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The Value of the Freezeout Option

Zohar Goshen
The Faculty of Law, The Hebrew University of Jerusalem,
and Columbia Law School

And

Zvi Wiener
The School of Business Administration, The Hebrew University of Jerusalem,
and Marshall School of Business, University of Southern California

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ABSTARCT

The value of the freezeout option is important in many legal policy issues concerning corporate law. In this article, we present, for the first time, a method for determining the value of the minority stock and the freezeout option. We price the freezeout option with two different sets of assumptions regarding the controlling shareholder informational advantage, using both an exogenous and endogenous stock prices in our pricing. The result of our model indicates that when using an exogenous market price to determine fair value, the freezeout option has a low value and the minority stock is only slightly discounted. This result implies that the use of publicly known information, *excluding market prices*, in determining a fair value for minority stocks will not cause expropriation of minority shareholders and will not lead to inefficiency in corporate and controlling owners' decisions. Additionally, our model shows that a complete reliance on market prices will lead to inability to positively price and trade minority shares. Any consistent and predictable use of market prices by the courts in the valuation process will cause a discount relative to the weight given to market prices in that process. Combining both results it can be said that, indeed, in an efficient market, prices do reflect fair value, but this is so *because courts do not heavily and consistently base their valuations on market prices.* Empirical studies support our results.

Introduction

Delaware's corporate law entitles a controlling shareholder to buy out -- or "freezeout" -- the minority shareholders.[1] Leaving aside the technical aspects of the freezeout, which is commonly performed through a merger, the most significant result is that the controlling shareholder is not just able to force the minority to sell their shares to her, but also determines the price of these shares. The right to freezeout, thus, carries the risk of minority shareholders expropriation. To counter that risk, the law offers protection to minority shareholders: a shareholder who is dissatisfied with the price offered for the shares in the merger is entitled to ask the court to determine the fair value of her shares. Without getting into details, this result is accomplished either by using the "appraisal right",[2] or by claiming a breach of the duty of loyalty, thereby initiating the "entire fairness" test.[3]

The controlling shareholder's freezeout right is, in fact, a call option on the minority shares for an indefinite time whose exercise price is determined by the option holder. However, given the appraisal right and the duty to meet the entire fairness standard, the exercise price should not be lower than the expected fair price of the shares in the court's valuation process.[4]

Regardless of the particular valuation principles, courts rely, among other things, on publicly known information about the corporation as a basis for the valuation process. One important piece of publicly known information is the market price of publicly traded corporations. The market price enters the valuation process in variety of ways: as the main

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[1] *See, e.g.,* Paramount Communications Inc., v. QVC Network Inc., 637 A.2d 34 (Del. 1994).

[2] *See,* Delaware General Corporations Law, Sec. 262(a) and (b).

[3] *See,* Weinberger v. UOP, 457 A.2d 701 (Del. 1983).

[4] *See,* Frank H. Easterbrook and Daniel R. Fischel, THE ECONOMIC STRUCTURE OF CORPORATE LAW, 148-150 (1991) (appraisal rights establish a floor under the freezeout price).

indication of the fair value;[5] as part of a "block" valuation that weighs and averages several different methods of valuation;[6] or as a component of a given valuation method.[7]

This reliance on publicly known information, however, might generate a risk of under-valuation of the minority shares even in efficient capital markets. This risk has two related possible sources. The first source is the controlling shareholder's *informational advantage*: since the controlling shareholder holds private information about the future value of the corporation, she can time the exercise of the option to her maximum benefit.[8] Indeed, whenever the expected future price is higher than the current market price, the option can be exercised. Given that courts are unable to reflect *the value of private information* held by the controlling shareholder in the valuation process, the valuation process might result in under-valuation of minority shares.[9] Of course, for the informational advantage to be valuable -- *i.e.*, not priced by the court -- it must extend beyond the time it takes to complete the freezeout. Otherwise, any private information that will become public during the execution of the freezeout will be part of the valuation process.

This initial under-valuation might lead to a second source of under-valuation: investors expecting to receive fair value based on a discounted price -- due to the existence of the

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[5] *See, e.g.,* Smith v. Shell Petroleum, Inc., 1990 WL 84218 (Del. Ch. 1990).

[6] Under the block method the appraiser computes separate values for market, earnings, and net assets, gives weight to each and adds them together. Although the Delaware supreme court has critiqued the block method in Weinberger v. UOP, *supra* note 3, it is still a permissible (but generally not used) method in Delaware and still being used in other states. *See*, John Coats IV, *"Fair Value" as an Avoidable Rule of Corporate Law: Minority Discounts in Conflict Transactions,* 147 U. PA. L. REV. 1251, FN. 44 (1999).

[7] For instance, using the market price movements to calculate beta to be used in a capital assets pricing model or extrapolating market rate of return on the business activity.

[8] We consider private future business plans or contemplated transactions by the controlling shareholder to be entrepreneurship and not an informational advantage. Only the ability to foresee ahead of the market the current ongoing sequence of events is considered informational advantage.

[9] *See,* Lucian A. Bebchuk & Marcel Kahan, *The "Lemons Effect" in Corporate Freeze-Outs*, forthcoming, in Randall Morck (ed.), Concentrated Ownership (U. Chi. Press).

freezeout option -- discount the stock market price further to reflect the discounted expected "fair value". Given that the now discounted market price will in turn influence the valuation process, leading to an even greater under-valuation of the fair value of the minority shares, the market price will be discounted further. This process, known as the *"lemons effect"*, will repeat itself until the stock price drops to the lower end of the expected range of values.[10]

Indeed, if minority shares are under-valued in cases of freezeouts, this fact will be reflected, ex ante, in the stock price. That is, minority shareholders will discount the price that they are willing to pay for the shares, in response to the risk of under-valuation, and thereby avoid the risk of expropriation. However, even in this case, it has been argued, under-valuation of the fair value of minority shares will result in inefficiency. To the extent that the freezeout option provides the controlling owner with a private benefit of control, she will attempt, ex post, to capture this benefit. These attempts will result in inefficiency due to: inefficient investment in search for private information; inefficient business decisions designed to increase the value of the option; and unexpected (not priced) instances of minority shareholders' expropriation.[11]

Against that view others have argued that in an efficient market minority shares are properly priced. Consequently, in capital markets where shares are traded frequently enough to have a market price, the pre-merger fair value is the pre-merger market price.[12] Thus, there is no need for complicated appraisal proceedings: every price above the pre-merger market price is a fair price. This view regards the risk of under-valuation due to the freezeout option as

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[10] Bebchuk and Kahan, *Id*. Easterbrook & Fischel, *supra note* 4, at pp. 154-55, also voiced such a concern. As well, opposing minority discounts, *See,* Coats, *supra* note 6.

[11] Bebchuk and Kahan, *Id*.

[12] Benjamin Hermalin & Alan Schwartz, *Buyouts in Large Companies*, 25 J. Leg. Stud. 351 (1996).

negligible.

Assuming the value of the firm in a legal regime prohibiting freezeouts to be the "fair value" of the firm, how serious is the risk of under-valuation given the right to freezeout? In other words, how valuable is the freezeout option? If the freezeout option has only minor value, neither of the evils mentioned above should be expected and the whole valuation process can indeed be simplified for firms having efficient pricing: every price above the pre-merger market price is a fair price. On the other hand, if the freezeout option is very valuable (i.e., the risk of under-valuation is substantial) inefficiency will result, and a proper policy for freezeout proceedings should be devised.

As the value of the freezeout option is critical to this debate, the importance of pricing the freezeout option becomes clear. Yet, pricing an option that is exercised based on private information regarding the future value of the stock is a complicated task. To date no such attempt was made. In this article, we present, for the first time, a method for determining the value of the minority stock and the freezeout option.

The result of our model indicates that as far as *the value of the private information* is concerned the freezeout option has a low value when the ability of the controlling shareholder to see into the future is limited. In this case, the minority stock is only slightly discounted. This result implies that the use of publicly known information, *excluding market prices*, in determining a fair value for minority stocks will not cause expropriation of minority shareholders and will not lead to inefficiency in corporate and controlling owners' decisions. However, as the informational advantage of the controlling shareholder is increasing, allowing her to see several months into the future, the value of the option becomes substantial resulting in a similar discount of the share price. This might suggest a need for a policy change regarding the appropriate method of valuation.

In our model, although the controlling shareholder holds the freezeout option for indefinite time, due to the nature of the freezeout transaction it can be exercised only once. Thus, the controlling shareholder will not exercise the option the first time that her private information indicates that the firm value is higher than the current market price. Rather, the controlling shareholder will attempt to capture the greatest expected divergence between the current price and the privately known future price. This strategy is aimed at the extreme cases of high expected values for the firm, leaving the rest of the expected range of values out of the reach of the freezeout option.

