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PIPEs: a Theory of Private vs. Public Placements in Public Firms*

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Abstract

This paper studies a public firm's investment decision and whether to raise the equity capital needed using the public market (SEO) or a private channel (PIPE, Private Investment in Public Equity). Issuing the security privately allows the firm to enjoy greater financial flexibility since funds can be raised faster due to less legal requirements and marketing efforts than a public offering (i.e. the shares do not need to be registered before they are sold). This greater financial flexibility also alleviates information asymmetries. However because they are initially illiquid, they carry a cost. The trade-off is therefore between liquidity and the value of financial flexibility. The model throws light on what firm characteristics determine the use of one market or the other, and on the optimal timing of investment decisions to better use financial flexibility. We then solve for the optimal private debt contract and show that, whithin private placements, the pecking order theory need not hold. The model explains empirical regularities, for instance, why do SEOs have negative abnormal returns around its announcement whereas abnormal returns for PIPEs are positive.

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1 Introduction

Corporate Finance studies the financing decisions of firms by looking at several features of the securities used by them to raise funds: their control rights vs. their cash-flow rights, whether they are long-term contracts vs. short-term ones, etc. After relaxing the assumptions under which the Modigliani-Miller theorem is expected to hold, modern corporate finance has developed different theories that help us understand such decisions and that focus mainly on the trade-off between debt and equity.[1] In this paper I study another dimension of such financing decisions, namely a public firm's trade-off regarding the use of a private market as opposed to a public market when new capital needs to be raised. By private placement I mean what is known as Private Investment in Public Equity (henceforth PIPE) whereas a public placement is commonly known as a Secondary Equity Offering (SEO.) In this paper I seek to answer what drives a public firm looking to raise capital to use one channel or the other. In order to do so, and as an intermediate goal, I make use of an optimal timing model of investing and financing decisions to help understand the drivers of such trade-off.

Both channels are quite distinct. The main features of a SEO are that the new shares issued need to be registered and marketed ex-ante, an underwriter is used and, as a consequence, it typically results in dispersed ownership. In a PIPE, on the other hand, the securities are offered in a manner not involving any general advertising or general solicitation (therefore underwriters are not used and the marketing effort is absent.) New shares are sold to a smaller group of sophisticated investors. Moreover, in a typical PIPE, the company uses an exemption from SEC registration requirements to issue investors common stock (or securities convertible to common stock) for cash. The company then registers the resale of the common stock issued in the private placement. Generally, investors must hold securities issued in a private placement for at least one year. However, because the company registers the resale of the newly issued shares, investors are free to sell them into the market as soon as the SEC declares the resale registration statement effective (typically within a few months of the closing of the placement.) PIPE deals can close within seven to ten days of receiving definitive purchase commitments whereas an SEO can take from three to nine months.[2]

When private placements started to develop they were typically used by small-to-medium sized companies, in a more or less desperate need to obtain capital but without access to more traditional sources. These placements were sometimes sold as a structured security whose terms and conditions

[1] Obviously the list is too long to name them all.
[2] See Dresner (2006).

created unwanted "death spirals" and granted them adjectives as "toxic convertibles". All this gave them the ugly duck tag from the financial community and prevented them to grow more widely. Lately, however, practitioners seem to agree that PIPEs are becoming a more mature market: larger firms are making use of it and most of the structured securities have become rare.[3] Given the current state of this market, the trade-off between private and public placements becomes more relevant. The paper's purpose is therefore to be able to throw light on such dimension of a typical public firm's financing.

The paper draws on an important advantage of a private placement, namely, greater financial flexibility. It has been argued that one of the main advantages of PIPEs is that it allows to raise money faster than other channels. Financial flexibility has been defined, in general, as the firm's ability to access and restructure its financing with low transaction costs.[4] In this paper, financial flexibility refers to a firm's ability to time investment and financial decisions in such a way that it maximizes gross returns by investing earlier and it also minimizes dilution costs caused by asymmetries of information. To be even more precise, we model the value of flexibility in a two-dimensional way: If funds are raised earlier this provides an advantage over competitors in the industry and expected profits from investing increase, or another way to put it, delaying the investment is costly. However, by delaying the investment and eliminating the time gap between announcing and obtaining the necessary funds, (sophisticated) investors are provided with a better signal about the firm's profitability. This alleviates potential dilution costs stemming from asymmetries of information. The firm, if possible, optimally chooses investment timing by taking into account both effects. The relative advantage of a private placement sits on the value created by this flexibility. In contrast, a public placement is more time consuming and because it is sold to a pool of relatively less sophisticated investors the potential benefits of a more precise signal cannot be enjoyed by the firm. The comparative advantage of a public placement, on the other hand, is the benefit of an ex-post more liquid newly issued stock. Equity issued in a public placement becomes immediately liquid (i.e., tradeable in a secondary market), whereas that issued in a private placement suffers from temporary illiquidity, as we have argued. In a scenario where investors might be shocked with liquidity needs, PIPEs create an illiquidity cost that needs to be priced in. This illiquidity in turn feeds back into the optimal timing problem by amplifying dilution costs.[5]

[3] For instance, in January 2007, KKR closed a $700 million PIPE with Sun Microsystems Inc. Elevation Partners invested $325 million via a PIPE into Palm Inc. in June 2007.

[4] See Gamba and Triantis (2008).

[5] Arguably, a bank loan would provide the same type of financial flexibility advantage as a private placement. There

We show that when firms choose a private placement an equilibrium exists for the optimal timing decision. We find that firms facing more asymmetric information delay the investment in order for the investors to enjoy a more precise signal about the firm's type which in turn lowers dilution costs. Firms dealing with a larger illiquidity cost also choose to invest later. This is because in a more illiquid market the mispricing factor caused by asymmetric information worsens, thus increasing the marginal benefit of delaying the announcement of the investment decision in order to capture the benefits of a more precise signal about the firm's profitability. By the same reason, investment scale also increases the marginal benefit of delaying investment more so than the direct cost. The effect of internal funds goes in the opposite way by the same argument. Finally, lowering the expected gross return from the new project makes the company invest sooner, the reason being that the marginal dilution costs are lower compared with the benefit of investing earlier in order to extract rents from competitors. In sum, if firms choose to issue a PIPE, firms in a more liquid market, with lower investment cost, more internal funds and lower expected profits from the investment will invest sooner. The model derives a closed-form solution for the PIPE discount due to illiquidity, which is proportional to the expected returns from the investment and a measure of how illiquid the market is. It shows how asymmetric information affects the discount.

Depending on the cost of illiquidity the equilibrium exhibits the use of a private placement or a public one instead. Intuitively, as long as illiquidity costs are not too large but asymmetries are important, firms will choose to place new equity issues privately. On the other hand, if asymmetric information is less important firms will tend to issue publicly since the value of financial flexibility is lower. Similarly, if competition is less important it is likely that firms will issue publicly. This is largely consistent with some empirical facts. First, PIPEs are more common among smaller and younger firms and also among firms where timing and competition matters (high-tech industries, pharmaceutical, biotech and medical devices). Second, Gomes and Phillips (2005) find that the probability of public firms issuing private over public is positively related with their measures of asymmetric information for all security types. This is also confirmed in Wu (2004).

We then generalize the analysis to debt contracts and show that within the private dimension of security issuance the pecking order might not hold. The main reason why this result arises is the endogeneity of dilution costs. Given that the firm might choose investment timing, this affects both

are two reasons why we leave bank loans out of the picture. First, the paper seeks to stress the private vs. public dimension of an equity issuance. Secondly, most companies issuing new equity might lack the collateral and/or the financial performance to qualify for bank loans, or might not be willing to increase leverage ratios. We will however deal with the debt/equity dichotomy in the last section of the paper.

dilution costs and the net present value of the project. Because with a debt contract the firm only makes profits if there is no default, the optimal investment timing will be affected by this, and it will likely have the firm invest earlier than with an equity contract, causing dilution costs to be higher and profits potentially lower.

This paper can be included as part of the large body of literature on corporate financing under asymmetric information (for a survey of articles see Tirole (2006).) Although this is the first paper, to my knowledge, that models a firm's choice between private and public placement of securities, the trade-off is somewhat related to the study of the decision to go public (see for instance, Subrahmanyam and Titman (1999), Chemmanur and Fulghieri (1999) among others.) Chemmanur and Fulghieri (1999) argue that by going public firms benefit from greater bargaining power against dispersed investors (who are better diversified that a venture capitalist) however it also carries higher information production costs. A part from the fact that we study the investment and financing decision of a firm that is already public, we do not focus on either information costs or diversification arguments.

The empirical literature on PIPEs can be broadly summarized with two main findings: one, that unlike SEOs, firms announcing PIPEs observe, on average, positive stock price effects; the other, that the new stock is sold at a discount (Wu (2004) reports median issue discounts of about 15 to 20 percent.) Consistent with our model, the issuance at a discount is widely attributed to illiquidity. The positive announcement effect is still a puzzle, at least from a theoretical perspective. The model helps better understanding the reason why that might be the case. In short, the reason why an SEO triggers a negative stock price reaction is that with asymmetric information, "good" firms might find unprofitable to invest if the mispricing factor is large enough. If only "bad" firms are willing to invest, announcing an SEO fully reveals a firm's type and the stock price drops. This assumes that the investment opportunity is expected. In our model, PIPEs alleviate the mispricing factor (dilution cost) and allow "good" firms to invest in a positive NPV project that would not be financed if only public placements were available. If that is the case, announcing a PIPE should not carry a negative stock price reaction. If additionally, the new project is not anticipated by the market, it can trigger a positive reaction. This is consistent with the findings in Gomes and Phillips (2005). They find that stock market returns around equity issues are negatively related with the degree of asymmetric information for public equity but the reverse holds for private equity offerings.

