or damaged relationships with stakeholders. We show that if the trade-off theory is to be empirically validated, it will be necessary for distress costs to be incurred *prior to bankruptcy.*

To address the first question, we begin with a benchmark case that considers bankruptcy to be the triggering event for financial distress. That is, for the time being, we follow Andrade and Kaplan (1998), Graham (2000), Molina (2005), Almeida and Philippon (2007) and ignore any financial distress costs that are incurred prior to bankruptcy. Estimating a structural model involving nearly 500,000 firm-quarter observations from 1970-2007, we find that the present value of expected financial distress costs incurred after bankruptcy comprises less than 1% of current firm value.[3] This differs from recent estimates by Almeida and Philippon (2007), which finds comparable averages in the range of 4-6%. Given such a large discrepancy, it is perhaps unsurprising that our estimates of bankruptcy costs are not large enough to offset the tax benefits of debt, whereas those found by Almeida and Philippon (2007) are.

The reason for the difference can be best understood with a simple numerical example. Take a one period model for a hypothetical firm currently worth $10 billion, of which $2 billion is financed with debt. Further, suppose that if the firm defaults (Almeida and Philippon's definition of the onset of financial distress), it will lose another 16.5% of its value at that time (the midpoint of Andrade and Kaplan's (1998) study of financially distressed LBO firms). Denoting the risk-adjusted probability of default q and assuming a risk-free rate of zero, Almeida and Philippon estimate expected bankruptcy costs as equal to $0.165q$, the product of Andrade and Kaplan's (1998) ex post estimate and the risk-adjusted default probability. For example, if q equals 10%, then the dollar price of expected bankruptcy costs would be ($10 billion)(.1)(.165) = $165 million. This would clearly be correct if q described the probability that the firm defaults *at its current value*, but in general, this is not so.

In the typical case, a firm loses all of its net worth, and more, before entering bankruptcy. In this case that corresponds to a value decline of $10 billion - $2 billion = $8 billion, bankruptcy occurring when value reaches $2 billion. Warner (1977) finds similar figures in his early analysis of railroad bankruptcies, finding that firm values dropped 79% on average from seven years prior to bankruptcy (Table 4, p. 392). More recently, Davydenko (2009) examines a large panel of bankruptcies across industries, finding that the average firm

---

[3]This takes as given that the cost given default is 16.5% of value (at default).

defaults at a value equal to roughly two-thirds of its total liabilities, which here would imply a loss of 86.7% necessary to engender bankruptcy. Applying Andrade and Kaplan's (1998) ex post fraction to this lower level of firm value generates much lower estimates of ex ante bankruptcy costs, and explains the difference between our results and those of Almeida and Philippon (2007).

The inability for bankruptcy costs alone to reconcile trade-off is not an artifact of the structural model we use, nor is it particularly sensitive to the model's parameters. For example, we vary the percentage of value expected to be destroyed in bankruptcy, considering both the upper (23%) and lower (10%) bounds of Andrade and Kaplan's (1998) study. Neither change alters the main conclusion. We also vary the *structure* of bankruptcy costs, allowing both a fixed and proportional component to account for the types of heterogeneity discussed in, e.g., Warner (1977) and Bris, Welch, and Shu (2006). While small firms are most affected – some even enough to reconcile the tax benefits – reasonably sized fixed costs of bankruptcy has a negligible effect for the typical firm. Finally, we experiment with different models for default. We find that only "liquidity-based" defaults that prevent distressed firms from raising capital are sufficient to generate large ex ante bankruptcy costs. While this may be a realistic model for, say, firms in economies without developed financial systems, the default probabilities predicted by such models are far higher than what we observe domestically, and thus, are not applicable to the typical U.S. firm (see also Davydenko (2009)).[4]

With this set of main results established, we move to our second set of analyses. Rather than consider only bankruptcy costs experienced near default, we now permit firms to experience debt-related value losses prior to declaring bankruptcy. Here, we still allow firms to experience costs after declaring bankruptcy, but also penalize them for getting sufficiently *close* to default. Importantly, this allows for firms to enter financial distress when their going concern value becomes questionable (such as losing key employees or risk-shifting), but well before entering bankruptcy. This means that financial distress occurs with not only a higher probability, but also over a larger range of values compared to losses incurred in bankruptcy. Both effects contribute to a higher present value of the total costs of debt.

Our analysis of financial distress costs prior to bankruptcy includes both a general case as well as a specific illustration. In the general case, we simply posit that a firm begins to "leak" value when its credit rating

---

[4]It is noteworthy that in our analysis, both tax benefits and deadweight costs of debt are adjusted for systematic risk. As shown by Almeida and Philippon (2007), the fact that "financial distress is more likely to occur in bad times" implies that an adjustment for the systematic risk of default is important. The same is true for a firm's expected tax benefits, which are more likely to be enjoyed in "good times," when a firm is highly profitable. By combining Graham's firm-specific tax rate estimates (obtained from his website) with a structural model that adjusts for risk, the tax benefits we measure are correct in a pricing sense. Thus, when we compare these to the expected financial distress and bankruptcy costs, the comparison is apples-to-apples.

falls below investment grade, but before defaulting. Here, we are intentionally agnostic about the specific mechanism responsible for the leak, be it lost customers (e.g., Zingales (1998), Opler and Titman (1994)), expropriation from workers (e.g., Sharpe (1994), Bronars and Deere (1991)), coordination problems with suppliers (Titman (1984), Banerjee, Dasgupta, and Kim (2008)), predation by competitors (e.g., Bolton and Scharfstein (1990)), etc. Regardless of the channel, we calibrate the model against the expected tax benefits, asking what financial distress leak rate is necessary (over and above a 16.5% bankruptcy cost) to balance the present values of risk-adjusted expected tax benefits and total financial distress costs. That is, given a process for firm value that pins down the expected tax benefits, here we ask how much annual value loss *given a downgrade below investment grade* would be required to offset them. Interestingly, we find that it does not take much. In the typical case, we find that very modest leak rates (in the range of 1%-2% annually) are sufficient to offset the tax benefits from an ex ante perspective, even for firms with low leverage ratios.

To be more concrete, we impose additional structure, and conclude by considering one particular type of financial distress cost that may be incurred prior to bankruptcy: risk shifting. We endow management with the ability to alter the firm's operating risk, i.e., selecting projects with higher variance (Jensen and Meckling (1976)). In the traditional sense, such risk shifting may destroy value, but is optimal from shareholders' point of view. As in the reduced form case with a pre-default leak rate, we characterize the shifts in asset risk which are necessary to offset the tax benefits of debt. For example, for a typical firm with an 'A' credit rating, an increase in the costs of financial distress associated with an increase in the annual asset volatility from 33% to 41% can balance the tax benefits of debt on an ex ante basis.

Together, these findings are directly relevant for ongoing debates that assess the empirical merits of trade-off theory. For much of its existence, trade-off theory has been battered by a body of evidence suggesting that bankruptcy costs are either too small, or not realized frequently enough, to justify the conservative debt usage we observe. Warner (1977) notes that from an ex ante perspective, his estimates of direct bankruptcy costs for railroads are "small indeed," especially in comparison to the tax savings due to debt. Andrade and Kaplan's (1998) analysis of LBO firms measures both indirect and direct costs incurred after bankruptcy, but arrive at a similar resolution: "the expected costs of financial distress for most public companies are modest if not minimal because the probability of financial distress is very small for most public companies (p. 1489)."

The results herein pertain directly to this claim. Opponents of trade-off theory can argue that even the most

liberal assumptions for bankruptcy losses are inadequate to explain observed leverage ratios.[5] At the same time, we show that relatively small pre-default distress costs – on the order of 1% to 2% of firm value per annum – can restore empirical support for the trade-off theory.

Our study relates directly to the literature measuring ex ante financial distress and bankruptcy costs, and by extension, to those that study optimal capital structure. Almeida and Philippon (2007) compare estimates of ex ante default costs when physical rather than risk-adjusted default probabilities are used. While our results point to smaller expected, risk-adjusted default losses, our main innovation is allowing distress costs to occur prior to bankruptcy. Graham, van Binsbergen, and Yang (2010) map out a forward-looking tax benefit function in the spirit of Graham (2000), and infer the manager's perceived "all-in" costs of debt including financial distress costs, bankruptcy costs, and agency considerations. Aside from considerable differences in methodology, van Binsbergen, Graham, and Yang exclude from their analysis firms with either very high or low leverage, firms of particular interest for our analysis. Korteweg (2009) measures the side effects of financing (tax benefits net of financial distress costs) by relying on constraints implied by Modigliani and Miller on firm values and their betas. As in van Binsbergen, Graham, and Yang (2010), his estimates combine bankruptcy and distress losses, and are, on average, about 5% of firm value. While the overall estimates in our paper can be reconciled with these, our main contribution is to distinguish between specific types of financial distress costs, and characterize conditions under which type are most relevant.

We organize the paper as follows. Immediately following, we describe our general methodology and strategy for estimating financial distress costs. Section 3 considers the benchmark case where we allow financial distress costs to accrue only when the firm defaults; here, we characterize default costs under a variety of default rules, assumptions for loss given default, and in different risk environments. Then, in Section 4, we develop closed-form solutions for a model that explicitly allows for pre-default financial distress losses, and estimate ex ante financial distress costs under this regime. Section 5 concludes.

2 Modelling financial distress costs

We start with a general framework of financial distress costs, and then consider specific cases throughout the remainder of the paper. Denote as $\varphi(\cdot)$ the realized cost of financial distress at any given point in time. Further, assume that $\varphi(\cdot)$ is a function of both the firm's time-t value, v_t, and the parameters that determine

---

[5]See Parsons and Titman (2009) for a recent survey of the cross-sectional and time series patterns of capital structure.

the firm's environment. Then, we can write the total ex ante, discounted costs of financial distress as

$$FDC_t = E_t^Q \left[\int_t^\infty e^{-rs} \varphi(v_s) \cdot ds \right], \tag{1}$$

where the superscript Q denotes that the time t expectation is taken with respect to risk-adjusted probabilities, which allows us to discount all future $(s : s > t)$ financial distress costs at the risk-free rate, r. The central question is how to parameterize the function $\varphi(\cdot)$.

A common assumption is to assume that all distress costs are incurred as a lump sum when the firm declares bankruptcy. Examples of reduced-form models that adopt this convention include Graham (2000), Molina (2005), and Almeida and Philippon (2007). This assumption is also common in the structural literature, e.g., Chen (2009), where default is triggered when unlevered firm value, v_t, reaches a lower bankruptcy threshold V_B for the first time. In this setting, the firm's distress costs are

$$\varphi(v_t) = \begin{cases} 0, & \text{prior to bankruptcy} \\ \alpha \cdot V_B & \text{at bankruptcy.} \end{cases} \tag{2}$$

We will first consider this benchmark specification, along with several variations for robustness, in Section 3. In this benchmark, firms do not experience financial distress costs prior to declaring bankruptcy. We will see that, although this assumption provides tractability, it implies extremely small present values for the costs of debt. Consequently, from this exercise we conclude that if firms experience financial distress only after entering bankruptcy, the trade-off theory has little empirical validity.

In Section 4, we relax the assumption that bankruptcy is the sole trigger for experiencing financial distress costs. Specifically, for a given set of parameters describing a firm and its capital structure, (i) there is a threshold asset value V_D above which the firm is rated investment grade and below which it is rated speculative grade; (ii) when the firm is rated speculative it experiences financial distress costs that are proportional to asset value at a constant continuous rate γ; (iii) when the firm's asset value reaches the lower threshold V_B for the first time, default occurs and additional bankruptcy costs (as before) are realized as a lump sum proportional to asset value at default, V_B. This can be summarized as

6

$$\varphi(v_t) = \begin{cases} 0, & \text{while the firm is rate investment grade} \\ \gamma \cdot v_t \cdot dt, & \text{while the firm is rate speculative grade} \\ \alpha \cdot V_B, & \text{at bankruptcy.} \end{cases} \tag{3}$$

Figure 1 provides a visual summary of the distinction between the models implied by Equations (2) and (3). Consider first the case without pre-bankruptcy financial distress costs, corresponding to the solid black line. The firm begins as an investment-grade entity, but is downgraded at time t_1, when asset value dips below the threshold V_D. It remains speculative-grade for a brief period until time t_2, when it is upgraded again. At t_3, it is downgraded a final time and ultimately defaults when the asset value reaches the default threshold V_B at time t_5. At this time, the firm would experience a bankruptcy loss with a present value of αV_B, calculated as of time t_5.

Consider now the dotted path, which corresponds to a firm that experiences financial distress costs governed by the process in Equation (3). This path reflects the effect of setting $\varphi(v_t) = \gamma \cdot v_t \cdot dt$ when the firm has been downgraded. Because of this cost, the firm is never upgraded again along this path, as it would have been at time t_2 if there had been no such cost. Moreover, the firm defaults *earlier* when the reduced asset value reaches V_B at time t_4 rather than t_5. This, of course, will increase the probability of the firm declaring bankruptcy, so that pre-default financial distress costs influence the present value of costs incurred at bankruptcy. The model we later develop explicitly takes this interdependence into account.

To summarize, we will separate financial distress costs into two mutually exclusive types, in Sections 3 and 4, respectively:

1. **Bankruptcy costs** (BC) are all financial distress costs, whether indirect or direct, that a firm incurs after it declares bankruptcy. We do not make the distinction (nor is it important for our purposes) between direct, out-of-pocket costs, and indirect impairments to value. What is relevant is *when* such losses occur - at or after bankruptcy. Except in robustness checks, these are taken as proportional to the firm's value at bankruptcy.

