USC FBE FINANCE SEMINAR
presented by: Tyrone Callahan
FRIDAY, January 16, 2004
10:30 am - 12:00 pm, **Room: JKP-202**

Speculative Markets With an Unknown Number of Insiders

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This Draft : December 2003

*McCombs School of Business, UT Austin. Portions of this research were completed while visiting the Marshall School of Business at USC. I am grateful to Andres Almazan, Aydogan Alti, Sheridan Titman and especially Dimitri Vayanos for helpful comments and suggestions, and to seminar participants at the 2003 Texas Finance Festival. All errors are my own.

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Abstract

This paper analyzes how uncertainty about the number of informed traders in a market alters the market characteristics. Increasing the uncertainty about the number of informed traders while holding the expected number of informed traders constant: (i) increases the residual price uncertainty in the market; (ii) increases the total expected volume of informed trade and profits earned by insiders; and (iii) significantly prolongs the impact of an information event by extending the expected trade horizon of insiders and the time over which market liquidity is depressed. These results are compared with those found in experimental asset markets with a similar information structure. The model confirms several of the broad experimental conclusions and provides new insights in some important dimensions, particularly with respect to the behavior of (unknown) monopolist insiders.

How should traders who expend resources to generate private information to use when trading allocate their resources? One branch of literature, beginning with Kyle (1985), offers a clear prescription: seek to acquire unique information from which one can earn monopoly rents. This branch of literature, among other things, provides a theoretical foundation for the intuition that, in financial markets, private information is valuable while public information is not. Public information, being already reflected in prices, doesn't lend itself to profitable trading strategies.[1] Private information, in constrast, allows one to trade strategically and earn positive profits (on average) while the price adjusts to incorporate the information (Kyle (1985)). The value of private information depends largely on whether it is known to one or many. Holden and Subrahmanyam (1992), for example, show that information shared by as few as two insiders may not yield any trading profits. Foster and Viswanathan (1996) show that traders with correlated private signals will trade aggressively on the common component of their signals and strategically on the unique component. Expected total insider profits are always lower than those of a monopolist insider, but they are the lowest when the insiders have highly correlated signals.[2]

It would appear to follow, then, that traders could maximize their collective profits by minimizing the competition among themselves by, for example, focusing their information gathering efforts in different arenas. This paper shows that this conclusion may be incorrect. While traders who acquire "information monopolies" will earn monopoly rents, higher profits

[1]Fama (1991) reviews the literature on market efficiency. In general, most U.S. financial markets show evidence of semi-strong form market efficiency. That is, trading strategies based on public information (including price histories) are rarely profitable.

[2]See also Back, Cao and Willard (2000) who solve the continuous time analog of the earlier discrete time models.

can be earned by traders who intentionally dilute their monopoly status by spreading their information gathering efforts across several arenas. For example, it can be more profitable for an analyst to follow several companies already being followed by other analysts than to focus her analytical efforts on a company with no existing analyst coverage.

This result is established by relaxing a key feature of previous theoretical work. Existing theoretical results have been derived in settings where market participants know the complete distribution of private information throughout the market. That is, traders and market makers know how many insiders there are, the quality of every insider's signal, and the relation among signals. Such structure is helpful for analytic tractability. In practice, dealers and traders may need to infer the distribution of information in the market. That is, dealers in general don't know whether a non-public information event has occured and must monitor the order flow for signs of informed trading. Insiders, for their part, are unlikely to know whether their information is known only to them, or known to other insiders as well. They monitor the order flow to assess their competitive position.

This paper shows, in a simple setting, how uncertainty about the number of informed traders changes the behavior of dealers and insiders. I focus specifically on the case when the possible number of informed traders is zero, one, or two. I view this as the most interesting case because past research has shown starkly different market characteristics when there are known to be no insiders versus a monopolist insider versus competing insiders. In the multi-period Kyle (1985) setting, an analog of which is studied in this paper, market liquidity, price efficiency, insider trade intensity, and insider profits all change in significant ways as the known number of insiders changes from zero to one to two. When it is common knowledge that there are no insiders, markets are infinitely liquid and price and price efficiency are constant. With one insider, the monopolistically informed trader maximizes expected profits by gradually revealing his information over the entire trade horizon. Market liquidity is constant over the trade horizon and price efficiency increases at a near constant rate. In contrast, two identically informed insiders will compete very aggressively and quickly impound their shared information into price. In this case the market price is very sensitive to order flow early on, but liquidity quickly increases thereafter. In the continuous trading limit, traders are infinitely aggressive in the first auction, prices instantly reflect the inside information, and insider profits are driven to zero (Back, Cao, and Willard (2000)).

My goal is to study the effects of uncertainty about the number of informed traders independent from the effects of changing the number of informed traders by fixing the expected number of insiders to one. Uncertainty about the number of insiders in the market changes as the market moves from one with a known monopolist insider to one in which there is an equal probability of there being no insiders or two insiders. In the general case presented here there may exist zero, one, or two insiders, but the expected number is always one.

I find that uncertainty about the number of insiders in the market plays a dual role. First, it changes the dynamics of competition between insiders. Duopolist insiders in my model compete less aggressively than would duopolists whose presence were common knowledge. Second, uncertainty about the number of insiders acts as an additional source of 'noise' in the market. The added noise, however, is greater for the market maker than for insiders because insiders each know of their own existence. Consequently, monopolist insiders generally infer their monopolist status before the market maker and are able to trade significantly more over the entire trade horizon than would a monopolist whose presence was common knowledge. Such undiscovered monopolists are expected to earn significantly higher profits than known monopolists. This leads to the interesting implication that potentially informed traders would prefer to be entered in an "information lottery" in which there were positive probabilities that the information would be revealed to zero, one, or two traders versus a lottery in which there was guaranteed to be a single winner with monopoly rights to the information. For example, my model implies that it is likely to be more profitable to be one of several analysts following a company, than to be the only analyst following a company, all else equal.

I find that market makers have difficulty determining the number of insiders in the market. It is particularly difficult for market makers to distinguish between no insiders being present and a single insider being present. This has several effects. First, when no insiders are present in the market (ex post), the chance that an information event may have occured and that there may be an insider trading in the market creates excess price volatility and deviations of price from fundamental value. Second, when there are insiders present in the market, the duration of an information event is expected to be significantly longer when the presence of insiders must be inferred by the market maker. That is, market liquidity is depressed significantly longer following an information event when the number of insiders is unknown versus known.

These findings confirm the broad conclusions reported by Schnitzlein (2002) based on the observation of experimental asset markets: Uncertainty about the number of insiders present in the market induces insiders to delay their trades and lowers the informational efficiency of the market. Nevertheless, theory produces several normative results not observed experimentally. The area in which the theory and experimental evidence differ most is regarding the trading volume and profits of (ex post) monopolist insiders. As mentioned, insiders will tend to infer their monopolist status before the market maker. When this happens, such undiscovered monopolists should optimally delay their trades more than has been observed experimentally. This allows the insider to remain concealed from the market maker and continue to trade over the entire trade horizon and earn significantly higher trading profits. In equilibrium, this behavior extends the impact of an information event and depresses market liquidity relative to the case with a known monopolist.

The paper proceeds as follows. Section 1 presents the model and the equilibrium solution. Section 2 discusses the model equilibrium in the context of some numerical examples. Section 3 compares the results with experimental evidence. Section 4 concludes. The appendix contains a proof of the equilibrium and the technical details of how market liquidity is defined in the model.

1 The Model

The model is similar in structure to Kyle (1985). A single risky asset is traded by three types of traders: informed traders with private information about the liquidation value of the risky asset; liquidity traders with exogenously determined needs for trade; and a competitive market maker. All agents are risk neutral. There are N rounds of trade. In each round, informed traders seek to maximize total expected future profits while the market maker sets price equal to her expectation of the liquidation value of the risky asset. Let $\tilde{v}$ be the liquidation value of the risky asset and $\tilde{u}_n$ represent the noise trader order flow in round n.

The model's primary focus is to better understand how uncertainty about the number of informed traders effects insider behavior and market price efficiency. From past research we know that changing the number of informed traders can significantly change the nature of the equilibrium. In particular, Holden and Subrahmanyam (1992) and Foster and Viswanathan (1996), among others, have shown stark differences between settings with a monopolistic insider and those with competing (i.e., multiple) insiders. To isolate the effect of uncertainty about the number of informed traders from the effect of changes in the number of informed traders, I keep constant the expected number of informed traders while changing the likelihood of deviations from the mean. Specifically, the expected number of informed traders will be fixed at one while the probabilities of the actual number of insiders being zero, one, or two change. In this way we can explore the boundary between monopolistic and competitive markets and how potential, yet uncertain, competition effects the market equilibrium.

The realized number of informed traders is chosen by nature. There is one informed trader with *ex ante* probability θ, there are two informed traders with *ex ante* probability $(1-\theta)/2$, and there are no informed traders with *ex ante* probability $(1-\theta)/2$. θ is common knowledge. As stated, the expected number of informed traders is always one so θ is a mean-preserving spread parameter of the distribution of insiders. When $\theta = 1$, it is common knowledge that there exists a single monopolist insider. That is, an analog of the single informed trader model of Kyle (1985) is nested within my model and corresponds with $\theta = 1$. Because the assumptions in my model depart in significant ways from those of Kyle (1985), $\theta = 1$ is an important benchmark against which to compare the results for $\theta \neq 1$. When $\theta = 0$ there is an equal probability that there exist no insiders or two insiders. In this case, an insider knows before trading begins that he will be competing with another, but the market maker will have to infer the existence of informed traders by observing the order flow. In general (i.e., when $0 < \theta < 1$), both the market maker and any insiders will have to observe the order flow to draw inferences about how many insiders are present. Of course, the market maker and insider inference problems are not symmetric since each insider has knowledge of their own existence.[3]

Prior to any trade informed traders receive one of two signals, $\Phi \in \{H, L\}$, about the liquidation value of the asset.[4] The signals are equally likely. The simplest interpretation is that $\tilde{v}$ has a binomial distribution and insiders receive perfect signals of its value: $\tilde{v} \in \{H, L\}$. Alternatively, one can consider a general distribution for $\tilde{v}$ and define $H = \mathrm{E}[\tilde{v}|\Phi = H]$ and $L = \mathrm{E}[\tilde{v}|\Phi = L]$. In either case, all insiders receive the same signal.

The quantity traded by informed trader $i = 1, 2$ in trading round n is $\tilde{x}_{i,n}$. $|\tilde{x}_{i,n}|$ is assumed to be common knowledge.[5] Each round the market maker sets price, $\tilde{p}_n$, based on the observed aggregate order flow $\tilde{\omega}_n$ where $\tilde{\omega}_n = \tilde{I}_1\tilde{x}_{1,n} + \tilde{I}_2\tilde{x}_{2,n} + \tilde{u}_n$. $\tilde{I}_i$ is an indicator

[3]The form of uncertainty about the number of insiders in the experimental setup studied by Schnitzlein (2002) is nested in the parameterization assumed here and corresponds with $\theta = 1/2$. Section 3 compares the experimental results of Schnitzlein with the analytical results of this paper.

[4]Dridi and Germain (2001) study a one-period model of a speculative market in which insiders receive bullish or bearish signals.

[5]This is for technical reasons. Lacking this assumption, the model has no Nash equilibrium. The assumption is economically innocuous in that it commits insiders to trade their equilibrium magnitude, but divulges nothing about the existence of insiders or the information they possess. An alternative (though more restrictive) assumption would be to require that orders be of a fixed size.

variable equal to one when informed trader i exists and zero otherwise.

Each round of trading occurs in two stages. In stage one, $\tilde{u}_n$ is realized and the informed trader(s) choose(s) the quantity to trade, $\tilde{x}_{i,n}$, given his signal and the history of past prices. Informed traders use past prices to update their belief about the existence of multiple informed traders. In the second stage the market maker observes the aggregate order flow and sets the price at which she is willing to clear the market.

For tractability I assume $\tilde{u}_n$ is uniformly distributed with zero mean. I set the width of the uniform distribution to $2W/\sqrt{N}$ so that the total variance of noise trade over all rounds of trade remains constant at $W^2/3$. An unfortunate side-effect of bounded liquidity trade is that it can create incentives for insiders to manipulate the market. That is, insiders may find it profitable to trade against their information in the short term so as to deceive other market participants and earn higher profits later on. I restrict insiders from doing so.[6,7]

[6]Examples of settings allowing the possibility of profitable insider manipulation include Allen and Gale (1992), Fishman and Hagerty (1995) and John and Narayanan (1997).

[7]In models with unbounded liquidity trade (e.g., normally distributed), manipulative trading strategies are suboptimal. Because the focus of this paper is to isolate, to the extent feasible, the impact on trading behavior of uncertainty about the number of insiders, I restrict insiders from engaging in market manipulation. This, of course, is consistent with the Security Exchange Act of 1934 which prohibits manipulating securities prices. The SEA of 1934 states, in part, that "It is unlawful for any person to effect, alone or with one or more other persons, a series of transactions in any security registered on a national securities exchange or in connection with any security-based swap agreement (as defined in section 206B of the Gramm-Leach-Bliley Act) with respect to such security creating actual or apparent active trading in such security, or raising or depressing the price of such security, for the purpose of inducing the purchase or sale of such security by others." From a modeling perspective, this allows me the tractibility of uniformly distributed liquidity trade while at the same time maintaining as much as possible comparability with the existing literature based on unbounded noise trade. The feasibility and impacts of manipulative trading strategies in the present setting are the topic of a related research effort.

1.1 Definition of the Equilibrium

In the N-period model, with 0, 1, or 2 insiders, a market maker, and noise traders, the equilibrium comprises a trading strategy vector, $X = (X_1, \ldots, X_N)$, and a price function vector, $P = (P_1, \ldots, P_N)$, such that:

1. The informed trader(s) maximize(s) profit:

$$\mathrm{E}[\tilde{\pi}_n(X,P)|\tilde{p}_1,\ldots,\tilde{p}_{n-1},\tilde{v}] \geq \mathrm{E}[\tilde{\pi}_n(X^*,P)|\tilde{p}_1,\ldots,\tilde{p}_{n-1},\tilde{v}] \quad \forall\, n=1,\ldots,N \text{ and } X^* \neq X$$

   where

$$\tilde{\pi}_n(X,P) = \sum_{\eta=n}^{N} (\tilde{v} - \tilde{p}_\eta)\tilde{x}_\eta$$

2. The market price is efficient:

$$\tilde{p}_n = \mathrm{E}[\tilde{v}|\tilde{\omega}_1,\ldots,\tilde{\omega}_n] \quad \forall\, n=1,\ldots,N.$$

Similar to Foster and Viswanathan (1996), and unlike Kyle (1985) and Holden and Subrahmanyam (1992), informed trader strategies will be a function of the price history. Informed traders look at the price history to update their beliefs about the number of informed traders in the market.

Proposition 1 *There exists an equilibrium for the described model characterized by an eleven-state Markov chain with a time-dependent transition probability matrix. The transition probabilities are functions of the insider trade policy and market maker price policy,* *both of which are state- and time-dependent and satisfy a set of difference equations.*

A proof and details of the equilibrium are in the appendix.

