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# Should We Expect Star Managers to Be Star Performers? (Dis)Incentive Effects of Fund Flows in Money Management

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# Should We Expect Star Managers to Be Star Performers? (Dis)Incentive Effects of Fund Flows in Money Management

## Abstract

In a dynamic portfolio choice framework, I study the investment policy of fund managers with superior information (or superior skills) when fund flows are an increasing and convex function of end-of-period relative performance. I solve for the optimal strategies in closed-form and show that informed managers will likely take contrarian (or anti-herding) positions based on their private information early in the period until they have achieved a desired outperformance margin. From then on, they hedge against relative underperformance by herding with their less informed (lower skill) peers. Depending on the sensitivity of flows to past performance, informed managers may invest more conservatively than without flow concerns and attain modest returns. Moreover, standard performance measures may either under- or overstate their risk-adjusted returns. Using a sample of top performers in the U.S. mutual fund industry I provide evidence supporting the model implied relation between funds herding behavior and the shape of their flow-performance relationship.

Keywords: Portfolio Delegation, Mutual Funds, Incomplete Information, Fund Flows, Herding, Performance Evaluation.

JEL Classification: C61, C63, D60, D81, D82, D83, G11, G23.

# 1 Introduction

A vast and growing literature in money management examines whether investors' flow-to-relative performance relationships give managers incentives to take risks in order to pretend superior investment skill.<sup>1</sup> In this paper, I turn this question around and explore how truly more skilled managers, which I model as having superior information about fundamentals, should respond to these same incentives. Within a standard portfolio choice framework, I solve for the optimal trading and performance of skilled managers when they compete for investors' money flows against a large pool of less informed peers that possibly learn this fundamentals over time. I show that, despite facing highly convex incentives, skilled managers' performance will not differentiate much from that of their less skilled peers in many situations. I relate these strategies to herding and contrarian behavior by informed managers, and explore its implications for performance evaluation. By estimating flow-performance relationships for a sample of U.S. mutual funds I argue that the model implications are of practical relevance for the study of mutual funds behavior.

Money flows impose short horizon concerns on managers as investors respond to frequently published—usually annual—fund rankings such as MorningStars, U.S. News, Forbes, Barron's, etc. Within such short horizon end-of-period returns may or may not agree with asset fundamentals. When a few managers have private information about these fundamentals, empirically documented flow-performance relationships provide these managers with complex incentives as their private information will most likely not be reflected in peers' performance. Typically, these relationships are an increasing and convex function of a fund performance relative to that of its peers above a certain threshold, but approximately insensitive to poor relative performance below this threshold.<sup>2</sup> On the one hand, the sensitivity of flows to relative past performance implies that by following their private signal skilled managers risk underperforming their peers and suffering money outflows by the end of the period. They may then choose conservative policies to hedge against this possibility. On the other hand, the low sensitivity of flows to very poor relative performance gives skilled managers limited liability-like incentives. They may then choose to take high risks in an attempt to beat their less skilled peers and get large money inflows by the end of the period. Whereas for a given flow-performance relationship the incentives to take risks *conditional* on an interim relative performance state should be independent of managers' information, the *a priori* probability of being in such state and the particular risks managers choose to take should depend on their particular informational advantage.

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<sup>1</sup>See, among others, Brown, Harlow, and Starks (1996), Chevalier and Ellison (1997), Taylor (2003), Kempf and Ruenzi (2008), Chen and Pennacchi (2009), and Huang, Sialm, and Zhang (2011).

<sup>2</sup>See e.g. Sirri and Tufano (1998), Huang, Wei, and Yan (2007) and Ivkovich and Weisbenner (2009) for mutual funds, and Ding, Getmansky, Liang, and Wermers (2008) for hedge funds.

To analyze the impact of these conflicting incentives over managers' decisions, I assume a simple financial market consisting of one risky asset (a stock) and a risk-less bond. The stock mean return is a random variable realized at the beginning of the investment period, while its volatility and the risk-free rate are constant and known. The asset management industry in my model has a large number of managers, a relatively small fraction of which are "informed" (skilled) managers that observe the realization of the stock mean return. This informational advantage could be interpreted as stock-picking ability in a multi-asset framework, and provides a tractable shortcut to modeling higher ability. The rest are "uninformed" (unskilled) managers that only know the mean return prior distribution and try to infer its value from the observation of prices over time.<sup>3</sup> Both types of managers have identical constant relative risk aversion (CRRA) preferences and dynamically allocate their funds' wealth between the two assets over a fixed investment period to maximize utility over end-of-period compensation. This compensation is proportional to the terminal value of assets under management, which in turn depends on a fund relative performance with respect to other funds through the flow-performance relationship. To capture different flow sensitivities at different levels of past performance according to the empirical evidence, I assume this relationship is flat as a function of relative performance up to a certain threshold, and then increases at a higher or lower rate depending on the flow elasticity controlling the convexity of the relationship.

I solve for the equilibrium strategies of all managers and characterize their dynamic trading and performance in closed-form. In a symmetric equilibrium in which managers of the same type choose identical strategies, a first implication of the model is that risk-shifting incentives have no effects on the strategies of the uninformed managers. Absent an informational advantage or, more broadly, absent heterogeneity in managers' preferences and incentives, individual attempts to outperform peers are too risky for these managers, who then invest as if they were trading for their own accounts. Uninformed managers' inference about the stock mean return drives their fund risk exposure, boosting it when markets boom and cutting it down when markets plummet. The resulting "trend chasing" behavior leads these managers to end up the period with either an aggressive risk-exposure in "bull" markets or with a very conservative risk-exposure in "bear" markets.

Informed managers' optimal portfolio includes a "contrarian" component that takes an aggressive opposite stance to uninformed managers' risk exposure (overweighting the stock when uninformed managers underweight it and vice versa when these overweight it), but also a "herding" component that partially replicates uninformed managers' portfolio. Since investors evaluate

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<sup>3</sup>The qualitative results in this paper do not hinge on the particular nature of the informational advantage, so similar results should be obtained when some managers have superior market-timing ability or other asymmetries in information exist.

relative performance at end-of-period, informed managers' dynamic trading strategy attaches a higher weight to the contrarian position early in the period when their informational advantage is largest, until the outperformance margin necessary to rank high is achieved. The contrarian position can increase the absolute risk of their portfolio in some situations, but decrease it significantly in others. Once on the upward-sloping portion of the flow-performance relationship, managers become highly risk-averse and hedge against the risk of underperforming to uninformed peers by increasing the weight to the herding component in their portfolios. Since different sensitivities of the flow-performance relationship to past performance translate into different desired outperformance margins by informed managers I show that, depending on the particular shape of this relationship, equally skilled managers may behave in opposite ways in response to the same market circumstances. By simulating the model over different economic conditions and fund-flow specifications, I derive testable implications regarding the expected herding or contrarian behavior of informed managers in terms of the tracking error volatilities of their portfolios. One such implication is that managers with high sensitivity to middle-range past performance should be expected to take lower risks and to be observed as herding managers in the data despite facing convex fund-flow relationships.

Informed managers' investment policy has an intuitive interpretation in terms of the payoff profiles at horizon they choose. Since uninformed managers' trend-chasing strategy results in a high performance in both extraordinarily good and bad market years, informed managers find it cheaper to outperform in "average" market years in which their uninformed peers learn more—their end-of-period estimated mean return is closer than their prior to the actual value. The resulting performance profile resembles the payoff of an option strategy that delivers gains when uninformed managers' returns do not fluctuate much but large losses in volatile times.

Due to the non-linearity of payoffs as a function of stock performance and their sensitivity to the specific shape of the flow-performance relationship, I next examine the ability of standard performance measures to gauge informed managers' risk-adjusted returns for different parameterizations. For the structural approach adopted in the paper the correct benchmark against which to compare these measures is households' certainty equivalent return to delegation. By simulating the model over all possible states for a wide range of flow-performance specifications I find that the Sharpe ratio and Jensen's alpha can be misleading indicators of risk-adjusted performance for many plausible economic settings, understating performance in some situations and overstating it in others. This problem is particularly pronounced for managers facing flows sensitive to high relative performance, a feature present in the flow-performance relationship estimated by several authors.<sup>4</sup>

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<sup>4</sup>This is the case, for instance, of the mutual funds in Ivkovich and Weisbenner (2009), and of the high-participation-cost funds in Huang, Wei, and Yan (2007).

I evaluate the model testable implications over a sample of actively managed U.S. equity mutual funds during the period 1981-2010. Although skilled managers are not directly observable in the data, I use the model predictions to obtain a sample of potentially better informed mutual funds. Specifically, the probability that informed managers perform better than average is higher than one half and relatively homogeneous for a large set of fund flow specifications according to the model simulations. Thus, I identify “informed” mutual funds in the sample with those that outperform the average funds in their objective category, irrespective of their outperformance margin, the greatest number of times as a proportion of their years in activity. In this way I seek to avoid the possibility of dropping out of the sample, if skilled managers instead were identified as those delivering the highest raw or (factor-based) risk-adjusted returns, truly informed but *herding* mutual funds.

Direct examination of the model implications would require inspection of how managers react, in terms of herding or contrarian behavior, to different flow-performance relationships. Since the theory is not conclusive about the factors determining the shape of this relationship, I follow an indirect approach instead and test whether herding and contrarian funds respond to different fund flow sensitivities to past performance. Based on different measures of average herding, I screen a group of “herding” and another of “contrarian” funds out of the sample of informed mutual funds. Controlling for fund characteristics, I then estimate different flow-performance relationships for each of these groups and evaluate whether the observed differences, if any, agree with the model empirical predictions. I estimate piecewise linear regressions allowing for separate flow sensitivities for the herding and contrarian groups to past performance. Consistent with the model predictions, I find that flows to herding funds are more sensitive to poor past performance, inducing significant relative concerns, while flows to contrarian funds display higher sensitivity on the top, leading to a more convex flow performance relationship. These findings are robust to using different criteria for selecting informed funds and for measuring average herding behavior.

I argue that the incentive effects of fund flows could help understand managers’ behavior in relation to two phenomena that have drawn the attention of several authors recently: institutional investors’ holdings of “bubble” stocks during the late 1990s, and “closet-indexing” by active managers.<sup>5</sup> Bayesian learning by the uninformed managers in the model leads them to increase significantly their exposure to stocks that have shown sustained abnormally high returns (relative to their true but unobservable mean rate) in the past, very much like is observed during bubble-like asset price developments. However, informed managers fully aware of this “bubble” in the model may still choose to “ride” it if their flow-performance relationship is sensitive enough to poor past performance or shows low convexity. Similarly, if the average (unskilled) fund manager

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<sup>5</sup>See e.g. Brunnermeier and Nagel (2004) and Dass, Massa, and Patgiri (2008) on institutional investors and bubbles, and Cremers and Petajisto (2009) on closet-indexing.

within an objective category follows a benchmark closely (i.e. is a closet indexer) and moreover money flows in this category are very sensitive to moderately poor past performance, the model predicts that a skilled/informed manager in the same category may choose to optimally herd with the average fund. Such a manager will look like a closet indexer and deliver low or insignificant Jensen's alpha in spite of its superior ability and of potentially creating positive net benefits to delegating investors. By contrast, the true ability of active and informed mutual funds (high return tracking error relative to the average fund) responding to highly convex fund flows may be overstated by performance measures that fail to capture the non-linear risks inherent in their option-like strategies.

This paper is closely related to the literature on risk-taking behavior of mutual funds in a tournament setting. Basak, Pavlova, and Shapiro (2007, 2008) find that managers subject to convex fund-flow to relative performance relationships gamble to finish ahead of their benchmark but only over a finite-range of interim performance.<sup>6</sup> Chen and Pennacchi (2009) argue that portfolio managers have no incentives to increase the *overall* volatility of their portfolios but only the variance of the tracking error, that is, the departure of their portfolios from the benchmark portfolio. Cuoco and Kaniel (2011) investigate the equilibrium asset pricing implications of commonly used management compensation contracts. Basak and Makarov (2011) analyze the strategic interactions that emerge from tournaments among managers for investors flows and find an equilibrium in which managers' policies are driven by chasing and contrarian behaviors in some situations, and by gambling behavior in others. Since the focus is on the risk-seeking incentives of portfolio managers, this literature assumes symmetric information both among managers and between managers and delegating households. Risk-shifting is then equivalent to pure gambling, and moreover households always derive negative returns from delegation. In order to reconcile these negative results with the vast size of the asset management industry in practice, an implicit assumption that delegation brings about large savings on transaction costs to households is needed. By incorporating asymmetric information among managers and between these and investors, risk shifting in my model can be interpreted as contrarian herding based on private information, and managers' informational advantage may make delegation valuable despite the risk aversion misalignment between managers and investors.<sup>7</sup> Furthermore, I show that industry equilibrium considerations may offset the risk-seeking incentives within a large pool of identical managers.

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<sup>6</sup>Although not in a tournament setting, Carpenter (2000) studies the incentives provided by option-like compensation within a dynamic portfolio choice approach and shows that option compensation does not strictly lead to greater risk seeking.

<sup>7</sup>In a two-stage model with constant absolute risk aversion (CARA) preferences, Maug and Naik (2011) show that performance concerns relative to peers in managerial contracts may induce informed managers to "go with the flow" and disregard their own superior information, a situation that nevertheless may be in the interest of delegating agents.

Herding behavior by informed portfolio managers with less informed agents in multi-period settings has been analyzed in models featuring career concerns. In a two-period setup, Scharfstein and Stein (1990) show that if managers worry about the markets perception about their ability, they could ignore useful private information in their portfolio choice and imitate the rest of managers. Froot, Scharfstein, and Stein (1992) argue that, because managers have short evaluation periods, they cannot afford to wait until their (eventually correct) private information is revealed and incorporated in asset prices. Short-term managers may then herd by acquiring the same information. In a model with differential reputation among financial intermediaries (FI), Villatoro (2009) finds equilibria in which FI with high reputation are prone to invest in information, whereas those with poor reputation imitate other FI's portfolio decisions. In all three cases, these models do not examine the effects of managers' benchmarking against their peers, and thus the implicit incentives they analyze differ from the non-linear flow to relative-performance relations in this paper. Moreover, their focus is a relatively long investment horizon, whereas this paper is about managers' portfolio choice *within* one of the many periods in career concerns models. An exception in the reputation concerns literature characterizing manager's within-period investment policy is Arora and Ou-Yang (2001). These authors develop a dynamic principal-agent model with complete information stressing the interplay between implicit incentives (given by reputation concerns) and explicit incentives (given by the compensation contract) in which the optimal portfolio policy suggests some herding behavior. My paper resembles the within-period analysis of these authors, but allows for convexities in the fund-flow relationship and for asymmetric information among managers.

The rest of the paper proceeds as follows. Section 2 sets up the financial markets, preferences and investment management industry structure. I solve for the equilibrium strategies and performance in Section 3. I derive testable predictions and implications for performance measurement in Section 4. I present empirical support for the model predictions in Section 5, and suggest other potential applications of the model results in Section 6. Section 7 concludes, while the technical details are summarized in Appendix A through C.

## 2 Model Setup

I consider an economy in which households  $h$  delegate their financial wealth to portfolio management companies over a certain horizon denoted by  $[0, T]$ .<sup>8</sup> I refer to these companies as mutual funds, although the analysis in this paper is equally applicable to hedge funds and other profes-

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<sup>8</sup> In general, this horizon extends over a sequence of periods  $[(n-1)T, nT]$ ,  $n \in \{1, 2, \dots, N\}$  and  $N < \infty$ , but I focus the analysis on only one of such periods (e.g. one calendar year).



sional money managers whose fund flows are sensitive to relative past performance. Delegation occurs at  $t = 0$  and no additional fund share purchases or redemptions take place until  $t = T$ . All agents have constant relative risk aversion (CRRA) preferences, with coefficient of relative risk aversion  $\gamma > 1$  for fund managers and  $\gamma_h > 1$  for households:<sup>9</sup>

$$u(w) = \begin{cases} \frac{w^{1-\tilde{\gamma}}}{1-\tilde{\gamma}} & \text{if } w \geq 0, \\ -\infty & \text{if } w < 0, \end{cases} \quad (1)$$

for  $\tilde{\gamma} \in \{\gamma, \gamma_h\}$ .

Although the delegation decision is exogenous to the model, it is grounded on the assumption that some fund managers have private information about asset returns and all of them observe asset prices throughout the investment period  $[0, T]$ , so households may find delegation valuable.<sup>10</sup>

## 2.1 Financial markets

Mutual funds have access to financial markets consisting of one risk-free and one risky assets, with prices  $\beta$  and  $S$  respectively. The risk-less asset can be a short-term bond or a bank account, whereas the risky asset can be a stock or any portfolio of risky assets (e.g. the market portfolio or other traded benchmark). Each mutual fund is an atomistic participant in the asset markets and takes asset price dynamics as exogenously given. The bond has initial price  $\beta_0 = 1$  and pays a constant interest rate  $r$  per unit time, such that its price dynamics are  $d\beta_t = r\beta_t dt$ . The stock has initial price  $S_0 = s$ , and dynamics given by the following SDE:

$$dS_t = S_t(\mu dt + \sigma dB_t), \quad (2)$$

where  $B$  is a standard Brownian motion process defined on a filtered probability space  $(\Omega, \mathcal{F}, P, \{\mathcal{F}\}_{t \leq T})$ , on the interval  $0 \leq t \leq T$ . All parameters are constant.

The risk-free rate  $r \geq 0$  and volatility  $\sigma > 0$  are observable by all mutual fund managers (and households), as are security prices. The stock mean return  $\mu$ , however, is the unobservable realization at  $t = 0$  of a random variable with normal distribution  $\mathbf{N}(r + \sigma m, \sigma^2 v_0)$ , for some given constants  $m$  and  $v_0 > 0$ . Equivalently, the ‘‘market price of risk’’  $\eta \equiv (\mu - r)/\sigma$  is an unobservable draw from a normal distribution  $\mathbf{N}(m, v_0)$  at  $t = 0$ .

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<sup>9</sup> The assumption that managers’ risk-tolerance is the same for informed and uninformed managers is for notational simplicity only. All qualitative results hold with different type of managers featuring different risk aversion.

<sup>10</sup> In a more general model, investors would be allowed to dynamically choose how much of their portfolio to hold directly and how much to hold indirectly through investment companies. For a model in which investors allocate money to mutual funds dynamically over time, see Hugonnier and Kaniel (2010).

## 2.2 Investment companies and fund flows

I assume an investment company industry consisting of a large pool of mutual funds that, except possibly for their managers' ability, are otherwise identical. These mutual funds can be seen as a relatively homogeneous group following the same objective category (e.g. the set of "Growth" or "Growth and Income" mutual funds) and regarded as close substitutes by delegating households who, in determining a fund share purchases or redemptions at  $t = T$ , compare this fund performance to that of all others in the same group. To isolate the effects of relative performance concerns on managers' asset allocation decisions, I assume moreover that this group of funds is sufficiently small relative to the overall asset markets and has not a significant impact on prices.

Fund managers can be skilled or unskilled. I model superior skill/investment ability as access to private information about the realization of the market price of risk  $\eta$  at  $t = 0$ . Therefore, all fund managers belong to either one of two sets: a set  $\mathcal{I}$  of *informed* managers (type " $I$ " or " $I$ -managers") that observe  $\eta$  (equivalently, the stock mean return  $\mu$ ) at  $t = 0$ , and a set  $\mathcal{U}$  of *uninformed* managers (type " $U$ " or " $U$ -managers") that do not observe  $\eta$  but have to infer its value from the information they have available. Since  $v_0 > 0$ , uninformed managers face parameter uncertainty. This informational structure could be interpreted as stock-picking ability by the skilled managers in a multi-asset framework, and provides a tractable shortcut to modeling higher ability for some managers while at the same time allowing for the possibility that the rest of them reduce their informational/ability disadvantage over time.<sup>11</sup> All  $I$ -managers are identical to each other, as are all  $U$ -managers. Furthermore, I assume that  $\mathcal{U}$  has unit measure ( $m(\mathcal{U}) = 1$ ) whereas  $\mathcal{I}$  has zero measure ( $m(\mathcal{I}) = 0$ ). This assumption is meant to reflect an industry structure where "star" (skilled/informed) managers are only a small fraction of the overall manager population following the same objective category. In particular, it will imply that  $I$ -managers' portfolio choices are conditioned by relative performance concerns with respect mostly to uninformed peers.

Each manager  $i$  of type  $J \in \{I, U\}$  dynamically chooses an investment policy  $\phi_t^J(i)$  representing the fraction of the fund wealth  $W_t^J(i)$ , or assets under management, to be allocated to the risky asset at time  $t$  (fund  $i$ 's *risk exposure*). Given initial wealth  $W_0^J(i) = w^J$ , managers' portfolio value processes follow:

$$dW_t^J(i) = W_t^J(i) (r + \phi_t^J(i)\sigma\eta) dt + W_t^J(i)\phi_t^J(i)\sigma dB_t, \quad (3)$$

for  $J \in \{I, U\}$ .

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<sup>11</sup>Proprietary research may lead fund managers to find profitable trading strategies that would characterize such managers as "informed". As this trading strategies becomes public (e.g. the "momentum" strategy) their informational advantage will shrink over time. Similarly, a reduction in the ability wedge between skilled and unskilled manager would occur naturally as the industry competitive forces drive the least skilled managers out of the market, only to be replaced by higher ability contestants.

I assume that each manager's portfolio decisions are unobservable by other managers, as are fund returns during  $[0, T]$ .<sup>12</sup> Thus, uninformed manager's available information  $\mathcal{F}_t^S$  at each moment of time  $t \in [0, T]$  is given by observed market prices up to  $t$ :  $\mathcal{F}_t^S \equiv \sigma \{S_u, 0 \leq u \leq t\}$ .