2. **Pre-bankruptcy costs** (PBC) are value declines experienced prior to default, but that are driven by the prospect of bankruptcy as a consequence of debt financing. These are intended to capture lost sales, underinvestment, increased predation by competitors driven by an increased threat of bankruptcy. Importantly, these costs are incurred *only* after a firm's value drops to the point where it loses its
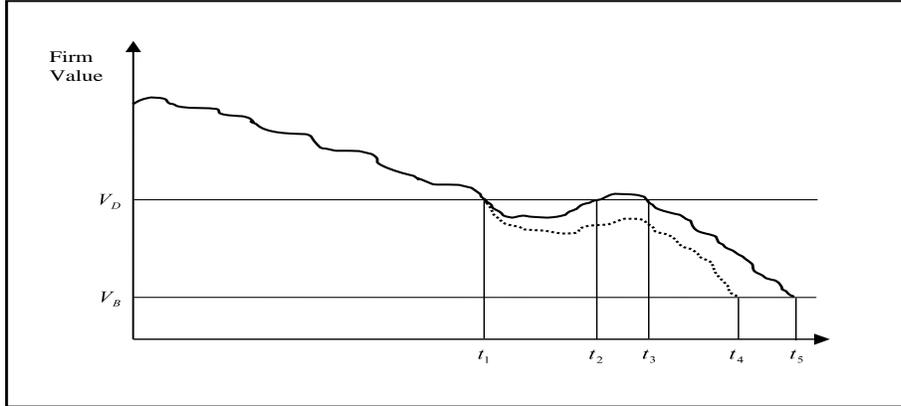
7

Figure 1: **Pre-default financial distress costs vs. bankruptcy costs** This figure shows two hypothetical paths of firm value as it enters financial distress. V_D and V_B represent the downgrade (to junk debt) and bankruptcy thresholds respectively. The solid black line corresponds to the case in which all costs are born at V_B. Prior to t_5, the firm's value evolves under a Geometric Brownian Motion with drift, after which it incurs a loss proportional to V_B. The dotted line shows what may happen if the firm begins to lose value (e.g., sales) after being downgraded at t_1. Ex ante, the financial distress costs associated with the dotted line are higher for two reasons: i) distress occurs sooner , and, ii) distress costs are incurred over a larger range of asset values.

investment grade credit rating, but before it defaults. By assumption, losses prior to this threshold are assumed to be unrelated to the firm's financial position, and are thus, not capitalized as financial distress costs.

In order to operationalize these assumptions and obtain a model capable of providing quantitative measures of a firm's default rates, security values, tax benefits and costs of financial distress, we follow the literature on structural models in assuming one factor lognormal dynamics for the firm's asset value. In the first specification of the function φ (Equation 2), our model collapses to the well-known Leland & Toft (1996) model, described in the appendix. In our second specification of the distress cost function (Equation 3), our model can be thought of as a modification of the framework used in the Leland (1998) presidential address. The appendix provides the technical derivation details. In both cases, we obtain closed-form solutions for the desired quantities.

3 Bankruptcy costs

The majority of the literature on financial distress costs focuses on those that occur at or near bankruptcy.[6] Consequently, the ongoing debate over the empirical relevance of the trade-off theory is based mostly on a comparison of ex ante tax benefits to ex ante bankruptcy costs. Graham (2000) is the first to explicitly make this comparison, concluding that in most cases, the tax benefits overwhelmingly outweigh these costs. More recent studies include Molina (2005) and Almeida and Philippon (2007), which, respectively, argue that when measured correctly, expected bankruptcy costs do offset the tax benefits on the margin. We revisit this issue in this section, and challenge their conclusions.

3.1 Estimation

Because we are concerned only with bankruptcy costs in this model, we can use equation (2) to value financial distress costs that occur at bankruptcy. Under such a specification, we need only two quantities: 1) a firm's probability of declaring bankruptcy at various future dates, and 2) the expected cash flows lost in these states (αV_B).

We estimate these quantities using the debt pricing model of Leland and Toft (1996). We do not repeat the theoretical results here, but present in the Appendix (Section 5) a brief overview of the technical details for the interested reader. Estimation of the model is standard (see, e.g., Leland (2004), Schaefer and Strebulaev (forthcoming)), so we relegate the estimation details also to the Appendix (Section 5). Most of the key inputs to the model (e.g., equity values, total leverage) are available from COMPUSTAT and CRSP; for firm-specific forward-looking tax rates, we rely on those provided on John Graham's website.[7]

Table 1 provides estimation output for over 452,000 firm-quarter observations from the years 1970-2007. We tabulate asset volatility (σ), current unlevered firm value (V_t), total debt, debt-to-value ($\frac{Debt}{V_t}$), bankruptcy threshold (V_B), proportional net worth at bankruptcy ($\frac{V_B}{Debt}$) and the ratio of the bankruptcy threshold to current value $\frac{V_B}{V_t}$. As seen, for most firms, their current values, V_t, are quite far from their values at bankruptcy, V_B, the point when bankruptcy costs are incurred. Thus, $1 - \frac{V_B}{V_t}$ is the percentage decline that

---

[6]A necessarily incomplete list of papers that do so includes Warner (1977), Ang, Chua, and McConnell (1982), Altman (1984), Weiss (1990), Tashjian, Lease, and McConnell (1996), Gilson (1997), Andrade and Kaplan (1998), Lubben (2000), and Welch (2007). Exceptions include Opler and Titman (1994), Maksimovic and Phillips (1998), and Titman and Tsyplakov (2007).

Risk quintiles	Highest risk	2	3	4	Lowest risk	All (median)	All (mean)
Distance to default	2.4	3.6	4.6	5.8	8.9	4.3	5.1
Asset risk, σ	0.42	0.36	0.33	0.31	0.25	0.32	0.41
Asset value, V_t ($\$M$)	35.6	82.2	155.7	270.3	454.2	135.2	1,917.4
Total Debt ($\$M$)	21.2	33.1	47.3	57.8	60.0	38.2	727.1
Debt to value $\left(\frac{Debt}{V_t}\right)$	0.60	0.40	0.30	0.21	0.13	0.28	0.38
Default boundary, (V_B) ($\$M$)	11.0	19.2	28.5	36.6	40.6	23.0	561.6
Boundary to debt, $\left(\frac{V_B}{Debt}\right)$	0.52	0.58	0.60	0.63	0.68	0.60	0.77
Boundary to value $\left(\frac{V_B}{V_t}\right)$	0.31	0.23	0.18	0.14	0.09	0.17	0.29
N	90,455	90,455	90,455	90,455	90,455	452,275	452,275

Table 1: **Firm level descriptives predicted by Leland and Toft (1996) model**. The columns are ordered by distance-to-default, defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$, where σ is asset volatility, V_t is unlevered asset value, and V_B is the endogenous bankruptcy threshold predicted by the model. Firms with the highest default risk (lowest distance-to-default) are shown far left, and decrease in default likelihood to the right. *Debt* is total liabilities from COMPUSTAT. N is the number of firm-quarter observations entering the estimation.

a firm at time t would have to sustain before suffering bankruptcy costs. Evidence from Warner (1977) and Davydenko (2009) indicate that on average, this fraction is between 80% and 90% of current value.

The values reported in the table are medians (unless otherwise indicated) for quintiles defined on distance-to-default (henceforth DTD).[8] The advantage of this metric is that it allows us to stratify simultaneously along financial leverage and unlevered volatility, even for firms without public debt ratings. Firms in the lowest quintile (category 1) have an average DTD of 2.4, which can be interpreted as being 2.4 asset value standard deviations away from defaulting over a one-year horizon. These firms have 5-year objective cumulative default rates of about 30%, which corresponds roughly to a firm rated single B by Moody's. By the same token, our highest DTD quintile (5) corresponds roughly to the Aa/Aaa category.

The leftmost column refers to the riskiest 20% of our firm-quarter observations, with credit quality increasing as one moves to the right. The first and second row report current firm value, unlevered and levered respectively. Both (expectedly) right-skewed, as is debt (column 3). The mean (median) unlevered firm value is \$1.9B (\$135M), with \$727.1M (\$38.2M) in total debt. The firms closest to default tend to be smaller, more leveraged (row 4), and more volatile (row 5). All of these patterns are well-known in the credit literature.

We are mainly interested in the bankruptcy threshold, V_B, as predicted by the LT model. As seen, the median ratio of V_B to V_t is roughly 0.17, indicating that the median firm would need to lose over 80% of its

---

[8]$DTD = \frac{ln(V_t)-ln(V_B)}{\sigma}$, where V_t is the firm's current asset value, V_B is the default threshold, and σ is the asset volatility.

value before incurring default losses. This is consistent with the large sample evidence in Davydenko (2009). Moreover, as seen in the second to last row, the bankruptcy threshold is on average lower than the firm's debt across all risk quintiles. This is because equity has option value; equity holders have an incentive to inject equity into a struggling firm if there is sufficient upside. However, Table 1 also makes clear that, even without such optionality, values at default are quite different from current firm values. For example, suppose that the firm defaults as soon as its value V drops below the face value of its total debt (similar to Merton (1974)). The third row indicates that, even in this case, the median firm would have to lose over 70% of its value before defaulting. For the remainder of the paper, we maintain the LT convention that default is triggered the first time when firm value drops below V_B, but for robustness also consider alternative default rules that yield similar results.

In addition to an estimate of the bankruptcy threshold, a sequence of risk-adjusted bankruptcy probabilities is also required in order to calculate ex ante bankruptcy costs. Table 2 shows both the 5-year and 10-year probabilities of bankruptcy predicted by the LT model, broken down by credit quality as in Table 1. The first and third rows of Table 2 are objective (historical) default probabilities, while the second and third rows are those adjusted for systematic risk. For comparison, directly underneath, we show Moody's estimates (1970-2001) of objective default probabilities as well as Almeida and Philippon's (2007) estimates of risk-adjusted default probabilities, each by Moody's credit rating: B, Ba, Baa, A, Aa/Aaa.

Two things are apparent in Table 2. First, the objective (physical) probabilities generated by the LT model, when stratified by risk quintiles, closely align with Moody's objective default probabilities. Although these objective probabilities, of course, do not enter into our pricing equations, we show them to emphasize that the model realistically captures the important determinants of default.[9] The second noteworthy observation is that the risk-adjusted default probabilities generated by the LT model are similar, in absolute terms, to those that Almeida and Philippon (2007) extract from credit spreads. This is surprising given the substantial differences in both methodology and sample firms (i.e., publicly rated versus all CRSP/COMPUSTAT firms).

The only remaining input is the proportional loss given default, α. In our first set of calculations, we choose the midpoint of Andrade and Kaplan's (1998) study of 31 highly leveraged firms that subsequently underwent financial distress. Although clustered in time and industry (mostly retail), their estimates of 10%-23% of value of default have been used extensively in the literature to estimate ex ante bankruptcy costs. In later sections, we provide alternative estimates of α, but here use this range to allow our results to be directly

---

[9]See Leland (2004) for a more detailed discussion regarding the ability of the Leland and Toft (1996) model to match objective default likelihoods.

Risk quintiles	Highest	2	3	4	Lowest
P_{t+5}	0.322	0.107	0.034	0.007	0.000
P_{t+10}	0.464	0.225	0.110	0.040	0.006
Q_{t+5}	0.399	0.180	0.077	0.021	0.001
Q_{t+10}	0.573	0.372	0.240	0.123	0.025
Moody's Rating	B	Ba	Baa	A	Aa/Aaa
$P^{Moody's}_{t+5}$	0.310	0.114	0.020	0.005	0.002
$P^{Moody's}_{t+10}$	0.465	0.215	0.052	0.016	0.009
Q^{AP}_{t+5}	0.349	0.211	0.114	0.071	0.011
Q^{AP}_{t+10}	0.625	0.392	0.209	0.127	0.042

Table 2: **Physical and Risk-Adjusted Bankruptcy Probabilities.** This table compares the objective and risk-adjusted default probabilities predicted by the Leland and Toft (1996) model to Moody's historical averages and to Almeida and Philippon's (2007) estimates, respectively. Unless otherwise specified, all values are medians. The columns are broken up by quintiles of distance to default (DTD) for the entire sample, where DTD is defined as $\frac{ln(V_t)-ln(V_B)}{\sigma}$. The first two rows present five- and ten-year objective default probabilities predicted by LT, P_{t+5} and P_{t+10}, respectively. The third and fourth rows present their risk-adjusted counterparts, Q_{t+5} and Q_{t+10}, respectively. For comparison, rows five and six show the objective default probabilities from Moody's over 1971-2001 (to be compared to rows one and two). Rows seven and eight are Almeida and Philippon's (2007) risk-adjusted probabilities of default. Rows five through eight are presented by Moody's credit rating. We amalgamate the Aaa and Aa credit rating categories and report averages of Moody's and Almeida and Philippon (2007) figures.

comparable to recent studies.

3.2   Primary Results

Table 3 displays our main results. The first three rows constitute the benchmark case, where we calculate the present value of bankruptcy costs assuming a 16.5% proportional loss in default. In the first row, firms default at V_B, the endogenous value predicted by the Leland and Toft (1996) model. Beginning first in the quintile of firms closest to default, we see that on average, bankruptcy costs comprise slightly more than 2% of current firm value. Moving across the table, this fraction steadily declines, so that for the least risky firms, bankruptcy costs appear negligible. Averaged across all risk groups, bankruptcy costs amount to less than 1% of current value.