Intuition about the nature of the equilibrium can be gained by considering a one-period version of the model in which there exist either zero or one insider with equal probability. Assume that the order flow from noise traders is uniformly distributed between -1 and 1 (i.e., $W = 1$). The liquidation value equals 0 or 1 with equal probability. Let x^H represent the equilibrium order of an informed trader with a high signal and x^L the order flow of an insider with a low signal. Symmetry dictates that $x^H = -x^L \equiv x$. In this case, the market maker must decide which of three uniform distributions the order flow came from: pure noise trade, distributed $U(-1, 1)$; noise trade plus trade from an insider with positive information, distributed $U(-1 + x, 1 + x)$; or, noise trade plus trade from an insider with negative information, distributed $U(-1 - x, 1 - x)$. The unconditional probability that the order flow represents only noise trade is 1/2. The unconditional probability of each of the other two order flow distributions is 1/4. Figure 1 shows the probability density function of the order flow.[8]

The order flow pdf has five distinct regions, labeled R_1 through R_5. The market maker sets her price based on which region the order flow falls in. If the observed order flow falls in region 1, it must be that there exists an insider with negative information and the market maker will set price equal to 0. Similarly, if the observed order flow falls in region 5, it must be that there exists an insider with positive information and the market maker will set

[8]The order flow pdf presumes that $x < 1$, which is confirmed in equilibrium. It is straightforward to show that $x = 6/7$ as depicted in the figure. Recall that the existence of the equilibrium requires that $|x|$ is common knowledge.

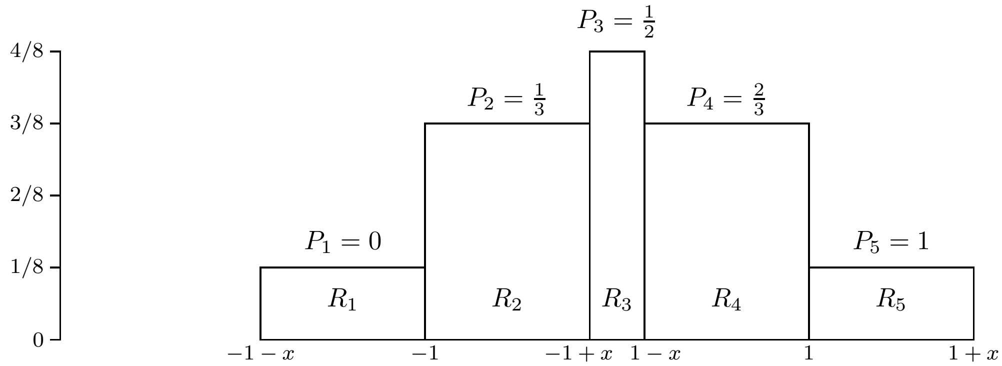

Figure 1: Order Flow Probability Density Function. The figure assumes there is one round of trade, noise trader order flow is distributed uniform (-1,1), there exist 0 or 1 insiders with equal probability, and the liquidation value of the risky asset is 0 or 1 with equal probability. x represents the equilibrium insider order flow conditional on the liquidation value being one. In equilibrium $x = 6/7$. The pdf is centered at 0. Equilibrium prices are listed above each region of the pdf.

price equal to 1. If the order flow falls in region 2, the market maker is sure that there does not exist an insider with positive information, but there may exist an insider with negative information or no insider at all. The market maker will set price (based on Baye's rule) to 1/3. If the observed order flow falls in region 3, close to zero, the market maker can draw no inference about which of the three possible order flow distributions the order flow came from and the market maker sets price equal to the unconditional expectation of 1/2. Lastly, if the observed order flow falls in region 4, the market maker is certain that there does not exist an insider with negative information and sets price equal to 2/3 after appropriately updating the likelihood that there exist no insiders versus one insider with positive information.

The equilibrium of the multiperiod model with zero, one, or two insiders has a similar form as above. In the first round there are seven, instead of five, order flow regions with unique prices. In total there are eleven possible states of the game, where each state is defined by the market maker and insider beliefs about the number of insiders present and the liquidation value of the asset. Each state has a unique step-function pricing rule similar to that illustrated in Figure 1. All order flows within a region generate the same updated beliefs because the uniform distribution of noise trade generates constant likelihood ratios within each region. The insider trading strategy depends on the game state and current round of trade. Because the pricing rule is a function of the equilibrium insider trading strategy, it also is a function of the current round of trade.

1.2 Definition of Market Liquidity

As shown, the model follows the basic approach of Kyle (1985) with specific modeling assumptions designed to make the model tractable when the number of informed traders is unknown. The departures from Kyle (1985) are an explicit restriction on manipulative trade and the specific distributional assumptions made: insider information has a binomial distribution ("bullish" or "bearish" signals) and noise trader order flow has a uniform distribution. One consequence of these assumptions is that the market maker's pricing rule is a step function. It is standard to measure inverse market liquidity as the sensitivity of the market maker's pricing rule with respect to changes in the order size. In this model defining a market liquidity parameter is more complicated because the pricing rule is not continuous in the order flow. The general approach is as follows (details are given in the appendix): I use as a measure of market liquidity the slope of a straight line connecting the two pricing rule segments bracketing the state price for each game state. Consider, for example, the pricing rule depicted in Figure 1. Before trade begins the price is equal to $P_3 = 1/2$, the unconditional expectation of the liquidation value. The price will remain equal to P_3 for order flows sufficiently close to zero. Orders flows greater than $1 - x$ will move the price to

$P_4 = 2/3$. Order flows less than $-1 + x$ will move the game to state 2 with price $P_2 = 1/3$. The market liquidity parameter in the one shot game is then defined as:

$$\lambda = \frac{P_4 - P_2}{(1-x) - (-1+x)} = \frac{1}{6(1-x)} = \frac{7}{6}.$$

Of course in the general model x is state- and time-dependent so the market liquidity parameter is state- and time-dependent as well.[9]

2 Numerical Examples

The analytic solution of the model is presented in the appendix. As is common in the literature, comparative statics of the model are illustrated via numerical example. This is much more efficient and enlightening than trying to understand the dynamics of the model via inspection of the equilibrium difference equations. I compute the equilibrium for a variety of parameter values, focusing on the comparative statics of the model with respect to θ, the probability there exists a single informed trader. Recall, that the expected number of informed traders is held constant at one. There is always an equal probability of there being zero or two informed traders. The baseline parameter values are: $H = 1$ and $L = 0$, $N = 50$, and $\tilde{u}_n$ distributed uniform $(-1/\sqrt{N}, 1/\sqrt{N})$. The only parameter that does something other than change the scale of the results (i.e., that is not completely without loss of generality) is N. Very small N (e.g., $N \leq 5$) mute some of richer dynamics of the model. Very large N compress the model dynamics into the beginning of the trading period,

[9]Some additional technical issues are discussed in the appendix.

making the model dynamics difficult to see in the figures even though all of the dynamics are present. There is nothing special about $N = 50$ other than that it is a nice intermediate value that suffers relatively little from either effect. The following subsections detail how changing θ impacts price efficiency, market liquidity, informed trade volume, the duration of an information event, and expected insider profits.

2.1 Price Efficiency

Price efficiency in each period in each state is measured in the standard way as the conditional variance of the liquidation value of the risky asset with respect to the market maker's information set: $\mathrm{V}[\tilde{v}|\tilde{\omega}_1, \ldots, \tilde{\omega}_n]$. The expected price efficiency for each period is taken as the expectation over the possible states for each period.

Figure 2, Panel A shows how the initial price uncertainty is expected to resolve for different values of θ. Remember that $\theta = 1$ corresponds to a known monopolist insider and $\theta = 0$ corresponds to an equal probability of there being no insiders or two insiders. The expected residual price uncertainty following the last round of trade is strictly decreasing in θ. This is a direct consequence of the increasing likelihood that there exist no informed traders. The more likely it is that there are no informed traders in the market, the more likely it is that none of the ex ante price uncertainty will be resolved and hence the higher is the expectation of the ex post residual uncertainty. It is more interesting to look at the residual price uncertainty conditional on the number of informed traders that actually exist in the market. Panel B of Figure 2 shows how price uncertainty evolves conditional on the realized number of informed traders being one. In this case, price uncertainty is expected to be driven to zero by the end of trade for any value of θ greater than zero.[10] However, uncertainty about the number of insiders delays the price discovery process. The higher the initial probability that there exist no insiders, the slower is the price discovery process. The market maker has stronger priors that the order flow is being driven purely by noise traders and therefore is slower to attribute order flow imbalances to informed trade. Of course, eventually the persistence of order flow imbalances will override the market maker's prior such that price moves to fundamental value, but this process takes significantly longer than when there exists a known number of insiders.

Figure 3, Panels A and B show how price uncertainty evolves conditional on the realized number of informed traders being zero and two, respectively. When there actually exist no informed traders (Panel A), the market maker discovers this over the course of the trading rounds and the final price uncertainty converges to the initial price uncertainty. During early trade rounds, however, particularly when there is a high initial probability that there exists a single informed trader, the market maker will mistake noise trade for informed trade and extract from the order flow information that isn't really there. Consequently, there can initially be a fairly large false sense of value discovery, from which the market maker later retreats. This results in non-monotonic price discovery patterns wherein the residual price uncertainty initially decreases and later increases. This (ex post) excess price volatility arises whenever the market maker perceives an ex ante probability of a private information event, but the event doesn't happen.

Based on Figure 2 and Panel A of Figure 3 we can conclude that, on average, uncertainty

[10] When $\theta = 0$ there can not exist a single informed trader.

about the number of informed traders will prolong the price discovery process. In Panel B of Figure 3, however, we see that this is not universally true. In some cases uncertainty about the number of informed traders can accelerate the price discovery process during early trading rounds. This occurs when it is very likely that there exists only one informed trader, but in fact there exist multiple informed traders. In this case both insiders trade almost as if they were known monopolists, which results in significantly more informed trade volume than if each informed trader knew of the others existence.[11] On average, however, insiders will determine the actual number of informed traders in the market more quickly than the market maker. Once this happens, the competing insiders will scale back their orders such that the price discovery process is once again delayed relative to what it would be in the case with a known number of insiders. In later rounds of trade the market is on average less informationally efficient than if there exists a known monopolist.

2.2 Informed Trade Volume

The above discussion indicates that changing θ has two effects on the equilibrium. First, it adds a new dimension of 'noise' to the market in the form of uncertainty about the number of informed traders. The amount of extra market 'noise' first increases and then decreases as θ changes from one to zero.[12] Second, changing θ changes the *potential* level of competition in the market. When $\theta = 1$ it is common knowledge that there exists a monopolist insider.

[11] In all cases, however, informed traders trade less than when it is common knowledge that two informed traders exist. That is, when $\theta = 0$ and informed traders know with certainty that they are duopolists, the market maker still doesn't know whether there exists zero or two informed traders. This causes the insiders to trade less aggressively than they would if their presence were known by the market maker as well.

[12] The maximum level of uncertainty about the number of informed traders in the market is at $\theta = 1/3$.

As θ decreases the probability that there exist two insiders increases and, hence, so does the potential for competition among insiders.

The net effect on insider behavior is not clear, a priori. Considered alone, increasing the potential for competition should influence insiders in two ways. First, the expected volume of informed trade should decrease. When the number of insiders is common knowledge, adding an additional insider decreases the trade volume of each insider while increasing the overall volume of informed trade. In this model, the expected number of insiders is always one, so increasing the potential for competition, all else equal, will decrease the expected volume of informed trade. The second effect of increased potential competition is an incentive for informed traders to trade earlier rather than later. If an insider knows he is a monopolist he will trade to exploit to the greatest degree his information advantage over the market maker. If an insider doesn't know if he is a monopolist, he must consider that the value of his informational advantage relies on his trading on the information no later than his competitor. On balance, then, an increased potential for competition should decrease the expected total amount of informed trade while at the same time increasing the relative proportion of informed trade occuring in the early trading rounds.

Next, consider θ's role in changing the amount of 'noise' in the market. This effect is not the same as increasing the level of liquidity trading. This is because increasing the level of liquidity trading has no direct effect on an informed trader's strategy. Increased liquidity trade hardens the market maker's inference problem, and therefore allows an informed trader to submit larger orders without revealing more information, but this simply 'scales' an insider's trading strategy without changing it's nature. Increasing uncertainty about the number insiders, in contrast, hardens both the market maker's and insider's inference problems. Therefore, at any point in time, increased 'noise' in the market should have a scaling effect as just described and, all else equal, more noise will allow an informed trader to trade more heavily for a given risk of information revelation. Acting alone, this effect would cause the amount of informed trade to be higher when the number of insiders is unknown. However, there is an offsetting effect coming from an informed trader's desire that the market maker believe there are no informed traders in the market, and that he (the informed trader) discover the presence of additional insiders before additional insiders find out about him (ignoring for the sake of discussion the competitive effects discussed above). This gives an informed trader an incentive to delay trade and 'sit' on his information during early rounds and trade more heavily during later rounds (relative to the case with a known number of insiders).

Figure 4 illustrates the net effect of these influences on informed trade. Panel A shows cumulative expected informed trade over the trading horizon. On balance we see that uncertainty about the number of informed traders delays informed trade in the early rounds at least to the monopolist level ($\theta = 1$). When uncertainty is very high (e.g., $\theta = 0.5$) trade in the early rounds is significantly delayed. The second striking result is that the overall amount of informed trade over the entire trading horizon differs greatly with θ. Panel B of Figure 4 shows the expected total informed trade volume for different values of θ. For $\theta < 1$ the expected total informed trade decreases as the potential for competition increases, in line with the reasoning discussed above. Also in line with the above discussion, the expected amount of informed trade when the number of insiders is unknown is always at least as great as when there is a known monopolist insider. In fact, there is a discontinuity at $\theta = 1$: adding even the smallest amount of uncertainty about the existence of a monopolist insider almost doubles the expected amount of informed trade! It turns out in equilibrium that it is very hard for the market maker to eliminate a slight doubt about the existence of a monopolist insider, but that the market maker doesn't care much if this is the case. The price becomes very efficient on average, in fact arbitrarily close to true value, and the market maker is arbitrarily close to certain that there exists a monopolist insider. The insider continues to trade, but these trades earn next to no profits because the price is so close to true value. The next subsection shows that this trading pattern induces a different time pattern of market liquidity when the number of insiders is unknown compared with when the number of insiders is known.

2.3 Market Liquidity

Figure 5 shows various features of the equilibrium market liquidity for different values of θ. Panel A plots the market liquidity parameter λ over time. Lower values of λ correspond with a deeper, more liquid market. Uncertainty about the number of informed traders in the market significantly changes the time pattern of market liquidity. Initially, markets are deeper because uncertainty about the number of informed traders adds a dimension of noise to the market. This added noise, however, delays the revelation through the market of value-relevant information making the market less deep during later trade rounds. Panel B of Figure 5 plots the log of the expected market λ. In Panel B it is clear that when θ equals 0 or 1 market liquidity increases at an exponential rate while when $0 < \theta < 1$ the rate of increase is less than exponential. As discussed above these discontinuities are related to the speed with which the number of insiders is revealed through the market. When $0 < \theta < 1$ the market maker takes much longer on average to disentangle what the order flow implies about terminal value of the risky asset versus what the order flow implies about the number of informed traders in the market. As a result, when $0 < \theta < 1$ the realized number of insiders is discovered much more slowly.

