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Optimal Financial Education

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Abstract

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When agents first become active investors in financial markets, they are relatively inexperienced. We focus on the incentives of economic agents to educate these individuals. A feature of the financial market arena is that the agents best positioned to educate the inexperienced themselves stand to earn trading profits at the expense of inexperienced agents. Owing to this phenomenon, we show that the equilibrium amount of financial education may not fully correct the biases of the inexperienced agents. Thus, biased agents may not be fully educated by those with the best financial knowledge. This result complements hindrances to learning due to the self-attribution bias. With monopolistic delivery of financial education, the equilibrium proportion of educated agents tends to decrease with the profit potential of the information possessed by sophisticated agents, suggesting a policy need to reduce the informational advantage of agents with privileged access to information. On the other hand, in a competitive setting, increasing the variance of information tends to increase the rents from trading and thus can decrease the equilibrium proportion of uneducated agents.

1 Introduction

The notion that irrational investors may be prevalent in financial markets has taken on increased impetus in recent years. For example, while early empirical studies by Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) suggest a significant positive cross-sectional relation between security betas and expected returns, supporting the capital asset pricing model (Sharpe (1964), Lintner (1965), and Mossin (1966)), more recently, Fama and French (1992) find that the relation between return and market beta is insignificant. This calls into question the empirical importance of links between risk and expected returns.

On the importance of characteristics other than those directly related to risk-return models, the evidence is quite compelling. The landmark study by Fama and French (1992) finds that size and the book/market ratio strongly predict future returns (returns are negatively related to size and positively to book/market). Fama and French (1993) provide evidence that a three-factor model based on factors formed on the size and book-market characteristics explains average returns, and argue that the characteristics compensate for distress risk. But Daniel and Titman (1997) argue that, after controlling for size and book/market ratios, returns are not strongly related to betas calculated based on the Fama and French (1993) factors (see, however, Davis, Fama, and French (2000) for a contrary view). More recently, Daniel and Titman (2006) argue that the book/market effect is driven by overreaction to that part of the book/market ratio not related to accounting fundamentals. The part of this ratio that is related to fundamentals does not appear to forecast returns, thus raising issues about the distress-risk explanation.

Brennan, Chordia, and Subrahmanyam (1998) find that investments based on book/market and size result in reward-to-risk ratios which are about three times as high as that obtained by investing in the market. These seem too large to be consistent with a rational asset pricing model. On balance, it seems reasonable to assert that the evidence on the predictability of returns from book/market ratios at least partially supports non-risk-based (i.e., behavioral) explanations. Thus, Daniel, Hirshleifer, and Subrahmanyam (2001) suggest that overconfidence induces overreaction, and that extreme book/market ratios represent overreactions to extreme private signals which are later corrected. Similarly, Barberis, Shleifer, and Vishny (1998) suggest that naïve extrapolation from past growth causes stock prices to overreact and reverse, resulting in return predictability from fundamental/price ratios. The work of Barber and Odean (2001), Brennan (1995), and Odean (1998, 1999) suggests that individual investors are indeed overconfident and may also be susceptible to the bias of loss aversion.

While in recent years, it has come to be accepted by financial economists that individual investors may indeed be overconfident or biased in other ways, what has not been studied is whether there are sufficient economic incentives on the part of other agents to induce these agents to become rational. One way in which agents may learn to be rational is to simply learn from their trading. However, Gervais and Odean (2001) point out that such activity can take a long time because agents do not update properly owing to a self-attribution bias. So the question is whether there is a role for direct financial education which may allow these agents to converge faster to the rational outcome.

Trading successfully in financial markets requires a certain degree of sophistication. For example, one must learn about rudimentary aspects like the importance of risk, the potential futility from trading on already-public information, and being aware of possible behavioral biases such as overconfidence and loss aversion. Naturally, being an experienced agent in financial markets confers an advantage. Now, one aspect of financial markets is that the agents in the best position to confer financial education to the unsophisticated are themselves traders on their own account.[1] What is the equilibrium level of financial education in this scenario?