In the second row, we show the results when α is applied to current value, V_t, instead of to value at bankruptcy, V_B. In every case, this calculation produces much higher estimates. In the riskiest group, ex

α	Value at Default	High risk	2	3	4	Low risk	All (median)	All (mean)
16.5%	V_B	0.021	0.009	0.005	0.002	0.000	0.004	0.009
16.5%	V_t	0.086	0.054	0.038	0.025	0.006	0.035	0.045
	Difference	**0.065**	**0.045**	**0.033**	**0.023**	**0.006**	**0.031**	**0.036**
10%	V_B	0.012	0.005	0.003	0.001	0.000	0.003	0.005
10%	V_t	0.054	0.034	0.023	0.015	0.003	0.027	0.022
	Difference	**0.042**	**0.029**	**0.020**	**0.014**	**0.003**	**0.024**	**0.017**
23%	V_B	0.029	0.013	0.007	0.003	0.001	0.007	0.013
23%	V_t	0.121	0.078	0.056	0.036	0.009	0.051	0.064
	Difference	**0.092**	**0.065**	**0.049**	**0.033**	**0.008**	**0.044**	**0.051**
α^*		0.21	0.33	0.43	0.54	0.73	0.43	0.69

Table 3: **Present value of bankruptcy costs proportional to current firm value.** This table shows the present value of bankruptcy costs, as a fraction of current firm value, for: 1) different proportional losses given default (α), and 2) different values at default. Columns are ordered as quintiles of default risk, with increasing distance-to-default (see Table 1 for a definition and summary statistics) moving from left to right. Overall sample medians and means are shown in the rightmost columns. The first three rows constitute the benchmark used by Almeida and Philippon (2007), which assumes that firms incur a one-time bankruptcy cost upon defaulting, proportional to 16.5% of the firm's value at that time. The first row uses the Leland and Toft (1996) model to predict the firm's value at bankruptcy, denoted V_B. The second row takes the firm's current value, V_t, as the default boundary. The third row shows the difference between rows 1 and 2. Rows 4-6 (7-9) make this same comparison, but assume that the proportional bankruptcy loss at default is 10% (23%). The last row shows the proportional loss at default, α^*, that would be required to offset tax benefits. Both bankruptcy costs and tax benefits are adjusted for systematic risk.

ante bankruptcy costs are in the neighborhood of 8%-9%, and remain substantial for all but the safest quintile. Averaged across all firm-quarters, capitalized bankruptcy costs comprise 4.5% of firm value, identical to the figure Almeida and Philippon (2007) report for the typical BBB firm (p. 2557). The range is also similar. Almeida and Philippon (2007) report ex ante bankruptcy costs of 9.54% for B rated firms (corresponding roughly to quintile 1), whereas Table 3 indicates 8.6%. One the other extreme, the least risky quintile (5) includes both AA and AAA firms, weighted toward the former, and indicates that only .6% of ex ante firm value is sacrificed to bankruptcy costs. Almeida and Philippon (2007) break up AA and AAA firms separately, reporting values of 1.84% and 0.32% respectively (Table IV, Panel A, p. 2571).

To better appreciate what these results imply for the trade-off theory, we revisit the calculation performed by Almeida and Philippon (2007), and compare the *change* in tax benefits to the *change* in bankruptcy costs across leverage categories. The idea behind this comparison is straightforward: if bankruptcy costs increase roughly as fast as do tax benefits when leverage increases, then at least in aggregate, trade-off is approximately correct. Taking a firm in the least risky quintile (roughly rated AA), consider a leverage increase sufficient to place it in the median quintile (roughly rated BBB). Given that tax benefits will increase in the neighborhood of 3-4 percent (Table 6, Almeida and Philippon (2007)), a similar increase in bankruptcy

costs is required for trade-off to hold. Inspection of Table 3 reveals that this occurs only in the second row (0.038-0.006 = 0.032), when we take 16.5% of the firm's *current* value, V_t, as the dollars lost in default. When we apply 16.5% to the firm's predicted value at default, V_B, the increase in bankruptcy costs (0.005) is far too small to offset the accompanying increase in tax savings.

The rows immediately below present the same estimates, except that instead of using Andrade and Kaplan's 16.5% midpoint for proportional bankruptcy costs, we use the lower (10%) and upper (23%) bounds. Of course, larger fractions of firm value destroyed in bankruptcy will increase their present value, which explains why the estimates in row 4 (5) are universally smaller than in row 7 (8). The more important point, however, is that even in the most aggressive case (α=23%), the present value of bankruptcy costs are modest, and imply debt sensitivities roughly one-fifth the size needed to balance the tax benefits.

Given these findings, a natural question is how big α would need to be to reconcile trade-off. The final row presents these values. We see that even in the riskiest quintile, the median firm would need to lose 21% of its value at default for its observed capital structure to be optimal. This just falls within Andrade and Kaplan's (1998) upper range. Proceeding across the table, this is no longer the case. Even in the second quintile, proportional losses of one-third are necessary, well outside the range documented by Andrade and Kaplan (1998), and far outside the range of even the highest estimates such as Opler and Titman (1994) or Phillips and Maksimovic (1998). Across all groups, the average firm would need to lose nearly half its value once default hits, in order to offset the tax benefits from an ex ante basis.[10]

3.3 Alternative Default Specifications

The analysis, up to this point, has been tailored to match the assumptions of previous studies, so as to allow a direct comparison. Here, we consider a number of robustness checks. The first issue is how we model the firm's decision to default. In Leland and Toft (1996), this decision is endogenous, the product of equityholders weighing the cost of servicing debt against the potential for a turnaround in the firm's prospects. At the default threshold, they are indifferent. Here, we consider two alternative default rules: 1) the firm defaults when it can no longer meet interest payments from retained earnings (i.e., equity cannot be issued to finance cash shortfalls), and 2) the firm declares bankruptcy at time t if it has negative net worth.

---

[10]In a recent paper, Glover (2010) performs a conceptually similar exercise and reports similar figures.

3.3.1 Liquidity Crisis

The policy we rely on in our framework assumes, like Geske (1979), Leland (1994, 1998) and others, that the firm defaults when shareholders no longer find it profitable to inject further capital to stave off bankruptcy. At this point, they exercise their limited liability option, and stop servicing debt. In fact, the endogenous default threshold that results in this setting is the lowest possible that respects the limited liability condition. This exposes the results to the criticism that our bankruptcy costs, which are taken as a proportion of firm value at bankruptcy, are unrealistically low. If firms near bankruptcy have difficulty raising funds to service debt, then bankruptcy may be declared at higher levels of asset value, and may bias downward our estimates of ex ante bankruptcy costs.

To address this possibility, we consider the case polar to ours that rules out any additional funding from shareholders. In other words, debt service has to be financed from internal cash flows only. This yields the following default condition:

$$(1 - \tau)C + \frac{P}{T} - d(V_{CF}) - \delta V_{CF} \quad = \quad 0,$$

where P is the total book value of outstanding debt and T is the maturity of newly issued bonds.[11] This equation states that the firm defaults when the net of tax debt service (interest $(1 - \tau)C$ and amortization $\frac{P}{T}$) minus proceeds of new debt issuance ($d(V_{CF})$) and cash flow from operations (δV_{CF}) no longer remains positive. When debt is issued just prior to default, the proceeds equal the recovery due

$$d(V_{CF}) \quad = \quad \frac{(1 - \alpha)}{T} \cdot V_{CF}.$$

Inserting this expression into the default condition yields the default threshold that prevails when shareholders cannot raise new capital to stave off distress,

$$V_{CF} \quad = \quad \frac{(1 - \tau)C + \frac{P}{T}}{\left(\frac{(1 - \alpha)}{T} + \delta \right)}.$$

Notice that this expression *only* depends on the cash flow rate and debt structure parameters, in contrast to the threshold when the firm has access to financial markets up to declaring bankruptcy, which in particular

---

[11]The ratio $\frac{P}{T}$ measures the amount of total outstanding principal P which is refinanced every year. For example if all new bonds are issued with a five year maturity then a fifth of the total debt is rolled over yearly. See Leland & Toft (1996) for details.

Default trigger	High risk	2	3	4	Low risk	Median	Mean
Cash flow shortfall ($V \leq V_{cf}$)	0.114	0.055	0.031	0.015	0.004	0.031	0.042
Negative net worth ($V \leq Debt$)	0.027	0.010	0.004	0.001	0.000	0.004	0.009
Immediate ($V \leq V_t$)	0.062	0.030	0.014	0.006	0.001	0.013	0.028

Table 4: **Ex-ante bankruptcy costs under alternative default triggers.** This table shows the present value of ex ante bankruptcy costs implied by different default specifications. Unless otherwise specified, all reported values are medians. Columns are ordered as quintiles of default risk, with increasing distance-to-default (see Table 1 for a definition and summary statistics) moving from left to right. In all cases, losses at bankruptcy are assumed to be 16.5% of the firm's value at bankruptcy. In the first row, the firm defaults when shareholders are prevented from issuing new shares, even if the firm's equity has positive value. Here, the firm defaults the first time its cash flows are less than its interest expenses, a threshold denoted as V_{cf}. The second row shows the results when bankruptcy is triggered by the firm have negative net worth, i.e., when firm value V is less than total liabilities, $Debt$. The corresponds to the Merton (1974) model. For comparison, the final column shows the results when ex post bankruptcy costs are calculated as a percentage of current firm value, V_t.

depends critically on volatility.

As seen in the first row of Table 4, limiting access to capital markets makes an enormous difference to the probability of bankruptcy, and consequently, to the present value of bankruptcy costs. For the most risky group, ex ante bankruptcy costs in the neighborhood of 11% of current firm value. Moving across the table, the values steadily decline, with less than 0.4% for the least risky groups. However, on average, bankruptcy costs accrue to over 4% of firm value, almost big enough to offset the tax benefits ex ante.

Given the stark differences between Table 3 and the first row of Table 4, it is useful to assess which default specification is most reasonable. First, recalling the results presented in Table 2, we see that the endogenous default rule in Leland and Toft (1996) yields good predictions of both physical and risk-adjusted probabilities at various horizons. By contrast, the risk-adjusted probabilities at the 5- and 10-year horizon implied in column 1 are 51% and 82%, respectively, nearly twice those implied by credit spreads (Table 2). Second, in the largest study of values at defaults, Davydenko (2009) finds that on average, firms default at approximately two-thirds of the face value of their liabilities. This is similar to the ratios of $\frac{V_B}{V_t}$ implied by the Leland and Toft model, but far less than those implied by Panel A. Thus, although an interesting exercise, Panel A would appear to apply to only a small subset of firms who, despite positive net worth, cannot access capital markets. For example, firms in developing financial economies with similar tax codes, but with only limited access to external finance, may default – and experience bankruptcy costs – at values high enough to offset tax benefits from an ex ante perspective.

3.3.2 Positive Net Worth Requirement

The Leland and Toft (1996) model is just one of many structural models that provides the information necessary to predict probabilities of and values at bankruptcy. We choose it because it balances not only tractability and realistic features of default, but also matches empirically observed default values (e.g., Davydenko (2009)). However, to provide some insight into the sensitivity of the results to our choice of model, in the next two rows of Table 4, we report the results of re-estimating ex ante bankruptcy costs, relying on Merton (1974).

The Merton model is the canonical structural model and, its limitations and recent extensions notwithstanding, has been used often in the past and recent literature. Compared to the LT model, there are two main differences. First, in Merton (1974), the firm is required to declare bankruptcy if its value drops below the face value of its total liabilities. That is, equity holders do not have the option of servicing equity out of their own pockets, hoping for a turnaround in the firm's fortunes. Second, it is based on a much simpler and more stylized model of capital structure - the firm is financed exclusively by one issue of discount debt, the non-payment of which is the only trigger for bankruptcy. The key output of the model is the cumulative risk-adjusted default probability for the debt horizon.

In this setting, extensions to multiple bankruptcy dates necessarily require a somewhat ad hoc implementation. For example, it is convenient to value a stream of cash flows at different dates simply as the value of a portfolio where each cash flow is valued individually using the Merton model.[12] However, such an approach ignores that, at least in the case of bankruptcy, the realization of each cash flow is conditional on the others not taking place - bankruptcy only occurs once. It is difficult to compute the appropriate conditional default probabilities if default is permitted to occur only at discrete times (in LT, it can occur at any instant).[13] Our approach is to approximate, for example, the probability of defaulting in the fifth year, conditional on surviving until the end of the fourth, as the difference between the cumulative four- and five-year survival probabilities.

As before, we present our results by default risk quintiles in Table 4. The results are quite similar to those seen with the LT model, even with a different specification for default. Compared to Table 3 (with $\alpha = 16/5\%$), the estimated ex ante bankruptcy costs are higher, but only slightly. For consistency, we also

---

[12]See, for example, Eom, Helwege, and Huang (2004).

[13]Geske (1977) solves this problem and highlights how it quickly becomes computationally intractable for more than a few payment dates.

calculate the ex ante bankruptcy costs when α is taken as a fraction of current value, V_t, rather than of value at bankruptcy (here total liabilities, LT). As before, this conditioning makes an enormous difference, leading to estimates of bankruptcy costs that are different by a factor of roughly five on average, and over an order of magnitude in some cases.

3.4 Fixed vs. Proportional Bankruptcy Costs

A reasonable criticism of our specification of bankruptcy costs so far is that although empirical work tends to report estimates as proportions of bankrupt firm value (or near this value), there may in reality be a mixture of fixed and proportional costs. For example, court and attorney fees are likely to be fairly similar across firms, while asset sale discounts may vary in proportion to firm size. See, for example, Warner (1977) and Bris, Welch, and Zhu (2006). A mix between fixed and proportional costs may have significantly different implications in terms of ex ante bankruptcy costs for firms that are heterogenous in size.

In order to address this criticism, we consider the following thought experiment. Let ex post bankruptcy costs be defined as $\alpha \cdot V_B + \phi$, where α is the proportional bankruptcy cost and ϕ is a fixed *dollar* cost.[14] Suppose that the empirical estimates reported in, for example, Andrade and Kaplan (1998) are accurate on average, but that they vary across firms depending on the mixture of fixed and proportional costs. To be more specific, we assume that all bankruptcy costs are realized at default, and are structured to average 16.5% of firm value at this time, so that $\alpha \cdot V_B + \phi = 0.165 \cdot V_B$.

Now consider the following two scenarios:

1. Bankruptcy costs are exclusively proportional, corresponding to $\alpha = 0.165, \phi = 0$. This is the scenario considered so far (Tables 3 and 4) and in most studies to date.