Panel C of Figure 5 shows how market liquidity during the first round of trade varies with uncertainty about the number of informed traders in the market. Market liquidity is lowest when it is equally likely that there exists two informed traders or zero informed traders. Initial market liquidity is highest when there is much uncertainty about the number of informed traders, not when there exists a single informed trader as may be expected. That is, the market is most liquid for intermediate values of θ. Recall that the expected number of insiders in the market is always one. All else equal, the market maker, in a sense, always expects the same amount of information to be present in the market. However, changing θ changes the amount of additional 'noise' in the market and changes the potential for competition in the market. First round market liquidity increases with the amount of 'noise' in the market and decreases with the potential for competition in the market. The additional 'noise' in the market is highest when $\theta = 1/3$ and the potential for competition increases linearly as θ decreases from one to zero. The interaction of these two effects produces the first round liquidity pattern shown in Figure 5, Panel C: liquidity is highest when the additional amount of noise is large (i.e., $\theta \approx 1/2$), but not at its maximum (i.e., $\theta = 1/3$), because at this point the increase in liquidity from adding more noise to the market is offset by the decrease in liquidity from raising the potential level of competition in the market.

2.4 Information Event Duration

Figure 6 shows the expected duration of an information event for different values of θ. The beginning of an information occurs when an insider may first be present in the market. This is time zero in the model, by definition. I define the end of an information as the time when price either equals true value, or when the market maker has determined with certainty that no insiders are present in the market and price returns to the ex ante expected value. Panel A shows for each trading round the probability that the information event has passed. Panel B shows how many trading rounds the information event is expected to endure. The event is expected to resolve itself most quickly when there is a known monopolist insider and only slightly more slowly when there is an equal probability of there being zero or two insiders. In either case, the market maker is expected to have sorted out within two or three trading rounds whether or not an insider is present and, if so, what the true value of the asset is.

The situation is very different when $0 < \theta < 1$. In this case the market maker requires ten to twelve trading rounds, on average, to sort out whether or not an insider is present. The implication is that insiders are very good at exploiting their informational advantage with respect to the number insiders present in the market. Specifically, because each insider is certain of his own presence in the market, he is able to determine relatively quickly whether he is alone or in competition with another insider. Once an insider knows that he is a monopolist, he does a very good job of concealing this fact from the market maker. When $\theta = 0$ and two insiders exist, each knows with certainty that the other exists. This creates a situation analogous to the prisoners' dilemma. Both insiders would be better off if they could commit to postponing their trade so that the market maker will believe that no insiders are present. However, if one insider is postponing his trade, the other insider has an incentive to trade in front of him. As a result, they both trade sooner rather than later and their presence is detected fairly early by the market maker.

2.5 Expected Insider Profits

Figure 7 summarizes the effect varying θ has on expected insider profits. Panel A shows how expected future profits decay through time and Panel B shows the start-of-trade expected total insider profits. Because the expected number of insiders is always one, these graphs also represent the expected profits per insider. The most striking result is that expected profits exceed monopoly profits over most of the parameter range. This implies that potentially informed traders would rather enter into an "information lottery" in which there were positive probabilities that the information would be revealed to zero, one, or two traders versus a lottery in which there was guaranteed to be a single winner with monopoly rights to the information. The benefit of uncertainty about the number of informed traders is that the market is more liquid, allowing one to trade more aggressively. The cost is that one may face competition. Expected insider profits are maximized when the marginal cost of increased potential competition offsets the marginal benefit of more uncertainty about the number of informed traders. The result is that start-of-trade expected insider profits are roughly proportional to first round market liquidity (Panel B) and maximized at $\theta \approx 1/2$.

The evolution of expected profits, however, does not mirror the evolution of expected market liquidity (Panel A). High expected total profits are expected to be earned more gradually over time than lower expected total profits, which are earned disproportionately during the early trade rounds. This is the case even though the market is less liquid in later trade rounds when expected total profits are high (e.g., $\theta \approx 1/2$). When there is a known monopolist informed trader ($\theta = 1$), or competing informed traders (if any) ($\theta = 0$), the market maker is expected to determine the liquidation value of the asset quickly (recall Figure 6). After the liquidation value is known, the market becomes infinitely liquid. When $0 < \theta < 1$, however, the market maker is expected to take much longer to determine precisely the liquidation value. This suppresses market liquidity in the later rounds while at the same time allowing insiders to continue to earn trading profits. In short, the most profitable situation for an insider is one in which the insider has determined his status as a monopolist, but the market maker has not. The likelihood of this situation arising is highest for intermediate values of θ.

3 Comparison with Experimental Evidence

Schnitzlein (2002) presents evidence from experimental asset markets with an unknown number of insiders. Coincidentally, and fortunately, his experimental design is similar to my model in important respects. In particular, he studies a market in which each of two potential insiders are present with (independent) probabilities of one half. This corresponds to my model with $\theta = 1/2$. Schnitzlein provides intuitive rationale and experimental evidence for seven experimental hypotheses. All but one of his hypotheses map into my theoretical framework.[13] My model provides theoretical support for the broad conclusions made by Schnitzlein. Namely, uncertainty about the number of insiders in the market induces insiders to delay their trades and makes it difficult for the market maker to figure out how many insiders are present. Nevertheless, my theoretical framework does not conform with all of the experimental evidence presented by Schnitzlein. I present additional theoretical insights about six of the hypotheses tested by Schnitzlein. The hypotheses consider the differences between three types of markets:

1. A market with a *known monopolist* insider. This corresponds to the present model with $\theta = 1$.

2. A market with *known duopolist* insiders. This does not map directly into the model as developed thus far. Recall that in the present model with $\theta = 0$ there is an ex ante fifty-fifty chance that there will be no insiders versus two insiders. Therefore, when insiders are present (ex post), the insiders know with certainty before trade begins that they are duopolists, but the market maker remains unaware of their presence. For this section of the paper I separately derive results for a market in which the presence of duopolist insiders is common knowledge.

3. A market with an unknown number of insiders. This corresponds to the present model with $\theta = 1/2$. There is a 25% chance that there are no insiders, a 50% chance that there is a single insider, and a 25% chance that there are two insiders. This is the general case in which both insiders and the market maker must infer how many insiders are

[13] In the experimental framework the time between trades is not fixed. This allows Schnitzlein to test the relation between market liquidity and inter-trade intervals. This type of comparative static doesn't exist in my model with fixed trade intervals.

present. Insiders in this market are referred to as *inferred monopolists* and *inferred duopolists* as appropriate.

Hypothesis 1 *Informational efficiency will be lower when the number and presence of insiders is unknown.*

Hypothesis 1 is supported by theory. (See Figures 2 and 3, previously discussed.) However, there is a caveat. Recall that Figure 3B shows that prices may be more informationally efficient over some intervals of the trading horizon when the number and presence of insiders in unknown.

Hypothesis 2 *Two insiders will compete less aggressively when the number of insiders is unknown.*

Hypothesis 2 is theoretically supported. Figure 8A shows per-insider trade volume when it is common knowledge that two insiders exist versus when the presence of two insiders must be inferred. Known duopolists trade more heavily overall and compete away their information earlier than inferred duopolists. Even after inferred duopolists learn of one another's presence, they compete less aggressively than known duopolists so that they can better exploit their informational advantage relative to the market maker.

Hypothesis 3 *Ex post monopolist insiders will behave similarly whether or not their presence is common knowledge.*

Hypothesis 3 is not theoretically supported. The hypothesis is based on the reasoning that, relative to a known monopolist, inferred monopolists have both an incentive to accelerate their trade as well as an incentive to delay their trade. The incentive to accelerate trade comes from a desire to front run the possibly present competing insider. The incentive to delay trade comes from the desire to hide one's presence from the market maker. Figure 8B compares expected insider trade volume for a known monopolist versus an inferred monopolist and shows that the incentive to delay trade dominates. Delaying trade during the early rounds allows the uncertain monopolist to determine his monopolist status without revealing himself to the market maker. Then, during the later trading rounds, the inferred monopolist continues to trade far longer than the known monopolist by optimally concealing his presence from the market maker. The net effect is that over the entire trade horizon the inferred monopolist is able to trade significantly more than the known monopolist and earn larger profits.

In the experimental asset markets studied by Schnitzlein (2002), the trade behavior and profits of known monopolists and inferred monopolists were similar. One possible explanation is that the incentive to accelerate trade may be stronger in the experimental market than in the theoretical model. The theoretical model assumes perfect rationality of all market participants. In this setting insiders optimally bide their time and scale back their trade volume in the early rounds until they determine if they are a monopolist or a duopolist. If an insider is concerned, as one may be in an experimental market, that a competing insider might irrationally trade too aggressively in the early rounds, then one also has an incentive to trade more aggressively in the early rounds. This could account for the observed behavior of inferred monopolists in the experimental evidence.

Hypothesis 4 *Insider behavior will be more stable (relative to the realized number of insid-*

ers) when the number of insiders is unknown.

Hypothesis 4 is partially supported by theory. Figure 9 shows expected cumulative informed trade (per insider) for a known monopolist versus a known duopolist (Panel A) and an inferred monopolist versus an inferred duopolist (Panel B). The hypothesis states that the trade behaviors in Panel B should be more similar than the trade behaviors in Panel A. Whether the hypothesis is supported depends on which aspect of trade behavior one considers. In Panel A we see that the trade behavior of a known duopolist differs significantly from that of a known monopolist during the early trade rounds. The known duopolist trades more aggressively. However, over the entire trade horizon, a known monopolist and known duopolist are expected to trade equivalent volumes.

The opposite holds for an inferred monopolist versus an inferred duopolist. Panel B shows that during the early trade rounds they trade the same. Of course they must because during the early trade rounds an insider doesn't know whether he is in fact a monopolist or a duopolist. During the later rounds the insider's trade behavior will differ significantly based on whether he determines himself to be monopolist or a duopolist. If he infers that he is a monopolist he will optimally conceal his presence and trade over the entire trade horizon. If he infers that he is duopolist he will scale back his trading to the optimal competitive level, but nevertheless the insiders are likely to reveal their information to the market maker sooner rather than later which eliminates the expectation of any trading in the later rounds.

Hypothesis 5 *The initial responsive of price to order flow (i.e., λ) will be: (1) higher when there are known to be two insiders versus when it is not known that there exist two insiders,* *(2) lower when there are known to be no insiders versus when it is not known that no insiders exist, and (3) relatively stable when there exists a monopolist insider, independent of whether the existence of the monopolist insider is common knowledge.*

Hypothesis 5 is partially supported by theory. Figure 10 shows the evolution of λ over time for known versus inferred duopolists (Panel A), known versus inferred lack of insiders (Panel B), and known versus inferred monopolists (Panel C). This hypothesis concerns the first round market λ which is the left-most point of each graph. Panels A and B confirm statements (1) and (2) of the hypothesis. Price is initially more responsive in a market with known duopolists and less responsive in a market with a known lack of insiders. Panel C of Figure 10 shows that statement (3) of the hypothesis is not supported by theory. In fact, the market maker will optimally set a pricing schedule that is significantly more responsive to order flow when there is a known monopolist versus when the presence of a monopolist must be inferred. This is for two reasons. First, if there are no insiders the price should not be responsive to order flow at all. It follows that the possibility of there being no insiders causes the market maker to lessen the responsiveness of the price schedule. Second, if there are two insiders there will be twice the level of informed order flow relative to uninformed order flow (all else equal) and once again the market maker is optimally less sensitive to changes in order flow. Therefore the possibility that there might exist two insiders also causes the market maker to lessen the responsiveness of the price schedule.

Hypothesis 6 *Over the trading interval, the responsiveness of price to order flow will: (1) decline more rapidly when there are known to be two insiders versus when it is not known that there exist two insiders, (2) be lower when there are known to be no insiders versus* *when it is not known that no insiders exist, and (3) evolve through time similarly whether or not the existence of a monopolist insider is common knowledge.*

Similar to Hypothesis 5, Hypothesis 6 is partially supported by theory. Figure 10 Panels A and B confirm the first two statements of the hypothesis. Panel C of Figure 10 shows that when there is a monopolist insider, price responsiveness to order flow does differ when the monopolist is known to exist versus when his presence must be inferred. The reasoning follows from the discussion of the previous hypothesis. It happens that when the existence of a monopolist insider is not common knowledge, the insider infers his status as a monopolist before the market maker does, on average. In this case the insider effectively conceals his presence and trades consistently over the trade horizon. Because the presence (and information) of the insider is not revealed, the market maker maintains a degree of price sensitivity to order flow throughout the trading rounds. In contrast, when an insider is known to exist, the market maker infers the insider's information within a few trading rounds (on average) and thereafter sets a price schedule that is completely insensitive to order flow.

4 Conclusion

We are aware (Holden and Subrahmanyam (1992), Foster and Viswanathan (1996)) that competition causes informed traders with similar or identical information to mitigate their strategic behavior. In previous models the characteristics and distribution of information among agents was known. I have studied the impact of uncertainty in the distribution of private information. I focused on the case when the expected number of insiders is always one, but there may be as few as zero or as many as two insiders present. I find that uncertainty about the number of insiders in the market plays a dual role: it changes the dynamics of competition between insiders and acts as an additional source of noise. Specifically, I find that uncertainty with respect to the number of informed traders in the market tends to:

- create excess price volatility when no insiders are present;
- prolong the price discovery process;
- increase the expected volume of informed trade;
- increase market liquidity following an information event;
- delay the trade of monopolist insiders; and
- increase expected insider profits.

These findings confirm many of the results established in experimental asset markets. They illustrate that some aspects of the market environment that have been ignored for analytical tractability are not innocuous. The model presented here also has important limitations. It does not incorporate many significant features incorporated in models with a known number of insiders. Chief among these are perhaps the analysis of risk-averse agents and consideration of heterogeneous information among insiders. These topics are left for further research.

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A Equilibrium Derivation

A.1 The General Case: $0 < \theta < 1$

The equilibrium is derived in two steps. The first step is to layout all the possible states of the game. This entails detailing the market maker and insider beliefs for every possible scenario. The market maker beliefs will define the security price in each state as well. Once the game states are well-defined, I solve for the optimal insider strategy in each state by optimizing over expected future profits. This is a tedious exercise. Unfortunately, I am not aware of a more elegant approach.

