

Debt Covenants and Distressed Equity Issuance: Optimal Financing in the Presence of Monitoring*

Tomasz Piskorski

Mark M. Westerfield

Columbia Business School

Marshall School of Business

December 2008

Abstract

We solve for the optimal dynamic contract when the agent can imperfectly divert cash flows and the principal can substitute direct monitoring for performance-based incentives. We show how to implement the contract using equity and debt. Debt covenants implement monitoring choices and equity issuance implements changes in performance sensitivity. When the firm reaches its endogenous borrowing limit, the firm defaults and the manager is terminated. Distressed equity issuance reduces the firm's default risk by raising funds after shortfalls and is coupled with increased monitoring to maintain incentive compatibility. Equity holders will choose the optimal equity issuance policy, contrary to the common intuitions underlying debt overhang and asset substitution arguments, because they do not wish their perpetually renewed call option on the firm to be terminated by bankruptcy. Our model helps to explain the frequency of distressed equity issues, complements the existing literature on the use of debt covenants and managerial compensation, and makes several new testable predictions.

*Piskorski can be reached at Columbia Business School, Broadway 3022, Uris Hall 810, New York City, NY 10069; tp2252@columbia.edu; <http://www4.gsb.columbia.edu/cbs-directory/detail/6335859/Piskorski>.

Westerfield can be reached at Finance and Business Economics, Bridge Hall 308, MC-0804, 3670 Trousdale Parkway, Los Angeles, CA 90089-0804; mwesterf@usc.edu; <http://www-rcf.usc.edu/~mwesterf/>.

We thank Ricardo Alonso, Harry DeAngelo, Kevin Murphy, Oguzhan Ozbas, Paul Tetlock, Neng Wang, Fernando Zapatero and the participants at the Marshall Seminar Series and the Columbia Seminar Series for their useful discussion and comments.

1 Introduction

Monitoring is an important feature of many corporate finance theories, and the recent empirical literature on debt covenants has shown that monitoring is a vital feature of debt.¹ As a firm experiences losses, it triggers covenants, and management finds itself increasingly restricted in what corporate policies it can use. Covenants can alleviate agency problems but they are potentially costly to the firm exactly because they restrict the set of management choices. Despite this, the role of monitoring has remained relatively unexamined in dynamic settings, including the dynamic theory of security design. The primary obstacle has been the lack of a parsimonious monitoring model.

Our paper makes three contributions. First, we present a parsimonious dynamic principal-agent model with monitoring, and we characterize the optimal contract. Second, we show that this contract can be implemented with debt and equity, where debt covenants implement monitoring choices and equity issuance implements changes in performance sensitivity allowed by monitoring. In doing so, we can explain patterns in both the use of debt covenants and equity issuance, and we make new testable predictions. Third, we show that the optimal equity issuance policy arises in a game with strategic actions by equity holders and management. After we present the model in the main text and describe its properties, we will discuss our empirical predictions in the conclusion (Section 7).

We begin with a dynamic agency problem similar to DeMarzo and Sannikov (2006) in which the project manager with limited liability can imperfectly divert cash flows for his own consumption.² To induce the manager to report cash flows accurately, the principal

¹Chava and Roberts (2008) and Chava, Kumar, and Warga (2008) demonstrate that covenants have an impact on corporate policies and that the effects are strongest in environments in which agency problems are greatest. Black, Carnes, Mosebach, and Moyer (2004) directly show the link between flexibility/monitoring and covenants in banks: as regulatory monitoring increases, the use of covenants in debt contracts declines. More generally, Dichev and Skinner (2002) show that covenant violations are extremely common, and Roberts and Sufi (2008b) show that violations do not generally lead to default and that the terms of the debt contract are frequently renegotiated. Instead, covenant violations grant control rights to debt-holders who force the firm to follow more restrictive financial policies (Roberts and Sufi (2008a)).

²We label the agency problem as diversion, but this is reduced form for other possible agency problems, including the misuse of corporate assets or the lack of costly effort. In the case of costly effort, we would say that the agent can spend effort to increase the value of a project, but in doing so he incurs an opportunity cost. In the securities implementation, the agent may also represent a controlling shareholder who can divert resources from the firm at the expense of minority shareholders.

can use standard performance-based incentives, where low performance leads to termination of the project but high performance leads to additional consumption for the agent. The manager adds value to the project, so termination of the project (default) is costly to the principal because he retains the now-less-valuable assets. We extend this setting by allowing the principal access to a stochastic monitoring technology that can detect diversion by the manager. This second technology represents an average profitability vs. monitoring trade-off: the principal can impose constraints that make diversion more difficult (less efficient) but only at the cost of reducing the overall profitability of the project. Using pay-for-performance incentives results in a higher average project profitability, but they also increase the likelihood of costly termination because the manager is receiving a higher share of the project's gains and losses. The optimal contract efficiently trades off these two ways of providing incentives to the manager.

We derive an optimal contract that maximizes the total surplus from the project. In our continuous time setting, the contract takes a tractable form. The manager's expected payoff from now until the termination of the contract (continuation utility) acts as a state variable that determines the nearness to default and the efficient trade-off between the two incentive technologies. When the project nears default, the principal becomes effectively more averse to volatility in the manager's continuation utility, and so the principal switches to the use of monitoring and away from performance-based promises. When the manager's continuation utility becomes large enough, the principal pays the manager out of the collection of promises.

We show how our dynamic contract can be implemented in a dynamic game with common equity, a line of credit (or short-term debt), and preferred stock. The firm maintains a line of credit to which the manager deposits the cash flows. When the draw on the line of credit reaches zero, the firm pays dividends, some of which go to the manager. These dividends are the manager's consumption. When the draw on the line of credit reaches its maximum, the firm can no longer absorb losses, and so it defaults. At the same time, dividends are sufficiently far away that the manager's continuation value is equal to his outside option, and so the manager leaves. By tracking the balance on the line of credit, one tracks the manager's continuation utility. When the firm enters distress – meaning it is near default – the optimal contract dictates the use of monitoring instead of performance-based incentives,

and so the manager’s continuation utility must become less sensitive to cash flow. The manager’s incentive not to divert cash is maintained because lessened cash flow sensitivity is coupled with reduced management flexibility (increased monitoring) through covenants. To make the line of credit less sensitive to cash flow, the firm raises capital by issuing equity as the firm enters distress and repurchasing equity as the firm leaves distress. In both cases, the new capital is reflected in a revised draw on the line of credit. We say that debt covenants restrict management’s flexibility to manage its existing projects which reduces those projects’ average profitability, matching the assumed cost of monitoring.

The securities implementation works because the equity holders, and the firm as a whole, prefer lower levels of volatility on the line of credit. Some volatility is always needed to incentivize the manager – the performance-based reward through equity to the manager for reporting accurately. However, monitoring through debt covenants substitute for performance based incentives, increasing welfare. With covenants, the firm can relax the performance-based reward to the manager, and it does so by raising equity in distress and repurchasing it after success. As we show, our implementation is Pareto optimal, and it is not only the outcome of an optimal contract with commitment but also of a decentralized game between equity holders and management.

In addition to matching existing facts and making predictions regarding the design of debt securities, our model provides an insight into an apparently unrelated phenomenon: equity issuance by firms that have a high likelihood of going bankrupt in the near future (distressed equity issuance).³ Equity issuance by distressed firms is very common overall: DeAngelo, DeAngelo, and Stulz (2008) report that among all secondary equity offerings (SEOs), over 40% are from firms with Altman Z-scores⁴ indicating financial distress. Other studies, such as Wruck (1989), Hertz, Lemmon, Linck, and Rees (2002), and Franks and Sanzhar (2006) have also emphasized the frequency and importance of private equity issuance when firms are in distress or near bankruptcy.

³Throughout the paper, we will use distress to mean that a firm is in significant danger of actual default (non-payment of obligations) rather than technical default (failing to meet a covenant restriction). We also exclude firms that are already in bankruptcy proceedings as being beyond the “distress” stage.

⁴From Altman (1968), the Z-score is a measure of financial distress. It is a successful predictor of corporate bankruptcy over a two year horizon and is created from accounting data. The most important inputs are capital turnover, earnings, and existing leverage.

Yet, standard incentive theory indicates that similar common equity issuance should not happen often. First, the debt overhang theory observes that while the cash infusion is coming from equity holders, many of the gains go to debt holders if the firm avoids bankruptcy. Since equity holders do not receive the full benefit of their actions, distressed equity issuance should occur much less often than is optimal. Second, if one considers equity to be a call option on the value of the firm, then equity holders might want to take risk, shifting the cost to debt holders (asset substitution). However, raising capital in distress reduces risk and dilutes any future payoff from success. Third, the ability to raise equity in distress dulls the incentives to avoid distress, and so a firm might like to commit not to raise distressed equity.

The key to our explanation of distressed equity issuance is that the static intuition of equity as a call option on the value of the firm is incorrect in a dynamic model. Instead of one call option on the value of the firm, the equity holder has a perpetually renewed call option. The future options have value because they reflect the ability of equity holders to take risk and receive payment in the future, but their existence is contingent on the firm maintaining operations. Since bankruptcy terminates the sequence of options, it creates an equity-specific cost of bankruptcy. First, when the firm is near default, a cash infusion to partially offset a shortfall – even one that goes directly to paying down debt – is valuable to equity holders because it pushes the sequence of options closer to being in-the-money. Thus, the debt overhang intuition does not apply because new funds reduce the probability of realizing the equity-specific cost of bankruptcy. Second, enough losses results in bankruptcy in which equity holders lose all of their future options. Even if a loss is absorbed by the line of credit, the firm moves closer to the equity-specific cost of bankruptcy. Thus, contrary to the intuition underlying asset substitution, equity holders act to avoid volatility.⁵ Third, when debt covenants are used, distressed equity issuance does not weaken incentives. This is because increased monitoring is used as a substitute for performance-based incentives

⁵Panageas and Westerfield (2008) show that a hedge fund manager with option-style compensation (high-water mark contracts) will have a concave value function. The key difference between that paper and this one is that in the portfolio management context, increasing risk comes from investing more in risky assets, and thus both requires capital and increases average return. The issues of incentivizing effort and debt overhang does not appear in their model. In our paper, volatility comes from incentive provision and does not require capital. DeMarzo and Sannikov (2006) also show that equity can have a concave value function in our setting without monitoring and equity issuance.

implied by default. In fact, by reducing cash flow volatility, the combination of distressed equity issuance and restrictive covenants increases the value of equity.

This paper belongs to the growing literature on dynamic optimal security design and contracting that began with Holmström and Milgrom (1987), Spear and Srivastava (1987), and Abreu, Pearce, and Stacchetti (1990), among many others. Sannikov (2008) demonstrates a recent treatment of continuous-time techniques for the principal-agent problem, while Sannikov and Skrzypacz (2007) provide a general characterization of the set of equilibria of repeated games where players learn information via a continuous-time stationary process that has a Brownian component and a Poisson component.⁶ The studies that are most closely related to ours are DeMarzo and Fishman (2007) and its continuous-time formulation by DeMarzo and Sannikov (2006) and Biais, Mariotti, Plantin, and Rochet (2007a).⁷ These papers study long-term financial contracting in a setting with privately observed cash flows, and show that the implementation of the optimal contract involves a credit line, long-term debt, and equity. In contrast, we consider an environment where the principal has access to monitoring technology that allows detection of diversion by the manager. We solve for the optimal contract in this setting and find that its implementation involves active stock issuance and repurchase as well as debt covenants. Our paper is also related to the contracting literature with costly state verification, as exemplified by Townsend (1979) in a two-period setting and Wang (2005) in a dynamic context. Unlike these papers, we set our problem in continuous time, and we do not focus on risk insurance. Instead, we consider a setting in which the relationship is costly to terminate. In addition, our monitoring technology allows for stochastic rather than deterministic detections.