2. A mixture where the fixed dollar cost is set at 5% of the median firm bankruptcy threshold, V_B, across the firms in our sample. We then set the proportional component to 11.5% of firm value at bankruptcy. The median firm bankruptcy threshold V_B is 26.5 million dollars, so the implied fixed dollar cost is $0.05 \cdot 26.5 = 1.32$ million dollars.[15]

---

[14]In the presence of fixed costs, the closed-form for the default threshold changes from that provided in Leland & Toft (1996). Letting the total default cost equal $\alpha \cdot V_B + \phi$, we obtain $V_B = \frac{\frac{C}{r}(\frac{A}{rT} - B) - \phi B - A\frac{P}{rT} - \tau C\frac{x}{r} - \phi x)}{(1 + \alpha x - (1-\alpha)B)}$, where A, B and x are constants.

[15]We rely on empirical evidence relating to direct bankruptcy costs to guide our choice of fixed costs. Warner (1977) reports direct costs for Chapter 11 bankruptcy for 11 railroads in the neighbourhood of 4% of the market value of the firm 1 year prior

	Estimate	Smallest	2	3	4	Largest	Median	Mean
Level	Bankruptcy costs	0.029	0.009	0.004	0.002	0.001	0.009	0.013
Level	Tax benefits	0.012	0.018	0.024	0.032	0.058	0.029	0.038
Sensitivity	Bankruptcy costs	0.079	0.046	0.030	0.021	0.012	0.042	0.045
Sensitivity	Tax benefits	0.088	0.111	0.132	0.150	0.174	0.100	0.060

Table 5: **Fixed and proportional bankruptcy costs.** This table shows levels and sensitivities for both bankruptcy costs and tax benefits, in the presence of fixed costs at bankruptcy. Results are presents as quintiles of size, and unless otherwise noted, all reported values are medians. A fixed cost of 5% of the median firm's value at default, or 0.05*$26.5 M = $1.32 M is assumed. On top of this, a proportional cost of 11.5% is assumed for all firms. The first row presents the present value of bankruptcy costs, as a fraction of current firm value. The second row shows the same calculation for tax benefits. The third and fourth row show debt sensitivities, calculated as numerical derivatives at the observed level of debt.

Table 5 reports on this exercise. Note that, in contrast to Tables 3 and 4, firms are categorized by size rather than distance-to-default. The introduction of fixed costs, unsurprisingly, has the strongest impact for small firms, comprising almost 3% of current firm value. As one considers larger firms in columns to the right, this fraction declines steadily, so that for the largest firms, bankruptcy costs are once again negligible. In the median case, bankruptcy costs including both a fixed and proportional component comprise slightly less than 1% of current firm value, virtually identical to the results we see in Table 3.

In the final two rows, we present the point sensitivities of both bankruptcy costs and tax benefits.[16] For example, the 0.079 coefficient in the third row of the first column indicates that for the median firm, a dollar increase in leverage would correspond to nearly eight cents in expected bankruptcy costs (assuming both fixed and proportional components). To assess the validity of trade-off, we can compare this to the sensitivity of tax benefits to debt, which the last row shows is 8.8 cents. Although not identical, inspection of the other columns makes clear that in the presence of fixed costs, trade-off works relatively well for small firms. Moreover, this is true only for the single parameterization we have analyzed; larger contributions for fixed costs will validate trade-off for a larger cross-section of firms, although when they become too substantial, the smallest firms will be *overleveraged.*

to default. Altman (1984) estimates these costs to be approximately 7.5%. Lubben (2000) studies legal costs and finds that in Chapter 11 bankruptcy the total about about 1.8% of the distressed firms total assets, corresponding to a typical outlay of about 700,000 dollars. We believe our fixed cost of $1.3 million to be reasonable in light of these estimates.

[16]In previous tables, the reader can infer a rough idea of these sensitivities by making comparisons across rows (which were stratified by risk), whereas a stratification by size does not permit this.

4   Financial Distress Prior to Bankruptcy

The preceding section indicates that if the costs of debt are limited to those incurred around bankruptcy, then the trade-off theory does not explain the leverage ratios across a set of typical firms – especially those with moderate to low leverage ratios. However, this assumption ignores that financial distress is, in most cases, a gradual rather than discrete process. Although there are instances where steep plummets in asset values lead to immediate and large costs of financial distress (e.g., the credit crisis of Fall 2008), it is more likely that such losses accumulate as a firm's core business deteriorates. For example, a firm may be forced to cut prices if customers question its going concern, or may have difficulty recruiting top employees afraid of being displaced in the event of distress or bankruptcy (Titman (1984)). Similarly, the investment policies of firms approaching financial distress may be distorted, either by the inability to raise capital for valuable projects (Myers (1977)), or by incentives to shift risk (Jensen and Meckling (1976)).

Despite their importance, such pre-bankruptcy distress costs are often ignored in ex ante calculations of financial distress.[17] At least part of the reason is that unlike bankruptcy losses, there is no single point at which more general financial losses begin to be incurred. Instead, these are experienced over a range of values and, moreover, with differing severities depending on how distressed the firm is at a given point in time. Consequently, while default probabilities play a crucial role in calculating bankruptcy losses, they play a minimal role in determining the financial distress costs incurred prior to bankruptcy.

Revisiting Figure 1, we see that when a firm experiences financial distress prior to bankruptcy, several aspects of the problem change. First, relative to the solid black line where default costs are incurred only at default (time t_5), the dotted line begins to incur distress costs at a higher value (V_D), and sooner (time t_1). This implies that the probability of incurring distress losses, $\Pr(V_t < V_D)$, will be strictly higher than the corresponding probability of defaulting, $\Pr(V_t < V_B)$. Second, the calculation of the expected value will be more complex than in Section 3; because pre-default financial distress costs are incurred from t_1 until t_5, proper integration over the relevant range of values is necessary. Finally, note that the dotted line hits the default boundary, V_B, at an earlier time (t_4) than does the black line (t_5), implying an important interdependence between pre- and post-bankruptcy losses. We will thus need a new sequence of risk-adjusted bankruptcy probabilities that takes this into account.

The presence of pre-bankruptcy financial distress costs also changes the firm's ex ante capital structure

---

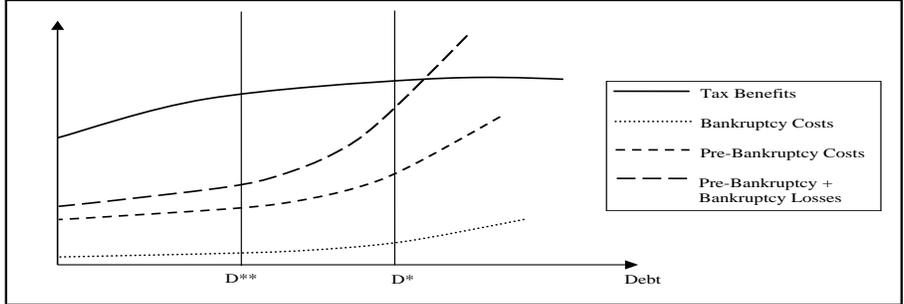[17]Titman and Tsyplakov (2007) is an exception.

Figure 2: **Optimal Leverage Ratios with and without Financial Distress Incurred Prior to Bankruptcy.** Above, we show the present value of tax benefits (solid black), present value of losses at bankruptcy (dotted), and present value of pre-bankruptcy distress losses (short dashed). D^* corresponds to the optimal debt level where all financial distress costs are incurred at bankruptcy. D^{**} corresponds to the optimal debt level where the firm incurs both losses at bankruptcy and incurs financial distress costs prior to bankruptcy.

decision, as shown graphically in Figure 2. The solid line shows a hypothetical tax benefit function which, while increasing with debt, is concave (as firms add debt, there is an increasing likelihood of not being profitable, which reduces the value of the interest deductions). Consider now the firm's optimal capital structure, provided it only faced bankruptcy costs, the present value of which is shown in fine dots. When the slope of the tax benefit equals the slope of this line (approximately D^*), the firm's levered value is maximized. However, when we add pre-bankruptcy financial distress costs (dotted line directly above bankruptcy costs), the firm's optimal capital structure will shift leftward. Now, the slope of the tax benefit must equate to the *sum* of the bankruptcy and pre-bankruptcy curve slopes, shown as D^{**}.

Modeling these issues requires us to develop new theoretical results prior to making new empirical estimates. The mechanics of the model are very similar to the original LT framework, but with one key departure. At a point V_D (described in more detail below), the firm begins to leak a constant percentage of its value due to lost sales, underinvestment, or other financial distress costs incurred prior to bankruptcy. Importantly, these losses are incurred as long as the firm's value is below this threshold, but if it emerges above this threshold, these will cease. This formulation captures the idea that in practice, firms can fully recover from financial distress if their fortunes sufficiently improve.

4.1 A model with financial distress prior to declaring bankruptcy

Operationally, calculating ex ante pre-bankruptcy financial distress costs requires us to answer two questions: 1) what point defines V_D, and 2) how fast do firms leak value when between V_D and V_B? For the first, we define the onset of financial distress to be when a firm's debt loses its investment grade status. This is admittedly arbitrary. However, working under the restriction that such an assumption must be made, this particular threshold has been shown to be relevant. Kisgen (2007) and Kaylan and Titman (2009) provide evidence that firms take active measures to protect their credit ratings, manipulating dividends and investment near upgrades and downgrades. This effect is magnified at the investment grade threshold. Furthermore, survey and anecdotal evidence suggests that firms value maintaining an investment grade credit rating (e.g., Graham and Harvey (2001)). Regardless of whether this specific threshold precisely identifies the onset of financial distress, our calculations provide a quantitative benchmark. Also, from a feasibility standpoint, there are few alternative choices of V_D for which there are readily available historical probabilities.

Once V_D is specified, Figure 2 makes clear that if we know the slopes of the dotted (bankruptcy costs) and solid black (tax benefits) lines, we can infer the required value for the second parameter (the key input to the medium dashed line) from the estimation. In other words, because the model already delivers the other two quantities, we can ask what pre-default leak rate will be sufficient as an offset. We now proceed by first presenting the theoretical results, and then by conducting the related empirical estimation.

Suppose that a firm is solvent so that we have $v_t > V_B$ where V_B is the bankruptcy threshold as before. Assume now that the dynamics of the firm's levered assets depend on whether the firm is above or below an "investment grade threshold", V_D, where $v_t > V_D > V_B$. Here, we wish to model how a firm, once it has been downgraded to speculative grade ($v_t < V_D$), continuously incurs a proportional loss in value, representing pre-default distress costs. The threshold V_B represents the point at which shareholders no longer wish to keep their option alive and stop servicing debt. The threshold V_D, on the other hand, does not involve a decision by any of the firm's stakeholders. We take V_D to be exogenous and determine the threshold empirically utilizing historical rating transition data.

When the firm is investment grade, the dynamics are

$$dv_t = (\mu - \delta)\, v_t dt + \sigma v_t dW_t,$$

22

and when the firm has been downgraded to speculative grade (i.e. $V_D > v_t > V_B$),

$$dv_t = (\mu - \delta - \gamma)\, v_t dt + \sigma v_t dW_t.$$

The parameter δ is the free cash flow rate which, together with potential equity issuance, can be used to service debt and possibly pay dividends. The novelty here is the parameter γ that captures a proportional loss in value as the firm approaches default, i.e., financial distress costs realized prior to bankruptcy. Whereas δ is paid out to claimholders, γ is a deadweight loss, reflecting factors such as lost consumer confidence, managerial distraction and other distress costs incurred prior to default.

4.1.1 Valuation

To incorporate pre-bankruptcy distress costs, we depart slightly from the Leland & Toft (1996) framework. The assumed maturity structure in Leland (1994b, 1998) simplifies derivations at minimal cost in terms of realism and thus for the quantitative output we require.[18] Following Leland's 1998 presidential address, we can write the debt value as the solution to the following PDE:

$$\frac{1}{2}\sigma^2 \frac{\partial^2 D}{\partial v^2} + (r - \delta)v_t \frac{\partial D}{\partial v} - rD + \frac{\partial D}{\partial t} + e^{-mt}(C + mP) = 0,$$

for $v_t > V_D > V_B$, and

$$\frac{1}{2}\sigma^2 \frac{\partial^2 D}{\partial v^2} + (r - \delta - \gamma)v_t \frac{\partial D}{\partial v} - rD + \frac{\partial D}{\partial t} + e^{-mt}(C + mP) = 0,$$

for $V_D > v_t > V_B$. The general solution to this is

$$D_{IG} = \frac{C + mP}{r + m} + \alpha_{1,IG} v^{y_{1,IG}} + \alpha_{2,IG} v^{y_{2,IG}}, \quad v_t > V_D > V_B \tag{4}$$

$$D_J = \frac{C + mP}{r + m} + \alpha_{1,J} v^{y_{1,J}} + \alpha_{2,J} v^{y_{2,J}}, \quad V_D > v_t > V_B \tag{5}$$

where the constants $y_{1,IG}, y_{2,IG}, y_{1,J}$, and $y_{2,J}$ are given in the appendix. To solve for the constants

---

[18]We have benchmarked the quantitative output from a version of our model, which has all the same features as Leland & Toft (1996) save for the debt maturity structure, and find no significant differences in the firm value components predicted by the two models. The difference between the models is that in Leland & Toft (1996), each debt contract in the capital structure has finite maturity; whereas in Leland (1998) and in our setting, debt has finite average maturity. Intuitively, bonds in the former case are straight bullet bonds, whereas in the latter they resemble a sinking fund structure.

$\alpha_{1,IG}, \alpha_{2,J}, \alpha_{1,IG}, \alpha_{2,J}$, we need to impose a number of value-matching and smooth-pasting boundary conditions. At the downgrade threshold V_D, we require that the solutions to (4) and (5) are equal

$$D_{IG}(V_D) \quad = \quad D_J(V_D). \tag{6}$$

Next, we require a smooth-pasting condition, which essentially requires that the instantaneous rate of return on debt does not change discretely at the investment-grade boundary. This can be thought of as a flow version of the value-matching condition, which rules out a discrete jump in value at the boundary - thus inducing a violation of no-arbitrage.

