

# INFLATION AND THE PRICE OF REAL ASSETS\*

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## Abstract

This paper considers price and quantity movements in the three major asset classes – real estate, equity, and nominal fixed income – in the postwar period. To understand these movements, we compute a sequence of temporary equilibria in a lifecycle model with heterogeneous agents and uninsurable nominal risk. A key input to the model is the joint distribution of asset endowments and income, which we take from household level data. We show that changes in inflation expectations, together with demographic shifts and changes in asset supply, help understand the experience of the 1970s.

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# I Introduction

Both the size and composition of US household sector wealth have changed dramatically over the postwar period. Figure 1 plots aggregate household wealth in the three major asset classes – real estate, equity, and nominal instruments, all as multiples of GDP. In both the 1960s and the 1990s, household wealth was high relative to GDP. In addition, the aggregate portfolio was relatively more tilted towards equity, as shown in Figure 2. In the 1970s and early 1980s – a period notable for relatively high inflation – wealth was lower relative to GDP, and to a greater extent invested in real estate. At the same time, the household sector’s *net* position in nominal instruments – that is, nominal assets minus nominal liabilities – has been relatively stable over the whole period, at least compared to positions in equity and real estate.

This paper develops a model of the joint determination of asset prices and household wealth portfolios. The key features of the model are household heterogeneity by age and wealth and the uninsurability of nominal risk. Equilibrium prices and portfolios are linked to (i) the distribution of income and initial asset endowments, measured from the Survey of Consumer Finances, (ii) the supply of assets to the household sector, taken from the Flow of Funds Accounts, and (iii) expectations about future prices. We show that the model captures a number of stylized facts about individual portfolio choice. We then use it to assess quantitatively whether inflation could have been responsible for Figures 1 and 2, and how its effect compares with demographic shifts and movements in asset supply. Our results point to a combination of these factors as the most plausible account.

Our approach is based on two building blocks. The first is a model of lifecycle savings and portfolio choice at the individual level. Households choose between three assets: equity, real estate and nominal bonds. They experience idiosyncratic labor income risk that cannot be insured using only the three available assets. In credit markets, they face collateral constraints – all borrowing must be backed by real estate – as well as a spread between borrowing and lending rates. Moreover, lending and borrowing are required to be in nominal terms – there is no riskfree asset. Inflation risk is thus also uninsurable.

The second building block is a one-period model of asset trading. Households of different

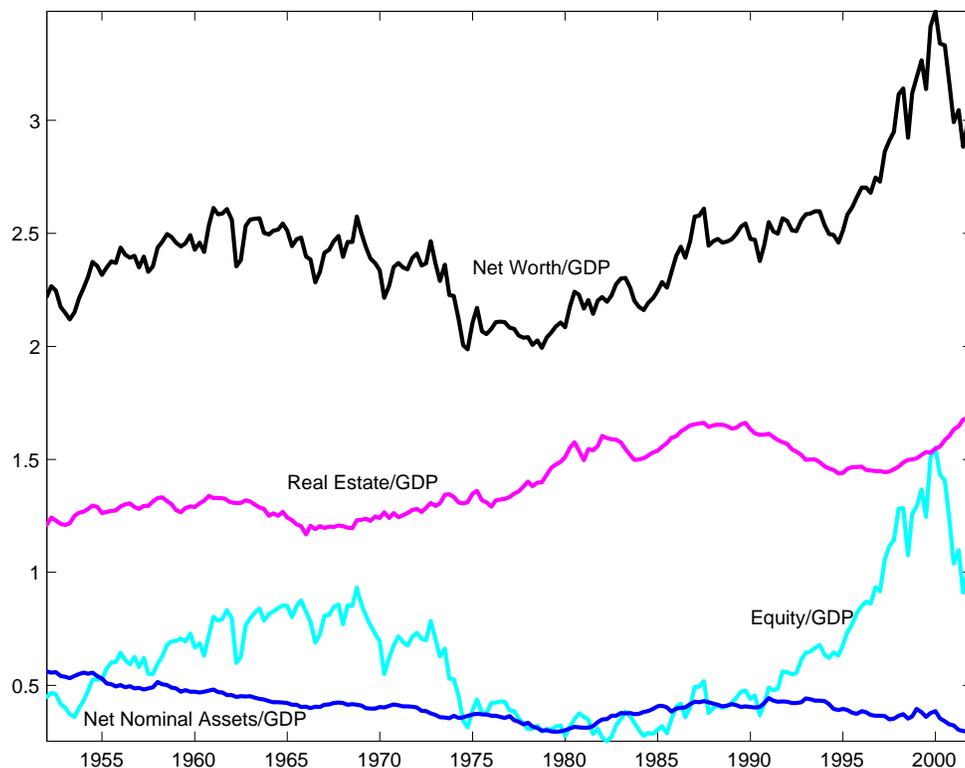


Figure 1: Aggregate wealth components (market value) divided by GDP, Flow of Funds & own computations, 1952:1-2003:4.

age enter a period with different endowments of the three assets as well as non-asset income. During the period they trade in asset and goods markets. Expectations about future returns and non-asset income are given exogenously. Prices clear markets in the current trading period to establish a “temporary equilibrium”, as in Grandmont (1977, 1982). In the asset and goods markets, households also interact with the rest of the economy, that is, the government, business and foreign sectors. In particular, the model allows all assets to be in nonzero net supply; supply can also change within a period as the household sector trades with other sectors.

We implement the model quantitatively for three 6-year periods, centered around benchmark years 1968, 1978 and 1995. The idea is to view Figure 2 as a sequence of temporary equilibria, of which three are explicitly computed. For each period, we measure (i) the joint distribution of initial asset endowments and income endowments, (ii) the supply of assets to the household sector by other sectors. Data for these model inputs come from the Flow of Funds accounts and the Survey of Consumer Finances. The other key model input is households’ expectations about future

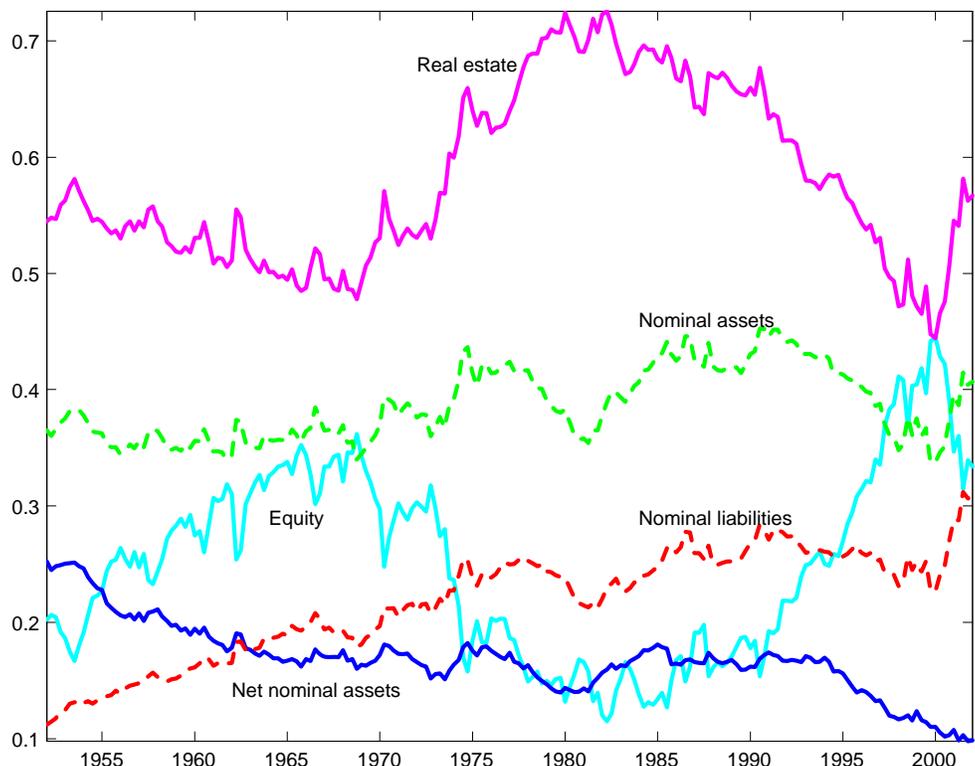


Figure 2: Aggregate portfolio shares, Flow of Funds & own computations, 1952:1-2003:4.

asset returns and income.

We select the discount factor and baseline beliefs so that the model replicates the aggregate wealth-to-GDP ratio, household sector portfolio weights, as well as the nominal interest rate in 1995. The baseline model then also matches price dividend ratios of equity and real estate. Moreover, it captures a number of stylized facts about the cross-section of households in the 1995 SCF. In particular, it generates hump-shaped cohort market shares in wealth, real estate and equity, as well as net nominal positions that increase – and real estate shares that decrease – with age and net worth. The main mechanism behind these facts is that agents who expect more future non-asset income are willing to build more risky portfolios.

To illustrate the sensitivity of asset prices to changes in expectations and asset supply, we perform a series of counterfactuals based on the 1995 calibration. The main result here is that events relevant to the stock market spill over much less into other asset markets than events relevant to the housing market, although stock prices are themselves more responsive to changes

in expectations. For example, a one percent increase in real stock return expectations over the next six years raises the price dividend ratio of stocks by 15%, but raises the nominal interest rate by only 10 basis points; it lowers the price-dividend ratio on housing by 3.5%. In contrast, a one percent increase in the expected return on real estate raises the house price by 7%, increases the nominal interest rate by 80 basis points, and also lowers stock prices by 13%. The reason for these results is that households can borrow against real estate, so that events in the housing market feed back more strongly to the credit market than news about stocks.

We then use the model to examine the 1970s. We show that, under baseline expectations, the model replicates the drop in aggregate wealth between 1968 and 1978. It attributes the dip in the wealth-GDP ratio to two effects. First, the entry of baby boomers into asset markets lowered the average saving rate. Second, capital losses from realized inflation lowered wealth and hence savings, especially for older households. At the same time, lower savings were not counteracted by a large increase in interest rates, because the outside supply of bonds to the household sector also fell. If we assume that the spread between borrowing and lending rate was 75 basis points higher in 1968 than in 1995, the model also generates the increase in gross borrowing and lending between these two years.

Under baseline expectations, the model cannot account for a portfolio shift towards housing. We thus explore the effects of changes in inflation expectations and quantify three different channels through which these changes affect the model predictions for the 1970s. First, we endow agents with higher mean expected inflation rates, taken from the Michigan survey of consumers. An interesting fact from the survey is that young households were expecting more inflation than old households in 1978. We show that with survey expectations, the model is consistent with a portfolio shift from stocks to houses that leaves the share of bonds unchanged. In addition, the heterogeneity of expectations leads to an increase in gross borrowing and lending. This effect in isolation, however, leads to a portfolio shift that is too small, while the increase in gross credit and the nominal interest rate are too large.

Second, we consider an increase in inflation uncertainty. This tends to depress nominal borrowing and lending, but does not have strong effects on portfolio shares. Third, we consider changes in conditional expected stock returns. This experiment is motivated by a number of studies on the

forecasting of stock returns using measures of expected inflation that appeared in the 1970s. We show that mild pessimism about stocks – consistent with what one would have derived from the regression results of Fama and Schwert (1977) – generates large portfolio shifts, but small changes in the nominal interest rate. Pessimism about stocks brought about by higher expected inflation thus helps account for the observed portfolio shift without upsetting other features of the model. We conclude that – through a combination of these three channels – inflation expectations mattered for prices and portfolios in the 1970s.

Section II discusses related literature. Section III presents the model. Section IV describes the quantitative implementation and documents properties of the model inputs, that is, the joint distribution of asset endowments and income as well as asset supply. Section V presents results under baseline expectations, both for aggregates and for the cross section of holdings. Section VI considers the effect of inflation. Section VII concludes.

## II Related Literature

Some of the effects of inflation that arise in our model have been discussed before. Feldstein (1980), Summers (1981) and Poterba (1991) have examined various ways in which the interaction of taxes and inflation can affect asset prices. One argument is that inflation lowers depreciation allowances based on historical cost accounting and hence real cash flows, which should in turn drive down stock prices. We consider this effect only indirectly, since we take real dividends as given, and find it to be relatively small. These authors have also argued that inflation changes lowers after tax returns on bonds and stocks more than those on real estate and hence might be responsible for the portfolio shift of the 1970s. We show that this effect contributes to the portfolio shift, although it cannot quantitatively generate all of it.

Previous literature has shown that demographics cannot account fully for changes in stock prices, if equity is the only long-lived asset in the model (Abel 2002, Geanakoplos et al. 2004). The effect of demographics on house prices in isolation has been considered by Mankiw and Weil (1989) and Ortalo-Magne and Rady (2005). In our model, with both equity and real estate present in nonzero net supply, demographics impact aggregate savings and hence the wealth-GDP ratio, but

it can also not account for the larger movements in the individual components of wealth, especially stocks.

There is a large literature on asset pricing models with heterogeneous agents and incomplete markets and/or borrowing constraints. For example, Constantinides and Duffie (1996), Heaton and Lucas (1996), Krusell and Smith (1998), Constantinides et al. (2002) and Storesletten et al. (2004) consider models with equity, riskless bonds and uninsurable income risk. Alvarez and Jermann (2001) and Lustig and van Nieuwerburgh (2005) consider models with complete markets, where income risk cannot be insured because of borrowing constraints or collateral constraints backed by real estate, respectively. The goal of these papers is to derive a stationary equilibrium of the model that matches empirical moments of returns such as the equity premium. The input to the model is typically a jointly stationary process for income and dividends, while the output compared to the data are moments of returns and macro aggregates, and sometimes also the cross section of consumption (Brav et al. 2002, Kocherlakota and Pistaferri 2005).