To address this question, we build a model of a financial intermediary who possesses a technology that allows the correction of the biases of individual investors and/or directs them away from useless information sources. The technology is disseminated in accordance with an exogenous profit function.[2] We show that the trading profits earned at the expense of inexperienced agents create a wedge between what is optimal for individual investors and what is optimal for the agents providing the education. Thus, the optimal amount of education partially, but not fully, moves the agents to-

[1]Such agents consist, for example, of financial institutions such as investment banks, traditional commercial banks, and pension funds.

[2]Several financial institutions have website sections devoted to financial education. See, for example, www.citigroup.com/citigroup/financialeducation/highlights041101.htm or www.bankofamerica.com/financialtools/. The profit function may be viewed as advertising revenue or additional website traffic generated by these public education sources.

wards rationality. That is, agents are not fully educated about their bias or lack of sophistication.

The preceding result arises because in financial markets for someone to make abnormal expected profits, somebody has to lose money on average. Hence, if the ones profiting are experienced, they do not have the incentive to remove the inexperience of the unsophisticated. Therefore, there is an interior optimum for the level of financial sophistication in society. We also show that in a dynamic setting, the experienced intermediary may delay the education of the irrational agent till after an initial round of trading that allows the earning of commission revenues.[3] The irrational agents may survive by earning positive overall expected profits because the possibility of being educated in a later round more than makes up for losing money in the initial rounds.

Our paper recognizes the aspect that the equilibrium degree of sophistication achieved by the unsophisticated itself depends on the trading rents earned by already-experienced agents at the experience of the unsophisticated. Thus, the decision to make agents sophisticated is not governed by the unsophisticated but by the experienced agents. This is contrast to the literature on information acquisition (e.g., Grossman and Stiglitz,

[3]There is an important literature on selling information in financial markets; see, e.g., Admati and Pfleiderer (1986, 1988, 1990), and Allen (1990). These papers examine whether and the extent to which information will be sold by rational informed agents to uninformed rational agents. Our focus instead is on the degree to which *irrational* agents may be educated not to trade on biases or mistaken beliefs, and thus on the equilibrium level of bias and the equilibrium proportion of educated (and thus, rational) agents in the economy. In addition, in much of this literature, the seller of does not trade on information in addition to selling it. The exception is Admati and Pfleiderer (1988) but in that paper the uninformed agents do not trade in the absence of information. In much of our paper, all agents, sophisticated or irrational, do trade in equilibrium. Further, we also consider a setting which allows for possible time-delays in education, in contrast to the static settings in the literature on information sales.

1980) wherein uninformed agents can choose to become informed on their own at a cost.

We find that when financial education is delivered by a monopolist, the equilibrium proportion of educated agents tends to decrease with the profit potential of the information possessed by sophisticated agents, suggesting a policy need to reduce the informational advantage of agents with privileged access to information. On the other hand, in a competitive setting, increasing the variance of information tends to increase the rents from trading and thus decrease the proportion of uneducated agents. This is because in the latter case, rents have to be competed away and any increase in rent at the expense of the uneducated simply increases the equilibrium proportion of the educated agents.

Our model also addresses the issue that the literature on financial markets has traditionally focused on explaining asset prices, while trading activity has attracted only peripheral attention. Empirical investigations of well-known asset pricing models such as the CAPM have centered only on the determinants of expected returns. Yet trading activity is a key feature of financial markets and, thus, warrants separate examination. Indeed, trading volumes are large in financial markets. For example, the NYSE website indicates that the annual share turnover rate in 2003 on the NYSE was about 99%, amounting to a total volume of about 350 billion shares. Assuming a per share value of $20 and a 50 basis point round-trip cost of transacting, this amounts to a transaction cost of $17.5 billion dollars that the investing public paid in 2003. The level of turnover in 2003 also represents approximately a 100% increase relative to its corresponding level in 1990 (see Chordia, Huh, and Subrahmanyam, 2006). The high levels of volume and the recent explosive growth in trading activity both deserve a comprehensive understanding by financial economists.