Further, we impose that debt becomes risk free as firm value tends to infinity,

$$\lim_{v \to \infty} (D_{IG}(v)) \quad = \quad \frac{C + mP}{r + m}, \tag{7}$$

and finally a value-matching condition at default,

$$D_J(V_B) \quad = \quad (1 - \alpha)V_B. \tag{8}$$

Consider now equations (4) and (7). Since $y_{1,IG} > 0$, we can infer that $\alpha_{1,IG} = 0$. Next consider equations (8) - value matching at default - and (5) . We can infer that

$$\alpha_{1,J} V_B^{y_{1,J}} + \alpha_{2,J} V_B^{y_{2,J}} - (1 - \alpha)V_B + \frac{C + mP}{r + m} \quad = \quad 0, \tag{9}$$

Next, consider value matching at the downgrade threshold (equation (6) above). It allows us to write

$$\alpha_{2,IG} V_D^{y_{2,IG}} - \alpha_{1,J} V_D^{y_{1,J}} - \alpha_{2,J} V_D^{y_{2,J}} \quad = \quad 0. \tag{10}$$

The smooth-pasting condition at downgrade can be written

$$y_{2,IG} \cdot \alpha_{2,IG} V_D^{y_{2,IG}-1} - y_{1,J} \cdot \alpha_{1,J} V_D^{y_{1,J}-1} - y_{2,J} \cdot \alpha_{2,J} V_D^{y_{2,J}-1} \quad = \quad 0. \tag{11}$$

Equations (9), (10) and (11) can now be solved for the three remaining unknowns: $\alpha_{2,IG}, \alpha_{1,J}, \alpha_{2,J}$. The solution to these three equations can be written in matrix form as

$$
\begin{bmatrix} \alpha_{2,IG} \\ \alpha_{1,J} \\ \alpha_{2,J} \end{bmatrix} = \begin{bmatrix} 0 & V_B^{y_1,J} & V_B^{y_2,J} \\ V_D^{y_2,IG} & -V_D^{y_1,J} & -V_D^{y_2,J} \\ y_{2,IG} \cdot V_D^{y_2,IG} & -y_{1,J} \cdot V_D^{y_1,J} & -y_{2,J} \cdot V_D^{y_2,J} \end{bmatrix}^{-1} \cdot \begin{bmatrix} (1-\alpha)V_B - \frac{C+mP}{r+m} \\ 0 \\ 0 \end{bmatrix}
$$

4.1.2 Valuing pre-bankruptcy financial distress costs

It is straightforward to solve for the values of the tax shield and bankruptcy costs in an analogous fashion. We report the resulting closed-form solutions in the appendix. The solution to the value of pre-bankruptcy financial distress costs also follows the same lines, so we only delineate the solution method. Similar to the valuation of debt, we can write the general solutions to the value of the pre-bankruptcy costs as

$$
\begin{aligned}
PBC_{IG} &= \phi_{1,IG} v^{x_1,IG} + \phi_{2,IG} v^{x_2,IG}, \quad v_t > V_D > V_B \\
PBC_J &= \frac{\gamma v}{\delta + \gamma} + \phi_{1,J} v^{x_1,J} + \phi_{2,J} v^{x_2,J}, \quad V_D > v_t > V_B.
\end{aligned}
$$

We apply the following boundary conditions (corresponding to pairs of value-matching and smooth-pasting conditions at the downgrade and default thresholds, respectively):

$$
\begin{aligned}
PBC_{IG}(V_D) &= PBC_J(V_D), \\
\frac{\partial PBC_{IG}(V_D)}{\partial v_t} &= \frac{\partial PBC_J(V_D)}{\partial v_t}, \\
PBC_J(V_B) &= 0, \\
\lim_{v \to \infty} (PBC_{IG}(v)) &= 0.
\end{aligned}
$$

The derivations and results are reported in the appendix.

4.2 Empirical results

Table 6 displays the results of calibrating the model to fit the downgrade probabilities for the median firm within each risk category. This is a similar exercise to that shown in Table 2, which reports the median results for the model without pre-bankruptcy financial distress. Here, the additional restriction of matching downgrade probabilities is accommodated by an additional parameter: the downgrade threshold, V_D. In the estimation procedure, V_D is set by requiring the model to match downgrade probabilities for the period

25

Risk quintiles	Highest	2	3	4	Lowest
"Equivalent Moody's rating"	B	Ba	Baa	A	Aa & Aaa
$Pr(default)^{model}$	0.311	0.095	0.031	0.005	0.000
$Pr(default)^{Moody's}$	0.242	0.096	0.018	0.006	0.002
$\frac{V_B}{V_t}$	0.37	0.27	0.21	0.15	0.10
$Pr(junk)^{model}$	0.963	0.766	0.190	0.054	0.008
$Pr(junk)^{Moody's}$	0.963	0.768	0.190	0.053	0.008
$\frac{V_D}{V_t}$	4.76	1.80	0.56	0.36	0.31

Table 6: **Calibration of model with pre-bankruptcy financial distress costs.** This table gives the results of calibrating a structural model, that allows pre-bankruptcy distress costs, to historical downgrade probabilities. The historical numbers are based on Moody's default experience 1970-2008. As before, we present median values within quintiles of distance-to-default (DTD). The first two rows compare the 5-year default probability predicted by the model to Moody's 5-year default probability estimates. The third row shows the media ratio of the bankruptcy threshold (V_B) to unlevered firm value (V_t). The fourth and fifth columns compare the model's predicted probability of being downgraded to junk to Moody's 5-year downgrade rates. The final column shows the median ratio of the downgrade (V_D) threshold to current unlevered firm value (V_t).

1970-2008 provided by Moody's.

Five-year default probabilities are best matched for firms of moderate default risk, i.e., columns 2 and 3, and are slightly too low (high) for firms of low (high) default risk. By construction, we obtain a near perfect fit for the five-year probabilities of losing an investment grade credit rating. Shown also are the downgrade thresholds, V_D, as percentages of current firm value. Ratios larger than one, trivially, are seen for risky firms that have already lost their investment grade ratings.[19]

With the model calibrated to match empirical default and downgrade frequencies, we are now in a position to determine the implied leak rate, γ^*, that when combined with bankruptcy losses, balances the tax benefits:

$$\frac{\delta PBC}{\delta P}\Big|_{\gamma=\gamma^*} + \frac{\delta BC}{\delta P} = \frac{\delta TB}{\delta P}. \tag{12}$$

In words, this expression characterizes the optimality condition shown in Figure 2. At the margin, an additional dollar of debt destroys as much in pre-bankruptcy (first term) and bankruptcy (second terms) losses, as it creates in additional tax savings (third term). All comparisons are in present values.

---

[19]Note that the 5-year historical default probabilities from Moody's differ slightly between Tables 2 and 6. In Table 2, our probabilities are based on the time period 1970-2001, which allow us to directly compare our results to Almeida and Philippon (2007) who also use this time period. Because that is no longer an objective here, we use the maximum available sample of 1970-2008.

γ	Highest risk		2		3		4		Lowest risk	
	$\equiv 0$	$\neq 0$	$\equiv 0$	$\neq 0$	$\equiv 0$	$\neq 0$	$\equiv 0$	$\neq 0$	$\equiv 0$	$\neq 0$
$\frac{\partial BC}{\partial P}$	9.6%	9.6%	8.2%	8.2%	6.7%	6.8%	4.9%	5.7%	2.2%	3.4%
$\frac{\partial TB}{\partial P}$	1.4%	1.4%	5.4%	5.4%	8.4%	8.2%	11.7%	10.9%	16.5%	15.2%
$\frac{\partial PBC}{\partial P}$	N/A	0.0%	N/A	0.0%	N/A	1.4%	N/A	5.2%	N/A	11.8%
γ^*	N/A	0.0%	N/A	0.0%	N/A	0.2%	N/A	1.6%	N/A	2.3%

Table 7: **Pre-bankruptcy financial distress and trade-off.** This table shows how large pre-bankruptcy financial distress costs need to be to offset the tax benefits. Columns are quintiles of increasing default risk, from left to right. The annualized, pre-default financial distress leak rate is denoted γ. For each risk group, we first present the benchmark case when $\gamma = 0$ for comparison, and then present the results when γ is allowed to be larger than zero. For this second case, we increase γ so that the sensitivity of tax benefits with respect to leverage ($\frac{\partial TB}{\partial P}$), equals the sum of the sensitivities of bankruptcy costs ($\frac{\partial BC}{\partial P}$) and pre-bankruptcy costs ($\frac{\partial PBC}{\partial P}$). If this comparison already favors bankruptcy costs, the required pre-default financial distress cost, γ^*, is zero (the first two columns).

To gauge the importance of these pre-bankruptcy distress costs, consider the sensitivities shown in Table 7. The first column within each risk class corresponds to the benchmark case, where we set $\gamma = 0$. As seen, this implies that for firms nearest to bankruptcy (particularly the most risky quintile), additional debt does not create value, even when pre-bankruptcy distress costs are ignored. This is not surprising, given that such firms are already close to defaulting, and have little if any taxable income to shield.

However, the picture changes as one moves from left to right. For virtually all firms with investment grade ratings (third group and higher), we would, in the absence of financial distress costs realized before bankruptcy, find direct evidence of debt conservatism. The typical A firm stands to gain more than twice what it loses (11.7 cents/dollar in tax savings vs. 4.9 cents/dollar in expected bankruptcy costs), with even more extreme differences for higher rated firms. Indeed, these are the firms for which the trade-off theory has traditionally had the most trouble - firms with moderate default probabilities, but with high taxable earnings. The reason is that bankruptcy costs are not realized frequently enough for such firms, and when they are, translate to trivial fractions of current value.

In the second column within each risk group (labeled as $\gamma \neq 0$), we allow the firm to incur financial distress costs prior to default, specifically when it dips below investment grade. We increase the annual leak rate, γ, until, from an ex ante perspective, the overall tax benefits and costs are offset. This exercise clearly has no relevance for firms closest to default, but does for investment grade firms. For the typical Baa firm shown

in the third column, an annual leak rate of only 0.2% after being downgraded to junk will close the gap in sensitivities. In other words, less than a half percent annual value penalty after being downgraded to junk means that the typical Baa firm is leveraged appropriately, according to a simple, static version of the trade-off theory. Moving further across the table, we see that for firms comparable to the typical A rated firm, 1.6% is necessary, whereas for the most highly rated firms, Aa and Aaa, a pre-default loss rate of 2.3% per annum is required to offset the tax benefits.[20]

It is also interesting to gauge the indirect effects of pre-bankruptcy distress costs on bankruptcy costs themselves, the type we considered in Section 3. This can be appreciated by comparing the sensitivities of bankruptcy costs and tax benefits, with and without pre-default distress costs. For example, for firms in risk category 4 (approximately A rated firms), the presence of small pre-bankruptcy distress costs increases the sensitivity of bankruptcy costs from 4.9% to 5.7%, while the comparable tax sensitivities decreases by approximately the same amount. Likewise, in the least risky quintile, the "sensitivity gap" between taxes and bankruptcy costs is shrunk by 2.5% (16.5% -15.2%+3.4%-2.2%), reducing the burden that pre-bankruptcy financial distress costs must bear to equate the overall sensitivities.

The reason, recalling Figure 1, is that pre-bankruptcy financial distress costs themselves contribute to higher default probabilities, and in so doing, increase the expected value of bankruptcy losses. To our knowledge, this point is novel both in a qualitative and quantitative sense. Because the firm is less likely to realize taxable income in such states, the present value of the tax shield (as well as its sensitivity to changes in debt) is also reduced. Together, this underscores an additional reason why the presence or absence of pre-bankruptcy financial distress matters for assessing optimal capital structure. Although the capitalized present value of such costs are important by themselves, they change the present values and sensitivities of other relevant quantities, making a joint consideration of all relevant trade-offs crucial.

The framework described above is intentionally generic, and does not distinguish between, e.g., underinvestment Myers (1977), damage to stakeholder relationships Titman (1984), or other types of distress. Here we are more specific, modeling a particular type of pre-bankruptcy financial distress cost: risk shifting. To do so, we develop what amounts to a special case of the Leland (1998) model for asset substitution.[21] Specifically,

---

[20]To gauge the amount of time spent in different rating regions by firms, we simulate 10000 asset value paths for the intermediate risk ("Baa") firm conditional on just having reached the speculative grade region ($v_t = V_D$). Of the simulated firms, those that subsequently default spend on average about four and a half years in the speculative region. Those that avoid bankruptcy and subsequently reemerge as investment grade firms spend on average 14 months incurring pre-default distress costs.)

[21]For simplicity, we abstract from his adjustment of the tax shield near default, and the possibility of calling debt to relever at an upper restructuring point.

we endow management, acting in the interests of shareholders, with the flexibility to endogenously alter the operating risk of the firm.[22]

Operationally, as in Leland (1998), we let shareholders operate the firm at two distinct risk levels. Denote the firm's normal level of operating risk as $\sigma^{initial}$, and a higher level as σ^{high}. Given these risk levels, we can then derive the point at the manager is indifferent between the two. Denote this level as V_S. This risk-shifting policy is, for a given firm environment, jointly determined with the default policy $V_B \leq V_S$.

Our experiment investigates how severe the asset substitution problem would have to be to justify current leverage ratios, given that other distress costs are realized only at bankruptcy. That is, note that in this exercise, $\gamma = 0$ by construction. Instead, the source of the financial distress costs is through the increased volatility, which increases the present value of bankruptcy costs.

Table 8 reports our findings for this exercise. As before, we categorize firms by quintiles of distance to default. We only report on the three highest quintiles, noting that for the two lower ones, the sensitivity of firm value to increases in leverage is negative (as in Section 4.2), even without the opportunity to increase risk. However, for the highest risk firms reported, the difference between the sensitivities of the tax shield and bankruptcy costs is modest but positive. The estimated current asset volatility is 33%, and an increase to 41% at an asset value 34% lower than the initial is required to bring the ex ante values of the tax benefits and bankruptcy costs into balance. This corresponds to the firm's asset volatility moving from the 52nd to the 62nd percentile in our full sample of firms.