Fund managers' compensation is due at horizon  $T$  and set in proportion to their funds' collected fees, which in turn consist of a fraction of their assets under management.<sup>13</sup> Depending on a fund performance relative to its peers, households are assumed to purchase or redeem additional fund shares according to a exogenously given *flow-to-relative performance relationship*. Letting  $Y$  represent the average performance of all mutual funds:  $Y_t \equiv \int_I W_t^I(i)di + \int_U W_t^U(i)di$  for  $t \in [0, T]$ , managers receive households' money flows at the rate  $f_T$  at the end of the period following the functional form in Basak and Makarov (2011):

$$f_T = k \mathbb{1}_{\{R_T^J(i) < \delta R_T^Y\}} + k \left( \frac{R_T^J(i)}{\delta R_T^Y} \right)^\alpha \mathbb{1}_{\{R_T^J(i) \geq \delta R_T^Y\}}, \quad (4)$$

where  $k, \alpha > 0$ ,  $R_T^J(i) \equiv W_T^J(i)/W_0^J(i)$ , and  $R_T^Y \equiv Y_T/Y_0$ ,  $J \in \{I, U\}$ . Note that  $f_T$  is always positive, with  $f_T > 1$  denoting additional inflows and  $f_T < 1$  denoting outflows. This relation resembles the convex payoff of a call option: for managers underperforming a fraction  $\delta \leq 1$  of the industry average return  $R_T^Y$ , fund flows are insensitive to relative performance and arrive (leave) at the constant rate  $k$ ; for those outperforming the same fraction of the industry average return, fund flows are *increasing* in performance relative to all other mutual funds (*peers*) at a rate  $k \left( \frac{R_T^J(i)}{\delta R_T^Y} \right)^\alpha$ . Since  $k$  is a scaling parameter that does not modify incentives in the margin, the two key parameters controlling the shape of the flow performance relationship are the *performance threshold*  $\delta$  and the *flow elasticity*  $\alpha$ . The performance threshold is the minimum relative performance that has an effect on end-of-period fund flows, whereas the flow elasticity controls fund flows rate of growth in the top performance region (i.e. relative performance above the threshold  $\delta$ ).

The specification (4) is meant to capture the flow-performance relationships empirically documented by e.g. Brown, Harlow, and Starks (1996), Chevalier and Ellison (1997), Sirri and Tufano (1998) and Ivkovich and Weisbenner (2009) for mutual funds, and Ding, Getmansky, Liang, and Wermers (2008) for hedge funds. These authors observe that investors withdraw money from poor performers at an approximately flat rate, but supply additional capital to good performers

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<sup>12</sup> This assumption prevents uninformed managers to learn about  $\mu$  from the observation of informed managers' investment policies throughout the period. Such restriction seems plausible for many institutions like hedge funds that are largely secretive about their trading strategies. While it may seem less realistic for mutual funds given the ready availability of their net asset value (NAV) figures, it becomes more plausible once we account for the many fees and expenses involved in NAV's determination, usually available with some delay (e.g. one month/quarter) and on a less frequent basis.

<sup>13</sup> This compensation reflects the prevailing fee structure in the mutual fund industry. According to Elton, Gruber, and Blake (2003), 98.3% of all U.S. bond and equity mutual funds and around 89.5% of the assets under management had fixed fees as of 1999.

at an increasingly higher rate. In general, their estimations imply a value of  $k$  less than one and a flow elasticity  $\alpha$  greater than one, with the relative performance threshold  $\delta$  taking values both above and below one depending on different fund characteristics. In all cases, this literature hints at a convex fund-flow relationship that is approximately insensitive to very poor past relative performance.<sup>14</sup>

Without loss of generality, I normalize both types of funds' initial wealth to be equal:  $w^I = w^U = w$ , so  $Y_0 = w$ . Defining  $\bar{\gamma} \equiv \gamma + \alpha(\gamma - 1)$ , (1) and (4) imply that manager  $i$  of type  $J \in \{I, U\}$  optimally chooses her fund risk-exposure  $\{\phi_t^J(i) : 0 \leq t \leq T\}$  to maximize expected utility of final wealth:

$$u(f_T W_T^J(i)) = \begin{cases} \frac{k^{1-\gamma}}{1-\gamma} (W_T^J(i))^{1-\gamma} & \text{if } W_T^J(i) < \delta Y_T \text{ (underperformance),} \\ \frac{k^{1-\gamma}}{1-\gamma} (W_T^J(i))^{1-\bar{\gamma}} (\delta Y_T)^{\bar{\gamma}-\gamma} & \text{if } W_T^J(i) \geq \delta Y_T \text{ (outperformance),} \end{cases} \quad (5)$$

subject to initial wealth  $w$  and the self-financing constraint (3). The objective function (5) makes it clear how, whenever  $W_T^J(i) \neq Y_T$ , the flow-performance relationship (4) affects manager  $i$ 's incentives: in those states of the economy in which an  $I$ -manager outperforms the industry average ( $W_T^I \geq \delta Y_T$ ), (i) her *effective* relative risk aversion increases to  $\bar{\gamma} > \gamma$ , and (ii) the increase is larger for more convex (higher  $\alpha$ ) relationships in the outperformance region.<sup>15</sup>

### 3 Optimal Investment Strategies

When managers choose their investment policies to solve (5), each one considers her peers' optimal policies as determinants of the industry average performance  $Y$ . Managers' rationality then requires finding the industry Nash equilibrium (if such equilibrium exists) in which manager  $i$ 's optimal strategy ( $i \in \mathcal{I} \cup \mathcal{U}$ ) is a best response to all other managers' optimal policies. Whereas such equilibrium (equilibria) may be hard to characterize in a more general setting, the assumed small number of informed managers ( $m(\mathcal{I}) = 0$ ) simplifies the analysis considerably.<sup>16</sup> In particu-

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<sup>14</sup> The convexity in the flow-to-relative performance relationship may be the result of rational investors learning managerial ability from past performance, along with managers' implicit option to abandon poorly performing strategies (Lynch and Musto (2003)), decreasing returns to scale in money management (Berk and Green (2004)), or costs of participating in mutual funds (Huang, Wei, and Yan (2007)).

<sup>15</sup> The increase in risk aversion occurs because changes in *actual* wealth are augmented by a flow rate  $k \left( \frac{R_T^I}{\delta R_T^U} \right)^\alpha > k$  in the outperformance region, but only by a flow rate  $k$  outside of it. The flow rate for a top performer is not only increasing in  $\alpha$ , but also itself increasing in wealth. Therefore, effective wealth fluctuates more in response to the same change in actual wealth in this region than in the underperformance region, raising manager's effective risk aversion.

<sup>16</sup> Equally important in order to find a Nash equilibrium in this setup is the assumption that uninformed managers do not observe informed managers' portfolio choice (or assets under management). This assumption rules out strategic interdependence, beyond that induced by the average industry performance  $Y$ , between the two types of managers.

lar, if a symmetric equilibrium exists in which all  $J$ -managers adopt the same investment policy  $\hat{\phi}_t^J$ , identical initial assets under management  $w$  imply that  $W^J(i) = W^J$  for  $J \in \{I, U\}$ . Then:

$$\begin{aligned} Y_t &= \int_{\mathcal{I}} W_t^I(i) di + \int_{\mathcal{U}} W_t^U(i) di \\ &= W_t^I m(\mathcal{I}) + W_t^U m(\mathcal{U}) \\ &= W_t^U, \end{aligned} \tag{6}$$

i.e. each fund's average peer performance in this economy is given by the performance of the *uninformed* managers. Equation (6) implies that  $U$ -managers' problem can be decoupled from that of  $I$ -managers' and their equilibrium strategies studied in isolation. If such equilibrium exists, the industry average  $Y$  in informed manager's problem is completely determined outside this problem and we can look for their optimal policies within the standard portfolio choice approach. The next two sections characterize an equilibrium in uninformed and informed managers' strategies. All proofs are given in Appendix A.

### 3.1 Peer Group Performance: Uninformed Managers Equilibrium

Uninformed managers have only partial information about the market price of risk  $\eta$  but can learn about it by observing realized stock returns over time. The information structure I assume is the same as that considered by, among others, Brennan (1998) and Cvitanic, Lazrak, Martellini, and Zapatero (2006), and represents a particular case of the incomplete information case in continuous-time studied by Detemple (1986, 1991), Dothan and Feldman (1986) and Gennotte (1986). Given prior  $\mathbf{N}(m, v_0)$  and flow of information  $\mathcal{F}_t^S$  as of time  $t \in (0, T]$ , uninformed managers seek to extract information about  $\eta$  by solving the following filtering problem:<sup>17</sup>

$$\begin{cases} dR_t = \frac{dS_t}{S_t} = (r + \sigma\eta)dt + \sigma dB_t, & R_0 = 0, & (\text{observation}) \\ d\eta = 0, & \eta \sim \mathbf{N}(m, v_0). & (\text{state}) \end{cases} \tag{7}$$

An application of the Kalman-Bucy filter (see e.g. Liptser and Shirayayev (2001)) to (7) allows us to characterize the distribution of  $\eta$  conditional on  $\mathcal{F}_t^S$  as Gaussian, with conditional mean  $\tilde{\eta}_t \equiv E[\eta | \mathcal{F}_t^S]$  and variance  $v_t \equiv E[(\eta - \tilde{\eta}_t)^2 | \mathcal{F}_t^S]$  satisfying:

$$\begin{cases} d\tilde{\eta}_t = v_t d\tilde{B}_t, \\ dv_t = -v_t^2 dt, \end{cases} \tag{8}$$

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<sup>17</sup>  $R$  is the stock "return" process, defined by  $R_t \equiv \int_0^t (S_u)^{-1} dS_u$ .

with  $\tilde{\eta}_0 = m$  and  $v_0$  as initial values for  $\tilde{\eta}$  and  $v$ , respectively.  $\tilde{B}$  is a standard Brownian motion with respect to  $\mathcal{F}_t^S$ , known as the *innovation process*:<sup>18</sup>

$$d\tilde{B}_t = \frac{1}{\sigma} [dR_t - (r + \sigma\tilde{\eta}_t)dt] = dB_t + (\eta - \tilde{\eta}_t)dt. \quad (9)$$

Under parameter uncertainty, markets are not complete with respect to the *true* states of the world. However, they are complete with respect to the *observable* states of the economy (a single risky asset  $S$  driven by a single Brownian motion  $\tilde{B}$ ). Indeed, we can rewrite each  $U$ -manager's optimization problem (5) in terms of observables as:

$$\max_{(\phi_t^U(i))_{0 \leq t \leq T}} \tilde{E} [u(f_T W_T^J(i))] \quad (10)$$

$$s.t. \quad dW_t^U(i) = W_t^U(i) (r + \phi_t^U(i)\sigma\tilde{\eta}_t) dt + W_t^U(i)\phi_t^U(i)\sigma d\tilde{B}_t, \quad (11)$$

and initial wealth  $w$ .  $\tilde{E}(\cdot)$  denotes the expectation with respect to an equivalent probability  $\tilde{P}$  under which  $\tilde{B}$  is a standard Brownian motion. Problem (10) can now be addressed in a full-information framework. Absent arbitrage opportunities, uninformed managers see financial markets as driven by a unique state-price deflator  $\tilde{\pi}$  with dynamics  $d\tilde{\pi}_t = -r dt - \tilde{\pi}_t \tilde{\eta}_t d\tilde{B}_t$ . The dynamic budget constraint (11) can be restated (see e.g. Karatzas and Shreve (1998)) as:

$$\tilde{E} [\tilde{\pi}_T W_T^U(i)] = w \quad (12)$$

and, using the martingale/duality approach of Cox and Huang (1989) and Karatzas, Lehoczky, and Shreve (1987), the dynamic optimization problem (10) can be solved as a static problem over final payoffs  $W_T^U(i)$ .

Let  $B_t^Q = \tilde{B}_t + \int_0^t \tilde{\eta}_s ds = B_t + \eta t$  denote the risk-neutral Brownian motion. Proposition 1 guarantees the existence of a symmetric equilibrium in the uninformed managers' strategies and characterizes the resulting optimal investment policies and wealth dynamics:

**Proposition 1.** *Let  $\tau \equiv T - t$ . Under the economy of Section 2, there exists a symmetric equilibrium in managers' investment policies in which, for  $i \in \mathcal{U}$  and  $t \in [0, T]$ , optimal fund value  $\hat{W}_t^U(i)$  and risk exposure  $\hat{\phi}_t^U(i)$  are given by:*

$$\hat{W}_t^U(i) = \hat{W}_t^U = (\lambda_U \tilde{\pi}_t)^{-\frac{1}{\gamma}} e^{-(1-\frac{1}{\gamma})r\tau} g_1 \left( \frac{1}{\gamma}, t, \tilde{\eta}_t; T \right), \quad (13)$$

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<sup>18</sup> The innovation process can be interpreted as the "surprise" component of observed returns, measured by the normalized deviation of returns from their conditional mean.

$$\hat{\phi}_t^U(i) = \hat{\phi}_t^U = \frac{1}{\gamma + (\gamma - 1)v_t\tau} \frac{\tilde{\eta}_t}{\sigma}, \quad (14)$$

with

$$\lambda_U = \left[ g_1 \left( \frac{1}{\gamma}, 0, m; T \right) / w \right]^\gamma, \quad (15)$$

and

$$g_1(\zeta, t, x; T) \equiv \sqrt{\frac{(1 + v_t(T - t))^{1-\zeta}}{1 + (1 - \zeta)v_t(T - t)}} \exp \left\{ -\frac{\zeta}{2} \frac{(1 - \zeta)(T - t)}{1 + (1 - \zeta)v_t(T - t)} x^2 \right\}, \quad (16)$$

where the conditional mean  $\tilde{\eta}$  and variance  $v$  of the market price of risk are given by:

$$\begin{cases} \tilde{\eta}_t = v_t \left( B_t^Q + \frac{m}{v_0} \right), \\ v_t = \frac{v_0}{1 + v_0 t}. \end{cases} \quad (17)$$

A comparison of equations (14) and (17) with Theorem 1 in Cvitanic, Lazrak, Martellini, and Zapatero (2006) reveals that, in a symmetric equilibrium, uninformed managers invest “as if” the flow-performance relationship were flat ( $f_T = 1$ ) and no risk-shifting incentives existed. Absent an informational advantage or, more broadly, absent heterogeneity in managers’ preferences and incentives, individual attempts to outperform peers are too risky for a risk-averse manager. Proposition 1 then highlights an important equilibrium implication of managers’ policies that can be overlooked from an individual portfolio choice perspective: highly convex payoffs do not ensure risk-shifting behavior by risk-averse fund managers *per se*. When they are subject to convex flow-performance relationships, managers may not only shift risk over a finite range of past performance (Basak, Pavlova, and Shapiro (2007)), but they may choose not to shift risk at all over *the whole range* of past performance if their peers have similar preferences and information.

$U$ -managers inference  $\tilde{\eta}$  about the market price of risk drives their fund risk exposure, boosting it when markets boom and cutting it down when markets plummet.<sup>19</sup> Since uncertainty  $v_t$  decays deterministically over time, their informational disadvantage with respect to informed managers also shrinks as time goes by. In both unexpectedly good and bad market years in which observed stock returns deviate significantly from fundamentals (as given by  $\mu$ ) by  $T$ , their “trend chasing” behavior implies that uninformed managers will end up the period with either an aggressive risk-exposure in “bull” markets or with a very conservative risk-exposure in “bear” markets. These learning and asset allocation decisions over time entail additional risks on informed managers’ payoffs.

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<sup>19</sup> Note that  $\hat{\phi}_t^U$  can be rewritten as:  $\frac{\tilde{\eta}_t}{\gamma\sigma} - \frac{(\gamma-1)v_t\tau}{\gamma+(\gamma-1)v_t\tau} \frac{\tilde{\eta}_t}{\gamma\sigma}$ . The first component is the standard (conditional) mean-variance efficient allocation to the risky asset scaled by  $U$ -managers’ coefficient of relative risk aversion  $\gamma$ , whereas the second component represents the manager’s hedging demand against unexpected changes in her *derived* investment opportunity set (future reassessments of the market price of risk  $\eta$ ). The hedging demand goes to zero as the end of the period approaches,  $t \rightarrow T$ , or as uncertainty  $v_t$  vanishes.

### 3.2 Informed Managers' Trading and Payoff Profile

Informed managers have access to all the information their uniformed peers have, so even without observing their asset allocation at every point in time they can solve for  $\hat{\phi}^U$  (and  $\hat{W}^U$ ) on their own. In addition, informed managers face complete information about asset returns: they form expectation  $E(\cdot)$  with respect to the actual (objective) probability measure  $P$ . They also observe past realizations of  $B$  and thus face complete markets with respect to the *actual* states of the economy, with a unique state price deflator  $\pi$  following dynamics  $d\pi_t = -r dt - \pi_t \eta_t dB_t$ . Once more, this allows us to re-express  $I$ -managers' dynamic budget constraint (3) as a static one:

$$E [\pi_T W_T^I(i)] = w, \quad (18)$$

and transform the dynamic optimization program (5) into a static problem over final payoffs  $W_T^I(i)$ , subject to  $U$ -managers' optimal wealth process (13) at  $t = T$ . Absent implicit incentives ( $f_T \equiv 1$ ), problem (5) subject to (18) is identical to that solved by Merton (1971) and results in a constant weight in the risky asset  $\hat{\phi}^J(i)$  as the optimal investment policy:

$$\hat{\phi}_t^I(i) = \phi^M \equiv \frac{\mu - r}{\gamma \sigma^2} = \frac{\eta}{\gamma \sigma}. \quad (19)$$

Henceforth I refer to  $\phi^M$  as the *Merton* policy. It represents the *normal* risk-exposure an informed manager would take if she were trading for her own account. In the presence of fund flow incentives ( $f_T \neq 1$ ) informed managers will hedge against fluctuations in fund flows and their portfolio will generally differ from the Merton policy. Proposition 2 characterizes  $I$ -managers' optimal fund value and investment strategy during  $[0, T]$  as a function of the state-price deflator  $\pi$  and  $U$ -managers' inferred market price of risk  $\tilde{\eta}$ :

**Proposition 2.** *Let  $\nu(t; T) \equiv \frac{1+v_t(T-t)}{1+(1-\theta)v_t(T-t)}$  and  $\theta \equiv \frac{\tilde{\gamma}-\gamma}{\tilde{\gamma}\gamma} < 1$ . Under the symmetric equilibrium for  $U$ -managers of Proposition 1, the optimal fund value of manager  $i \in \mathcal{I}$  at time  $t \in [0, T]$  is given by:*

$$\begin{aligned} \hat{W}_t^I(i) = \hat{W}_t^I = & (\lambda_I \pi_t)^{-\frac{1}{\tilde{\gamma}}} Z(\gamma, \tau) [\mathcal{N}(d_{1,t}) + 1 - \mathcal{N}(d_{2,t})] + (\lambda_I \pi_t)^{-\frac{1}{\tilde{\gamma}}} Z(\tilde{\gamma}, \tau) g_1(\theta, t, \tilde{\eta}_t; T) g_2(t, \tilde{\eta}_t) \\ & \times \exp \left\{ \frac{\theta \tau}{1 + (1 - \theta) v_t \tau} \frac{\eta}{\tilde{\gamma}} \left( \tilde{\eta}_t + \frac{\eta}{2\tilde{\gamma}} v_t \tau \right) \right\} [\mathcal{N}(d_{3,t}) - \mathcal{N}(d_{4,t})], \end{aligned} \quad (20)$$



and her optimal risk-exposure is given by:

$$\begin{aligned} \hat{\phi}_t^I(i) = \hat{\phi}_t^I = & \omega_t \left( \frac{\eta}{\gamma\sigma} + \frac{1}{\sigma\sqrt{\tau}} \frac{\mathcal{N}'(d_{2,t}) - \mathcal{N}'(d_{1,t})}{\mathcal{N}(d_{1,t}) + 1 - \mathcal{N}(d_{2,t})} \right) \\ & + (1 - \omega_t) \left( \nu(t; T) \frac{\eta}{\bar{\gamma}\sigma} + \frac{\theta}{1 + (1 - \theta)v_t\tau} \frac{\tilde{\eta}_t}{\sigma} + \frac{1}{\sigma} \sqrt{\frac{\nu(t; T)}{\tau}} \frac{\mathcal{N}'(d_{4,t}) - \mathcal{N}'(d_{3,t})}{\mathcal{N}(d_{3,t}) - \mathcal{N}(d_{4,t})} \right), \end{aligned} \quad (21)$$

where the Lagrange multiplier  $\lambda_I$  solves  $E \left[ \tilde{\pi}_T \hat{W}_T^I \right] = w$ ,  $\mathcal{N}(\cdot)$  is the standard normal cumulative distribution function,

$$\omega_t \equiv \frac{(\lambda_I \pi_t)^{-\frac{1}{\gamma}} Z(\gamma, \tau) (\mathcal{N}(d_{1,t}) + 1 - \mathcal{N}(d_{2,t}))}{\hat{W}_t^I}, \quad 0 \leq \omega_t \leq 1, \quad (22)$$

and

$$\begin{aligned} g_2(t, x; T) & \equiv A_1 \sqrt{(1 + v_0 t)^{-\theta}} \exp \left\{ \theta r T + \frac{\theta}{2} \left( \frac{x^2}{v_t} - \frac{m^2}{v_0} \right) \right\}, \quad Z(\zeta, t) \equiv \exp \left\{ -\frac{\zeta-1}{\zeta} \left( r + \frac{\eta^2}{2\zeta} \right) t \right\}, \\ d_{1,t} & \equiv \frac{\frac{\eta - \tilde{\eta}_t}{v_t} + \frac{\gamma-1}{\gamma} \eta \tau - \varphi(\lambda_I)}{\sqrt{\tau}}, \quad d_{2,t} \equiv d_{1,t} + 2 \frac{\varphi(\lambda_I)}{\sqrt{\tau}}, \\ d_{3,t} & \equiv \frac{\frac{\eta - \nu(t; T) \tilde{\eta}_t}{v_t} + \frac{\bar{\gamma} - \nu(t; T)}{\bar{\gamma}} \eta \tau + \varphi(\lambda_I)}{\sqrt{\nu(t; T) \tau}}, \quad d_{4,t} \equiv d_{3,t} - 2 \frac{\varphi(\lambda_I)}{\sqrt{\nu(t; T) \tau}}, \\ \varphi(x) & \equiv \frac{1}{\sqrt{v_T}} \sqrt{\frac{(m-\eta)^2}{v_0} + 2 \ln(x^{-1} A_0 \sqrt{1 + v_0 T})}, \end{aligned}$$

for the positive constants  $A_0$  and  $A_1$  as given in the proof.

Since all  $I$ -managers share the same information and start from identical assets under management, they all choose the same portfolios (21) and attain identical performance (20). We can interpret informed managers strategy in terms of “herding”, “no-herding” and “contrarian” behavior with respect to uninformed managers’ strategies. In the present setup, I consider that  $I$ -managers herd with  $U$ -managers whenever their weight in the risky asset is between the normal (Merton) risk exposure and  $U$ -managers’ risk exposure, overweighting the risky asset (relative to normal) when  $U$ -managers are over-exposed to risk and vice versa when  $U$ -managers are under-exposed. Conversely, I consider that  $I$ -managers show a *contrarian* behavior if they adopt a lower-than-normal risk exposure when  $U$ -managers overweight the risky asset and a higher-than-normal risk exposure when  $U$ -managers underweight this asset. *No-herding* represents the remaining situation in which  $I$ -managers just follow the Merton policy without concerns about  $U$ -managers’ risk exposure.