The paper proceeds as follows. In Section 2, we introduce the model primitives and define the objective functions. We solve for the optimal contract in Section 3, and we discuss its properties in Section 4. We then show how the contract can be implemented with common securities in Section 5. Section 6 describes the strategic motivations of the various securities

⁶See also Biais, Mariotti, Plantin, and Rochet (2007b), He (2007b), Piskorski and Tchisty (2007), among other, for studies of dynamic agency problems in Brownian-Poisson and pure Poisson settings.

⁷Other related papers include the formulations of Tchisty (2006), Clementi and Hopenhayn (2006), Biais, Mariotti, Plantin, and Rochet (2007b), He (2007a), Sannikov (2007), Piskorski and Tchisty (2007), and Philippon and Sannikov (2007).

holders. We conclude with a discussion of our empirical predictions in Section 7. All proofs are in the appendix.

2 Setup

2.1 The Project and Participants

Time is continuous and infinite. The principal (an investor or a group of investors) is risk neutral, has unlimited capital, and values a stochastic cumulative cash flow $\{f_t\}$ as

$$E \left[\int_0^\infty e^{-rt} df_t \right], \quad (1)$$

where r is the investor's discounts rate.

The manager is also risk neutral, has limited wealth, and values a stochastic cumulative consumption flow $\{C_t\}$ as

$$E \left[\int_0^\infty e^{-\gamma t} dC_t \right]. \quad (2)$$

We assume that $\gamma > r$, reflecting the assumption that a borrowing-constrained, undiversified manager has a higher intertemporal marginal rate of substitution than an unconstrained, diversified investor.

The manager can operate a project starting from date $t = 0$. An investment opportunity requires an initial investment of $I > 0$. The manager's initial wealth is Y_0 ; we assume that $0 \leq Y_0 < I$ so that the manager must obtain outside funds to finance the project.

A standard Brownian motion $Z = \{Z_t, t \geq 0\}$ drives the cumulative cash flow process, Y_t , for the project. We assume that Y_t evolves according to

$$dY_t = \mu_t dt + \sigma dZ_t, \quad (3)$$

where μ_t is the average value of the project and σ is the project's volatility. We assume that $\mu_t \in (-\infty, \bar{\mu}]$, with its value determined by the principal, as described below.

The manager has the ability to secretly and inefficiently divert cash flows from the project for his or her own consumption. The manager receives dY_t from the project, and chooses to report and pass on $d\hat{Y}_t \leq dY_t$ to the principal. If the manager misreports, he consumes $\alpha (dY_t - d\hat{Y}_t)$, where $\alpha \in (0, 1)$ measures the degree of inefficiency in cash diversion.

The principal's choice of μ_t gives him some ability to determine if $dY_t \neq d\hat{Y}_t$. Specifically, let

$$m_t = \bar{\mu} - \mu_t \tag{4}$$

denote the value loss associated with constraining the manager. This loss can represent either a direct cost of monitoring or a loss in average profitability from imposing rules on the manager and reducing his or her flexibility. Note that $m_t \geq 0$. By choosing to monitor or constrain the manager, the principal gains access to a Poisson process that hits with intensity $\lambda(m_t, dY_t - d\hat{Y}_t)$. We assume that

$$\lambda(0, dY_t - d\hat{Y}_t) = \lambda(m_t, 0) = 0, \tag{5}$$

$$\left. \frac{\partial \lambda(m_t, dY_t - d\hat{Y}_t)}{\partial d\hat{Y}_t} \right|_{d\hat{Y}_t \leq dY_t} = -m_t, \tag{6}$$

In the above (5) implies that the principal must accept lower average profitability to catch any stealing and that the principal never detects stealing if there isn't any; (6) implies that the probability that the principal detects diversion is proportional to the intensity of monitoring and to the magnitude of stealing.

There are three relevant outside values. The principal receives L if the manager is replaced. If replacement (or default) occurs as a result of insufficient performance, the manager receives R , which reflects the manager's outside options. However, the manager suffers more in reputation if replacement occurs because of fraud: in this case the manager receives $R^f < R$, reflecting decreased outside options.⁸

⁸Rigorously, the principal can offer the agent an outside value greater than R^f , and he is not forced to terminate the agent. However, because our monitoring technology does not have false positives, the principal will always choose the maximum punishment allowable.

In this model, the manager's only active decision is whether to divert cash from the project to private consumption. This does not mean that the manager adds no value. The match between the principal and manager improves their joint opportunities, and so the sum of the participants' outside options is less than the continuation value of the project, making termination inefficient:

$$\gamma R + rL < \bar{\mu}. \quad (7)$$

2.2 The Contract

If the manager and principal agree to start the project, the funds needed to start it in the amount of $I - Y_0$ are transferred from the principal to the manager at time 0. The principal commits to the terms of the contract.

Let $N = \{N_t, t \geq 0\}$ be a standard compound Poisson process with time- t intensity $\lambda(m_t, dY_t - d\hat{Y}_t)$, which counts stealing detections. Let $\mathcal{F} = \{\mathcal{F}_t; t \geq 0\}$ be the filtration generated by (Z, N) . Let $\hat{Y} = \{\hat{Y}_t \leq Y_t : t \geq 0\}$ be the manager's report of his income, which is (Z, N) -measurable. We assume that the manager's consumption (payment) process $C = \{dC_t \geq 0 : t \geq 0\}$ and the monitoring process $m = \{m_t \geq 0 : t \geq 0\}$ are (\hat{Y}, N) -measurable.

There are three relevant stopping times. Let τ^q be the time that the manager chooses to quit, after we he receives his outside option of R . Let τ^d be the time at which the contract terminates and the manager receives R . Let τ^f be the time at the principal fires the manager and the manager receives R^f . As the firing of the manager can happen only if he is caught stealing, we require that $\tau^f \geq \min\{t \geq 0 : N_t = 1\}$. τ^q must be (Z, N) -measurable, while τ^d and τ^f are (\hat{Y}, N) -measurable. In addition, we require that C be such that the manager's utility, $E^{\hat{Y}} \left[\int_0^{\tau^d} e^{-\gamma s} dC_s \mid \mathcal{F}_t \right]$ is square-integrable for all \hat{Y} .

The contract is optimal if it maximizes the expected profit of the principal in the class of all contracts that are incentive compatible and provides the manager with his initial expected utility V_0 . More formally,

Definition 1 *The contract $\xi = \{\tau^d, \tau^f, C, m\}$, specifies a termination time of the relation-*

ship $\tau^d \leq \infty$, a firing time for the manager $\tau^f \leq \infty$, the consumption of the manager C , and the monitoring choice m , all as a function of the manager's reports and the observed stealing.

The contract is incentive compatible if the manager wishes to report accurately and does not quit early.

The contract is optimal if it is incentive compatible and maximizes the principal's profit subject to the constraint that the manager receives his expected utility of V_0 .

The definition of the optimal contract implies that it maximizes the total surplus. This will be useful when we implement the contract with securities issued by a firm.

3 The Optimal Contract

In this subsection, we formulate the dynamic moral hazard problem and determine the optimal contract. Methodologically, our approach is based on continuous-time techniques used by DeMarzo and Sannikov (2006), extended to a setting with a monitoring technology. We will discuss the properties of the optimal contract in Section 4.

3.1 The Manager's Problem

We begin by describing the manager's continuation utility. At any time t , we are interested in the value that the manager places on the remainder of the contract. For any reporting strategy and quitting time (\hat{Y}, τ^q) , that value is

$$V_t(\hat{Y}, \tau^q) = E \left[\int_t^{\min(\tau^d, \tau^f, \tau^q)} e^{-\gamma(s-t)} dC_s \right. \\ \left. + e^{-\gamma(\min(\tau^d, \tau^f, \tau^q)-t)} (R - 1_{\tau^f < \min(\tau^d, \tau^q)}(R - R^f)) | \mathcal{F}_t \right]. \quad (8)$$

This equation just means that the manager collects consumption up until the time he quits or the contract is terminated. At that point, the manager collects the outside value given by how the contract ended – in default, after the manager quit, or with fraud and involuntary termination.

Before continuing, define the change in the manager's continuation value when he or she is caught cheating to be

$$\psi_t \leq V_t - R^f, \quad (9)$$

where $\psi_t > 0$ is a punishment/loss. This means that the maximum that the principal can punish the agent for misreporting is $V_t - R^f$.

Then, to understand incentive compatible contracts, we must know how V_t evolves and under what conditions the manager truthfully reports income (does not steal or divert cash). The following proposition summarizes the manager's problem:

Proposition 1 *In any contract, for $t \leq \min(\tau^d, \tau^f, \tau^q)$, the manager's continuation value evolves as*

$$dV_t(\hat{Y}, \tau^q) = \gamma V_t dt - dC_t(\hat{Y}, N) - \psi_t dN_t^{\lambda(m_t, dY_t - d\hat{Y}_t)} + \beta_t (d\hat{Y}_t - \mu_t dt). \quad (10)$$

The contract is incentive compatible if and only if

$$\beta_t \geq \alpha + \psi_t \left. \frac{\partial \lambda(m_t, dY_t - d\hat{Y}_t)}{\partial d\hat{Y}_t} \right|_{d\hat{Y}_t = dY_t} = \alpha - \psi_t m_t. \quad (11)$$

If the manager accurately reports cash flows and does not quit early, his continuation utility evolves as

$$dV_t = \gamma V_t dt - dC_t(Y, N) + \beta_t (dY_t - \mu_t dt) \quad (12)$$

for $0 \leq t \leq \tau^d$.

The first equation (10) describes how the manager's continuation value must evolve over time. The first two terms reflect the fact that the principal must be consistent in his promises: if the principal does not award the manager consumption today, then the manager's future consumption must grow at a rate equal to the manager's discount rate of γ . The next term reflects any punishment the manager receives after being caught stealing. The manager's

expected loss from punishment is equal to the magnitude of punishment, ψ_t , times the probability of being caught, $\lambda(m_t, dY_t - d\hat{Y}_t)$. Thus, the manager's value to stealing an additional dollar is the manager's consumption minus the change in the expected loss from punishment, or $\alpha - \psi_t m_t$. The final term in (10) reflects the manager's reward for accurately reporting the cash flow (not stealing): for every dollar he turns over, the manager receives a portion, β_t , as compensation. Equation (11) shows that in order to induce the manager to act correctly, his marginal reward from accurate reporting must at least equal his marginal benefit from stealing.

When the manager reports accurately, his continuation value simplifies to (12). The Poisson process that detects stealing does not report false positives, and the principal always induces the manager to report accurately in equilibrium. Thus, any punishment is an out-of-equilibrium event.

3.2 The Principal's Problem

Here we provide an intuitive derivation of the profit maximizing way to deliver to the manager his continuation utility of V_t . The proof of Proposition 2 formalizes our discussion below.

Let $F(V)$ be the highest expected payoff the principal can obtain from an incentive compatible contract that provides the manager with utility equal to V . To simplify our discussion we assume that the function F is concave and C^2 .

We start by observing that the principal can always make a lump sum payment to the manager of dC . This moves the manager from a continuation utility of V to $V - dC$. Such a transfer is efficient only if $F(V) \leq F(V - dC) - dC$, and so we have no transfers when

$$F'(V) \geq -1.$$

Define V^c as the lowest value of V such that $F'(V) = -1$. Then, it is optimal to set the manager's consumption so that

$$dC(V_t) = \max(V_t - V^c, 0). \tag{13}$$

These transfers keep the manager's continuation utility below V^c .