For the safer second quintile, firms appear more underleveraged - the sensitivity of the tax benefits is more than twice as high as that of the bankruptcy costs. The initial asset volatility is estimated at 30.8% (48th percentile) and an increase to 54.1% (75th percentile), at a switching threshold V_S corresponding to a 28% drop in asset value, is necessary to offset the tax benefits. For the safest firms in the third reported quintile, where the tax benefits are almost 5 times as sensitive to book leverage changes than bankruptcy costs, initial asset risk needs to more than double (from 25% to 67%, or from the 37th to the 84th percentiles respectively) at a risk-shifting threshold about 28% lower than the initial asset value.

That we allow for asset substitution indirectly affects bankruptcy costs in that the option value of equity grows following the risk increase. Thus the bankruptcy thresholds drop, in particular for the firms with stronger risk-shifting incentives.

---

[22]In a recent paper, Eisdorfer (2008) provides empirical evidence on risk shifting by distressed firms.

DTD	4.6	4.6	5.8	5.8	8.9	8.9
σ	33.0%	41.5%	30.8%	54.1%	25.0%	67.2%
$\frac{\partial BC}{\partial P}$	6.7%	6.9%	4.9%	6.2%	2.2%	5.6%
$\frac{\partial TB}{\partial P}$	8.4%	6.9%	11.7%	6.2%	16.5%	5.6%
Difference	**1.7%**	**0.0%**	**6.8%**	**0.0%**	**14.3%**	**0.0%**

Table 8: **Risk shifting and trade-off.** This table show how large increases in asset volatility would need to be in order for bankruptcy costs and tax benefits to equate. All values are medians for quintiles ranked by distance to default (DTD). For each risk group, we first present the sensitivities of bankruptcy costs ($\frac{\partial BC}{\partial P}$) and tax benefits ($\frac{\partial TB}{\partial P}$) to leverage, given the actual asset volatility (σ) observed in the data. Then, we increase σ until these sensitivities equate.

In summary, for lower tier investment grade names, allowing for moderate risk-shifting opportunities is, in conjunction with 16.5% proportional bankruptcy costs, sufficient to justify current observed leverage ratios. However, for the higher grade firms, more significant risk increases are necessary. As risk shifting is not likely to be the only source of pre-bankruptcy financial distress costs, these greater than plausible levels of required risk shifting merely indicate that other sources of such costs are required to help offset tax benefits.

5   Conclusion

Financial distress and bankruptcy costs are generally thought to be the chief drawbacks to debt. However, financial economists are perplexed as to why highly profitable firms such as Amgen, Intel, and ExxonMobil appear to pay billions in taxes that could be avoided by a simple equity-for-debt swap. Under the assumption that target leverage ratios matter for firm values, there are two possible scenarios.[23] The first is that such firms are, in fact, leveraged too conservatively. This is a consequence, perhaps, of entrenched managers who extract private benefits from shareholders by choosing a suboptimally low debt levels (e.g., Berger, Ofek, and Yermack (1997)), or due to the very high cost to firms of adjusting their capital structures (Leary and Roberts (2005)). The second possibility is that although firms may select optimal debt levels, we can

---

[23]We take as given that firms have leverage targets. However, there exists substantial debate as to the importance of target leverage ratios at all. On the one hand, a family of cross-sectional studies (e.g., Bradley, Jarrell, and Kim (1984), Rajan and Zingales (1995), Frank and Goyal (2003)) suggests that leverage ratios are predictable from firm characteristics that proxy for the costs and benefits of leverage. For example, firms with high effective marginal tax rates have high leverage (Graham (1996)), while those tangible assets easily redeployed in bankruptcy choose low leverage (Titman and Wessels (1988)). On the other hand, time-series tests have not been as favorable to the trade-off theory, as evidenced by relatively low "speeds of adjustment" toward leverage targets documented by, for example, Shyam-Sunder and Myers (1999) and Flannery and Rangan (2006). Finally, evidence of market timing (e.g., Baker and Wurgler (2002) or indifference (Welch, 2004) suggests that firms may not adhere to target leverage ratios at all. For a more detailed discussion of these issues, see Parsons and Titman (2008).

measure the tax benefits more accurately than the offsetting costs. Accordingly, when the distress costs are measured correctly, the balance implied by the trade-off is restored (e.g., Molina (2005), Almeida and Philippon (2007)).

This study addresses the second possible scenario. To do so, we present empirical estimates of ex ante financial distress costs that occur: 1) at (or as of) bankruptcy, and 2) before bankruptcy. This distinction is important because, in general, ex post studies of financial distress begin tracking firm value after it defaults, or attempts to restructure its debt obligations. Although such studies (e.g., Andrade and Kaplan (1998)) can imply considerable proportional losses given bankruptcy, these amount to trivial percentages of ex ante firm value. In particular, our estimates suggest that extreme losses in bankruptcy - some three to four times greater than what has been estimated to date – are necessary to reconcile the predictions of the trade-off theory with observed leverage ratios. For example, recent estimates by Bris, Welch, and Zhu (2006) find a range of 0-20% for default costs as a function of the firm's value at default, which stands in sharp contrast to the required ratios implied by our estimates (50% or more).

However, when we permit firms to experience financial distress costs prior to bankruptcy, the picture changes. For example, if we impose a 1% annual financial distress loss on a firm that has recently lost its investment grade credit rating (but long before it defaults), these capitalized costs are sufficient to offset the tax benefits of debt on an ex ante basis. Because such modest pre-bankruptcy distress losses are sufficient, it seems very unlikely that the trade-off theory could ever be formally disproved. Specifically, the estimates of pre-bankruptcy distress losses would need to be both very small and very precisely estimated to reject the trade-off theory. Thus we find that under mild assumptions about pre-bankruptcy distress costs, firms do not, in fact, appear overly conservative in their use of debt.

Furthermore, by showing that losses following bankruptcy are not the dominant consideration for ex ante leverage decisions, our results call for a shift in attention toward the types of costs (e.g., stakeholder relationships, underinvestment, risk shifting) likely to be incurred as the mere possibility of bankruptcy increases.

References

[1] Almeida, H., Philippon, T., 2007, The Risk-Adjusted Cost of Financial Distress, *Journal of Finance* 62, 2557-2586.

[2] Altman, E., Kishore, V., 1996, Almost Everything You Wanted to Know about Recoveries on Defaulted Bonds, *Financial Analysts Journal* 52, 57-64.

[3] Andrade G. and Kaplan S., 1998, How Costly is Financial (not Economic) Distress? Evidence from Highly Leveraged Transactions that Become Distressed, *Journal of Finance* 53, 1443-1493.

[4] Altman, E., 1984, A Further Empirical Investigation of the Bankruptcy Cost Question, *Journal of Finance* 39, 1067-1089.

[5] Asquith, P., Gertner, R., and Scharfstein, D., 1994, Anatomy of financial distress: An examination of junk bond issuers, *Quarterly Journal of Economics* 109, 625658.

[6] Banejee, S., Dasgupta, S., and Kim, Y. 2008, "Buyer-Supplier Relationships and the Stakeholder Theory of Capital Structure," *Journal of Finance* 63 (5), 2507-2552

[7] Bharath, S., Shumway, T., 2008, Forecasting Default with the Merton Distance to Default Model, *Review of Financial Studies*, forthcoming.

[8] Baker, M., Wurgler, J., 2002, Market Timing and Capital Structure, *Journal of Finance* 57(1), 1-32.

[9] Berger, P., Ofek, E., Yermack, D., 1997, Managerial Entrenchment and Capital Structure Decisions, *Journal of Finance* 52(4), 1411-1438.

[10] Berndt, A., Douglas, R., Duffie, D., Ferguson, M., Schranz, D., 2008, Measuring Default Risk Premia from Default Swap Rates and EDFs, working paper.

[11] van Binsbergen, J., Graham J. and J. Yang, 2010, The Cost of Debt, working paper.

[12] Black, F., Cox, J., 1976, Valuing Corporate Securities: Some Effects of Bond Indenture Provisions, *Journal of Finance*, 31, 351-367.

[13] Bolton, P., Scharfstein, D., 1990, A Theory of Predation Based on Agency Problems in Financial Contracting, *The American Economic Review* 80(1), 93-106.

[14] Bradley, M., Jarrell, G., Kim, E., 1984, On the Existence of an Optimal Capital Structure: Theory and Evidence, *Journal of Finance* 39(3), 857-878.

[15] Brander, J., Lewis, T., 1986, Oligopoly and Financial Structure: The Limited Liability Effect, *The American Economic Review* 76(5), 956-970.

[16] Brennan, M., Schwartz, E., 1978, Corporate Income Taxes, Valuation, and the Problem of Optimal Capital Structure, *Journal of Business* 51.

[17] Bris, A., Welch I., and Zhu N., 2006, The Costs of Bankruptcy. *Journal of Finance* 61-3, June 2006, 1253-1303.

[18] Broadie, M., Chernov, M., Sundaresan, 2007, Optimal Debt and Equity Values in the Presence of Chapter 7 and Chapter 11, *Journal of Finance*, 38, 1341-1377.

[19] Chen, H., 2008, Macroeconomic conditions and the puzzles of credit spreads and capital structure, working paper.

[20] Davydenko, S., 2007, When Do Firms Default? A Study of the Default Boundary, working paper.

[21] Denis, David and Diane Denis, 1995, Causes of Financial Distress Following Leveraged Recapitalizations, *Journal of Financial Economics* 37, 129-157.

[22] Elton, Edwin, Martin Gruber, Deepak Agrawal, and Christopher Mann, 2001, Explaining the Rate Spread on Corporate Bonds, *Journal of Finance* 56, 247-278.

[23] Eisdorfer, A., 2008, Empirical Evidence of Risk Shifting in Financially Distressed Firms, *Journal of Finance* 63 (2), 609-637.

[24] Elkamhi, R., Ericsson, J., 2009, Time Varying Risk Premia in Corporate Bond Markets, working paper.

[25] Eom Y., Helwege J., Huang J., 2004, Structural Models of Corporate Bond Pricing, *Review of Financial Studies* 17, 499-544.

[26] Fischer, E., Heinkel, R., Zechner, J., 1989, Dynamic capital structure choice: Theory and tests, *Journal of Finance* 44, 1940.

[27] Flannery, M., Rangan, K., 2006, Partial Adjustment Toward Target Capital Structure, *Journal of Financial Economics* 79, 469-506.

[28] Frank, M., Goyal, V., 2003, Testing the Pecking Order of Capital Structure, *Journal of Financial Economics* 15, 2-33.

[29] Frank, M., Goyal, V., 2007, Capital Structure Decisions: Which Factors are Reliably Important?, working paper.

[30] Geske R., 1977, The Valuation of Corporate Liabilities as Compound Options, *Journal of Financial and Quantitative Analysis*, 541-552.

[31] Gilson, Stuart, 1997, Transaction Costs and Capital Structure Choice: Evidence from Financially Distressed Firms, *Journal of Finance* 52, 161-196.

[32] Glover B., 2010, The Expected Cost of Default, *Working paper*.

[33] Graham, J., 2000, How Big Are the Tax Benefits of Debt?, *Journal of Finance* 55, 1901-1942.

[34] Graham, J., 1996, Debt and the Marginal Tax Rate, *Journal of Financial Economics* 41, 41-73.

[35] Graham, J., Harvey, C., 2001, The theory and practice of corporate finance: Evidence from the field, *Journal of Financial Economics* 60, 187-243.

[36] Hennessy, C., Whited, T., 2007, How Costly is External Financing? Evidence from a Structural Estimation, *Journal of Finance* 62, 1705-1745.

[37] Hennessy, C., Levy, A., Whited, T, 2007,Testing Q Theory with Financing Frictions, *Journal of Financial Economics* 83, 691-717

[38] Hovakimian, A., Kayhan, A., Titman, S., 2008, How do managers target their credit ratings? A study of credit ratings and managerial discretion, working paper.

[39] Jensen, M., Meckling, W., 1976, Theory of the Firm: Managerial Behavior, Agency Costs, and Capital Structure, *Journal of Financial Economics* 3, 305-360.

[40] Jones, E., Mason, S., Rosenfeld, E., 1984, Contingent claims analysis of corporate capital structures: An empirical investigation, *Journal of Finance* 39, 611627.

[41] Jones, E., Mason, S., Rosenfeld, E., 1985, Contingent Claims Valuation of Corporate Liabilities: Theory and Empirical Tests, University of Chicago Press.

[42] Korteweg, A., The Net Benefits to Leverage, forthcoming, Journal of Finance.

[43] Larrain, B., Yogo, M., 2008, Does Firm Value Move Too Much to be Justified by Subsequent Changes in Cash Flow?, *Journal of Financial Economics* 87(1), 200226.

[44] Leary, M, Roberts, M., Do Firms Rebalance Their Capital Structures?, *Journal of Finance* 60, 2575 - 2619

[45] Leland, Hayne, 1994, Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance* 49, 1213-1252.

[46] Leland, H., 2004, Predictions of default probabilities in structural models of debt, *Journal of Investment Management* 2, 520.

[47] Leland, H., Toft, K., 1996, Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads, *Journal of Finance* 51, 987-1019.

[48] Lemmon, M., Roberts, M., Zender, Z., 2008, Back to the Beginning: Persistence and the Cross-Section of Corporate Capital Structure, *Journal of Finance* 63, 1575-1608.

[49] Longstaff, F., Schwartz, E., 1995, A simple approach to valuing risky fixed and floating rate debt, *Journal of Finance* 50, 789819.

[50] Kim, I., Ramaswamy, K., Sundaresan, S., 1993, Does default risk in coupons affect the valuation of corporate bonds? A contingent claims model, *Financial Management* 22, 117131.