Our paper differs from these studies because of our focus on nominal risk. It also differs in the empirical implementation, where we use observed household asset positions both as an input to the model and as a target of the analysis. In addition, we do not derive a stationary equilibrium that is compared to empirical moments, but consider instead asset prices and holdings at specific dates. In this respect, our approach is similar to that of McGrattan and Prescott (2004), who look at the effect of taxes on stock prices, Nakajima (2005) who considers the effect of precautionary savings on house prices and Campbell and Hercowitz (2005) who study the effect of credit market deregulation on debt levels. However, existing studies that focus on low frequency movements in the economy typically compare steady states or stationary equilibria at different parameter values, while we use the temporary equilibrium concept of Grandmont (1977, 1982).

Portfolio choice with housing has been considered by Flavin and Yamashita 2002, Fernandez-Villaverde and Krueger 2005, Campbell and Cocco 2005. Cocco (2005) and Yao and Zhang (2005) study intertemporal problems with three assets that are similar to the problem solved by our households. General equilibrium OLG models with housing have been considered by Chambers et al. (2003) and Yang (2005). These papers are also interested in the cross section of house ownership. While they consider a shorter period length and thus study the cross section in more

detail than we do, they abstract from aggregate risk which is important for our application.

### III Model

The model describes the household sector's planning and asset trading in a single time period  $t$ .

#### A. Households

Households enter the period with assets and debt accumulated earlier. During the period, they earn labor income, pay taxes, consume and buy assets. Labor income is affected by uninsurable idiosyncratic income shocks. Households can invest in three types of assets: long-lived equity and real estate as well as short lived nominal bonds. There is no riskless asset and markets are incomplete.

##### *Planning Horizon*

Consumers alive at time  $t$  differ by endowment of assets and numeraire good as well as by age. Differences in age are represented by differences in planning horizon: the idea is that all agents expect to reach a certain age. We now describe the problem of a typical consumer with a planning horizon of  $T - t > 0$  periods beyond the current period  $t$ .

##### *Preferences*

Consumers care about two goods, housing services and other (non-housing) consumption which serves as the numeraire. A consumption bundle of  $s_t$  units of housing services and  $c_t$  units of numeraire yields utility

$$(1) \quad C_t = c_t^\delta s_t^{1-\delta}.$$

Preferences over (random) streams of consumption bundles  $\{C_t\}$  are represented by the recursive

utility specification of Epstein and Zin (1989). Utility at time  $t$  is defined as

$$(2) \quad U_t = \left( C_t^{1-1/\sigma} + \beta E_t \left[ U_{t+1}^{1-\gamma} \right]^{\frac{1-1/\sigma}{1-\gamma}} \right)^{\frac{1}{1-1/\sigma}},$$

where  $U_T = C_T$ . Here  $\sigma$  is the intertemporal elasticity of substitution,  $\gamma$  is the coefficient of relative risk aversion towards timeless gambles and  $\beta$  is the discount factor. The expectation operator takes into account that the agent will reach the next period only with an age-specific survival probability.

### *Equity*

Shares of equity can be thought of as trees that yield some quantity of numeraire good as dividend. A consumer enters period  $t$  with an endowment of  $\bar{\theta}_t^e \geq 0$  units of trees. Trees trade in the equity market at the cum-dividend price  $\tilde{p}_t^e$ ; they cannot be sold short. A tree pays  $d_t^e$  units of dividend during period  $t$  *after* trading in the equity market has taken place. The ex-dividend price is denoted  $p_t^e$ . We summarize consumers' expectations about prices and dividends beyond period  $t$  by specifying expectations about returns. In particular, we assume that consumers expect to earn a (random) real return  $R_{\tau+1}^e$  by holding equity between any two periods  $\tau$  and  $\tau + 1$ , where  $\tau \geq t$ .

To make our timing conventions for period  $t$  more concrete, consider a consumer who enters period  $t$  with an endowment  $\bar{\theta}_t^e$ . He can sell his shares for  $\tilde{p}_t^e \bar{\theta}_t^e$  or buy new shares at the price  $\tilde{p}_t^e$ . Suppose trading in the stock market leaves the consumer with  $\theta_t^e$  shares. He immediately receives  $d_t^e \theta_t^e$  units of numeraire as dividend, which can be consumed (or sold in the numeraire good market) in  $t$ . In addition, the consumer expects to receive  $R_{t+1}^e p_t^e \theta_t^e$  units of numeraire once he sells his shares in period  $t + 1$ .

### *Real Estate*

Real estate – or houses – may be thought of as trees that yield housing services. A consumer enters period  $t$  with an endowment of  $\bar{\theta}_t^h \geq 0$  units of houses. Houses trade at the cum-dividend price  $\tilde{p}_t^h$  during period  $t$ ; they cannot be sold short. To fix units, we assume that one unit of real estate (also referred to as one house) yields one unit of housing services in period  $t$ . As with equity, this dividend arrives *after* trading in the real estate market has occurred. Moreover, every house

requires a maintenance cost of  $m$  units of numeraire, also after trading.

Housing services can be either consumed or sold. There is a perfect rental market, where housing services can be rented at the price  $p_t^s$ . If a consumer buys  $\theta_t^h$  units of real estate, he obtains a dividend  $(p_t^s - m)\theta_t^h =: d_t^h\theta_t^h$  and the ex-dividend value of his property is  $(\tilde{p}_t^h - d_t^h)\theta_t^h =: p_t^h\theta_t^h$ . Alternatively, a houseowner may directly consume the housing services provided by his house. In either case, we assume that consumers form expectations about future returns on housing and rental prices  $\{R_\tau^h, p_\tau^s\}_{\tau>t}$ .

### *Borrowing and Lending*

Consumers can borrow or lend by buying or selling one period discount nominal bonds. A consumer enters period  $t$  with an endowment of  $\bar{b}_t$  units of numeraire that is due to past borrowing and lending in the credit market. In particular,  $\bar{b}_t$  is negative if the consumer has been a net borrower in the past. In period  $t$ , consumers can buy or sell bonds at a price  $q_t$ . A consumer expects every bond bought to pay  $1/\pi_{t+1}$  units of numeraire in period  $t+1$ . Here  $\pi_{t+1}$  is random and may be thought of as the expected change in the dollar price of numeraire. This is a simple way to capture that debt is typically denominated in dollars.<sup>1</sup> For every bond sold, the consumer expects to repay  $(1+\xi)/\pi_{t+1}$  units of numeraire in period  $t+1$ , where  $\xi > 0$  is an exogenous credit spread.<sup>2</sup> Bond sellers – borrowers – face a collateral constraint: the value of bonds sold may not exceed a fraction  $\phi$  of the ex-dividend value of all real estate owned by the consumer. For periods  $\tau > t$ , consumers form expectations about the (random) real return on bonds  $\{R_\tau^b\}$ . They believe that  $R_\tau^b = 1/q_\tau\pi_{\tau+1}$  is the (ex post) real lending rate, and that  $R_\tau^b(1+\xi)$  is the (ex post) real borrowing rate.

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<sup>1</sup>To see why, consider a nominal bond which costs  $q_t$  dollars today and pays of \$1 tomorrow, or  $1/p_{t+1}^c$  units of numeraire consumption. Now consider a portfolio of  $p_t^c$  nominal bonds. The price of the portfolio is  $q_t$  units of numeraire and its payoff is  $p_t^c/p_{t+1}^c = 1/\pi_{t+1}$  units of numeraire tomorrow. The model thus determines the price  $q_t$  of a nominal bond in \$.

<sup>2</sup>One way to think about the organization of the credit market is that there is a financial intermediary that matches buyers and sellers in period  $t$ . In period  $t+1$ , the intermediary will collect  $(1+\xi)/\pi_{t+1}$  units of numeraire from every borrower (bond seller), but pay only  $1/\pi_{t+1}$  to every lender (bond buyer), keeping  $\xi/\pi_{t+1}$  for itself. We do not model the financial intermediary explicitly since we only clear markets in period  $t$ .

### Non-Asset Income

Consumers are endowed with an age-dependent stream of numeraire good  $\{y_\tau\}_{\tau=t}^{t+T}$ . Here income should be interpreted as the sum of labor income, transfer income, and income on illiquid assets such as private businesses.

### Budget Set

The consumer enters period  $t$  with an endowment of trees and houses  $(\bar{\theta}_t^h, \bar{\theta}_t^e)$  as well as an endowment of  $y_t + \bar{b}_t$  from non-asset income and past credit market activity. At period  $t$  prices, initial wealth is therefore

$$(3) \quad \bar{w}_t = \tilde{p}_t^h \bar{\theta}_t^h + \tilde{p}_t^e \bar{\theta}_t^e + \bar{b}_t + y_t.$$

To allocate this initial wealth to consumption and purchases of assets, the consumer chooses a plan  $a_t = \{c_t, s_t, \theta_t^h, \theta_t^e, b_t^+, b_t^-\}$ , where  $b_t^+ \geq 0$  and  $b_t^- \geq 0$  denote the amount of bonds bought and sold, respectively. It never makes sense for a consumer to borrow and lend simultaneously, that is,  $b_t^+ \geq 0$  implies  $b_t^- = 0$  and vice versa.

The plan  $a_t$  must satisfy the budget constraint

$$(4) \quad c_t + p_t^s s_t + w_t = \bar{w}_t,$$

where terminal wealth is defined as

$$w_t = p_t^h \theta_t^h + p_t^e \theta_t^e + q_t b_t^+ - q_t b_t^-.$$

To formulate the budget constraint for periods beyond  $t$ , it is helpful to define the ex-dividend value of the consumer's stock portfolio in  $t$  by  $w_t^e = p_t^e \theta_t^e$ , the consumer's real estate portfolio by  $w_t^h = p_t^h \theta_t^h$  as well as the values of a (positive or negative) bond portfolio,  $w_t^{b+} = q_t b_t^+$  and  $w_t^{b-} = q_t b_t^-$ . For periods  $\tau > t$ , the consumer chooses plans  $a_\tau = \{c_\tau, s_\tau, w_\tau^h, w_\tau^e, w_\tau^{b+}, w_\tau^{b-}\}$  subject

to

$$\begin{aligned}
& c_\tau + p_\tau s_\tau + w_\tau^h + w_\tau^e + w_\tau^{b+} + R_\tau^b (1 + \xi) w_{\tau-1}^{b-} \\
(5) \quad & = R_\tau^h w_{\tau-1}^h + R_\tau^e w_{\tau-1}^e + R_\tau^b w_{\tau-1}^{b+} + w_\tau^{b-} + y_\tau
\end{aligned}$$

We denote the consumer's overall plan by  $a = \left( a_t, \{a_\tau\}_{\tau=t+1}^{t+T} \right)$ . This plan is selected to maximize utility (2) subject to (3)-(5).

### *Taxes*

In some of our examples below, we will assume proportional income taxes as well as capital gains and dividend taxes. This will not change the structure consumer's problem, just the interpretation of the symbols. In particular, labor income, dividends and returns will have to be interpreted as their after-tax counterparts. Their precise form will be discuss in the calibration section below.

### *Oldest Consumers*

The consumers described so far have planning horizons  $T - t > 0$ . We also allow consumers with planning horizon  $T - t = 0$ . These consumers also enter period  $t$  with asset and numeraire endowments that provide them with initial wealth  $\bar{w}_t$ , as in (3). However, they do not make any savings or portfolio decisions. Instead, they simply purchase numeraire and housing services in the period  $t$  goods markets to maximize (1) subject to the budget constraint

$$c_t + p_t^s s_t = \bar{w}_t.$$

## **B. Equilibrium**

Suppose that there is a finite number of consumers, indexed by  $i$ , with different initial endowment vectors  $(\bar{\theta}_t^h(i), \bar{\theta}_t^e(i), y_t(i) + \bar{b}_t(i))$  and planning horizons  $T(i) - t$ .

### *The Rest of the Economy*

To close the model and regulate the supply of assets exogenous to the household sector, we

introduce a rest-of-the-economy (ROE) sector. It may be thought of as a consolidation of the business sector, the government and the rest of the world. The ROE sector is endowed with  $f_t^e$  trees and  $f_t^h$  houses in period  $t$ . Here  $f_t^e$  could be negative to represent repurchases of shares by the corporate sector. In addition, the ROE enters period  $t$  with an outstanding debt of  $\bar{B}_t$  units of numeraire, and it raises  $D_t$  units of numeraire by borrowing in period  $t$ . The surplus from these activities is

$$(6) \quad C_t^{ROE} = \tilde{p}_\tau^h f_t^h + \tilde{p}_t^e f_t^e + D_t - \bar{B}_t.$$

If  $C_t^{ROE}$  is positive, it is consumed by the ROE sector. More generally, the ROE sector is assumed to have “deep pockets” out of which it pays for any deficit if  $C_t^{ROE} < 0$ .