A.1.1 Game State Definitions

The game comprises eleven possible states. Let the game start in state 3 (so named because it is the third and central order flow region at the start of the game – see Figure A1). Let $\Pr_S^{MM}(M,\Phi)$ represent the market maker's belief (i.e. probability) in state S that there exist M insiders with signal Φ. Let $\Pr_S^I(M|\Phi)$ represent an insider's belief (i.e. probability) in state S that there exist M insiders given signal Φ. Last, let $x_{n,S}^{\Phi}$ be the insiders order in round n in state S given signal Φ. After insiders receive their signals, but before trade begins, the following beliefs are held:

State 3 Beliefs		
$\Pr_3^{MM}(0,L) = \frac{1-\theta}{4}$	$\Pr_3^{MM}(0,\cdot) = \frac{1-\theta}{2}$	$\Pr_3^{I}(0\|L) = 0$
$\Pr_3^{MM}(0,H) = \frac{1-\theta}{4}$	$\Pr_3^{MM}(1,\cdot) = \theta$	$\Pr_3^{I}(0\|H) = 0$
$\Pr_3^{MM}(1,L) = \frac{\theta}{2}$	$\Pr_3^{MM}(2,\cdot) = \frac{1-\theta}{2}$	$\Pr_3^{I}(1\|L) = \frac{2\theta}{1+\theta}$
$\Pr_3^{MM}(1,H) = \frac{\theta}{2}$	$\Pr_3^{MM}(\cdot,L) = \frac{1}{2}$	$\Pr_3^{I}(1\|H) = \frac{2\theta}{1+\theta}$
$\Pr_3^{MM}(2,L) = \frac{1-\theta}{4}$	$\Pr_3^{MM}(\cdot,H) = \frac{1}{2}$	$\Pr_3^{I}(2\|L) = \frac{1-\theta}{1+\theta}$
$\Pr_3^{MM}(2,H) = \frac{1-\theta}{4}$		$\Pr_3^{I}(2\|H) = \frac{1-\theta}{1+\theta}$

These beliefs completely define state 3. Now consider a round of trade when the game is in state 3. Figure A1 illustrates the distribution of possible order flows for each combination of number of insiders (0, 1, or 2) and signals (H or L). For example, if there is one insider who has received an H signal, the total order flow is uniformly distributed over the interval $[-W + x_{n,3}^H, W + x_{n,3}^H]$. Unconditionally, the total order flow must fall within the interval $[-W - 2x_{n,3}^H, W + 2x_{n,3}^H]$.[14] There are seven distinct ways in which beliefs can evolve and each corresponds with a different region in the interval $[-W - 2x_{n,3}^H, W + 2x_{n,3}^H]$. I will derive the updated market maker and insider beliefs for the range of positive order flows ($\omega > 0$). Beliefs for negative order flows ($\omega < 0$) are symmetric.

First, suppose the order flow is positive, but less than $W - 2x_{n,3}^H$: In this region beliefs remain as they are because the probability that the order flow came from any of the possible distributions is equally likely. At the other extreme, if $\omega > W$ the market maker perfectly infers that the insider information is H. I define this as state 6. State 6 is completely characterized by the following beliefs:

[14] Here I have applied the equilibrium condition that in state 3 the insider orders will be symmetric in the signal. That is the insider order given a high signal will be equal in magnitude, but opposite in sign to the insider order given a low signal: $x_{n,3}^H = -x_{n,3}^L > 0$. The value of $x_{n,3}^H$ will be derived following the state definitions.

State 6 Beliefs		
$\Pr_6^{MM}(0,L) = 0$	$\Pr_6^{MM}(0,\cdot) = 0$	$\Pr_6^{I}(0\mid L) = 0$
$\Pr_6^{MM}(0,H) = 0$	$\Pr_6^{MM}(1,\cdot) = \text{N/A}$	$\Pr_6^{I}(0\mid H) = 0$
$\Pr_6^{MM}(1,L) = 0$	$\Pr_6^{MM}(2,\cdot) = \text{N/A}$	$\Pr_6^{I}(1\mid L) = \text{N/A}$
$\Pr_6^{MM}(1,H) = \text{N/A}$	$\Pr_6^{MM}(\cdot,L) = 0$	$\Pr_6^{I}(1\mid H) = \text{N/A}$
$\Pr_6^{MM}(2,L) = 0$	$\Pr_6^{MM}(\cdot,H) = 1$	$\Pr_6^{I}(2\mid L) = \text{N/A}$
$\Pr_6^{MM}(2,H) = \text{N/A}$		$\Pr_6^{I}(2\mid H) = \text{N/A}$

N/A means that the beliefs are off-equilibrium and/or don't effect the equilibrium (i.e., any probability between 0 and 1 inclusive is acceptable). For $\omega \in [W - 2x_{n,3}^H, W - x_{n,3}^H]$, it is clear that there do not exist two insiders with low signals, but any other combination is equally likely. I define this as state 4, which is characterized by the following beliefs:

State 4 Beliefs		
$\Pr_4^{MM}(0,L) = \frac{1-\theta}{3+\theta}$	$\Pr_4^{MM}(0,\cdot) = \frac{2(1-\theta)}{3+\theta}$	$\Pr_4^{I}(0\mid L) = 0$
$\Pr_4^{MM}(0,H) = \frac{1-\theta}{3+\theta}$	$\Pr_4^{MM}(1,\cdot) = \frac{4\theta}{3+\theta}$	$\Pr_4^{I}(0\mid H) = 0$
$\Pr_4^{MM}(1,L) = \frac{2\theta}{3+\theta}$	$\Pr_4^{MM}(2,\cdot) = \frac{1-\theta}{3+\theta}$	$\Pr_4^{I}(1\mid L) = 1$
$\Pr_4^{MM}(1,H) = \frac{2\theta}{3+\theta}$	$\Pr_4^{MM}(\cdot,L) = \frac{1+\theta}{3+\theta}$	$\Pr_4^{I}(1\mid H) = \frac{2\theta}{1+\theta}$
$\Pr_4^{MM}(2,L) = 0$	$\Pr_4^{MM}(\cdot,H) = \frac{2}{3+\theta}$	$\Pr_4^{I}(2\mid L) = 0$
$\Pr_4^{MM}(2,H) = \frac{1-\theta}{3+\theta}$		$\Pr_4^{I}(2\mid H) = \frac{1-\theta}{1+\theta}$

Lastly, if $\omega \in [W - x_{n,3}^H, W]$ it is clear that there do not exist any insiders with low signals, but there may exist no insiders or insiders with high signals. This is defined as state 5 and it is characterized by the following beliefs:

State 5 Beliefs		
$\Pr_5^{MM}(0,L) = \frac{1-\theta}{3-\theta}$	$\Pr_5^{MM}(0,\cdot) = \frac{2(1-\theta)}{3-\theta}$	$\Pr_5^{I}(0\mid L) = \text{N/A}$
$\Pr_5^{MM}(0,H) = \frac{1-\theta}{3-\theta}$	$\Pr_5^{MM}(1,\cdot) = \frac{2\theta}{3-\theta}$	$\Pr_5^{I}(0\mid H) = 0$
$\Pr_5^{MM}(1,L) = 0$	$\Pr_5^{MM}(2,\cdot) = \frac{1-\theta}{3-\theta}$	$\Pr_5^{I}(1\mid L) = \text{N/A}$
$\Pr_5^{MM}(1,H) = \frac{2\theta}{3-\theta}$	$\Pr_5^{MM}(\cdot,L) = \frac{1-\theta}{3-\theta}$	$\Pr_5^{I}(1\mid H) = \frac{2\theta}{1+\theta}$
$\Pr_5^{MM}(2,L) = 0$	$\Pr_5^{MM}(\cdot,H) = \frac{2}{3-\theta}$	$\Pr_5^{I}(2\mid L) = \text{N/A}$
$\Pr_5^{MM}(2,H) = \frac{1-\theta}{3-\theta}$		$\Pr_5^{I}(2\mid H) = \frac{1-\theta}{1+\theta}$

Now consider a (following) round of trade beginning in any of states so far defined. If the game is in state 0 or state 6 it will stay in state 0 or 6, respectively, because the inside information has been completely revealed. States 0 and 6 are terminal states. On the other hand, if the game has remained in state 3, the analysis is as above. It remains to derive how the game evolves from states 1, 2, 4, and 5. I will treat states 4 and 5 explicitly. The game evolves from states 1 and 2 symmetrically to states 5 and 4.

In state 4 there are four possible order distributions and six distinct ways in which beliefs can evolve. Each path of evolution corresponds with a different region in the interval $[-W - x_{n,2}^H, W + 2x_{n,4}^H]$. Here I am maintaining the convention of stating all results in terms of orders placed by an insider with a high signal because these orders are always positive. In state 4 the order placed by an insider with a high signal will not be equal in magnitude and opposite in sign to that placed by an insider with a low signal. However, the symmetry of the game implies that the order placed by an insider with a high signal in state 2 will be equal in magnitude and opposite in sign to that placed by an insider with a high signal in state 4. This is why $-W - x^H_{n,2}$ (and not $-W - x^H_{n,4}$) is the lower support of the above distribution. The order flow distributions and regions are shown in Figure A1. I will derive the updated market maker and insider beliefs for each region. Beginning with an order flow falling in $[-W + 2x^H_{n,4}, W - x^H_{n,2}]$: In this region beliefs remain as they are (i.e., the game remains in state 4) because the probability that the order flow came from any of the possible distributions is equally likely. At the other extreme, if $\omega < -W$ or $\omega > W$, the market maker perfectly infers the inside information and the game goes to States 0 or 6 (previously defined), respectively. When $\omega \in [-W, -W + x^H_{n,4}]$, the market maker knows that insiders could not have received a high signal, but there may be a single insider with a low signal or no insiders. I define this as state 41 and it is characterized by the following beliefs:

State 41 Beliefs		
$\Pr^{MM}_{41}(0,L) = \frac{1-\theta}{2}$	$\Pr^{MM}_{41}(0,\cdot) = 1-\theta$	$\Pr^{I}_{41}(0\|L) = 0$
$\Pr^{MM}_{41}(0,H) = \frac{1-\theta}{2}$	$\Pr^{MM}_{41}(1,\cdot) = \theta$	$\Pr^{I}_{41}(0|H) = $ N/A
$\Pr^{MM}_{41}(1,L) = \theta$	$\Pr^{MM}_{41}(2,\cdot) = 0$	$\Pr^{I}_{41}(1\|L) = 1$
$\Pr^{MM}_{41}(1,H) = 0$	$\Pr^{MM}_{41}(\cdot,L) = \frac{1+\theta}{2}$	$\Pr^{I}_{41}(1|H) = $ N/A
$\Pr^{MM}_{41}(2,L) = 0$	$\Pr^{MM}_{41}(\cdot,H) = \frac{1-\theta}{2}$	$\Pr^{I}_{41}(2\|L) = 0$
$\Pr^{MM}_{41}(2,H) = 0$		$\Pr^{I}_{41}(2|H) = $ N/A

If $\omega \in [-W + x^H_{n,4}, -W + 2x^H_{n,4}]$, the market maker knows that there are not two insiders. This is defined as state 42 and it is characterized by the following beliefs:

State 42 Beliefs		
$\Pr^{MM}_{42}(0,L) = \frac{1-\theta}{2(1+\theta)}$	$\Pr^{MM}_{42}(0,\cdot) = \frac{1-\theta}{1+\theta}$	$\Pr^{I}_{42}(0\|L) = 0$
$\Pr^{MM}_{42}(0,H) = \frac{1-\theta}{2(1+\theta)}$	$\Pr^{MM}_{42}(1,\cdot) = \frac{2\theta}{1+\theta}$	$\Pr^{I}_{42}(0\|H) = 0$
$\Pr^{MM}_{42}(1,L) = \frac{\theta}{1+\theta}$	$\Pr^{MM}_{42}(2,\cdot) = 0$	$\Pr^{I}_{42}(1\|L) = 1$
$\Pr^{MM}_{42}(1,H) = \frac{\theta}{1+\theta}$	$\Pr^{MM}_{42}(\cdot,L) = \frac{1}{2}$	$\Pr^{I}_{42}(1\|H) = 1$
$\Pr^{MM}_{42}(2,L) = 0$	$\Pr^{MM}_{42}(\cdot,H) = \frac{1}{2}$	$\Pr^{I}_{42}(2\|L) = 0$
$\Pr^{MM}_{42}(2,H) = 0$		$\Pr^{I}_{42}(2\|H) = 0$

Lastly, if $\omega \in [W - x^H_{n,2}, W]$ the market maker knows that insiders could not have received a low signal, but there may exist no insiders or insiders with high signals. In this case beliefs are identical to state 5, defined above.

Beginning from state 5 there are three possible order distributions and four distinct ways in which beliefs can evolve. Each path of evolution corresponds with a different region in the interval $[-W, W + 2x^H_{n,5}]$. The order flow distributions and regions are shown in Figure A1. I will describe the updated market maker and insider beliefs for each region. Beginning with an order flow falling in $[-W + 2x^H_{n,5}, W]$: In this region beliefs remain as they are (i.e., the game remains in State 5) because the probability that the order flow came from any of the three possible distributions is equally likely. At the other extreme, if $\omega > W$, the market maker perfectly infers that the inside information is high and the game goes to state 6. When $\omega \in [-W, -W + x^H_{n,5}]$, the market maker knows that there are no insiders. I define this as state 50 and it is characterized by the following beliefs:

State 50 Beliefs		
$\Pr_{50}^{MM}(0,L) = \frac{1}{2}$	$\Pr_{50}^{MM}(0,\cdot) = 1$	$\Pr_{50}^{I}(0\|L) = \text{N/A}$
$\Pr_{50}^{MM}(0,H) = \frac{1}{2}$	$\Pr_{50}^{MM}(1,\cdot) = 0$	$\Pr_{50}^{I}(0\|H) = \text{N/A}$
$\Pr_{50}^{MM}(1,L) = 0$	$\Pr_{50}^{MM}(2,\cdot) = 0$	$\Pr_{50}^{I}(1\|L) = \text{N/A}$
$\Pr_{50}^{MM}(1,H) = 0$	$\Pr_{50}^{MM}(\cdot,L) = \frac{1}{2}$	$\Pr_{50}^{I}(1\|H) = \text{N/A}$
$\Pr_{50}^{MM}(2,L) = 0$	$\Pr_{50}^{MM}(\cdot,H) = \frac{1}{2}$	$\Pr_{50}^{I}(2\|L) = \text{N/A}$
$\Pr_{50}^{MM}(2,H) = 0$		$\Pr_{50}^{I}(2\|H) = \text{N/A}$

State 50 is a terminal state because the game will always remain in state 50 once it has been reached. Lastly, if in state 5 the market maker observes $\omega \in [-W + x_{n,5}^H, -W + 2x_{n,5}^H]$, the market maker knows there is at most one insider and that the insider has a high signal if he exists. This is defined as state 51 and it is characterized by the following beliefs:

State 51 Beliefs		
$\Pr_{51}^{MM}(0,L) = \frac{1-\theta}{2}$	$\Pr_{51}^{MM}(0,\cdot) = 1-\theta$	$\Pr_{51}^{I}(0\|L) = \text{N/A}$
$\Pr_{51}^{MM}(0,H) = \frac{1-\theta}{2}$	$\Pr_{51}^{MM}(1,\cdot) = \theta$	$\Pr_{51}^{I}(0\|H) = 0$
$\Pr_{51}^{MM}(1,L) = 0$	$\Pr_{51}^{MM}(2,\cdot) = 0$	$\Pr_{51}^{I}(1\|L) = \text{N/A}$
$\Pr_{51}^{MM}(1,H) = \theta$	$\Pr_{51}^{MM}(\cdot,L) = \frac{1-\theta}{2}$	$\Pr_{51}^{I}(1\|H) = 1$
$\Pr_{51}^{MM}(2,L) = 0$	$\Pr_{51}^{MM}(\cdot,H) = \frac{1+\theta}{2}$	$\Pr_{51}^{I}(2\|L) = \text{N/A}$
$\Pr_{51}^{MM}(2,H) = 0$		$\Pr_{51}^{I}(2\|H) = 0$

This completes the description of all the possible game states. Figure A1 summarizes the possible order flow distributions and state transition regions for each state. Tables A1 to A5 present unconditional and conditional transition probability matrices for the game.