In addition, the manager quits if $V_t \leq R$. Thus, as long as the relationship continues, we can assume $V \in (R, V^c]$.

Next, we characterize the optimal choice of process (β_t, ψ_t, m_t) , where β_t determines the sensitivity of the manager's continuation utility to his report, ψ_t determines intensity of punishment after fraud is detected, and m_t determines the intensity of monitoring. Since the contract must be incentive compatible, (β_t, ψ_t, m_t) must satisfy the conditions of Proposition 1.

The manager's continuation utility evolves according to (12), and so Ito's lemma implies

$$dF(V_t) = \gamma V_t F'(V_t) dt + \frac{1}{2} \beta_t^2 \sigma^2 F''(V_t) dt + \beta_t \sigma F'(V_t) dZ_t.$$

Since the principal also receives cash flows of $\bar{\mu} - m_t$ we have a standard Hamilton-Jacobi-Bellman equation:

$$\begin{aligned} rF(V_t) = & \max & & \left[\bar{\mu} - m_t + \gamma V_t F'(V_t) + \frac{1}{2} \beta_t^2 \sigma^2 F''(V_t) \right]. & (14) \\ & \beta_t \geq & \alpha - \psi_t m_t & \\ & \psi_t \leq & V_t - R^f & \\ & m_t \geq & 0 & \end{aligned}$$

Because F is concave, β_t takes the minimum incentive compatible value, and so

$$\beta_t = \alpha - \psi_t m_t.$$

In addition, increasing ψ_t serves only to reduce the total value of β_t . This is because a large punishment allows a small monitoring intensity to be an effective deterrent, and because monitoring never falsely reports fraud that is not present. Thus, the principal sets

$$\psi_t = \psi_t^* = V_t - R^f.$$

and fire the manager immediately following the first fraud detection. This leaves us with the

final Hamilton-Jacobi-Bellman equation

$$rF(V_t) = \max_{m_t \geq 0} \left[\bar{\mu} - m_t + \gamma V_t F'(V_t) + \frac{1}{2} (\alpha - \psi_t^* m_t)^2 \sigma^2 F''(V_t) \right]. \quad (15)$$

Taking the derivative of the right hand side of (15) (RHS), we have

$$\frac{dRHS}{dm_t} = -1 - \psi_t^* (\alpha - \psi_t^* m_t) \sigma^2 F''(V_t).$$

Monitoring is used only when $\left. \frac{dRHS}{dm_t} \right|_{m_t=0} > 0$, which is whenever

$$F''(V_t) < -\frac{1}{\alpha \sigma^2 \psi_t^*}.$$

The optimal choice of m_t satisfies

$$m_t = \frac{1}{\sigma^2 F''(V_t) \psi_t^{*2}} + \frac{\alpha}{\psi_t^*}, \quad (16)$$

whenever that m_t is positive.

By plugging the values for β , ψ and m back into (14), we can fully described the behavior of $F(V)$ on the set $V \in [R, V^c]$. However, we still need boundary conditions. First, we have default after low performance: $F(V_{\tau,d}) = L$.⁹ Second, we have smooth pasting when the manager is paid after success. Thus, $F'(V^c) = -1$. We also have an optimality condition for the undetermined V^c , meaning the second derivatives match at the boundary and so $F''(V^c) = 0$. Plugging in the conditions at V^c gives us

$$rF(V^c) = \bar{\mu} - \gamma V^c.$$

We formalize these findings in the following proposition:

Proposition 2 *The optimal contract that delivers to the manager the value V_0 takes the following form:*

⁹The principal also receives L if he fires the agent, but this never happens in equilibrium.

(i) If $V_t \in [R, V^c]$, V_t evolves as (12), with $\psi_t^* = V_t - R^f$, and $\beta_t = \alpha - \psi_t^* m(V_t)$. In addition,

- when $V_t \in [R, V^c)$, $dC_t = 0$,
- when $V_t = V^c$ the transfer dC_t cause V_t to reflect at V^c .
- If $V_0 > V^c$ an immediate transfer $V_0 - V^c$ is made.

(ii) The optimal intensity of monitoring satisfies:

$$m_t = m(V_t) = \begin{cases} 0 & \text{for } F''(V_t) \geq -\frac{1}{\alpha\sigma^2(V_t - R^f)} \\ \frac{\alpha}{V_t - R^f} + \frac{1}{(V_t - R^f)^2 \sigma^2 F''(V_t)} & \text{for } F''(V_t) < -\frac{1}{\alpha\sigma^2(V_t - R^f)} \end{cases}. \quad (17)$$

(iii) The principal's expected utility at any time t is given by the function $F(V_t)$, which solves

$$rF(V_t) = \bar{\mu} - m(V_t) + \gamma V_t F'(V_t) + \frac{1}{2} [\alpha - (V_t - R^f)m(V_t)]^2 \sigma^2 F''(V_t), \quad (18)$$

for $V_t \in [R, V^c]$. The boundary conditions are $F'(V_t) = -1$ for $V_t \geq V^c$, and $rF(V^c) = \bar{\mu} - \gamma V^c$.

(iv) If $\tau^d < \infty$, then $F(V) = L$, and the relationship is terminated the first time V_t hits R . If the stealing is detected the manager is replaced and receives utility of R^f .

We discuss the properties of this contract in Section 4.

3.3 Hidden Savings

The optimal contract of Sections 2 and 3 is unchanged if the manager is allowed to privately save at any interest rate less than or equal to γ . First, it is weakly inefficient for the manager to save on his private account as any such contract can be improved by having the principal save and make direct transfers to the manager. Therefore, the optimal contract with no private savings yields at least as much utility to the principal as any incentive compatible contract when the manager is allowed to privately save. Second, the risk neutrality of the

manager implies that he or she would prefer not to save because the return on savings is less than the manager's discount rate of γ . In addition, the manager's continuation value grows at rate γ , and so the manager always prefers to save through the principal rather than through private means. Thus, the contract above is fully incentive compatible, even with hidden savings by the manager, and the contract above delivers as much to the principal as it would if savings were allowed.

4 Compensation, Default, and Monitoring

The driving force in the principal's choice of compensation and monitoring is the fact that the principal is effectively risk averse, and this is strictly a result of the dynamic nature of the contract. Even though the principal is risk neutral with respect to consumption, he faces a risk of inefficient default. Without maximal monitoring ($\beta_t > 0$), the manager's continuation value must be continually adjusted in proportion to the size of the shock to cash flow in order to ensure incentive compatibility. Thus, a series of purely random negative shocks to cash flow results in the manager's continuation value falling below his outside option. The manager then simply walks away, and the project is terminated with a loss of efficiency (7). The effective risk aversion of the principal is measured by the concavity of the value function.

If we were to consider only the risk of default, the principal would always prefer to postpone payments to the manager because doing so increases the manager's continuation value from future consumption and thus lowers the likelihood of costly default. However, the manager's discount rate is higher than the principal's discount rate ($\gamma > r$), and so delaying payment to the manager is always inefficient. Thus, the principal faces a trade-off between risk and efficiency and chooses to delay payment to the manager until he is exactly indifferent between the efficiency and risk considerations ($F''(V^c) = 0$ and $F'(V^c) = -1$).

If we were to consider only the risk of default, the principal would prefer to monitor the manager so that pay-for-performance incentives were unnecessary ($\beta_t = 0$). In that case, the manager's continuation utility would not decline and default would be impossible. However, monitoring is costly, and so the principal balances the cost of monitoring against the risk cost of pay-for-performance incentives.

There are several qualitative changes to the contract that result from the addition of monitoring. The first is that default does not always occur in equilibrium. If the principal ever chooses to monitor so intensively that variable pay is zero ($\beta_t = \alpha - \psi_t m_t = 0$), then the manager's continuation utility, which has a positive drift (12), cannot fall below the point of maximal monitoring. Thus, the manager's continuation utility never falls below his outside option, and the project is never terminated. Default is purely optional for the principal. Such an example is illustrated in figure 1.

Next, we discuss how the principal chooses whether to allow default. The two key values are the effectiveness of monitoring and the loss to the principal in default. Default is inefficient, and so the principal is willing to pay a cost to avoid it. However, monitoring is not uniformly effective because its deterrent value depends on how much the manager has to lose. If the principal can punish the manager greatly (R^f is much less than R), either because reputation is important or because the manager has posted some kind of bond, then the principal can effectively deter stealing at only a small cost (11). In contrast, when the manager cannot be effectively punished ($R^f \approx R$), the principal must spend much more to deter stealing. This cost of monitoring may not be worth spending if the loss to default is small.

The following proposition formalizes the intuitions regarding default:

Proposition 3 *There exists L^* such that the optimal contract exhibits default at $V_t = R$ if and only if $L > L^*$. Moreover L^* is non-increasing in R^f .*

Finally, we want to understand the timing of the principal's choice of monitoring. Monitoring is expensive, and so the principal may choose to monitor only when default is near and has an otherwise high probability of occurring. In contrast, punishment is most effective when the manager still has a high continuation value, and so monitoring is least costly when the project is far from default. The principal chooses to pay the manager when the "risk-aversion" with respect to default is zero ($F''(V^c) = 0$). Thus, when the project is farthest from default, the value function is less sensitive to risk and so the principal never chooses to monitor (17). At this point, the desire to delay monitoring is always dominant.

Despite the fact that the principal never monitors when $V_t = V^c$, it is not the case that monitoring must increase monotonically as V declines from V^c to R . When punishment is very effective, the principal may choose maximum monitoring (as in Proposition 3) when $V_t = R$. However, when punishment is not effective, the principal chooses not to monitor near default. That is, the principal may become more effectively risk averse as default becomes more likely, but still choose to monitor less. The reason is that the effectiveness of punishment is declining. One possible result is that the principal chooses to monitor at intermediate levels of V , but not near $V = R$ or near V^c . To see this, consider the case in which $R^f \approx R$. In this case, monitoring is ineffective at $V \approx R$ because the principal cannot reduce the agent's continuation utility any further. Thus, there is no monitoring near default, no matter how much the principal wishes to avoid default.

Similarly, in cases in which the principal's ability to punish is neither very strong nor very weak, the principal may partially monitor the manager near default so that $\alpha > \beta(V_t = R) > 0$.

We illustrate several features of the optimal contract with two different parameter sets in figure 1. That figure contains two solutions with identical parameters, save that the dashed blue solution corresponds to a case of more effective punishment ($R^f = 0$) than the solid green solution ($R^f = 1$). As a result, the dashed blue solution exhibits no default in equilibrium with $\beta(R) = 0$, while the solid green solution has positive monitoring and default in equilibrium with $\beta(R) > 0$.