#### *Aggregate Asset Supply*

We normalize initial endowments of equity and real estate such that there is a single tree and a single house outstanding:

$$\sum_i \bar{\theta}_t^h(i) = \sum_i \bar{\theta}_t^e(i) = 1.$$

In addition, we assume that initial endowments from past credit market activity are consistent, in the sense that every position corresponds to some offsetting position, either by a household or by the ROE sector:

$$\sum_i \bar{b}_t(i) = \bar{B}_t.$$

#### *Equilibrium*

An equilibrium consists of a vector of prices for period  $t$ ,  $(\tilde{p}_t^h, \tilde{p}_t^e, q_t, p_t^s)$ , a surplus for the ROE sector  $C_t^{ROE}$ , as well as a collection of consumer plans for period  $t$ ,  $\{a_t(i)\} = \{c_t(i), s_t(i), \theta_t^h(i), \theta_t^e(i), b_t^+(i), b_t^-(i)\}$  such that

(1) for every consumer, the plan  $a_t(i)$  is part of an optimal plan  $a(i) = (a_t(i), \{a_\tau(i)\}_{\tau=t+1})$  for that maximizes utility 2 given consumer  $i$ 's endowment, planning horizon, and expectations about future prices and returns;

(2) markets for all assets and goods clear:

$$\begin{aligned}
\sum_i \theta_t^h(i) &= 1 + f_t^h, \\
\sum_i \theta_t^e(i) &= 1 + f_t^e, \\
q_t \sum_i b_t^+(i) &= D_t + q_t \sum_i b_t^-(i), \\
\sum_i c_t(i) + m \theta_t^h + C_t^{ROE} &= \sum_i y_t(i) + d_t^e (1 + f_t^e), \\
\sum_i s_t(i) &= \sum_i \theta_t^h(i).
\end{aligned}$$

In addition to market clearing conditions for stocks, bonds and numeraire, there are two market clearing conditions for housing: one for the asset “real estate” and one for the good “housing services”. The first equation ensures that the total demand for houses equals their total supply. The fifth equation ensures that the fraction of houses that owners set aside as investment real estate – that is, selling services in the rental market – is the same as the fraction of housing services demanded in the rental market. As is common in competitive models, one of the five market clearing conditions is redundant, as it is implied by the sum of consumers’ budget constraints, the definition of  $C_t^{ROE}$  and the other four market clearing conditions. Solving for equilibrium prices thus amounts to solving a system of four equations in the four prices  $\tilde{p}_t^h$ ,  $\tilde{p}_t^e$ ,  $q_t$  and  $p_t^s$ .

### C. Discussion of the Assumptions

#### *Temporary Equilibrium*

In our setup, we do not require structural knowledge – that is we do not assume that there is a “true” probability distribution of fundamentals and a “true” model that all agents know and use for forecasting. In particular, household expectations are not required to anticipate structural change that takes place later. Some of the changes that are relevant for thinking about asset markets – different monetary policy regimes, for example – are hard to foresee, or, more generally, hard to form probabilities about. Structural knowledge is easier to obtain when fundamentals change in regular ways and when agents have plenty of data and experience about the link between fundamentals

and prices.

### *Nonnegative Net Worth*

Few households have negative net worth. Table 1 documents that the percentage of negative net worth households has always been between 4% and 7%. Table 1 also shows that the net worth of these households is moderate. For example, the average net worth was -11K Dollars in 2001. These numbers suggest that the most important reason for household borrowing is not consumption smoothing. Instead, young households “borrow to gamble” — they borrow to be able to buy more risky assets, such as housing.

TABLE 1: NEGATIVE NET WORTH HOUSEHOLDS

	1962 SCF				1995 SCF				2001 SCF			
	avg.	29	53	77	avg.	29	53	77	avg.	29	53	77
% of households	4%	10%	3%	2%	7%	18%	4%	2%	7%	20%	5%	2%
net worth (in \$)	-.4K	-.4K	-.8K	-1.3K	-9K	-9K	-13K	-8K	-11K	-10K	-9K	-1K

NOTE: Row 1 reports the percentage of households with negative net worth in the U.S. population based on different SCFs. Row 2 reports the average net worth of these households in dollars.

### *The Role of Housing*

In both our model and in reality, housing plays a dual role: housing can be used for consumption and investment. In its role as consumption good, housing is different from other consumption, because it enters the utility function separately and has a different price  $p_t^s$ . In its role as asset, housing is different from other assets, because of its return properties (which will be discussed later), and because it can serve as collateral. In reality, the two roles are often connected, because the amount of housing services consumed is equal to the dividend paid by the amount of housing held in the portfolio. In our model, we abstract from this issue (at least for now) for several reasons. First, most homeowners not only own their primary residence, but also some investment real estate (such as time shares, vacation homes, secondary homes, etc.) Hence, there is no tight link between consumption and investment for these households. Second, U.S. households are highly mobile. As a consequence, houses are owned by the same owner on average for only seven years, which is roughly

the same length as a period in our model. Moreover, the decision to move is often more or less exogenous to households (for example, because of job loss, divorce, death of the spouse, etc.)

## IV Quantitative Implementation

The inputs needed for implementing the model are (i) the joint distribution of asset endowments and income, (ii) aggregate supply of assets to the household sector from other sectors, (iii) expectations about labor income and asset returns and (iv) parameter values for preferences and the credit market. This section describes where we obtain these inputs. We start with a description of what a model period corresponds to in the data.

### *Timing*

The length of a period is six years. We assume that consumers expect to live for at most 10 such periods, where the first period of life corresponds roughly to the beginning of working life. In any given period, we consider 11 age groups of households (<23, 24-29, 30-35, 36-41, 42-47, 48-53, 54-59, 60-65, 66-71, 72-77, >77) who make portfolio choice decisions. For ease of comparison with other models, we nevertheless report numbers at annual rates.

In the time series dimension, we divide the period 1959-2003 into 8 six-year periods, 1959-64, 1965-70, 1970-75, 1975-80, 1980-85, 1986-91, 1992-97 and 1998-2003. In our exercises below, we calculate temporary equilibria for three of these periods, namely 1965-70, 1975-80 and 1992-95. To construct asset endowment distributions, we use data on asset holdings from the respective precursor periods. Moreover, we pick the income distribution by specifying a stochastic process for individual income, and then forecasting the whole income distribution one period ahead. This approach allows to capture the correlation between income and *initial* asset holdings that is implied by the joint distribution of income and wealth.

Since the model compresses what happens over a six year span into a single date, prices and holdings are best thought of as averages over the period. However, individual level data is not available at high frequencies. To capture the wealth and income distribution during a period, we have chosen the above intervals so that every period that contains a Survey of Consumer Finances

contains one in the 4th year of the period, in particular, the surveys we use are 1962, 1983, 1989, 1995 and 2001 (some periods also contain a second survey in their 1st year). We use the 4th year survey to infer income and asset holdings where possible.

## A. Assets and Income: Definitions

We map the three assets in the model to three broad asset classes in US aggregate and household statistics. For the long-lived assets, we need estimates of household holdings at both the aggregate and the individual level, as well as a measure of aggregate dividends and new purchases of the asset by households. For nominal bonds, we need both net and gross positions, as well as interest rates. We also need a concept of after-tax non-asset income from the data to fit the role of labor income in the model.

Our main data sources are the Flow of Funds Accounts (FFA) for aggregates and the Survey of Consumer Finances (SCF) for individual positions. To make these data sets comparable, we must ensure that aggregates match. As shown by Antoniewicz (2004), the match is good for most asset classes in both 1989 and 1995, after a few adjustments. However, our own computations show that the match for nominal assets is bad for the 1962 SCF. For some classes of assets, especially short-term deposits, the SCF aggregates are only about 50% of the FFA aggregates. Apparent underreporting of short-term nominal assets is also present in later SCFs, but is less severe. To achieve a comparable time series of positions, we assume throughout that the FFA aggregates are correct and that individual positions in the SCF suffer from proportional measurement error. We then multiply each individual position by the ratio of the FFA aggregate and the SCF aggregate for the same asset class.

### *Long-lived assets*

We identify *equity* with shares in corporations held and controlled by households. We include both publicly traded and closely held shares, and both foreign and domestic equity. We also include shares held indirectly through investment intermediaries if the household can be assumed to control the asset allocation (into our broad asset classes) himself. We take this to be true for mutual funds and defined contribution (DC) pensions plans. For these intermediaries, while the fund manager

determines the precise composition of the portfolio, the household typically makes the decision about equity versus bonds by selecting the type of fund.

We thus consolidate mutual funds and DC pension funds. For example, when households own a mutual fund, an estimate of the part of the fund invested in stocks is added to stock holdings. In contrast, we do not include equity held in defined benefit (DB) pension plans, since the portfolios of these plans are not controlled by households themselves. Instead, DB plans are treated as a tax-transfer system sponsored by the rest of the economy (in practice, the corporate sector or the government). We also do not include noncorporate business, which is treated partly as real estate and partly as labor income, as described below.

We construct an annual series for the aggregate value of household sector equity holdings. Our starting point is the published series in the FFA. We cannot use that series directly, since it contains (i) the market value of the equity component of foreign direct investment (that is, equity positions by foreigners in excess of 10% of shares in a US corporation) and (ii) the market value of equity held by DB pension funds. We estimate the equity component of FDI using data on the International Investment Position from the Bureau of Economic Analysis. Shares held by defined benefit pension funds are available from the FFA. Our series is obtained by subtracting (i) and (ii) from the FFA series on household equity holdings.

We derive estimates of net new equity purchased by households using a similar correction of FFA numbers. Finally, our concept of dividends equals dividends received by households from the National Income and Product Accounts (NIPA) less dividends on their holdings in DB pension plans. We use the numbers on value, dividends and new issues to calculate price dividend ratios and holding returns on equity. The properties of the return series are discussed in Section V below. For household-level positions, we use the Survey of Consumer Finances, which also contains direct holdings of publicly-traded and closely-held shares, as well as an estimate of equity held indirectly through investment intermediaries.

Our concept of *residential real estate* contains owner-occupied housing, directly held residential investment real estate, as well as residential real estate recorded in the FFA/NIPA as held indirectly by households through noncorporate businesses. This concept contains almost all residential real

estate holdings, since very few residential properties are owned by corporations. To construct holdings of tenant-occupied residential real-estate at the individual level, we start from the SCF numbers and then add a proportional share of the household's noncorporate business position. This share is selected so that our aggregate of tenant-occupied real estate over all households matches the corresponding value from the FFA. We take housing dividends to be housing consumption net of maintenance and property tax from NIPA. For net purchases of new houses, we use aggregate residential investment from NIPA. As with equity, the annual series for holdings, dividends and new issues give rise to a return series, discussed below.

### *Nominal Positions*

Our concept for a household's *bond* holdings is its net nominal position, that is, the market value of all nominal assets minus the market value of nominal liabilities. As for equity, holdings include not only direct holdings, but also indirect holdings through investment intermediaries. To calculate market value, we use the market value adjustment factors for nominal positions in the U.S. from Doepke and Schneider (2004). In line with our treatment of tenant-occupied real estate, we assign residential mortgages issued by noncorporate businesses directly to households. At the individual level, we assign a household mortgages in proportion to his noncorporate business position, again with a share selected to match the aggregate value of residential mortgages from the FFA.

When considering *gross* nominal positions, we must take into account the fact that some netting of positions occurs at the individual level. A typical household will have both a mortgage loan or credit card and a savings account or bonds in a pension fund. However, the model has only one type of nominal asset, and a household can be either long or short that asset. If we were to match the gross aggregates from the FFA or SCF in our model, this would inevitably lead to net positions that are too large. Instead, we sort SCF households into borrowers and lenders, according to whether their net nominal position is negative or not. The numbers for gross borrowing and lending are then calculated as minus the sum of net nominal positions of borrowers as well as the sum of net positions of all lenders, respectively.

Table 2 summarizes these gross nominal positions after individual netting from the SCF and compares them to those in the FFA. Both the FFA numbers and our estimates reflect a steady

increase in borrowing by the household sector. At the same time, both sets of numbers show a reduction in nominal asset holdings in the 1970s followed by an increase between 1978 and 1995. Throughout, individual netting reduces gross lending by roughly one third, while it reduces gross borrowing by slightly more than half.

TABLE 2: GROSS BORROWING AND LENDING (%GDP)

	1968		1978		1995	
	lending	borrowing	lending	borrowing	lending	borrowing
FFA aggregates	88	47	84	51	107	68
SCF after indiv. netting	61	20	56	23	70	30

### *Non-Asset Income*

Our concept of non-asset *income* comprises all income that is available for consumption or investment, but not received from payoffs of one of our three assets. We start by constructing an aggregate measure of such income from NIPA. Of the various components of worker compensation, we include only wages and salaries, as well as employer contributions to DC pension plans. We do not include employer contributions to DB pension plans or health insurance, since these funds are not available for consumption or investment. However, we do include benefits disbursed from DB plans and health plans. Also included are transfers from the government. Finally, we subtract personal income tax on non-asset income.

Non-asset income also includes dividends from noncorporate business except those attributable to residential real estate. To construct the latter concept of noncorporate dividends, we use the aggregate price-dividend ratio of housing to estimate the housing dividend provided by an individual's private business. Our approach essentially splits up the noncorporate sector into a real estate component that is very capital intensive and relies heavily on debt finance, and a rest that is much more labor intensive. Indeed, the capital stock of the noncorporate nonfinancial sector in 1995 was \$3.6trn, of which \$2.4trn was residential real estate.

Given the aggregate series of income, we apply the same conventions to individual income in the SCF to the extent possible. A problem is that the SCF does not report employer contributions

to DC plans and only reports pretax income for all items. To address this issue, we apply a proportional tax rate to pretax non-asset income reported in the SCF, where the tax rate is chosen such that aggregate non-asset income is equal to its counterpart from NIPA. The outcome is an income distribution that matches with NIPA at the aggregate level.