Informed managers’ optimal risk exposure (21) combines a mean-variance portfolio  $\omega_t \frac{\eta}{\gamma\sigma} + (1 - \omega_t) \left( \nu(t; T) \frac{\eta}{\bar{\gamma}\sigma} + \frac{\theta}{1 + (1 - \theta)v_t\tau} \frac{\tilde{\eta}_t}{\sigma} \right)$  and risk-shifting components  $\frac{\omega_t}{\sigma\sqrt{T-t}} \frac{\mathcal{N}'(d_{2,t}) - \mathcal{N}'(d_{1,t})}{\mathcal{N}(d_{1,t}) + 1 - \mathcal{N}(d_{2,t})}$  and  $\frac{1 - \omega_t}{\sigma} \sqrt{\frac{\nu_1(t)}{T-t}} \frac{\mathcal{N}'(d_{4,t}) - \mathcal{N}'(d_{3,t})}{\mathcal{N}(d_{3,t}) - \mathcal{N}(d_{4,t})}$ . The mean-variance portfolio assigns a (time- and state-dependent) weight  $\omega_t$  to the Merton (*no-herding*) policy  $\frac{\eta}{\gamma\sigma}$ , and a weight  $1 - \omega_t$  to a *herding* composite

$\nu(t; T) \frac{\eta}{\bar{\gamma}\sigma} + \frac{\theta}{1+(1-\theta)v_t\tau} \frac{\tilde{\eta}_t}{\sigma}$ . The weight  $\omega_t$  is increasing in  $I$ -managers' conditional probability of trailing behind their uninformed peers by the end of the period,  $\mathcal{N}(d_{1,t}) + 1 - \mathcal{N}(d_{2,t})$ . As *underperforming* becomes more likely,  $I$ -managers tilt their portfolio towards the Merton policy they would choose if they were trading for their own account. As *outperforming* becomes more likely,  $1 - \omega_t$  rises and they tilt their portfolio towards the sum of the two mean-variance portfolios  $\nu(t; T) \frac{\eta}{\bar{\gamma}\sigma}$  and  $\frac{\theta}{1+(1-\theta)v_t\tau} \frac{\tilde{\eta}_t}{\sigma}$  that make up the herding composite. The first of these portfolios corresponds to the risk exposure of a fully informed investor with time-varying coefficient of relative risk aversion  $\bar{\gamma}/\nu(t; T)$ , which is affected by informed managers' uncertainty  $v_t$ ; the second corresponds to the (conditional) mean-variance allocation to the risky asset chosen by an uninformed manager with coefficient of relative risk aversion  $\frac{\tilde{\gamma}\gamma}{\tilde{\gamma}-\gamma} > \gamma$ . A positive weight to the herding portfolio then has the interpretation that informed managers *herd* with their uninformed peers and disregard their own superior information to some extent.<sup>20</sup>

The risk-shifting components depend on uninformed managers' current and initial estimation error  $\eta - \tilde{\eta}_t$  and  $\eta - m$  through the stochastic coefficients  $d_{1,t}$  to  $d_{4,t}$ . In general, the sum of these components is positive when  $U$ -managers underweight the risky asset and negative in the opposite case. These risk-shifting components can then be interpreted as a *contrarian* portfolio representing a large long or short position in the risky asset depending on whether the uninformed managers under- or overweight this asset, respectively, relative to the normal risk exposure. Informed managers shift risk not by pure gambling but by taking large bets in the direction suggested by their private information.

Figure 1 illustrates the dynamic strategy of informed managers as a function of uninformed managers' inference error  $\tilde{\eta}_t - \eta$ . Results are in terms of managers' interim excess (relative to the Merton policy) risk exposure as of the first month ( $t = 1/12T$ ) and after the third quarter ( $t = 3/4T$ ), with the solid lines corresponding to informed managers' policies when they are subject to typical flow-performance relationships as estimated by the literature. A first observation is that, under different circumstances, the same manager can exhibit herding, no-herding and contrarian behavior during the same year. Moreover, contrarian strategies can either *increase* overall portfolio risk (risk over-exposure) or *decrease* it (risk under-exposure) depending on uninformed managers' estimation error. This is shown in Panel B of Figure 1: after a good market performance during the first three quarters,  $U$ -managers infer a higher-than-actual market price of risk and overweight the stock in their portfolios accordingly.  $I$ -managers attach a higher probability to the event that such market performance reverts back to its true value  $\eta$  before  $T$ , and thus expects to profit

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<sup>20</sup>From managers' objective (5) we see that herding is induced by the convexity of the flow-performance relationship in the outperformance region, where an augmented effective risk-aversion encourages managers to lock-in their outperformance margin by sticking close to their peer group. Accordingly, the extent of herding is increasing in the flow elasticity parameter  $\alpha$ , with a fraction of the informed manager's portfolio perfectly replicating uninformed peers' position ( $\theta \rightarrow \gamma$ ) when  $\alpha \rightarrow \infty$ .

on the mistake of the  $U$ -managers by taking a contrarian position. When  $U$ 's policy is close to the optimal with full information ( $\tilde{\eta}_t - \eta$  close to 0), the only way to increase the chances of outperforming by the required margin  $\delta$  is to take a large short position (relative to the Merton policy) that results in the U-shape to the right of 0. A symmetric arguments explains  $I$ -managers' excessive risk exposure after bad market years, resulting in the hump-shaped policy to the left of 0.

Second, informed managers take contrarian positions with higher probability after the first month ( $t = 1/12T$ ) than after the third quarter ( $t = 3/4T$ ). This behavior is consistent with informed managers taking aggressive positions early in the year, when their information advantage relative to uninformed managers is largest, to herd with them later after the outperformance margin necessary to rank high is achieved.

Third, informed managers can take either a conservative herding or a risky contrarian stance in response to the *same circumstances* depending on the particular flow-performance relationship they face. That is, identically skilled managers subject to the slightly different flow-performance relationships of Figure 1 can take the opposite risk exposure under the same economic conditions. I look into informed managers' policies as a function of the flow-performance parameters in more detail in Section 4.

Equation (20) shows that informed managers' interim performance is a non-linear function of the state variable  $\tilde{\eta}$ . Corollary 1 relates these managers' end-of-period performance to uninformed managers' (normalized) error correction from the initial estimation error  $\frac{(\tilde{\eta}_T - \eta)^2}{v_T} - \frac{(m - \eta)^2}{v_0}$ :

**Corollary 1.** *Under the symmetric equilibrium for  $U$ -managers of Proposition 1,  $I$ -managers' optimal terminal fund value  $\hat{W}_T^I$  is given by:*

$$\hat{W}_T^I = \begin{cases} (\lambda_I \pi_T)^{-\frac{1}{\gamma}} \leq \underline{\delta} \hat{W}_T^U, & \text{if } \frac{(\tilde{\eta}_T - \eta)^2}{v_T} - \frac{(m - \eta)^2}{v_0} > 2 \ln (\lambda_I^{-1} A_0 \sqrt{1 + v_0 T}), \\ (\lambda_I \pi_T)^{-\frac{1}{\gamma}} g_2(T, \tilde{\eta}_T; T) \geq \bar{\delta} \hat{W}_T^U, & \text{if } \frac{(\tilde{\eta}_T - \eta)^2}{v_T} - \frac{(m - \eta)^2}{v_0} \leq 2 \ln (\lambda_I^{-1} A_0 \sqrt{1 + v_0 T}), \end{cases} \quad (23)$$

where:

$$\underline{\delta} \equiv (1 + \alpha)^{-\frac{1+\alpha}{\alpha}} \left( \frac{\tilde{\gamma}}{\gamma} \right)^{\frac{\tilde{\gamma}}{\tilde{\gamma}-\gamma}} \delta < \delta, \quad \bar{\delta} \equiv (1 + \alpha)^{-\frac{1}{\alpha}} \left( \frac{\tilde{\gamma}}{\gamma} \right)^{\frac{\gamma}{\tilde{\gamma}-\gamma}} \delta > \delta. \quad (24)$$

Whether  $I$ -managers opt to beat or to lose to  $U$ -managers depends more on the *extent of learning* by the latter and less so on the *direction* of the overall market. Indeed, informed managers underperform their peers whenever the uninformed managers' error correction from the initial estimation error is small enough (less than  $2 \ln (\lambda_I / (A_0 \sqrt{1 + v_0 T}))$ ).<sup>21</sup> Otherwise they end up beating their peers. Note that underperformance happens in *both good and bad states*, as long

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<sup>21</sup> In all numerical examples considered in this paper, the outperformance regions is non-empty because  $\lambda_I$  is such that  $\frac{(m - \eta)^2}{v_0} + 2 \ln (\lambda_I^{-1} A_0 \sqrt{1 + v_0 T}) > 0$ .

as the stock return during the period is extreme (good or bad) enough, relative to the true distribution, to deviate  $U$ -managers inference of  $\eta$  far away from the actual value.

This result implies the a priori counterintuitive prediction that, the higher  $U$ -managers' estimation error by the end of the period  $(\tilde{\eta}_T - \eta)^2/v_T$ , and consequently the *less* they learn, the more likely it is that an informed manager *underperforms*. Conversely, an informed manager *outperforms* when  $U$ -managers learn throughout the period so that by  $T$  their estimation error has shrunk relative to (or at least has not deviated much from) their initial error  $(m - \eta)^2/v_0$ .<sup>22</sup> The intuition is that peer group's performance imposes additional risk on informed managers: even though they could achieve a superior performance if they were evaluated in absolute terms, a shock to prices that drives the rest of the market's inference about the mean return further away from the true value induces losses on these managers' portfolio in the very short term. Depending on uninformed managers' speed of learning (i.e. depending on their prior uncertainty), informed managers may expect those losses to revert or to even worsen before  $T$ .<sup>23</sup>

The argument is illustrated in Panel A of Figure 2, displaying informed managers' terminal assets under management in excess of uninformed managers' that result from following the trading policies of Figure 1. Depending on the particular flow-performance relationship they face,  $I$ -managers' optimal policy (23) delivers both higher and lower end-of-period returns than the normal policy in the outperformance region. Since managers' skill (informational advantage) is the same across the depicted payoff profiles, end-of-year performance may be a poor indicator of managers' true ability, over-estimating managers' ability for some fund-flow relations and under-estimating it for others. The over-estimation problem seems unlikely to be tackled by risk-adjustment based on linear factor models (e.g. CAPM) since informed managers' strategy is highly non-linear in the economy driving state variable. I examine the implications of informed managers' strategies for performance measurement in more detail in Section 4.2.

This non-linearity is the result of the particular option strategy that informed managers' trading policy (21) replicates, as Panel B of Figure 2 shows. When the same payoff profiles (and density function over states) of Panel A are plotted as a function of uninformed managers' terminal fund wealth  $\hat{W}_T^U$  it becomes clear that investing in an informed manager's fund looks like simultaneously selling an out-of-the-money put and an out-of-the-money call (digital) options with maturity  $T$  and  $\hat{W}_T^U$  as underlying.<sup>24</sup> A delegating investor collects positive excess returns over delegation to uninformed managers when these managers' returns do not fluctuate much, but suffers large nega-

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<sup>22</sup>In any case, a high initial error makes it more likely that by the end of the period  $U$ -managers have learned about  $\eta$  and thus that an  $I$ -manager ends up ahead.

<sup>23</sup>This effect is similar to that found in the literature on limits of arbitrage (see, e.g., Shleifer and Vishny (1997)), where shocks pushing prices further away from fundamentals (rendering arbitrage opportunities even more profitable) may end up hurting an arbitrageur's performance during a finite investment horizon.

<sup>24</sup>In the financial jargon, such a strategy is known as a (short) "strangle" or "top vertical combination".

tive excess returns in volatile times. Exactly how high excess returns are and how much volatility can turn relative profits into losses depends on the particular flow-performance relationship the informed manager faces: for the gray solid line of Figure 2 the strategy delivers frequent small excess returns (resembling a “nickel-picking” strategy) and rare large losses, whereas for the dark solid line the strategy delivers less frequent though larger excess returns, along with more frequent but smaller losses.

The next section derives the implications of these strategies on the utility of delegating households.

### 3.3 Households’ Derived Utility

For a realization  $\eta$  at  $t = 0$  of the market price of risk, households’ expected utility  $u_J(\eta)$  of delegating wealth  $w$  to a type  $J$  manager ( $J \in \{I, U\}$ ) can be computed as the expectation, with respect to the true probability  $P$ , of households’ utility over the final wealth  $\hat{W}_T^J$ . Proposition 3 summarizes the results for delegation to the uninformed and informed managers of sections 3.1 and 3.2:

**Proposition 3.** *Let  $\hat{\gamma} \equiv \frac{\gamma_h - 1}{\gamma}$  and  $\check{\gamma} \equiv \frac{\gamma_h - 1}{\bar{\gamma}}$ . For a realized market price of risk  $\eta$ , under the symmetric equilibrium for  $U$ -managers of Proposition 1 households’ expected utility from delegating to  $U$ -managers is given by:*

$$u_U(\eta) \equiv E \left[ \frac{\left( \hat{W}_T^U \right)^{1 - \gamma_h}}{1 - \gamma_h} \right] = \frac{\lambda_U^{\hat{\gamma}} e^{-\hat{\gamma} r T}}{1 - \gamma_h} \sqrt{\frac{(1 + v_0 T)^{1 + \hat{\gamma}}}{1 + (1 + \hat{\gamma}) v_0 T}} \times \exp \left\{ \frac{\hat{\gamma}(1 + \hat{\gamma})}{1 + (1 + \hat{\gamma}) v_0 T} \frac{m^2}{2} T - \frac{\hat{\gamma} v_0 T}{1 + (1 + \hat{\gamma}) v_0 T} \eta \left( \frac{\eta}{2} T + \frac{m}{v_0} \right) \right\}, \quad (25)$$

and their expected utility from delegating to  $I$ -managers is given by:

$$u_I(\eta) \equiv E \left[ \frac{\left( \hat{W}_T^I \right)^{1 - \gamma_h}}{1 - \gamma_h} \right] = \frac{e^{-\hat{\gamma} r T}}{1 - \gamma_h} \left\{ \lambda_I^{\hat{\gamma}} e^{-\hat{\gamma}(1 - \hat{\gamma}) \frac{\eta^2}{2} T} [\mathcal{N}(d_1^0) + 1 - \mathcal{N}(d_2^0)] \right. \\ \left. + \lambda_I^{\check{\gamma}} A_1^{1 - \gamma_h} \sqrt{\frac{(1 + v_0 T)^{\theta(\gamma_h - 1)}}{1 + \theta(\gamma_h - 1) v_T T}} \exp \left\{ -\check{\gamma}(1 + \check{\gamma} \theta v_T T) \frac{\eta^2}{2} T - \frac{\theta(\gamma_h - 1) T}{1 + v_0 T} \left( \eta m - \frac{m^2}{2} \right) \right. \right. \\ \left. \left. + \frac{\theta^2}{2} \frac{(\gamma_h - 1)^2 v_T^2 T}{1 + \theta(\gamma_h - 1) v_T T} \left( \eta T + \frac{m}{v_0} + \frac{\eta}{\check{\gamma} \theta v_T} \right)^2 \right\} [\mathcal{N}(d_3^0) - \mathcal{N}(d_4^0)] \right\}, \quad (26)$$

where:

$$d_1^0 \equiv \frac{\frac{\eta}{v_T} + (\tilde{\gamma} - 1)\eta T - \frac{m}{v_0} - \varphi(\lambda_I)}{\sqrt{T}}, \quad d_2^0 \equiv d_1^0 + 2\frac{\varphi(\lambda_I)}{\sqrt{T}},$$

$$d_3^0 \equiv \frac{\frac{\eta}{v_T} + \frac{(\tilde{\gamma}-1)\eta T - \frac{m}{v_0}}{1+\theta(\gamma_h-1)v_T T} + \varphi(\lambda_I)}{\sqrt{T/(1+\theta(\gamma_h-1)v_T T)}}, \quad d_4^0 \equiv d_3^0 - 2\frac{\varphi(\lambda_I)}{\sqrt{T/(1+\theta(\gamma_h-1)v_T T)}}.$$

These results allow for the computation of the certainty equivalent returns an investor may expect to collect from delegation, as detailed in Appendix C. I use these certainty equivalent as benchmark to analyze the implications of informed managers' strategies on performance evaluation in Section 4.2.

## 4 Herding Behavior and Performance Measurement

In this section I simulate uninformed and informed managers' optimal investment strategies for a broader range of flow-performance relationship specifications, under different economic settings. Reported results correspond to the baseline and alternative parameterizations described in Appendix B, with market, preference and fund-flow parameters set to match typical values during the period 1980-2006 and to be in line with parameterizations used by previous authors. The analysis is in terms of *expected* values, averaging outcomes over the joint distribution of all random variables according to the numerical procedure detailed in Appendix C. The baseline parameterization implies a potential advantage of informed over uninformed managers of 40 basis points (bps) in terms of households' certainty equivalent returns and of 70 bps in terms of (log) returns over the period, or equivalently a 14.5% increase in the Sharpe ratio.<sup>25</sup> Section 4.1 examines herding or contrarian behavior by informed managers as a function of the performance threshold  $\delta$  and the flow elasticity  $\alpha$  and derives the model testable implications to be examined in Section 5. Section 4.2 looks into the relation between fund flows and informed manager's performance, with emphasis on the ability of standard performance measures to adjust for the risks informed managers take.

### 4.1 Fund Flows and Herding: Testable Implications

We saw in Section 3.2 that contrarian managers shift risk by undertaking large long or short positions in the risky asset whenever their uninformed peers are under- or over-exposed to risk, respectively, relative to the Merton policy. As a result, end-of period returns exceed or fall short of uninformed managers' returns by a large margin for contrarian managers but by a very small

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<sup>25</sup>These figures correspond to the excess performance informed managers would achieve by following the Merton policy.

margin for herding managers. This observation points to the distance between informed managers' and uninformed managers' returns as a candidate measure of herding behavior. Motivated by the empirical analysis of the next section, I look at tracking error volatility  $\sigma(R_T^I - R_T^U)$  (hereafter just "tracking error") as such distance. A low value of tracking error is then indicative of herding behavior, while a high value is suggestive of contrarian behavior.

Panels A to D of Figure 3 illustrate the effects of  $\delta$  and  $\alpha$  on the extent of herding by the informed manager, with different panels corresponding to the baseline and alternative parameterizations of Appendix B. In general, all figures show a non-monotonic relationship between tracking error and either  $\delta$  or  $\alpha$ , with contrarian behavior increasing in both parameters until a maximum is reached and decreasing afterwards. For fixed  $\alpha$ , this hump-shaped pattern is not surprising given that observed contrarian behavior should be highest when outperforming is a likely event but only towards the end of the period. Only for moderately high values of the threshold ( $1 < \delta < 1.05$  in the baseline parameterization) being a top performer is possible but requires informed managers to trade aggressively on their private information throughout the whole period in order to achieve the necessary outperformance margin.

For medium to high values of  $\delta$  herding is decreasing in the flow elasticity  $\alpha$ , as can be expected from the higher incentives to take risks that a more convex compensation structure induces. However, Figure 3 also shows that herding is actually *increasing* in  $\alpha$  for low values of the performance threshold ( $\delta < 0.93$  in the baseline parameterization). This result implies that a flow-performance relationship rewarding large inflows to top performers can actually make informed managers choose not to differentiate much from the "crowd" despite having the informational advantage to place on the top, due to the high risk aversion of managers on the upward-sloping part of the fund-flow relationship.<sup>26</sup>

The model main testable implications can then be stated as follows: for informed managers subject to flow-performance relationships with

1. low thresholds  $\delta$ , average herding behavior should be increasing in flow elasticity  $\alpha$ .
2. medium to high thresholds  $\delta$ , average herding behavior should be increasing in flow elasticity  $\alpha$ .

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<sup>26</sup>The ambiguous effect of  $\alpha$  on the extent of herding by informed managers is due to the interplay of two opposite effects. On the one hand, a higher convexity of the flow-performance relationship on the top increases informed managers' incentives to deviate from the crowd in order to be a "star" performer and enjoy large inflows. This can be seen from expressions (24) in Section 3.2: how much managers are willing to deviate in order to place on the top is controlled by the ratio  $\bar{\delta}/\underline{\delta} = \gamma/(\gamma - \alpha/(1 + \alpha))$ , which is independent of  $\delta$  but increasing in  $\alpha$ . But once managers reach the upward-sloping part of the fund-flow relationship they become highly risk averse and seek to lock in their outperformance by sticking close to their uninformed peers. As a result, whether average herding is increasing or decreasing in alpha depends on how likely it is to outperform and thus on the threshold  $\delta$ .

3. medium to high flow elasticity  $\alpha$ , average herding behavior should be decreasing in the performance threshold  $\delta$ .

I provide empirical support for predictions 2 and 3 in Section 5.

## 4.2 Implications for Performance Measurement

The discussion about managers' end-of-period performance in Section 3.2 suggests that empirically estimated flow-performance relationships may distort performance measurement based exclusively on the extent of outperformance relative to peers, for at least two reasons: (i) lack of success in distinguishing informed from uninformed managers for low values of  $\delta$  or  $\alpha$ , and (ii) potential overstatement of informed manager's true informational advantage for medium to high values of these parameters. I examine next the ability of two standard performance measures, the Sharpe ratio and Jensen's alpha, to assess the desirability from delegating investors' point of view of informed managers' policies.

Panel A of Figure 4 plots the average excess certainty equivalent return (CER) that households would attain by delegating their wealth to an informed instead of to an uninformed manager.<sup>27</sup> CER is the correct risk-adjusted benchmark against which to compare different performance measures because it takes full account of delegating households' risk-preferences. For low flow-elasticity  $\alpha$ , informed managers deliver positive excess CER to households for all performance thresholds  $\delta$ . However, as the convexity in flow-performance relationship rises the contrarian policies adopted by informed managers subject to medium to high  $\delta$  make households bear too much risk, delivering negative CER in this region. Moreover, managers subject to middle-range thresholds  $\delta$  (most aggressive contrarian policies) deliver the lowest CER for each degree of convexity  $\alpha$ .