The two solutions do not have all of the apparently obvious relationships. First, when the principal has a lesser ability to punish the manager ($R^f = 1$), monitoring is less effective on a marginal basis, and yet the principal actually monitors more intensively ($m_{R^f=1}(V) \geq m_{R^f=0}(V)$). Thus, even though money spent on monitoring and the effectiveness of punishment are technological complements when giving incentives – since the manager's expected loss is the product of the two – less effective punishment can be coupled with more intensive monitoring. The principal with the lesser ability to punish has less effective monitoring, and so the value to the contract is lower and he does not prevent default. This means that he is sensitive to the risk of default and is actually more risk averse (more negative $F''(V)$) over most of the range of V in our example, and so he chooses to monitor

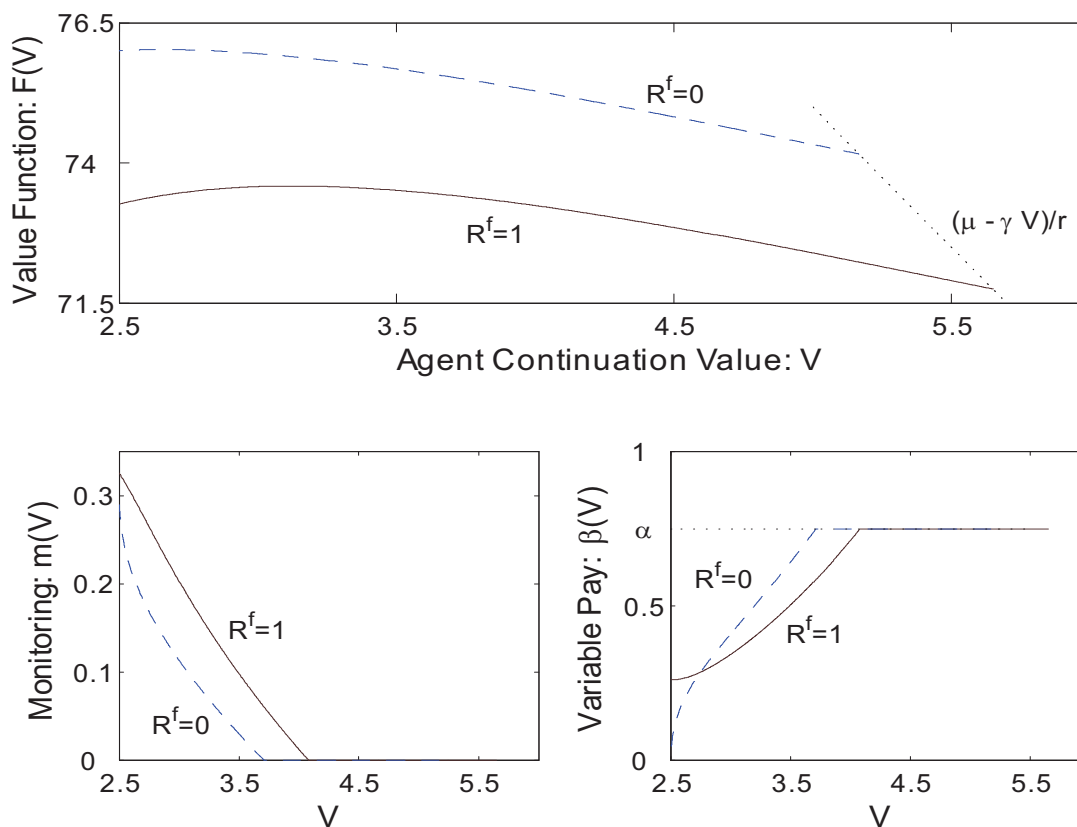


Figure 1: We plot the principal's value function, the level of monitoring, and the level of variable pay, all as a function of the agent's continuation value. For these plots, we set $\mu = 1$, $\sigma = 1$, $R = 2.5$, $r = .01$, $\gamma = .05$, $L = 73.25$, and $\alpha = .75$. The solid brown line uses $R^f = 1$, while the dashed blue line uses $R^f = 0$. The solid brown solution exhibits default in equilibrium, while the dashed blue solution does not.

more. In contrast, the principal with the greater ability to punish is less risk averse over most of the range of V , only becoming very sensitive to risk very near the manager's outside option. At this point, monitoring is still effective and so the principal can cheaply prevent default. This ability to avoid default cheaply is what allows him to be less averse to risk over most of the range of V .

In the next section we show how our contract can be implemented with common securities.

5 An Implementation of the Contract

In this section and the next, we will show how our optimal contract can be implemented using common financial securities. First, in this section, we demonstrate a solution in which all participants, with the exception of the manager, commit to the desired actions. We will describe the optimal actions and the payoffs to the various securities. Second, in the next section, we will show how our solution can be decentralized to a setting with strategic equity issuance. We will describe the incentives of the various parties and show why they are willing to take the desired actions.

5.1 Securities

The implementation uses three securities: common stock, preferred stock, and a line of credit. The manager, who has a partial common equity stake, observes the cash flows from operations and has the ability to divert them to his own consumption. As with the contract, the manager's ability to divert funds is reduced from for many possible agency problems. For example, the manager could misuse corporate assets, undertake less profitable activities with a private benefit, or simply not exert effort.

The central security in the implementation is the line of credit because we use the balance on the line of credit to keep track of the history of cash flows. When the manager does not divert cash flows ($d\hat{Y}_t$), they are turned over to the firm and used to pay off any balance on the line of credit (M_t). The line of credit is also used to pay preferred stock dividends (dD_t^{Pref}) and common stock dividends (dD_t). In addition, the firm can choose to issue or repurchase (common) equity with proceeds to the firm of dEQ_t . We assign the line of credit an interest rate of γ . Thus, the balance on the line of credit evolves as

$$dM_t = \gamma M_t dt + dD_t^{Pref} + dD_t - dEQ_t - d\hat{Y}_t. \quad (19)$$

The line of credit has a maximum (\bar{M} , determined endogenously below), and once the maximum is reached, the firm defaults and the assets are claimed by the holder of the line of credit. The line of credit also comes with certain covenants attached. Those covenants are

a function of the draw on the line of credit (the firm's overall amount of leverage), and they allow the holder of the line of credit to monitor the firm and restrict management's flexibility. If the manager chooses to divert funds, there is a chance that doing so is observed by the line of credit holder through a covenant violation, in which case the holder of the line of credit gains control of the firm's assets. The use of covenants to restrict management's flexibility comes at a cost to the firm of $m_t dt$: less flexibility corresponds to lower average profitability. The chance that diversion triggers a covenant violation and also the take-over of the firm by the line of credit holder is given by a Poisson process with intensity $\lambda(m_t, dY_t - d\hat{Y}_t)$. The properties of λ are given in the contracting model (Equations 5 and 6).

The manager has outside options equivalent in value to R if the firm defaults. However, the manager has his outside options reduced to R^f if he is caught diverting funds. The value of the firm's assets in the hands of the holder of the line of credit is L .

To continue, we need only define the relevant dividend and equity issuance levels and observe that our securities induce the manager to accurately report all dividends:

Proposition 4 *Assume the following cash flows, where the constant V^c and the processes $\mu_t = \bar{\mu} - m^*(V^c - \alpha M_t)$ and $\beta^*(V^c - \alpha M_t)$ are given by the results of Proposition 2:*

- The balance on the **line of credit** follows (19), has a maximum $\bar{M} = \frac{1}{\alpha}(V^c - R)$, and an interest rate γ .
- The **preferred equity** holders receive a dividend flow of $dD_t^{Pref} = \mu_t dt - \frac{\gamma}{\alpha} V^c dt$.
- The **common equity** holders receive a dividend flow dD_t which causes M_t to reflect at zero ($dD_t = \max(-M_t, 0)$).
- **Equity issuance** (and repurchases) are so that $dEQ_t = \left(\frac{\beta_t}{\alpha} - 1\right) (d\hat{Y}_t - \mu_t dt)$.
- The **manager** receives a constant fraction α of all common equity dividends.
- The debt covenants dictate managerial flexibility (the level of **monitoring**) equal to $m^*(V^c - \alpha M_t)$, and diversion triggers a covenant violation with probability $\lambda(m_t, dY_t - d\hat{Y}_t)$.

Then the manager reports cash flows accurately and his continuation value is related to the draw on the line of credit by

$$V_t = V^c - \alpha M_t. \quad (20)$$

5.2 Discussion

To understand why this security setup for the manager is incentive compatible, let us plug in the various cash flows to see how the draw on the line of credit evolves. At this point, we assume that $M_t > 0$, so that there are no common stock dividends. Then, we have

$$\begin{aligned} dM_t &= \overbrace{\gamma M_t dt}^{\text{Interest}} + \overbrace{\mu_t dt - \frac{\gamma}{\alpha} V^c dt}^{\text{Preferred Stock}} - \overbrace{\left(\frac{\beta_t}{\alpha} - 1\right) (d\hat{Y}_t - \mu_t dt)}^{\text{Equity Issuance}} - \overbrace{d\hat{Y}_t}^{\text{Cash Flow}} \\ &= \gamma M_t dt - \frac{\gamma}{\alpha} V^c dt - \frac{\beta_t}{\alpha} (d\hat{Y}_t - \mu_t dt), \end{aligned} \quad (21)$$

which we can compare to the evolution of the manager's continuation value when $dC_t = 0$, given by (12):

$$dV_t = \gamma V_t dt + \beta_t (d\hat{Y}_t - \mu_t dt).$$

The two processes show that if we make the connection in (20), then the law of motion for the draw on the line of credit and for the manager's continuation value coincide. In the optimal contract, the manager's continuation value increased when he reported a positive cash flow, and this moved the manager closer to the consumption point $V = V^c$. In the securities implementation, the draw on the line of credit declines when the manager reports a positive cash flow, and this moves the manager closer to $M_t = 0$, which is when the manager collects his α share of dividends. At the same time, the maximum draw on the line of credit, \bar{M} , forces default exactly when $V_t = R$, which is when the firm would dissolve under the optimal contract. The other parameters of the implementation are set so as to maintain the conformity between the security design and the optimal contract. The preferred stock coupon payments are made so as to absorb enough cash so that the line of credit has the

right incentive properties. Dividends paid to preferred stock match consumption paid to the manager, so that $dC_t = \alpha dD_t$.

Next, notice that the ability of the holder of the line of credit to take over the firm after a covenant violation fulfills the role of punishing the manager after diversion is detected in the optimal contract. In the optimal contract, the manager's continuation value declines by an amount $V_t - R^f$. In the implementation, the holder of the line of credit takes ownership of the firm's assets, and the manager is fired, receiving his outside option of R^f . Both procedures result in the same punishment to the manager for the same amount of diversion.

To complete incentive compatibility, we must understand the interaction of monitoring and equity issuance. In the optimal contract, monitoring is useful because it enables the principal to shift away from the risk of pay-for-performance incentives. More monitoring means that the manager's continuation utility is less volatile, making default less likely and improving the principal's welfare. In the implementation, we have a similar mechanism. Let us consider a time in which the firm is using partial monitoring, so that $\beta = \frac{1}{2}\alpha$. When the manager reports dY_t to the firm, the line of credit would adjust by the full amount, absent any equity issuance. To keep the manager's continuation value from changing by the full amount, we require that the line of credit does not reflect the full gain or loss from dY_t . Thus, the firm raises funds by selling equity or spends funds by re-purchasing equity in order to partially offset the cash flow from operations. If $\beta = \frac{1}{2}\alpha$, then equity issuance is $-\frac{1}{2} \left(d\hat{Y}_t - \mu_t dt \right)$, meaning that the firm offsets half of the surprise cash flow from operations with equity issuance or re-purchases. Monitoring allows the firm to raise capital without destroying incentives. This ability allows the firm's financial variables (in this case, M_t) to be less volatile, and so it increases the value of the firm as a whole.

Equity issuance has an important feature in this model: it is *conditional*. The firm decides on an equity issuance policy, summarized by β , and then observes cash flows to see whether to issue or re-purchase stock. This policy maintains incentive compatibility, and it has the property that as the firm loses money and enters financial distress, it issues shares. Similarly, it repurchases shares as it exits distress.