Focusing on the *value of the private information*, our initial basic model uses an exogenous stock price to price the freezeout option, avoiding the possibility of a "lemons effect". To illustrate this point, assume that there are two countries: country A in which freezeout is allowed and country B in which freezeout is restricted. In country B, the stock will be traded for its full value, reflecting the whole range of expected probabilities of values. In country A, the stock will be discounted to reflect the value of the freezeout option. In our model, investors in country A expect the court determining the fair value to draw the market price from (hypothetical) country B.

On the other hand, when the stock price is endogenous the court determining the fair value of investors in country A draws the market price from country A, in which the stock is traded and a freezeout is allowed. Investors expecting a discounted "fair value" further discount the stock price. It is this feature that gives rise to the "lemons effect".

Indeed, to allow us to evaluate the *value of the "lemons effect"* and offer a complete analysis of the freezeout option, we present an extended general model that allows us to price the option based on endogenous stock price as well. The result of our model indicates that in this case the price of the minority shares will drop all the way to zero. This result indicates that the risk of a discounted fair price due to the use of market prices is real. A complete reliance on market prices as indication of fair values is impossible. Moreover, every partial, but systematic, use of market prices will result in partial discount *in all traded shares of all companies that are subject to freezeout* relative to the weight given to market prices by the courts. Assume, for instance, the use of a block valuation: the greater the weight that will be assigned to market prices the greater the discount of the fair price. Similarly, *consistently* assigning a given weight to market prices in every valuation will result in greater discount *in all traded shares that are subject to freezeout,* than in the case of *randomizing* the use of market prices in few of the cases. In the latter case, the expected effect of the discount due to the use of market prices is lower.

Combining both results it can be said that, indeed, in an efficient market, prices do reflect fair value, but this is so *because courts do not heavily and consistently base their valuations on market prices.* In other words, the more courts will insert market prices into their valuations in a more *consistent* and *predictable* manner, the more discounted the market price will be, and vice versa. Indeed, ironically, the less reliance courts put on market prices, the more accurate are market prices as a reflection of fair value. But, if courts will increase their use of market prices in their valuations, the quality of market prices as indication of fair value will deteriorate.

Obviously, minority stocks are trading with positive prices. Given the above results of our model, this can be explained by the fact that courts do not *consistently* assign high weight to the market price in the valuation process, thereby leaving some positive value for the stock. Courts, indeed, rely on financial and accounting data that are *independent* of the firm's market price alongside *randomize* use of *differing low weights* of market prices. Investors' expectation of the courts to use the independent data is similar to an expectation to use exogenous stock price in the valuation process. Once investors hold such expectations, minority stock price is only slightly discounted. Again, although in such a case the market price is the equivalent of an exogenous price *it cannot be consistently used* in the valuation process without changing its nature. Empirical studies support the view that market prices are not heavily discounted due to the freezeout option. In other words, the value of the option is determined by the value of the *private information* held by the controlling owner, and not by her ability to capture the drop in the stock price due to the *"lemons effect*." The policy conclusion of our model is, thus, that courts should preserve their practice of not heavily and consistently basing their valuations on market prices.[13]

The article is organized as follows: In the first part we price the freezeout option using a basic initial model with two different sets of assumptions regarding the controlling shareholder informational advantage. In the second part we price the freezeout option using an extended general model that allows us to use exogenous and endogenous stock prices in our pricing, as well as a mix of both. This model is able to capture the "lemons effect". In the third part we discuss the model's implications for the debate surrounding the value of the freezeout option. In the fourth part we discuss some empirical studies that can supports our results. The fifth part is the conclusion.

I. A Basic Model

[13] This suggests, as well, that both sides to this debate are both right and wrong. Those who argue that market prices cannot be used because market prices are discounted, are right in that, indeed, market prices cannot be relied upon, but wrong in that market prices are not discounted. Those who argue that market prices are not discounted and can be used as a fair price, are right in that, indeed, market prices are not discounted, but are wrong in that market prices cannot be used as fair price.

1. The Institutional Setting

We assume a corporation with a controlling shareholder who holds over 50% of the shares. This control position could be achieved through different alternatives. For instance: the controlling owner never sold more than 50% of the shares to the market; or, the controlling owner bought several blocks from different institutional investors outside of the market; or, the controlling owner just completed an earlier stage of a partial tender offer. Whatever alternative placed the controlling owner in her position is irrelevant to our analysis. We are concerned with the stage in which there is *only* one person -- the controlling shareholder -- who could freezeout the minority shareholders. In this stage there is no other party that can launch a hostile takeover and get control of the corporation, nor there is any third party that can *force* the minority shareholders to sell him their shares on his terms. Otherwise, absent such level of control, possible *bidding competition* or the need to achieve *shareholders consent* to tender their shares reduces the risk of minority shareholders' expropriation. In other words, we are concerned with the stage in which the controlling shareholder is the holder of *the sole and uncontestable option to freezeout*. We focus on this stage because this is the case that presents the greatest risk of expropriation for minority shareholders. This is the essence of the freezeout option.

The controlling shareholder has an informational advantage due to private information. We consider only the ability to foresee ahead of the market the ongoing sequence of events as informational advantage, while foreknowledge of private future business plans or contemplated transactions by the controlling shareholder is considered as entrepreneurship and not an informational advantage. In other words, the informational advantage relates to the *current value* of the company, and not to contemplated business activities that will create *future value.*[14] Defining the informational advantage in this way, allows us to focus on the claim that due to the freezeout option share prices are discounted relative to the *current value* of the firm in the absence of a freezeout. Since the informational advantage is only related to the pre-merger current value, the valuation process does not involve any part of future value. [15]

Given the freezeout option, on any given day (t_0) the controlling shareholder can decide to affect a freezeout and buy the minority shares. The exercise price of the option is the market price on the decision date, t_0. That is, we assume here that the court will use the pre-freezeout price as the measure of the fair price.[16] Once the decision to exercise the option is made, the freezeout is completed immediately after the decision (t_1). Of course, in reality it takes much longer, normally several months, to complete a freezeout. However, since any information revealed during that period loses its value as private information, the only valuable informational advantage is a foreknowledge regarding information that will be revealed after the completion of the freezeout. Thus, we are considering only the informational advantage that extends beyond the time the freezeout is completed.

We further assume that the market price of the shares is determined according to the set of available public information, and information as to current value is being disclosed to

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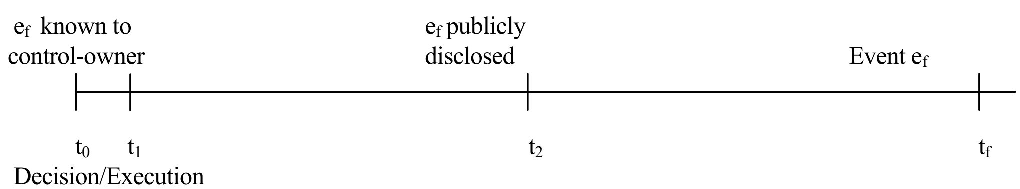
[14] For example, assume that the company has a major R&D project. On a given day, the market estimates, based on public information, that there is a 50% probability of success. On that same day, the controlling shareholder received private information according to which the probability of success is 99%. This informational advantage relates to the current value of the company and as such is an informational advantage. On the other hand, if the controlling shareholder contemplates starting a new R&D project after completing the freezeout, this information relates to future value that we consider as entrepreneurship and not informational advantage.

[15] As well, we avoid the debate over whether minority shareholders are entitled to receive part of the gain that the transaction (e.g., the merger) created when determining fair value.

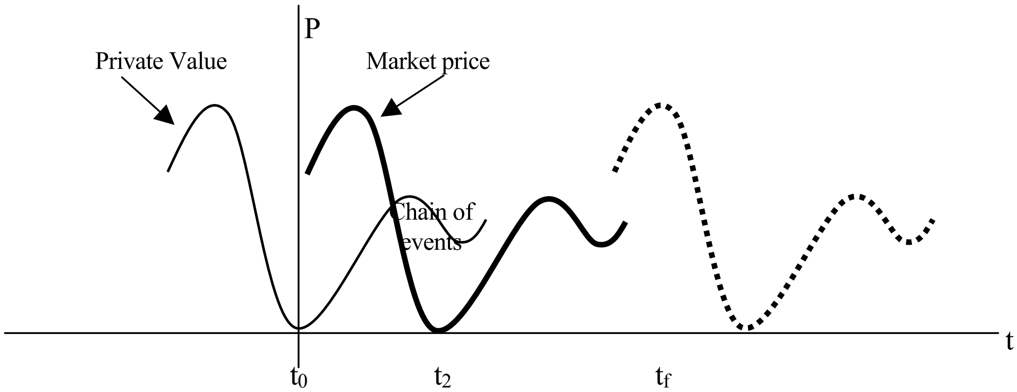
[16] In practice courts do not use the pre-freezeout price as the sole measure of fair value, and, as well, rely on other non-market measures such as accounting information and private appraisals by experts. However, as we are trying to find the effect of a rule according to which courts will be using the pre-freezeout price as the sole measure of fair value, adopting this assumption is useful for illustrating both the effect of doing so and the effect of using non-market measures to determine fair value.

the market on a continuous basis following the chain of changing business events. However, the information about any future event (e_f) to take place at time t_f, is known to the controlling shareholder at time t_0, while it is being disclosed to the market only at time t_2. Thus, while the market price on t_0 is determined based only on public information, the controlling shareholder's private value will depend, as well, on her private information regarding the events at time t_f. At time t_2 the information regarding the future events (e_f) is publicly disclosed and the market price equals the controlling shareholder's private value at time t_0. Nonetheless, at time t_2 the controlling shareholder will already have a different private value.