While the preceding studies have added much to our understanding of volume, they do not address the generally high levels of volume, nor do they consider why trading volume has increased so substantially in recent times. One rationale for high levels of volume is that portfolio-rebalancing or risk-sharing among investors leads to trading activity. De Bondt and Thaler (1995) argue, however, that such reasons alone would probably not result in the turnover of shares observed in reality and note that "the high trading volume observed in financial markets is perhaps the single most embarrassing fact to the standard finance paradigm." Indeed, Tkac (1999) shows that real-world volume exceeds that indicated by rational portfolio-rebalancing for a vast majority of traded stocks. Further, simple differences of opinion alone cannot give rise to volume (Milgrom and Stokey, 1982). Another rationale for volume is that irrational traders submit random trades with possibly little impact on the price, and thus create liquidity (Kyle, 1985). In this instance, it is food for thought as to why so many dollars are squandered on trading volume if it truly is a wasteful activity. Thus, the potential explanations for volume leave room for alternative rationales to be developed.

In this paper we propose that trading volume arises, in part, because agents learn about the validity of different sources of information. When an agent first decides to become a financial market investor, there potentially are scores of information sources at his disposal. These sources include email spams, internet bulletin boards, analyst recommendations, and several technical signals available on finance websites. The agent does not generally know which source is potentially useful.[4] Thus, in order to have an opportunity to trade successfully on useful information, the agent must take initial chances on potentially useless sources of information, which involve initial payments of commissions to intermediaries. Through subsequent education, the agent then is able to arrive at an assessment of what is an accurate signal;[5] this activity creates volume. We solve for an equilibrium in a dynamic market where such a phenomenon takes place.[6]

This paper is organized as follows. Section 2 analyzes the equilibrium where agents are educated not to be overconfident. Section 3 considers noise trading–i.e., an extreme case of overconfidence where agents trade on a signal that is pure noise, and a scenario where some agents are educated about the uselessness of the signal. Section 4 extends the model to a dynamic setting. Section 5 addresses the long-run survival of unsophisticated agents. Section 6 concludes.

[4]The assumption here is that many retail investors are not sophisticated enough to use statistical techniques such as regression analyses and the like to discover potentially useful sources of information. See, for example, Benartzi and Thaler (2001), Lo, Repin, and Steenbarger (2005), or Hong, Stein, and Yu (2007) for evidence regarding investor naïveté about financial markets.

[5]We postulate that an agent is not able to assess the usefulness of an information source unless he trades on it.

[6]In our model the agents who learn from experience that about whether their signals are valid interact with rational agents who have valid information at all times; a contrasting approach to learning is that in which a representative agent learns about the dividend process; see Guidolin and Timmermann (2007). There are other models of trading where agents interact of course; see Hommes (2006) for an excellent survey.

2 Education as Bias Reduction

Consider a security that pays off an amount $\theta + \epsilon$ at date 2, and is traded at date 1. There are two types of agents who trade the security. The first type are rational agents who observe the realization of θ and have a hedgeable endowment of w. The second class are those who do not observe the realization of θ but infer it from market prices. These agents are interchangeably termed individual investors or unsophisticated investors. The variables θ, ϵ, and w are mutually independent and normally distributed with mean zero. Throughout the paper, v_X represents the variance of X.

In this section, the individual investors agents are overconfident and underassess the variance of ϵ.[7] Specifically their subjective estimate of this variance is $v_c < v_\epsilon$. For now, we solve for the rational expectations equilibrium taking the value of v_c to be fixed. Later we will endogenize this choice by allowing the rational agent to provide access to a learning technology to the irrational agent. Both groups of agents have unit mass and have negative exponential utility with risk aversion coefficient R. The standard equilibrium in this market requires the following market-clearing condition to be satisfied:

$$\frac{\theta - P}{Rv_\epsilon} + \frac{E(\theta|P) - P}{R[v_c + v_(\theta|P)]} = w \tag{1}$$

From the above equation it is clear that from the perspective of the uninformed, observing P is equivalent to observing $\tau = \theta - Rv_\epsilon w$. Thus, conditioning on P is equivalent

[7]See Odean (1998) or Daniel, Hirshleifer, and Subrahmanyam (1998) for a review of the extensive experimental evidence supporting overconfidence.

to conditioning on w. Thus, we have:

$$E(\theta|P) = E(\theta|\tau) = k_1\tau$$

where

$$k_1 \equiv \frac{v_\theta}{v_\theta + R^2 v_\epsilon^2 v_w}\tau.$$

and

$$\text{var}(\theta|P) = k_2 \equiv \frac{R^2 v_\epsilon^2 v_w v_\theta}{R^2 v_\epsilon^2 v_w + v_\theta}.$$