[51] Kisgen, D., 2008, Do Firms Target Credit Ratings or Leverage Levels?, *Journal of Financial and Quantitative Analysis*, forthcoming.

[52] Lewis, C., Firm-Specific Estimates of the Ex-ante Bankruptcy Discount, working paper, Vanderbilt University.

[53] Maksimovic, V. 1986, Optimal Financial Structure in a Stochastic Oligopoly, Ph.D. dissertation, Harvard University.

[54] Merton, R., 1974, On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance*, p. 449-470.

[55] Myers, S., 1977, Determinants of Corporate Borrowing, *Journal of Financial Economics* 5, 147-175.

[56] Molina, Carlos, 2005, Are Firms Underleveraged? An Examination of the Effect of Leverage on Default Probabilities, *Journal of Finance* 60, 1427-1459.

[57] Opler, Tim and Sheridan Titman, 1994, Financial Distress and Corporate Performance, *Journal of Finance* 49, 1015-1040.

[58] Parrino, R., Weisbach, M., 1999, Measuring Investment Distortions Arising from Stockholder-Bondholder Conflicts, *Journal of Financial Economics* 53, 3-42.

[59] Parsons, C., Titman, S., 2008, Empirical Capital Structure: A Review, working paper.

[60] Warner, Jerold, 1977, Bankruptcy Costs: Some Evidence, *Journal of Finance* 32, 337-347.

[61] Rajan, R., Zingales, L., 1995, What Do We Know about Capital Structure? Some Evidence from International Data, *Journal of Finance* 50(5), 1421-1460.

[62] Ross, S., 1985, Debt and Taxes and Uncertainty, *Journal of Finance* 40, 637-58.

[63] Saita, L., 2006, The puzzling price of corporate default risk, working paper.

[64] Sharpe, S., 1994 Financial Market Imperfections, Firm Leverage, and the Cyclicality of Employment, *The American Economic Review* 84(4), 1060-1074.

[65] Shyam-Sunder, L. Myers, S., 1999, Testing Static Tradeoff against Pecking Order Models of Capital Structure, *Journal of Financial Economics* 51, 219-243.

[66] Stohs, M., Mauer, D., 1994, The determinants of corporate debt maturity structure, working paper.

[67] Schaefer, S., Strebulaev, I., Structural Models of Credit Risk are Useful: Evidence from Hedge Ratios on Corporate Bonds, forthcoming, *Journal of Financial Economics.*

[68] Strebulaev, I., 2007, Do Tests of Capital Structure Theory Mean What They Say?, *Journal of Finance*, 62(4), 1747-1787.

[69] Strebulaev, I., Yang, B., 2006, The Mystery of Zero-Leveraged Firms, working paper.

[70] Titman, S., 1984, The Effect of Capital Structure on a Firm's Liquidation Decision, *Journal of Financial Economics* 13(1), 137-151.

[71] Titman, S., Wessels, R., 1988, The Determinants of Capital Structure, *Journal of Finance* 43, 1-19.

[72] Titman, S., Tsyplakov, S., 2007, A Dynamic Model of Optimal Capital Structure, *Review of Finance* 11, 401-451.

[73] Warner, J., 1977, Bankruptcy costs: Some evidence, *Journal of Finance* 32, 337347.

[74] Welch, I., 2004, Capital Structure and Stock Returns, *Journal of Political Economy* 112(1), 106-131.

A.1. Technical details of Leland and Toft (1996) and estimation

A firm has unlevered assets with value V_t. As in Merton (1974), Black and Cox (1976), Brennan and Schwartz (1978) and other papers on structural credit risk models, the dynamics of V_t^U can be described using a continuous diffusion model with constant proportional volatility. The objective and risk-adjusted dynamics of the unlevered firm value can be written as

$$dV_t = (\mu - \delta)V_t dt + \sigma V_t dW_t, \tag{13}$$
$$dV_t = (r - \delta)V_t dt + \sigma V_t d\tilde{W}_t, \tag{14}$$

where μ is the expected continuously compounded growth rate for V, r is the risk-free interest rate, and δ is the payout ratio, or proportion of value paid out to security holders. The volatility of the unlevered assets is σ, and W_t and $\tilde{W}_t$ are Brownian motions under objective and risk-adjusted probabilities, respectively.

Additionally, the firm has debt B_t, which generates both a tax shield (TS) and ex ante financial distress costs (FDC). In the LT model, the firm operates with a stationary debt structure. This debt structure consists of a continuum of bonds with maturities ranging from immediate to an upper bound T, which corresponds to the maturity of newly issued debt. As a bond matures, it is replaced by a new bond with the same coupon, initial maturity and principal. Should the fortunes of a firm decline, the amount raised by reselling a nominally unchanged bond must clearly decrease. The difference between the amount raised and that necessary to withdraw the old bond will be funded by shareholders if they find it optimal to keep the firm afloat. New bonds are issued and old bonds retired at a rate of $p = (\frac{P}{T})$ per year, where P is the total principal value of all outstanding bonds. This condition ensures that as long as the firm remains solvent, at any time during the life of the firm, the total outstanding debt principal will be invariant and equal to P. These latter assumptions also ensure that the total coupon C paid for all outstanding bonds is also stationary.

Levered firm value, V^L, is equal to the value of its unlevered assets (V_t^U) plus the tax shield (TS) less the costs of financial distress (FDC), or

$$V_t^L = V_t + TS(V_t) - FDC(V_t)$$
$$= E(V_t) + D(V_t). \tag{15}$$

In equation (15), we are primarily interested in quantifying the ex ante distress costs FDC. In our estimation, we will exploit the closed-form solution for equity in order to estimate asset values, V_t^U, and their volatility, σ, the key unobservable inputs to the analytical expressions for the components of levered firm value.

The value of the tax shield is given by

$$TS(V_t) = \frac{\tau C}{r} \cdot \left(1 - \left(\frac{V_t}{V_B} \right)^x \right),$$

and the ex ante distress costs are given by

$$FDC(V_t) = LGD \cdot \left(\frac{V_t}{V_B} \right)^x,$$

37

where LGD is given by $\alpha \cdot V_B$.

The value of debt is given by

$$D(v_t) = \frac{C}{r} + \left(N - \frac{C}{r}\right)\left(\frac{1 - e^{-rM}}{rM} - I(v_t)\right) + \left((1 - \alpha)L - \frac{C}{r}\right)J(v_t).$$

The default boundary is given by

$$V_B = \frac{\frac{C}{r}\left(\frac{A}{rM} - B\right) - \frac{AP}{rM} - \frac{\tau C x}{r}}{1 + \alpha x - (1 - \alpha)B},$$

where

$$A = 2ye^{-rM}\phi\left[y\sigma\sqrt{M}\right] - 2z\phi\left[z\sigma\sqrt{M}\right]$$
$$- \frac{2}{\sigma\sqrt{M}}n\left[z\sigma\sqrt{M}\right] + \frac{2e^{-r\Upsilon}}{\sigma\sqrt{M}}n\left[y\sigma\sqrt{M}\right] + (z - y)$$

$$B = -\left(2z + \frac{2}{z\sigma^2 M}\right)\phi\left[z\sigma\sqrt{M}\right] - \frac{2}{\sigma\sqrt{M}}n\left[z\sigma\sqrt{M}\right] + (z - y) + \frac{1}{z\sigma^2 M},$$

and $n[\cdot]$ denotes the standard normal density function.

The components of the debt formulae are

$$I(v) = \frac{1}{r\Upsilon}\left(i\left(v\right) - e^{-r\Upsilon}j(v)\right),$$

$$i(v) = \phi\left[h_1\right] + \left(\frac{v}{L}\right)^{-2a}\phi\left[h_2\right],$$

$$j(v) = \left(\frac{v}{L}\right)^{-y+z}\phi\left[q_1\right] + \left(\frac{v}{L}\right)^{-y-z}\phi\left[q_2\right],$$

and

$$J(v) = \frac{1}{z\sigma\sqrt{M}}\left(\begin{array}{c} -\left(\frac{v}{L}\right)^{-a+z}\phi\left[q_1\right]q_1 \\ +\left(\frac{v}{L}\right)^{-a-z}\phi\left[q_2\right]q_2 \end{array}\right).$$

. Finally,

$$q_1 = \frac{-b - z\sigma^2 M}{\sigma\sqrt{M}},$$

$$q_2 = \frac{-b + z\sigma^2 M}{\sigma\sqrt{M}},$$

$$h_1 = \frac{-b - y\sigma^2 M}{\sigma\sqrt{M}},$$

$$h_2 = \frac{-b + y\sigma^2 M}{\sigma\sqrt{M}},$$

$$y = \frac{r - \beta - 0.5\sigma^2}{\sigma^2},$$

$$z = \frac{\sqrt{y^2\sigma^4 + 2r\sigma^2}}{\sigma^2},$$

$$x = y + z,$$

and
$$b = \ln\left(\frac{v}{L}\right).$$

As a result, equity can be valued simply as

$$E(V_t) = V_t + TS(V_t) - BC(B_t) - D(V_t). \tag{16}$$

A.2. Estimation

The advantage of a structural model such as LT is that it simultaneously provides closed-form expressions for FDC, default probabilities, equity values and volatilities. The two key inputs to the model, asset value V_t and volatility σ, cannot be observed. Instead, they are inferred by requiring the model to fit observables. We implement the model by for each firm quarter by matching the model-implied total equity value and equity volatility with the observed market capitalization and trailing historical volatility. This procedure is widely used in industry and academic research.[24] Prior to this procedure, a number of additional model parameters need to be determined.

- Payout rate (δ). This parameter determines how much cash flow is available at each point in time to service debt and pay dividends. One way to estimate it would be to write $\delta = \frac{IE+DIV}{V_t} = \frac{IE}{TL}lev + \frac{DIV}{E}(1-lev)$, where IE denotes interest expenses, DIV dollar dividends paid by the firm, TL total liabilities and $lev = \frac{TL}{TL+E} = \frac{TL}{V}$. This is feasible as long as we are willing (in this step) to proxy V_t by the sum of the market capitalization and total liabilities. We carry out this calculation for the subset of firms for which we have the required data. We then take the average of 2.9% and impose it across all firms. This number is in the range of estimates provided in recent work on payout rates by Larrain and Yogo (2008).

- Debt structure (C, P, T). We use total liabilities to proxy for P, the total principal of debt, and assume for simplicity that the aggregate coupon rate on debt to be the risk-free rate, and thus $C = r \cdot N$. Although this may seem a strong assumption, note that many forms of debt do not pay interest and that the rates on the others depend on the maturity. An alternative used in the literature is to assume that C can be proxied by the coupon rate on corporate debt. This is appealing as it would link the coupon rate to market data. However, it will tend to bias the rate upwards as corporate bonds tend to be issued as long-term instruments, and term structures of credit spreads tend to be upward sloping. In our case, this is not a viable approach as using bond data would greatly limit the size of our data set. An alternative approach, which is also limited by data considerations, would be to use actual figures for interest expenses. For the subset of our data for which these are available, we find that the ratio of interest expense to total liabilities is about 5.2% annually. The average risk free rate use in the estimation is 6.6%. There is limited data available for debt maturity. We use an average of 6.76 years which corresponds to a value of $T = 3.38$ for the maturity of newly issued debt in the Leland and Toft (1996) model.[25]

- Taxes. We obtain a sequence of firm-specific, forward-looking marginal effective tax rates from the website of John Graham, and then use these as inputs into the LT model.[26] We are able to match over 85% of the firms in our sample. For each matched firm, we average its forward-looking effective tax rate (after interest expense) over time, and use this for every observation for that firm. While this

---

[24]See for example Jones, Mason and Rosenfeld (1984, 1985) and Bharath and Shumway (2008).

[25]This number is taken from an empirical study by Stohs and Mauer (1994).

[26]Firm-specific tax rates are available upon request at http://faculty.fuqua.duke.edu/jgraham.

procedure will pick up differences in average tax rates, it does not capture the type of state-dependent considerations considered in Graham (1996). Extending the LT model to allow for state-dependent tax rates is a complex task that lies beyond the scope of this paper. In particular, allowing the default boundary in LT to be time-dependent, would considerably complicate the present analysis. The sample average is 19.96%.

- Risk-free rate. We use the 5-year Treasury rate.

- *Ex post* distress costs (α). For this parameter, we use the upper and lower bounds reported in Andrade and Kaplan (1998)-23% and 10%-as well as the midpoint, 16.5%.

- Corporate tax rate. We use average firm-level tax rates obtained from John Graham's web site.

Once these have been determined, we rely on the following two equations:

$$
\begin{aligned}
E_{mkt} &= E_{mod}(V_t; \sigma) \\
\sigma_E &= \frac{V_t}{E_{mod}(V_t; \sigma))} \frac{\partial E_{mod}(V_t; \sigma)}{\partial V_t} \sigma
\end{aligned}
$$

where subscripts *mod* and *mkt* denote model and market values, respectively. Estimates of value and volatility $\left(\hat{V}_t, \hat{\sigma}\right)$ are obtained by a straightforward numerical solution to the two above equations.