Due to the assumed short time between t_0 and t_1 no additional information, either public or private, influences the market price or the private value. As there is no change either in the market price or in the private value, for the purposes of our model, we consider t_0 to be equal to t_1—that is, the decision to exercise the option and the execution are done simultaneously. The time sequence will look as follow:



The relationship between the chain of future events, the market price, and the controlling shareholder's private value will look as follow:



The relationship between the chain of future events,
the market price, and the controlling shareholder's private value

The market price is delayed relative to the controlling shareholder's private value. The extent by which the market price will be shifted relative to the controlling shareholder's private value, will depend on the period between t_0 and t_2. That is, the time it takes to the private information to become public. This is the period that represents the informational advantage of the controlling shareholder -- how far to the future can she see ahead of the market. In the following section we assume this period to be one day, and later on we assume this period to change from one day to one hundred days. In both cases, however, we refer to the *informational advantage that extends beyond the time it takes to complete the freezeout.* Given her informational advantage at time t_0, the controlling shareholder decides whether or not to exercise the freezeout option, based on the difference between the market price and her private value.[17] As we show next, the decision is a consequence of the value of the freezeout option.

[17] Of course, exercising the option can be done due to reasons other than capturing the value of the option. For instance, the controlling shareholder might be interested in engaging in a new project that she does not want to share with minority shareholders. *See,* Hermalin & Schwartz, *supra* note 12. However, as we are trying to explore the potential for expropriation of minority shareholders due to the mere existence of the freezeout option and the use of the pre-freezeout price as the sole measure of fair value, we examine the pure case of exercising the option for the purpose of shifting value from minority shareholders in the absence of any other synergies.

2. Pricing A One Day Option

Let's assume that markets are efficient, and posit that the pre-merger market price is the fair price. The freezeout option is, then, a call option for an indefinite time to buy a share at current market price (t_0). If the controlling shareholder has no information about the future price, and the demand curve for the shares is perfectly elastic, this option is valueless as the controlling shareholder can buy the shares on the market for the same price without this option.

But, if the controlling shareholder has private information about the future price of the share, the option will be valuable. To illustrate the core idea of our model, lets assume the option holder can see one day ahead of the market, and can complete the freezeout immediately. In this case, she has a call option to buy a share at today's market price (t_0), while only she knows tomorrow's market price (t_2). This informational advantage provides the option-holder the benefit of foresight; she knows two values, today's market price and tomorrow's market price (her private value), before she decides whether or not to exercise the option.

The option-holder, however, will not exercise the option the first time it gets into the money, i.e., as soon as tomorrow's market price is greater than today's market price. Rather, since the option is indefinite but can be used only once, the option-holder will wait for the time when the difference in expected value between the price today and the price tomorrow is large enough to maximize her profit. In other words, the option-holder will attempt to capture the extreme changes in value, *i.e.*, the highest range of the expected probabilities of the firm's values. Of course, because the extreme-expected-values will be captured by the option the share's price will endure a drop equal to the value of the option. The share and the option, together, reflect the whole range of expected probabilities of values for the corporation. Thus, the value of the option is subtracted from the potential value of the share without the freezeout option.

We will start the pricing of the freezeout option with a simple model.[18] First, we assume that the stock price follows a random process with changes in the price distributed normally over a short time horizon. In fact, this assumption is similar to an arithmetic Brownian motion, i.e. "random walk" (and not geometric as in Black-Scholes), which is a reasonable assumption for a short time-horizon.[19]

Second, it should be noted that since there is no explicit time dependence, the option's price does not depend on calendar time.[20] Third, the option is homogeneous in price (there is no fixed strike), so its price is linear in the stock price.[21] We can always normalize the stock price at $1. The price of the option is a function of the distribution of price changes, interest rates, time interval (one day in the current settings) and is proportional to the stock price.

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[18] The freezeout option is equal to a perpetual call option to buy a share today for an exercise price equal to yesterday's market price. In other words, it is possible to reverse the time line: the option holder can exercise the option at time t_2 for an exercise price equal to the market price at t_0. The freezeout option is equal to this option due to the continuous nature of time: today's and tomorrow's market prices will soon become yesterday's and today's market prices, respectively. Here, too, the option-holder has the benefit of hindsight. The option-holder knows two values -- yesterday's (t_0) market price and today's (t_2) market price -- before she decides whether or not to exercise the option. Here, too, the option-holder will attempt to capture the largest expected difference between today's price and yesterday's price. In short, the two options are equal since both options enable the holder to capture the same price difference between t_0 and t_2 while knowing both values.

[19] The standard Black & Scholes assumptions view return on a stock as normally distributed, while we have assumed that the absolute changes in the stock price are normally distributed. We have also developed a model with lognormal distribution (normally distributed returns). For short time horizons (between one and one hundred days) the numerical results are similar.

[20] The option is perpetual and its value depends on the market price of the stock. It cannot explicitly depend on time. If at two differing moments the stock prices are the same, then the option value must be the same as well.

[21] This means that if the stock price grows twice and all other relevant variables remain the same, then the option's price will also grow twice. For example, if we will measure the stock price in cents, instead of dollars, then the option's price will be 100 times more (but expressed in cents). Another example can be a split. Note that for regular options one need to increase both stock price and strike price in order to get linearity, in our case everything is governed by the stock price only.

Denote the value of the option by v, which is then equal to the expected payoff relative to the risk-neutral probability measure.[22] The payoff is defined by the realized price change (the price difference between t_0 and t_2). The optimal exercise strategy is to exercise the option as soon as the price change is greater than the cash value of the option but to keep it alive otherwise.

Denote the risk neutral probability density distribution of the price change x by $\rho(x)$. Two events are possible: either the price jump is above v (then the option will be exercised) or the price jump is below v and the option is then worth more alive than dead:

$$ve^{r\tau} = v\int_{-\infty}^{v} \rho(x)dx + \int_{v}^{+\infty} x\rho(x)dx$$

Denote the yearly drift and volatility of the stock price by μ and σ.[23] The corresponding drift and volatility of the price changes in time τ (τ =1 day = 1year /365) is then $\mu\tau$ and $\sigma\sqrt{\tau}$. Since we assume the price changes to be distributed normally, we can use the standard cumulative normal:[24]

$$\int_{-\infty}^{v} \rho(x)dx = N\left(\frac{v-\mu\tau}{\sigma\sqrt{\tau}}\right)$$

And correspondingly:

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[22] If there is no arbitrage and no transactions costs, then there exist risk-neutral probabilities that guarantee that the price is equal to the discounted expected payoff under these probabilities. *See,* Michael J. Harrison & David M. Kreps, *Martingales And Arbitrage in Multiperiod Securities Markets,* 20 J.Econ.Theory 381 (1979); Michael J. Harrison & S. R. Pliska, *Martingales and Stochastic Integrals in the Theory of Contingent Trading,* in STOCHASTIC PROCESSES AND THEIR APPLICATIONS, V.11, 261 (1981).

[23] The drift μ is the expected annual return (increase of the price), and volatility σ is the standard deviation of this (uncertain) future price.

[24] N(x) is the probability of a normally distributed random variable to have value equal or below x, for example N(0)=0.5.

$$\int_{v}^{+\infty} x\rho(x)dx = \mu\tau\left(1 - N\left(\frac{v-\mu\tau}{\sigma\sqrt{\tau}}\right)\right) + \frac{\sigma\sqrt{\tau}}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{v-\mu\tau}{\sigma\sqrt{\tau}}\right)^2}$$

The option pricing equation becomes:

$$ve^{r\tau} = \mu\tau + (v-\mu\tau)N\left(\frac{v-\mu\tau}{\sigma\sqrt{\tau}}\right) + \frac{\sigma\sqrt{\tau}}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{v-\mu\tau}{\sigma\sqrt{\tau}}\right)^2}$$

Its solution is not analytic, but can be easily found numerically.