A summary of the empirical evidence is in order so that we can link our results with the data. Hertzell, Lemmon, Linck and Rees (2002) is one of the latest papers confirming that while public issues, on average, are associated with negative stock-price effects, private issues are associated with positive.

As mentioned earlier, Gomes and Phillips (2005) find that the probability of public firms issuing private over public is positively related with their measures of asymmetric information for all security types (this is also confirmed in Wu(2004).) They also find that stock market returns around equity issues are negatively related with the degree of asymmetric information for public equity but the reverse holds for private equity offerings. The authors claim that it is evidence consistent with the view that public investors believe that private investors produce or obtain valuable information, and learning about private investments is more valuable when there is higher degree of asymmetric information.

Wruck and Wu (2007) find that relationships create value in private placements. Of particular importance are new relationships created as part of the transaction. Price discounts are smaller when a relationship is involved. This holds for both pre-existing and newly formed relationships. This is consistent with the idea that investors play a certification role. Innovative firms, as measured in terms of patent applications, make placements at a relatively small discount. In contrast, firms in new economy industries make placements at a substantial discount.

Wu (2004) also finds that private placements investors do not engage in more monitoring than public offerings investors. This further supports our assumption that (sophisticated) investors might benefit from signals in a private placement but do not necessarily engage in active monitoring or information production. He also finds that discounts for private placements sold to managers are higher than discounts for private placements in which managers do not participate. Our model assumes that managers are aligned with existing (old) shareholders and therefore does not make prediction in that regard.

Finally, Brophy, Ouimet and Sialm (2004) focus on the differences between structured and traditional PIPEs. Hedge funds invest the most in the first type. In the short term, the average PIPE studied has an average abnormal return of 3.87 percent over a ten-day window around the closing date of the deal, confirming previous results in private placements by Wruck (1989). Furthermore, consistent with HLLR they find the (counter-intuitive) result that PIPE issuers, on average, experience negative long-term returns after positive announcement returns. The poor long-term performance of companies issuing PIPEs confirms Barclay, Holderness and Sheehan (2003) and Wu (2004),who document that investors in private placements are typically passive and do not appear to increase firm value through monitoring. They find no abnormal performance in firms issuing traditional PIPEs sold to non-hedge fund investors, and significant negative abnormal performance following the PIPE issuance if the traditional security is bought by a hedge fund.

2 The Model

2.1 Investment Technology, Information and Security Choice

The model has three dates, indexed $t = 0$, 1 and 2. We simplify the model by assuming a discount rate of 0. At $t = 0$, a public firm with available (i.e., liquid) cash-flows A has an investment opportunity. Such investment opportunity can be thought of as a "deepening investment." The investment requires a deterministic outflow of funds I. This investment cost must be paid at time $x \in [0, 1]$, that is, at some time between date 0 and date 1. The investment timing x might be an endogenous decision of the firm, depending on the type of security chosen by them. At $t = 0$ the firm also faces the decision whether to raise capital using the public market (e.g., through a Secondary/Seasoned Equity Offering, or SEO) or placing the security privately (via a Private Placement in Public Equity, or PIPE).

There are two states of nature, success (S) and failure (F.) In each state, gross firm value is R^S and R^F respectively. Denote $\Delta R \equiv R^S - R^F > 0$. The information about the probability of each state occurring is asymmetric. As common in principal-agent models, managers possess superior information. In particular, firms can be of two types, indexed H and L. If the project is undertaken and an SEO is used, an $H - type$ firm generates R^S with probability $v_H = v + \Delta_H v + \Delta v/2$. Its prior probability is γ. On the other hand, $L - type$ firms yield R^S with probability $v_L = v + \Delta v/2$ and prior probability $1 - \gamma$. We assume that $\Delta v > 0$ and $\Delta_H v > 0$ and $v + \Delta v + \Delta_H v < 1$. At $t = 0$, this information is known to managers but not to shareholders. In contrast, if the company decides to use a private placement, an $H - type$ firm generates R^S with probability $v_H = v + \Delta_H v + \Delta v(1 - \frac{x^2}{2})$ and an $L - type$ firm yields R^S with probability $v_L = v + \Delta v(1 - \frac{x^2}{2})$. In sum, the expected net present value is given by

$$\begin{aligned} \mathbb{E}[R_i] &= R^F + v_i \Delta R - I, \text{ where } i = \{H, L\}, \\ v_H &= v + \Delta_H v + \Delta v(1 - \frac{x^2}{2}) \text{ and } v_L = v + \Delta v(1 - \frac{x^2}{2}),\ x \in [0, 1]. \end{aligned}$$

If, on the other hand, the firm decides not to undertake the deepening investment firm values are

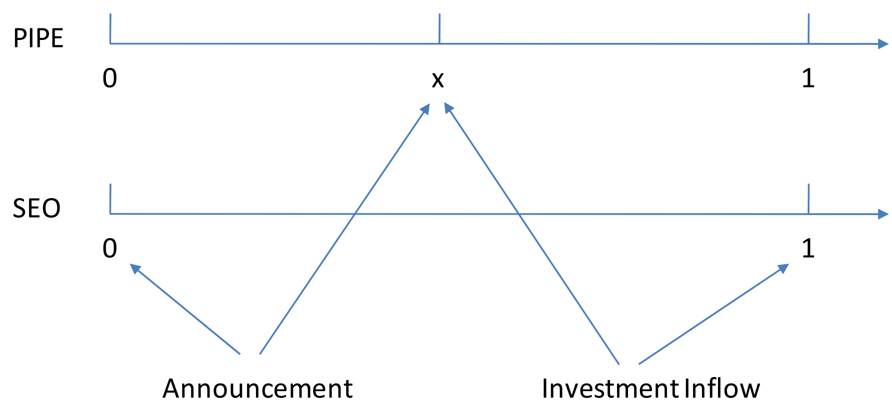
$$R_H^0 = R^F + (v + \Delta_H v)\,\Delta R \text{ and } R_L^0 = R^F + v\Delta R.$$

Moreover, we shall assume that the expected net present value for low profitability firm raising funds in the public market is positive. That is,

Assumption 1. (Positive Expected SEO NPV) $\frac{1}{2}\Delta\nu\Delta R > I - A$.

Before proceeding with the description of the model, it is worth justifying the difference in expected returns depending on the channel used to raise funds. A key feature of the paper is to stress the greater flexibility, in terms of raising money, a private placement (PIPE) entails over a public offering (SEO). The main idea is that it takes time to raise funds through an SEO. Public offerings are usually underwritten and marketed by an investment bank, and have to fulfill several SEC requisites that take time. In sum, an SEO is more time consuming than a PIPE. In order to capture this idea, we assume that if the firm uses an SEO, a gap exists between the time the investment is announced/decision is made ($t = 0$) and the time the funds are received by the company ($t = 1$). This gap is not present in case the firm decides to issue a PIPE. Figure 1 below illustrates this.

Figure 1. Timing differences between SEO and PIPE



The lack of financial flexibility which we define as the existence of a lag between deciding to invest and obtaining the funds for the actual investment is costly in two ways.

First, by raising the funds only a period after the decision is made ($t = 1$) the expected profitability is lower. The economic reason for this assumption is that the later you invest the more likely it is that a competitor will be able to reap the benefits from the deepening investment. Therefore, we implicitly have in mind an industry model where similar firms simultaneously undertake similar investments, and there exists a first mover advantage. We model this by assuming that investing away from $t = 0$ carries a cost. We assume that this cost is proportional to the expected returns on the investment project and equal to $\frac{x^2}{2}\Delta v \Delta R$. This seems to be critical and important precisely in industries where PIPEs are more commonly used, in high-technology sectors, Chemicals or Biotechnology idustries (see, for instance, Hertzel et al. (2002) and Wu (2004).)

Secondly, deciding early on the investment prevents the firm from the benefits of having sophisticated investors acquire more information about the company. In other words, the later the announcement is made the more likely some signal about the firm's profitability will provide information to sophisticated investors. Delaying the investment decision away from $t = 0$ carries out an informational advantage. We argue that the closest to $t = 1$ the investment decision is made, the less severe the information asymmetry and hence dilution cost is. In particular, the firm's choice of the investment timing affects the informativeness of a signal about the project's quality, a signal observed only by sophisticated investors. The way this is advantageous for a PIPE issuance comes from the fact that the decision on the amount of shares to be issued in a public offering is taken at $t = 0$ due to the relatively lower financial flexibility assumed and because it is precisely sophisticated investors those who invest in PIPEs, whereas the proportion of sophisticated investors in SEOs is relatively lower.[6] To model this formally, we focus on a functional form for the probability that a particular signal (high or low) will convey information for a given firm type, $P(s = s_i \mid v_i)$, that satisfies the following properties, summarized in the following definition.

Definition *We call a Proportionally Increasingly Informative (P.I.I.) signal a probability function $P(s = s_i \mid v_i)$ that satisfies the following properties:*

i) It is an affine function of the investment decision time, x;

ii) the signal is completely uninformative at $x = 0$ and completely informative at $x = 1$. That is, the posterior probability function after Bayesian updating are

$$P(v_H \mid s = s_H, x = 0) = \gamma, \; P(v_L \mid s = s_H, x = 0) = 1 - \gamma \tag{1}$$

$$p(v_H \mid s = s_H, x = 1) = 1, \; P(v_L \mid s = s_H, x = 1) = 0.$$

[6] At the issuance stage we do not distinguish between informed and uninformed investors in the sense that we assume no information production about the project's quality/return. Note that this set-up is different from Fulghieri and Lukin (2001), where informed investors learn private information before the issuance.

In the following lemma we show that such signal exists and we characterize its form.