One might also consider apparently similar policies in which the firm issues shares once in distress to avoid bankruptcy. Let us consider an unconditional policy that would have the

firm issue stock worth X on the first occasion in which $M_t = \bar{M}$, thus avoiding bankruptcy. Then, the manager faces the same situation with unconditional stock issuance as he faces if the firm has a line of credit with a maximum of $\bar{M} + X$ and does not issue stock. This is not incentive compatible. To see why, let us calculate the managers total gains from stealing the entire line of credit:

$$\begin{array}{rcc}
 \text{Consumption directly from the line of credit} & & \text{Outside option} \\
 \underbrace{\alpha (\bar{M} + X)} & + & \underbrace{R} \\
 \text{Consumption from equilibrium equity issuance} & & \text{Expected loss from being caught stealing} \\
 + \alpha \int_0^{\bar{M}+X} \left(1 - \frac{\beta(M)}{\alpha}\right) dM & - & \int_0^{\bar{M}+X} \psi(M)m(M)dM
 \end{array}$$

Incentive compatibility for monitoring and equity issuance implies that the last two terms sum to zero (since $\beta_t = \alpha - \psi_t m_t$). Thus, the manager's total gain from stealing is $\alpha (\bar{M} + X) + R = \alpha \times \left(\frac{1}{\alpha} (V^c - R) + X\right) + R = V^c + \alpha X$, which is greater than the maximum continuation value if he operates the firm legitimately (V^c). Since the firm defaults only after more losses than the optimal contract would allow, the manager has an increased incentive to steal so as to realize those "extra losses" as consumption.

Unconditional stock re-purchases also destroy value. They are equivalent to reducing the maximum draw on the line of credit, and they result in the firm defaulting too soon. Since default destroys value, it is optimal to delay it as long as incentives allow.

Lastly, the value of preferred stock coupons fall as debt covenants restrict the flexibility of the manager. This means that the firm is able to preserve capital by not paying coupons exactly when the firm is most constrained by the presence of debt covenants.¹⁰

Next, we show that the various policies we outlined in this section are actually optimal policies in a setting with strategic players.

¹⁰In our current implementation, preferred stock is paid all of the "left-over" cash flows from the firm that are not allocated to common equity or the line of credit. These cash flows match most closely those of preferred stock because they decline in value as the firm experiences losses. However, one could lower the preferred stock coupon by an amount X and pay that amount to long term debt holders in perpetuity (until default). Thus, our firm has an unexploited capacity for long term debt. We leave this out of our implementation to preserve the simplicity of the exposition.

6 An Implementation with Strategic Equity Issuance

6.1 Decision Rights

In this section, we will outline a decentralized setting in which the policies in section 5 are optimal. To do that, we must allocate decision rights. We preserve the manager's ability to divert cash flows and add that

- The manager decides the firm's dividend policy.
- The common stockholders decide the firm's equity issuance policy.
- The parameters of the line of credit, including covenants, are decided when the line of credit is first formed.

The manager's ability to decide the firm's dividend policy simply gives the manager another way to "steal" from the firm by mis-timing dividends. Previously, in order to steal, the manager would report negative cash flows, and consume an α fraction of those flows, net of equity issuance and monitoring. Now, the manager can simply issue common stock dividends, of which the manager receives a fraction α , and bypass both the equity issuance and the monitoring. We could also allow the manager to choose the equity issuance policy, as long as either 1) the common stockholders could veto anything that is not incentive compatible, or 2) the market would refuse to buy securities from a firm that has a non-incentive compatible equity issuance policy.¹¹

We will treat the market as a continuum of risk neutral investors with discount rate r . Thus, we assume that the market values equity and debt strictly as the expected present value of all future cash flows. With equity, we have to account for the changing number of shares and the possible cash flow from issuance and re-purchasing activity, in addition to dividends. Since the market is made up of a continuum of risk neutral investors, we have

¹¹The second reason is more intuitively relevant since non-incentive compatible policies deliver diversion and an equity value of zero. However, we would like to keep our game simple to avoid as much as possible the need to forecast strategic expectations of a market player. Thus, we will avoid the issue by giving the common stockholder's decision rights over equity issuance. This also allows us to analyze the common corporate finance intuitions of debt overhang and asset substitution in our setting.

perfect aggregation, and we can look at the cash flows associated with equity holders as a whole, rather than the cash flows associated with a particular share of stock. We will treat the problem of a continuum of risk neutral investors as the problem of one large price-taking risk neutral investor with whom the firm transacts to issue and re-purchase equity.

We also need to add a decision space for equity issuance:

Assumption 1 *Equity issuance and re-purchases (dEQ_t) can be decomposed into*

$$dEQ_t = d\Delta_t + \delta_t \sigma dZ_t \tag{22}$$

where Δ_t and δ_t are predictable, integrable, Z -adapted processes.

This assumption makes equity issuance a semi-martingale, a very general class of integrable processes.¹² In this setting, Δ_t corresponds to unconditional equity issuance, while δ_t measures conditional equity issuance. The equity issuance policy amounts to the choice of $\{\Delta_t, \delta_t\}$.

For our description to be complete, we need an additional boundary condition. With default, we need to divide up the principal's default value between common equity, preferred equity, and the line of credit. To be consistent with the absolute priority rule, we assume that the line of credit has the senior claim and receives L in default.¹³

Assumption 2 *Default occurs with positive probability in the optimal contract. The value of common and preferred equity in default is zero and the value of the line of credit in default is L .*

This is also the empirically relevant case because default is a possibility for most companies.

¹²For a more detailed explanation of semimartingales, including why they are “the largest possible class with respect to which one may ‘reasonably’ integrate all bounded predictable processes”, please see Jacod and Shiryaev (2003). For example, all integrable predictable processes and integrable Lévy processes are semi-martingales.

¹³When monitoring is so intense that there is no default, then the value of equity is not tied down by an outside value. In that case, we have a degree of freedom relating to the sharing rule between the line of credit and equity. If we denote the value function for the line of credit as $H(M_t)$ and the value function for common equity as $G(M_t)$, then in the case of no default, we have $\frac{1}{1-\alpha}G(M_t) + H(M_t) = \frac{\gamma}{r\alpha}V^c$. This result comes from the fact that a $1 - \alpha$ fraction of every dividend paid out of the line of credit is received by the equity holders in perpetuity and that $\frac{\gamma}{\alpha}V^c$ is what is left after the preferred stock receives its dividend.

Next, we show that the various policies we outlined in our centralized implementation are actually optimal policies in a decentralized setting.

6.2 Equilibrium

We can now demonstrate why the manager and equity holders take the desired actions. To that end, we provide a lemma regarding the ability to negotiate the terms of the line of credit and preferred stock:

Lemma 1 *The desired terms for the line of credit and preferred stock can be agreed to when the firm is founded.*

This lemma is simply a result of the fact that the debt contract is a commitment, and the debt market is competitive, so the firm-value-maximizing debt contract can always be agreed on.¹⁴ This is important because it means that the equity holders take the flexibility implied by debt covenants as given.

Next, we show that the manager and the equity holders choose the equilibrium and incentive compatible level of equity issuance:

Proposition 5 *There is an equilibrium in which common equity holders choose the optimal level of equity issuance, the manager chooses not to steal, and the manager chooses to pay out the optimal level of dividends. This equilibrium implements the optimal contract. In this equilibrium, the value of common outside equity is concave in the draw on the line of credit.*

The key players in this equilibrium are the common equity holders because they are the ones that set incentives and determine firm value through the level of equity issuance. The equity holders behave correctly because the value of common equity is concave in the draw on the line of credit. This means that they prefer the minimum incentive compatible level of volatility on the line of credit, as prescribed by the optimal contract. This concavity is surprising because equity – through the optimal dividend policy – appears to have an option-like cash flow. Whenever the company enjoys enough success that the draw on the line of

¹⁴The value of the line of credit, and hence how much money is raised by the firm in agreeing to the specific terms of the line of credit, depends on the equity issuance policy that the firm will later enact.

credit hits zero, equity holders are paid a fraction of any continuing positive cash flow. This means that successes result in payout, but losses are absorbed by the line of credit. However, equity does not have just one option on the draw on the line of credit (or the value of the firm). Instead, they have a sequence of options on the value of the firm – if equity is not paid a dividend in this period, they can still be paid a dividend in the future. As continuing losses are absorbed by the line of credit, more and more successes are necessary before any dividends are paid, and so those dividends move farther and farther into the future. As a result, the value of equity declines. If the firm experiences enough losses, the line of credit is eventually drawn to its maximum, and the firm defaults. This creates an equity-specific loss due to bankruptcy: the loss of any future options. To avoid this loss, equity holders prefer to avoid excessive risk.

We plot the market values of the three assets in figure 2. As stated in Proposition 5, the value of common equity is concave. In addition, the value of the line of credit is strictly increasing in the draw on the line of credit. This is not true in all economies, and it does not mean that the holder of the line of credit prefers to lend as much money to the firm as possible or to drive the firm into bankruptcy. The value of the line of credit is a continuation (future) value, and it does not take into account the cost to obtaining the future payments. For example, if the slope of the value function is .9, then for every dollar the line of credit lends to the firm, the holder of that line loses \$1 in cash and gains \$.9 in value. As long as the slope is between zero and one, the holder of the line of credit simultaneously loses money on new loans and is also willing to enforce bankruptcy provisions (moving to default without lending additional money).

The concavity intuition runs directly counter to the commonly held belief that equity holders would prefer to engage in asset substitution. This is because the asset substitution intuition is a static intuition: gains from risk are captured by equity holders in continued operation while losses are captured by debt holders in bankruptcy. In a dynamic model, equity holders lose in bankruptcy as well because they lose any possibility of future dividends. In fact, in our model, equity holders could easily choose to increase the volatility of the firm's value: they could simply reverse the optimal equity issuance policy and issue equity after a gain and re-purchase equity after a loss. The result would be that the volatility of the draw

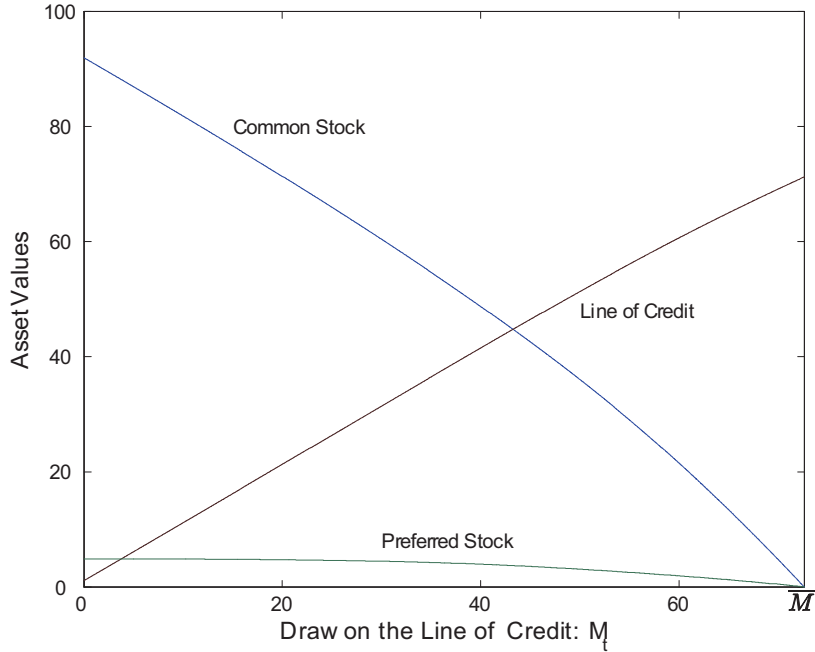


Figure 2: We plot the value of common stock, preferred stock, and the line of credit, all as a function of the draw on the line of credit. For these plots, we set $\mu = 1$, $\sigma = 5$, $R = 10$, $r = .01$, $\gamma = .011$, $L = 71.23$, and $\alpha = .75$. To make the plot more instructive, we made the cost of monitoring equal to $10 \times m_t$. With these values, the line of credit is risky, with a small loss in default, while preferred stock and common stock have no value in default.

on the line of credit would be higher than the volatility of the firm's cash flows. As we have shown, this is a value-destroying policy for the firm as a whole and for equity alone.