The probability of the exercise is:

$$\int_{v}^{+\infty} \rho(x)dx = 1 - N\left(\frac{v-\mu\tau}{\sigma\sqrt{\tau}}\right)$$

The numerical results are presented below.[25] As a starting point we choose r = 10%, σ = 40%, μ = 15%. The price of this option is then \$0.059. The probability of an exercise tomorrow is 0.0026.

We plot below several graphs that show how the price of this option changes with changes in the parameters (interest rate,[26] drift and volatility).

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[25] Calculated with *Mathematica*.

[26] Note that a typical European call option increases in value when interest rates increase. However the freezeout option is perpetual and does not have a fixed expiration date. At any time there is a probability that the option will be exercised during the next period. As empirical data and our model show the expected time to exercise is typically long – of the order of several years. Thus the freezeout option depends on the interest rates almost as a bond with the expected payment to occur in a few years. This leads to a decline in the price of the freezeout option as interest rates increases in a way similar to a long bond.

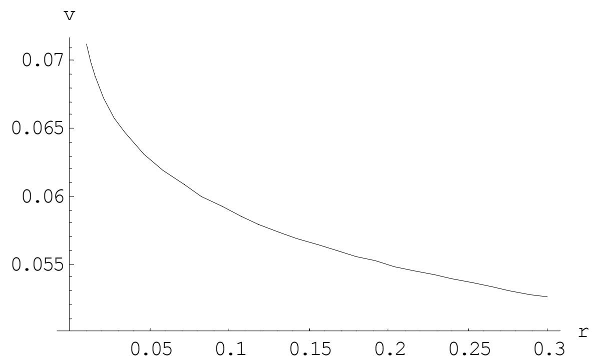

Figure 1.
Value of a 1 day option when interest
rates change from 1% to 30%, μ=15%, σ=40%.

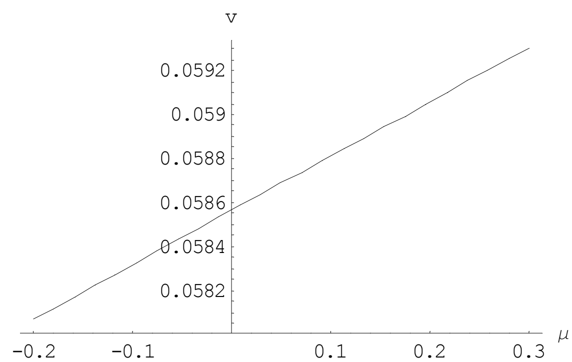

Figure 2.
Value of a 1 day option when drift μ
changes from -20% to 30%, r=10%, σ=40%.

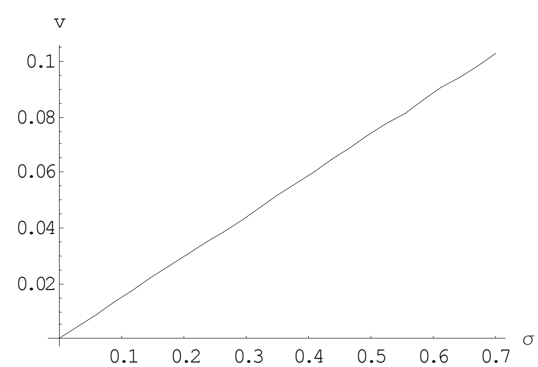

Figure 3.
Value of a 1 day option when volatility σ
changes from 0% to 70%, μ=15%, r=10%.

3. Pricing A Multi-day Option

The results suggest that the value of the option is not as high as might be expected. However, we priced an option for a single day, while the controlling shareholder might have a greater information advantage. Whether the option holder has a wider range of values to choose from (*i.e.*, greater information advantage) does make a difference. The option will have greater value if the option-holder can look to the future and see the share's prices for several days in advance before deciding whether or not to exercise. This foresight will increase the option-holder's ability to capture the largest difference between today's price and future prices. Thus, the longer the horizon (t_0 to t_2), the greater the value of the option. Using the same method, we price an option that allows the option holder to exercise the option, while looking forward for up to 100 days (beyond the time it takes to complete the freezeout). Each day the option holder can exercise the option for a price equal to today's stock price (t_0) while knowing the prices 100 days ahead of the market (t_2).[27]

Furthermore, above we priced the option assuming that the option holder knows, every day with certainty, the whole range of future prices within her horizon (t_0 to t_2). In reality it seems more plausible that the controlling shareholder will have private information about the future price for only part of the time. Therefore, we add a probability factor q to our pricing, which reflects the probability with which the option holder knows the future prices within her horizon. It is clear that the higher the probability, the more valuable the option.

The freezeout option allows its owner to buy the stock at the price registered today (t_0) while knowing the future price some time (τ) ahead of the market, but it is active with

[27] As explained earlier, *supra* note 18, this option is equal to an option that allows the option holder to exercise the option, while looking backward for up to 100 days.

some probability (q) and inactive with probability (1-q). The idea is that the controlling shareholder has important information about future price changes (e_f) with some probability and can then buy shares back from other stockholders at the current price. We assume that there is zero correlation between the information and the jump size. This is a useful assumption, even though in many cases the controlling shareholder will be informed of significant events first. However, as long as q is a parameter, this can be incorporated in a risk-neutral version of q (in other words q is an adjusted probability).

In addition, we assume here a European type option.[28] If the controlling shareholder has the information for a forthcoming period of length τ, but is prohibited from freezingout and then immediately reselling the stock, then the only information that matters is the information for the longest time horizon. In other words, if the private information predicts a price increase followed by a decline, the freezeout option should not be exercised.

Given the above assumptions, there are two possible events: either the price jump is above v and, with probability q, the option will be exercised, or the price change is not big enough to exercise the option. Since ρ is the risk-neutral probability measure, we can equate the future value of the current price with the expected payoff:

$$ve^{r\tau} = v\int_{-\infty}^{v} \rho(x)dx + q\int_{v}^{+\infty} x\rho(x)dx + (1-q)v\int_{v}^{+\infty} \rho(x)dx .$$

The option pricing equation becomes:

$$ve^{r\tau} = vN\left(\frac{v-\mu\tau}{\sigma\sqrt{\tau}}\right) + \left((1-q)v + q\mu\tau\right)\left(1 - N\left(\frac{v-\mu\tau}{\sigma\sqrt{\tau}}\right)\right) + q\frac{\sigma\sqrt{\tau}}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{v-\mu\tau}{\sigma\sqrt{\tau}}\right)^2}$$

Or:

[28] This means that the option can be exercised at maturity only. In fact the freezeout option can be exercised at

$$ve^{r\tau} = vN\left(\frac{v-\mu\tau}{\sigma\sqrt{\tau}}\right) + \left((1-q)v + q\mu\tau\right)N\left(\frac{-v+\mu\tau}{\sigma\sqrt{\tau}}\right) + q\frac{\sigma\sqrt{\tau}}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{v-\mu\tau}{\sigma\sqrt{\tau}}\right)^2}.$$

Its solution is not analytic, but can be easily found numerically. The probability of the exercise is: $q\int_v^{+\infty}\rho(x)dx = q\left(1-N\left(\frac{v-\mu\tau}{\sigma\sqrt{\tau}}\right)\right) = qN\left(\frac{-v+\mu\tau}{\sigma\sqrt{\tau}}\right)$.

We plot below several graphs that show how the price of this option changes when the parameters (interest rate, drift and volatility) change.

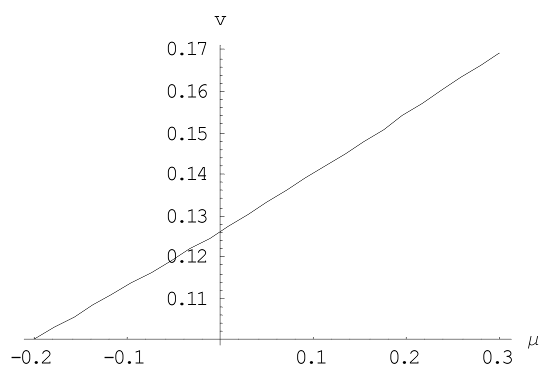

Figure 4.
Value of a 100 days option when drift μ changes from -20% to 30%, r=10%, σ=40%, q=0.1.

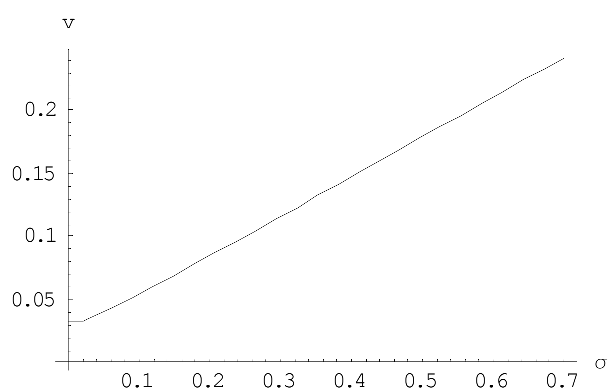

Figure 5.
Value of a 100 days option when volatility σ changes from 0% to 70%, μ=15%, r=10%, q=0.1.