Lemma 1. *There exists a P.I.I. signal probability function. It is given by*

$$P(s = s_H \mid v_H) = k + (1-k)x \text{ and}$$
$$P(s = s_H \mid v_L) = k - kx, \ \forall k \subset [0,1].$$

The posterior probabilities given such probability functions are

$$P(v_H \mid s = s_H) = \frac{\gamma\left[k + (1-k)x\right]}{k(1-x) + \gamma x} \text{ and}$$
$$P(v_L \mid s = s_H) = \frac{(1-\gamma)(1-x)k}{k(1-x) + \gamma x}.$$

Proof. See Appendix.

In figure 2 we plot the implied posteriors for these probability functions (fixing $\gamma = \frac{1}{2}$.) The straight-line posterior corresponds to a convenient subcase where $k = \gamma$. We will use this probability function throughout the paper, for tractability reasons. It is immediate to see that the posterior probability corresponding to $k = \gamma$ simplifies to

$$P(v_H \mid s = s_H) = \gamma + (1-\gamma)x,$$
$$P(v_L \mid s = s_H) = (1-\gamma)(1-x).$$

[Figure 2 about here]

Armed with this convenient expression for the signal's probability as a function of the time the firm announces and raises funds, we proceed with the description of the rest of the model's ingredients.

As we have just argued, because issuing securities to the public requires time, funds can only be raised at $t = 1$ and not before, however the decision must be made in the initial period. If, at $t = 0$, the firm decides to use the public market, it announces the issuance of α new shares to new investors. On the other hand, a PIPE allows a firm to raise the amount as soon as $t = 0$ if the firm finds it optimal to do so. Hence, the investment decision is made simultaneously to obtaining the investment funds. This assumption captures the advantage of the private market we seek to highlight in this paper: greater financial flexibility.

The other important element in the model is liquidity needs by investors. The benefit of raising funds faster comes at a cost. Private placements allow firms to issue new stock prior to registering it. But, because the stock is typically unregistered when the deal takes place, it is illiquid for a certain period of time (investors are not able to resell it in the secondary market.) This temporary illiquidity depends on the exact type of private placement used. In particular, restricted private placements (known as Regulation D) place restrictions on both the way these securities can be resold and the time until they can be sold in the public market: two years. Even the so-called Registered placements, which incorporate effective registration statements that cover the resale of securities, have some degree of illiquidity. In the model we group all private placements under one type and simply assume that, compared to public placements, they initially suffer from illiquidity. In the next subsection we describe the exact modelling of this liquidity issue.

2.2 Agents and Markets' Description

In order to model liquidity we use the approach by Gorton and Pennachi (1990) and Bolton and Von Thadden (1998), among others. Shareholders/Investors have liquidity needs in the future, which arise either in period 1 or period 2, such that some investors (denoted "early" consumers) derive utility from consumption at $t = 1$ and the rest derive utility from consuming at $t = 2$ ("late" consumers.) Investors are risk neutral. At $t = 0$, however, these needs are unknown to everyone and such uncertainty is not realized until $t = 1$. Since investors do not know exactly when will they like to consume at the time they buy their shares, it is apparent that some trade will occur in the second period: early consumers would like to sell their share in the firm and buy the consumption good whereas late consumers might want to buy those shares in exchange for their period 2 consumption good endowment in order to consume more at $t = 2$. Let us introduce some additional notation. At $t = 1$, the proportion of investors that are early consumers can take the value e_h or e_l with prior probability q_h and q_l respectively. We assume that $e_h > e_l$ and denote $\overline{e} \equiv q_h e_h + q_l e_l$. In addition to their share in the firm, investors receive and endowment of c units of the consumption good (one can think of it as money.) Each unit of this consumption good can either be consumed at $t = 1$ or stored to yield a certain return of one unit of the consumption good at $t = 2$.

Figure 3 below summarizes the sequence of events and decisions in the model.

Figure 3. Timing for the Model

Date	**0**	**x**	**1**	**2**
Event		Signal s_i	Liquidity needs	Returns
Decisions	Raise capital?		e_j realized	R_i realized
	What market?	Issue shares if PIPE	Investment I if SEO	Payoffs
	Decide shares if SEO	Investment I if PIPE	Trading	

In order to solve for the equilibrium of the issuance model we start off solving for the trading part of the model. We denote P_{ij} the stock price at date 1 when the firm's investment opportunity is of the $i - type$ and the proportion of early consumers is e_j. It is obvious that at $t = 1$ early consumers wish to sell their stock and purchase the consumption good whereas late consumers are the only ones from whom early consumers can buy endowment of the consumption good. Therefore late consumers will end up buying some or all of their period 1 endowment of the consumption good. The following assumption guarantees no storage in equilibrium, that is, that late consumers will sell their consumption endowment to early consumers instead of storing it. It is done without loss of generality since the total illiquidity cost is not altered by it.

Assumption 2. $\mathbb{E}[R_L] < \frac{c(1-e_l)}{e_l}$.

Given these liquidity needs by investors, the choice of security type by the firm will affect the trading outcome and the equilibrium utilities of investors/shareholders. The crucial difference between a public offering and a private placement is that in the case a PIPE is used, new investors that happen to be early consumers are not allowed to sell their new stock in exchange for the consumption good. We start the next section by solving the trading problem in the second period and highlighting the main difference between both markets.

3 Equilibrium

3.1 The Illiquidity Deadweight Loss

The following Lemma summarizes the main result from solving the trading model.

Lemma 2 *If the firm issues a PIPE, investors experience a deadweight loss as a result of the illiquidity of the newly-issued shares. This aggregate cost is proportional to the expected returns on* *the project, the expected measure of early consumers and the proportion of shares issued:*

$$\mathbb{E}_0\left[c_1 + c_2 \mid \text{PIPE}\right] = c + \mathbb{E}_0\left[R \mid \text{PIPE}\right] - \alpha\bar{e}\mathbb{E}_0\left[R \mid \text{PIPE}\right] = c + (1 - \alpha\bar{e})\,\mathbb{E}_0\left[R \mid \text{PIPE}\right],$$

$$\text{where } \bar{e} = q_h e_h + q_l e_l.$$

Proof. See Appendix. ■

Some comments are in order. The result in Lemma 2 allows us to obtain in a closed-form solution the illiquidity cost from first principles. In particular, such cost is larger the larger the proportion of new shares sold to PIPE investors, the larger the expected return on capital and the larger, on average, the proportion of early consumers. Note that this proportion can be quite sizeable in the case of restricted offerings since the period with resale restrictions can be as long as two years. We will use $\bar{e}$ as a measure of the importance of such illiquidity cost. The result above also allows us to make the decision problem tractable since the illiquidity cost is proportional to α, the fraction of new shares issued.

3.2 The Public Placement

First, we solve for an equilibrium assuming that all a firm can do is to raise funds using a public placement. This will serve as a benchmark towards solving for the private placement's equilibrium as well as for the optimal issuance policy to be executed by the firm.

In the absence of arbitrage, the price at which the new equity is sold should be such that (new) investors, in expectation, do not make positive profits. Total expected returns available to them are given by the expected returns from the investment, that is, $\alpha\mathbb{E}_0[R]$.

On the other hand, by definition, the amount raised in the offering is $I - A$, therefore an $i - type$ firm's individual rationality constraint for new investors is

$$\alpha_i \mathbb{E}_0[R_i] \geq I - A. \tag{2}$$

It is important to highlight that the expectations operator in the expressions above will depend on the nature of the equilibrium, assuming that agents update their prior probabilities according to Bayes' rule. Note also that the constraint is alleviated whenever available cash flows (A) are high, the number of shares issued is large and the initial investment outflow is low.

The last piece that remains to be described in order to fully characterize the equilibrium is the decision on the number of new shares that need to be issued. We assume that managers act on behalf of existing shareholders, therefore they seek to maximize $(1-\alpha_i)\mathbb{E}_0[R_i] - A$. This means that a firm of type i solves the following program:

$$\max_{\alpha_i}(1-\alpha_i)\mathbb{E}_0[R_i] - A$$
$$\textit{subject to } (2)$$

We summarize the equilibrium issuance in the public market in the following proposition.

Proposition 1 (SEO Equilibrium) *If dilution costs are not too large so that $H-$type firms invest, there exists a unique SEO pooling equilibrium. It is characterized in the following way:*

i) At $t=0$, both types of firms decide to issue a proportion of new shares equal to

$$\alpha^* = \frac{I-A}{R^F + (v + \gamma\Delta_H v + \frac{1}{2}\Delta v)\Delta R}.$$

ii) $H-$type firms make profits

$$R^F + (v + \Delta_H v + \frac{1}{2}\Delta v)\Delta R - I - \frac{(1-\gamma)\Delta_H v(I-A)}{R^F/\Delta R + v + \gamma\Delta_H v + \frac{1}{2}\Delta v},$$

iii) $L-$type firms make profits

$$R^F + (v + \frac{1}{2}\Delta v)\Delta R - I + \frac{\gamma\Delta_H v(I-A)}{R^F/\Delta R + v + \gamma\Delta_H v + \frac{1}{2}\Delta v}.$$

If on the other hand, dilution costs are large, the unique equilibrium is one in which $H-$type firms do not invest in the project and $L-$type firms do, with firm values being

$$R^F + (v + \Delta_H v)\Delta R \textit{ and } R^F + (v + \frac{1}{2}\Delta v)\Delta R, \textit{ respectively.}$$

Proof. *See appendix.* ■

The proposition above is just a straightforward application of Myers and Majluf (1984). On the one hand, because both types of firms are credit-worthy (assumption 1) the only equilibrium possible is a pooling one. Separation is not possible. As it has been argued in the literature, mispricing affects negatively high-profit firms and positively lower-profit types since they are able to masquerade with more profitable types. The equilibrium therefore exhibits *underinvestment*: by looking at the $H-type$ firm payoff, only those with non-negative values will accept going ahead with the project. Such requirement is stronger than requiring a non-negative net present value since as we just mentioned mispricing affects them negatively. Finally it is worth stressing that mispricing is a more severe problem the less the internal funding available to the firm (A), and the more severe the adverse selection problem is. This is captured by the proportion of $L-type$ firms $(1-\gamma)$ times a normalized difference in returns $(\Delta_H v/(R^F/\Delta R+v+\gamma\Delta_H v+\frac{1}{2}\Delta v))$. We now describe the equilibrium investment decision if a private placement is used.