## B. Measuring the Distribution of Endowments

Consumers in our model are endowed with both assets and non-asset income. To capture decisions made by the cross-section of households, we thus have to initialize the model for every period  $t$  with a *joint* distribution of asset endowment and income. We derive this distribution from asset holdings and income observed in the previous period  $t - 1$ . Limits on data availability imply that we have to resort to different approaches for the different years. For 1995, the data situation is best, since we can use the SCF in the 4th year of period  $t$  together with the SCF from the 4th year of period  $t - 1$ . We describe our strategy first for this case. We then explain how it is modified for earlier years where less data are available.

### *Approximating the distribution of households*

In principle, one could use all the households in SCF and update them individually. This would lead to a large number of agents and consequently a large number of portfolio problems would have to be solved. We simplify by approximating the distribution of endowments and income with a small number of household types. First, we sort households into the same nine age groups described at the beginning of this section. Within each age group, we then select 6 subclasses of SCF households. We start by extracting the top 10% of households by net worth. Among the bottom 90% net worth, we divide by homeowner and renter. We then divide homeowners further into “borrowers” and “lenders.” Here a household is a borrower if his net nominal position – nominal asset minus nominal liabilities – is negative. We further subdivide each homeowner category into high/low wealth-to-income ratios.

The above procedure splits up households into  $9 \times 6 = 54$  different cells. We assume that all households that fall into the same cell are identical and compute asset positions at the cell level. The SCF survey weights determine the cell population. Naturally, the procedure loses some

features of the true distribution due to aggregation. However, it ensures that key properties of the distribution that we are interested in are retained. In particular, because very few among the top 10% are net nominal borrowers, gross borrowing and lending are very close in the true and the approximating distributions. In addition, the approximating distributions retains asset positions conditional on age as well as conditional on wealth when net worth is split as top 10% and the rest, our key measures of concentration described.

*Asset endowments for a transiting individual household*

Consider the transition of an individual household’s asset position from period  $t - 1$  into period  $t$ . We have treated both stocks and houses as long-lived trees and we normalize the number of trees carried into the period by consumers to one. We can thus measure the household’s endowment of a long-lived asset from its share in total market capitalization of the asset in period  $t - 1$ . The SCF does not contain consumption data. Using the language introduced in the discussion of the budget constraint (4), we can thus measure either initial or terminal wealth in a given period, but not both. We assume that terminal wealth can be directly taken from the survey. The initial supply of assets is normalized to one, so that the initial holdings of housing  $\bar{\theta}_t^h$  and stocks  $\bar{\theta}_t^e$  are the agent’s *market shares* in period  $t - 1$ . For each long-lived asset  $a = h, e$ , suppose that  $w_{t-1}^a(i)$  is the market value of investor  $i$ ’s position in  $t - 1$  in asset  $a$ . Now we can measure household  $i$ ’s initial holdings as

$$\begin{aligned} \bar{\theta}_t^a(i) &= \theta_{t-1}^a(i) = \frac{w_{t-1}^a(i)}{\sum_i w_{t-1}^a(i)} = \frac{p_{t-1}^a \theta_{t-1}^a(i)}{p_{t-1}^a \sum_i \theta_{t-1}^a(i)} \\ &= \text{market share of household } i \text{ in period } t - 1. \end{aligned}$$

*Updating Nominal Positions*

For the nominal assets, the above approach does not work since these assets are short-term in our model. Instead, we determine the market value of nominal positions in period  $t - 1$  and update it to period  $t$  by multiplying it with a nominal interest rate factor. In particular, suppose that

$w_{t-1}^b(i)$  is the market value of investor  $i$ 's net nominal positions in  $t - 1$  and that

$$\begin{aligned}\bar{\theta}_{t-1}^b(i) &= \frac{w_{t-1}^b(i)}{\sum_i w_{t-1}^b(i)} \\ &= \text{market share of household } i \text{ in period } t - 1.\end{aligned}$$

We define the initial holdings of bonds for household  $i$  as

$$\begin{aligned}\bar{b}_t(i) &= (1 + i_{t-1}) \frac{w_{t-1}^b(i)}{\text{GDP}_t} \\ &= (1 + i_{t-1}) \frac{w_{t-1}^b(i)}{\sum_i w_{t-1}^b(i)} \frac{\sum_i w_{t-1}^b(i)}{\text{GDP}_{t-1}} \frac{\text{GDP}_{t-1}}{\text{GDP}_t}.\end{aligned}$$

Letting  $g_t$  denote real GDP growth and  $D_t$  the aggregate net nominal position as a fraction of GDP, we have

$$\bar{b}_t(i) \approx \bar{\theta}_{t-1}^b(i) D_{t-1} (1 + i_{t-1} - g_t - \pi_t).$$

This equation distinguishes three reasons why  $\bar{b}_t(i)$  might be small in a given period. The first is simply that the household's nominal investment in the previous period was small. Since all endowments are stated relative to GDP, all current initial nominal positions are also small if the economy has just undergone a period of rapid growth. Finally, initial nominal positions are affected by surprise inflation over the last few years. If the nominal interest rate  $i_{t-1}$  does not compensate for realized inflation  $\pi_t$ , then  $\bar{b}_t^i$  is small (in absolute value). Surprise inflation thus increases the negative position of a borrower, while it decreases the positive position of a borrower. As an interest factor for a positive (lending) position, we use an average of 6-year bond rates between the 4th year of period  $t$  and the 4th year of period  $t - 1$ . We add a spread for the borrowing rate. The spread is 2% for 1995 and 2.75% for 1968 and 1978, for reasons described in the calibration section V below.

### *Forecasting Income*

The final step in our construction of the joint income and endowment distribution is to specify the marginal distribution of non-asset income. Here we make use of the fact that income is observed in period  $t - 1$  in the SCF. We then assume that the transition between  $t - 1$  and  $t$  is determined by a stochastic process for non-asset income. We employ the same process that agents in the model

use to forecast their non-asset income, described in the next subsection. If the assumption were true, and if there were a large number of identical individuals in every cell, then our discretization implies that households in a cell should split up into nine different cells in the following period, with fractions provided by the probabilities of the income process. This is what we assume. As a result, the distribution of agents in period  $t$  is approximated by  $9 \times 6 \times 9 = 486$  different cells. For each cell, we know the endowment of assets as well as income, and we have a set of population weights that sums to the total population.

#### *Non-transiting households*

The previous discussion has covered only households who transit from period  $t$  into period  $t+1$ . We also need to take into account the creation and destruction of households between  $t-1$  and  $t$ . In years where successive SCFs are available, we calculate “birthrates” and “deathrates” for households directly by comparing these surveys. We assume that exiting households receive no labor income, but sell their assets and consume the proceeds, while entering households start with zero assets and the average labor income of their cohort. This is a simplified view that does not do justice to the many different reasons why households form and dissolve, and how wealth is passed along among households. However, we view it as a useful benchmark.

#### *Time periods without two successive SCFs*

For periods before 1980, the above strategy cannot be executed as is, because we do not have two consecutive SCFs. For the period 1965-70, the 1962 SCF is used to determine the initial endowment and income distribution. The only difficulty here is the adjustment of exiting and entering households. We use data from the Census Bureau on the evolution of household populations to gauge the size of exiters and entrants. The average labor income of the entering cohort is then estimated by multiplying per capita income of the young in the 1962 SCF by the growth rate of aggregate per capita labor income.

For the period 1975-80, we do not have SCF information for period  $t-1$ . As for the 1960s, the updating of population weights is performed using Census data. To estimate the cross sectional distribution of endowments and income, we start from the 1962 distribution and its division of

households into cells and modify cell holdings to obtain a new distribution. In particular, we calculate the unique distribution such that, for stocks, real estate, nominal assets, nominal debt and income, *(i)* aggregates match the 1973 aggregates from the FFA, *(ii)* the share of an individual cell member’s holdings in the aggregate holdings is the same as in 1962 and *(iii)* the share of all cell members’ holdings in their respective cohort aggregates is the same as in 1962.

Condition *(i)* and *(ii)* imply that per capita holdings or income within a cohort changes in order to account for differences in demographics while simultaneously matching aggregates. Condition *(iii)* imposes that the cross section conditional on age is the same in the two years. The reason for using the 1962 distribution as the starting point rather than, say the 1989 distribution, is that the 1973 aggregates – especially gross nominal assets – appear more similar to 1962 than to the 1980s. Once we have a distribution of positions at the cell level for 1973, we proceed as above to generate an updated distribution for 1978.

### C. Distributions for 1968, 1978 and 1995

Figure 3 provides summary information on asset endowment and income distributions in the three trading periods we consider below. The trading periods are identified in the figure by their respective fourth year: 1968, 1978 and 1995. The top left panel provides population weights by cohort. Cohorts are identified on the horizontal axis by the upper bound of the age range. In addition, the fraction of households that exit during the period are offset to the far right.

The different years can be distinguished by the line type: solid with circles for 1968, dashed with squares for 1978 and dotted with diamonds for 1995. Using the same symbols, the top right panel shows house endowments (light lines) and stock endowments (dark lines) by age cohort, while the bottom left panel shows initial net nominal positions as a percent of GDP. Finally, the bottom right panel shows income distributions. Here we plot not only non-asset income, but also initial wealth not invested in long-lived assets, in other words,

$$E_t = d_t^h \bar{\theta}_t^h + d_t^e \bar{\theta}_t^e + \bar{b}_t + y_t.$$

This aggregate will be useful to interpret the results below.

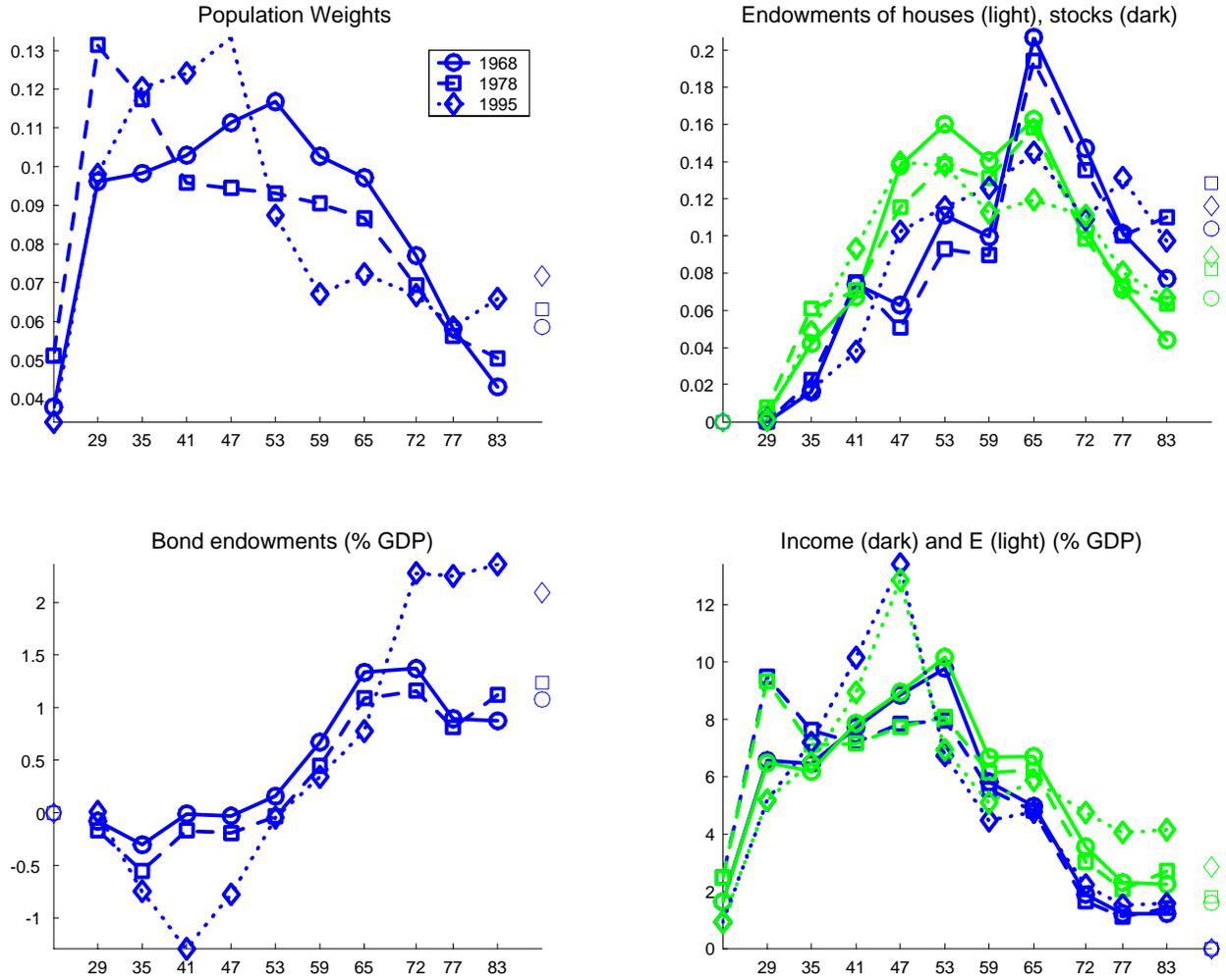


Figure 3: Asset endowment and income distributions in 1968, 1978 and 1995. *Top left panel:* Population weights by cohort, identified on the horizontal axis by the upper bound of the age range. Exiting households during the period are on the far right. *Top right panel:* House endowments (light lines) and stock endowments (dark lines) by age cohort. *Bottom left panel:* initial net nominal positions as a percent of GDP. *Bottom right panel:* Income distributions.