Sharpe ratios and Jensen's alpha are widely used measures to evaluate the performance of asset managers on a risk-adjusted basis. Panels B and C of Figure 4 show that these measures do a poor job at adjusting for the kind of risks informed managers take.<sup>28</sup> First, both fail to reflect the *relative* performance of equally informed managers subject to different fund-flow relations: for each level of flow elasticity  $\alpha$ , the Sharpe ratio and Jensen's alpha are highest for middle-range values of the performance threshold  $\delta$ , precisely where households enjoy the lowest benefits of delegation according to their excess CER in this region. Jensen's alpha is even *increasing* in the convexity  $\alpha$  for the middle region of  $\delta$ , whereas households' CER are *decreasing* instead in this region.

Second, both the Sharpe ratio and Jensen's alpha may fail to capture, even in *absolute* terms, the attractiveness from investors' viewpoint of informed managers' policies. Indeed, delegation to

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<sup>27</sup>See Appendix C for details on the computation of the excess CER in the model simulations.

<sup>28</sup>See Appendix C for details on the computations of these measures in the model simulations.



funds with performance thresholds in a neighborhood of 0.97 is most appealing according to the positive and high value of both measures although it is against households' interest according to the corresponding negative excess CER. The converse problem is also true in the case of Jensen's alpha: the negative values of this measure for high performance thresholds and moderate flow elasticity indicate that households do better by delegating to uninformed rather than to informed managers. However, households would actually be better off by delegating to informed managers according to the corresponding positive excess CER. This result implies that Jensen's alpha may not only under-estimate the risks taken by informed managers under certain flow-performance relationships, but also *over-estimate* these risks under other flow-performance specifications.

In non-tabulated results, I confirm that the potential inaccuracy of these performance measures remains a problem across different relative risk aversion misalignment between managers and delegating households ( $\gamma_h = 2, 8$ ) and for different levels of initial uncertainty ( $v_0 = 0.063^2, 0.317^2$ ). The reason lies in the specific option-like payoff replicated by informed managers' strategy, as argued in Section 3.2. In consequence, assessing higher-order moments in managed portfolio returns distribution is key in adjusting for risk, as suggested by e.g. Fung and Hsieh (2001) in the context of hedge funds. More generally, the results in this section point to the importance for performance evaluation of jointly estimating managers' incentives, risk-preferences and skills within a structural approach, in the spirit of Kojien (2010). In the next section I argue that the concerns about traditional performance measures are of practical relevance in the mutual fund industry given the shape the flow-performance relationships skilled managers may face.

## 5 Empirical Analysis

Led by the model main predictions, in this section I test the hypothesis that observed herding or contrarian behavior by mutual fund managers with superior information is an optimal response to different flow-performance relationships. I test this hypothesis over a sample of actively managed U.S. equity mutual funds, following three steps:

1. Identify potentially informed mutual funds in the sample.
2. Estimate extent of herding by the informed funds with the average fund in their respective objective category, and rank these funds according to their measured herding during the sample.
3. Test whether highest herding funds are subject to different flow-performance relationships than lowest herding (contrarian) funds, and whether this differences agree with the implications in Section 4.1.

I obtain information about mutual fund returns, total net assets, net asset values and characteristics from the Center for Research in Security Prices (CRSP) Survivor Bias Free Mutual Fund Database. I also obtain market excess returns and the 30-day Treasury bill rate ( $r_f$ ) from CRSP’s Fama-French, Momentum and Liquidity dataset. The sample consists of monthly data over the period January 1981 through December 2010. Since my focus is on open-ended, actively managed domestic equity funds, I apply the following screens. First, I exclude all funds classified as “index-based” or “index” funds as of 2008. Second, I exclude all funds with (Lipper) objective categories different from growth, growth and income, equity income and income, to facilitate comparison with the prior literature. Third, I omit all fund-month observations with total net assets less than \$5 million. The resulting total sample consists of 7,656 mutual fund share classes,<sup>29</sup> although data availability reduces this number to 3,060 mutual funds for steps 1 and 2 above. I next describe my approach to each of steps 1 to 3:

**1. Sample of “informed” managers:** One of the main implications of the model is that some informed managers may choose to perform just slightly better than average. Therefore, identifying privately informed managers only with the top mutual funds in a performance ranking in any given period (with or without factor-based risk-adjustment) is likely to miss truly informed funds from the sample. In order to circumvent this difficulty, I use the model predictions regarding informed managers’ probability of outperforming in any given period. Panel D of Figure 4 plots the average probability that informed managers’ total period returns ( $R_T^I$ ) are higher than those of their uninformed peers ( $R_T^U$ ), for the different flow-performance specifications in Section 4.2. We see that, for almost all plausible fund-flow relationships, informed managers’ performance is relatively homogeneous in this dimension: the probability that they perform better than average is greater than 0.5.<sup>30</sup> This result suggests identifying informed mutual funds in my sample with those that consistently rank in the top half (irrespective of their exact position) of the annual performance ranking for their respective objective category. More precisely, for each year and objective category I rank all funds according to their annual raw return and assign them a continuous rank ( $Rank$ ) ranging from 0 (worst) to 1 (best). Funds with  $0.45 \leq Rank \leq 0.55$  are considered “median performers” whereas funds with  $Rank > 0.55$  are considered “outperformers”, for that year-

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<sup>29</sup>Different share classes reflecting differing fee structures for the same mutual fund are treated as stand-alone mutual funds by CRSP. Since my goal in this section is the estimation of the flow-performance relationships, I follow Huang, Wei, and Yan (2007) and Huang, Wei, and Yan (2011) and perform my analysis at the fund-share level in order to capture the reaction of investors with possibly dissimilar tastes to similar performance. In any case, I control for the fee structures of different share classes in the flow regressions. I refer to fund-shares as mutual funds henceforth.

<sup>30</sup>The pattern in Panel D of Figure 4 is robust to different value of the initial informational advantage  $v_0$ , as long  $v_0 > 0$ : the probability of outperforming falls as  $v_0$  shrinks, but remain greater than 0.5 in most cases. Informed managers’ probability of outperforming is smaller than 0.5 only for those subject to flow performance relationships with very high thresholds ( $\delta \geq 1.05$ ) or with high thresholds but low flow elasticity ( $\delta \geq 1.025$ ,  $\alpha < 1$ ).

objective class.<sup>31</sup> For all funds ranked during at least 5 years in the sample, I then compute the proportion of years in their life that they were outperformers and sort all funds in ascending order based on this proportion. Finally, I pick the top 30% as the “informed” mutual funds in my analysis. I chose this threshold in an attempt to keep a fair balance between the sample size for the estimation of the flow-performance relationships on the one hand, and the fraction of “uninformed” managers expected to be incorrectly screened as “informed” on the other. A lower threshold favors a more accurate estimation, but turns the sample of informed managers noisier, and vice versa.

Table 1 reports summary statistics for the overall sample and for the top 30% mutual funds according to the criterion above. Top performers are on average larger and achieve higher returns (both in terms of raw returns and in terms of Jensen’s alpha with respect to the median performer in the respective objective class) without a significant increase in volatility. However, none of these differences are statistically significant. In particular, top performers are not above-average systematic risk-takers (as measured by average beta), so their higher proportion of outperformance years seems not attributable to a higher load on risk factors. Moreover, the average mutual fund in the overall sample outperforms with a 46% probability, whereas the average top 30% mutual fund outperforms with 65% probability.<sup>32</sup> Since the average fund in the latter group was ranked in 11.2 years, the percentage of average but “lucky” mutual funds that are expected to be screened as top 30% performers is 11.9%.<sup>33</sup> Whereas this fraction suggests that the potential influence of non-informed managers on my results in Step 3 may be non-negligible, the robustness of these results to a stricter selection criterion (including only the top 20% performers) in the robustness checks at the end of this section suggests that these results are not driven by pure chance.

**2. Herding measures:** I compute three alternative herding measures following Chevalier and Ellison (1999) and Arora and Ou-Yang (2001). The first,  $HerdTrackErr_i$ , is based on fund  $i$ ’s tracking error of monthly returns relative to the median performer within  $i$ ’s objective category for each year:

$$TrackErrVol_i = \sqrt{\frac{1}{T_i} \sum_{t=1}^{T_i} (r_{i,t} - r_{i,t}^{med})^2}, \quad (27)$$

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<sup>31</sup>I use this selection criterion in order to construct a meaningful portfolio of average/median performers for each year-objective category. The returns on this portfolio are used to compute herding measures in step 2 below.

<sup>32</sup>From Panel D of Figure 4, informed managers subject to performance thresholds lower than 1.02 and all values of flow elasticity will in general outperform with 65% or higher probability. This parameter subset gives rise to enough heterogeneity in expected herding/contrarian behavior according to Panels A-D of Figure 3, so there is no reason to expect either herding or contrarian mutual funds to dominate the top 30% performers sample *a priori*.

<sup>33</sup>This approximate proportion can be computed as  $f(7; 11, 0.46)$ , where  $f(k; n, p)$  is the binomial probability of  $k$  successes in  $n$  trials when the probability of success is  $p$ .

where  $r_{i,t}$  is the monthly return of fund  $i$  in month  $t$ ,  $r_{i,t}^{med}$  is the monthly return of a portfolio of median performers within  $i$ 's objective category during the same year (all funds in  $i$ 's objective style with  $0.45 \leq Rank \leq 0.55$ ), and  $T_i$  is the total number of monthly observations for fund  $i$ . The second measure,  $HerdBeta_i$ , is based on fund  $i$ 's beta relative to the median performer in  $i$ 's objective class:  $\beta_i^{med} \equiv |\beta_i - 1|$ , where  $\beta_i$  is the usual regression-based measure of systematic risk-taking relative to  $r_i^{med}$ , over the entire sample:

$$r_{i,t} - r_t^f = \alpha_i + \beta_i \left( r_{i,t}^{med} - r_t^f \right) + \xi_i. \quad (28)$$

The third measure,  $HerdCorr_i$ , is based on the correlation coefficient between fund  $i$ 's monthly returns and those of the median performer in  $i$ 's objective category over the entire sample:  $\rho_{i,med} \equiv corr(r_i, r_i^{med})$ . The herding measures  $HerdTrackErr_i$ ,  $HerdBeta_i$  and  $HerdCorr_i$  then result from subtracting the median of, respectively,  $TrackErrVol_i$ ,  $\beta_i^{med}$  and  $\rho_{i,med}$  for all funds in  $i$  objective class from each of these variables. These measures proxy for managers' boldness in the sense of departing from the typical portfolio ( $HerdTrackErr$  and  $HerdCorr$ ), or in the sense of taking above- or below-average systematic risk ( $HerdBeta$ ). One should expect more herding corresponding to lower  $HerdTrackErr$  and  $HerdBeta$  on the one hand, and to higher  $HerdCorr$  on the other hand. The use of these three returns-based measures over portfolio holding measures will be beneficial whenever managers actively trade *within* quarters (the frequency at which most mutual funds' portfolio holdings data are available) and whenever managers take offsetting positions in assets other than equities or bonds (e.g. long or short positions in derivative to hedge their risk exposure). Since the availability of monthly returns in CRSP database improves significantly starting from 1990, I compute the herding measures over the sample covering January 1990 through December 2010 so as to avoid noisy measurement of the portfolio of median performers for each year-objective style prior to 1990. I then rank all informed mutual funds as screened in Step 1 according to measured herding and identify the bottom and top 33% herding groups as the contrarian and herding mutual funds, respectively. The dummy variable *Contrarian* equals 1 for the group displaying, alternatively, the highest values for  $HerdTrackErr$  or  $BetaCorr$ , or the lowest values for  $HerdCorr$ , and 0 otherwise. I refer to "contrarian" and "herding" groups those with  $Contrarian = 1$  and  $Contrarian = 0$ , respectively.

Table 2 presents summary statistics for the contrarian and herding top funds. Even though differences are not statistically significant, contrarians seem to be smaller mutual funds with higher expense ratios, achieving higher Sharpe ratios and Jensen's alpha (with respect to the median portfolio in their respective styles), without a corresponding fall in the probability of outperforming. We see that herding funds according to  $HerdTrackErr$  are also lower  $HerdBeta$  and higher  $HerdCorr$  funds, signaling herding behavior in both cases, and similarly for the herding

mutual funds according to the latter two measures. Herding funds then appear to take both lower systematic and unsystematic risk, delivering a high comovement with the median performers in their objective styles.

**3. Flow-performance relationships:** I estimate informed mutual funds’ flow-performance relationships following closely the methodology in Sirri and Tufano (1998). Their approach consists in regressing a mutual fund flow rate on its previous year performance allowing for a piecewise linear relation between the two, and on a set of non-performance-related control variables. I define the annual flow rate into a fund as:

$$Flow_{i,t} = \frac{TNA_{i,t} - TNA_{i,t-1}(1 + R_{i,t})}{TNA_{i,t-1}(1 + R_{i,t})}, \quad (29)$$

where  $R_{i,t}$  is the return of fund  $i$  during year  $t$  and  $TNA_{i,t}$  is fund  $i$ ’s total net asset value at the end of year  $t$ . (29) reflects the growth rate of assets under management after adjusting for appreciation of the mutual fund assets, following the definition of  $f_T$  in Section 2.2.<sup>34</sup> To avoid the impact of mutual funds mergers and stock splits leading to extreme values of flows, I exclude all funds merging with other funds during the sample and I filter out the top and bottom 2.5% of the net flow data.

The regression equation is:

$$\begin{aligned} Flow_{i,t} = & a + b_1 Size_{i,t-1} + b_2 CategoryFlow_{i,t} + b_3 Volatility_{i,t-1} + b_4 Age_{i,t} \\ & + b_5 ExpenseRatio_{i,t} + b_6 Low_{i,t-1} + b_7 4thPerfQuint_{i,t-1} \\ & + b_8 3rdPerfQuint_{i,t-1} + b_9 2ndPerfQuint_{i,t-1} + b_{10} Top_{i,t-1} \\ & + b_{11} Contrarian_i + b_{12} Contrarian_i * Low_{i,t-1} + \dots + b_{16} Contrarian_i * Top_{i,t-1} + \epsilon_{i,t}, \end{aligned} \quad (30)$$

where  $Size_i$  is the natural log of fund  $i$ ’s TNA,  $CategoryFlow_i$  is the aggregate flow rate into fund  $i$ ’s category,  $Volatility_{i,t}$  is the standard deviation of fund  $i$ ’s monthly returns in year  $t$ ,  $Age_i$  is the natural log of  $1 +$  fund  $i$ ’s age,  $ExpenseRatio_i$  is fund  $i$ ’s expense ratio, and  $Contrarian_i$  is  $i$ ’s “contrarian” dummy variable of Step 2. Performance quintiles are constructed as follows:  $Low_{i,t-1} \equiv \min(Rank_{i,t-1}, 0.2)$ ,  $4thPerfQuint_{i,t-1} \equiv \min(Rank_{i,t-1} - Low_{i,t-1}, 0.2)$ , and so forth, up to  $Top = Rank_{i,t-1} - Low_{i,t-1} - \dots - 2ndPerfQuint_{i,t-1}$ . Following the literature I also estimate an alternative 3-piece specification that combines the three middle quintiles in one:  $Mid_{i,t-1} \equiv \min(Rank_{i,t-1} - Low_{i,t-1}, 0.6)$ . According to the testable implications of Section

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<sup>34</sup>This definition of fund flows is the same used by Huang, Sialm, and Zhang (2011) and Huang, Wei, and Yan (2011)

4.1, we would expect funds in the herding group to be more sensitive to the bottom and middle relative performance pieces (quintiles *Low* to *3rdPerfQuint*, or *Low* and *Mid* pieces), consistent with a lower threshold, or to be less sensitive on the top (*Top* fractional rank), consistent with a less convex relationship.

Each year, I run cross-sectional regressions to estimate equation (30) and compute the means and *t*-statistics from the time series of coefficient estimates following Fama and MacBeth (1973). In order to control for the high autocorrelation of mutual fund flows (see DelGuercio and Tkac (2002)), reported Fama-MacBeth *t*-statistics are calculated using Newey and West (1987) autocorrelation and heteroskedasticity consistent standard errors. Results are reported in Tables 3 and 4 for the quintile and 3-piece specifications, respectively.

Consistent with the model predictions, the estimated flow-performance relationships are significantly different for the herding and contrarian groups, as reflected by the statistically significant coefficients for the interaction terms *Contrarian \* 2ndPerfQuint* and *Contrarian \* Mid* across the regressions for all three herding measures. In particular, the contrarian group is less sensitive to the second performance quintile (even at the 1% significance level in the case of *HerdBeta* regression) and weakly more sensitive to the top quintile (significant at the 5% level for *HerdBeta*) in Table 3, and less sensitive to middle-range performance in Table 4. The differences are economically significant: according to Table 4, for instance, an increase from the 40th to the 50th ranking percentile will lead herding mutual funds to an expected increase in fund flows next year of 3.5%, 3.1% or 3.8% depending on the herding measure considered, whereas otherwise identical contrarian fund flows will rise by only 1.5%, 1.4% and 1.8%. These estimates lead to a more convex flow-performance relationship for the contrarian group. Moreover, the negative coefficients for the term *Contrarian<sub>i</sub> \* 4thPerfQuint<sub>i,t-1</sub>* suggest that these are higher-threshold mutual funds in terms of the analysis of Section 4. Panels A and B in Figure 6 illustrate this point, by depicting the multivariate 5- and 3-piece flow-performance relationships of herding and contrarian informed mutual funds according to the tracking error measure *HerdTrackErr*.<sup>35</sup> The graphs suggest: (i) fund flows for herding mutual funds are roughly as sensitive to very poor as to mid-range performance, delivering a linear relationship between flows and performance in these regions, and (ii) contrarian mutual funds face a more convex relation between flows and performance.

As robustness checks, I re-estimate equation (30) and its 3-piece counterpart for different alternatives to steps 1 to 3 above: (i) herding measures of step 3 computed over returns *gross of expense ratios* instead of over net of expense ratios as reported by CRSP; (ii) “informed” mutual funds defined as those belonging to the top quintile in the outperformance probability ranking

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<sup>35</sup>The pictures show the relation between expected flow rate and lagged relative performance for the average fund in each herding category, i.e. substituting average values for all included control variables into the estimated equations.

of step 1.<sup>36</sup> Alternative (i) is meant to capture true managerial ability more closely, following Cohen, Coval, and Pastor (2005).<sup>37</sup> Alternative (ii) imposes a stricter requirement for a mutual fund to be screened as “informed”, reducing the importance of “uninformed but lucky” mutual funds in the sample. Estimates for the quintile specifications with herding groups classified by *HerdTrackErr* are reported in Table 5. In general, we see that the same differences between the fund-flow relationships of herding and contrarian groups survive the different specifications: herding funds face higher sensitivity to middle-range relative performance but lower sensitivity on the top, leading to a less convex flow performance relationship that is consistent with their more conservative trading.<sup>38</sup>

## 6 Other Applications

**Institutional investors and “bubbles”.** The optimal investment policy (21) implies that a manager with superior information but relative concerns with respect to less informed peers will not always adopt a contrarian stance against peer’s suboptimal strategies (“lean against the wind”). This is true in the case of bubble-like asset price developments as well, which in the current setup would manifest as a sustained increase in the stock price  $S$  throughout the period way above the expected appreciation according to its true mean return  $\mu$ . Such a situation would correspond, e.g. to an estimation error  $\tilde{\eta}_t - \eta = 0.07$  by the uninformed managers in Panel B of Figure 1, resulting in risk over-exposure by these managers. We see that whereas some informed managers (dark solid line) would trade aggressively against uninformed funds and under-expose their portfolios to market risk, others (gray solid line) would herd with their uninformed peers and over-expose their portfolios to market risk. The model implications are relevant in light of the recent empirical evidence on institutional investors’ holdings of “bubble” stocks during the apparent technology bubble of the late 1990s<sup>39</sup> and suggests an alternative channel, i.e. the incentive effects of fund flows, behind some institutions’ over-exposure to “bubble” stocks during this period.

**Active management and closet indexing** Cremers and Petajisto (2009) find that many ac-

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<sup>36</sup>In non-tabulated results, I also checked the robustness of these empirical findings to different definitions of the portfolio of average performers ( $0.49 \leq Rank \leq 0.51$  and  $0.4 \leq Rank \leq 0.6$ ), as well as different thresholds for selecting contrarian and herding mutual funds (top and bottom 20% in the herding ranking).

<sup>37</sup>To obtain gross returns, I add the annual expense ratio divided by 12 to the monthly returns in that year. Fee data is sparse prior to 1998 in CRSP Mutual Funds database, which is why my main results are presented in terms of net returns despite the arguments in favor of using gross returns instead.

<sup>38</sup>Estimates for the 3-piece specification and for the herding classification of informed funds according to *HerdBeta* and *HerdCorr* are available from the author upon request. In all cases, the results follow the same patterns as in Tables 3 through 5.

<sup>39</sup>See e.g. Brunnermeier and Nagel (2004) and Dass, Massa, and Patgiri (2008).

tively managed U.S. equity mutual funds have holdings that are similar to those of their benchmarks, and point to the importance of distinguishing between funds that are truly active from those that are “closet index funds”. They also find that funds whose holdings are most different from their benchmarks have higher Jensen’s alpha than their benchmarks. The results in Section 4.1 suggest that, if the average (unskilled) fund manager within an objective category follows a benchmark closely (i.e. is a closet indexer) and moreover money flows in this category are very sensitive to moderately poor past performance, a skilled (informed) manager in the same category may choose to optimally herd with the average fund. Such a manager will look like a “mediocre” manager and generate a too small Jensen’s alpha according to the results in Section 4.2 in spite of its superior ability and of potentially delivering positive net benefits to delegating investors. By contrast, the true ability of active and informed mutual funds (high return tracking error) responding to highly convex fund flows may be overstated by performance measures that fail to capture the non-linear risks inherent in their option-like strategies.

## 7 Concluding Remarks

Within a standard portfolio choice framework, I analyze the incentive effects of increasing and convex fund flows on the trading strategies of fund managers with superior information. Fund flows provide these managers with complex incentives, inducing “limited liability” on the bottom relative performance region and concerns relative to less informed peers on the top. I show how the interplay of these effects can lead informed managers to take positions contrarian to their peers in some situations, but to herd with them and disregard their private information in others. I argue that for many plausible flow-performance relationships skilled managers’ performance will not differentiate much from their unskilled peers’ despite facing highly convex incentives. I show that standard performance measures may fail to adjust for the risks informed managers take, resulting in performance under- or over-statement in many situations. Using a sample of U.S. mutual funds I provide evidence supporting the model implied relation between herding behavior and the shape of the flow-performance relationship. I suggest that the developed framework can be used to study the trading strategies of sophisticated institutional investors during bubble-like asset price developments, and to understand the extent of closet-indexing in actively managed mutual funds.