3.3 The Private Placement

We now turn towards describing the equilibrium of the model when the channel available to both firms is the private one (and only that.) Because one of the main features of such channel is that equity can be issued and sold before registering the securities, it allows firms to raise capital more flexibly. The downside of this greater flexibility in timing investment decisions is that the security cannot be traded until registration is completed.[7] In Lemma 2 we claimed that this translates into investors paying an illiquidity cost that is proportional to the expected return from investing, the expected proportion of early consumers and the proportion of new shares sold. We used the measure of early consumers ($\overline{e}$) as an index of how important market illiquidity is.

Managers now choose both investment timing and the amount of shares issued to new investors in order to maximize existing shareholder's value, subject to the contract being incentive-compatible and acceptable by both parties. That is, in a pooling equilibrium, they seek to

$$\max_{\{\alpha,x\}}(1-\alpha)\left[R^F+(v+\Delta_H v+\Delta v(1-\frac{1}{2}x^2))\Delta R\right]-A$$
$$\textit{subject to}\ \mathbb{E}_0[\alpha(1-\overline{e})R]\geq I-A,$$

where $q\equiv P(v_H\mid s=s_H)=\gamma+(1-\gamma)x$ and the individual rationality constraint takes into account the illiquidity cost that the new issuance faces $(\mathbb{E}_0[\alpha\overline{e}R].)$

By the same argument stressed in the previous section, and since both firm types have positive

[7] As we have mentioned, in practice this amounts to a varying amount of time depending on the terms of the issuance. It can take from an average of 40 trading days provided that the issuance incorporates effective registration statements that cover the resale of securities to a period of two years in the case of restricted placements.

NPV investment projects, the more profitable firms cannot separate from the rest, and the unique equilibrium is again a pooling one. The following proposition characterizes the equilibrium.

Proposition 2 *If funds are raised using the PIPE market, there exists a pooling equilibrium in which*

i) at $t = x^*$ firms issue

$$\widehat{\alpha} = \frac{I - A}{(1-\overline{e})\left(R^F + \left[v + q(x^*)\Delta_H v + (1 - \frac{1}{2}x^{*2})\Delta v\right]\Delta R\right)}$$

where x^* is given implicitly by the first-order condition

$$-\Delta v \Delta R x^* + \frac{(1-\gamma)\Delta v_H (I-A)\left[R^F/\Delta R + v + \Delta_H v + (1 - x^* + \frac{1}{2}x^{*2})\right]}{(1-\overline{e})(R^F/\Delta R + v + q(x^*)\Delta_H v + (1-\frac{1}{2}x^{*2})\Delta v)^2} = 0,$$

$$q(x^*) = \gamma + (1-\gamma)x^*$$

ii) $H-type$ and $L-type$ firms' values are, respectively,

$$R^F + (v + \Delta_H v + (1 - \frac{1}{2}x^{*2})\Delta v)\Delta R - I - \frac{\overline{e}}{1-\overline{e}}(I - A)$$
$$- \frac{(1 - q(x^*))\Delta_H v (I - A)}{(1-\overline{e})\left(R^F/\Delta R + v + q(x^*)\Delta_H v + (1 - \frac{x^{*2}}{2})\Delta v\right)} \text{ and}$$
$$R^F + (v + (1 - \frac{1}{2}x^{*2})\Delta v)\Delta R - I - \frac{\overline{e}}{1-\overline{e}}(I - A)$$
$$+ \frac{q(x^*)\Delta_H v (I - A)}{(1-\overline{e})\left(R^F/\Delta R + v + q(x^*)\Delta_H v + (1 - \frac{x^{*2}}{2})\Delta v\right)}.$$

By inspecting the value of the firm in the proposition above, one can distinguish three elements: the net increase in value due to the new investment, an illiquidity cost (which is invariant to firm's type) and a mispricing term as a result of the asymmetry of information. Even in the absence of asymmetric information, the firm does not appropriate the entire net present value of the new project. This "distortion" is created by the illiquidity cost. It generates a new source of underinvestment. Second, just like in the public offering equilibrium, underinvestment caused by the asymmetry of information is also present. However, the mispricing term is now somewhat alleviated because the firm announces to invest after $t = 0$ (in an interior solution) conveying a more precise signal on the expected profitability of the firm. At the margin, this lower mispricing margin equals the direct cost of delaying investment.

The second thing to note is that, as is immediate from the proposition above, $L-type$ firms face underinvestment as well due to the illiquidity discount whereas $H-type$ firms exhibit underinvestment from two sources: illiquidity and asymmetric information.

Finally, illiquidity affects the agency cost as well. In particular, by increasing it. Intuitively, the illiquidity cost increases the actual number of shares that are needed to fund the same investment scale, since the mispricing is larger when more shares are needed, the total mispricing cost increases as a result.

It is also important to understand how the optimal investment timing is affected by the parameters of the model. The following proposition summarizes such comparative statics.

Proposition 3 (Comparative Statics) *The optimal investment timing x^* increases with illiquidity costs ($\overline{e}$), investment scale (I) and asymmetric information ($(1-\gamma)\Delta_H v$). It decreases with the amount of available liquid assets (A), the profitability of the new project ($\Delta v \Delta R$) and an increase in the returns for the low state (R^F) holding ΔR constant.*

Proof. *See Appendix.* ■

Illiquidity affects investment timing in the following way: more illiquid shares worsen the mispricing factor caused by asymmetric information, thus increasing the marginal benefit of delaying the announcement of the investment decision in order to capture the benefits of a more precise signal. Investment scale also increases the marginal benefit of delaying investment more so than the direct cost. This is so because increasing the investment scale makes mispricing more costly, raising the marginal benefit of delaying investments away from $x^* = 0$. In a similar vein, the extend to which asymmetric information matters also makes the firm delay the investment in order for investors to have a more precise signal and ameliorate the mispricing cost. On the other hand, the effect of liquid assets (A) goes in the opposite way by the same argument just explained: more liquid assets lower the mispricing term. If the marginal cost of delaying investment goes down, we will optimally raise funds later to benefit from a more precise signal. In a similar vein, if the loss under default is lower (R^F higher), while keeping ΔR constant, that is, an equal increase in R^F and R^S will make the company invest sooner, the reason being, again, that the marginal dilution cost is lower, and it pays to invest sooner in order to rip off some rents from competitors.

An interesting case is the one in which Δv is small compared to $\Delta_H v$, which implies that investing earlier is relatively less valuable (smaller cost of delaying the investment.) Given the first-order condition, as $\Delta v \to 0$, $x^* \to 1$ and the mispricing term vanishes since the signal is totally informative. In this limiting case firms would choose to issue a PIPE. We will formally develop the comparison between a PIPE and a public placement in the next subsection.

Before we turn to one of the main results of the paper we first derive an expression for the illiquidity discount attached to a security issued using a private placement.

Proposition 4 (Illiquidity Discount) *The expected per-share illiquidity discount in a PIPE is given by*

$$\overline{e}\left(R^F + \left[v + q(x^*(\overline{e}))\Delta_H v + (1 - \frac{x^*(\overline{e})^2}{2})\Delta v\right]\Delta R\right).$$

Proof. *See Appendix.* ■

The proposition above shows how the discount at which shares are sold under a PIPE is the compounded effect of a direct channel as a result of the illiquidity of the stock (the first term in the product above, $\overline{e}$) and an information asymmetry effect since illiquidity increases the agency cost of equity, affecting the optimal investment timing. Increasing $\overline{e}$ raises the cost proportionally to the expected returns to the investment (direct effect.) On the other hand, it also delays the investment. If the asymmetry of information problem is important enough ($(1-\gamma)\Delta_H v > \Delta v$) an increase in $\overline{e}$ further raises the discount, above and beyond the direct effect. On the other hand, though, if the asymmetry is mild the indirect effect might end up making the discount lower. This result is consistent with the findings of Wruck and Wu (2007). They find that relationships create value in private placements in the sense that price discounts are smaller when a relationship is involved. A relationship in our model would translate into a lower asymmetry of information and would mitigate the direct effect of illiquidity, making the discount lower. Also, to the extend that the investment technology exhibits constant or increasing returns to scale (that is, a larger I would increase ΔR at least proportionally) the model predicts that a larger issuance size would also carry a larger illiquidity discount. As we will see next, this helps explaining why a public placement dominates larger issuances.

3.4 The Issuance Decision

In the two previous sections we have characterized the equilibrium that would arise in each of the markets assuming they where the only alternative available. Of particular importance were the properties of a private placement in terms of how and when a firm uses its financial flexibility depending upon the different parameters of the model. We now put the two types of issuance together and assess the firm's investment decision. This amounts to deciding which market to use to raise funding in order to maximize existing shareholders' value.

In order to solve for this problem we use the results from proposition 1 and 2. However we have to consider two other potential separating equilibria where the two types issue different securities. These hybrid equilibria are: one where $H-type$ firms choose to issue a PIPE whereas $L-type$ firms prefer an SEO, and a reversed one where firms switch the market around. We find that these hybrid do not arise as a PBE of the program, essentially because $L-type$ firms will find more convenient to pool and benefit from mispricing, and therefore we are left with the following result.

Proposition 5 (Issuance decision) *There exists a cutoff $\widetilde{e}$ such that:*

i) For $\overline{e} \leq \widetilde{e}$, the unique equilibrium is one in which both types issue a private placement of equity in order to raise funds for its investment opportunity,

ii) for $\overline{e} > \widetilde{e}$, firms use a public offering.