A.1.2 Optimal Insider Trade Strategy

Given the state definitions, the insider trade strategy is solved by backward recursion, beginning with the terminal states: state 0, state 6, and state 50. First, define the following variables: let P_S be the market price in state S; let $\pi_{n,S}^{\Phi}$ be the expected insider profits from trade in rounds n through N conditional on having signal Φ and being in state S at the beginning of round n; and, let $\Pr_n^{\Phi}(S|S')$ be an insider's assessment of the probability of being in state S at the start round $n+1$ given that the game is in state S' prior to round n and the insider signal is Φ. Then, for the terminal states:

$$\pi_{n,0}^{L} = 0 \;\Rightarrow\; x_{n,0}^{L} = 0 \;\; \forall\, n$$
$$\pi_{n,6}^{H} = 0 \;\Rightarrow\; x_{n,6}^{H} = 0 \;\; \forall\, n$$
$$\pi_{n,50}^{\Phi} = \text{N/A} \;\Rightarrow\; x_{n,50}^{\Phi} = \text{N/A} \;\; \forall\, n$$

That is, insiders can not earn profits in any of the terminal states and they will not trade.[15]

[15] If the game arrives in state 50 no insiders exist. Insider orders are not well-defined in this case. Under the derived equilibrium, the game will never reach state 0 if $\Phi = H$ or state 6 if $\Phi = L$. Insiders in states 0 with a low signal and state 6 with a high signal can trade any amount and earn zero profits, so technically there are multiple equilibria. I set insider trades to zero because this would be the unique equilibrium with, for example, the addition of ϵ transaction costs.

For the remaining states, I first solve for the optimal insider strategy assuming the insider has a high signal and then proceed by symmetry. The game will never reach state 41 if there exists an insider with a high signal. In state 51 an insider will only earn positive future profits if the game remains in state 51. Therefore, the insider's maximization is:

$$\max_{x^H_{n,51}} \pi^H_{n,51} = \max_{x^H_{n,51}} \Pr{}^H_n(51|51)[(H - P_{51})x^H_{n,51} + \pi^H_{n+1,51}]. \tag{A1}$$

P_{51} is determined from the market maker's beliefs to be $[(1+\theta)H + (1-\theta)L]/2$. $\Pr^H_n(51|51)$ is determined from the insider beliefs, order flow distributions, and state transition regions to be $(1 - x^H_{n,51}/2W)$. After substituting, the solution to the insider's optimization is:

$$x^H_{n,51} = W - \frac{\pi^H_{n+1,51}}{(1-\theta)(H-L)}. \tag{A2}$$

Starting with the boundary condition $\pi^H_{N+1,51} = 0$, one uses equations (A1) and (A2) to solve for $\pi^H_{n,51}$ and $x^H_{n,51}$ for all $n = 1 \ldots N$.

Given expected insider profits and optimal trade strategy for all periods in states 6, 50, and 51, one can solve the insider's problem for state 5. In state 5, the insider will earn future profits if the game remains in state 5 or moves to state 51. The insider's maximization problem is:

$$\max_{x^H_{n,5}} \pi^H_{n,5} = \max_{x^H_{n,5}} \left\{ \Pr{}^H_n(51|5)[(H - P_{51})x^H_{n,5} + \pi^H_{n+1,51}] + \Pr{}^H_n(5|5)[(H - P_5)x^H_{n,5} + \pi^H_{n+1,5}] \right\}. \tag{A3}$$

P_5 is determined from the market maker's beliefs to be $[2H + (1-\theta)L]/(3-\theta)$. $\Pr^H_n(51|5)$ and $\Pr^H_n(5|5)$ are determined from the insider beliefs, order flow distributions, and state transition regions to be:

$$\Pr{}^H_n(51|5) = \frac{\theta x^{H\,\prime}_{n,5}}{(1+\theta)W},$$
$$\Pr{}^H_n(5|5) = \frac{2W - x^H_{n,5} - x^{H\,\prime}_{n,5}}{2W}$$

where $x^{H\,\prime}_{n,5}$ is an insider's inference about the order of the other insider, should another insider exist. By symmetry, $x^{H\,\prime}_{n,5}$ will equal $x^H_{n,5}$ in equilibrium, but the insider treats $x^{H\,\prime}_{n,5}$ as fixed in his optimization. After substituting, the solution to the insider's optimization is:

$$x^H_{n,5} = \frac{2(1+\theta)W}{3+\theta^2} - \frac{(1+\theta)(3-\theta)\pi^H_{n+1,5}}{(1-\theta)(3+\theta^2)(H-L)}. \tag{A4}$$

Starting with the boundary conditions $\pi^H_{N+1,S} = 0$ for all S, one uses equations (A3) and (A4) and the previous results for state 51 to solve for $\pi^H_{n,5}$ and $x^H_{n,5}$ for all $n = 1 \ldots N$.

One can also solve the insider's problem for state 42 given expected insider profits and optimal trade strategy for all periods in states 6, 50, and 51. In state 42, the insider (with a high signal) will earn future profits if the game remains in state 42 or moves to state 51. The insider's maximization problem is:

$$\max_{x^H_{n,42}} \pi^H_{n,42} = \max_{x^H_{n,42}} \left\{ \Pr^H_n(51|42)[(H - P_{51})x^H_{n,42} + \pi^H_{n+1,51}] + \Pr^H_n(42|42)[(H - P_{42})x^H_{n,42} + \pi^H_{n+1,42}] \right\} \quad (A5)$$

P_{42} is determined from the market maker's beliefs to be $(H + L)/2$. $\Pr^H_n(51|42)$ and $\Pr^H_n(42|42)$ are determined from the insider beliefs, order flow distributions, and state transition regions to be:

$$\Pr^H_n(51|42) = \frac{{x^H_{n,42}}'}{2W},$$
$$\Pr^H_n(42|42) = \frac{2W - x^H_{n,42} - {x^H_{n,42}}'}{2W}$$

Where (analogous to before) ${x^H_{n,42}}'$ will equal $x^H_{n,42}$ in equilibrium, but is treated as fixed during the insider's optimization. After substituting, the solution to the insider's optimization is:

$$x^H_{n,42} = \frac{2}{2+\theta}\left[W - \frac{\pi^H_{n+1,42}}{(H-L)}\right]. \quad (A6)$$

Starting with the boundary conditions $\pi^H_{N+1,S} = 0$ for all S, one uses equations (A5) and (A6) and the previous results for state 51 to solve for $\pi^H_{n,42}$ and $x^H_{n,42}$ for all $n = 1 \ldots N$.

The insider's optimization problem for states 2 and 4 can now be solved. Proceeding as before, the insider's maximization problems are:

$$\begin{aligned} &\max_{x^H_{n,2}} \pi^H_{n,2} \\ &= \max_{x^H_{n,2}} \left\{ \Pr^H_n(51|2)[(H - P_{51})x^H_{n,2} + \pi^H_{n+1,51}] \right. \quad (A7) \\ &\quad \left. + \Pr^H_n(42|2)[(H - P_{42})x^H_{n,2} + \pi^H_{n+1,42}] + \Pr^H_n(2|2)[(H - P_2)x^H_{n,2} + \pi^H_{n+1,2}] \right\}. \\ &\max_{x^H_{n,4}} \pi^H_{n,4} \\ &= \max_{x^H_{n,4}} \left\{ \Pr^H_n(5|4)[(H - P_5)x^H_{n,4} + \pi^H_{n+1,5}] \right. \quad (A8) \\ &\quad \left. + \Pr^H_n(42|4)[(H - P_{42})x^H_{n,4} + \pi^H_{n+1,42}] + \Pr^H_n(4|4)[(H - P_4)x^H_{n,4} + \pi^H_{n+1,4}] \right\}. \end{aligned}$$

where

$$P_2 = \frac{(1+\theta)H + 2L}{3+\theta}$$
$$P_4 = \frac{2H + (1+\theta)L}{3+\theta}$$
$$\Pr^H_n(51|2) = \frac{x^H_{n,4}}{2W}$$

$$\begin{aligned}
\Pr_n^H(42|2) &= \frac{x_{n,4}^H}{2W} \\
\Pr_n^H(2|2) &= \frac{2W - x_{n,2}^H - 2x_{n,4}^H}{2W} \\
\Pr_n^H(5|4) &= \frac{x_{n,2}^H}{2W} \\
\Pr_n^H(42|4) &= \frac{\theta x_{n,4}^{H\,\prime}}{(1+\theta)W} \\
\Pr_n^H(4|4) &= \frac{2W - x_{n,2}^H - x_{n,4}^H - x_{n,4}^{H\,\prime}}{2W}
\end{aligned}$$

are determined from the market maker's beliefs, insider beliefs, order flow distributions, and state transition regions. These optimizations are solved jointly because they are interdependent. After substituting, and doing the necessary calculus and algebra[16] the solutions to the insider optimizations for states 2 and 4 are:

$$\begin{aligned}
x_{n,2}^H &= \frac{(3-\theta)}{2(18+10\theta+3\theta^2-6\theta^3-\theta^4)}\Big[(10+7\theta+3\theta^2-3\theta^3-\theta^4)W \\
&\quad -\frac{(3+\theta)(3+3\theta+2\theta^2)}{(H-L)}\pi_{n+1,2}^H + \frac{(3+\theta)(1+\theta)(2+\theta+\theta^2)}{2(H-L)}\pi_{n+1,4}^H\Big] \qquad \text{(A9)} \\
x_{n,4}^H &= \frac{2(1+\theta)}{(18+10\theta+3\theta^2-6\theta^3-\theta^4)}\Big[2(3-\theta^2)W + \frac{\theta(3+\theta)}{(H-L)}\pi_{n+1,2}^H \\
&\quad -\frac{(3+\theta)(3-\theta)}{(H-L)}\pi_{n+1,4}^H\Big] \qquad \text{(A10)}
\end{aligned}$$

Starting with the boundary conditions $\pi_{N+1,S}^H = 0$ for all S, one uses equations (A7), (A8), (A9), and (A10) and the previous results for states 5 and 51 to solve for $\pi_{n,2}^H$, $\pi_{n,4}^H$, $x_{n,2}^H$, and $x_{n,4}^H$ for all $n = 1 \ldots N$.[17]

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[16]Details available upon request.

[17]For n very close to N it happens that $x_{n,2}^H + 2x_{n,4}^H > 2W$. This violates a presumption of the above formulation. In this case the appropriate joint optimization becomes:

$$\begin{aligned}
&\max_{x_{n,2}^H} \pi_{n,2}^H \\
&\quad = \max_{x_{n,2}^H} \left\{\Pr_n^H(51|2)[(H-P_{51})x_{n,2}^H + \pi_{n+1,51}^H] + \Pr_n^H(42|2)[(H-P_{42})x_{n,2}^H + \pi_{n+1,42}^H]\right] \\
&\max_{x_{n,4}^H} \pi_{n,4}^H \\
&\quad = \max_{x_{n,4}^H} \left\{\Pr_n^H(5|4)[(H-P_5)x_{n,4}^H + \pi_{n+1,5}^H]\right. \\
&\qquad \left. +\Pr_n^H(51|4)[(H-P_{51})x_{n,4}^H + \pi_{n+1,51}^H] + \Pr_n^H(42|4)[(H-P_{42})x_{n,4}^H + \pi_{n+1,42}^H]\right\}.
\end{aligned}$$

where all values are as previously given except

$$\Pr_n^H(42|2) = \frac{2W - x_{n,2}^H - x_{n,4}^H}{2W}$$

Lastly, the insider's optimization problem for state 3, the initial state, can be solved. In state 3 the insider's optimization problem is:

$$\begin{aligned}
&\max_{x^H_{n,3}} \; \pi^H_{n,3} \\
&= \max_{x^H_{n,3}} \Big\{ \Pr{}^H_n(2|3)[(H - P_2)x^H_{n,3} + \pi^H_{n+1,2}] + \Pr{}^H_n(3|3)[(H - P_3)x^H_{n,3} + \pi^H_{n+1,3}] \\
&\quad + \Pr{}^H_n(4|3)[(H - P_4)x^H_{n,3} + \pi^H_{n+1,4}] + \Pr{}^H_n(5|3)[(H - P_5)x^H_{n,3} + \pi^H_{n+1,5}] \Big\}.
\end{aligned} \tag{A11}$$

where

$$\begin{aligned}
P_3 &= \frac{H+L}{2} \\
\Pr{}^H_n(2|3) &= \frac{\theta {x^H_{n,3}}'}{(1+\theta)W} \\
\Pr{}^H_n(3|3) &= \frac{2W - x^H_{n,3} - 3{x^H_{n,3}}'}{2W} \\
\Pr{}^H_n(4|3) &= \frac{{x^H_{n,3}}'}{2W} \\
\Pr{}^H_n(5|3) &= \frac{{x^H_{n,3}}'}{2W}
\end{aligned}$$

are determined from the market maker's beliefs, insider beliefs, order flow distributions, and state transition regions (and all other values have been derived in previous steps). After substituting, the solutions to the insider optimization for state 3 is:

$$x^H_{n,3} = \frac{2(1+\theta)(3+\theta)(3-\theta)}{33 + 9\theta + 7\theta^2 - \theta^3}\left[W - \frac{\pi^H_{n+1,3}}{H-L}\right] \tag{A12}$$

$$\begin{aligned}
\Pr{}^H_n(5|4) &= \frac{2W - x^H_{n,4} - {x^H_{n,4}}'}{2W} \\
\Pr{}^H_n(42|4) &= \frac{\theta(2W - x^H_{n,2} - x^H_{n,4})}{(1+\theta)W} \\
\Pr{}^H_n(51|4) &= \frac{\theta(x^H_{n,2} + x^H_{n,4} + {x^H_{n,4}}' - 2W)}{(1+\theta)W}.
\end{aligned}$$

The solutions to the insider optimizations for states 2 and 4 are then:

$$\begin{aligned}
x^H_{n,2} &= \frac{1}{(6 - 6\theta + 14\theta^2 - 9\theta^3 + \theta^4)}\Big[2(3-\theta)(1 - \theta + 2\theta^2 - \theta^3)W \\
&\quad - \frac{2(3 - 3\theta + 4\theta^2 - 2\theta^3)}{(H-L)}\pi^H_{n+1,42} + \frac{\theta(1+\theta)(3-\theta)}{(H-L)}\pi^H_{n+1,5} - \frac{2\theta^2(3-\theta)}{(H-L)}\pi^H_{n+1,51}\Big] \\
x^H_{n,4} &= \frac{2}{(6 - 6\theta + 14\theta^2 - 9\theta^3 + \theta^4)}\Big[(2 + \theta^2 - \theta^3)W - \frac{\theta(2-\theta)(3-\theta)}{(H-L)}\pi^H_{n+1,42} \\
&\quad - \frac{(1+\theta)(3-\theta)}{(H-L)}\pi^H_{n+1,5} + \frac{2\theta(3-\theta)}{(H-L)}\pi^H_{n+1,51}\Big].
\end{aligned}$$