The assumed informational advantage by skilled managers in my model is arguably simplistic. In the real world, managers may have private information not only about assets mean returns but also about their volatility, about their correlation structure in a multi-asset framework, or about the relationship between asset returns and the economy state variables if managers are market-timers. However, the model’s qualitative results should still hold in all these cases, and we could



expect more informed managers to herd with less informed peers in many situations even for more sophisticated information structures or distribution of skills among the managerial population.

A natural direction for future research would be to analyze the general equilibrium implications of informed and uninformed managers' trading strategies in response to convex fund flows on asset prices. Although the task looks challenging, any progress in this area could prove fruitful in addressing the deeper question of how managers' implicit incentives affect the informative role of asset prices and the overall efficiency of capital markets.

# Appendix

## A Proofs

I start by stating two auxiliary lemmas that are used repeatedly throughout the remaining proofs.

**Lemma 1.** For  $\kappa \in \mathbb{R}$ ,  $0 \leq t \leq u$ , we have

$$e^{\frac{\kappa}{2} \int_t^u \tilde{\eta}_s^2 ds - \kappa \int_t^u \tilde{\eta}_s dB_s^Q} = \sqrt{(1 + v_t(u-t))^\kappa} e^{\frac{\kappa}{2} \frac{\tilde{\eta}_t^2}{v_t} - \kappa \frac{v_u}{2} (B_u^Q - B_t^Q + \frac{\tilde{\eta}_t}{v_t})^2}. \quad (31)$$

*Proof.* First, note that the differential equation for  $v$  in (8) is of the Ricatti type, whose solution is given by (see e.g. Gennotte (1986), p.741):

$$v_t = \frac{v_0}{1 + v_0 t}. \quad (32)$$

We can now solve the differential equation for  $\tilde{\eta}$  in (8). Since  $v_t > 0$  for all  $t$  when  $v_0 > 0$ , we can write:

$$\frac{d\tilde{\eta}_t}{v_t} = d\tilde{B}_t = dB_t^Q - \tilde{\eta}_t dt$$

Using (32) and integrating both sides between 0 and  $t$  gives:

$$\frac{\tilde{\eta}_t}{v_t} - \frac{m}{v_0} - \int_0^t \tilde{\eta}_s ds = B_t^Q - B_0^Q - \int_0^t \tilde{\eta}_s ds,$$

i.e.

$$\tilde{\eta}_t = v_t \left( B_t^Q + \frac{m}{v_0} \right). \quad (33)$$

Next, following Cvitanic, Lazrak, Martellini, and Zapatero (2006) denote  $E_t \equiv \frac{v_0}{2} (B_t^Q)^2 + mB_t^Q$ . An application of Itô's Lemma gives:

$$dE_t = (v_0 B_t^Q + m) dB_t^Q + \frac{v_0}{2} dt \Rightarrow (v_0 B_t^Q + m) dB_t^Q = dE_t - \frac{v_0}{2} dt. \quad (34)$$

We have:

$$\begin{aligned} \kappa \int_t^u \tilde{\eta}_s dB_s^Q &= \kappa \int_t^u v_s \left( B_s^Q + \frac{m}{v_0} \right) dB_s^Q \\ &= \kappa \int_t^u \frac{v_s}{v_0} dE_s - \frac{\kappa}{2} \int_t^u v_s ds \\ &= \frac{\kappa}{v_0} \int_t^u v_s dE_s - \ln \left( \frac{1 + v_0 u}{1 + v_0 t} \right)^{\frac{\kappa}{2}}, \end{aligned} \quad (35)$$

where the last equality follows from direct integration of the expression for  $v_t$  in (17). Integrating by

parts,

$$\begin{aligned}
\int_t^u v_s dE_s &= v_u \left( \frac{v_0}{2} (B_u^Q)^2 + mB_u^Q \right) - v_t \left( \frac{v_0}{2} (B_t^Q)^2 + mB_t^Q \right) \\
&\quad + \int_t^u \left( \frac{v_0}{2} (B_s^Q)^2 + mB_s^Q \right) v_s^2 ds \\
\Rightarrow \frac{\kappa}{v_0} \int_t^u v_s dE_s &= \kappa \frac{v_u}{v_0} \left( \frac{v_0}{2} (B_u^Q)^2 + mB_u^Q \right) - \kappa \frac{v_t}{v_0} \left( \frac{v_0}{2} (B_t^Q)^2 + mB_t^Q \right) \\
&\quad + \frac{\kappa}{2} \int_t^u \tilde{\eta}_s^2 ds - \frac{\kappa}{2} \left( \frac{m}{v_0} \right)^2 \int_t^u v_s^2 ds.
\end{aligned} \tag{36}$$

Direct integration gives  $\int_t^u v_s^2 ds = -\frac{\kappa}{2} \left( \frac{m}{v_0} \right)^2 (v_u - v_t)$ . Plugging the result in (35), we get:

$$\begin{aligned}
\frac{\kappa}{2} \int_t^u \tilde{\eta}_s^2 ds - \kappa \int_t^u \tilde{\eta}_s dB_s^Q &= \ln \left( \frac{1 + v_0 u}{1 + v_0 t} \right)^{\frac{\kappa}{2}} \\
&\quad - \frac{\kappa}{2} \left[ v_u \left( (B_u^Q)^2 + 2 \frac{m}{v_0} B_u^Q \right) - v_t \left( (B_t^Q)^2 + 2 \frac{m}{v_0} B_t^Q \right) \right] \\
\Rightarrow e^{\frac{\kappa}{2} \int_t^u \tilde{\eta}_s^2 ds - \kappa \int_t^u \tilde{\eta}_s dB_s^Q} &= \sqrt{\left( \frac{1 + v_0 u}{1 + v_0 t} \right)^{\kappa}} \\
&\quad \times e^{-\frac{\kappa}{2} \left( \frac{m}{v_0} \right)^2 (v_u - v_t) - \frac{\kappa}{2} \left[ v_u \left( (B_u^Q)^2 + 2 \frac{m}{v_0} B_u^Q \right) - v_t \left( (B_t^Q)^2 + 2 \frac{m}{v_0} B_t^Q \right) \right]}
\end{aligned} \tag{37}$$

Using the expressions in (17) for  $\tilde{\eta}_t$  and  $v_t$  we finally get, after some algebraic manipulation, equation (31).  $\square$

**Lemma 2.** Let  $z \sim \mathbf{N}(0, \sigma_z^2)$ , and let  $\rho, c, \bar{z} \in \mathbb{R}$ . We have:

$$(i) \ E \left[ e^{\rho z} \mathbf{1}_{\{z \leq \bar{z}\}} \right] = e^{\frac{\rho^2 \bar{z}^2}{2}} \mathcal{N} \left( \frac{\bar{z} - \rho \sigma_z^2}{\sigma_z} \right),$$

$$(ii) \ E \left[ e^{-\rho(z-c)^2} \mathbf{1}_{\{z \leq \bar{z}\}} \right] = \frac{e^{-\frac{\rho c^2}{1+2\rho\sigma_z^2}}}{\sqrt{1+2\rho\sigma_z^2}} \mathcal{N} \left( \frac{\bar{z} - \frac{2\rho\sigma_z^2}{1+2\rho\sigma_z^2} c}{\sigma_z / \sqrt{1+2\rho\sigma_z^2}} \right),$$

where  $\mathcal{N}(\cdot)$  is the standard normal cumulative distribution function.

*Proof.* Follows from direct integration against the normal density, using the change of variables  $\tilde{z} = \frac{z - \rho\sigma_z^2}{\sigma_z}$

for part (i) and  $\tilde{z} = \frac{z - \frac{2\rho\sigma_z^2}{1+2\rho\sigma_z^2}}{\sigma_z / \sqrt{1+2\rho\sigma_z^2}}$ ,  $\tilde{z}_l = \frac{\bar{z} - \frac{2\rho\sigma_z^2}{1+2\rho\sigma_z^2} c}{\sigma_z / \sqrt{1+2\rho\sigma_z^2}}$  for part (ii).  $\square$

**Proof of Proposition 1.** The expressions for  $\tilde{\eta}$  and  $v$  in (17) are given in the proof of Lemma 1 above (equations (33) and (32)). To prove (13) and (14) we first show that a symmetric (Nash) equilibrium in final payoffs  $\hat{W}_T^U$  exists and characterize it.

We first conjecture that such an equilibrium exists and proceed to characterize it. By (6),  $\delta Y_T = W_T^U$ . Each  $U$ -manager seeks to solve (10) subject to (12). The martingale/duality approach cannot be applied directly to this problem because the objective function is locally non-concave in a neighborhood

of  $W_T^U(i) = \delta Y_T = \delta W_T^U$ ,<sup>40</sup> where we used (6) in the last equality. The first step is then to construct the concavification  $\tilde{u}(\cdot)$  of  $u(\cdot)$  (i.e. the smallest concave function  $\tilde{u}(w)$  satisfying  $\tilde{u}(w) \geq u(w)$  for all  $w \geq 0$ ), restate and solve the original problem (10) in terms of  $\tilde{u}(\cdot)$ , and then verify that the solutions of the non-concave problem can be derived from those of the concavified problem. Applying this approach gives each  $U$ -manager's optimal terminal wealth as (see Proposition 1 in Basak and Makarov (2011)):

$$\hat{W}_T^U(i) = \begin{cases} (\lambda_U(i)\tilde{\pi}_T)^{-\frac{1}{\gamma}} & \text{if } \lambda_U(i)\tilde{\pi}_T > b(\delta\hat{W}_T^U), \quad \text{(I)} \\ (1+\alpha)^{\frac{1}{\bar{\gamma}}} (\lambda_U(i)\tilde{\pi}_T)^{-\frac{1}{\bar{\gamma}}} (\delta\hat{W}_T^U)^{\frac{\bar{\gamma}-\gamma}{\bar{\gamma}}} & \text{if } \lambda_U(i)\tilde{\pi}_T \leq b(\delta\hat{W}_T^U), \quad \text{(II)} \end{cases} \quad (38)$$

where  $\lambda_U(i)$  is the Lagrange multiplier attached to the  $i$  manager's time- $T$  budget constraint (12) solving  $w = \tilde{E} [\tilde{\pi}_T \hat{W}_T^U(i)]$ , and  $b(x) \equiv (1+\alpha)^{\gamma\frac{1+\alpha}{\alpha}} \left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\bar{\gamma}}{\bar{\gamma}-\gamma}} x^{-\gamma}$ . In region (I) we have:

$$\hat{W}_T^U(i) < (1+\alpha)^{-\frac{1+\alpha}{\alpha}} \left(\frac{\bar{\gamma}}{\gamma}\right)^{\frac{\bar{\gamma}}{\bar{\gamma}-\gamma}} \delta\hat{W}_T^U < \delta\hat{W}_T^U, \quad (39)$$

whereas in region (II):

$$\hat{W}_T^U(i) \geq (1+\alpha)^{-\frac{1}{\alpha}} \left(\frac{\bar{\gamma}}{\gamma}\right)^{\frac{\bar{\gamma}}{\bar{\gamma}-\gamma}} \delta\hat{W}_T^U > \delta\hat{W}_T^U. \quad (40)$$

This implies that in any symmetric equilibrium in which  $\hat{W}_T^U(i) = \hat{W}_T^U$  for all  $i \in \mathcal{U}$ , whether all  $U$ -managers end up the period in region (I) or in region (II) depends on  $\delta$ . More precisely,

- if  $\delta \leq 1$  only  $\hat{W}_T^U \geq \delta\hat{W}_T^U$  can be true, so if an equilibrium exists it occurs in region (II), where all  $U$ -managers *outperform* in all economic states:

$$\begin{aligned} \hat{W}_T^U(i) &= (1+\alpha)^{\frac{1}{\bar{\gamma}}} (\lambda_U(i)\tilde{\pi}_T)^{-\frac{1}{\bar{\gamma}}} (\delta\hat{W}_T^U)^{\frac{\bar{\gamma}-\gamma}{\bar{\gamma}}} \\ \Rightarrow \hat{W}_T^U(i) &= \left(\frac{\lambda_U(i)}{\kappa}\tilde{\pi}_T\right)^{-\frac{1}{\bar{\gamma}}}, \end{aligned} \quad (41)$$

where  $\kappa \equiv (1+\alpha)\delta^{\bar{\gamma}-\gamma}$ .

- if  $\delta > 1$  only  $\hat{W}_T^U < \delta\hat{W}_T^U$  can be true, so if an equilibrium exists it occurs in region (I), where all  $U$ -managers *underperform* in all economic states:

$$\hat{W}_T^U(i) = (\lambda_U(i)\tilde{\pi}_T)^{-\frac{1}{\gamma}}. \quad (42)$$

For this to be a candidate equilibrium it has to be true that  $\lambda_U(i) = \lambda_U$  for all  $i \in \mathcal{U}$ . This is indeed the case when  $\delta \leq 1$ , since:

$$\begin{aligned} w &= \tilde{E} [\tilde{\pi}_T \hat{W}_T^U(i)] = \tilde{E} \left[ \tilde{\pi}_T \left(\frac{\lambda_U(i)}{\kappa}\tilde{\pi}_T\right)^{-\frac{1}{\bar{\gamma}}} \right] \\ \Rightarrow \lambda_U(i)/\kappa &= \lambda_U/\kappa = \left\{ \tilde{E} \left[ \tilde{\pi}_T^{1-\frac{1}{\bar{\gamma}}} \right] / w \right\}^{\bar{\gamma}}, \end{aligned} \quad (43)$$

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<sup>40</sup>Moreover, it is non-differentiable at this point.

and similarly (setting  $\kappa \equiv 1$ ) for the case  $\delta > 1$ . We next show that, given:

$$\hat{W}_T^U = \begin{cases} (\lambda_U/\kappa\tilde{\pi}_T)^{-\frac{1}{\gamma}}, & \lambda_U/\kappa = \left\{ \tilde{E} \left[ \tilde{\pi}_T^{1-\frac{1}{\gamma}} \right] / w \right\}^\gamma & \text{if } \delta \leq 1 \\ (\lambda_U\tilde{\pi}_T)^{-\frac{1}{\gamma}}, & \lambda_U = \left\{ \tilde{E} \left[ \tilde{\pi}_T^{1-\frac{1}{\gamma}} \right] / w \right\}^\gamma & \text{if } \delta > 1, \end{cases} \quad (44)$$

no  $U$ -manager has incentives to deviate and adopt a different policy  $\tilde{W}_T^U(i) \neq \hat{W}_T^U$ . Consider first the case  $\delta \leq 1$ . From (38) and (44):

$$\hat{W}_T^U(i) = \begin{cases} (\lambda_U(i)\tilde{\pi}_T)^{-\frac{1}{\gamma}} & \text{if } \lambda_U(i) > (1+\alpha)\gamma^{\frac{1+\alpha}{\alpha}} \left(\frac{\gamma}{\tilde{\gamma}}\right)^{\frac{\gamma\tilde{\gamma}}{\tilde{\gamma}-\gamma}} \delta^{-\gamma} \frac{\lambda_U}{\kappa}, & \text{(I')} \\ (\lambda_U(i)/\lambda_U)^{-\frac{1}{\gamma}} (\lambda_U/\kappa\tilde{\pi}_T)^{-\frac{1}{\gamma}} & \text{if } \lambda_U(i) \leq (1+\alpha)\gamma^{\frac{1+\alpha}{\alpha}} \left(\frac{\gamma}{\tilde{\gamma}}\right)^{\frac{\gamma\tilde{\gamma}}{\tilde{\gamma}-\gamma}} \delta^{-\gamma} \frac{\lambda_U}{\kappa}, & \text{(II')} \end{cases} \quad (45)$$

Conditions in (I') and (II') are state-independent, so the manager ends up either in region (I') or in region (II') depending on the value of  $\lambda_U(i)$ . If region (II') is true,

$$\begin{aligned} w &= \tilde{E} \left[ \tilde{\pi}_T \hat{W}_T^U(i) \right] = (\lambda_U(i)/\lambda_U)^{-\frac{1}{\gamma}} \tilde{E} \left[ \tilde{\pi}_T \left( \frac{\lambda_U}{\kappa} \tilde{\pi}_T \right)^{-\frac{1}{\gamma}} \right] \\ &= (\lambda_U(i)/\lambda_U)^{-\frac{1}{\gamma}} w \\ &\Leftrightarrow \lambda_U(i) = \lambda_U. \end{aligned}$$

Since  $(1+\alpha)\gamma^{\frac{1+\alpha}{\alpha}} \left(\frac{\gamma}{\tilde{\gamma}}\right)^{\frac{\gamma\tilde{\gamma}}{\tilde{\gamma}-\gamma}} \delta^{-\gamma}/\kappa = (1+\alpha)^{\frac{\tilde{\gamma}}{\alpha}} \left(\frac{\gamma}{\tilde{\gamma}}\right)^{\frac{\gamma\tilde{\gamma}}{\tilde{\gamma}-\gamma}} / \delta^{\tilde{\gamma}} > 1$ , it holds that  $\lambda_U(i) = \lambda_U \leq (1+\alpha)\gamma^{\frac{1+\alpha}{\alpha}} \left(\frac{\gamma}{\tilde{\gamma}}\right)^{\frac{\gamma\tilde{\gamma}}{\tilde{\gamma}-\gamma}} \delta^{-\gamma} \frac{\lambda_U}{\kappa}$  so region (II') is indeed true and  $\hat{W}_T^U(i) = \hat{W}_T^U = (\lambda_U/\kappa\tilde{\pi}_T)^{-\frac{1}{\gamma}}$ . A similar argument shows that  $\tilde{W}_T^U(i) = \hat{W}_T^U = (\lambda_U\tilde{\pi}_T)^{-\frac{1}{\gamma}}$  if  $\delta > 1$ . Therefore, (44) represents indeed a symmetric Nash equilibrium in  $U$ -managers' policies. Moreover, this is the *unique* symmetric equilibrium.

We can finally prove equations (13) and (14). For all values of  $\delta$ , (44) can be summarized as:

$$\hat{W}_T^U = (\lambda_U\tilde{\pi}_T)^{-\frac{1}{\gamma}}, \quad \lambda_U = \left\{ \tilde{E} \left[ \tilde{\pi}_T^{1-\frac{1}{\gamma}} \right] / w \right\}^\gamma. \quad (46)$$

Under the risk-neutral measure the deflated wealth process is a martingale, so for  $i \in \mathcal{U}$  and  $t \in [0, T]$  the optimal wealth is given by:

$$\hat{W}_t^U(i) = \hat{W}_t^U = e^{-r(T-t)} E_t^Q \left[ \hat{W}_T^U \right] = e^{-r(T-t)} (\lambda_U\tilde{\pi}_t)^{-\frac{1}{\gamma}} E_t^Q \left[ \left( \frac{\tilde{\pi}_T}{\tilde{\pi}_t} \right)^{-\frac{1}{\gamma}} \right] \quad (47)$$

where  $E_t^Q(\cdot)$  denotes the expectation under the risk-neutral measure  $Q$  conditional on  $\mathcal{F}_t^S$ , and the state-price deflator is:

$$\tilde{\pi}_t = e^{-rt + \frac{1}{2} \int_0^t \tilde{\eta}_s^2 ds - \int_0^t \tilde{\eta}_s dB_s^Q}. \quad (48)$$

Therefore,

$$\begin{aligned}\hat{W}_t^U &= (\lambda_U \tilde{\pi}_t)^{-\frac{1}{\gamma}} e^{-\left(1-\frac{1}{\gamma}\right)r(T-t)} E_t^Q \left[ e^{-\frac{1}{2\gamma} \int_t^T \tilde{\eta}_s^2 ds + \frac{1}{\gamma} \int_t^T \tilde{\eta}_s dB_s^Q} \right] \\ &= (\lambda_U \tilde{\pi}_t)^{-\frac{1}{\gamma}} e^{-\left(1-\frac{1}{\gamma}\right)r(T-t)} \sqrt{(1+v_t(T-t))^{-\frac{1}{\gamma}}} E_t^Q \left[ e^{-\frac{1}{2\gamma} \frac{\tilde{\eta}_t^2}{v_t} + \frac{v_T}{2\gamma} (B_T^Q - B_t^Q + \frac{\tilde{\eta}_t}{v_t})^2} \right],\end{aligned}\quad (49)$$

where the last equality follows from applying Lemma 1 with  $\kappa = -\frac{1}{\gamma}$  and  $u = T$  to the expectation on the RHS. Using part (ii) of Lemma 2 with  $z = B_T^Q - B_t^Q$ ,  $\sigma_z^2 = T - t$ ,  $\rho = -\frac{v_T}{2\gamma}$ ,  $c = -\frac{\tilde{\eta}_t}{v_t}$ , and  $\bar{z} \rightarrow \infty$  to compute this expectation:

$$\hat{W}_t^U = (\lambda_U \tilde{\pi}_t)^{-\frac{1}{\gamma}} e^{-\left(1-\frac{1}{\gamma}\right)r(T-t)} \times \sqrt{\frac{(1+v_t(T-t))^{1-\frac{1}{\gamma}}}{1 + \left(1-\frac{1}{\gamma}\right)v_t(T-t)}} \exp \left\{ -\frac{1}{2\gamma} \frac{(1-\frac{1}{\gamma})(T-t)}{1 + \left(1-\frac{1}{\gamma}\right)v_t(T-t)} \tilde{\eta}_t^2 \right\}.\quad (50)$$

Equation (15) for the Lagrange multiplier follows from solving for  $\lambda_U$  in (50) for  $t = 0$  with  $\hat{W}_0^U = w$ . In order to derive the investment policy (14) replicating the optimal portfolio value (13), note that this can be rewritten as  $\hat{W}_t^U = f(t, \tilde{\eta}_t; T)$ , with  $d\tilde{\eta}_t = -v_t \tilde{\eta}_t + v_t dB_t^Q$  and  $f \in C^{1,2}$ . Applying Itô's Lemma the diffusion term of  $d\hat{W}_t^U$  is:  $v_t \frac{\partial d\hat{W}_t^U}{\partial \tilde{\eta}_t}$ .