Moreover, when asymmetries of information are important ($\Delta_H v$ is large) firms tend to issue PIPEs in order to invest later and reduce the mispricing cost. Finally, PIPEs are more beneficial when available internal funds (A) are low. This is because potential dilution costs are larger. This suggests that smaller firms or firms that are more financially constrained will tend to pick a private placement first, SEOs might dominate for firms with more available cash flows.

Proof. *See Appendix.* ■

The result contained in the proposition above is intuitive. It suggests the following ranking of markets: if expected illiquidity is important, a public firm should use a public placement, otherwise it should use a private placement. If asymmetric information is important, PIPEs might rank first, since its flexibility provides a mechanism to lower expected dilution costs. This result is consistent with studies showing that firms are more likely to issue a PIPE when asymmetries of information are larger (see Gomes and Phillips (2005)). Last but not least, the result of the proposition above relating the lack of internal funds and the higher likelihood of issuing a PIPE is consistent with empirical studies in different aspects. First, the existing literature shows that SEO issuances are larger in the amount raised than PIPEs and that they are also done by larger companies. To the extend that smaller companies might have less available cash flows this would be consistent with our model. Apart, of course, from the fact that smaller companies tend to be newer and might be exposed to higher asymmetries of information, something that we have just commented upon.

4 Private Debt

As we mentioned in the introduction, the paper primarily focuses on the private/public dimension of equity financing. However, private placements are also made with debt instruments. In this section I solve for the optimal debt-like contract. I then analyze whether the private/public dimension of the problem alters the well-known pecking-order theory that arises in a public market (Myers and Majluf (1984)). We will argue that within private placements, the pecking order (internal funds first, then debt, then equity) need not hold. In this analysis, it is worth noting that debt contracts are assumed to face the same illiquidity problem than equity, that is, a debt contract pays off at $t = 2$. If investors are hit by a liquidity shock they are not able to sell the claim and will face a loss. In expectation, that needs to be taken into account in the same way as it has in the previous section.

We start analyzing the case with safe debt. That is, let us first assume that profits, net of the illiquidity cost, in the low state are just equal to the investment amount needed from outside investors, $R^F(1-\bar{e}) = I - A$. The following proposition summarizes the main result when the equilibrium is compared to a private placement of equity.

Proposition 6 (Riskless Debt) *If* $(1-\gamma)\Delta_H v > \Delta v$, *using private debt allows the firm to raise funds faster. That is ,* $x^*_{PE} > x^*_{PD}$. *Also, profits might be larger than with an equity contract. However, if the asymmetry of information is less important, there exists cases in which an equity contract allows for a faster investment and provides larger profits.*

Proof. *See Appendix.* ■

The intuition behind the proposition above goes as follows. It is well-known that when asymmetric information is important, the mispricing becomes larger in an equity-like contract, a contract that is more sensitive to such problem, that is the reason why debt dominates equity. On the other hand, when such problem is less acute, the result might not hold. The reason is that dilution costs are now endogenous, given that the firm can choose investment timing, which affects both dilution costs and the NPV of the project. Because with a debt contract the firm only makes profits if the success state (R^S) is realized this affects the optimal investment timing, which will likely have the firm invest earlier than with an equity contract, causing dilution costs to be higher and profits potentially lower. Next proposition will be more precise on the instances when this is more likely. It is also worth noting the effect of illiquidity. In a debt contract, the illiquidity cost is not shared with the firm in the lower state but solely beard by the investor. In order to satisfy the participation constraint, the compensation given to the investor in the good state must increase, this is the state in which the firm makes non-zero profits. This in turn affects the choice of x in a way that might lower the potential benefits of a debt contract.

Proposition 7 (Risky Debt) *More generally, within a private placement of securities, the pecking order might not hold. Under some conditions private equity dominates private debt. These scenarios would involve, for instance, projects where the recovery rate (net of illiquidity costs) is low ($R^F(1-\bar{e})/(I-A)$ small) and/or the normalized difference in profitability between states, $\Delta R/R^F$, is low.*

The result above illustrates a relative disadvantage of a debt-like contract and also predicts that privately placed equity is more likely to be valuable (compared to a debt contract) when the recovery rate in case of default is low or the difference in profitability between both states is low. As explained above for the case of safe debt, the reason is that dilution costs are now endogenous, given that the firm can choose investment timing. Because with a debt contract the firm only makes profits if the success state (R^S) is realized this might cause the firm to invest earlier than with an equity contract, raising dilution costs and hurting profits. This is more likely to happen if the recovery rate is low, that is, when R^F compared to $I-A$ is small. The reason is that, in this case, the two types of contracts differ more from each other in terms of their option structure (an equity contract resembles the payoff of a call option, a debt contract one of a put). Moreover, when illiquidity costs are relevant, debt has the undesirable feature of punishing the investor relatively more in the default state. Unlike an equity contract where part of the illiquidity cost is shared with the firm. A company does not pay any of that cost in such state since all payoffs go to new (debt) investors. Because of that, the firm needs to raise the face value of debt, increasing the dilution cost. An equity contract might then be more desirable. This distortion is more likely to benefit equity when the recovery rate under default is small. In those cases the distortion from the illiquidity in the market creates a more severe disadvantage for debt. Figure 4 depicts the value functions for both private debt and equity contracts in two numerical cases, each of which reflect the dominance of either debt or equity in a private placement.

[Figure 4 about here]

This result finds some confirmation in the data as well. Gomes and Phillips (2005) find support for a pecking order of security issuance conditional upon the firm issuing in the public market (the probability of issuing equity declines with asymmetric information.) They also find that, conditional on the firm issuing in the private market, there is a partial reversal of such sensitivity.

Finally, it is worth commenting on the fact that this is not the first paper arguing that the pecking order can be reversed. Another example is Fulghieri and Lukin (2001), who argue that firms might prefer to issue a more information-sensitive security in order to encourage information production by specialized investors thus inducing higher issuing prices (in their paper the distinction between public and private markets is absent.) Although we share the fact that dilution costs are endogenous, the mechanisms are quite distinct. They argue that information gathering is done costly by specialized (informed) investors in a market microstructure model where a market maker adds up demand from informed and uninformed. Our model, in contrast, does not rely on a market microstructure model but argues that lower dilution costs can be attained by placing the security privately given that in this case only informed investors participate and the security is flexible enough to capture and benefit from signals about the project's quality. Also, in our model, the reversal of the pecking order only happens within the private placement of securities.

5 Analysis and Empirical Predictions

5.1 On the Stock Price Announcement Effect of PIPEs

The potential reversal of the pecking order explained in the previous section (see Proposition 7) helps explaining the empirical regularity that PIPEs do not usually carry a negative stock price reaction, unlike SEOs. The argument behind negative stock price reactions to (public) equity issuances has been that if dilution costs are important, high-profitability borrowers might decide not to invest. If only low-profitability borrowers do invest, announcing an equity issuance is "bad" news. In this model, however, since private placements lower the asymmetry of information with an adequate timing of the investment decision, this allows high-profitability firms that otherwise would have not invested to invest in a positive net present value project. If that is the case, announcing an unexpected investment (and notice that it should not be expected by the market otherwise it would have been priced already) financed via a private placement carries only a positive effect that captures the positive net present value of the project and no negative signal since both types of firms issue the security. Essentially a private placement loosens the participation constraint for $H-type$ companies (that is, it partially solves the underinvestment problem) allowing both types of companies to invest, something that might not be possible if the only channel available to them was a public placement. This is consistent with the empirical findings explained in the introduction which report a positive abnormal return around the announcement of a PIPE. This theory argues that the positive announcement effect comes from the possibility of undertaking positive NPV projects by both better and worse firms. Figure 5 illustrates this argument: as long as illiquidity costs are not too high (left part of the diagram), a private placement might alleviate the participation constraint of high-profit firms, thus precluding from an equilibrium where only low profitability firms invest. To explain the positive abnormal returns to PIPEs we need to further assume that they are unexpected investment opportunities, which makes sense given the private nature of the issuance. In an SEO, on the other hand, it is more likely that the investment is expected, which is the usual assumption made to explain the negative stock price reaction.

[Figure 5 about here]

5.2 On the Effects of Sarbanes-Oxley

Our model can also be used to extract some policy implications. The model argues that SEOs are less flexible because the SEC legal requirements that firms need to undertake in order to make a public placement create a gap between the investment decision and the implementation of such investment. On the other hand, in a private placement this can be done simultaneously. One can then interpret SOX as increasing the lack of flexibility of public placements, which in turn raises the relative advantage of PIPEs. The model then predicts that it would be more likely that companies would use private placements after SOX was passed, other things equal. Although we are not aware of any formal test of this prediction, this can be seen as consistent with the fact that in the past few years larger companies have started using more private placements. Larger firms are more likely to be at the margin between both types of channels and making the public one more costly through additional lack of flexibility might have caused an increased use of PIPEs among larger companies. This could also explain the increased use of this type of security after 2001.

6 Concluding Remarks

This paper presents a theory that explains the decision of public firms whether to use a private placement or a public one. A central element of such a theory is to model the value of financial flexibility that private placements possess. In short, private placements time investment decisions in order to benefit from competitors and to reduce potential dilution costs due to asymmetric information.

We find that when illiquidity costs are important, SEOs dominate, however when they are not, PIPEs are chosen. We show that when asymmetry of information is relevant or available cash-flows are low, PIPEs dominate instead. We then introduce private debt into the picture and show that within the private placement of securities the traditional pecking order might be reversed, with equity ranking before debt. Finally we show how the model can explain some empirical regularities, such as the current increase in the use of these types of securities and the positive stock price effect when PIPEs are announced. The paper opens up additional research on this area, a natural follow-up question is, for instance, how the choice of public or private placements is done by companies in a dynamic model, that is, when in the life-cycle of a company one type of security dominates the other?

References

[1] Bolton, P. and E. Von Thadden (1998): "Blocks, Liquidity and Corporate Control'" *Journal of Finance* vol.53, No.1, 1-25.