Our conditional equity issuance policy runs counter to the debt overhang intuition. The conditional issuance policy sells shares after a loss and re-purchases them after a gain. This means that after a loss that makes bankruptcy more likely, equity holders agree to transfer funds to debt holders. In common static models, this does not make sense because the limited liability implies that equity holders would like to leave as many of the losses with the debt holders as possible. Even if such a transfer were efficient for the firm, equity holders would never agree. In our model, however, this is part of the monitoring mechanism. Because of debt covenants already in place, the equity holders agree to the *conditional* transfers implied by the equity issuance policy.

In addition, our dynamic valuation results run directly counter to the common intuition regarding debt overhang. In our model, equity issuance is used to pay down debt and is thus a direct transfer to debt holders. Yet, absent any incentive compatibility concerns, equity holders would often prefer to unconditionally issue stock (set $d\Delta_t > 0$). This is because when bankruptcy is close enough, equity holders are very sensitive to the potential loss of their future dividends, and so money inside the firm becomes more valuable to equity holders than money outside the firm.¹⁵ The fact that any money raised is directly transferred to debt holders is still true, but the value to avoiding bankruptcy and to continuing the operation of the firm with the potential for future dividends is worth the risk that the firm will eventually default anyway. The reason that equity holders do not raise funds through an unconditional stock issuance is that it removes incentives for the manager to report accurately, as shown in Section 5.

Finally, because the equity issuance policy is combined with monitoring, it does not weaken incentives, and so the manager does not steal. Taken together, we see that distressed equity issuance is optimal.

7 Conclusion

In this paper, we have presented a model of optimal contracting in which the principal has access to two incentive technologies: pay-for-performance incentives and direct monitoring. We have shown how to implement that contract using equity and debt. Debt covenants implement monitoring choices while equity issuance implements changes in performance sensitivity. The key theoretical link is the one between monitoring and covenants, and it implies the ability and desire to issue distressed equity when a firm has a cash shortfall. Thus, we have contributed a parsimonious model of dynamic monitoring, an explanation for patterns in debt covenants, and an explanation for distressed equity issuance.

Our paper is consistent with broad financing and compensation patterns:

¹⁵Let $G(M_t)$ be the value of equity as a function of the draw on the line of credit. Dividend policy implies that $G'(0) = -(1 - \alpha)$, and Proposition 5 implies that $G'''(M_t) < 0$. In many solutions, there exists an M^* such that $G'(M^*) = -1$. In those solutions, the common equity holders would prefer to pay down the line of credit with their own funds.

- **Covenants:** covenants are frequently based on leverage ratios and outstanding debt (Chava and Roberts (2008) and Nini, Smith and Sufi (2008)), covenants are ubiquitous and violations are common (Dichev and Skinner (2002)), covenant violations have a strong effect on the amount and terms of debt financing (Roberts and Sufi (2008a) and Roberts and Sufi (2008b)), and covenants are used more frequently and have a larger impact on corporate policies when agency problems are strongest (Chava and Roberts (2008) and Chava, Kumar, and Warga (2008)).
- **Equity Issuance:** We have also shown the link between leverage, monitoring, and incentives can explain distressed equity issuance, consistent with the findings of DeAngelo, DeAngelo, and Stulz (2008).
- **Compensation:** More leverage, conditional on industry and project type, should result in a lower pay-for-performance sensitivity for managers (consistent with the empirical results of Gilson and Vetsuypens (1993) and Ortiz-Molina (2006)).

In addition, we make several new predictions (based on propositions 2, 3, and 4 and the associated discussions), that are untested to our knowledge:

- **Equity Issuance/Repurchase:** Firms that issue equity in distress should repurchase it (or pay special dividends) as they recover¹⁶, with firms that issued more also repurchasing more. Equity issuance in distress should be higher for firms with higher leverage and more restrictive covenants.
- **Covenants:** Lower recovery rates in default should lead to more restrictive covenants, a lower likelihood of actual default (as opposed to technical default), and more frequent re-capitalizations.

¹⁶Recall here that we use distress to mean the danger of bankruptcy, so this prediction does not apply to firms leaving chapter 11.

A Proofs

A.1 Proof of Proposition 1

First, we will find a convenient state space for the recursive representation of this problem. For this purpose, define the manager's total expected utility received under the contract ξ conditional on his information at time t , from consumption and termination utility, if the manager reports truthfully and does not quit early ($\tau^q = \tau^d$):

$$V_t(Y) = E^Y \left[\int_0^{\min(\tau^d, \tau^f)} e^{-\gamma t} dC_t + e^{-\gamma \min(\tau^d, \tau^f)} (R - 1_{\tau^f < \tau^d} (R - R^f)) | \mathcal{F}_t \right].$$

The process $V = \{V_t, \mathcal{F}_t; 0 \leq t < \tau\}$ is a square-integrable F_t -martingale because we assumed $E^{\hat{Y}} \left[\int_0^{\tau^d} e^{-\gamma s} dC_s | \mathcal{F}_t \right]$ is square-integrable. Then, by the martingale representation theorem for Lévy processes, we have that there exists \mathcal{F}_t -predictable processes $(\beta, \psi) = \{(\beta_t, \psi_t); 0 \leq t \leq \min(\tau^d, \tau^f)\}$ such that

$$\begin{aligned} V_t(Y) &= V_0 + \int_0^t e^{-\gamma s} \beta_s \sigma dZ_s - \int_0^t e^{-\gamma s} \psi_s (dN_s^\lambda - \lambda(m_t, d\hat{Y}_t - dY_t = 0) ds) \\ &= V_0 + \int_0^t e^{-\gamma s} \beta_s \sigma dZ_s \end{aligned}$$

The jump integral is zero under truth telling (5), and so $\tau^f = \infty$.

When the manager chooses any report \hat{Y} and decides to quite at any time $\tau^q \leq \tau^d$, he receives the expected utility, $V_0(\hat{Y})$, which equals

$$\begin{aligned} V_0(\hat{Y}) &= E[V_{\min(\tau^d, \tau^q, \tau^f)}(\hat{Y})] \\ &= V_0 + E \left[\int_0^{\min(\tau^d, \tau^f, \tau^q)} e^{-\gamma t} \left((\alpha - \beta_t) (dY_t - d\hat{Y}_t) - \psi_t dN_t(m_t, dY_t - d\hat{Y}_t) \right) | \mathcal{F}_0 \right]. \end{aligned}$$

Now let's define the manager's continuation utility at time t for a given reporting strategy \hat{Y} and quitting time $\tau^q \leq \tau^d$ as

$$\begin{aligned} V_t(\hat{Y}, \tau^q) &= E \left[\int_t^{\min(\tau^d, \tau^f, \tau^q)} e^{-\gamma(s-t)} dC_s \right. \\ &\quad \left. + e^{-\gamma(\min(\tau^d, \tau^f, \tau^q)-t)} (R - 1_{\min(\tau^d, \tau^q) < \tau^f} (R - R^f)) | \mathcal{F}_t \right]. \end{aligned}$$

It follows that for $t \leq \min(\tau^d, \tau^f, \tau^q)$ we have

$$V_{\min(\tau^d, \tau^f, \tau^q)}(\hat{Y}, \tau^q) = \int_0^{\min(\tau^d, \tau^f, \tau^q)} e^{-\gamma s} dC_s + e^{-\gamma t} V_t.$$

which, together with the martingale representation for V , implies the following law of motion

of the manager's continuation utility until time $t \leq \min(\tau^d, \tau^f, \tau^q)$:

$$dV_t(\hat{Y}, \tau^q) = \gamma V_t dt - dC_t(\hat{Y}) + \beta_t(d\hat{Y}_t - (\bar{\mu} - m_t)dt) - \psi_t dN_t(m_t, dY_t - d\hat{Y}_t). \quad (23)$$

When the manager reports truthfully, we have

$$dV_t = \gamma V_t dt - dC_t + \beta_t(dY_t - (\bar{\mu} - m_t)dt). \quad (24)$$

Representation (23) leads us to the formulation of incentive compatibility: It is incentive compatible for the manager to report truthfully ($d\hat{Y}_t = dY_t$) and not quit early ($\tau^q = \tau^d$) if and only if

$$\beta_t \geq \alpha + \psi_t \frac{\partial \lambda(m_t, dY_t - d\hat{Y}_t)}{\partial d\hat{Y}_t} \Bigg|_{d\hat{Y}_t = dY_t} = \alpha - \psi_t m_t, \quad (25)$$

for all \hat{Y} and $0 \leq t \leq \min(\tau^d, \tau^f)$.

If the manager steals $dY_t - d\hat{Y}_t$ at time t , he gains immediate utility from consuming of $\alpha(dY_t - d\hat{Y}_t)$ and changes his future expected payoff by

$$\beta_t(d\hat{Y}_t - dY_t) - E_t(\psi_t dN_t(m_t, dY_t - d\hat{Y}_t)) = \beta_t(d\hat{Y}_t - dY_t) - \psi_t \lambda(m_t, dY_t - d\hat{Y}_t).$$

The total value of stealing is thus equal to

$$(\alpha - \beta_t) (dY_t - d\hat{Y}_t) - \psi_t \lambda(m_t, dY_t - d\hat{Y}_t).$$

If (25) does not hold, we have

$$(\beta_t - \alpha) - \psi_t \frac{\partial \lambda(m_t, dY_t - d\hat{Y}_t)}{\partial d\hat{Y}_t} \Bigg|_{d\hat{Y}_t = dY_t} < 0$$

and so the manager benefits from decreasing his report from dY_t . Conversely, if (25) holds, given (5)-(6), we have

$$\beta_t - \alpha - \psi_t \frac{\partial \lambda(m_t, dY_t - d\hat{Y}_t)}{\partial d\hat{Y}_t} \Bigg|_{d\hat{Y}_t < dY_t} \geq 0,$$

and so it is optimal for a manager to report $d\hat{Y}_t = dY_t$.

To address the quitting time, observe that if the manager truthfully reports, then the definition of incentive compatibility implies that if the manager does not quit early ($\tau^q = \tau^d$) he has continuation utility of at least R , which what he would get by quitting early. \square

A.2 Proof of Proposition 2

Let F be a C^2 function (in V) that solves the differential equation from Proposition 2. We start by showing that the function F is strictly concave in V over $[R, V^c]$.

Lemma 2 *The function F is strictly concave over $[R, V^c]$.*

Proof Let $\bar{V}^m = \sup\{V_t \in [R, V^c] : F''(V_t) < -\frac{1}{\alpha\sigma^2(V_t - R^f)}\}$. We first start by showing the strict concavity of the function F over $[\bar{V}^m, V^c]$. We observe that $m(V_t) = 0$ over $[\bar{V}^m, V^c]$ and thus we have

$$rF(V) = \bar{\mu} + \gamma VF'(V) + \frac{1}{2}\alpha^2\sigma^2 F''(V). \quad (26)$$

Taking the derivative with respect to V , we obtain

$$rF'(V) = \gamma VF''(V) + \gamma F'(V) + \frac{1}{2}\alpha^2\sigma^2 F'''(V).$$

Since $F''(V^c) = 0$ and $F'(V^c) = -1$, we obtain

$$0 < (\gamma - r) = \frac{1}{2}\alpha^2\sigma^2 F'''(V^c),$$

and there exists an ϵ so that $F''(V) < 0$ on $[V^c - \epsilon, V^c]$. Next, (26) implies

$$\frac{1}{2}\alpha^2\sigma^2 F''(V) = rF(V) - \bar{\mu} - \gamma VF'(V).$$

So, $F'(V) > -1$ and $rF(V) < \bar{\mu} - \gamma V$ are sufficient for $F''(V) < 0$.