Two demographic changes are apparent from the figure. First, the baby boom makes the two youngest cohorts relatively larger in the 1978 cross section than in the other two years. By 1995, the boomers have aged so that the 42-47 year olds are the now strongest cohort. This shift of population shares is also reflected in the distribution of income in the bottom right panel. Second, the relative size of the oldest group has become larger over time. Recently, a lot of retirement income comes from assets, so that the share of  $E_t$  of the elderly groups has also increased a lot. A key difference between the 1968 and 1978 distributions is thus that the latter places more weight

on households who tend to save little: the oldest and, especially, the youngest. While the 1995 distribution also has relatively more weight on the elderly, it emphasizes more the middle-aged rather than the young.

The comparison of stock and house endowments in the top right panel reveals that housing is more of an asset for younger people. For all years, the market shares of cohorts in their thirties and forties are larger for houses than for stocks, while the opposite is true for older cohorts. By and large, the market shares are however quite similar across years. In contrast, the behavior of net nominal positions relative to GDP (bottom right hand panel) has changed markedly over time. In particular, the amount of intergenerational borrowing and lending has increased: young households today borrow relatively more, while old households hold relatively more bonds.

#### **D. Asset Supply**

The endowment of the ROE sector consists of new equity issued during the trading period. The factor  $f^e$  states this endowment relative to total market capitalization in the model. We thus use net new corporate equity divided by total household holdings of corporate equity. We obtain the corresponding measure for housing by dividing residential investment by the value of residential real estate. The top panel of Figure 4 plots both quarterly series of these numbers. The calibration of the model uses six-year aggregates.

The initial nominal position of the ROE sector is taken to be minus the aggregate (updated) net nominal position of the household sector. Finally, the new net nominal position of the ROE sector in period  $t$  – in other words, the “supply of bonds” to the household sector – is taken to be minus the aggregate net nominal positions from the FFA for period  $t$ . This series is reproduced in the bottom panel of Figure 4.

#### **E. Baseline expectations**

The previous subsections have described actual income and holdings in a trading period  $t$ . In addition, the model requires agents’ expectations about returns and income in the future.

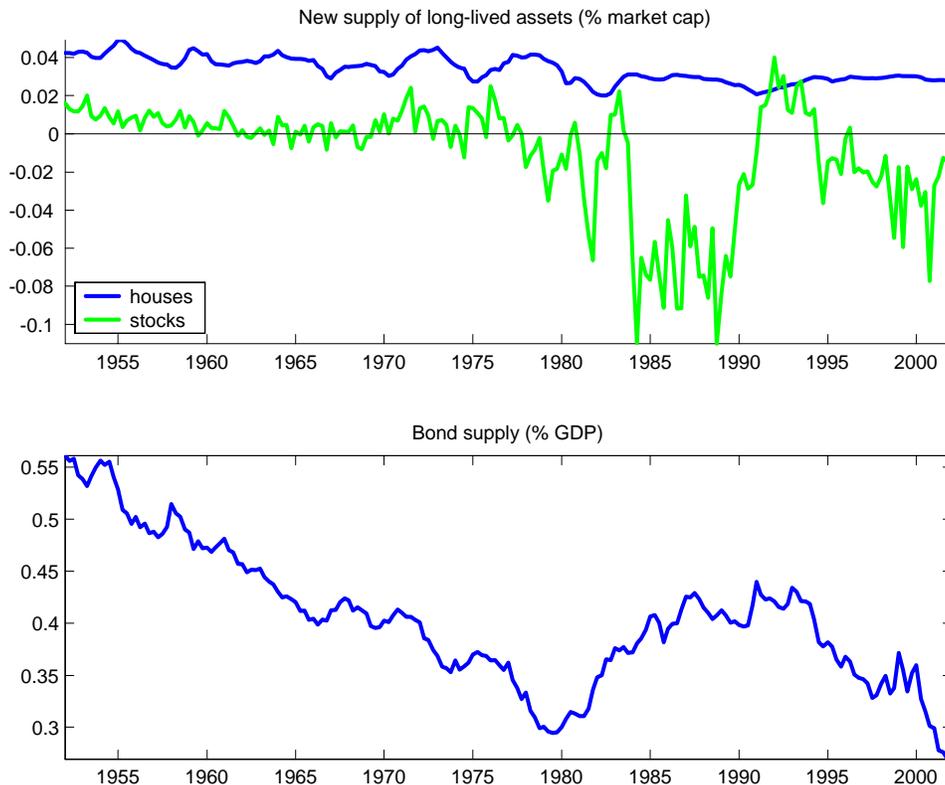


Figure 4: *Top panel:* Net new corporate equity as a percent of total household holdings of corporate equity and residential investment as percent of value of residential real estate. Data are quarterly at annual rates. *Bottom panel:* Net nominal position of household sector as a percent of GDP.

### *Non-Asset Income*

We specify a stochastic process to describe consumer expectations about after-tax income. The functional form for this process is motivated by existing specifications for labor income that employ a deterministic trend to capture age-specific changes in income, as well as permanent and transitory components. In particular, following Zeldes (1989) and Gourinchas and Parker (2001), we assume that individual income  $Y^i$  is

$$Y_t^i = G_t A_t P_t^i U_t^i$$

which has a common component  $G_t$ , an age profile  $A_t$ , a permanent idiosyncratic component  $P_t^i$  and a transitory idiosyncratic component  $U_t^i$ .

The growth rate of the common component  $G_t$  is equal to the growth rate of aggregates, such as GDP and aggregate income, in the economy. It is common to specify the transitory idiosyncratic

component as lognormally distributed

$$\ln U_t^i = N\left(-\frac{1}{2}\sigma_u^2, \sigma_u^2\right),$$

so that  $U_t^i$  is i.i.d with mean one. The permanent component  $P_t^i$  follows a random walk with mean one. The permanent component solves

$$\ln P_t^i = \ln P_{t-1}^i + \varepsilon_t^i - \frac{1}{2}\sigma_\varepsilon^2,$$

where  $\varepsilon_t^i$  are normal shocks with zero mean and standard deviation  $\sigma(\varepsilon_t^i)$ . In our numerical procedures, we discretize the state process using Gauss–Hermite quadrature with three states.

We estimate the age profile  $G_t$  as average income in each age-cohort from the SCF:

$$\frac{1}{\#a} \sum_{i \in a} Y_t^i = A_t G_t \frac{1}{\#a} \sum_{i \in a} P_t^i,$$

with  $\text{plim} \frac{1}{\#a} \sum_{i \in a} P_t^i = 1$ . Table 3 reports the profile relative to the income of the youngest cohort.

TABLE 3: INCOME AGE PROFILE

29	35	41	47	53	59	65	71	77	88+
2.04	2.51	3.17	3.80	4.56	3.81	3.00	1.93	1.42	1.17

NOTE: Income age profiles estimated from the SCF. The numbers represent the average cohort income relative to the average income of the youngest cohort ( $\leq 23$  years).

We obtain an estimate of the variance of permanent shocks by computing the cross-sectional variance of labor income for each cohort before retirement ( $<65$  years) and then regressing it on a constant and cohort age. The intercept of this regression line is .78, while the annualized slope coefficient is .014.<sup>3</sup> We thus set  $\sigma_\varepsilon^2 = 1.4\%$  and time-aggregate this variance for our six-year periods. This number is in line with more sophisticated estimations of labor income processes, which tend

<sup>3</sup>Of course, this simple approach uses only the cross-section and thus potentially confounds age and time or cohort effects. However, when we rerun the regression with SCF wages, using the similar sample selection criteria as Storesletten et al. (2004), our results are close to what these authors find for 1995 from an analysis with panel data on wages.

to produce estimates between 1% and 2% per year.

Typical estimates of the variance of temporary shocks  $\sigma_u^2$  are 2-10 times larger than those of the variance of permanent shocks. Moreover, several studies have shown that the variance of temporary shocks to log wages has increased since the 1970s. For example, Heathcote et al.(2004) show that the variance of log wages increased from about .05 in the 1970s to .07 in 1995. To capture this increase in temporary income risk, we adopt their numbers, thus assuming that the variance of hours is constant. We set  $\sigma_u^2 = .05$  for the years 1968 and 1978 and  $\sigma_u^2 = .07$  for 1995.<sup>4</sup> In all years, agents in the model assume that this variance is fixed forever. Finally, we determine the variance of the first draw of permanent income from the intercept of our regression line. For the earlier years, we scale down this initial variance by the relative change in the permanent component of income for the youngest agents in Heathcote et al. This is a simple way to accommodate changes in income due to education over time. Sensitivity checks have shown that the initial draw of permanent income does not matter much for the results, since it does not directly affect the portfolio problem.

Most labor income studies focus on pre-retirement income. There are major challenges to obtaining variance estimates for retirement income. For example, older households tend to experience large shocks to health expenditures, which are included as NIPA income if they are disbursed by health plans. These shocks contain both transitory and permanent components (see the estimates reported in Appendix A, Skinner, Hubbard and Zeldes 1994). Since these shocks are hard to measure at the household level, we could try to ignore their variances and assume that household receive a safe stream of income during retirement. However, this implies that precautionary savings drop dramatically as soon as the household enters retirement in the absence of such shocks. This prediction is not consistent with the household savings data from the SCF. For this reason, we apply the above shocks to income at any age, including retirement.

### *Returns and aggregate growth*

We assume that consumers believe real asset returns and aggregate growth to be serially independent over successive six year periods. Moreover, when computing an equilibrium for a given

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<sup>4</sup>The fixed effects only matter for the updating of the income distribution, as explained in the previous section. The model's results are not sensitive to the magnitude of these effects. For example, the results based on our model are unchanged when we use the estimates provided by Heathcote et al. (2004).

period  $t$ , we assume that returns are identically distributed for periods beyond  $t + 1$ . We will refer to this set of beliefs – to be described below – as *baseline beliefs*. However, in our exercises we will allow beliefs for returns between  $t$  and  $t + 1$  to differ. For example, we will explore what happens when expected inflation is higher over the next six year period. We discuss the latter aspect of beliefs below when we present our results. Here we focus on how we fix the baseline.

To pick numbers for baseline beliefs, we start from empirical moments. Table 4 reports summary statistics on ex-post realized pre-tax real returns on fixed income securities, residential real estate and equity, as well as inflation and growth. These returns are measured over six year periods, but reported at annualized rates. Since we work with aggregate portfolio data from the FFA, we construct returns on corporate equity and residential real estate directly from FFA aggregates.

TABLE 4: SUMMARY STATISTICS

$r_t^b$	$r_t^h$	$r_t^e$	$\pi_t$	$g_t$
Means				
2.68	4.81	8.51	4.01	2.19
Standard Deviations/Correlations				
3.24	-0.02	0.56	-0.04	0.33
-0.02	3.31	-0.04	-0.13	0.38
0.56	-0.04	22.87	-0.52	0.20
-0.04	-0.13	-0.52	5.60	-0.40
0.33	0.38	0.20	-0.40	1.31
Sharpe Ratios				
	0.45	0.27		

NOTE: The table reports annualized summary statistics of six-year log real returns. Below the means, the matrix has standard deviations on the diagonal and correlations on the off-diagonal. The last row contains the Sharpe ratios. The log inflation rate  $\pi_t$  is computed using the CPI, while  $g_t$  is the log growth rate of GDP multiplied by the factor 2.2/3.3 to match the mean growth rate of consumption.

Baseline beliefs assume that the payoff on bonds  $1/\pi_{t+1}$  is based on a (net) inflation rate  $\pi_{t+1} - 1$  with a mean of 4% per year, and that the volatility of  $\pi_{t+1}$  is the same as the unconditional volatility of real bond returns, about 1.3% per year. To obtain capital gains from period  $t$  to  $t + 1$ , we take the value of total outstandings from the FFA in  $t + 1$ , and subtract the value of net new issues (or, in the case of real estate, new construction.) To obtain dividends on equity in period  $t$ , we

use aggregate net dividends. To obtain dividends on real estate, we take total residential housing sector output from the NIPA, and subtract materials used by the housing sector. For bond returns, we use a six year nominal interest rate derived by extrapolation from the term structure in CRSP, and subtract realized inflation, measured by the CPI. Here growth is real GDP growth.

The properties of the equity and bond returns are relatively standard. The return on bonds has a low mean of 2.7% and a low standard deviation of 3.2%. The return on stocks has a high mean of 8.5% and a standard deviation of 23%. What is less familiar is the aggregate return on residential real estate: it has a mean and standard deviation in between the other two assets. It is apparent that the Sharpe ratio of aggregate housing is much higher than that on stocks.

In principle, we could use the numbers from Table 4 directly for our benchmark beliefs. However, this would not capture the tradeoff faced by the typical individual household. Indeed, the housing returns in Table 4 are for the aggregate housing stock, while real estate is typically a non-diversified investment. It is implausible to assume that investors were able to pick a portfolio of real estate with return characteristics as in Table 4 at any time over our sample period. Instead, the typical investor picks real estate by selecting a few properties local markets.