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any time, but this is captured by the perpetual nature of the option.

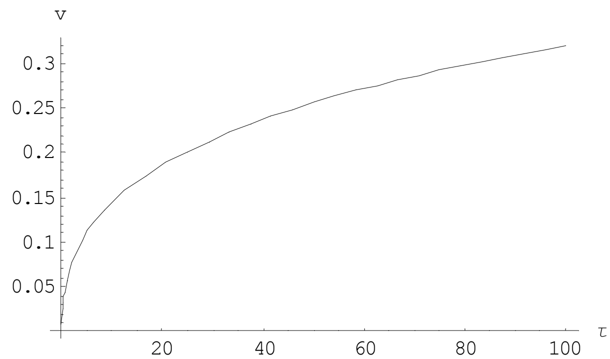

Figure 6.
Value of a multi-day option when the time horizon t changes from 1 to 100 days, μ=15%, σ=40%, r=10%, q=1.

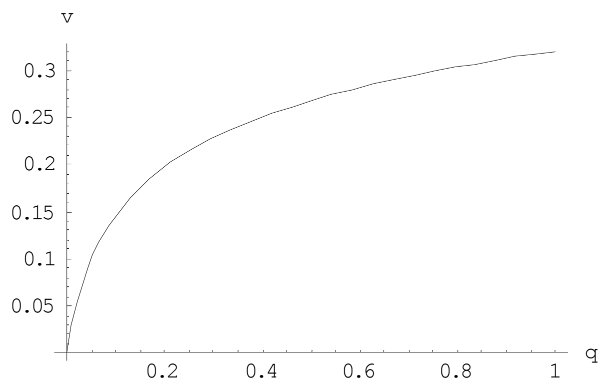

Figure 7.
Value of a 100 days option when the probability q changes between 0 and 100%, μ=15%, σ=40%, r=10%.

According to this model the price of a "one day freezeout option" (one day *beyond the several months it takes to complete the freezeout)* ranges from 4% to 6% of the stock price. This result is robust relative to all major parameters of the model as demonstrated in the figures above. Once the information advantage is increased beyond a few days, the value of the "multi-days freezeout option" becomes substantial: with two months information advantage, *beyond the several months it takes to complete the freezeout,* the option value is above 15% of the stock price.

As stated earlier, this basic model uses an exogenous stock price that avoids the under-valuation risk generated by the "lemons effect". Next, to complete the analysis, we present a more general model that will allow us to price the freezeout option based on either exogenous share prices, endogenous share prices, or a mix thereof.

II. An Extended General Model

We consider a company that distributes all profits and losses (P&L) to shareholders. The price of the whole company does not change and we set it equal to \$1. In addition we assume that the profits and losses over a short time horizon are distributed normally (i.e., follow the standard Arithmetic Brownian Motion). The longer the time horizon, the greater the mean of P&L and the greater the standard deviation of the distribution. For instance, a daily expected profit of 0.06% will translate into a yearly expected profit of 15%, and a daily volatility of 2.5% can be translated into a yearly volatility of 40% (drift is linear in time and volatility is proportional to the square root of time). Denote by μ the infinitesimal drift mean and by σ the infinitesimal standard deviation of the cash payoff. This means that over a time interval τ the cash flow is distributed $N\left(\mu\tau, \sigma\sqrt{\tau}\right)$, normally with a mean of $\tau\mu$, and a standard deviation of $\sigma\sqrt{\tau}$. The downside of this assumption is that we ignore the limited liability principle, since an outcome of a normal variable can be a large negative number (exceeding \$1 by absolute value). In a realistic world this situation should lead to bankruptcy but we ignore this possibility, because under reasonable assumptions ($r=10\%$, $\sigma=40\%$ and $\tau=2$ months), the probability of this event is less than $2.4 \cdot 10^{-10}$. For a shorter time interval the probability is even smaller.

Assume that there are two countries with two identical firms, but with different legal systems. The companies are of the same size, in the same industry, and have the same clients and risk characteristics. If they were owned by a single owner then there should not be any difference between them.

In the first country there are no freezeouts. There are two equivalent shares of the company each share is worth \$0.5. The future profits and losses are unknown at any moment. The profits and losses are random variables with some distribution, and we assume that this distribution, under risk-neutral probabilities, over a short horizon is $N\left(\mu\tau, \sigma\sqrt{\tau}\right)$.

The current value of the whole company -- \$1 -- must be equal to the discounted expected future payoff: $1 = (1 + \mu\tau)\, e^{-r\tau}$, this can be written also as $\mu\tau = e^{r\tau} - 1$. Since μ is the expected profit in the risk-neutral world, there is no risk premium and the discounting is by the risk free rate r.

In the second country there are also two shares but of different types, A and B. Denote their prices correspondingly by a and b. Together A and B constitute the whole company, thus $a+b=1$ at any moment, since the company distributes all its profits (and losses) immediately. Denote this uncertain profit/loss by z. In the second country the owner of the share A can force the owner of share B to sell him the share B for a price K that will be defined later. Due to the no-arbitrage assumption the $\mu\tau = e^{r\tau} - 1$ relation is again satisfied.

Using the standard pricing technique we can write for the A share:

$$ae^{r\tau} = \int_{-\infty}^{z^*}\left(a + \frac{z}{2}\right)\rho(z)dz + \int_{z^*}^{+\infty}(a + b - K + z)\rho(z)dz \ ,$$

and for the B share:

$$be^{r\tau} = \int_{-\infty}^{z^*}\left(b + \frac{z}{2}\right)\rho(z)dz + \int_{z^*}^{+\infty}K\rho(z)dz \ .$$

These two equations mean that the owner of each share calculates the expected payoff and compares it with the discounted current price.[29] For the owner of the *A* share the future payoff consists of the following scenarios. If the random variable *z* (company's profit) is small, she keeps her share *a* and receives exactly one half of the profits (losses). Otherwise, if the profits are high enough (above *z**) it is better to exercise the freezeout option and she will force the owner of the *B* share to sell her the *B* share, thereby receiving all the profits (*z*) and both shares (*a*+*b*), but paying the strike price *K*. Similarly the owner of the *B* share has the share and half of the profits in all cases when *z* is below *z**, otherwise he loses his *B* share and his portion of the profits, but is paid the amount *K*. The integration sign shows that each event is taken with its probability and then summed up.

The optimal exercise decision is determined by the value of *z**. We can view the share *A* as a combination of a simple share (like in the first country) and a call option on the *B* share. Since the freezeout option held by the owner of share *A* creates a possibility that part of the future profits will shift from *B* to *A*, the share *A* has a price $a > 0.5$ (the price in the state without freezeout) and the price of share *B* is $b < 0.5$.

The owner of *A* makes her decision based on a comparison of an option alive and dead. The equation she arrives at is:

$$a + \frac{z^*}{2} = a + b - K + z^*$$

This equation means that if she decides not to use the option she gets her share *a* and half of profits. If she exercises the option she receives both shares and all profits but must pay *K*. At the optimal exercise point she would be indifferent between these two possibilities. The optimal exercise price is: $z^* = 2(K - b)$. Using this result, we next consider the following pricing cases:

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[29] The discount factor $e^{r\tau}$ is moved to the left side of the equation.

the first with an exogenous stock price, the second with an endogenous stock price, and the third a mix of both.

1. Pricing with An Exogenous Stock Price

In the case that an exogenous stock price is used by courts for pricing, the strike price K is set based on the share price in the first country (without freezeout). For simplicity we set it equal to the price in the no freezeout state $K=1/2$. The equations become:[30]

$$\begin{cases} ae^{r\tau} = \int_{-\infty}^{z^*} \left(a + \frac{z}{2} \right) \rho(z) dz + \int_{z^*}^{+\infty} \left(1 - \frac{1}{2} + z \right) \rho(z) dz \\ be^{r\tau} = \int_{-\infty}^{z^*} \left(b + \frac{z}{2} \right) \rho(z) dz + \frac{1}{2} \int_{z^*}^{+\infty} \rho(z) dz \end{cases}$$

A numerical solution of this system is straightforward. We provide some numerical results for different horizons τ (this is the time period – in yearly terms -- that the owner of share A can see into the future):

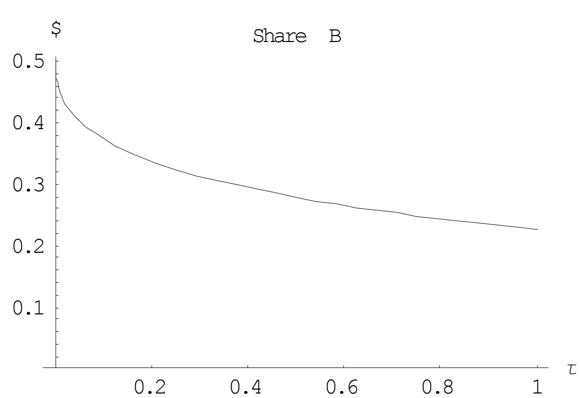

Figure 8.