Observe that if $F''(\text{all } V \in (\hat{V}, V^c - \frac{\epsilon}{2})) < 0$, then $F'(\hat{V}) > -1$ (since $F'(V^c) = -1$).

Second, observe that if $F'(\text{all } V \in (\hat{V}, V^c - \frac{\epsilon}{2})) > -1$, then $rF(\hat{V}) < \bar{\mu} - \gamma\hat{V}$ (since $F(V^c) = \bar{\mu} - \gamma V^c$). Since $F''(V) < 0$ and $F'(V) > -1$ on the set $[V^1 - \epsilon, V^1]$, then $F''(V) < 0$ and $F'(V) > -1$ on the set $[\bar{V}^m, V^c]$ follows by induction. Specifically, define $\tilde{V} = \sup\{V | F''(V) \geq 0 \text{ or } F'(V) \leq -1\}$, and assume such a point exists in $[\bar{V}^m, V^c - \epsilon]$. Since $F''(V) < 0$ and $F'(V) > -1$ on $(\tilde{V}, V^c - \frac{\epsilon}{2}]$, we have a contradiction.

If $\bar{V}^m = R$, that is there is no monitoring at the optimal contract the argument is complete.

If $\bar{V}^m > R$, then continuity there exists $\epsilon' > 0$ such that F is strictly concave over $[\bar{V}^m - \epsilon', \bar{V}^m]$.

Note also that by definition, a function solving the ODE in the monitoring region satisfies

$$F''(V_t) < -\frac{1}{\alpha\sigma^2(V_t - R^f)}.$$

Since $V_t \geq R > R^f$, we have that the solution is strictly concave in the monitoring region. Once we exit the monitoring region, then by continuity the solution is strictly concave on

the left-hand side boundary of the monitoring region \underline{V}^m . Reapplying successively the above argument establishes strict concavity of F for all $V \in [R, V^1]$. ■

The remainder of the proof proceeds as any standard verification theorem. For any incentive compatible contract, we define

$$G_t = \int_0^t e^{-rs} (dY_s - dC_s) + e^{-rt} F(V_t),$$

where V_t evolves according to (12). The process G is such that G_t is F_t -measurable and

$$e^{rt} dG_t = dY_t - dC_t + dF(V_t) - rF(V_t).$$

From Ito's lemma using (24) we have

$$dF(V_t) = (\gamma V_t dt - dC_t) F'(V_t) + \frac{1}{2} \beta_t^2 \sigma^2 F''(V_t) dt + \beta_t \sigma F'(V_t) dZ_t, \quad (27)$$

and $dY_t = \mu dt + \sigma dZ_t$. Thus,

$$\begin{aligned} e^{rt} dG_t &= \left[\bar{\mu} - m_t + \gamma V_t F'(V_t) + \frac{1}{2} \beta_t^2 \sigma^2 F''(V_t) \right] dt \\ &\quad - rF(V_t) - (1 + F'(V_t)) dC_t + (\sigma + \beta_t \sigma F'(V_t)) dZ_t \end{aligned}$$

where we remember that the incentive compatibility is equivalent to $\beta_t \geq \alpha + \psi_t m_t$, where $\psi_t \geq R^f - V_t$.

Substituting from (18) for $rF(V_t)$ in the above yields

$$\begin{aligned} e^{rt} dG_t &= \left[m(V_t) - m_t + \frac{1}{2} \sigma^2 \left(\beta_t^2 - [\alpha + (R^f - V_t)m(V_t)]^2 \right) F''(V_t) \right] dt \\ &\quad - (1 + F'(V_t)) dC_t + (\sigma + \beta_t \sigma F'(V_t)) dZ_t, \end{aligned}$$

whenever $V \in [R, V^c]$.

Observe that for any policies, we have

$$\left[m(V_t) - m_t + \frac{1}{2} \sigma^2 \left(\beta_t^2 - [\alpha + (R^f - V_t)m(V_t)]^2 \right) F''(V_t) \right] \leq 0$$

and

$$-(1 + F'(V_t)) dC_t \leq 0$$

with equality for the optimal policies. The first equation comes from the specification of $m(V_t)$ and the second comes from $F' \geq -1$ and $dC_t \geq 0$. Thus, we have

$$F(V_0) = G_0 \geq \mathbb{E} [G_{\min(\tau^d, \tau^f)}]$$

where $G_\infty = \int_0^\infty e^{-rs} (dY_s - dC_s)$ since $\lim_{t \rightarrow \infty} e^{-rt} F(V_t) = 0$. Since we have equality for the desired optimal policies, any other policies deliver less in gains to the principal. □

A.3 Proof of Proposition 3

We start by observing that for sufficiently high L , the optimal contract features a default in equilibrium (in particular for $L \geq \frac{\bar{\mu}}{r}$ an immediate termination of the project would be optimal).

Next, suppose that we have a solution that exhibits no default. It must be the case that $F_L(R) \geq L$, where F_L denotes the principal's value function in a problem with liquidation value of L . Consider a problem with a new $L' < L$. If there is default, we would have $F_{L'}(R) = L'$. This cannot be optimal because the principal could simply choose the F_L contract to obtain $F_{L'}(R) = F_L(R) \geq L > L'$. Therefore, if there exists L such that the optimal contract has no default, then there is no default for any project with $L' < L$.

To complete the proof we need to show that the optimal contract has no default for sufficiently small L . First, we show that for sufficiently small L there is a positive amount of monitoring in the neighborhood of R . To see this suppose that the optimal contract implies no monitoring in the neighborhood of R . Note that this implies $F(R) = L$. From the ODE describing the principal's value function, we have that

$$F''(R) = \frac{2(rL - \bar{\mu} - \gamma R F'(R))}{\alpha^2 \sigma^2} \leq \frac{2(rL - \bar{\mu} + \gamma R)}{\alpha^2 \sigma^2},$$

where the inequality follows from the fact that $F'(V) \geq -1$. If we assume $L < \frac{\bar{\mu} - \frac{1}{2(R-R^f)} - \gamma R}{r}$, then we have

$$F''(R) = \frac{2(rL - \bar{\mu} - \gamma R F'(R))}{\alpha^2 \sigma^2} < -\frac{1}{\alpha^2 \sigma^2 (R - R^f)},$$

which contradicts the assumption of no monitoring in the neighborhood of R .

Next, consider the HJB Equation which determines m_t (17):

$$0 = \max_{m_t \geq 0} \left[-rF(V_t) + \bar{\mu} - m_t + \gamma V_t F'(V_t) + \frac{1}{2} (\alpha - \psi_t m_t)^2 \sigma^2 F''(V_t) \right]$$

Solving for the first order condition for m_t , plugging it back in, and solving for $F''(V_t)$ yields

$$F''(V_t) = -\frac{1}{2\sigma^2 \psi_t (\alpha - \gamma V_t \psi_t F'(V_t) - \psi_t (\bar{\mu} - rF(V_t)))} \quad (28)$$

for the region in which $m_t > 0$. Since $F(V_t)$ is concave and since $\psi_t = V_t - R^f > 0$, we have

$$0 \geq -\frac{\alpha}{\psi_t} + \gamma V_t F'(V_t) + \bar{\mu} - rF(V_t)$$

Since $F'(V_t) \geq -1$, we also have

$$-\frac{\alpha}{\psi_t} + \gamma V_t F'(V_t) + \bar{\mu} - rF(V_t) \geq -\frac{\alpha}{\psi_t} - \gamma V_t + \bar{\mu} - rF(V_t)$$

Combining the first and second inequalities and solving for $F(V_t)$, we have

$$F(V_t) \geq \frac{\bar{\mu}}{r} - \frac{\alpha}{r\psi_t} - \frac{\gamma}{r}V_t = \frac{\bar{\mu}}{r} - \frac{\alpha}{r(V_t - R^f)} - \frac{\gamma}{r}V_t$$

Consider $V_t = R$. If

$$L < \frac{\bar{\mu}}{r} - \frac{\alpha}{r(R - R^f)} - \frac{\gamma}{r}R$$

then it must be the case that $F(R) > L$, and so we must have a solution with no default. It also directly follows from above that the default cutoff value L^* is non-increasing in R^f (since the principal must be weakly better off as R^f increases). \square

A.4 Proof of Proposition 4

Let \hat{Y} be a manager's reporting strategy under the proposed implementation. Define the process \tilde{V} as

$$\begin{aligned} \tilde{V}_0 &= V^c - \alpha M_0, \\ d\tilde{V}_t &= -\alpha dM_t + (R^f - \tilde{V}_t)dN_t^{\lambda(m(\tilde{V}_t), dY_t - d\hat{Y}_t)} \end{aligned} \quad (29)$$

where M_t is a credit line balance, which evolves according to (19) provided that $M_t \leq \bar{M}$, as defined in Proposition 4. It follows from (19), (29), and the definition of securities in Proposition 4 that

$$\begin{aligned} d\tilde{V}_t(\hat{Y}) &= \gamma\tilde{V}_t dt + \left[\alpha - (\tilde{V}_t - R^f)m(\tilde{V}_t) \right] (d\hat{Y}_t - (\bar{\mu} - m_t)dt) \\ &\quad - (\tilde{V}_t - R^f)dN_t^{\lambda(m(\tilde{V}_t), dY_t - d\hat{Y}_t)} - \alpha dD_t, \end{aligned} \quad (30)$$

For $t \leq \min(\tau^d, \tau^f)$ define a process V as

$$V_t = \int_0^t e^{-\gamma s} dC_s + e^{-\gamma t} \tilde{V}_t, \quad (31)$$

which evolves as

$$e^{\gamma t} dV_t = dC_t + d\tilde{V}_t - \gamma\tilde{V}_t dt.$$

Noting that $dC_t = \alpha[dY_t - d\hat{Y}_t] + \alpha dD_t$, (30) and (31) yield

$$e^{\gamma t} dV_t = \alpha[dY_t - d\hat{Y}_t] + \left[\alpha - (\tilde{V}_t - R^f)m(\tilde{V}_t) \right] (d\hat{Y}_t - (\bar{\mu} - m_t)dt) - (\tilde{V}_t - R^f)dN_t^{\lambda(m(\tilde{V}_t), dY_t - d\hat{Y}_t)}.$$

It follows from the above (and the fact that $dY_t - d\hat{Y}_t \geq 0$) that the process V is a supermartingale up to time $\min(\tau^d, \tau^f)$ where $\tau^d = \inf \{t : \tilde{V}_t = V^c\} = \inf \{t : M_t = \bar{M}\}$. Using this and the fact that by definition $\tilde{V}_{\tau^d} = R$, we have that for any reporting strategy of the

manager, \hat{Y} ,

$$\begin{aligned} R + \alpha[\bar{M} - M_0] &= V_0 = \tilde{V}_0 \geq E^{\hat{Y}} [V_{\min(\tau^d, \tau^f)}] \\ &= E^{\hat{Y}} \left[\int_0^{\min(\tau^d, \tau^f)} e^{-\gamma s} dC_s + e^{-\gamma \min(\tau, \tau^d)} (R - 1_{\tau^f < \tau^d} (R - R^f)) \right]. \end{aligned} \quad (32)$$

The right-hand-side of (32) represents the expected utility of the manager under any reporting \hat{Y} , given the terms of the implementation. This utility is bounded by $R + \alpha(\bar{M} - M_0)$, where M_0 is the initial draw on the credit line. If the manager reports cash flows truthfully, $d\hat{Y}_t = dY_t$, then V is a martingale, which means that (32) holds with equality and the manager's expected utility is $R + \alpha[\bar{M} - M_0]$. Thus, this is the optimal strategy for the manager.