Existing evidence suggests that the volatility of house returns at the metro area, and even at the neighborhood or property level are significantly higher than returns at the national aggregate. For example, Caplin et al. (1997) argue that 1/4 of the overall variance is aggregate, 1/4 is city-component, and 1/2 is idiosyncratic. Tables 1A and 1B in Flavin and Yamashita (2002) together with Appendix C in Piazzesi et al. (2005) confirm this decomposition of housing returns. As a simple way to capture this higher property-level volatility, we add idiosyncratic shocks to the variance of housing returns that have volatility equal to 3.5 times aggregate volatility.<sup>5</sup> Finally, we assume that expected future real rents are constant. This ignores the volatility of real rent growth, which is small, at around 2% per year.

### *Taxes on investment*

Investors care about after-tax real returns. In particular, taxes affect the relative attractiveness

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<sup>5</sup>Since the volatility of housing is measured imprecisely, we chose the precise number for the multiplicative factor such that the aggregate share of housing in the model roughly matches the FFA data. The resulting factor is 3.5, close to the rule-of-thumb factor of 4.

of equity and real estate. On the one hand, dividends on owner-occupied housing are directly consumed and hence not taxed, while dividends on stocks are subject to income tax. On the other hand, capital gains on housing are more easily sheltered from taxes than capital gains on stocks. This is because many consumers simply live in their house for a long period of time and never realize the capital gains. Capital gains tax matters especially in inflationary times, because the nominal gain is taxed: the effective real after tax return on an asset subject to capital gains tax is therefore lower when inflation occurs.

To measure the effect of capital gains taxes, one would ideally like to explicitly distinguish realized and unrealized capital gains. However, this would involve introducing state variables to keep track of past individual asset purchase decisions. To keep the problem manageable, we adopt a simpler approach: we adjust our benchmark returns to capture the effects described above. For our baseline set of results, we assume proportional taxes, and we set both the capital gains tax rate and the income tax rate to 20%. We define after tax real stock returns by subtracting 20% from realized net real stock returns and then further subtracting 20% times the realized rate of inflation to capture the fact that nominal capital gains are taxed. In contrast, we assume that returns on real estate are not taxed.

## V Supply, Demographics and Asset Prices

In this section, we compute equilibria at baseline expectations for 1968, 1978 and 1995. This isolates the effect of changes in supply and the wealth distribution on asset prices. We then compare the performance of the model in the cross-section of households to actual observations from the 1995 SCF.

### *Baseline parameters*

The previous section delivers distributions of asset endowments and income for each year. It remains to choose preference and credit market parameters. We fix an intertemporal elasticity of substitution of  $\sigma = .5$ , a standard value in the literature, and let  $\delta = .86$ , the share of housing services in aggregate consumption in the data. This expenditure share does not vary much over the

lifecycle and across households (Piazzesi et al. 2005, Appendix B). For the year 1995, we assume that there is a 2% per year spread between borrowing and lending interest rates. Early on, credit markets were less developed and gross credit was thus smaller. To capture this, we set the spread to 2.75% for the earlier years. In addition, we select the borrowing constraint parameter  $\alpha = .8$ . This implies a maximal loan-to-value ratio of 80%, where “value” is the ex-dividend value of the house.

Finally, we assume reasonable values for preference parameters. We set the coefficient of relative risk aversion to  $\gamma = 5$  and the discount factor to  $\beta = \exp(-0.025 \times 6)$ . Since  $\gamma$  is low, agents do not want to hold bonds when faced with historical Sharpe ratios on stocks and housing. To avoid this counterfactual implication, we assume that agents view long-lived assets as riskier than indicated by their historical moments. This idea of “low aversion against high perceived risk” can be captured by scaling the historical return variances from Table 4 with some factor. This scaling can be interpreted as a consequence of Bayesian learning about the premium on equity and housing. To generate reasonable portfolio shares, we use a factor of three.

An alternative strategy would be to work with agents who have “high aversion against low perceived risk.” In this case, agents base their portfolio choice on the historical variances from Table 4, but are characterized by high risk aversion,  $\gamma = 25$ , and high discounting,  $\beta = \exp(-0.07 \times 6)$ . The high  $\gamma$  is needed to lower the portfolio weight on bonds, while the low  $\beta$  is needed to reduce the precautionary savings motive in the presence of uninsurable income shocks. While the tables below report results based on agents with “low aversion against high perceived risk,” we would get comparable results based on this alternative parametrization.

### *Baseline results for 1995 aggregates*

Table 5 reports facts for the year 1995 and model results on aggregates for the 1995 calibration. The table also performs some counterfactuals to illustrate the sensitivity of asset prices to changes in expectations and asset supply. By comparing rows 1 and 2 in Table 5, we can see that the model roughly matches aggregate portfolio weights and the aggregate wealth-to-GDP ratio. The main results from the counterfactuals in rows 3-6 are that events relevant to the stock market spill over much less into other asset markets than events relevant to the housing market, although stock

prices are themselves more responsive to changes in expectations. For example, row 3 illustrates that a one percent increase in real stock return expectations over the next six years raises the price dividend ratio of stocks by 15%, but raises the nominal interest rate by only 10 basis points; it lowers the price-dividend ratio on housing by 3.5%. In contrast, row 4 shows that a one percent increase in the expected return on real estate raises the house price by 7%, increases the nominal interest rate by 80 basis points, and also lowers stock prices by 13%.

TABLE 5: COUNTERFACTUALS, 1995 DISTRIBUTION

experiment	wealth/ GDP	portfolio weights			lend./ GDP	borr/ GDP	PD ratios		interest rate nominal
		bonds	housing	stocks			housing	stocks	
(1) —1995 data —	2.51	.15	.59	.26	.70	.31	20.4	23.9	.061
(2) baseline	2.51	.15	.60	.25	.70	.31	20.8	22.7	.061
(3) stock exp. +1%	2.54	.15	.57	.28	.70	.31	20.1	26.2	.062
(4) house exp. +1%	2.50	.15	.64	.21	.71	.32	22.1	19.2	.069
(5) stock supply -5%	2.93	.13	.62	.25	.78	.41	25.3	27.1	.055
(6) house supply -5%	3.39	.11	.64	.25	.85	.47	29.8	31.8	.051

NOTE: The first row reports the aggregate portfolio weights on bonds, housing and stocks from Figure 2; the gross borrowing and lending numbers from Section A., the wealth-to-GDP ratio from Figure 1; the price-dividend ratios for housing and stocks together with the nominal 6-year interest rate. The remaining rows report the results computed from the model for 1995 with baseline beliefs. The relevant counterfactuals are described in the first column of the table and in the text.

Table 5 also shows what happens – other things equal – when the supply of long-lived assets drops by 5%. For stocks, the experiment in row 4 reduces the flow of new equity from zero, the 1995 value, to -5%. This corresponds to a phase of substantial repurchases of stocks by corporations, such as that observed in the 1980s, or alternatively to a period where foreigners buy equity from domestic households. For houses, the experiment in row 5 reduces housing investment from its 1995 value of 4% to -1%.

In both cases, the reduction in the amount of savings vehicles available to households increases the price of all savings vehicles: stock and house prices rise, while the nominal interest rate falls.

In addition, there is an increase in gross borrowing by households: households themselves thus replace some of the supply of savings vehicles withdrawn by the rest of the economy. The difference between the two experiments is that the response to a change in house supply is stronger for *both* prices as well as for credit. This underlines again the fact that the role of housing as collateral leads to stronger spillovers from the housing market to other markets.

## A. Lifecycle Savings and Portfolios

Since preferences are homothetic and all constraints are linear, the optimal savings rate and portfolio weights depend only on age and the ratio of initial wealth – that is asset wealth plus non-asset income – to the permanent component of non-asset income. For simplicity, we refer to the latter ratio as the wealth to income ratio. Figure 5 plots agents decisions as a function of this wealth-to-income ratio.

### *Savings*

The bottom right panel shows the ratio of terminal wealth to initial wealth, that is, the savings rate out of initial wealth. Savings are always positive, since the borrowing constraint precludes strategies that involve negative net worth. Investors who have more income in later periods than in the current period thus cannot shift that income forward by borrowing. In this sense, there is no borrowing for “consumption smoothing” purposes: all current consumption must instead come out of current income or from selling initial asset wealth. If initial wealth is very low relative to income, all assets will be sold and all income consumed, so that the investor enters the next period with zero asset wealth.

The bottom right panel also illustrates how the savings rate changes with age. There are two relevant effects. On the one hand, younger investors have a longer planning horizon and therefore tend to spread any wealth they have over more remaining periods. This effect by itself tends to make younger investors save more. On the other hand, the non-asset income profile is hump-shaped, so that middle-aged investors can rely more on labor income for consumption than either young or old investors. This tends to make middle-aged investors save relatively more than other investors.

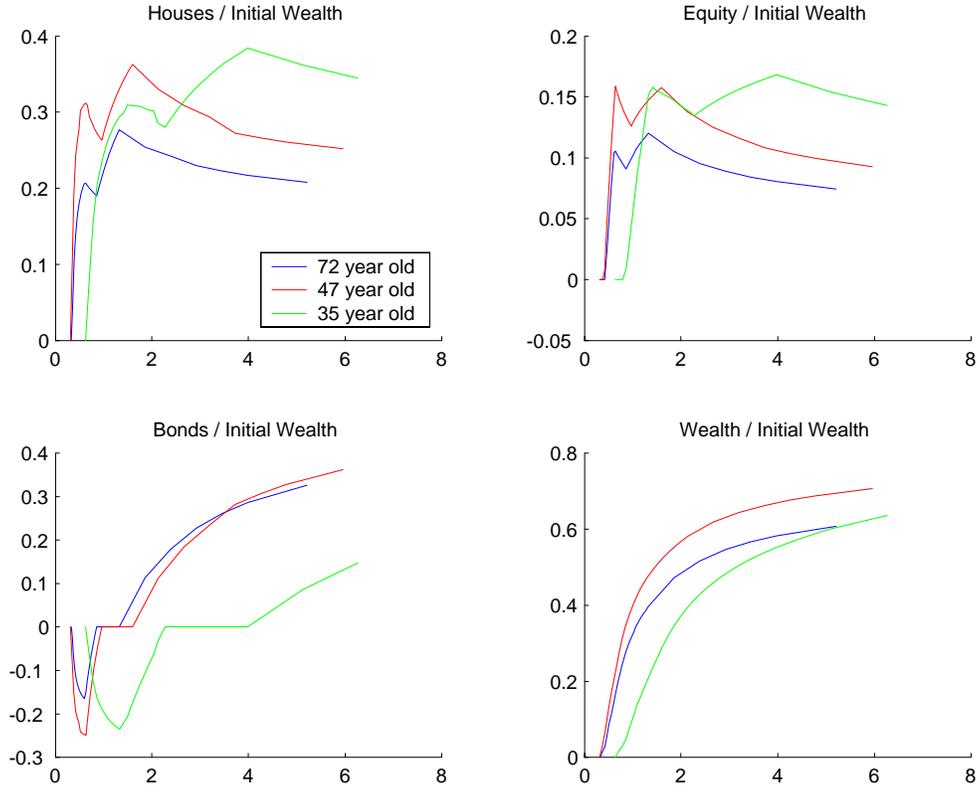


Figure 5: Asset holdings and terminal wealth, both as fractions of initial wealth, plotted against the initial wealth-to-income ratio. Age groups are identified by maximum age in the cohort.

The first effect dominates when labor income is not very important, that is, when the wealth-to-income ratio is high. The figure shows that at high wealth-to-income ratios, the savings rate of the 29-35 year old group climbs beyond that of the oldest investor group. It eventually also climbs below the savings rate of the 48-53 year old group. The second effect is important for lower wealth-to-income ratios, especially in the empirically relevant range around 1-2, where most ratios lie in the data. In this region, the savings rate of the middle-aged is highest, whereas both the young and the old save less. Among the latter two groups, the young save the least when their wealth-to-income ratio is low.

### *Borrowing and Leverage*

Rather than enable consumption smoothing, borrowing serves to construct a leveraged portfolio. The bottom left panel of Figure 5 shows that investors who are younger and have lower wealth-to-income ratios tend to go short in bonds. The top panels show that the borrowed funds are used

to build leveraged positions of houses and also stocks. In contrast, investors who are older and have higher wealth-to-income ratios tend to go long in all three assets. Along the wealth-to-income axis, there is also an intermediate region where investors hold zero bonds. This region is due to the credit spread: there exist ratios where it is too costly to leverage at the high borrowing rate, while it is not profitable to invest at the lower lending rate.

The reason why “gambling” with leverage decreases with age and the wealth-to-income ratio is the presence of labor income. Effectively, an investor’s portfolio consists of both asset wealth and human wealth. Younger and lower wealth-to-income households have relatively more human wealth. Moreover, the correlation of human wealth and asset wealth is small. As a result, households with a lot of labor income hold riskier strategies in the asset part of their portfolios. This effect has also been observed by Heaton and Lucas (2000) and Cocco (2005).

#### *Stock v. House Ownership*

For most age groups and wealth-to-income ratios, investment in houses is larger than investment in stocks. This reflects the higher Sharpe ratio of houses as well as the fact that houses serve as collateral while stocks do not. The latter feature also explains why the ratio of house to stock ownership is decreasing with both age and wealth-to-income ratio: for richer and older households, leverage is less important, and so the collateral value of a house is smaller.

The model can currently not capture the fact that the portfolio weight on stocks tends to increase with the wealth-to-income ratio. While it is true in the model that people with higher wealth-to-income own more stocks relative to housing, they also hold much more bonds relative to both of the other assets. As a result, their overall portfolio weight on stocks actually falls with the wealth-to-income ratio. Experimentation with alternative beliefs has shown that if stocks are relatively less attractive, it is possible to obtain a stock share in the portfolio that increases with wealth-to-income for intermediate levels of the latter. The behavior of the portfolio weight on stocks implies that the model produces typically too little concentration of stock ownership.