Under the risk-neutral measure,  $d\hat{W}_t^U$  satisfies the self-financing constraint:

$$d\hat{W}_t^U = \hat{W}_t^U r dt + \hat{W}_t^U \hat{\phi}_t^U \sigma dB_t^Q,\quad (51)$$

Equating diffusion terms:

$$\hat{\phi}_t^U = \frac{v_t}{\hat{W}_t^U \sigma} \frac{\partial d\hat{W}_t^U}{\partial \tilde{\eta}_t}.\quad (52)$$

Substituting the derivative of (50) with respect to  $\tilde{\eta}_t$  in (52) gives the optimal risk exposure (14).  $\square$

**Proof of Proposition 2.** Following the same approach as in the derivation of (46) above, the optimal terminal wealth of manager  $i \in \mathcal{I}$  is:

$$\hat{W}_T^I(i) = \begin{cases} (\lambda_I(i) \pi_T)^{-\frac{1}{\gamma}} & \text{if } \lambda_I(i) \pi_T > b \left( \delta \hat{W}_T^U \right), & (I) \\ (1 + \alpha)^{\frac{1}{\gamma}} (\lambda_I(i) \pi_T)^{-\frac{1}{\gamma}} \left( \delta \hat{W}_T^U \right)^{\frac{\tilde{\gamma}-\gamma}{\tilde{\gamma}}} & \text{if } \lambda_I(i) \pi_T \leq b \left( \delta \hat{W}_T^U \right), & (II) \end{cases}\quad (53)$$

where  $\lambda_I(i)$  is the Lagrange multiplier attached to the  $i$  manager's time- $T$  budget constraint (18) solving  $w = E \left[ \pi_T \hat{W}_T^I(i) \right]$  and  $b(x)$  is given in the proof of Proposition 1. It is immediate to see that, since initial wealth  $i$  is the same for all  $I$ -managers, we have  $\lambda_I(i) = \lambda_I$  and thus  $\hat{W}_T^I(i) = \hat{W}_T^I$  in (53) for all  $i \in \mathcal{I}$ . In order to express regions (I) and (II) in terms of the state variable  $\tilde{\eta}_T$  and the model parameters, note that  $\pi_T = e^{-(r+\frac{\eta^2}{2})T-\eta B_T} = e^{(\frac{\eta^2}{2}-r)T-\eta B_T^Q}$ , and that  $U$ -managers' optimal terminal wealth is given by (46) with:

$$\tilde{\pi}_T = e^{-rT + \frac{1}{2} \int_0^T \tilde{\eta}_s^2 ds - \int_0^T \tilde{\eta}_s dB_s^Q} = \sqrt{1 + v_0 T} e^{-rT + \frac{m^2}{2v_0} - \frac{v_T}{2} (B_T^Q + \frac{m}{v_0})^2},\quad (54)$$

where the last equality follows from the application of Lemma 1 with  $\kappa = 1, t = 0, u = T$ . Therefore,

region (I) can be rewritten (omitting the index  $i$ ) as:

$$\begin{aligned}
& \lambda_I \pi_T > b \left( \delta \hat{W}_T^U \right) \\
& \Leftrightarrow \lambda_I e^{(\frac{\eta}{2}-r)T-\eta B_T^Q} > (1+\alpha) \gamma^{\frac{1+\alpha}{\alpha}} \left( \frac{\gamma}{\tilde{\gamma}} \right)^{\frac{\tilde{\gamma}}{\tilde{\gamma}-\gamma}} \delta^{-\gamma} \lambda_U \sqrt{1+v_0 T} e^{\frac{m^2}{2v_0} - \frac{v_T}{2} (B_T^Q + \frac{m}{v_0})^2} \\
& \Leftrightarrow \lambda_I e^{(\frac{\eta}{2}-r)T-\eta(\frac{\tilde{\eta}}{v_T} - \frac{m}{v_0})} > (1+\alpha) \gamma^{\frac{1+\alpha}{\alpha}} \left( \frac{\gamma}{\tilde{\gamma}} \right)^{\frac{\tilde{\gamma}}{\tilde{\gamma}-\gamma}} \delta^{-\gamma} \lambda_U \sqrt{1+v_0 T} e^{\frac{m^2}{2v_0} - \frac{1}{2} \frac{\tilde{\eta}^2}{v_T}} \\
& \Leftrightarrow \frac{(\tilde{\eta}_T - \eta)^2}{v_T} - \frac{(m - \eta)^2}{v_0} > 2 \ln \left[ \lambda^{-1} A_0 \sqrt{1+v_0 T} \right], \tag{55}
\end{aligned}$$

for  $A_0 \equiv \left( \frac{(1+\alpha)\frac{1+\alpha}{\delta}}{\delta} \right)^\gamma \left( \frac{\gamma}{\tilde{\gamma}} \right)^{\frac{\tilde{\gamma}}{\tilde{\gamma}-\gamma}} \lambda_U > 0$ . Region (II) is just the relative complement in  $\mathbb{R}$  of region (I). Note that by letting  $\theta \equiv \frac{\tilde{\gamma}-\gamma}{\tilde{\gamma}} < 1$ , plugging (46) and (54) in the second part of (53) and rearranging we obtain the alternative expression for  $I$ -managers' wealth in region (II) of Corollary 1:

$$(1+\alpha)^{\frac{1}{\tilde{\gamma}}} (\lambda_I \pi_T)^{-\frac{1}{\tilde{\gamma}}} \left( \delta \hat{W}_T^U \right)^{\frac{\tilde{\gamma}-\gamma}{\tilde{\gamma}}} = (\lambda_I \pi_T)^{-\frac{1}{\tilde{\gamma}}} g_2(T, \tilde{\eta}_T; T), \tag{56}$$

where:

$$g_2(t, x; T) \equiv A_1 \sqrt{(1+v_0 t)^{-\theta}} \exp \left\{ \theta r T + \frac{\theta}{2} \left( \frac{x^2}{v_t} - \frac{m^2}{v_0} \right) \right\}, \tag{57}$$

and  $A_1 \equiv (1+\alpha)^{\frac{1}{\tilde{\gamma}}} \delta^{\frac{\tilde{\gamma}-\gamma}{\tilde{\gamma}}} \lambda_U^{-\theta} > 0$ .

We can now derive the interim performance (20). Under the risk-neutral measure the deflated wealth is a martingale, so using (53) the optimal wealth  $\hat{W}_t^I$  for all  $t \in [0, T]$  is given by:

$$\begin{aligned}
\hat{W}_t^I &= e^{-r(T-t)} E_t^Q \left[ \hat{W}_T^I \right] \\
&= e^{-r(T-t)} \left\{ \lambda_I^{-\frac{1}{\tilde{\gamma}}} E_t^Q \left[ \pi_T^{-\frac{1}{\tilde{\gamma}}} \mathbf{1}_{\mathcal{R}_I} \right] + (1+\alpha)^{\frac{1}{\tilde{\gamma}}} \lambda_I^{-\frac{1}{\tilde{\gamma}}} E_t^Q \left[ \pi_T^{-\frac{1}{\tilde{\gamma}}} \left( \delta \hat{W}_T^U \right)^{\frac{\tilde{\gamma}-\gamma}{\tilde{\gamma}}} \mathbf{1}_{\mathcal{R}_{II}} \right] \right\} \tag{58}
\end{aligned}$$

where  $\mathcal{R}_I$  denotes the underperformance region in (55):

$$\begin{aligned}
& \frac{(\tilde{\eta}_T - \eta)^2}{v_T} > \frac{(m - \eta)^2}{v_0} + 2 \ln \left[ \lambda^{-1} A_0 \sqrt{1+v_0 T} \right] \\
& \Leftrightarrow \left| \frac{\tilde{\eta}_T - \eta}{\sqrt{v_T}} \right| > \sqrt{\frac{(m - \eta)^2}{v_0} + 2 \ln \left[ \lambda^{-1} A_0 \sqrt{1+v_0 T} \right]} = \sqrt{v_T} \varphi(\lambda_I) \\
& \Leftrightarrow \left\{ \frac{\tilde{\eta}_T}{v_T} - \frac{\tilde{\eta}_t}{v_t} < \frac{\eta}{v_T} - \frac{\tilde{\eta}_t}{v_t} - \varphi(\lambda_I) \right\} \cup \left\{ \frac{\tilde{\eta}_T}{v_T} - \frac{\tilde{\eta}_t}{v_t} > \frac{\eta}{v_T} - \frac{\tilde{\eta}_t}{v_t} + \varphi(\lambda_I) \right\}, \tag{59}
\end{aligned}$$

and  $\mathcal{R}_{II}$  is the outperformance region  $\mathbb{R} \setminus \mathcal{R}_I$ . The first expectation on the RHS of (58) is:

$$\begin{aligned}
E_t^Q \left[ \pi_T^{-\frac{1}{\tilde{\gamma}}} \mathbf{1}_{\mathcal{R}_I} \right] &= \pi_t^{-\frac{1}{\tilde{\gamma}}} E_t^Q \left[ \left( \frac{\pi_T}{\pi_t} \right)^{-\frac{1}{\tilde{\gamma}}} \mathbf{1}_{\mathcal{R}_I} \right] \\
&= \pi_t^{-\frac{1}{\tilde{\gamma}}} e^{\frac{1}{\tilde{\gamma}} \left( r - \frac{\eta^2}{2} \right) (T-t)} E_t^Q \left[ e^{\frac{\eta}{\tilde{\gamma}} (B_T^Q - B_t^Q)} \mathbf{1}_{\mathcal{R}_I} \right] \\
&= \pi_t^{-\frac{1}{\tilde{\gamma}}} e^{\frac{1}{\tilde{\gamma}} \left( r - \frac{\gamma-1}{2\tilde{\gamma}} \eta^2 \right) (T-t)} \left[ \mathcal{N}(d_{1,t}) + 1 - \mathcal{N}(d_{2,t}) \right], \tag{60}
\end{aligned}$$

where the last equality follows from the applying part (i) of Lemma 2 to the expectation on the RHS, with:  $z = B_T^Q - B_t^Q = \frac{\tilde{\eta}_T}{v_T} - \frac{\tilde{\eta}_t}{v_t}$ ,  $\sigma_z^2 = T - t$ ,  $\rho = \frac{\eta}{\gamma}$ ,  $\bar{z} = \frac{\eta}{v_T} - \frac{\tilde{\eta}_t}{v_t} \mp \varphi(\lambda_I)$ , and:

$$d_{1,t} \equiv \frac{\frac{\eta}{v_T} - \frac{\tilde{\eta}_t}{v_t} - \frac{\eta}{\gamma}(T-t) - \varphi(\lambda_I)}{\sqrt{T-t}} = \frac{\frac{\eta - \tilde{\eta}_t}{v_t} + \frac{\gamma-1}{\gamma}\eta\tau - \varphi(\lambda_I)}{\sqrt{\tau}}, \quad (61)$$

$$d_{2,t} \equiv \frac{\frac{\eta}{v_T} - \frac{\tilde{\eta}_t}{v_t} - \frac{\eta}{\gamma}(T-t) + \varphi(\lambda_I)}{\sqrt{T-t}} = d_{1,t} + 2\frac{\varphi(\lambda_I)}{\sqrt{\tau}}. \quad (62)$$

The second expectation on the RHS of (58) is:

$$E_t^Q \left[ \pi_T^{-\frac{1}{\gamma}} \left( \delta \hat{W}_T^U \right)^{\frac{\gamma-\gamma}{\gamma}} \mathbf{1}_{\mathcal{R}_{II}} \right] = \lambda_U^{-\theta} \delta^{\frac{\gamma-\gamma}{\gamma}} \pi_t^{-\frac{1}{\gamma}} \tilde{\pi}_t^{-\theta} E_t^Q \left[ \left( \frac{\pi_T}{\pi_t} \right)^{-\frac{1}{\gamma}} \left( \frac{\tilde{\pi}_T}{\tilde{\pi}_t} \right)^{-\theta} \mathbf{1}_{\mathcal{R}_{II}} \right]. \quad (63)$$

We have:

$$\begin{aligned} E_t^Q \left[ \left( \frac{\pi_T}{\pi_t} \right)^{-\frac{1}{\gamma}} \left( \frac{\tilde{\pi}_T}{\tilde{\pi}_t} \right)^{-\theta} \mathbf{1}_{\mathcal{R}_{II}} \right] &= e^{\frac{1}{\gamma} \left( r - \frac{\eta^2}{2} \right) (T-t)} E_t^Q \left[ e^{\frac{\eta}{\gamma} (B_T^Q - B_t^Q)} \left( \frac{\tilde{\pi}_T}{\tilde{\pi}_t} \right)^{-\theta} \mathbf{1}_{\mathcal{R}_{II}} \right] \\ &= \sqrt{(1 + v_t(T-t))}^{-\theta} e^{\frac{1}{\gamma} \left( r - \frac{\eta^2}{2} \right) (T-t) - \frac{\theta}{2} \frac{\tilde{\eta}_t^2}{v_t}} \\ &\quad \times E_t^Q \left[ e^{\frac{\eta}{\gamma} (B_T^Q - B_t^Q) + \frac{\theta v_T}{2} (B_T^Q - B_t^Q + \frac{\tilde{\eta}_t}{v_t})^2} \mathbf{1}_{\mathcal{R}_{II}} \right] \\ &= \sqrt{(1 + v_t(T-t))}^{-\theta} e^{\frac{1}{\gamma} \left( r - \frac{\eta^2}{2} \right) (T-t) - \frac{\theta}{2} \frac{\tilde{\eta}_t^2}{v_t} + \frac{\theta v_T}{2} \left[ \frac{\tilde{\eta}_t^2}{v_t} - \left( \frac{\tilde{\eta}_t}{v_t} + \frac{\gamma}{\gamma-\gamma} \frac{\eta}{v_T} \right)^2 \right]} \\ &\quad \times E_t^Q \left[ e^{\frac{\theta v_T}{2} (B_T^Q - B_t^Q + \frac{\tilde{\eta}_t}{v_t} + \frac{\gamma}{\gamma-\gamma} \frac{\eta}{v_T})^2} \mathbf{1}_{\mathcal{R}_{II}} \right] \end{aligned} \quad (64)$$

Using part (ii) of Lemma 2 for  $z = B_T^Q - B_t^Q$ ,  $\sigma_z^2 = T - t$ ,  $\rho = -\frac{\theta v_T}{2}$ ,  $c = -\frac{\tilde{\eta}_t}{v_t} + \frac{\gamma}{\gamma-\gamma} \frac{\eta}{v_T}$ , and  $\bar{z} = \frac{\eta}{v_T} - \frac{\tilde{\eta}_t}{v_t} \pm \varphi(\lambda_I)$ , the expectation on the RHS is:

$$E_t^Q \left[ e^{\frac{\theta v_T}{2} (B_T^Q - B_t^Q + \frac{\tilde{\eta}_t}{v_t} + \frac{\gamma}{\gamma-\gamma} \frac{\eta}{v_T})^2} \mathbf{1}_{\mathcal{R}_{II}} \right] = \frac{e^{\frac{1}{2} \frac{\theta v_T}{1-\theta v_T(T-t)} \left( \frac{\tilde{\eta}_t}{v_t} + \frac{\gamma}{\gamma-\gamma} \frac{\eta}{v_T} \right)^2}}{\sqrt{1 - \theta v_T(T-t)}} [\mathcal{N}(d_{3,t}) + 1 - \mathcal{N}(d_{4,t})], \quad (65)$$

where:

$$d_{3,t} \equiv \frac{\frac{\eta}{v_T} - \frac{\tilde{\eta}_t}{v_t} - \frac{\theta v_T}{1-\theta v_T(T-t)} \left( \frac{\tilde{\eta}_t}{v_t} + \frac{\gamma}{\gamma-\gamma} \frac{\eta}{v_T} \right) + \varphi(\lambda_I)}{\sqrt{(T-t)/(1-\theta v_T(T-t))}} = \frac{\frac{\eta - \tilde{\eta}_t}{v_t} + \frac{\gamma-1}{\gamma}\eta\tau - \varphi(\lambda_I)}{\sqrt{\tau}}, \quad (66)$$

$$d_{4,t} \equiv \frac{\frac{\eta}{v_T} - \frac{\tilde{\eta}_t}{v_t} - \frac{\theta v_T}{1-\theta v_T(T-t)} \left( \frac{\tilde{\eta}_t}{v_t} + \frac{\gamma}{\gamma-\gamma} \frac{\eta}{v_T} \right) - \varphi(\lambda_I)}{\sqrt{(T-t)/(1-\theta v_T(T-t))}} = d_{3,t} - 2\frac{\varphi(\lambda_I)}{\sqrt{\frac{\tau}{1-\theta v_T(T-t)}}}. \quad (67)$$



Using the formulas in (64) and (65):

$$\begin{aligned}
E_t^Q \left[ \pi_T^{-\frac{1}{\bar{\gamma}}} \left( \delta \hat{W}_T^U \right)^{\frac{\bar{\gamma}-\gamma}{\bar{\gamma}}} \mathbf{1}_{\mathcal{R}_{II}} \right] &= \lambda_U^{-\theta} \delta^{\frac{\bar{\gamma}-\gamma}{\bar{\gamma}}} \pi_t^{-\frac{1}{\bar{\gamma}}} \tilde{\pi}_t^{-\theta} \\
&\times \sqrt{(1 + v_t(T-t))^{-\theta}} e^{\frac{1}{\bar{\gamma}} \left( r - \frac{\eta^2}{2} \right) (T-t) - \frac{\theta}{2} \frac{\tilde{\eta}_t^2}{v_t} + \frac{\theta v_T}{2} \left[ \frac{\tilde{\eta}_t^2}{v_t^2} - \left( \frac{\tilde{\eta}_t}{v_t} + \frac{\gamma}{\bar{\gamma}-\gamma} \frac{\eta}{v_T} \right)^2 \right]} \\
&\times \frac{e^{\frac{1}{2} \frac{\theta v_T}{1-\theta v_T(T-t)} \left( \frac{\tilde{\eta}_t}{v_t} + \frac{\gamma}{\bar{\gamma}-\gamma} \frac{\eta}{v_T} \right)^2}}{\sqrt{1 - \theta v_T(T-t)}} [\mathcal{N}(d_{3,t}) + 1 - \mathcal{N}(d_{4,t})]. \tag{68}
\end{aligned}$$

Plugging (60) and (68) in 58 and collecting like-terms:

$$\begin{aligned}
\hat{W}_t^I &= (\lambda_I \pi_t)^{-\frac{1}{\bar{\gamma}}} Z(\gamma, \tau) [\mathcal{N}(d_{1,t}) + 1 - \mathcal{N}(d_{2,t})] + (\lambda_I \pi_t)^{-\frac{1}{\bar{\gamma}}} Z(\bar{\gamma}, \tau) g_1(\theta, t, \tilde{\eta}_t; T) g_2(t, \tilde{\eta}_t; T) \\
&\times \exp \left\{ \frac{\theta \tau}{1 + (1-\theta)v_t \tau} \frac{\eta}{\bar{\gamma}} \left( \tilde{\eta}_t + \frac{\eta}{2\bar{\gamma}} v_t \tau \right) \right\} [\mathcal{N}(d_{3,t}) - \mathcal{N}(d_{4,t})], \tag{69}
\end{aligned}$$

where  $g_1(\cdot)$  is given in Section 3.1 and  $Z(\zeta, t) \equiv \exp \left\{ -\frac{\zeta-1}{\zeta} \left( r + \frac{\eta^2}{2\zeta} \right) t \right\}$ .

In order to derive the investment policy (21) replicating the optimal portfolio value (20), note that this can be rewritten as  $\hat{W}_t^I = f(t, \tilde{\eta}_t; T)$ , with  $d\tilde{\eta}_t = -v_t \tilde{\eta}_t + v_t dB_t^Q$  and  $f \in C^{1,2}$ . Applying Itô's Lemma the diffusion term of  $d\hat{W}_t^I$  is:  $v_t \frac{\partial d\hat{W}_t^I}{\partial \tilde{\eta}_t}$ .

Under the risk-neutral measure,  $d\hat{W}_t^I$  satisfies the self-financing constraint:

$$d\hat{W}_t^I = \hat{W}_t^I r dt + \hat{W}_t^I \hat{\phi}_t^I \sigma dB_t^Q, \tag{70}$$

Equating diffusion terms:

$$\hat{\phi}_t^I = \frac{v_t}{\hat{W}_t^I \sigma} \frac{\partial d\hat{W}_t^I}{\partial \tilde{\eta}_t}. \tag{71}$$

Substituting the derivative of (69) with respect to  $\tilde{\eta}_t$  in (71) gives the optimal risk exposure (21).  $\square$

**Proof of Corollary 1.** Equation (23) follows from equations (53) to (57) in the proof of Proposition 2. Expressions (24) for the minimum underperformance and outperformance margins follow from Proposition 1 in Basak and Makarov (2011).  $\square$

**Proof of Proposition 3.** The Brownian motion  $B$  under the actual probability  $P$  is related to the risk-neutral Brownian motion  $B^Q$  through:  $B_t = B_t^Q - \eta t$ . Delegation to an uninformed manager results in final wealth  $\hat{W}_T^U$  as given by (46) and (54), where  $\tilde{\pi}_T$  can be rewritten as:

$$\tilde{\pi}_T = \sqrt{1 + v_0 T} e^{-rT + \frac{m^2}{2v_0} - \frac{v_T}{2} \left( B_t + \eta t + \frac{m}{v_0} \right)^2}. \tag{72}$$

Letting  $\hat{\gamma} = \frac{1-\gamma_h}{\gamma}$ , households' expected utility from delegating to  $U$ -managers is then:

$$E \left[ \frac{\left( \hat{W}_T^U \right)^{1-\gamma_h}}{1-\gamma_h} \right] = \frac{\lambda^{\hat{\gamma}U}}{1-\gamma_h} E \left[ \tilde{\pi}_T^{\hat{\gamma}U} \right]$$

$$= \frac{\lambda^{\hat{\gamma}U} e^{-\hat{\gamma}rT}}{1-\gamma_h} \sqrt{\frac{(1+v_0T)^{1+\hat{\gamma}}}{1+(1+\hat{\gamma})v_0T}} \exp \left\{ \frac{\hat{\gamma}}{2} \frac{m^2}{v_0} - \frac{\hat{\gamma}v_T}{1+\hat{\gamma}v_T} \frac{\left( \eta T + \frac{m}{v_0} \right)^2}{2} \right\}, \quad (73)$$

where the last equality follows from applying part (ii) of Lemma 2 for  $z = B_T, \rho = \frac{\hat{\gamma}v_T}{2}, c = -(\eta t + \frac{m}{v_0})$  and  $\bar{z} = +\infty$ . Rearranging, we get the formula in (25).