For the final statement of the theorem, observe that incentive compatibility and truth-telling, combined with both parts of (29) yields $V_t = V^c - \alpha M_t$. \square

A.5 Proof of Proposition 5

First, we prove that the value of equity is concave in the implementation given in proposition 4.¹⁷ Since the equilibrium allocations are the same in the centralized and decentralized implementations, this implies that the value of equity is concave in the equilibrium given in proposition 5.

The total cash flow received by equity holders equals dividends minus the proceeds from equity issuance, and so the value of equity is

$$G(M_t) = E \left[\int_0^\tau e^{-rt} ((1 - \alpha)dD_t - dEQ_t) \right] = E \left[\int_0^\tau e^{-rt} (1 - \alpha)dD_t \right]$$

where $\tau = \min(\tau^d, \tau^f)$ and $E[dEQ_t] = E \left[\left(1 - \frac{\beta_t}{\alpha}\right) \sigma dZ_t \right] = 0$. Given the optimal dividend payment policy it follows that $G'(M) < 0$ for all $M \in [0, \bar{M}]$. Taking the law of motion of M_t as given (21), we have that

$$0 = -rG(M) + \left(\gamma M_t - \frac{\gamma}{\alpha} V^c \right) G'(M) + \frac{1}{2} \beta(M)^2 \sigma^2 G''(M) \quad (33)$$

Since outside equity receives a $(1 - \alpha)$ fraction of dividends and nothing in default, the boundary conditions are $G'(0) = -(1 - \alpha)$ and $G(\bar{M}) = 0$.

Let $M_t = \bar{M}$. Then we have

$$\frac{1}{2} \beta(M)^2 \sigma^2 G''(\bar{M}) = -(\gamma \bar{M} - \frac{\gamma}{\alpha} V^c) G'(\bar{M}).$$

¹⁷This result was proven for the case of no monitoring in DeMarzo and Sannikov (2006). In that setting, there is no raising of capital, and thus the question of why firms are able to issue equity in distress is not addressed.

Using $V^c = \alpha\bar{M} + R$ (20), we have that

$$\frac{1}{2}\beta(M)^2\sigma^2G''(\bar{M}) = -(\gamma\bar{M} - \frac{\gamma}{\alpha}(\alpha\bar{M} + R))G'(\bar{M}) = \frac{\gamma}{\alpha}RG'(\bar{M}) < 0.$$

Since $G''(\bar{M}) < 0$ and $G''(M)$ is continuous, then to show concavity of G , we need only to show that $G''(M_t) = 0$ and $G'''(M_t) < 0$ are jointly impossible.¹⁸ From (33) we have that whenever $G''(M_t) = 0$ that

$$\beta(M)^2\sigma^2G'''(M_t) = 2(r - \gamma)G'(M_t) > 0,$$

and so $G'''(M_t) > 0$. Thus, G is concave, and $G'(M_t) \leq -(1 - \alpha)$.

To proceed we outline the manager's strategy. First, if equity issuance is optimal, then the manager chooses not to steal and pay out the optimal level of dividends, achieving the manager's continuation value. The value of this strategy is given by the equivalence between the optimal contract payoffs to the manager and the payoff implied by the set of securities defined in Proposition 4 and the fact that the manager is indifferent between any two dividend payout policies. In addition, assume that off-equilibrium, if the equity issuance policy is not incentive compatible, the manager steals (or pays dividends suboptimally). If the equity holders choose to do an unconditional stock issuance ($d\Delta_t > 0$), the manager immediately pays out the proceeds as dividends. If the equity holders decide to re-purchase stock ($d\Delta_t < 0$), the manager immediately pays out any remaining draw on the line of credit as in dividends. Note that these off equilibrium strategies do not affect the manager's continuation value because the manager's continuation value going forward in equilibrium is $V_t - R = \alpha(\bar{M} - M_t)$, and the manager receives an α fraction of dividends. Thus, these strategies weakly dominate any others.

Now we consider the strategy of the common equity holders. In equilibrium, $G'' < 0$ and $G' \leq -(1 - \alpha)$ for all $M \in [0, \bar{M}]$. The common equity holders strictly prefer to avoid unconditional equity issuance ($d\Delta_t > 0$) because any proceeds are immediately paid out in dividends, and common equity holders pay all of the proceeds but receive only an $(1 - \alpha)$ fraction of the dividends. The common equity holders also strictly prefer to avoid any unconditional equity repurchases ($d\Delta_t < 0$) because the market pays only the (now reduced) dividend value of the shares. After equity re-purchase, the manager immediately liquidates the firm by paying dividends, so the value to common equity upon the announcement of repurchases is $(1 - \alpha)(\bar{M} - M_t)$. From our conditions on G , we have $G(M_t) > (1 - \alpha)(\bar{M} - M_t)$, and so the equity holders are strictly worse off. Thus, we must have $\Delta_t = 0$.

Since G is concave it is optimal to chose the smallest incentive compatible level of sensitivity of credit line to cash flow reports. This smallest incentive compatible level is the optimal equity issuance policy, and so $\delta_t = \left(1 - \frac{\beta(M_t)}{\alpha}\right)$. In response, the manager chooses the correct stealing and dividends policy. \square

¹⁸Strictly, we are interested in the left and right derivatives of $G''(M)$. However, these are potentially unequal only when $\beta(M) = 0$, and explicitly differentiating (33) shows that the left and right derivatives of $G''(M)$ coincide when $G''(M) = 0$.

References

- Abreu, Dilip, David Pearce, and Ennio Stacchetti, 1990, Toward a Theory of Discounted Repeated Games with Imperfect Monitoring, *Econometrica* 58, 1041-1063.
- Altman, Edward I., 1968, Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy, *Journal of Finance* 23, 589-609.
- Biais, Bruno, Thomas Mariotti, Guillaume Plantin, and Jean-Charles Rochet, 2007a, Dynamic Security Design: Convergence to Continuous Time and Asset Pricing Implications, *Review of Economic Studies* 74, 345-390.
- Biais, Bruno, Thomas Mariotti, Guillaume Plantin, and Jean-Charles Rochet, 2007b, Accident Risk, Limited Liability and Dynamic Moral Hazard, Working paper, University of Toulouse.
- Black, Ervin L., Thomas A. Carnes, Michael Mosebach, and Susan E. Moyer, 2004, Regulatory monitoring as a substitute for debt covenants, *Journal of Accounting and Economics* 37, 367-391.
- Chava, Sudheer, Praveen Kumar, and Arthur Warga, 2008, Managerial Agency and Bond Covenants, Working paper, Texas A&M University.
- Chava, Sudheer, and Michael Roberts, 2008, How Does Financing Impact Investments? The Role of Debt Covenants, *Journal of Finance* 63, 2085-2121.
- Clementi, Gian Luca, and Hugo A. Hopenhayn, 2006, A Theory of Financing Constraints and Firm Dynamics, *Quarterly Journal of Economics* 121, 229-265.
- DeAngelo, Harry, Linda DeAngelo, and René M. Stulz, 2008, Seasoned Equity Offerings, Market Timing, and the Corporate Lifecycle, Working paper, University of Southern California.
- DeMarzo, Peter M., and Michael J. Fishman, 2007, Optimal Long-Term Financial Contracting, *Review of Financial Studies* 20, 2079-2128.

- DeMarzo, Peter M., and Yuliy Sannikov, 2006, A Continuous-Time Agency Model of Optimal Contracting and Capital Structure, *Journal of Finance* 61, 2681-2724.
- Dichev, Ilia D., and Douglas J. Skinner, 2002, Large Sample Tests of the Debt Covenant Hypothesis, *Journal of Accounting Research* 40, 1091-1123.
- Franks, Julian R., and Sergey V. Sanzhar, 2006, Evidence on Debt Overhang from Distressed Equity Issues, Working paper, London Business School.
- Gilson, Stuart C., and Michael R. Vetsuypens, 1993, CEO Compensation in Financially Distressed Firms: An Empirical Analysis, *Journal of Finance* 48, 425-458.
- He, Zhiguo, 2007a, Optimal executive compensation when firm size follows geometric Brownian motion, *Review of Financial Studies*, forthcoming.
- He, Zhiguo, 2007b, Dynamic Compensation Contracts with Private Savings, Working paper, University of Chicago.
- Hertzel, Michael, Michael Lemmon, James S. Linck, and Lynn Rees, 2002, Long-Run Performance Following Private Placements of Equity, *Journal of Finance* 57, 2595-2617.
- Holmström, Bengt, and Paul Milgrom, 1987, Aggregation and Linearity in the Provision of Intertemporal Incentives, *Econometrica* 55, 303-328.
- Jacod, Jean, and Albert N. Shiryaev, 2003, *Limit Theorems for Stochastic Processes*, Springer-Verlag (Berlin), Second Edition.
- Nini, Greg, David C. Smith, and Amir Sufi, 2008, Creditor Control Rights and Firm Investment Policy, Working paper, University of Pennsylvania.
- Ortiz-Molina, Hernan, 2006, Executive compensation and capital structure: The effects of convertible debt and straight debt on CEO pay, *Journal of Accounting and Economics* 43, 69-93.
- Panageas, Stavros, and Mark M. Westerfield, 2008, High-Water Marks: High Risk Appetites? Convex Compensation, Long Horizons, and Portfolio Choice, *Journal of Finance*, forthcoming.

- Philippon, Thomas and Yuliy Sannikov, 2007, Real Options in a Dynamic Agency Model, with Applications to Financial Development, IPOs, and business risk, Working paper, New York University.
- Piskorski, Tomasz, and Alexei Tchisty, 2007, Optimal Mortgage Design, Working paper, Columbia University.
- Roberts, Michael, and Amir Sufi, 2008a, Control Rights and Capital Structure: An Empirical Investigation, *Journal of Finance*, forthcoming.
- Roberts, Michael, and Amir Sufi, 2008b, Renegotiation of Financial Contracts: Evidence from Private Credit Agreements, *Journal of Financial Economics*, forthcoming.
- Sannikov, Yuliy, 2007, Agency problems, screening and increasing credit lines, working paper, University of California at Berkeley.
- Sannikov, Yuliy, 2008, A Continuous-Time Version of the Principal-Agent Problem, *Review of Economic Studies* 75, 957-984.
- Sannikov, Yuliy, and Andrzej Skrzypacz, 2007, The role of information in repeated games with frequent actions, Working paper, Princeton University.
- Spear, Stephen E, and Sanjay Srivastava, 1987, On Repeated Moral Hazard with Discounting, *Review of Economic Studies* 54, 599-617.
- Tchisty, Alexei, 2006, Security Design with Correlated Hidden cash flows: The Optimality of Performance Pricing, Working paper, New York University.
- Townsend, Robert M, 1979, Optimal Contracts and Competitive Markets with Costly State Verification, *Journal of Economic Theory* 21, 265-293.
- Wang, Cheng, 2005, Dynamic Costly State Verification, *Economic Theory* 25, 887-916.
- Wruck, Karen Hopper, 1989, Equity Ownership Concentration and Firm Value: Evidence from Private Equity Offerings, *Journal of Financial Economics* 23, 3-28.