Preliminary results on a version of the model with owner-occupied and rental housing suggests that the latter feature helps along several of the dimensions where the current version is still lacking. In particular, if housing services can be consumed more cheaply when there is owner

occupation, there is an additional reason for young people to hold relatively more houses. This will also contribute to making stock ownership more concentrated.

## B. The Cross Section of Asset Holdings

Figure 6 plots predicted portfolio weights and market shares for various groups of households for 1995, given baseline beliefs. The panels also contain actual weights and market shares for the respective groups from the 1995 Survey of Consumer Finances. It is useful to compare both portfolio weights and market shares, since the latter also require the model to do a good job on savings behavior. Indeed, defining aggregate initial wealth  $\bar{W} = \sum_i \bar{w}(i)$ , the market share of, say, houses for a household  $i$  can be written as

$$\theta^h(i) = \frac{\alpha^h(i) \bar{w}(i)}{\sum_i \alpha^h(i) \bar{w}(i)} = \frac{\alpha^h(i)}{\sum_i \alpha^h(i) \frac{\bar{w}(i)}{\bar{W}}} \frac{\bar{w}(i)}{\bar{W}} = \frac{\alpha^h(i)}{\bar{\alpha}^h} \frac{\bar{w}(i)}{\bar{W}},$$

where  $\alpha^h(i)$  is household  $i$ 's portfolio weight and  $\bar{\alpha}^h$  is the aggregate portfolio weight on houses. A model that correctly predicts the cross section of portfolio shares will therefore only correctly predict the cross section of market shares if it also captures the cross section of terminal wealth. The latter in turn depends on the savings rate of different groups of agents.

The first row of Figure 6 documents savings behavior by cohort and wealth level. The top left panel plots terminal wealth as a fraction of GDP at the cohort level (blue/black lines) for the model (dotted line) and the data (solid line). It also shows separately terminal wealth of the top decile by net worth (green/light gray lines), again for the model and the data. This color coding of plots is maintained throughout the figure, so that a “good fit” means that the lines of the same color are close to each other.

The top left panel shows that model does a fairly good job at matching terminal wealth. One exception is the very oldest group of savers who save too little in the model. The model also captures skewness of the distribution of terminal wealth and how this skewness changes with age. The top 10% by net worth own more than half of total terminal wealth, their share increasing with age. In the model, these properties are inherited in part from the distributions of endowment and

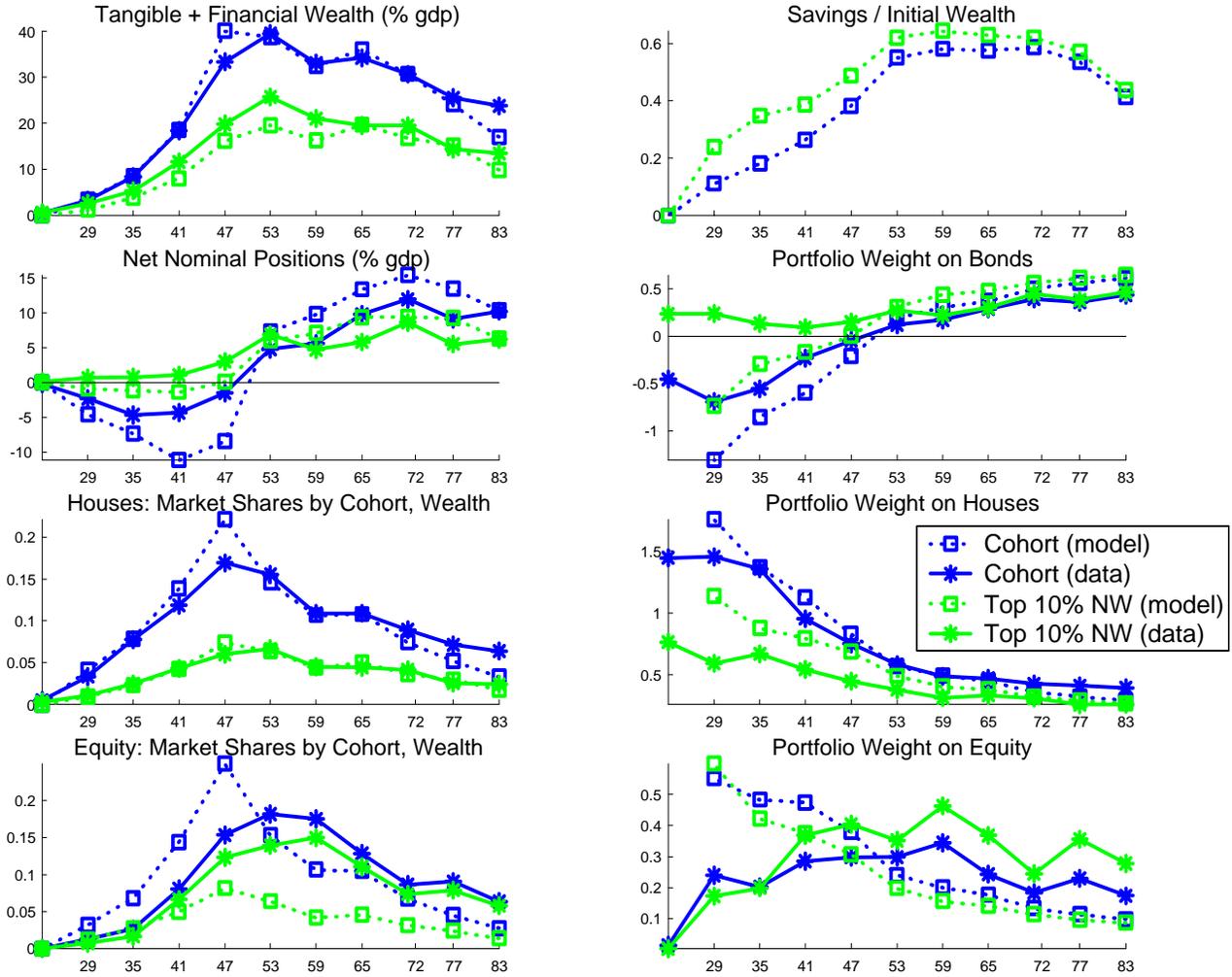


Figure 6: Market shares and portfolio weights for 1995

labor income. However, it is also the case that richer agents save more out of initial wealth. This feature is apparent from the top right panel of Figure 6 which reports savings rates by cohort and net worth. It obtains because (i) the rich have higher ratios of initial wealth relative to current labor income, and (ii) the savings rate is increase with the wealth-to-income ratio, as explained in the previous subsection.

In the data, the main difference in portfolio weights by age is the shift from houses into bonds over the course of the life cycle. This is documented in the right column of Figure 6. Young agents borrow in order to build leveraged positions in houses. In the second panel, their portfolio weights become positive with age as they switch to being net lenders. The accumulation of bond portfolios

makes houses – shown in the third panel on the right – relatively less important for older households. The model captures this portfolio shift fairly well. Intuitively, younger households “gamble” more, because the presence of future labor income makes them act in a more risk tolerant fashion in asset markets.

The left column shows the corresponding cohort aggregates. Nominal positions relative to GDP (second panel on the left) are first negative and decreasing with age, but subsequently turn around and increase with age so that they eventually become positive. These properties are present both in the model and the data. On the negative side, the model somewhat overstates heterogeneity in positions by age: there is too much borrowing – and too much investment in housing – by young agents. In particular, the portfolio weights for the very youngest cohort are too extreme. However, since the wealth of this cohort is not very large, its impact on aggregates and market shares is small.

For houses (third panel on the left), the combination of portfolio and savings choices generate a hump shape in market share. While younger agents have much higher portfolio weights on real estate than the middle-aged, their overall initial wealth is sufficiently low, so that their market share is lower than that of the middle-aged. Another feature of the data is that the portfolio shift from housing to bonds with age is much less pronounced for the rich. The model also captures this feature, as shown by the green/gray lines in the second and third rows of the figure. The intuition again comes from the link between leverage and the wealth-to-income ratio: the rich are relatively asset-rich (high wealth-to-income) and thus put together less risky asset portfolios, which implies less leverage and lower weights on housing.

The panels in the last row of Figure 6 plot market shares and portfolio weights for equity. This is where the model has the most problems replicating the SCF observations. Roughly, investment in stocks in the model behaves “too much” like investment in housing. Indeed, the portfolio weight is not only decreasing with age after age 53, as in the data, but it is also decreasing with age for younger households. As a result, while the model does produce a hump-shaped market share, the hump is too pronounced and occurs at too young an age. In addition, the model cannot capture the concentration of equity ownership in the data: the rich hold relatively too few stocks.

### C. Baseline results on aggregates for other years

Panels A and B of Table 6 compare the baseline model to the data not only for 1995, but also for 1968 and 1978. For 1968, the baseline model captures the fact that the wealth-GDP ratio as well as the nominal interest rate were both slightly lower than their counterparts in 1995. It accounts for most of the difference because the variance of income shocks was lower in 1968, which implies that less precautionary savings lowers wealth and interest rates. The model captures only a small fraction of the portfolio shift towards housing that took place between 1968 and 1995. As a result, it cannot explain the observed increase in house prices, although it does produce a drop in the price dividend ratio on stocks.

TABLE 6: BASELINE RESULTS

year	beliefs	wealth/ GDP	portfolio weights			lend./ GDP	borr/ GDP	PD ratios		interest rate nominal
			bonds	housing	stocks			housing	stocks	
PANEL A: DATA										
1968		2.41	.17	.51	.32	.61	.20	20.1	32.3	.057
1978		2.08	.16	.68	.17	.56	.23	24.4	18.6	.084
1995		2.51	.15	.59	.26	.70	.31	20.4	23.9	.061
PANEL B: MODEL										
1968	baseline	2.44	.16	.59	.25	.60	.19	23.6	24.8	.059
1978	baseline	2.09	.16	.59	.25	.53	.20	21.6	28.1	.062
1995	baseline	2.51	.15	.60	.25	.70	.31	20.8	22.7	.061

NOTE: Panel A reports the aggregate portfolio weights on bonds, housing and stocks from Figure 2; the gross borrowing and lending numbers from Section the SCF of 1962, the constructed numbers for 1978 and the SCF of 1995; the wealth-to-GDP ratio from Figure 1; the price-dividend ratios for housing and stocks together with the nominal 6-year interest rate. Panel B reports the results computed from the model with baseline beliefs.

With a spread of 2.75% between borrowing and lending rates, the model matches gross borrowing and lending in 1968. A fairly small drop in the spread – 75 basis points – is thus sufficient to account for the increase in the volume of credit. Changing the spread has otherwise little effect on

the equilibrium. This is illustrated in Table 7, where we collect a set of counterfactuals designed to provide intuition for the baseline results in Table 6. The second row of Table 7 recomputes the equilibrium for 1968 with a spread of 2%.

Household portfolios in 1978 were very different from those in 1968 or 1978: wealth as a percent of GDP was much smaller, and there was a strong portfolio shift from stocks into houses. The model with baseline expectations held fixed delivers the first fact, but not the second. The wealth to GDP ratio drops to about twice GDP in both the model and the data. However, the portfolio allocation in the model remains essentially the same as in 1995. As a result, the price dividend ratios of houses falls and that of houses rises, in contrast to what happened in the data.

Three changes in fundamentals are important for the drop in the wealth-to-GDP ratio in 1978. The first is the change in the distribution of endowments illustrated in Figure 3 of Section C. The special feature of the 1978 endowment distribution is that a larger fraction of the funds available for investment resides with the very youngest and oldest cohorts. As shown in the last section, the model predicts that these cohorts have small savings rates, which leads to lower wealth-to-income ratios and pushes interest rates up. This effect is reinforced by the effects of low ex post real interest rates, which also reduce the ratio of initial wealth to income and hence savings. A counteracting force is the reduction in bond supply documented in Figure 4 which tends to lower interest rates and slightly raises the wealth-GDP ratio. Taken together, these effects produce relatively stable interest rates and a low wealth-GDP ratio.

Table 7 reports experiments that consider the role of each of these three factors in isolation, leaving the others fixed at their 1968 values. The experiment in row 3 (1978 bond supply) retains the whole 1968 distribution of households, and changes only the supply of new bonds to the 1978 value, a reduction of 25%. Since the supply of bonds drops, this experiment is similar to the counterfactual experiments that reduce the house and stock supplies in Table 6 above.

The results are also similar: a reduction in the supply of any savings vehicle – here bonds – raises the prices of all savings vehicles; it thus raises the wealth to GDP ratio and lowers the nominal interest rate. Lower nominal interest rates in turn lead to more borrowing within the household sector – borrowing households supply more bonds now, mitigating some of the shortfall of bond

supply from the rest of the rest of the economy. Because of the collateral constraint, borrowing goes along with more investment in housing relative to stocks – house prices increase proportionately more than stock prices and there is a portfolio shift from bonds into housing.