[2] Dresner, S. (editor) (2006): *PIPEs: a Guide to Private Investments in Public Equity.* Bloomberg L.P., New York, NY.

[3] Gamba A. and Triantis (2008): "The Value of Financial Flexibility," *Journal of Finance* vol. 68, No.6.

[4] Gomes, A. and G. Phillips (2005): "Why do Public Firms Issue Private and Public Securities?" Working Paper.

[5] Gorton, G. and G. Pennachi (1990): "Financial Intermediation and Liquidity Creation," *Journal of Finance* vol. 45, No.1, 49-71.

[6] Hertzel, M., M. Lemmon, J. Linck and L. Rees (2002): "Long-Run Performance following Private Placements of Equity," *Journal of Finance* vol. 57, No.6, 2595-2617.

[7] Fulghieri, P. and D. Lukin (2001): "Information production, Dilution Costs and Optimal Security Design," *Journal of Financial Economics* 61, 3-41.

[8] Myers, S and N. Majluf (1984): "Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have," *Journal of Financial Economics* 13, 187-221.

[9] Subraymanyan A. and S. Titman (1999): "The Going-Public Decision and the Development of Financial Markets," *Journal of Finance* vol. 54, No.3, 1045-1082.

[10] Tirole, J. (2006): *The Theory of Corporate Finance.* Princeton University Press, Princeton, NJ.

[11] Wruck, K. (1989): "Equity Ownership Concentration and Firm Value," *Journal of Financial Economics* 23, 3-28.

[12] Wruck, K. and Y. Wu (2007): "Closeness Counts: The Role of Relationships in Private Placements of Equity," Working Paper SSRN.

[13] Wu Y. (2004): "The Choice of Equity-Selling Mechanisms," *Journal of Financial Economics* 74, 93-119.

Appendix

Proof of Lemma 1

Start defining $P(s = s_H \mid v_H) = a + bx$ and $P(s = s_H \mid v_L) = a' + b'x$. Then, $P(s = s_L \mid v_H) = 1 - a - bx$ and $P(s = s_L \mid v_L) = 1 - a' - b'x$. Applying Bayes' theorem we have that

$$P(v_H \mid s_H) = \frac{(a+bx)\gamma}{(a+bx)\gamma + (a'+b'x)(1-\gamma)}.$$

Using the definition provided for the signal and the conditions in (1) we impose $P(v_H \mid s_H, x = 0) = \gamma$ and $P(v_H \mid s_H, x = 1) = 1$ and substituting in the expression above yields $a = a'$ and $a' + b' = 0$. Similarly,

$$P(v_L \mid s_L) = \frac{(1-a'-b'x)(1-\gamma)}{(1-a-bx)\gamma + (1-a'-b'x)(1-\gamma)}.$$

Again imposing the conditions required in the definition, $P(v_L \mid s_L, x = 0) = 1-\gamma$ and $P(v_L \mid s_L, x = 1) = 1$ and substituting in the expression above yields $a = a'$ and $a + b = 1$. We are therefore left with three equations and four unknowns. We use the degree of freedom left to set $a = \gamma$, which implies $b = 1 - \gamma$ and $b' = -\gamma$. It is immediate to realize that these values correspond to the posterior probabilities that appear in the Lemma.■

Proof of Lemma 2

In order to show the result of Lemma 1 we start solving for the prices in each state that will clear the market. First, since we assume $R_i > \frac{c(1-e_l)}{e_l}$, $\forall i$, there is no storage in equilibrium, that is, late consumers supply all the capital good to early consumers. The market clearing condition equates the demand for the consumption good with supply. Thus,

$$e_j P_{ij} = c(1 - e_j)$$

in the case the firm does a public offering, or

$$(1-\alpha) e_j P_{ij} = c(1 - e_j)$$

in the case the firm issues a PIPE. The equation above captures the fact that only old shareholders with liquid shares are able to sell their stock if needed, whereas new shareholders are not able to sell them (they are however able to buy new shares in the secondary market if they need to.) The illiquidity of new shares raises the price, other things equal. We now turn to the calculation of expected utilities in both cases. We start with the aggregate expected utility in the case of a public offering:

$$\mathbb{E}_0\left[c_1 + c_2 \mid SEO,\ i = H\right] = q_h\left[e_h(c + P_{hH}) + (1 - e_h)(\mathbb{E}_0[R_H \mid SEO] + c\mathbb{E}_0[R_H \mid SEO]/P_{hH})\right] +$$
$$+q_l\left[e_l(c + P_{lH}) + (1 - e_l)(\mathbb{E}_0[R_H \mid SEO] + c\mathbb{E}_0[R_H \mid SEO]/P_{lH})\right] = c + R^s_H$$

On the other hand, solving the same expression in the case of a PIPE yields

$$\mathbb{E}_0\left[c_1 + c_2 \mid PIPE,\ i = H\right] = q_h\left[e_h(1 - \alpha)(c + P_{hH}) + e_h\alpha c + (1 - w_h)(\mathbb{E}_0[R_H \mid PIPE] + e\mathbb{E}_0[R_H \mid PIPE]/P_{hH})\right]$$
$$+q_l\left[e_l(1 - \alpha)(c + P_{lH}) + e_l\alpha c + (1 - e_l)(\mathbb{E}_0[R_H \mid PIPE] + c\mathbb{E}_0[R_H \mid PIPE]/P_{lH})\right]$$
$$= c + \mathbb{E}_0[R_H \mid PIPE] - \alpha\bar{e}\mathbb{E}_0[R_H \mid PIPE].$$

■

Proof of Proposition 1. The proof of this proposition is parallel to solving for a pooling equilibrium under asymmetric information. The proof is available upon request but it is standard in these models. For a general treatment of the game see Tirole (2006).

Proof of Proposition 2

The program to be solved by the $H - type$ company in a pooling equilibrium is as follows:

$$\max_{\{\alpha,x\}}\ (1-\alpha)\left[R^F + \left(v + \Delta_H v + (1 - \frac{1}{2}x^2)\Delta v\right)\Delta R\right]$$
$$s.t.\ \alpha\left(R^F + \left(v + q\Delta_H v + (1 - \frac{1}{2}x^2)\Delta v\right)\Delta R\right) \geq \frac{I - A}{1 - \bar{e}}$$
$$\alpha \leq 1$$

Using the participation constraint for the investor one can rewrite the problem as

$$\max_{\{x\}}\ R^F + \left(v + \Delta_H v + (1 - \frac{1}{2}x^2)\Delta v\right)\Delta R - I - \bar{e}\frac{I - A}{1 - \bar{e}} - \frac{(1-q)\Delta_H v(I - A)}{(1-\bar{e})\left(R^F + \left(v + q\Delta_H v + (1 - \frac{1}{2}x^2)\Delta v\right)\Delta R\right)}$$

which yields the F.O.C.

$$-\Delta v\Delta Rx + \frac{(1-\gamma)\Delta_H v\Delta R(I - A)\left[R^F + \left(v + \Delta_H v + \left(1 - x + \frac{1}{2}x^2\right)\Delta v\right)\Delta R\right]}{(1-\bar{e})\left(R^F + \left(v + q\Delta_H v + \left(1 - \frac{1}{2}x^2\right)\Delta v\right)\Delta R\right)^2} = 0.$$

In order to prove that a solution exists we will apply the Intermediate Value theorem.

First, the FOC above is a continuous function in the domain of x. At $x = 0$ the FOC reads

$$\frac{(1-\gamma)\Delta_H v \Delta R(I-A)\left[R^F + (v + \Delta_H v + \Delta v)\Delta R\right]}{(1-\overline{e})\left(R^F + (v + \gamma\Delta_H v + \Delta v)\Delta R\right)^2} > 0.$$

On the other hand at $x = 1$ we have

$$-\Delta v \Delta R + \frac{(1-\gamma)\Delta_H v \Delta R(I-A)}{(1-\overline{e})\left(R^F + \left(v + \Delta_H v + \frac{1}{2}\Delta v\right)\Delta R\right)} < 0.$$

To show why the last inequality holds, let us assume for a contradiction that

$$\begin{aligned}
-\Delta v \Delta R + \frac{(1-\gamma)\Delta_H v \Delta R(I-A)}{(1-\overline{e})\left(R^F + \left(v + \Delta_H v + \frac{1}{2}\Delta v\right)\Delta R\right)} &> 0. \\
\Delta v \Delta R &< \frac{(1-\gamma)\Delta_H v \Delta R(I-A)}{(1-\overline{e})\left(R^F + \left(v + \Delta_H v + \frac{1}{2}\Delta v\right)\Delta R\right)} \\
\Delta v \Delta R(1-\overline{e})\left(R^F + \left(v + \Delta_H v + \frac{1}{2}\Delta v\right)\Delta R\right) &< (1-\gamma)\Delta_H v \Delta R(I-A) \\
\Delta v \Delta R(1-\overline{e})\left(v + \Delta_H v + \frac{1}{2}\Delta v\right) &< (1-\gamma)(I-A)\Delta_H v
\end{aligned}$$

a contradiction since $v + \Delta_H v + \frac{1}{2}\Delta v > \Delta_H v$ and $(1-\overline{e})\Delta v \Delta R > (I-A)$. Regarding the last inequality even though we only assume $\frac{1}{2}\Delta v \Delta R > (I-A)$, in an interior solution profits must be non-negative and so for $x = 1$

$$(1-\frac{1}{2}x^2)\Delta v \Delta R - \frac{I-A}{1-\overline{e}} - A \geq 0 \Rightarrow (1-\overline{e})(1-\frac{1}{2}x^2)\Delta v \Delta R - (I-A) \geq 0 \Rightarrow (1-\overline{e})\Delta v \Delta R - (I-A) > 0.$$

Therefore there exists an $x^* \in (0,1)$ such that the first-order condition is satisfied. After some algebra one can also show that the second-order condition is satisfied. The proof of that is available upon request.