A.3. A model of pre-default distress costs

The tax shield and default costs

The value of the tax shield can be solved for in an analogous way to the value of debt, to leave us with the following solution:

$$
\begin{aligned}
TS_{IG} &= \frac{\tau C}{r} + \beta_{1,IG} v^{x_{1,IG}} + \beta_{2,IG} v^{x_{2,IG}}, \quad v_t > V_D > V_B, \\
TS_J &= \frac{\tau C}{r} + \beta_{1,J} v^{x_{1,J}} + \beta_{2,J} v^{x_{2,J}}, \quad V_D > v_t > V_B,
\end{aligned}
$$

with

$$
\begin{bmatrix} \beta_{2,IG} \\ \beta_{1,J} \\ \beta_{2,J} \end{bmatrix} = \begin{bmatrix} 0 & V_B^{x_{1,J}} & V_B^{x_{2,J}} \\ V_D^{x_{2,IG}} & -V_D^{x_{1,J}} & -V_D^{x_{2,J}} \\ x_{2,IG} \cdot V_D^{x_{2,IG}} & -x_{1,J} \cdot V_D^{x_{1,J}} & -x_{2,J} \cdot V_D^{x_{2,J}} \end{bmatrix}^{-1} \cdot \begin{bmatrix} -\frac{\tau C}{r} \\ 0 \\ 0 \end{bmatrix},
$$

where

$$
\begin{aligned}
x_{1,IG} &= \frac{-(r - \delta - \frac{1}{2}\sigma^2) + \sqrt{(r - \delta - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}, \\
x_{2,IG} &= \frac{-(r - \delta - \frac{1}{2}\sigma^2) - \sqrt{(r - \delta - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}, \\
x_{1,J} &= \frac{-(r - \delta - \gamma - \frac{1}{2}\sigma^2) + \sqrt{(r - \delta - \gamma - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}, \\
x_{2,J} &= \frac{-(r - \delta - \gamma - \frac{1}{2}\sigma^2) - \sqrt{(r - \delta - \gamma - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}.
\end{aligned}
$$

For default costs, similar calculations yield

$$
\begin{aligned}
DC_{IG} &= \eta_{1,IG} v^{x_{1,IG}} + \eta_{2,IG} v^{x_{2,IG}}, \quad v_t > V_D > V_B \\
DC_J &= \eta_{1,J} v^{x_{1,J}} + \eta_{2,J} v^{x_{2,J}}, \quad V_D > v_t > V_B
\end{aligned}
$$

with

$$
\begin{bmatrix} \eta_{2,IG} \\ \eta_{1,J} \\ \eta_{2,J} \end{bmatrix} = \begin{bmatrix} 0 & V_B^{x_{1,J}} & V_B^{x_{2,J}} \\ V_D^{x_{2,IG}} & -V_D^{x_{1,J}} & -V_D^{x_{2,J}} \\ x_{2,IG} \cdot V_D^{x_{2,IG}} & -x_{1,J} \cdot V_D^{x_{1,J}} & -x_{2,J} \cdot V_D^{x_{2,J}} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \alpha V_B \\ 0 \\ 0 \end{bmatrix}.
$$

Pre-default distress costs

Similar to the valuation of debt we can write the general solutions to the value of the pre-default distress costs as

$$
\begin{aligned}
PDC_{IG} &= \phi_{1,IG} v^{x_{1,IG}} + \phi_{2,IG} v^{x_{2,IG}}, \quad v_t > V_D > V_B, \\
PDC_J &= \frac{\gamma v}{\delta + \gamma} + \phi_{1,J} v^{x_{1,J}} + \phi_{2,J} v^{x_{2,J}}, \quad V_D > v_t > V_B.
\end{aligned}
$$

We apply the following boundary conditions (corresponding to pairs of value-matching and smooth-pasting conditions at the downgrade and default thresholds respectively)

$$
\begin{aligned}
PDC_{IG}(V_D) &= PDC_J(V_D), \\
\frac{\partial PDC_{IG}(V_D)}{\partial v_t} &= \frac{\partial PDC_J(V_D)}{\partial v_t}, \\
PDC_J(V_B) &= 0, \\
\lim_{v \to \infty} (PDC_{IG}(v)) &= 0.
\end{aligned}
$$

The default condition yields

$$\frac{\gamma v}{\delta + \gamma} + \phi_{1,J} V_B^{x_1,J} + \phi_{2,J} V_B^{x_2,J} = 0,$$

while value matching at the downgrade threshold V_D gives us

$$\phi_{2,IG} V_D^{x_2,IG} - \phi_{1,J} V_D^{x_1,J} + \phi_{2,J} V_D^{x_2,J} - \frac{\gamma V_D}{\delta + \gamma} = 0.$$

The smooth-pasting condition at downgrade (after multiplying both sides by V_D) yields

$$x_{2,IG} \cdot \phi_{2,IG} V_D^{x_2,IG} - x_{1,J} \cdot \phi_{1,J} V_D^{x_1,J} - x_{2,J} \cdot \phi_{2,J} V_D^{x_2,J} - \frac{\gamma V_D}{\delta + \gamma} = 0$$

The solution in matrix form can be written as

$$\begin{bmatrix} \phi_{2,IG} \\ \phi_{1,J} \\ \phi_{2,J} \end{bmatrix} = \begin{bmatrix} 0 & V_B^{x_1,J} & V_B^{x_2,J} \\ V_D^{x_2,IG} & -V_D^{x_1,J} & -V_D^{x_2,J} \\ x_{2,IG} \cdot V_D^{x_2,IG} & -x_{1,J} \cdot V_D^{x_1,J} & -x_{2,J} \cdot V_D^{x_2,J} \end{bmatrix}^{-1} \cdot \begin{bmatrix} -\frac{\gamma V_B}{\delta + \gamma} \\ \frac{\gamma V_D}{\delta + \gamma} \\ \frac{\gamma V_D}{\delta + \gamma} \end{bmatrix}.$$

A.4. Risk shifting

In this section, we consider a model without the explicit pre-default distress costs described above (i.e., $\gamma = 0$), where management representing shareholders elects to increase business risk at a lower threshold V_{RS}. The model in this section is a simplified version of Leland (1998) so details are kept to a minimum.

In normal conditions, the firm operates at a risk level σ according to the following dynamics:

$$dv_t = (\mu - \delta) v_t dt + \sigma v_t dW_t.$$

As the firm approaches financial distress (potentially well before actual default), the shareholders will wish to see risk increased to σ_H, when $V_{RS} \geq v_t \geq V_B$. The remaining parameters are as defined above.

Valuing debt with risk shifting

Debt values satisfy the following two partial differential equations:

$$\frac{1}{2}\sigma^2 \frac{\partial^2 D}{\partial v^2} + (r - \delta)v_t \frac{\partial D}{\partial v} - rD + \frac{\partial D}{\partial t} + e^{-mt}(C + mP) = 0, \tag{17}$$

for $v_t > V_{RS} > V_B$, and

$$\frac{1}{2}\sigma_H^2 \frac{\partial^2 D_{RS}}{\partial v^2} + (r - \delta)v_t \frac{\partial D_{RS}}{\partial v} - rD_{RS} + \frac{\partial D_{RS}}{\partial t} + e^{-mt}(C + mP) = 0, \tag{18}$$

for $V_{RS} > v_t > V_B$.

The general solution to this is

$$D = \frac{C + mP}{r + m} + \alpha_1 \cdot v^{z_1} + \alpha_2 \cdot v^{z_2}, \quad v_t > V_{RS} > V_B \tag{19}$$

$$, D_{RS} = \frac{C + mP}{r + m} + \alpha_{1,RS} \cdot v^{z_{1,RS}} + \alpha_{2,RS} \cdot v^{z_{2,RS}}, \quad V_{RS} > v_t > V_B. \tag{20}$$

where the constants $z_1, z_2, z_{1,RS},$ and $z_{2,RS}$ are given by

$$z_1 = \frac{-(r - \delta - \frac{1}{2}\sigma^2) + \sqrt{(r - \delta - \frac{1}{2}\sigma^2)^2 + 2(r + m)\sigma^2}}{\sigma^2},$$

$$z_2 = \frac{-(r - \delta - \frac{1}{2}\sigma^2) - \sqrt{(r - \delta - \frac{1}{2}\sigma^2)^2 + 2(r + m)\sigma^2}}{\sigma^2},$$

$$z_{1,RS} = \frac{-(r - \delta - \frac{1}{2}\sigma_H^2) + \sqrt{(r - \delta - \frac{1}{2}\sigma_H^2)^2 + 2(r + m)\sigma_H^2}}{\sigma_H^2},$$

$$z_{2,RS} = \frac{-(r - \delta - \frac{1}{2}\sigma_H^2) - \sqrt{(r - \delta - \frac{1}{2}\sigma_H^2)^2 + 2(r + m)\sigma_H^2}}{\sigma_H^2}.$$

To obtain the α constants, we to need to impose boundary conditions analogous to those for the pre-default distress cost model described above. At the risk shifting threshold we impose value-matching and smooth-pasting conditions. The last two conditions are the values of debt at default and that debt approaches the value of risk-free debt as firm value tends to infinity.

$$D(V_{RS}) = D_{RS}(V_{RS}), \tag{21}$$

$$\frac{\partial D(V_{RS})}{\partial v_t} = \frac{\partial D_{RS}(V_{RS})}{\partial v_t}, \tag{22}$$

$$D_{RS}(V_B) = (1 - \alpha)V_B, \tag{23}$$

$$\lim_{v \to \infty} (D(v)) = \frac{C + mP}{r + m}. \tag{24}$$

Consider now equations (19) and (24). Since $z_1 > 0$, we can infer that $\alpha_1 = 0$. Next consider equation (23) - the default condition. We obtain

$$\alpha_{1,RS} V_B^{z_{1,RS}} + \alpha_{2,RS} V_B^{z_{2,RS}} - (1 - \alpha)V_B + \frac{C + mP}{r + m} = 0.$$

Next, consider value-matching at the risk-shifting threshold

$$\frac{C+mP}{r+m} + \alpha_2 \cdot V_{RS}^{z_2} = \frac{C+mP}{r+m} + \alpha_{1,RS} \cdot V_{RS}^{z_1,RS} + \alpha_{2,RS} \cdot V_{RS}^{z_2,RS},$$

or

$$\alpha_2 \cdot V_{RS}^{z_2} - \alpha_{1,J} \cdot V_{RS}^{z_1,RS} - \alpha_{2,RS} \cdot V_{RS}^{z_2,RS} = 0,$$

and smooth pasting at the risk-shifting point

$$z_2 \cdot \alpha_2 V^{z_2-1} - z_{1,RS} \cdot \alpha_{1,RS} \cdot V_{RS}^{z_1,RS-1} - z_{2,RS} \cdot \alpha_{2,J} \cdot V_D^{z_2,RS-1} = 0.$$

These three equations can now be solved for the three remaining unknowns: $\alpha_2, \alpha_{1,RS}, \alpha_{2,RS}$. As before we can write the solution in matrix form

$$
\begin{bmatrix} \alpha_2 \\ \alpha_{1,J} \\ \alpha_{2,J} \end{bmatrix} =
\begin{bmatrix} 0 & V_B^{z_1,RS} & V_B^{z_2,RS} \\ V_{RS}^{z_2} & -V_{RS}^{z_1,RS} & -V_{RS}^{z_2,RS} \\ z_2 \cdot V_{RS}^{z_2} & -z_{1,RS} \cdot V_{RS}^{z_1,RS} & -z_{2,RS} \cdot V_{RS}^{z_2,RS} \end{bmatrix}^{-1}
\cdot
\begin{bmatrix} (1-\alpha)V_B - \frac{C+mP}{r+m} \\ 0 \\ 0 \end{bmatrix}.
$$

The tax shield and default costs

The value of the tax shield can be written

$$TS = \frac{\tau C}{r} + \beta_1 v^{h_1} + \beta_{2,IG} v^{h_2}, \quad v_t > V_{RS} > V_B,$$

$$TS_{RS} = \frac{\tau C}{r} + \beta_{1,RS} v^{h_1,rS} + \beta_{2,RS} v^{h_2,RS}, \quad V_{RS} > v_t > V_B,$$

with

$$
\begin{bmatrix} \beta_2 \\ \beta_{1,RS} \\ \beta_{2,RS} \end{bmatrix} =
\begin{bmatrix} 0 & V_B^{h_1,RS} & V_B^{h_2,RS} \\ V_{RS}^{h_2} & -V_{RS}^{h_1,RS} & -V_{RS}^{h_2,RS} \\ h_2 \cdot V_{RS}^{h_2} & -h_{1,RS} \cdot V_{RS}^{h_1,RS} & -h_{2,RS} \cdot V_{RS}^{h_2,RS} \end{bmatrix}^{-1}
\cdot
\begin{bmatrix} -\frac{\tau C}{r} \\ 0 \\ 0 \end{bmatrix},
$$

and

$$
h_1 = \frac{-(r - \delta - \frac{1}{2}\sigma^2) + \sqrt{(r - \delta - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2},
$$

$$
h_2 = \frac{-(r - \delta - \frac{1}{2}\sigma^2) - \sqrt{(r - \delta - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2},
$$

$$
h_{1,RS} = \frac{-(r - \delta - \frac{1}{2}\sigma_H^2) + \sqrt{(r - \delta - \frac{1}{2}\sigma_H^2)^2 + 2r\sigma_H^2}}{\sigma_H^2},
$$

$$
h_{2,RS} = \frac{-(r - \delta - \frac{1}{2}\sigma_H^2) - \sqrt{(r - \delta - \frac{1}{2}\sigma_H^2)^2 + 2r\sigma_H^2}}{\sigma_H^2}.
$$

For default costs, we obtain

$$
\begin{aligned}
DC &= \eta_1 v^{h_1} + \eta_2 v^{h_2}, \quad v_t > V_{RS} > V_B, \\
DC_{RS} &= \eta_{1,RS} v^{h_{1,RS}} + \eta_{2,RS} v^{h_{2,RS}}, \quad V_{RS} > v_t > V_B,
\end{aligned}
$$

with

$$
\begin{bmatrix} \eta_2 \\ \eta_{1,J} \\ \eta_{2,J} \end{bmatrix} =
\begin{bmatrix}
0 & V_B^{h_{1,RS}} & V_B^{h_{2,RS}} \\
V_{RS}^{h_2} & -V_{RS}^{h_{1,RS}} & -V_{RS}^{h_{2,RS}} \\
h_2 \cdot V_{RS}^{h_2} & -h_{1,RS} \cdot V_{RS}^{h_{1,RS}} & -h_{2,RS} \cdot V_{RS}^{h_{2,RS}}
\end{bmatrix}^{-1}
\cdot
\begin{bmatrix} \alpha V_B \\ 0 \\ 0 \end{bmatrix}.
$$