When households delegate their portfolio to an informed manager, utility over final wealth depends crucially on whether the  $I$ -manager underperforms or outperforms the benchmark. As of  $t = 0$ , (59) tells us that the first will be the case when:

$$\left\{ \frac{\tilde{\eta}_T}{v_T} - \frac{m}{v_0} < \frac{\eta}{v_T} - \frac{m}{v_0} - \varphi(\lambda_I) \right\} \cup \left\{ \frac{\tilde{\eta}_T}{v_T} - \frac{m}{v_0} > \frac{\eta}{v_T} - \frac{m}{v_0} + \varphi(\lambda_I) \right\}$$

$$\Leftrightarrow \left\{ B_T^Q < \frac{\eta}{v_T} - \frac{m}{v_0} - \varphi(\lambda_I) \right\} \cup \left\{ B_T^Q > \frac{\eta}{v_T} - \frac{m}{v_0} + \varphi(\lambda_I) \right\}$$

$$\Leftrightarrow \left\{ B_T < \frac{\eta}{v_T} - \left( \frac{m}{v_0} \eta T \right) - \varphi(\lambda_I) \right\} \cup \left\{ B_T > \frac{\eta}{v_T} - \left( \frac{m}{v_0} \eta T \right) + \varphi(\lambda_I) \right\} \equiv \mathcal{R}_I^0, \quad (74)$$

with outperformance occurring in  $\mathcal{R}_{II}^0 \equiv \mathbb{R} \setminus \mathcal{R}_I^0$ . Letting  $\check{\gamma} \equiv \frac{\gamma_h-1}{\gamma}$  and noting that  $\pi_T = e^{-(r+\frac{\eta^2}{2})T-\eta B_T}$  under  $P$ , households' derived utility can then be computed as:

$$E \left[ \frac{\left( \hat{W}_T^I \right)^{1-\gamma_h}}{1-\gamma_h} \right] = \frac{1}{1-\gamma_h} \left\{ E \left[ \left( \lambda_I \pi_T \right)^{\frac{\gamma_h-1}{\gamma}} \mathbf{1}_{\mathcal{R}_I} \right] + E \left[ \left( (1+\alpha)^{\frac{1}{\check{\gamma}}} \lambda_I^{-\frac{1}{\check{\gamma}}} \pi_T^{-\frac{1}{\check{\gamma}}} \left( \delta \hat{W}_T^U \right)^{\frac{\check{\gamma}-\gamma}{\check{\gamma}}} \right)^{1-\gamma_h} \mathbf{1}_{\mathcal{R}_{II}^0} \right] \right\}$$

$$= \frac{\lambda^{\hat{\gamma}U} e^{-\hat{\gamma} \left( r + \frac{\eta^2}{2} \right) T}}{1-\gamma_h} E \left[ e^{-\check{\gamma} \eta B_T} \mathbf{1}_{\mathcal{R}_I^0} \right]$$

$$+ \frac{A_1^{1-\gamma_h} \sqrt{(1+v_0T)^{\theta(\gamma_h-1)}} e^{-\check{\gamma} \left( r + \frac{\eta^2}{2} \right) T + \theta(\gamma_h-1) \frac{m^2}{2v_0}}}{1-\gamma_h}$$

$$\times E \left[ e^{-\frac{\theta(\gamma_h-1)v_T}{2} \left( B_t + \eta t + \frac{m}{v_0} \right)^2 - \check{\gamma} \eta B_T} \mathbf{1}_{\mathcal{R}_{II}^0} \right] \quad (75)$$

Finally, using exactly the same steps as in the proof of Proposition 2 to compute the expectations in (75) we arrive at the formulas in Proposition 3.  $\square$

## B Model parameterization

Except otherwise noted, the investment horizon is assumed to be  $T = 1$  year. I identify the risk-less asset  $\beta$  with the 3-month U.S. Treasury bill, and the stock  $S$  with a broad-based market portfolio. The *baseline* scenario is as follows. I assume the real risk-less interest rate  $r$  is 3%. For the prior market price of risk  $m$  and market volatility  $\sigma$ , I use historical estimates during the sample January 1980-December

2006. This corresponds to a recent and relatively long period for which the hypothesis of normality of annual returns cannot be rejected.<sup>41</sup> Following Brennan and Xia (2001), I set the prior for the market excess return  $\mu - r$  equal to the sample mean return of the Fama and French (1996) market portfolio during the period, 8.1%. The corresponding standard deviation of the market portfolio,  $\sigma$ , equals 15.8%.

I set the prior variance  $v_0$  equal to (the square of) the standard error of the sample mean market price of risk. This standard error equals 0.192 for the period 1980-2006 and corresponds to a standard error for the mean return of 3%, in line with baseline values used by the prior literature. As alternative scenarios, I also examine the cases  $v_0 = 0.063^2$  (corresponding to a standard error for the mean return of 1%) and  $v_0 = 0.3167^2$  (corresponding to a standard error for the mean return of 5%).

I assume baseline coefficients of relative risk aversion equal to 5 for both managers ( $\gamma$ ) and households ( $\gamma_h$ ) in order to approximately match the median curvature parameter found by Kimball, Sahm, and Shapiro (2007) from the Health and Retirement Survey using hypothetical income gambles. This value is also in line with the empirical estimates in Kojien (2010) in a similar setup. This author highlights a high dispersion in managerial risk attitude, and moreover a strong correlation between ability and risk aversion. Because more skilled managers correspond to those with complete information in the present setup, I favor a moderately high coefficient of risk aversion in the numerical analysis of Section 4, but consider both lower (minimum= 2) and higher (maximum= 8) values as alternative specifications.

Without loss of generality, I set funds' initial  $w$  equal to 1. For the flow-performance relationship, I allow the performance threshold  $\delta$  to vary between 0.9 and 1.1.<sup>42</sup> For the performance rankings of mutual funds in my sample, the average minimum return relative to the median in deciles 9 (second lowest performance) is -8.8%, whereas the average minimum return for the top decile is 7.8%, implying that the range considered for  $\delta$  is wide enough to include most plausible performance thresholds specifications in empirically estimated fund flows. With the same purpose, I allow the flow elasticity  $\alpha$  to vary between 0.5 and 3.

## C Numerical Simulations

The model outcomes shown in figures 3 to 5 result from simulating equations (13)-(17), (20)-(22) and (25)-(26) for all possible economic states according to the baseline and alternative parameterizations in Appendix B, and averaging their values over their joint probability distribution.

More precisely, I first simulate a grid of possible realizations of the market price of risk  $\eta$  at  $t = 0$  over the support of its prior distribution  $\mathbf{N}(m, v_0)$ . For each realization  $\eta$  I compute the plotted variable as follows:

**Tracking error, Sharpe ratios and probability of outperformance:** I first simulate a grid of realizations of the Brownian motion process  $B$  as of  $t = T$  over the support of  $\mathbf{N}(0, T)$ , and compute end-of-period log returns for the informed and uninformed managers using equations (13) and (20). Tracking error variance  $TE(\eta)$  and probability of outperformance  $p^O(\eta)$  are then the averages over  $\mathbf{N}(0, T)$  of  $(r_T^I - r_T^U)^2$  and  $\mathbb{1}_{\{r_T^I > r_T^U\}}$ , where  $r_T^J \equiv \log(R_T^J)$  for  $J \in \{I, U\}$ .<sup>43</sup> Analogously, manager  $J$ 's Sharpe ratio

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<sup>41</sup> The Jarque-Bera test of normality results in a  $p$ -value of 16.6% during this period.

<sup>42</sup> Unlike Basak and Makarov (2011), setting  $\delta < 1$  does not raise non-existence or multiplicity of equilibrium concerns in my model because it does not feature the strategic interactions in their model.

<sup>43</sup> Averages are approximated by Gaussian quadrature of the corresponding mathematical expectation, similar to (78) below.

$SR^J(\eta)$  is the average of  $r_T^J - r$  divided by the standard deviation of  $r_T^J$ , whereas the excess Sharpe ratio is the difference  $SR^I(\eta) - SR^U(\eta)$ .

**Certainty equivalent returns (CER):** The certainty equivalent return  $CER(u_J(\eta))$  for the derived utility  $u_J$  ( $J \in \{I, U\}$ ) of delegating to a  $J$ -manager computed in Section 3.3 is given by:

$$CER(u_J(\eta)) = \frac{1}{w} [(1 - \gamma_h)u_J(\eta)]^{\frac{1}{1-\gamma_h}}. \quad (76)$$

I denote  $\rho(\eta)$  the risk-less return differential from delegating to an informed instead of to an uninformed manager:  $\rho(\eta) = CER(u_I(\eta))/CER(u_U(\eta)) - 1$ .

**Jensen's alpha:** I approximate managers' realized Jensen's alpha during the period by Monte Carlo simulations. First, I simulate  $M = 10,000$  paths of  $K = 52$  periods (weeks) of the Brownian motion  $B$ . Letting  $\Delta \equiv T/K$ , the market premium  $r_t^S - r\Delta$  ( $r_t^S \equiv \log(S_t/S_{t-\Delta})$ ) represents the only risk factor for the simple financial market structure in this setup. I then estimate Jensen's alpha as the intercept in the CAPM regression:

$$r_t^J - r\Delta = \alpha^J(\eta) + \beta^J(\eta)(r_t^S - r\Delta) + \epsilon, \quad (77)$$

where  $r_t^J \equiv \log(R_t^J/R_{t-\Delta}^J)$  and  $J \in \{I, U\}$ , over the time-series  $t = 1, \dots, K$  for the  $m$ -th path of  $B$ . Informed managers' excess alpha for the realized market price of risk  $\eta$  is then the average over the  $M$  paths of  $B$  of  $\alpha^J(\eta) - \alpha^U(\eta)$ .<sup>44</sup>

The final step consists in averaging these variables over  $\mathbf{N}(m, v_0)$  using Gaussian quadrature over the grid of  $\eta$ . The reported values in figures 3 to 5 are computed as:

$$\bar{\rho} \equiv E^\eta[x] = \int_{-\infty}^{\infty} x(\eta) \frac{e^{-\frac{(\eta-m)^2}{2v_0}}}{\sqrt{2\pi v_0}} d\eta, \quad (78)$$

where  $x$  equals, alternatively, tracking error,  $\rho(\eta)$ , excess Sharpe ratio, probability of outperformance or Jensen's alpha.<sup>45</sup>

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<sup>44</sup>I check the reliability of the Monte Carlo estimates by increasing, alternatively, the number of paths  $M$  to 50,000 and the number of periods  $K$  to 252. In all cases, the relative gain in accuracy is less than 1%.

<sup>45</sup>For a fine enough grid of  $\eta$ , the expectation in (78) can be approximated with arbitrary precision.

## D Tables and Figures

	All MFs (N=3060)		Top 30%	
	Mean	Std Dev	Mean	Std Dev
TNA (\$ mill)	<b>922</b>	3,876	<b>1,584</b>	5,542
Age	<b>17.6</b>	14.0	<b>19.8</b>	15.6
Expense Ratio (%)	<b>1.22</b>	0.58	<b>1.11</b>	0.56
Annual Return (%)	<b>7.65</b>	20.32	<b>9.21</b>	20.77
Std dev of Return (% monthly)	<b>4.18</b>	2.03	<b>4.27</b>	2.01
Jensen's alpha <sup>1</sup> (% monthly)	<b>0.02</b>	0.18	<b>0.15</b>	0.17
Beta <sup>1</sup>	<b>0.01</b>	0.16	<b>0.04</b>	0.18
Position in Return Ranking	<b>0.51</b>	0.29	<b>0.60</b>	0.28
Proportion of Outperforming Years	<b>0.46</b>	0.17	<b>0.65</b>	0.09
Number of years ranked	<b>11.4</b>	5.0	<b>11.2</b>	5.4

Table 1: **Summary statistics.** The table reports summary statistics over the period 1981-2010 for mutual funds ranked in at least 5 years. Annual rankings for each objective category are computed over raw returns and normalized to be between 0 and 1. (1) Jensen's alpha and beta are computed over monthly returns with respect to a portfolio of median performers comprised of mutual funds with performance rank between 0.45 and 0.55. A fund is considered to *outperform* in a year if its performance rank that year is greater than 0.55. For each mutual fund, the proportion of outperforming years is the ratio of number of years outperformed to the number of years ranked. Based on the proportion of outperforming years, all funds are sorted in ascending order. The top 30% are selected as the "informed" mutual funds in the analysis.

Herding measured by:	HerdTrackErr				HerdBeta				HerdCorr			
	Contrarian Group		Herding Group		Contrarian Group		Herding Group		Contrarian Group		Herding Group	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
TNA (\$ mill)	<b>821</b>	2,431	<b>1,866</b>	7,670	<b>1,214</b>	4,230	<b>1,621</b>	7,670	<b>1,288</b>	4,325	<b>1,760</b>	7,670
Age	<b>18.0</b>	11.6	<b>19.0</b>	18.7	<b>17.7</b>	12.1	<b>22.6</b>	18.7	<b>21.1</b>	14.3	<b>16.2</b>	18.7
Expense Ratio (%)	<b>1.32</b>	0.52	<b>0.81</b>	0.50	<b>1.27</b>	0.57	<b>1.01</b>	0.50	<b>1.31</b>	0.48	<b>0.85</b>	0.50
Annual Return (%)	<b>10.03</b>	23.02	<b>8.27</b>	18.54	<b>9.58</b>	22.43	<b>8.71</b>	18.54	<b>10.17</b>	21.41	<b>8.00</b>	18.54
Std dev of Return (% monthly)	<b>4.77</b>	2.24	<b>3.93</b>	1.79	<b>4.55</b>	2.24	<b>4.11</b>	1.79	<b>4.41</b>	2.20	<b>4.05</b>	1.79
Position in Return Ranking	<b>0.62</b>	0.33	<b>0.59</b>	0.22	<b>0.61</b>	0.32	<b>0.59</b>	0.22	<b>0.61</b>	0.32	<b>0.59</b>	0.22
Proportion of Outperforming Years	<b>0.66</b>	0.10	<b>0.65</b>	0.08	<b>0.66</b>	0.09	<b>0.64</b>	0.08	<b>0.65</b>	0.09	<b>0.65</b>	0.08
Jensen's alpha (% monthly)	<b>0.23</b>	0.23	<b>0.09</b>	0.08	<b>0.19</b>	0.22	<b>0.12</b>	0.08	<b>0.24</b>	0.22	<b>0.09</b>	0.08
TrackErrorStdDev (% monthly)	<b>1.25</b>	0.74	<b>-0.46</b>	0.23	<b>0.89</b>	0.88	<b>-0.07</b>	0.23	<b>1.15</b>	0.82	<b>-0.40</b>	0.23
BetaCorr	<b>0.12</b>	0.17	<b>-0.03</b>	0.06	<b>0.15</b>	0.14	<b>-0.06</b>	0.06	<b>0.07</b>	0.13	<b>-0.01</b>	0.06
HerdCorr	<b>-0.09</b>	0.06	<b>0.01</b>	0.02	<b>-0.06</b>	0.06	<b>-0.02</b>	0.02	<b>-0.10</b>	0.05	<b>0.01</b>	0.02

Table 2: **Summary statistics for top performers.** This table shows summary statistics for the top 30% performers over the period 1981-2010. *HerdTrackErr*, *HerdBeta* and *HerdCorr* are the return-based measures of herding described in Section 5, computed over the sample 1990-2010. Top performer mutual funds are ranked in ascending order according to the value of each of these measures. The bottom and top 33% are, respectively, the “Herding” and “Contrarian” funds for *HerdTrackErr* and *HerdBeta*, and conversely the “Contrarian” and “Herding” funds according to *HerdCorr*. The rest of the variables reported are as in Table 1.

Herding measured by	HerdTrackErr	HerdBeta	HerdCorr
Low	0.308 (0.82)	0.245*** (3.49)	0.348 (0.99)
4thPerfQuint	0.467** (2.86)	0.173 (1.36)	0.467** (2.94)
3rdPerfQuint	0.303*** (4.33)	0.213 (1.28)	0.251*** (3.23)
2ndPerfQuint	0.380* (1.94)	0.547*** (3.39)	0.576* (2.08)
Top	1.426*** (3.49)	0.945*** (4.49)	0.862 (1.59)
Contrarian	0.037 (0.73)	-0.038 (-1.14)	0.062 (1.14)
Low * Contrarian	0.008 (0.02)	0.151 (0.65)	-0.058 (-0.14)
4thPerfQuint * Contrarian	-0.278 (-0.95)	0.083 (0.36)	-0.333 (-1.17)
3rdPerfQuint * Contrarian	0.137 (0.61)	-0.048 (-0.19)	0.226 (1.04)
2ndPerfQuint * Contrarian	-0.704** (-2.21)	-0.588*** (-3.70)	-0.814* (-2.03)
Top * Contrarian	0.306 (0.55)	0.443** (2.66)	0.686 (1.29)
Adjusted R <sup>2</sup>	0.183	0.190	0.191
Number of observations	4298	4413	4357

Table 3: **Flow-performance relationships: 5-piece specification.** This table examines the sensitivity of fund flows to relative performance for the “Herding” and “Contrarian” top performance mutual funds in Table 2. All funds in each objective category are annually ranked according to their annual raw return and assigned a continuous rank (*Rank*) ranging from 0 (worst) to 1 (best). Performance quintiles are computed as follows:  $Low_{i,t-1} \equiv \min(Rank_{i,t-1}, 0.2)$ ,  $4thPerfQuint_{i,t-1} \equiv \min(Rank_{i,t-1} - Low_{i,t-1}, 0.2)$ , and so forth, up to  $Top = Rank_{i,t-1} - Low_{i,t-1} - \dots - 2ndPerfQuint_{i,t-1}$ . Each year a piecewise linear regression is performed by regressing net fund flows on funds performance quintiles following the specification (30). Control variables include:  $Size_{i,t-1}$  (natural log of fund *i*’s TNA),  $CategoryFlow_{i,t}$  (aggregate flow rate into fund *i*’s category),  $Volatility_{i,t-1}$  (standard deviation of fund *i*’s monthly returns in year *t* - 1),  $Age_{i,t}$  (natural log of 1 + fund *i*’s age),  $ExpenseRatio_{i,t-1}$  (fund *i*’s expense ratio), and  $Contrarian_i$  (equals 1 or 0 depending on whether *i* is a “Contrarian” or “Herding” fund according to the herding measure in the corresponding column). Time-series average coefficients and the Fama and MacBeth (1973) *t*-statistics (in parenthesis) calculated with Newey-West robust standard errors are reported. \*, \*\* and \*\*\* denote significance at the 10%, 5% and 1% level, respectively.

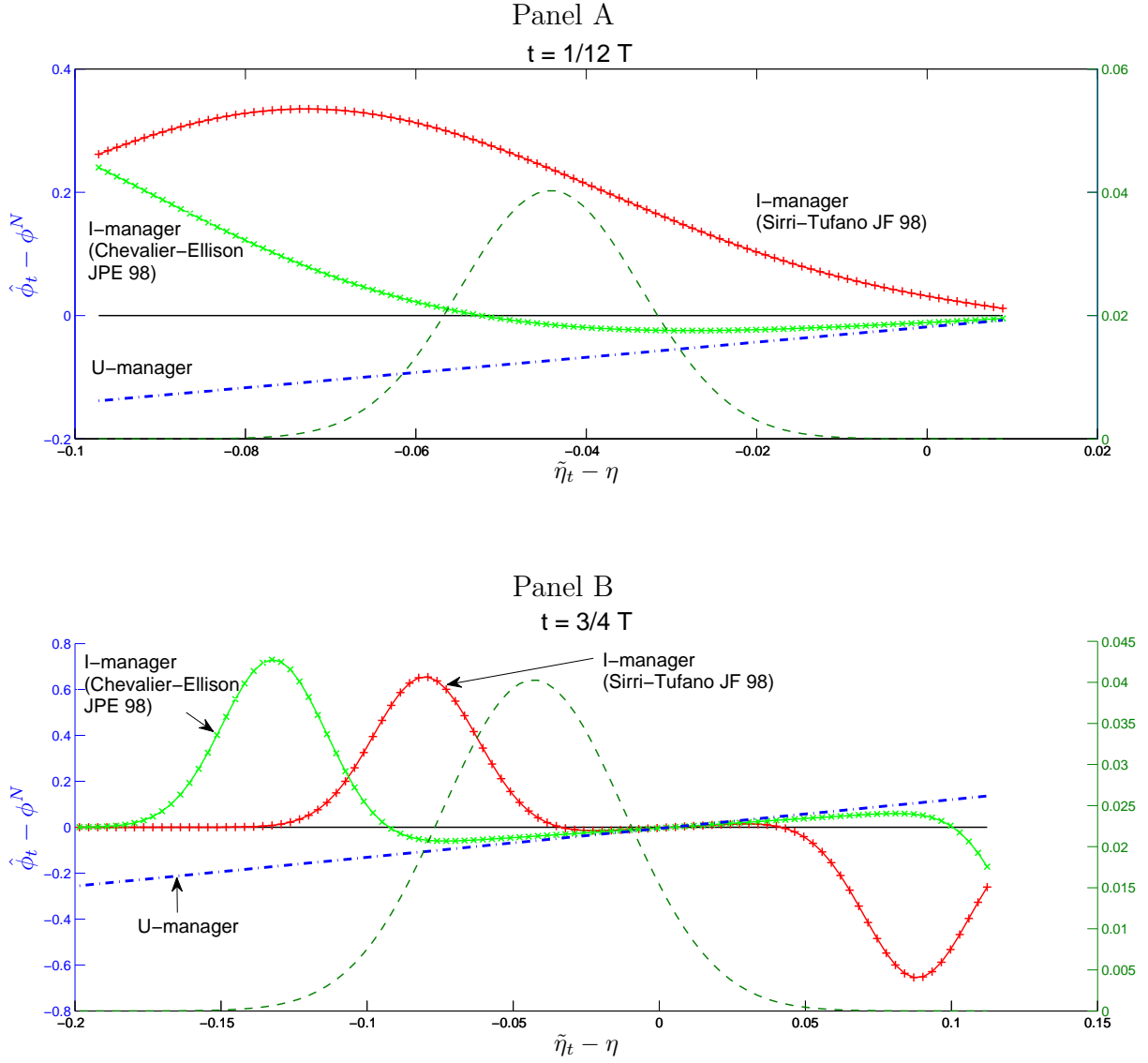
Herding measured by	HerdTrackErr	HerdBeta	HerdCorr
Contrarian	0.05 (1.61)	-0.061** (-2.43)	-0.134 (-1.21)
Low	0.485* (2.17)	0.069 (0.51)	0.372 (1.07)
Mid	0.346*** (5.14)	0.311*** (4.86)	0.382*** (4.36)
Top	1.653*** (5.86)	1.135*** (5.41)	1.284** (2.85)
Low * Contrarian	-0.088 (-0.33)	0.466** (2.60)	-0.03 (-0.08)
Mid * Contrarian	-0.191* (-2.10)	-0.174** (-2.35)	-0.200* (-2.10)
Top * Contrarian	-0.186 (-0.56)	0.101 (0.50)	-0.017 (-0.04)
Adjusted R <sup>2</sup>	0.182	0.184	0.188
Number of observations	4298	4413	4357

Table 4: **Flow-performance relationships: 3-piece specification.** This table examines the sensitivity of fund flows to relative performance for the “Herding” and “Contrarian” top performance mutual funds in Table 2. All funds in each objective category are annually ranked according to their annual raw return and assigned a continuous rank ( $Rank$ ) ranging from 0 (worst) to 1 (best). Performance segments are computed as follows:  $Low_{i,t-1} \equiv \min(Rank_{i,t-1}, 0.2)$ ,  $Mid_{i,t-1} \equiv \min(Rank_{i,t-1} - Low_{i,t-1}, 0.6)$ , and  $Top = Rank_{i,t-1} - Low_{i,t-1} - Mid_{i,t-1}$ . Each year a piecewise linear regression is performed by regressing net fund flows on funds performance segments. Control variables include:  $Size_{i,t-1}$  (natural log of fund  $i$ 's TNA),  $CategoryFlow_{i,t}$  (aggregate flow rate into fund  $i$ 's category),  $Volatility_{i,t-1}$  (standard deviation of fund  $i$ 's monthly returns in year  $t - 1$ ),  $Age_{i,t}$  (natural log of 1 + fund  $i$ 's age),  $ExpenseRatio_{i,t-1}$  (fund  $i$ 's expense ratio), and  $Contrarian_i$  (equals 1 or 0 depending on whether  $i$  is a “Contrarian” or “Herding” fund according to the herding measure in the corresponding column). Time-series average coefficients and the Fama and MacBeth (1973)  $t$ -statistics (in parenthesis) calculated with Newey-West robust standard errors are reported. \*, \*\* and \*\*\* denote significance at the 10%, 5% and 1% level, respectively.