Given that $L - type$ firms are creditworthy, the only possible equilibrium is a pooling one, this result is standard in these models. See, for a general treatment, Tirole (2006).■

Proof of Proposition 3 (Comparative Statics)

Define $\Phi'(x^*) = 0$ the first-order condition of the program and z a parameter of the model. Using the implicit function theorem it is easy to show that

$$\frac{dx^*}{dz} = -\Phi''(x^*)^{-1}\frac{\partial \Phi'(x^*)}{\partial z}.$$

Given that $\Phi''(x^*) < 0$ at a maximum,

$$sign\left[\frac{dx^*}{dz}\right] = sign\left[\frac{\partial \Phi'(x^*)}{\partial z}\right].$$

The result of the proposition follows after applying this result. That is,

$$\frac{\partial \Phi'(x^*)}{\partial A} = -\frac{(1-\gamma)\Delta v_H \Delta R\left[R^F + (v + \Delta_H v + (1 - x^* + \frac{1}{2}x^{*2})\Delta v)\Delta R\right]}{(1-\overline{e})(R^F + (v + \Delta_H v + (1 - \frac{1}{2}x^{*2})\Delta v)\Delta R)^2} < 0$$

$$\frac{\partial \Phi'(x^*)}{\partial I} = \frac{(1-\gamma)\Delta v_H \Delta R\left[R^F + (v + \Delta_H v + (1 - x^* + \frac{1}{2}x^{*2})\Delta v)\Delta R\right]}{(1-\overline{e})(R^F + (v + \Delta_H v + (1 - \frac{1}{2}x^{*2})\Delta v)\Delta R)^2} > 0$$

$$\frac{\partial \Phi'(x^*)}{\partial \overline{e}} = \frac{(1-\gamma)\Delta v_H \Delta R(I-A)\left[R^F + (v + \Delta_H v + (1 - x^* + \frac{1}{2}x^{*2})\Delta v)\Delta R\right]}{(1-\overline{e})^2(R^F + (v + \Delta_H v + (1 - \frac{1}{2}x^{*2})\Delta v)\Delta R)^2} > 0$$

$$\frac{\partial \Phi'(x^*)}{\partial \Delta v \Delta R} = -Ix^* - \frac{1}{(1-\overline{e})(R^F + (v + \Delta_H v + (1 - \frac{1}{2}x^{*2})\Delta v)\Delta R)^3}\{$$

$$(1-\gamma)\Delta v_H \Delta R(I-A)\left[\left(1 - \frac{1}{2}x^*\right)\left(R^F + (v + \Delta_H v + (1 - x^* + \frac{1}{2}x^{*2})\Delta v)\Delta R\right) - \left(1 - x^* + \frac{1}{2}x^{*2}\right)\right]\} < 0,$$

where the last sign follows since $\left(1 - \frac{1}{2}x^*\right) > \left(1 - x^* + \frac{1}{2}x^{*2}\right)$ and $\left(R^F + (v + \Delta_H v + (1 - x^* + \frac{1}{2}x^{*2})\Delta v)\Delta R\right) > 1$.

In order to show the effect of $(1-\gamma)\Delta_H v$ on x^*, we redefine $y \equiv (1-\gamma)\Delta_H v$ so that $\Delta_H v = y/(1-\gamma)$ and $q\Delta_H v = \left(\frac{\gamma}{1-\gamma} + x\right)y$. Also let us denote $a \equiv R^F/\Delta R + v + y/(1-\gamma) + \left(1 - x + \frac{1}{2}x^2\right)\Delta v$ and $b \equiv R^F/\Delta R + v + \gamma/(1-\gamma)y + xy + \left(1 - \frac{1}{2}x^2\right)\Delta v$. Then

$$\frac{\partial \Phi'(x^*)}{\partial y} = \frac{(I-A)}{(1-\overline{e})}\left[\frac{a}{b^2} + \frac{y/(1-\gamma)}{b^2} - \frac{2\left(\gamma/(1-\gamma) + x\right)ya}{b^3}\right]$$

After rearranging terms we have

$$sign\left[\frac{\partial \Phi'(x^*)}{\partial y}\right] = sign\left[b\left(a + \Delta_H v\right) - 2aq\Delta_H v\right] = sign\left[ba - \Delta_H v\left(2aq - b\right)\right] > 0$$

since $a > \Delta_H v$ and $b > 2aq - b$. To show how the last inequality holds we rewrite $b > 2aq - b$ as

$$\begin{aligned} b &> aq \\ R^F/\Delta R + v + \Delta v(1 - \frac{x^2}{2}) &> q\left(R^F/\Delta R + v\right) + q\Delta v(1 - x + \frac{x^2}{2}) \end{aligned}$$

which is true since $1 \geq q$ and $1 - \frac{x^2}{2} \geq 1 - x + \frac{x^2}{2}$.■

Proof of Proposition 4. Direct by substitution, using proposition 2.

Proof of Proposition 5

The proof of this proposition is done in several steps. To prove the first part, note that from Propositions 1 and 2 we have that the SEO and PIPE profits for an $H-type$ firm are

$$\begin{aligned}
\pi_{SEO} &= R^F + (v + \Delta_H v + \frac{1}{2}\Delta v)\Delta R - I - \frac{(1-\gamma)\Delta_H v(I-A)}{R^F/\Delta R + v + \gamma\Delta_H v + \frac{1}{2}\Delta v} \text{ and} \\
\pi_{PIPE} &= R^F + (v + \Delta_H v + (1 - \frac{1}{2}x^{*2})\Delta v)\Delta R - I - \frac{\overline{e}}{1-\overline{e}}(I-A) \\
&\quad - \frac{(1-q(x^*))\Delta_H v(I-A)}{(1-\overline{e})\left(R^F/\Delta R + v + q(x^*)\Delta_H v + (1-\frac{x^{*2}}{2})\Delta v\right)}
\end{aligned}$$

respectively. At $\overline{e} = 0$,

$$\pi_{PIPE} = R^F + (v + \Delta_H v + (1 - \frac{1}{2}x^{*2})\Delta v)\Delta R - I - \frac{(1-q(x^*))\Delta_H v(I-A)}{R^F/\Delta R + v + q(x^*)\Delta_H v + (1-\frac{x^{*2}}{2})\Delta v} > \pi_{SEO}$$

since $1 - \frac{1}{2}x^{*2} > \frac{1}{2}$ and $\frac{(1-q(x^*))\Delta_H v(I-A)}{R^F/\Delta R + v + q(x^*)\Delta_H v + (1-\frac{x^{*2}}{2})\Delta v} < \frac{(1-\gamma)\Delta_H v(I-A)}{R^F/\Delta R + v + \gamma\Delta_H v + \frac{1}{2}\Delta v}$ because $x^* < 1$ and $q(x^*) > \gamma$. On the other hand, π_{PIPE} is a continuos function of $\overline{e}$ and $\pi'_{PIPE}(\overline{e}) < 0$. Since for $\overline{e}$ arbitrarily close to 1, π_{PIPE} becomes non-positive, there exists an $\widetilde{e}$ such that for $\overline{e} \leq \widetilde{e}$, the unique pooling equilibrium consists of choosing a private placement, whereas $\overline{e} > \widetilde{e}$ means that the $H-type$ firm will choose public placement and the $L-type$ firm will follow.

For the second part of the proposition, note that dilution costs are proportional to $\Delta_H v$. The larger $\Delta_H v$ is the more likely that the firm will delay investing in order for investors to have a more precise signal. This will create a larger gap between the dilution costs under an SEO and those of a PIPE, benefiting the later as long as $\overline{e}$ is not too large. Even when $\overline{e}$ is large, if Δv is small the firm will be able to lower dilution costs enough to benefit a private placement as well, despite illiquidity costs.

Finally there are two other equilibrium candidates. These would be those in which each type of firm chooses a different type of placement: $H-type$ choosing a private placement and $L-types$ a public one, or $H-types$ choosing a public placement and $L-types$ a private one. These hybrid separating outcomes cannot be an equilibrium since given what $H-type$ firms choose and $L-type$ firm will always have the incentive to mimic what an $H-type$ does and receive higher profits than the profits from a separating equilibrium (which are the same as the profits under perfect information.)

To show that the larger $\Delta_H v$ the more likely the firm will issue a PIPE, we only need to notice that using proposition 3, the larger $(1-\gamma)\Delta_H v$ the larger x^*. This means that the value of flexibility is more important, since the mispricing cost, compared to the one that the firm faces in a SEO, is lower. Provided that the illiquidity cost ($\bar{e}$) is not too high, the firm chooses a PIPE in order to take advantage of the flexibility. The same reasoning goes behind the fact that the higher A the more likely the firm will issue an SEO: the value of flexibility is less important since the larger A is the lower the misprincing cost is.■

Proof of Proposition 6 and 7

Let's first solve the program under private debt financing. The simplified program for an $H-type$ firm, once we take into account the fact that the payment to the investor in the case the lower state of the world occurs is R^F is as follows:

$$\max_{\{R_i^S,x\}} \left(v+\Delta_H v+(1-\frac{1}{2}x^2)\Delta v\right)(R^S-R_i^S)$$
$$s.t. \left(v+q\Delta_H v+(1-\frac{1}{2}x^2)\Delta v\right)R_i^S+\left(1-\left(v+q\Delta_H v+(1-\frac{1}{2}x^2)\Delta v\right)\right)R^F \geq \frac{I-A}{1-\bar{e}}$$
$$R_i^S \geq 0$$

Using the PC constraint for the investor one can rewrite the problem as

$$\max_{\{x\}} \left\{R^F+\left(v+\Delta_H v+(1-\frac{1}{2}x^2)\Delta v\right)\Delta R-I-\bar{e}\frac{I-A}{1-\bar{e}} -\frac{(1-q)\Delta_H v(I-A-(1-e)R^F)}{(1-e)\left(v+q\Delta_H v+(1-\frac{1}{2}x^2)\Delta v\right)}-(1-q)\Delta_H vR^F\right\}$$

which yields the F.O.C.

$$-\Delta v\Delta Rx+\frac{(1-\gamma)\Delta_H v(I-A-(1-e)R^F)\left[v+\Delta_H v+\Delta v\left(1-x+\frac{1}{2}x^2\right)\right]}{(1-e)\left(v+q\Delta_H v+(1-\frac{1}{2}x^2)\Delta v\right)^2}+(1-\gamma)\Delta_H vR^F=0$$

The existence of an equilibrium is parallel to the proof of Proposition 2. The payoffs can be obtained after substitution of the constraint in the objective function.