Starting with the boundary conditions $\pi^H_{N+1,S} = 0$ for all S, one uses equations (A11) and (A12) and the previous results for states 2, 3, and 5 to solve for $\pi^H_{n,3}$ and $x^H_{n,3}$ for all $n = 1 \ldots N$.[18]

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[18]For n very close to N it happens that $x^H_{n,3} > W/2$. This violates a presumption of the above formulation. In this case, if $W/2 < x^H_{n,3} < 2W/3$ the appropriate optimization is:

$$\begin{aligned} \max_{x^H_{n,3}} \ \pi^H_{n,3} \\ &= \max_{x^H_{n,3}} \ \{\Pr^H_n(2|3)[(H-P_2)x^H_{n,3} + \pi^H_{n+1,2}] + \Pr^H_n(42|3)[(H-P_{42})x^H_{n,3} + \pi^H_{n+1,42}] \\ &\quad + \Pr^H_n(4|3)[(H-P_4)x^H_{n,3} + \pi^H_{n+1,4}] + \Pr^H_n(5|3)[(H-P_5)x^H_{n,3} + \pi^H_{n+1,5}]\}. \end{aligned}$$

where all values are as previously defined except

$$\begin{aligned} \Pr^H_n(2|3) &= \frac{\theta(2W - 2{x^H_{n,3}}' - x^H_{n,3}}{(1+\theta)W} \\ \Pr^H_n(42|3) &= \frac{\theta(3{x^H_{n,3}}' + x^H_{n,3} - 2W}{(1+\theta)W} \\ \Pr^H_n(4|3) &= \frac{2W - 2{x^H_{n,3}}' - x^H_{n,3}}{2W}. \end{aligned}$$

The solution to the insider optimization for state 3 is then:

$$x^H_{n,3} = \frac{(3-\theta)}{9 + 22\theta - 9\theta^2 + 2\theta^3}\left[2(1+3\theta)W - \frac{(1+\theta)(3+\theta)}{H-L}\pi^H_{n+1,4} + \frac{2\theta(3+\theta)}{H-L}(\pi^H_{n+1,42} - \pi^H_{n+1,2})\right]$$

If $x^H_{n,3} > 2W/3$ the appropriate optimization is:

$$\begin{aligned} \max_{x^H_{n,3}} \ \pi^H_{n,3} \\ &= \max_{x^H_{n,3}} \ \{\Pr^H_n(42|3)[(H-P_{42})x^H_{n,3} + \pi^H_{n+1,42}] + \Pr^H_n(51|3)[(H-P_{51})x^H_{n,3} + \pi^H_{n+1,51}] \\ &\quad + \Pr^H_n(5|3)[(H-P_5)x^H_{n,3} + \pi^H_{n+1,5}]\}. \end{aligned}$$

where all values are as previously defined except

$$\begin{aligned} \Pr^H_n(51|3) &= \frac{\theta(2{x^H_{n,3}}' + x^H_{n,3} - 2W)}{(1+\theta)W} \\ \Pr^H_n(42|3) &= \frac{\theta(2W - {x^H_{n,3}}' - x^H_{n,3})}{(1+\theta)W} \\ \Pr^H_n(5|3) &= \frac{2W - {x^H_{n,3}}' - x^H_{n,3}}{2W}. \end{aligned}$$

The solution to the insider optimization for state 3 is then:

$$x^H_{n,3} = \frac{1}{3 - 3\theta + 10\theta^2 - 4\theta^3}\left\{2(1 + 2\theta^2 - \theta^3)W - \frac{3-\theta}{H-L}\left[2\theta(\pi^H_{n+1,51} - \pi^H_{n+1,42}) - (1+\theta)\pi^H_{n+1,5}\right]\right\}.$$

A.2 Special Cases: $\theta \in \{0, 1\}$

When $\theta \in \{0,1\}$ the state-space degenerates. When $\theta = 1$ the market contains a single informed trader with certainty and the eleven states degenerate to just three: states 0, 3, and 6. When $\theta = 0$ the market is equally likely to contain no informed traders or two informed traders, and the eleven states degenerate to six: states 0, 1, 3, 5, 6 and 50. The state definitions from the previous section still hold, but the recursion equations to solve for the insider trade strategy are simplified. Specifically, when $\theta = 1$, the recursion equations to solve for $\pi^H_{n,3}$ and $x_{n,3}$ simplify to:

$$\begin{aligned}
&\max_{x^H_{n,3}} \ \pi^H_{n,3} \\
&= \max_{x^H_{n,3}} \ \Pr^H_n(3|3)[(H - P_3)x^H_{n,3} + \pi^H_{n+1,3}]
\end{aligned} \tag{A13}$$

where

$$\Pr^H_n(3|3) = \frac{2W - x^H_{n,3} - {x^H_{n,3}}'}{W}.$$

The solutions to the insider optimization for State 3 becomes:

$$x^H_{n,3} = \frac{2}{3}\left(W - \frac{\pi^H_{n+1,3}}{H-L}\right) \tag{A14}$$

Starting with the boundary condition $\pi^H_{N+1,3} = 0$, one solves for $\pi^H_{n,3}$ and $x^H_{n,3}$ for all $n = 1 \ldots N$ using equations (A13) and (A14).

When $\theta = 0$ the recursion equations to solve for $\pi^H_{n,5}$ and $x^H_{n,5}$ simplify to:

$$\begin{aligned}
&\max_{x^H_{n,5}} \ \pi^H_{n,5} \\
&= \max_{x^H_{n,5}} \ \Pr^H_n(5|5)[(H - P_5)x^H_{n,5} + \pi^H_{n+1,5}]
\end{aligned} \tag{A15}$$

where

$$\Pr^H_n(5|5) = \frac{2W - x^H_{n,5} - {x^H_{n,5}}'}{2W}.$$

The solutions to the insider optimization for state 5 becomes:

$$x^H_{n,5} = \frac{2W}{3} - \frac{\pi^H_{n+1,5}}{H-L} \tag{A16}$$

Starting with the boundary condition $\pi^H_{N+1,5} = 0$, one solves for $\pi^H_{n,5}$ and $x^H_{n,5}$ for all $n = 1 \ldots N$ using equations (A15) and (A16). The recursion equations to solve for $\pi^H_{n,3}$ and $x^H_{n,3}$ simplify to:

$$\begin{aligned}
&\max_{x^H_{n,3}} \ \pi^H_{n,3} \\
&= \max_{x^H_{n,3}} \ \left\{\Pr^H_n(3|3)[(H - P_3)x^H_{n,3} + \pi^H_{n+1,3}] + \Pr^H_n(5|3)[(H - P_5)x^H_{n,3} + \pi^H_{n+1,5}]\right\}
\end{aligned}$$

where

$$\Pr_n^H(3|3) = \frac{2W - x_{n,3}^H - 3x_{n,3}^{H\,\prime}}{2W}$$
$$\Pr_n^H(5|3) = \frac{x_{n,3}^{H\,\prime}}{W}.$$

The solutions to the insider optimization for state 3 becomes:

$$x_{n,3}^H = \frac{6}{11}\left(W - \frac{\pi_{n+1,3}^H}{(H-L)}\right).$$

$\pi_{n,3}^H$ and $x_{n,3}^H$ for all $n = 1 \ldots N$ are derived as before.[19]

A.3 Definition of the Market Liquidity Parameter λ

It is standard to measure inverse market liquidity as the sensitivity of the market maker's pricing rule with respect to changes in the order size. In my model defining a market liquidity parameter is more complicated because the pricing rule is not continuous in the order flow. The general approach I adopt is to measure inverse market liquidity as the slope of a straight line connecting the two pricing rule segments bracketing the state price for each game state. For example, when the game is in state 3, the price is P_3. The game will remain in state 3 for order flows sufficiently close to zero. Orders flows greater than $W - 2x_{n,3}^H$ will move the game to state 4 with price P_4. Order flows less than $-W + 2x_{n,3}^H$ will move the game to state 2 with price P_2. The market liquidity parameter in state 3, λ_3, is then defined as $(P_4 - P_2)/(2W - 4x_{n,3}^H)$.

The general approach given above is somewhat oversimplified. Some of the pricing rule discontinuities disappear at $\theta = 0$ or 1. That is, the pricing rule discontinuities have discontinuities and this makes defining market liquidity slightly more difficult because care must be taken to define market liquidity so that it is continuous in θ. I address this problem as follows. For each state I determine three different values of market liquidity: One for $\theta = 0$, one for $\theta = 1$, and one for $0 < \theta < 1$. Call these λ_0, λ_1, and λ_θ. For each state I then define the quadratic function $\lambda(\theta) = \alpha + \beta\theta + \gamma\theta^2$ where α, β, and γ are determined such that $\lambda(0) = \lambda_0$, $\lambda(1) = \lambda_1$, and $\max_\theta \lambda(\theta) = \lambda_\theta$.

[19] For n very close to N it happens that $x_{n,3}^H > W/2$. This violates a presumption of the above formulation. In this case the appropriate optimization is:

$$\max_{x_{n,3}^H} \pi_{n,3}^H = \max_{x_{n,3}^H} \Pr_n^H(5|3)[(H - P_5)x_{n,3}^H + \pi_{n+1,5}^H]$$

where

$$\Pr_n^H(5|3) = \frac{2W - x_{n,3}^H - x_{n,3}^{H\,\prime}}{2W}.$$

The solutions to the insider optimization for state 3 becomes:

$$x_{n,3}^H = \frac{2}{3}W - \frac{\pi_{n+1,5}^H}{(H-L)}.$$

This procedure generates α's, β's, and γ's for each state of the following form:

$$\begin{aligned}\alpha &= a \\ \beta &= 2\left[b - a + \sqrt{(b-a)(b-c)}\right] \\ \gamma &= a + c - 2b - 2\sqrt{(b-a)(b-c)}\end{aligned}$$

where a, b, and c for each state are as follows:

State(s)	a	b	c
3	$(P_4 - P_2)/(2W - 4x^H_{3,n})$	$(P_4 - P_2)/(2W - 4x^H_{3,n})$	$(P_5 - P_1)/(2W - 2x^H_{3,n})$
2, 4	$(P_6 - P_{42})/(2W - 2x^H_{4,n})$	$(P_5 - P_{42})/(2W - x^H_{2,n} - 2x^H_{4,n})$	$(P_5 - P_{41})/(2W - x^H_{2,n} - x^H_{4,n})$
1, 5	$(P_6 - P_{51})/(2W - 2x^H_{5,n})$	$(P_6 - P_{51})/(2W - 2x^H_{5,n})$	0
41, 51	0	$(P_6 - P_{50})/(2W - x^H_{51,n})$	0
42	0	$(P_{51} - P_{41})/(2W - 2x^H_{42,n})$	$(P_{51} - P_{41})/(2W - 2x^H_{42,n})$

In states 0, 6, and 50, $\lambda = 0$.

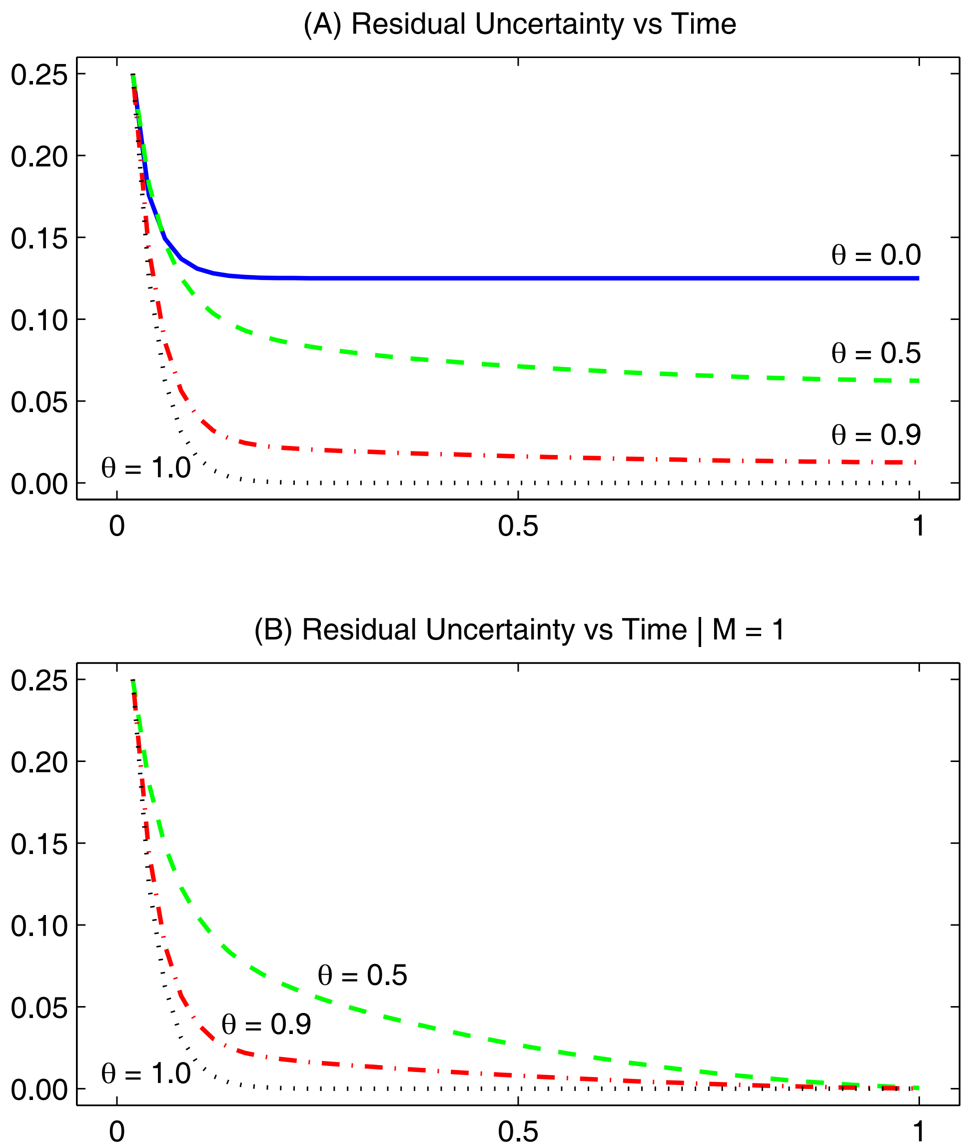


Figure 2: Unconditional and conditional residual uncertainty for different values of θ. Other parameters are held constant: $N = 50$, $H = 1$, and $L = 0$. Per round noise trade is distributed uniform$(-1/\sqrt{N}, 1/\sqrt{N})$. The top panel shows the unconditional expected error variance of the price over time. The lower panel shows the expected error variance of the price over time conditional on the realized number of informed traders being one.