TABLE 7: ADDITIONAL EXPERIMENTS FOR 1968 AND 1978

year	experiment	wealth/ GDP	portfolio weights			lend./ GDP	borr/ GDP	PD ratios		int. rate nom.
			bonds	housing	stocks			housing	stocks	
1968	(1) baseline	2.44	.16	.59	.25	.60	.19	23.6	24.8	.059
1968	(2) spread = 2%	2.44	.16	.60	.24	.66	.25	23.8	24.4	.063
1968	(3) 1978 bond supply	2.52	.13	.62	.25	.57	.24	25.5	26.4	.056
1968	(4) 1978 bond endowm.	2.32	.18	.58	.24	.58	.17	22.0	23.4	.061
1978	(5) baseline	2.09	.16	.59	.25	.53	.20	21.6	28.1	.062
1978	(6) 1968 bond market	2.18	.15	.60	.25	.55	.22	22.9	29.3	.060

NOTE: The table reports model results for various years and counterfactuals.

The experiment in row 4 (1968 bond endowments) retains the 1968 bond supply as well as the 1968 distribution of income, house and stock endowments. However, it constructs bond endowments by updating bond holdings with an interest rate factor that is about 3% lower than the factor used in the 1968 baseline case. The new factor thus lowers the initial wealth of lenders and increases the initial wealth of borrowers; its value is selected to make aggregate payoffs on bonds as a percent of GDP as low as in 1978, where inflation significantly reduced the ex post real interest rate.

The effect of this experiment is the opposite of a reduction in asset supply: a reduction in bond payoffs lowers initial wealth and thus reduces the demand for all savings vehicles and lowers all prices as well as the wealth-GDP ratio. The reason is that the household sector on aggregate is a net lender so that for the majority of households (in wealth-weighted terms) the wealth-income ratio and the savings rate go down. Of course, at the same time, borrower households experience an increase in their wealth-income ratio, so that the distribution of wealth-income ratios becomes less dispersed. This explains the drop in gross credit volume from the experiment.

Line 6 (1968 bond market) uses the 1978 distribution, but fixes the bond supply at its 1968 value. In addition, it increases the bond endowments by raising the interest rate factor by about 3%, so that the aggregate bond endowments is also at the higher 1968 value. This is a way to isolate the effect of the income and endowment distribution from the other two factors. The result is a drop in the wealth-GDP ratio that is twice as large as the drop caused by lower bond endowments alone. It is driven by the lower savings rates of the 1978 population.

## VI The Effects of Inflation

In this section, we use our model to explore the effects of inflation on the price of real assets. The idea is to see whether the effects of (i) expected inflation, (ii) inflation uncertainty and (iii) lower expected stock returns can help us understand why stock prices fell in the 1970s while house prices rose. Key statistics for the various scenarios are reported in Table 8. To facilitate quantitative assessments and comparisons, we repeat the main facts in the first row of Table 8 and the “baseline” exercise based on baseline beliefs in the second row. Real interest rates in the last row are model-implies ex ante real interest rates. For experiments with heterogeneous inflation expectations, we report the range of these rates in the population.

The third row with “hi inflation expectations” increases the mean inflation rate from 4% to 7%, which is the median forecast in the Michigan survey for inflation over the next 5 years. (The 7% forecast from the Michigan survey is comparable with the corresponding long-horizon inflation forecast from the Livingston Survey of professional forecasters, which is 6.9%.) Figure 7 plots the median forecasts, which are available starting in 1979 together with median forecasts for different age cohorts. Interestingly, median forecasts by older households during the great inflation tend to be lower than forecasts by younger households. Our lifecycle model offers a natural laboratory for studying the impact of these heterogenous expectations – the fourth row with “heterogeneous inflation expectations” in Table 5 is based on these cohort forecasts from the Michigan Survey.

The fifth row isolates the effects of heterogeneous expectations, by not taking into account tax effects. Here bond returns are computed based on survey expectations, but after-tax stock returns are computed based on a common forecast of 4% and are thus equal to expected stock returns in

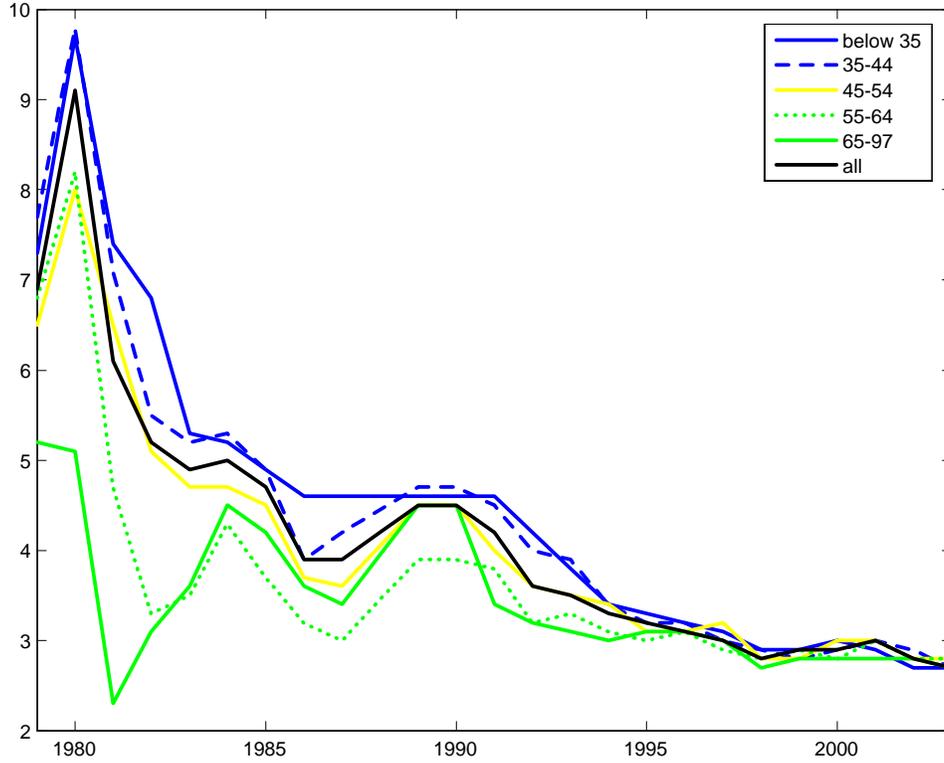


Figure 7: Median inflation expectations over the next 5 years, Michigan Survey of Consumers.

the baseline case. The case of “hi inflation volatility” in row 6 increases the volatility of  $\pi_{t+1}$  by doubling the baseline volatility. Finally, the seventh row considers the case that higher inflation lowers expected future stock returns. The remaining rows consider combinations of these scenarios.

Table 8 illustrates that changes expected inflation both increase interest rates and have significant – and opposite – effects on the value of stocks and houses. The effects are driven by the nonneutrality of capital gains taxes. In particular, households face after-tax real returns on equity  $(1 - \tau) r_t^e - \tau \pi_t$  and bonds  $(1 - \tau) r_t^b - \tau \pi_t$ , while the real return on housing  $r_t^h$  is untaxed. In the case of “hi inflation expectations”, capital gains taxes imply a larger tax burden  $\tau \pi_t$ . Since the tax burden only affects taxable assets, households will shift their portfolio towards housing, which is not taxed. As a consequence, equity and bonds become less valuable, while housing will become more valuable.

The effect on house prices is reinforced by the tax-deductibility of mortgages, since the portfolio shift away from bonds implies that households want to borrow and invest even more into housing.

Indeed, row 3 of Table 8 shows a slight increase in gross borrowing and lending due to the increased real tax subsidy associated with mortgages. The increase in credit volume occurs even though the real interest rate increases.

Table 8: The Effects of Inflation

beliefs	wealth/ GDP	portfolio weights			lend./ GDP	borr./ GDP	PD ratios		interest rate	
		bonds	housing	stocks			housing	stocks	nom.	real
(1) —1978 data —	2.08	.16	.68	.16	.56	.23	24.4	18.6	.084	
(2) baseline	2.09	.16	.59	.25	.53	.20	21.6	28.1	.062	.022
(3) hi infl exp	2.05	.16	.62	.22	.54	.21	22.0	24.6	.098	.029
(4) hetero infl exp	2.05	.16	.63	.21	.65	.32	22.5	23.1	.099	.019-.049
(5) w/o tax effects	2.08	.16	.61	.23	.65	.32	21.9	26.2	.099	.019-.049
(6) hi infl volatility	2.08	.16	.59	.25	.50	.17	21.4	27.3	.066	.026
(7) stock exp – 1.5%	2.02	.16	.64	.20	.52	.19	22.5	21.3	.060	.020
hetero infl exp combined with										
(8) stock exp - 1%	2.00	.16	.67	.17	.66	.32	22.9	18.6	.098	.018-.058
(9) and 3x infl vol	2.00	.16	.68	.16	.58	.25	23.1	18.1	.104	.024-.054

NOTE: Rows 1 and 2 of this table repeat the results from Table 5. Row 3 with “hi inflation expectations” reports results when households expect inflation to be 7%. Row 4 with “heterogeneous inflation expectations” reports results when different cohorts have different inflation expectations measured using the Michigan Survey of Consumers. Row 5 uses the same expectations as row 4 when computing bond returns, but after tax stock returns based on a 4% inflation rate for all cohorts. Row 6 with “high inflation volatility” reports results when expectations are based on a 2 times higher conditional volatility of inflation. Rows 7 report results based on expectations that lower mean pretax stock returns by 1.5%. Rows 8 combines survey expectations (Row 4) with 1% lower mean pretax stock returns. Row 9 combines row 8 with 3 times higher conditional volatility of inflation.

Row 4 in Table 8 considers the case of “heterogenous inflation expectations,” where we allow households’ expectations to differ by age cohort and measure these expectational differences using the Michigan Survey. Here, older households, who tend to have higher wealth-income ratios and thus save more, anticipate inflation to be lower than younger households, who tend to be borrowers.

Older households are therefore happy to lend at nominal rates that are viewed as bargain by young households. As a result, gross borrowing and gross lending goes up, which is the main difference to the case of homogeneous expectations. The other implications are similar: stocks and bonds become cheaper, while houses become more expensive.

Some of the effects in row 4 are due to the tax effect as in row 3 and thus have nothing directly to do with heterogeneity. To isolate the role of heterogeneity, row 5 reports an experiment in which the after real stock returns is held at its baseline value. Heterogeneity is seen to induce a portfolio shift also in the absence of tax effects. Row 6 in Table 8 considers the “hi inflation volatility” case, where households view inflation as more uncertain. This case has little effect on the allocation between real assets, but increases the real interest rate and lowers the volume of credit.

In row 7 of Table 8, we examine a third channel through which changes in inflation expectations affect asset prices: lower stock return expectations. To get an idea about the plausible order of magnitude for pessimism induced by inflation, we use the study of Fama and Schwert (1977), who document that measures of expected inflation are significant predictors of stock returns. The regression results in their Table 6 represent real-time forecasts of returns on a variety of assets – stocks, housing, and bonds – based on data available at that time. Their results indicate that a one percentage-point increase in expected inflation lowers the forecast of real stock returns by roughly 6 percentage points over the following year, but leaves the forecasts of real housing returns unchanged. Assuming that today’s inflation forecasts do not predict stock returns beyond the next year, the 1.5 percentage point increase in expected inflation measured by the Michigan Survey would lower expected real stock returns over the next 6 years by roughly 1.5 percentage points.

Row 7 of Table 8 reports results for return expectations that are 3 percentage points lower than the historical mean. As households lower their return expectations for stocks, other assets become relatively more attractive and thus valuable. As in the counterfactual experiments in Table 5, the price-dividend ratios of stocks and housing are highly sensitive to households’ subjective equity return, while interest rates are not. Hence, this scenario is able to generate large movements in the price of real assets, while bond returns remain relatively stable. If households lower their stock return expectations by 3 percentage points, the model actually overpredicts the movements in price-dividend ratios observed in the data.

While lower stock return expectations can explain the price movements in real assets, they do not explain the runup in nominal rates. Therefore, the results suggest that a combination of lower stock expectations and some change in inflation expectations is needed to explain the 1970s. The effect of combining the three channels is illustrated in the last two rows of Table 8. Experimentation with different specifications has shown that – fixing the observed survey expectations – mean stock returns affect mostly the portfolio weights and have a small negative effect on the interest rate, but do not strongly affect the volume of credit. In contrast, changes in uncertainty affect mostly the volume of credit, but do not cause strong portfolio shifts.

This suggests that we can pick mean stock returns to match the portfolio weight. The required decrease in mean stock returns is only one percentage point, shown in Row 8. Row 9 increases volatility to bring the volume of credit closer to the observed numbers. This combination comes close to matching all the portfolio weights. At the parameter values of Row 9, the model also generates high house prices and actually slightly overpredicts the drop in stock prices. The main shortcoming is that the nominal interest rate is still too high. Nevertheless, we conclude from these experiments that though the three channels described, inflation expectations could have played a significant role for asset prices in the 1970s.

## VII Conclusion

In this paper, we have combined aggregate data from the Flow of Funds with household-level data from successive SCF cross sections. This approach allows us to measure the income and asset endowment distribution across households at the beginning of each trading period. To explicitly capture nonstationarities, we consider a sequence of temporary equilibria of this heterogeneous agent economy. There are three assets – housing, stocks and nominal bonds. There is no riskless asset, so that market are incomplete. During the 1970s, households anticipate higher inflation and view inflation as more uncertain. In particular, we document that young households adjusted their inflation forecasts more than old agents. These changes in inflation expectations make housing more attractive, because of capital gains taxes on stocks and mortgage deductibility. Moreover, agents interpret higher inflation expectations as bad news for future stock returns. Taken together,

these effects can then explain the opposite movements of house and stock prices in the 1970s.

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