Claim 1. If $R^F(1-\bar{e})=I-A$ and $(1-\gamma)\Delta_H v>\Delta v$, $x^*_{PD}<x^*_{PE}$.

In order to show this result, we first substitute the safe debt assumption into the first-order condition derived before, to obtain that

$$x^*_{PD}=\frac{\Delta_H vR^F(1-\gamma)}{\Delta v\Delta R}.$$

Note $x^*_{PD} < 1$ requires $R^F/\Delta R > (1-\gamma)\Delta_H v/\Delta v$ which demands that $R^F/\Delta R > 1$, otherwise $x^* = 1$. First, if $R^F/\Delta R < 1$, then $x^*_{PD} = 1$. Substitute this value of x into the first-order condition for the private equity contract. This yields the expression

$$\begin{aligned}
& -\Delta v\Delta R + \frac{(1-\gamma)\Delta_H v R^F \left[R^F/\Delta R + \left(v + \Delta_H v + \left(1 - x + \frac{1}{2}x^2\right)\Delta v\right)\right]}{\left(R^F/\Delta R + \left(v + q\Delta_H v + \left(1 - \frac{1}{2}x^2\right)\Delta v\right)\right)^2} \\
> \quad & -\Delta v\Delta R + \frac{(1-\gamma)\Delta_H v \Delta R \left[R^F/\Delta R + \left(v + \Delta_H v + \left(1 - x + \frac{1}{2}x^2\right)\Delta v\right)\right]}{\left(R^F/\Delta R + \left(v + q\Delta_H v + \left(1 - \frac{1}{2}x^2\right)\Delta v\right)\right)^2} \\
= \quad & \Delta R \left(-\Delta v + \frac{(1-\gamma)\Delta_H v \left[R^F/\Delta R + \left(v + \Delta_H v + \left(1 - x + \frac{1}{2}x^2\right)\Delta v\right)\right]}{\left(R^F/\Delta R + \left(v + q\Delta_H v + \left(1 - \frac{1}{2}x^2\right)\Delta v\right)\right)^2}\right) > 0
\end{aligned}$$

The last inequality follows from the fact that $(1-\gamma)\Delta_H v > \Delta v$ which implies $\frac{R^F/\Delta R + \left(v + \Delta_H v + \left(1 - x + \frac{1}{2}x^2\right)\Delta v\right)}{\left(R^F/\Delta R + \left(v + q\Delta_H v + \left(1 - \frac{1}{2}x^2\right)\Delta v\right)\right)^2} > 1$ and the term in parenthesis positive. This implies that $x^*_{PE} > x^*_{PD}$. Secondly, if $R^F/\Delta R > (1-\gamma)\Delta_H v/\Delta v (> 1)$ then $x^*_{PD} = \frac{\Delta_H v R^F (1-\gamma)}{\Delta v \Delta R}$. Again following the same steps we can substitute this expression into the first-order condition for the private equity issuance, yielding

$$\begin{aligned}
& -(1-\gamma)\Delta_H v R^F + \frac{(1-\gamma)\Delta_H v R^F \left[R^F/\Delta R + \left(v + \Delta_H v + \left(1 - x + \frac{1}{2}x^2\right)\Delta v\right)\right]}{\left(R^F/\Delta R + \left(v + q\Delta_H v + \left(1 - \frac{1}{2}x^2\right)\Delta v\right)\right)^2} \\
= \quad & (1-\gamma)\Delta_H v R^F \left(-1 + \frac{R^F/\Delta R + \left(v + \Delta_H v + \left(1 - x + \frac{1}{2}x^2\right)\Delta v\right)}{\left(R^F/\Delta R + \left(v + q\Delta_H v + \left(1 - \frac{1}{2}x^2\right)\Delta v\right)\right)^2}\right) > 0.
\end{aligned}$$

This concludes the proof of claim 1. The rest of the proof follows from realizing that if $(1-\gamma)\Delta_H v < \Delta v$ then $\frac{R^F/\Delta R + \left(v + \Delta_H v + \left(1 - x + \frac{1}{2}x^2\right)\Delta v\right)}{\left(R^F/\Delta R + \left(v + q\Delta_H v + \left(1 - \frac{1}{2}x^2\right)\Delta v\right)\right)^2}$ might become lower than 1. If that is the case, when evaluating the optimal timing under a debt contract into the first-order condition of the equity contract might yield a negative value, indicating that $x^*_{PE} < x^*_{PD}$.■

Figure 2. Posterior Probabilities Implied by the P.I.I. function

The figure plots numerically the posterior distribution for the high type, that is

$$P(v_H|\ s = s_H) = \frac{\gamma[k+(1-k)x]}{k(1-x)+\gamma x},$$

as a function of investment timing x, for $\gamma = \frac{1}{2}$. Each line represents the posterior distribution for discrete increments in the value of $k \in [0.5, 10]$. The straight line corresponds to $k = 0.5$, the case when $k = \gamma$ which is adopted in the paper.

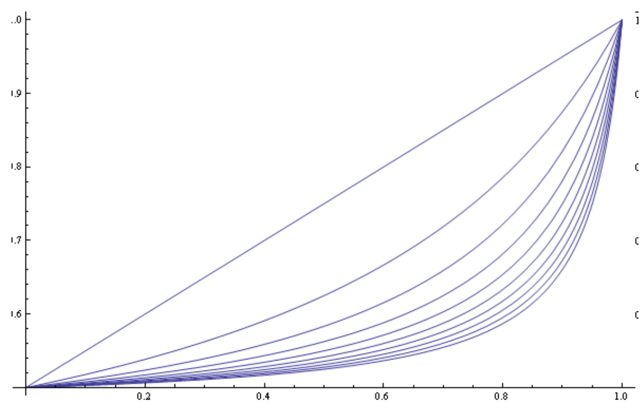


Figure 4. The Pecking Order and the Reverse Pecking Order

Top row: Numerical plot of the firm's objective function with parameter values: $\gamma = .5$, $v = .35$, $\Delta v = .25$, $\Delta_H v = .35$, $\Delta R = 15$, $R^F = 1$, $I = 3$, $A = 0$, $\bar{e} = 0.05$. Left-hand figures represent debt contracts whereas right-hand figures represent equity contracts. Firm values are 0.1939 and 0.1881 for equity and debt respectively. Bottom row: Same parameter values except for $v = .25$, $\Delta_H v = .4$, $\Delta R = 17$, $\bar{e} = .125$. Firm values are 0.0492 and 0.0475 for debt and equity respectively.

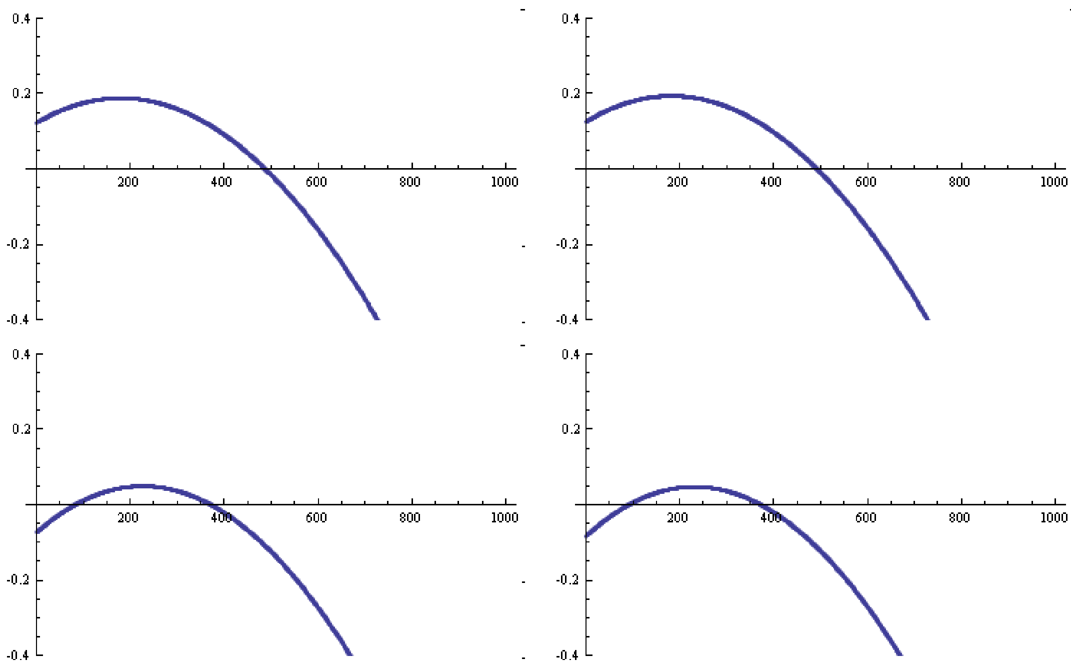


Figure 5. Stock Price Reaction of an Expected/Unexpected Issuance

The figure below illustrates the regions in which the model predicts that announcing an issuance will entail a positive or neutral reaction (blue, left region) or a negative reaction (red, top right region).