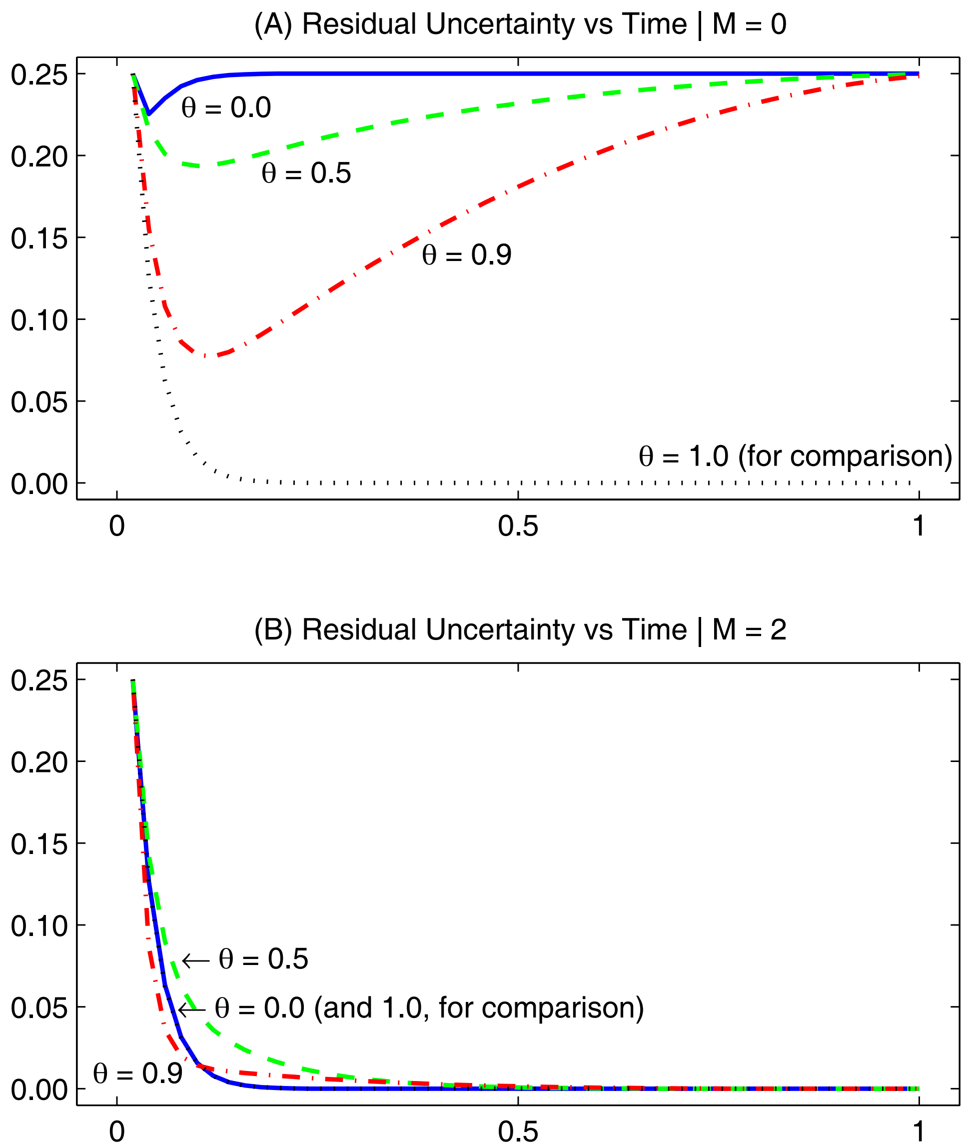


Figure 3: Conditional residual uncertainty for different values of θ. Other parameters are held constant: $N = 50$, $H = 1$, and $L = 0$. Per round noise trade is distributed uniform$(-1/\sqrt{N}, 1/\sqrt{N})$. The top panel shows the expected error variance of the price over time conditional on the realized number of informed traders being zero. The lower panel shows the expected error variance of the price over time conditional on the realized number of informed traders being two.

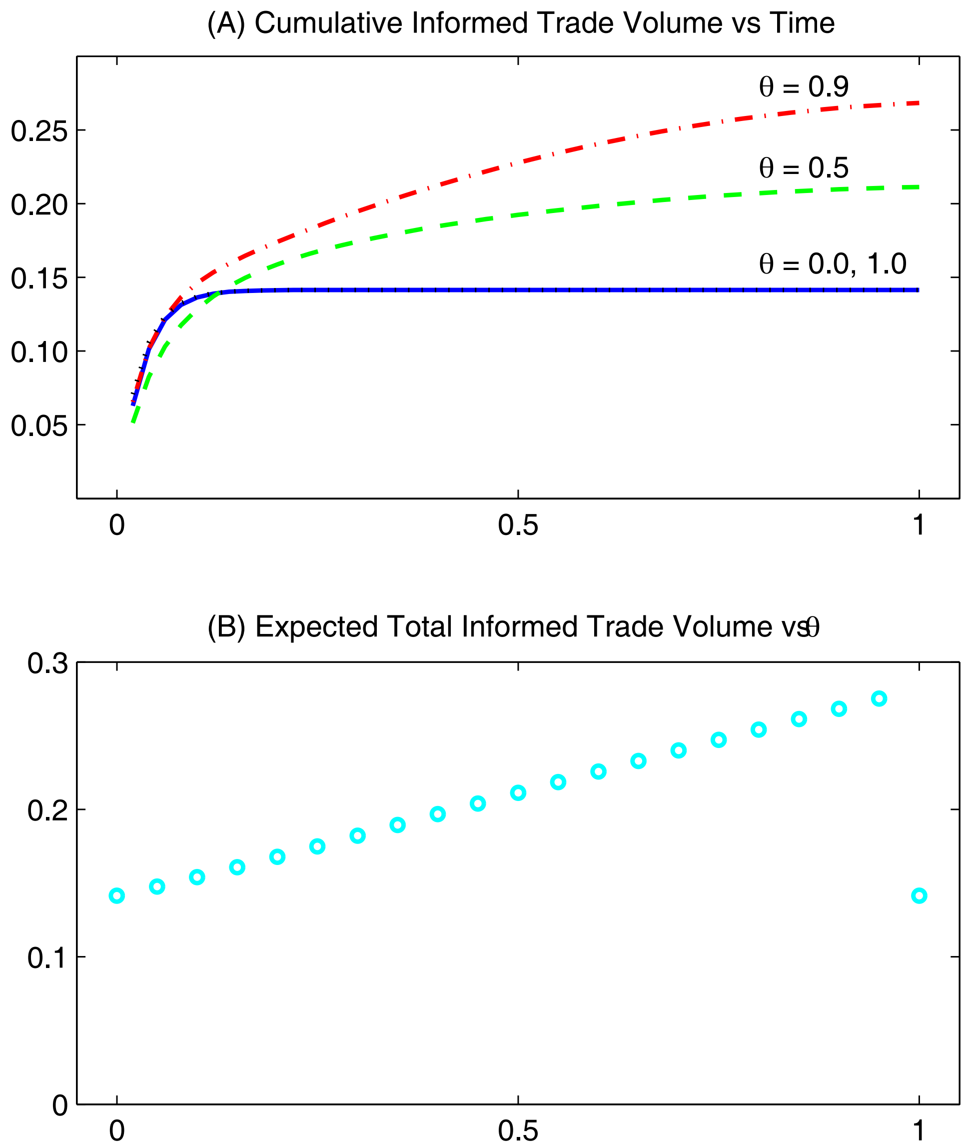


Figure 4: Informed trade volume for different values of θ. Other parameters are held constant: $N = 50$, $H = 1$, and $L = 0$. Per round noise trade is distributed uniform$(-1/\sqrt{N}, 1/\sqrt{N})$. The top panel shows the expected cumulative informed trade volume over time. The lower panel shows the total expected informed trade volume.

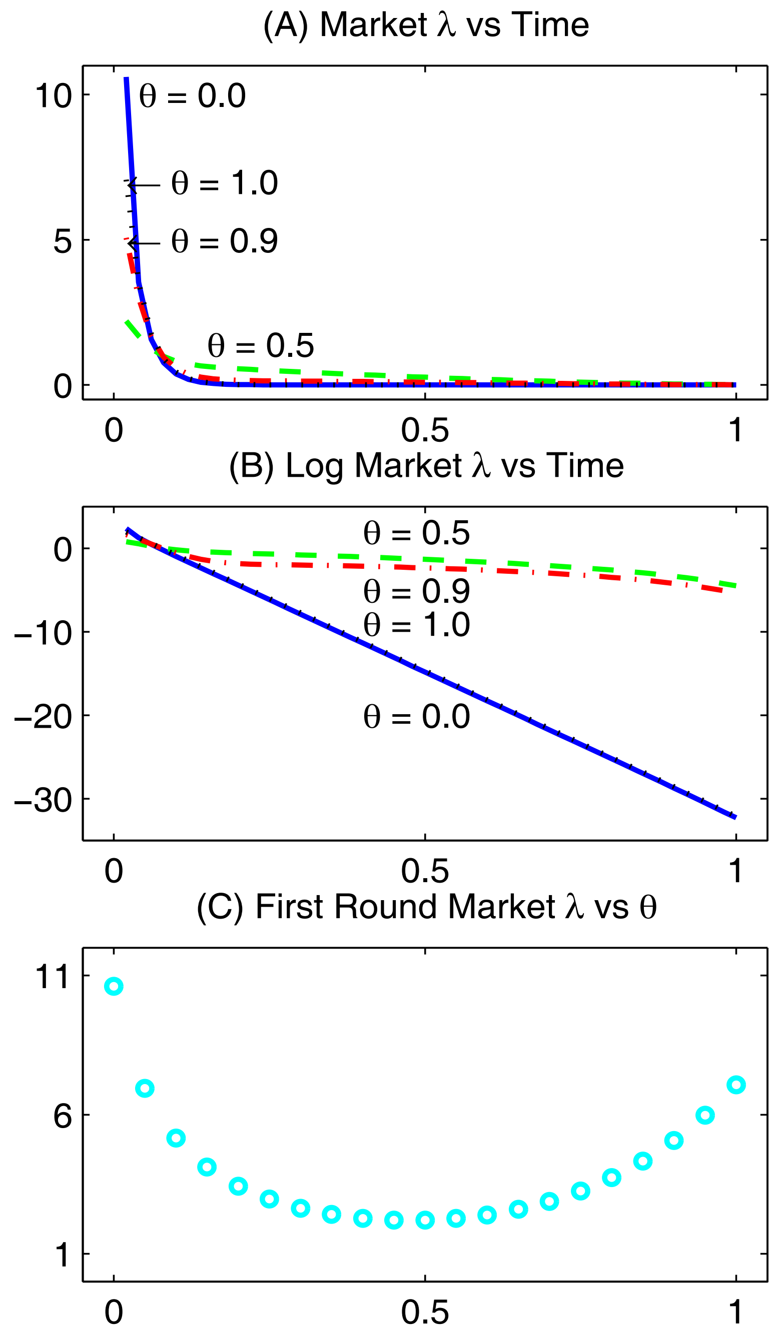
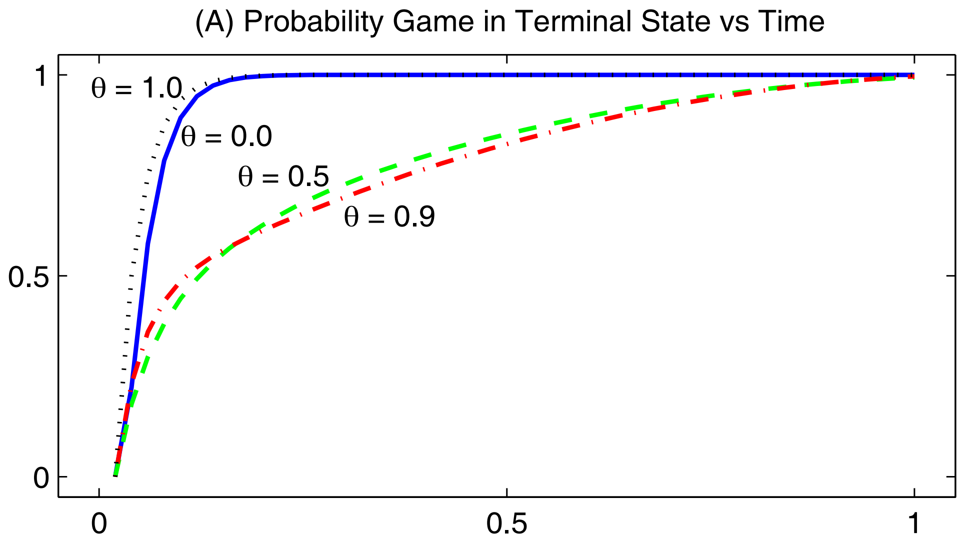


Figure 5: Market λ (inverse of market depth) for different values of θ. Other parameters are held constant: $N = 50$, $H = 1$, and $L = 0$. Per round noise trade is distributed uniform$(-1/\sqrt{N}, 1/\sqrt{N})$. The top panel shows the expected market λ over time. The second panel shows the log of the expected market λ over time. The last panel shows the market λ in the first round of trade for different values of θ.



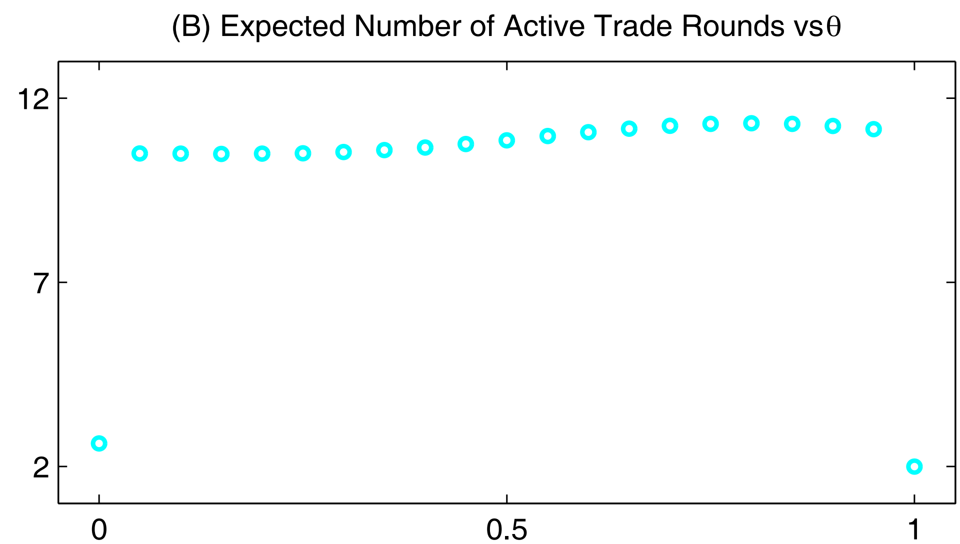


Figure 6: Market impact duration of an information event for different values of θ. Other parameters are held constant: $N = 50$, $H = 1$, and $L = 0$. Per round noise trade is distributed uniform$(-1/\sqrt{N}, 1/\sqrt{N})$. The top panel shows probability over time that the game has arrived in a terminal state. The lower panel shows the number of trading rounds insiders are expected to be active in the market following an information event.