[30] The implicit solution is developed in the appendix.

Price of the *B* share for *K=0.5* as a function of τ (in years)
when *r=10%* and σ*=40%.*

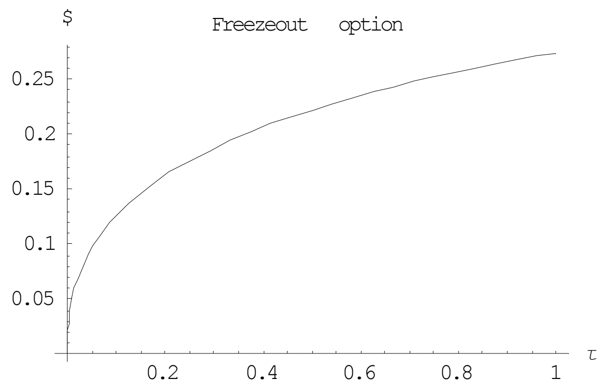

Figure 9.
The value of the freezeout option for *K=0.5* as a function of τ (in years)
when *r=10%* and σ*=40%.*

When τ tends to zero the value of the *B* share approaches the no-freezeout price $0.5 and the value of the option goes to 0. The results are similar to the results of the basic model.[31]

2. Pricing with An Endogenous Stock Price

In the case that an endogenous stock price is used by courts for pricing, the strike price *K* is set based on the price paid for this share (of type *B* in the second country). We consider first the case: *K=b*. We show below that in this case there is no equilibrium price. Set *K=b* and consider the equation for the share *B*. First note that under this condition *z*=0*. Then the pricing equation becomes:

$$be^{r\tau} = \int_{-\infty}^{0} \left(b + \frac{z}{2} \right) \rho(z) dz + b \int_{0}^{+\infty} \rho(z) dz \,,$$

or, it can be written as:

[31] The value of the option in the extended model is about half of its value in the basic model. This is a technical difference due to the fact that the price of the share in the basic model is normalized to one dollar while the B share in the extended model is about half a dollar.

$$be^{r\tau} = b + \int_{-\infty}^{0} \frac{z}{2} \rho(z)dz .$$

Note that the left hand side is strictly bigger than *b* since $\tau > 0$, and the right hand side is strictly less than *b* since the first term is *b* and the second term contains only losses ($z<0$ since the upper limit of integration is 0). Thus, there is no solution and there is no positive equilibrium price for this security.

Intuitively it is clear that the owner of share *A* will exercise it immediately when the company has a profit ($z^*=0$). Whoever will buy the share *B* will either suffer a loss or will have to give up his share in exchange for the price he has originally paid. Nobody will invest a positive amount of money in such a share.

3. Pricing with a Mix of Endogenous and Exogenous Market Prices

To make the model more realistic we consider an extension that allows a mix between the two approaches. In this case, the court can use either an exogenous market price (the value is taken from the no-freezeout-country) or an endogenous market price (the value is taken from the same market where the freezeout option is exercised). The court can use either method with certainty, with fixed probability, or randomize with different probabilities. The appropriate system of equations takes the following form:

$$\begin{cases} ae^{r\tau} = \int_{-\infty}^{z^*} \left(a + \frac{z}{2} \right) \rho(z)dz + \int_{z^*}^{+\infty} \left(1 - \left(0.5\lambda + (1-\lambda)b\right) + z\right) \rho(z)dz \\ be^{r\tau} = \int_{-\infty}^{z^*} \left(b + \frac{z}{2} \right) \rho(z)dz + \left(0.5\lambda + (1-\lambda)b\right) \int_{z^*}^{+\infty} \rho(z)dz \end{cases}$$

Here we use a mixed payoff $0.5\lambda + (1\text{-}\lambda)b$. Where some times decisions are based on the first model, sometimes on the second. The new parameter λ represents the probability that the payment will be 0.5 (*i.e.* an exogenous undiscounted market price) while (1-λ) represents the probability that the payment will equal the market price *b* (*i.e.* an endogenous discounted market price). An alternative explanation can be that the decisions are certain but always give some weight λ to the exogenous price and weight (1-λ) to the endogenous price, as in a block method valuation. In addition we assume that the owner of share A has the relevant information (about future performance) with some probability q. Then the equations are:

$$\begin{cases} ae^{r\tau} = \int_{-\infty}^{z^*}\left(a+\frac{z}{2}\right)\rho(z)dz + q\int_{z^*}^{+\infty}\left(1-(0.5\lambda+(1-\lambda)b)+z\right)\rho(z)dz + (1-q)\int_{z^*}^{+\infty}\left(a+\frac{z}{2}\right)\rho(z)dz \\ be^{r\tau} = \int_{-\infty}^{z^*}\left(b+\frac{z}{2}\right)\rho(z)dz + q(0.5\lambda+(1-\lambda)b)\int_{z^*}^{+\infty}\rho(z)dz + (1-q)\int_{z^*}^{+\infty}\left(b+\frac{z}{2}\right)\rho(z)dz \end{cases}$$

In this case, the relevant information is available with probability q, which reduces the value of the freezeout option. The example below shows the value of the share B, when *r=10%,* σ*=40%,* λ*=0.4, q=50%* and τ varies:

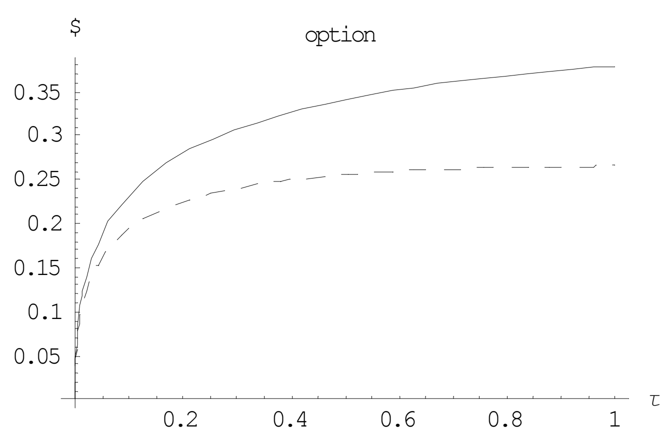

Figure 10.
The value of the freezeout option for *K=0.4(0.5)+0.6(b)* (i.e., with probability λ*=40%* the price will be set as 0.5 and with probability 1-λ=60% it will be set equal to b) as a function of τ (in years) when *r=10%,* and σ*=40%* the solid line is for q=1 and the dashed one for q=0.5.

As the information advantage increases - either in the probability q of knowing the future or in the length of time looking into the future -- the option becomes more valuable. Similarly, the greater the probability that the court will use an endogenous market price, or the greater the weight the court will give to an endogenous market price, the greater the value of the option. Thus, a random infrequent use of market prices as indication of fair value will not result in a large discount due to the lemons effect. But, relative to the degree of the information advantage of the controlling shareholder the minority shares will still be discounted. Moreover, every partial, but systematic, use of market prices will result in partial discount *in all traded shares of all companies that are subject to freezeout* relative to the weight given to market prices by the courts. Similarly, *consistently* assigning a given weight to market prices in every valuation will result in greater discount *in all traded shares that are subject to freezeout,* compared with the case of *randomizing* the use of market prices in few of the cases.

Combining both results it can be said that, indeed, in an efficient market, prices do reflect fair value, but this is so *because courts do not heavily and consistently base their valuations on market prices.* In other words, the more courts will insert market prices into their valuations in a more *consistent* and *predictable* manner, the more discounted the market price will be, and vice versa. Indeed, ironically, the less reliance courts put on market prices, the more accurate are market prices as a reflection of fair value. But, if courts will increase their use of market prices in their valuations, the quality of market prices as indication of fair value will deteriorate.

III. The Model's Implications

Given the result of our option pricing, the debate over the value of the freezeout option becomes a debate over which assumptions best reflect reality. If one assumes that courts are using the equivalent of an exogenous market price, and that the controlling shareholder has a short horizon of future prices and low probability of knowing them, then the freezeout option has very little value (about 4% of the share price). Indeed, given the length of time it takes to complete a freezeout and the uncertainty associated with such a transaction, it is unlikely that anyone will even try to capture a short horizon informational advantage.[32] Moreover, given that in freezeout mergers the normal premium paid is substantially greater than the value of the option, the source of this premium is not the exploitation of the minority through the freezeout mechanism.[33] Rather, the premium could come from a new project which the minority has no right to claim[34] or some other source of efficiency.[35] Therefore, the use of publicly known information, *other than market price*, would not distort the fair value determination. Moreover, such a low value of the freezeout option will not justify, from the controlling shareholder point of view, entering inefficient investments or investing in information as to future prices.