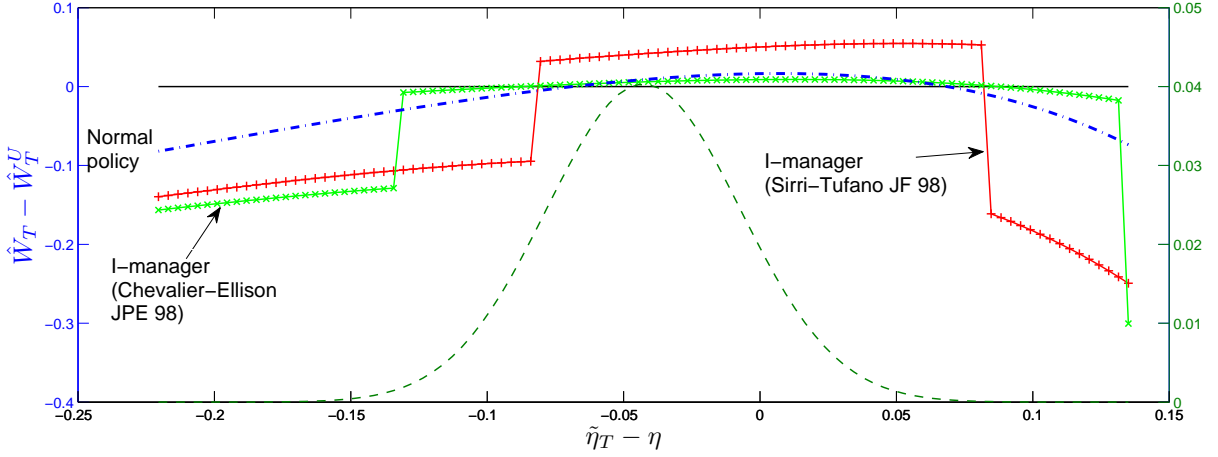
	Herding computed over gross returns	Top 20% performers
Low	0.719** (2.43)	0.37 (0.90)
4thPerfQuint	0.407** (2.63)	0.562*** (4.13)
3rdPerfQuint	0.285*** (5.17)	0.149 (1.17)
2ndPerfQuint	0.440** (2.62)	0.480** (3.04)
Top	1.488*** (3.22)	1.534** (2.98)
Contrarian	0.115** (2.27)	0.003 (0.06)
Low * Contrarian	-0.336 (-0.85)	0.116 (0.23)
4thPerfQuint * Contrarian	-0.285 (-1.08)	-0.833* (-2.01)
3rdPerfQuint * Contrarian	0.116 (0.53)	0.599* (2.02)
2ndPerfQuint * Contrarian	-0.724* (-1.91)	-0.736** (-3.02)
Top * Contrarian	0.262 (0.47)	0.29 (0.61)
Adjusted R <sup>2</sup>	0.187	0.183
Number of observations	4241	2963

Table 5: **Flow-performance relationships: robustness checks.** This table examines the sensitivity of fund flows to relative performance for top performance mutual funds in Table 1 classified as “Herding” or “Contrarian” according to whether they are in the bottom or top 33% of a ranking based on the values of *HerdTrackErr*. In the second column, the sample includes the same top 30% performers as in Tables 3 and 4, but *HerdTrackErr* is computed over returns *gross of* expense ratios. In the third column, *HerdTrackErr* is computed over fund returns net of expense ratios as in Tables 3 and 4, but the sample includes only top 20% performers according to their proportion of outperforming years. All funds in each objective category are annually ranked according to their annual raw return and assigned a continuous rank (*Rank*) ranging from 0 (worst) to 1 (best). Performance quintiles are computed as follows:  $Low_{i,t-1} \equiv \min(Rank_{i,t-1}, 0.2)$ ,  $4thPerfQuint_{i,t-1} \equiv \min(Rank_{i,t-1} - Low_{i,t-1}, 0.2)$ , and so forth, up to  $Top = Rank_{i,t-1} - Low_{i,t-1} - \dots - 2ndPerfQuint_{i,t-1}$ . Each year a piecewise linear regression is performed by regressing net fund flows on funds performance quintiles following the specification (30). Control variables include:  $Size_{i,t-1}$  (natural log of fund *i*’s TNA),  $CategoryFlow_{i,t}$  (aggregate flow rate into fund *i*’s category),  $Volatility_{i,t-1}$  (standard deviation of fund *i*’s monthly returns in year *t* - 1),  $Age_{i,t}$  (natural log of 1 + fund *i*’s age),  $ExpenseRatio_{i,t-1}$  (fund *i*’s expense ratio), and  $Contrarian_i$  (equals 1 or 0 depending on whether *i* is a “Contrarian” or “Herding” fund according to the value of *HerdTrackErr*). Time-series average coefficients and the Fama and MacBeth (1973) *t*-statistics (in parenthesis) calculated with Newey-West robust standard errors are reported. \*, \*\* and \*\*\* denote significance at the 10%, 5% and 1% level, respectively.

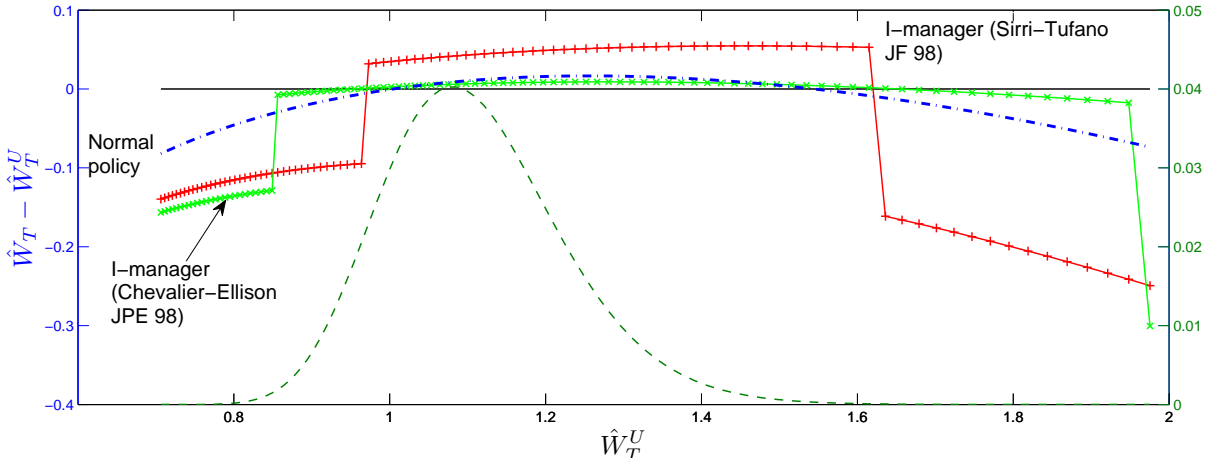


**Figure 1: Managers' interim risk exposure.** Panels A and B illustrate informed and uninformed managers' risk exposure  $\hat{\phi}^I$  and  $\hat{\phi}^U$  in excess of the *normal* risk exposure  $\phi^M$  as of different times  $t$ . For the informed manager, risk exposures are shown under two different parameterizations for the flow-performance relationship: (i)  $\delta = .985$  and  $\alpha = 1.7$  (approximately matching the relationship estimated by Sirri and Tufano (1998)), and (ii)  $\delta = .94$  and  $\alpha = 2.5$  (approximately matching the relationship estimated by Chevalier and Ellison (1997) for “old” mutual funds). The dashed line represents the probability of the different states  $\tilde{\eta}_t - \eta$ . Results correspond to a realized market price of risk  $\eta = 0.557$  at  $t = 0$ . The rest of the parameters follow the baseline parameterization in Appendix B:  $T = 1, r = 3\%, \sigma = .0158, m = 0.513, v_0 = 0.192^2, \gamma = 5$ .

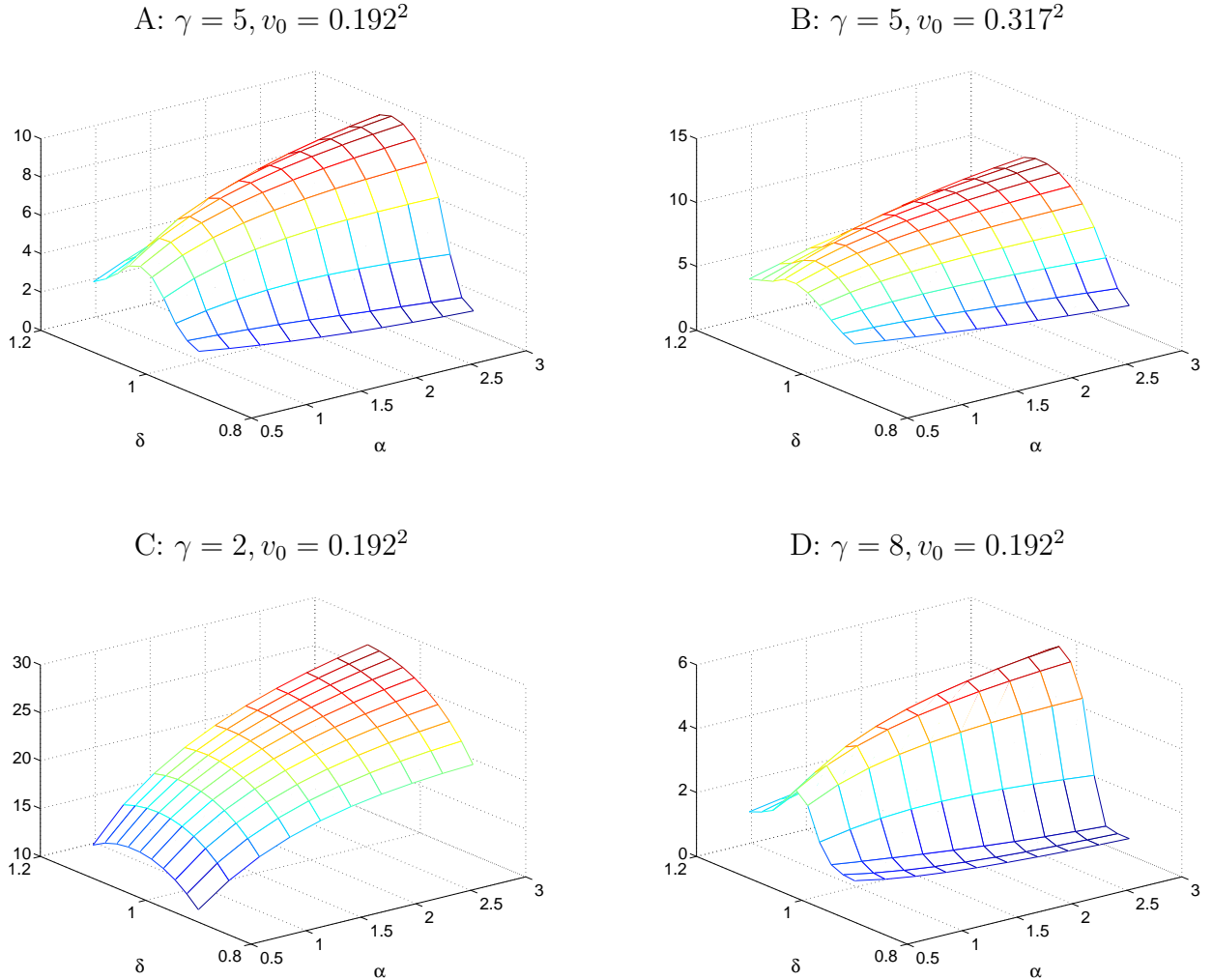
Panel A: Final payoffs as a function of uninformed manager's end-of-period inference error



Panel B: Final payoffs as a function of uninformed manager's end-of-period fund value

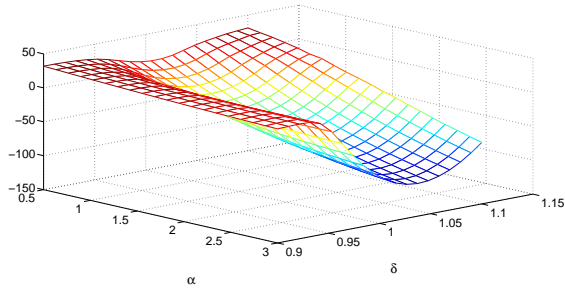


**Figure 2: Optimal Payoff Profiles.** Panels A and B illustrate informed managers' end-of-period excess payoffs  $\hat{W}_T^I - \hat{W}_T^U$  over uninformed managers'. For the informed manager, final payoffs are shown under two different parameterizations for the flow-performance relationship: (i)  $\delta = .985$  and  $\alpha = 1.7$  (approximately matching the relationship estimated by Sirri and Tufano (1998)), and (ii)  $\delta = .94$  and  $\alpha = 2.5$  (approximately matching the relationship estimated by Chevalier and Ellison (1997) for "old" mutual funds). The dashed line represents the probability of the different states  $\tilde{\eta}_T - \eta$  (Panel A) and  $\hat{W}_T^U$  (Panel B). Results correspond to a realized market price of risk  $\eta = 0.557$  at  $t = 0$ . The rest of the parameters follow the baseline parameterization in Appendix B:  $T = 1, r = 3\%, \sigma = .0158, m = 0.513, v_0 = 0.192^2, \gamma = 5$ .

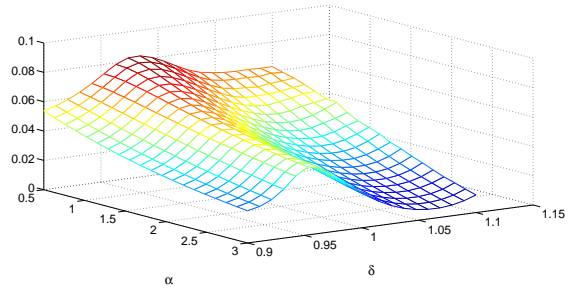


**Figure 3: Average tracking error (% annual).** Panels A through D plot the standard deviation of the difference in annual (log) returns between informed and uninformed managers for different flow-performance relationships as given by their flow elasticity  $\alpha$  and performance threshold  $\delta$ . Standard deviations are computed over the actual (objective) joint probability distribution of the market price of risk  $\eta$  and the driving Brownian motion  $B$  following the numerical procedure in Appendix C. The different panels correspond to different economic setup as determined by managers' relative risk aversion coefficient  $\gamma$  and uninformed managers' initial uncertainty  $v_0$ , corresponding to the baseline and alternative parameterizations of Appendix B ( $T = 1$ ,  $r = 0.03$  and  $\sigma = 0.158$  across all panels).

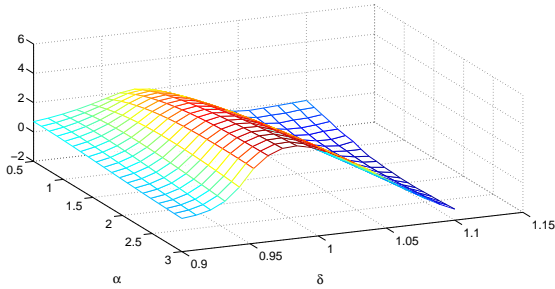
A: Certainty equivalent returns (bps)



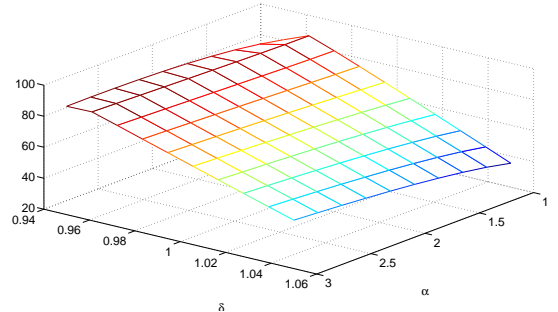
B: Sharpe ratios



C: Jensen's alpha (%)

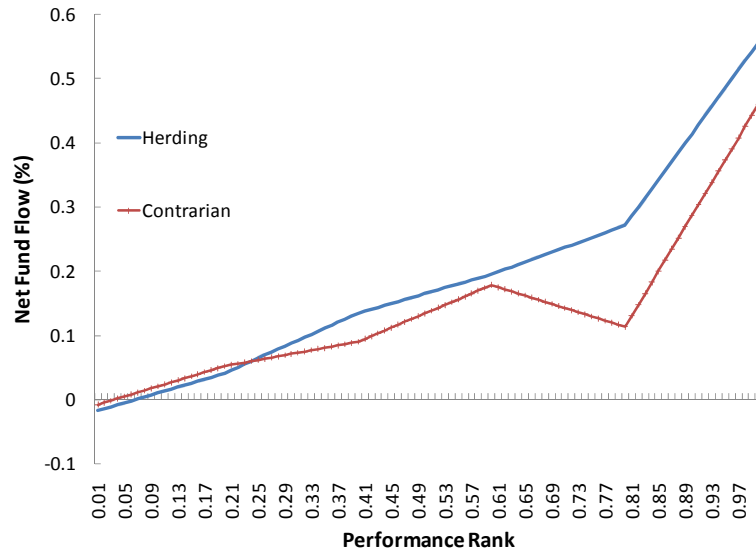


D:  $Prob(R_T^I > R_T^U)$  (%)

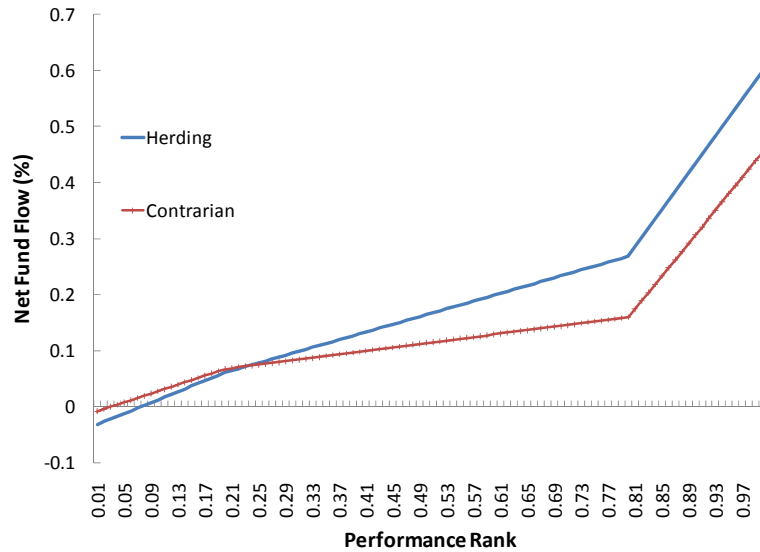


**Figure 4: Informed managers' performance in excess of uninformed managers'.** Panels A through C plot simulated average risk-adjusted performance of informed managers in excess of uninformed managers' as a function of the flow elasticity  $\alpha$  and performance threshold  $\delta$  in their flow-performance relationships, according to three measures: delegating households' certainty equivalent returns, Sharpe ratios and Jensen's alpha. Panel D plots the average probability (in %) that an informed manager's fund end-of-period return  $R_T^I$  is higher than that of an uninformed manager,  $R_T^U$ , as a function of the same parameters  $\alpha$  and  $\delta$ . Averages are computed over the actual (objective) joint probability distribution of the market price of risk  $\eta$  and the driving Brownian motion  $B$  following the numerical procedure in Appendix C. Results correspond to the baseline parameterization in Appendix B:  $T = 1, r = 3\%, \sigma = .0158, m = 0.513, v_0 = 0.192^2, \gamma = 5 = \gamma_h$ .

Panel A: 5-piece specification



Panel B: 3-piece specification



**Figure 6: Flow-performance relationships.** Panels A and B illustrate the flow-performance relationships estimated in Tables 3 and 4 for the top 30% mutual funds classified as “Herding” or “Contrarian” according to their values of *HerdTrackErr* (second column in each table). The graphs show the relation between expected flow rate and lagged relative performance for the average fund in each herding category, i.e. substituting average values for all included control variables into the estimated equations.

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