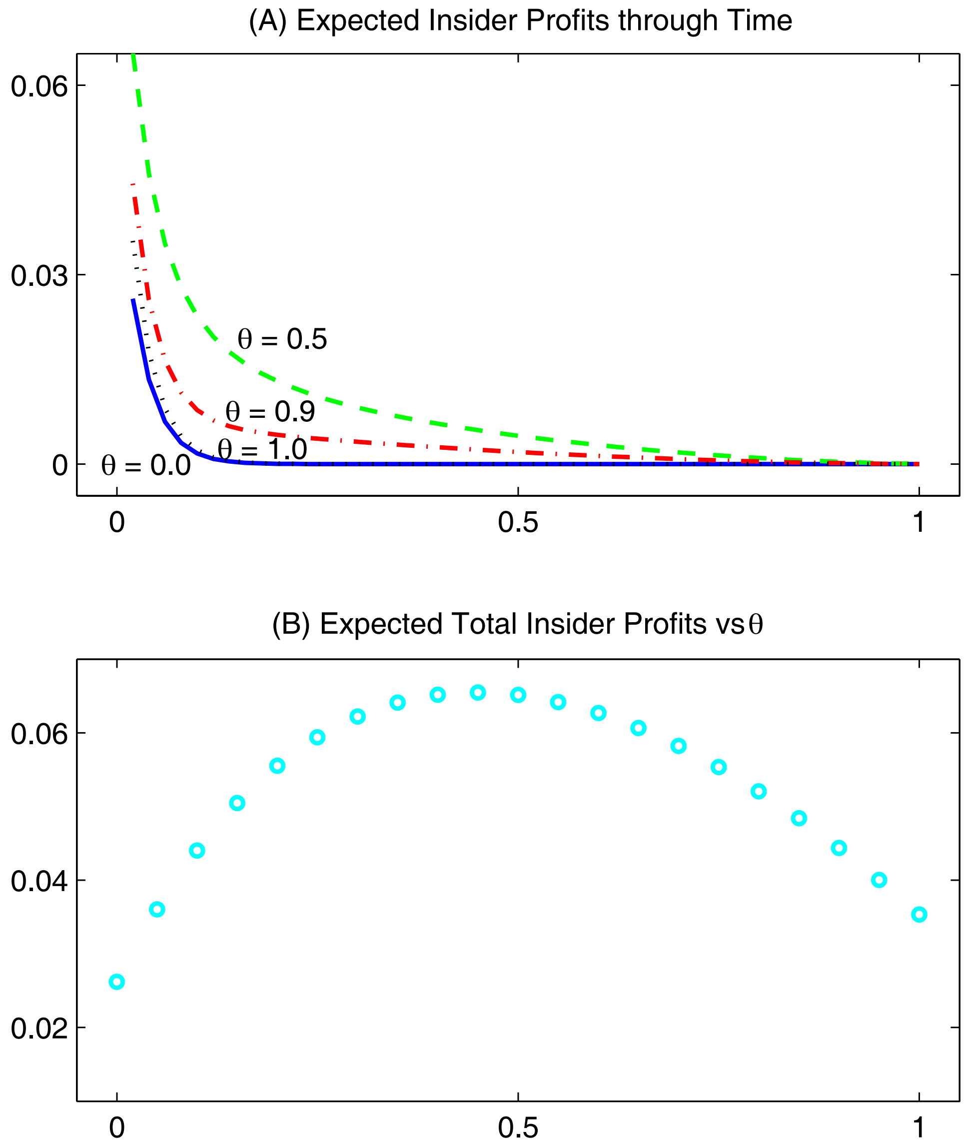


Figure 7: Expected insider profits for different values of θ. Other parameters are held constant: $N = 50$, $H = 1$, and $L = 0$. Per round noise trade is distributed uniform$(-1/\sqrt{N}, 1/\sqrt{N})$. The top panel shows how the expected future profits of insiders decay over time. The lower panel shows the total expected insider profits as a function of θ.

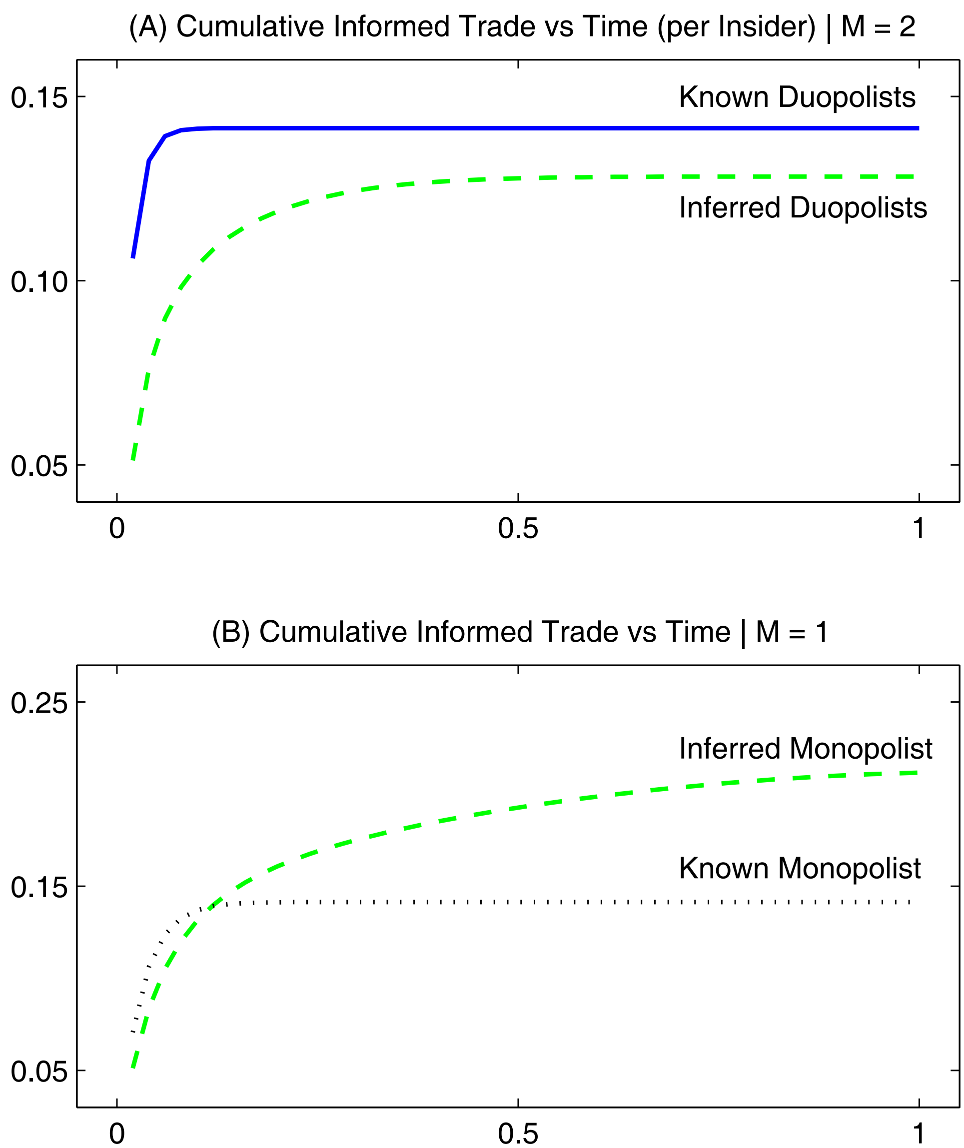


Figure 8: Conditional cumulative informed trade volume (per insider) over time when the number of insiders is known versus inferred. $\theta = 0.5$ for the case where the number of insiders is inferred. Other parameters are held constant: $N = 50$, $H = 1$, and $L = 0$. Per round noise trade is distributed uniform$(-1/\sqrt{N}, 1/\sqrt{N})$. The top panel shows per insider trade volume when there exist two insiders. The lower panel shows insider trade volume when there exists one insider.

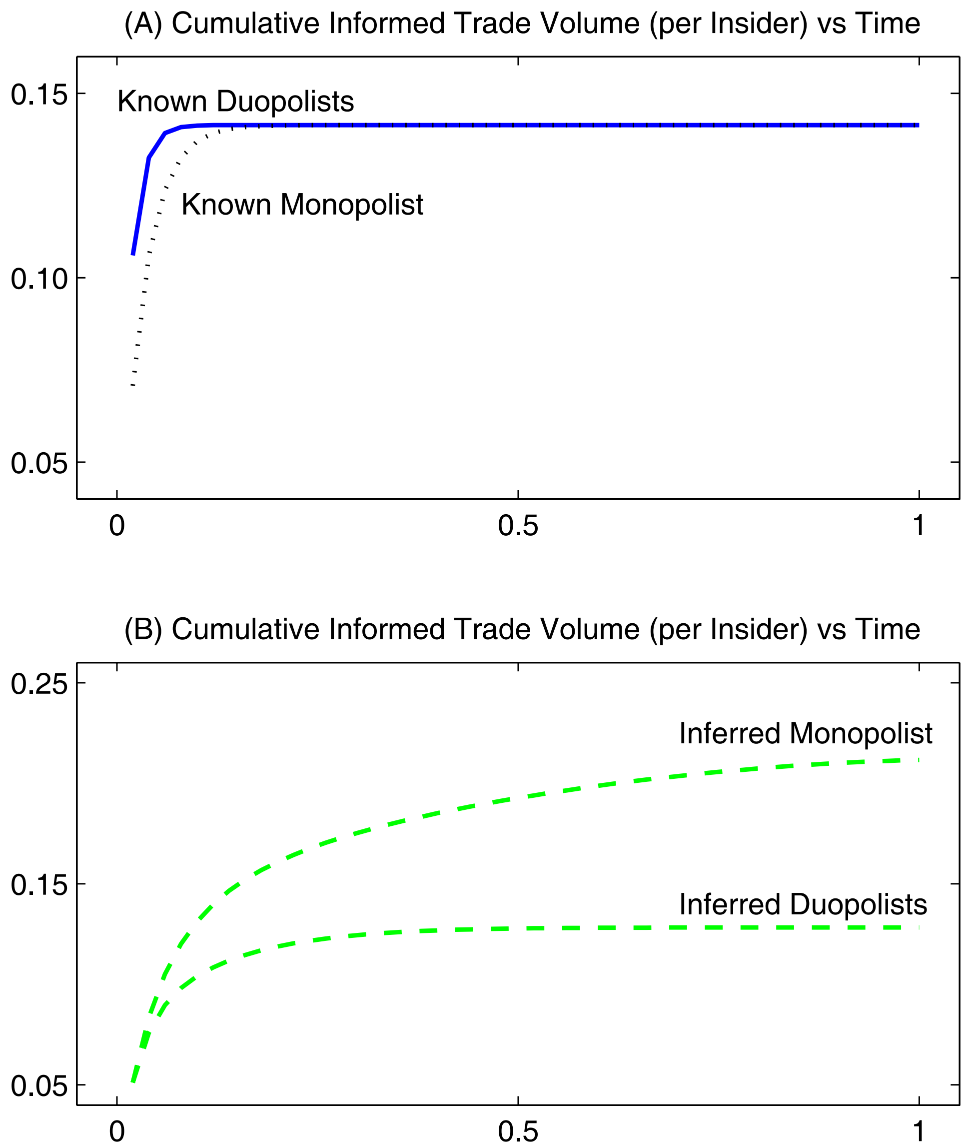


Figure 9: Conditional cumulative informed trade volume (per insider) over time for one versus two insiders. The top panel shows per insider trade volume when the number of insiders is common knowledge. The lower panel shows per insider trade volume when the number of insiders is inferred. $\theta = 0.5$ for the case where the number of insiders is inferred. Other parameters are held constant: $N = 50$, $H = 1$, and $L = 0$. Per round noise trade is distributed uniform$(-1/\sqrt{N}, 1/\sqrt{N})$.

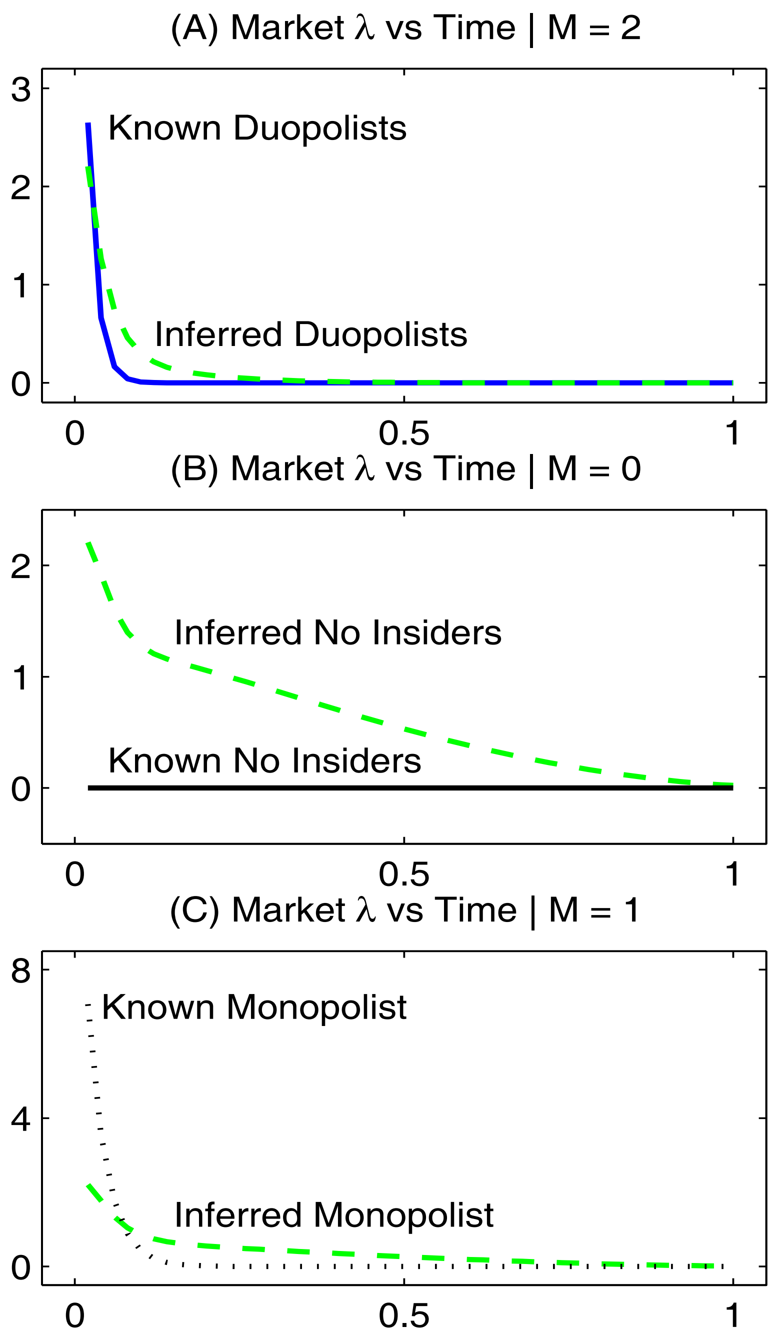


Figure 10: Conditional expected market λ over time when the number of insiders is known versus inferred. $\theta = 0.5$ for the case where the number of insiders is inferred. Other parameters are held constant: $N = 50$, $H = 1$, and $L = 0$. Per round noise trade is distributed uniform$(-1/\sqrt{N}, 1/\sqrt{N})$. Panel A shows the expected market λ when there exist two insiders. Panel B shows the expected market λ when there exist no insiders. Panel C shows the expected market λ when there exists one insider.