However, if one assumes that the controlling shareholder has a long horizon of future prices (extending long after the completion of the freezeout), which she can foresee with a high probability, the freezeout option is very valuable (can reach more than 25% of the share price) *even when courts are not using market prices in their valuations*. Given that in some freezeout mergers the premium is lower than the value of the freezeout option, this might reflect an

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[32] *See,* Hermalin and Schwartz, *supra* note 12 (arguing that firms will be unable to exploit temporary undervaluations).

[33] *See, e.g.,* Harry DeAngelo, Linda DeAngelo & Edward M. Rice, *Going Private: Minority Freezeouts and Shareholder Wealth*, 27 J. Law & Econ. 367 (1984) (average premium paid to minority shareholders is 56%).

[34] *See,* Hermalin and Schwartz, *supra note 12.*

[35] *See, e.g.,* DeAngelo, DeAngelo & Rice, *supra note 33* (eliminating the costs attendant to the regulation of public ownership); Eli Ofek, *Efficiency Gains in Unsuccessful Management Buyouts,* 49 J. Finance 637 (1994) (reducing agency costs); Frank H. Easterbrook & Daniel R. Fischel, *Corporate Control Transactions*, 93 Yale L.J. 698 (1982) (listing several efficiency gains).

exploitation of the minority through the use of the freezeout mechanism.

It can be argued that the assumptions should differ relative to the specific corporation. In some industries it is more plausible that the controlling shareholder will have longer horizons of future prices, or greater probability of knowing them, than in other industries. For instance, in a corporation operating an established supermarket chain, it is reasonable that the controlling shareholder will not have an advantage over market analysts in foreseeing future prices. On the other hand, in the high-tech industry it seems more likely that the controlling shareholder will have greater probability of foreseeing future prices than market analysts.

Nonetheless, it seems unlikely that the controlling shareholder will have an advantage over market analysts in most cases. In a mature corporation, for example, there is a greater probability of both foreseeing future prices and foreseeing them for a long horizon, but this effect works for outsiders as well as for insiders. Thus, it is easier for market analysts to erode the insider's advantage by pricing the information into today's price. On the other hand, in the high-tech industry, the controlling shareholder has an advantage regarding the probability of knowing future prices, but, given the nature of the industry, her horizon is limited. As well, the advantage of the controlling shareholder is limited by the need to foresee future prices beyond the time it takes to complete the freezeout. Given that normally it takes several months to complete a freezeout, it is unlikely that the controlling shareholder will have the ability to foresee future prices for several months beyond the freezeout completion date. This suggests that if *courts' will place limited reliance on market prices* the freezeout options will not be very valuable. Therefore, as a policy recommendation the courts should limit their reliance on market prices to random and infrequent occasions.

IV. Empirical Support

According to our model, the value of the freezeout option is low even when its price is based on publicly known information (excluding market prices). If courts are not putting much reliance on market prices, it should not be expected that freezeout options would lead to minority shareholders' exploitation or to inefficiency in corporate and controlling owners' decisions. Indeed, the empirical findings suggest both that courts are not consistently and heavily relying on market prices -- thereby avoiding the lemons effect -- and that as far as the informational advantage is concerned the option value is low.

One way to empirically test the value of the freezeout option may be drawn from a comparison between two kinds of corporations: a majority-owned firm and a diffusely held firm. The freezeout option is viable only in a majority-owned firm, and the market will reflect its value by discounting the stock price. On the other hand, in a diffusely held firm, the freezeout option does not exist. However, there is a probability that a diffusely held firm will transform into a majority-owned firm -- e.g., through stock accumulation or through a tender to the majority of the stocks -- and the freezeout option will be born. The market will discount the stock price to reflect this probability.

It is reasonable to assume that the discount applied to a majority-owned firm -- in which the existence of the option is certain -- will be greater than the discount applied to a diffusely held firm -- in which there is only a probability that the option will materialized. Thus, if the option has a great value we should find that stocks in majority-owned firms are traded at a discount relative to stocks in diffusely held firms.

Stocks of majority-owned firms, however, do not trade at a discount relative to stocks of diffusely held firms, as found in several studies measuring the impact of large block ownership on firms' market-to-book ratio: the ratio of the market value of the firm to the replacement costs of its assets – a ratio known as Tobin's q. Morck, Shleifer and Vishny found that for 371 Fortune 500 firms, the market-to-book ratio increases when managerial stock holdings went from 0% to 5%, decreases between 5% and 25%, and increases above that. [36] This result suggests that the freezeout option has very little value. If the freezeout option had a great value, we would expect that as managerial holdings increase market-to-book ratio would decrease due to the increased probability that a diffusely held firm would transform into a majority-owned firm.[37] Similarly, Holderness and Sheehan found *no significant difference* in the book-to-market ratios for paired sample of majority-owned and diffusely held firms.[38]

The studies using Tobin's q are important for our purposes. Investors in a majority owned firm are aware of the freezeout option and will, ex ante, discount the price of the minority stock to a level that should provide them with a return equal to an investment in a diffusely held firm. Therefore, finding equal returns will not be indicative of the freezeout option. The use of Tobin's q avoids this problem by testing discounts relative to an accounting non-market measure – assets' replacement costs. Some of the following studies do not avoid the ex-ante-discounting problem, and their value should be assessed in light of the equal results for the two kinds of firms found in the studies using Tobin's q.

Interestingly enough, the stocks of majority-owned firms are even issued at a premium relative to diffusely held firms. Schipper and Smith, who studied the performance of equity

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[36] Randall Morck, Andrei Shleifer & Robert Vishny, *Management Ownership and Market Valuation: An Empirical Analysis*, 20 J. Fin. Econ. 293 (1988).

[37] But *see*, John MeConnell & Henri Servaes, *Additional Evidence on Equity Ownership and Corporate Value*, 27 J. Fin. Econ. 595 (1990) (for the one year studied the market-to-book ratio increased until top management owned 40% or 50% of the stock, and declined thereafter).

[38] Clifford Holderness & Dennis Sheehan, *The Role of Majority Shareholders in Publicly Held Corporations: An Exploratory Analysis,* 20 J. Fin. Econ. 317 (1988).

carve-outs announced between 1965 and 1983,[39] found that the initial percentage returns on the stock of the new subsidiaries was much lower than those observed in studies of public offerings generally.[40] That is, in newly issued stocks of a majority-owned firm the issuer could offer a lower discount on its stocks relative to public offerings generally. Although this study measured relative returns – thus susceptible to the ex-ante-discounting problem – the finding of unequal returns in the opposite direction suggests that investors do not discount shares in majority owned firms, due to the freezeout option, relative to diffusely held firms.

Furthermore, we can expect that once a freezeout merger is effected in a majority-owned firm, the premium paid for the minority stocks would be lower relative to the premium paid to shareholders in a merger of a diffusely held firm. The lower premium would be expected for both a firm that went public as a majority-owned firm and a firm that transformed from a diffusely held firm into a majority-owned firm. In fact, in both cases, the discount -- due to the existence of the freezeout option -- reflects this expected low premium. Once the majority owner has paid for the option -- either in the form of discount to newly issued minority stocks or in the form of expenses to create a majority block in a diffusely held firm -- she will wish to make use of this option and pay a lower premium in the freezeout merger.

The empirical findings, however, reveal that premiums paid for the two kinds of firms are substantially similar, supporting the result of our model that the freezeout option has a low value. The equal premium relative to diffusely held firms was found for both firms that went public as a majority-owned firm and firms that transformed from a diffusely held firm into a majority-owned firm. Klein, Rosenfeld and Beranek found that parent firms' announcements

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[39] A carved-out occurs when parent firm sells partial ownership interest in a subsidiary to the public. Usually, the parent firm retains at least half of the common stock and thus controls the carved-out subsidiary. A carved-out subsidiary is thus a firm that went public as a majority-owned firm.

[40] Katherine Schipper and Abbie Smith, *A Comparison of Equity Carved-Outs and Equity Offerings: Share*

of reacquisition of their carved-out subsidiaries are associated with positive abnormal returns for public shareholders which approximate those earned by target firms in arms-length mergers and acquisitions. Additionally, after a parent firm sell-off its interest in the carved-out subsidiary, in most cases minority shareholders are being bought out for the same price.[41] Holderness and Sheehan paired diffusely held firms and majority-owned firms, and found that minority shareholders in majority-owned firms receive approximately the same premium for their shares as shareholders in diffusely held firms.[42] DeAngelo, DeAngelo and Rice examined Management Buy-Outs – acquisitions that involve informational advantage similar to a freezeout -- and found that the returns to public shareholders were substantially the same whether the buyer had control or not.[43]

Similarly, we would expect the frequency of freezeout mergers and other reorganizations to be greater than the frequency of mergers and other control transactions in diffusely held firms. The motives for mergers and other control transactions in diffusely held firms are substantially the same as in majority-owned firms, while the latter have an additional motive -- to exercise the freezeout option when the controller receives favorable private information. Indeed, Holderness and Sheehan found that, for paired majority-owned and diffusely held firms over the seven years followed, 36% of the majority shareholders redeemed the minority's shares, while only 29% of the paired firms reorganized over the same period. Similarly, Morck, Shleifer and Vishny found that the probability of a Fortune 500 firm

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Price Effects and Corporate Restructuring, 15 J. Fin. Econ. 153 (1986).

[41] April Klein, James Rosenfeld & William Beranek, *The Two Stages of An Equity Carve-Out and the Price Response of Parent and Subsidiary Stock*, 12 Managerial and Decision Economics, 449 (1991).

[42] Clifford Holderness & Dennis Sheehan, *Constraints on Large-Block Shareholders*, National Bureau of Economic Research, p. 28 (Conference on Concentrated Ownership, June 1998).

[43] DeAngelo, DeAngelo and Rice, *supra note 33*, at 393.

being acquired between 1981 and 1985 increased with the percentage of common stock owned by its top two managers.[44]

The increase in the frequency of reorganizations in majority-owned firms can be due to either a high value of the freezeout option or decreased transaction costs. If the freezeout option has a great value, its exercise should increase the frequency of reorganization. However, even if the freezeout option has a low value, the ownership of a large block of shares makes it easier for the majority shareholder to complete a reorganization relative to a shareholder holding a small fraction of a diffusely held firm. Thus, it is hard to conclude from these studies how much of the difference in the frequency of reorganizations is associated with the value of the option.

A final important caveat should be noted. The two different governance structures have a different mixture of two types of agency problems and thus might have different levels of agency costs that should be taken into account. In a diffusely held firm there is a severe agency problem between shareholders and managers, and a negligible agency problem between majority shareholders and minority shareholders. In a majority owned firm the controlling owner minimizes the effect of the agency problems vis-à-vis the managers but simultaneously aggravate the agency problem between the majority shareholder and the minority shareholders. Indeed, the following empirical studies attempted to reveal the relative efficiency of the two different governance structures. For our purposes, however, it is unnecessary to resolve the question of the relative efficiencies of the differing structures.

Whatever the relative *initial* levels of agency costs of the two differing governance structures are, the empirical studies would still provide us with the supplementary effect of the

[44] Randall Morck, Andrei Shleifer & Robert Vishny, *Characteristics of Targets of Hostile and Friendly Takeover,* in A.J. Auerbach, ed., CORPORATE TAKEOVERS: CAUSES AND CONSEQUENCES 101 (University of Chicago Press, 1988).

option. Since the studies reveal that the two governance structures do not have substantially differing levels of efficiency, it is very indicative of the value of the freezeout option. In other words, if the level of agency costs is similar for both structures then the empirical studies suggest that *the freezeout option is not valuable enough to change that result*. If the assumption is that a majority owned firm has higher agency costs relative to a diffusely held firm, then the empirical studies suggest that such effect cannot be identified and even *the freezeout option is not valuable enough to aggravate the problem* to a discernible level. Only when the assumption is that the agency costs of majority owned firms are lower than the agency costs of diffusely held firms, the result of the empirical studies is blurred. Finding equal levels of efficiency suggests that the freezeout option might countered the difference in agency costs levels in full, but what was the initial level of agency costs is unclear. With this caveat in mind we next conclude.

V. Summary

The freezeout option allows a majority shareholder to buy-out the minority for a price determined by the majority. The law protects minority shareholder by requiring the majority to pay a “fair price” for the minority shares. Courts determine the fair value to be paid to minority shareholders based on publicly known information. However, the majority shareholder can as well exercise the option when she holds favorable private information. Thus, it is claimed that using publicly known information undervalues minority stocks. Moreover, the initial under-valuation leads to a chain reaction resulting in a substantial market price discounting – a “lemons effect” – and greater under-valuation. Consequently, inefficiency will result. This claim suggests that the freezeout option is very valuable. Others have claimed that market prices in an efficient market are not discounted due to the freezeout option, and thus market prices can be used as a measure of fair value. This claim suggests that the freezeout option has a negligible value.

We presented a model that enabled us to price the freezeout option. Our model indicates that, as far as the value of private information is concerned, the freezeout option has a low value. This result implies that the use of publicly known information, *excluding market prices*, in determining a fair value for minority stocks will not cause expropriation of minority shareholders and will not distort efficiency. However, as far as the value of the "lemons' effect" is concerned, our model show that a complete reliance on market prices will lead to inability to positively price and trade minority shares. In other words, the discount is real and severe. Any consistent and predictable use of market prices by the courts will cause a discount in the "fair value" relative to the weight given to market prices in the valuation process.

Courts, however, do not heavily and consistently rely on market prices in their valuations. Consequently, prices are more accurate and low value of the freezeout option should be expected. Indeed, stocks of majority-owned firms are not traded at a discount relative to stocks of diffusely held firms. Moreover, in reorganizations and other control transactions, the premium received by minority shareholders in majority-owned firms is similar to the premium paid to shareholders in diffusely held firms.

Appendix

$$\begin{cases} ae^{r\tau} = \int_{-\infty}^{z^*} \left(a + \frac{z}{2}\right)\rho(z)dz + \int_{z^*}^{+\infty} (a + b + z - K)\rho(z)dz \\ be^{r\tau} = \int_{-\infty}^{z^*} \left(b + \frac{z}{2}\right)\rho(z)dz + K\int_{z^*}^{+\infty} \rho(z)dz \\ a + \frac{z^*}{2} = a + b + z^* - K \end{cases}$$

Note that the last two equations can be solved regardless of the first equation. Then in order to find *a* we can use *a+b=1* relationship. Note also that due to the no arbitrage summing the first two equations we get $a+b+\mu_0\tau=e^{r\tau}$.

$$\begin{cases} be^{r\tau} = \int_{-\infty}^{z^*} \left(b + \frac{z}{2}\right)\rho(z)dz + K\int_{z^*}^{+\infty} \rho(z)dz \\ z^* = 2(K - b) \end{cases}$$

Using simple transformations this can be written as:

$$\begin{cases} be^{r\tau} = bN\left(\frac{z^* - \mu}{\sigma}\right) + \frac{1}{2}\int_{-\infty}^{z^*} z\rho(z)dz + K\left(1 - N\left(\frac{z^* - \mu}{\sigma}\right)\right) \\ z^* = 2(K - b) \end{cases}$$

Or using $\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x} ze^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2} dz = \mu N\left(\frac{x-\mu}{\sigma}\right) - \frac{\sigma}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$$be^{r\tau} = bN\left(\frac{z^* - \mu}{\sigma}\right) + \frac{1}{2}\mu N\left(\frac{z^* - \mu}{\sigma}\right) - \frac{\sigma}{2\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z^*-\mu}{\sigma}\right)^2} + K\left(1 - N\left(\frac{z^* - \mu}{\sigma}\right)\right)$$

Then using $\mu=\mu_0\tau$ and $\sigma = \sigma_0\sqrt{\tau}$ we have:

$$be^{r\tau} = bN\left(\frac{z^*-\mu_0\tau}{\sigma_0\sqrt{\tau}}\right) + \frac{1}{2}\mu_0\tau N\left(\frac{z^*-\mu}{\sigma_0\sqrt{\tau}}\right) - \frac{\sigma_0\sqrt{\tau}}{2\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{z^*-\mu_0\tau}{\sigma_0\sqrt{\tau}}\right)^2} + K\left(1-N\left(\frac{z^*-\mu_0\tau}{\sigma_0\sqrt{\tau}}\right)\right)$$

And further with $\mu_0\tau=e^{r\tau}-1$, $z^*=2(K-b)$

$$be^{r\tau} = N\left(\frac{2(K-b)-\left(e^{r\tau}-1\right)}{\sigma_0\sqrt{\tau}}\right)\left(b+\frac{\left(e^{r\tau}-1\right)}{2}\right) - \frac{\sigma_0\sqrt{\tau}}{2\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{2(K-b)-\left(e^{r\tau}-1\right)}{\sigma_0\sqrt{\tau}}\right)^2} + K\left(1-N\left(\frac{2(K-b)-\left(e^{r\tau}-1\right)}{\sigma_0\sqrt{\tau}}\right)\right)$$

General Formulas

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x} ze^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}dz = \mu N\left(\frac{x-\mu}{\sigma}\right) - \frac{\sigma}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{x}^{+\infty} ze^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}dz = \mu\left(1-N\left(\frac{x-\mu}{\sigma}\right)\right) + \frac{\sigma}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$