All this implies that the equilibrium price P equals

$$P = \alpha_1\theta + \alpha_2 w$$

where

$$\alpha_1 = \frac{k_1 + v_\epsilon + k_2 + v_c}{k_2 + v_c + v_\epsilon}$$

and

$$\alpha_2 = -\frac{Rv_\epsilon(k_1 v_\epsilon + k_2 + v_c)}{k_2 + v_c + v_\epsilon}.$$

It can easily be shown that the loading on θ is decreasing in the degree of overconfidence (i.e., α_1 is increasing in v_c) because less overconfident agents trade less intensely in equilibrium which causes informed agents to trade more intensely in equilibrium. What is of interest is the expected utility of the rational and irrational investors, which, in turn, implies certainty equivalents.

We begin by stating the following lemma, which is a standard result on multivariate normal random variables (see, for example, Brown and Jennings, 1989).

Lemma 1 *Let $Q(\chi)$ be a quadratic function of the random vector χ: $Q(\chi) = C + B'\chi - \chi' A\chi$, where $\chi \sim N(\mu, \Sigma)$, and A is a square, symmetric matrix whose dimension corresponds to that of χ. We then have*

$$E[\exp(Q(\chi))] = |\Sigma|^{-\frac{1}{2}}|2A + \Sigma^{-1}|^{-\frac{1}{2}} \times$$

$$\exp\left(C + B'\mu + \mu' A\mu + \frac{1}{2}(B' - 2\mu' A')(2A + \Sigma^{-1})^{-1}(B - 2A\mu)\right). \tag{2}$$

The ex ante utility of the agents is derived by an application of Lemma 1. Define $\lambda = [\theta\ \epsilon\ w]$ and let Σ denote the variance matrix for this vector. Subscript the rational agents as S and the overconfident agents as O. Then, for agent class j, $j = S, O$, we can construct the square, symmetric matrix A_j such that $RW_j = \lambda' A_j \lambda$.[8] Noting then that the ex ante expected utility is given by $EU = E[-\exp(-RW_j)]$, we can apply Lemma 1 with $\mu = 0$, $C = 0$, and $B = 0$. The agent's ex ante utility thus becomes

$$EU = E[-\exp(-\lambda A_j \lambda')] = -|\Sigma|^{-\frac{1}{2}}|2A_j + \Sigma^{-1}|^{-\frac{1}{2}} = -|2A_j\Sigma + I|^{-\frac{1}{2}}.$$

Note that $W_S = (\theta + \epsilon)(x + w) - xP$, and $W_O = y(\theta + \epsilon - P)$. From these it is easy to construct the relevant matrices A_S and A_O. Plugging the actual quantities for A in the above expression yields the expected utility.

We then have our first proposition.[9]

Proposition 1 *The expected utility of the rational agents is increasing in the degree of overconfidence of the uninformed agents.*

[8] Indeed, this matrix can be constructed for any problem where the wealth consists of a linear function of a "product of normals" and we will use this observation later in the paper while omitting the specific details.

[9] Propositions 1 and 2 are proved in the appendix.

Thus, from the perspective of gains from trade, it pays for the informed to have the uninformed as overconfident as possible. Our next proposition is as follows:

Proposition 2 *The expected utility of the uninformed agents is decreasing in their degree of overconfidence.*

Hence, the uninformed lose utility on average because they are overconfident. This suggests that if the technology required to become unbiased is in the possession of the rational agents, there is a natural incentive to leave the uninformed irrational.

Now suppose the rational agent has access to a technology that allows for financial education. Also allow for an exogenous profit function that is increasing in this education.[10] As mentioned in Footnote 2 this profit function can be viewed as additional advertising revenue or website traffic generated by this education. Thus, these profits can viewed as the rents earned by the intermediary for educating the population about finance at large, not just the agents that trade in the model. We assume that the profit is increasing in the amount of the overconfidence reduction, so that the higher the v_c, the higher is the profit accruing from education.[11] Denote this profit function as $\pi(v_c)$, with $\pi'(v_c) > 0$. Then the agent maximizes

$$-exp[-R\pi(v_c)] \times |2A_S\Sigma + I|^{-\frac{1}{2}}$$

[10] A question arises as to whether the profit from education may be endogenized by having the irrationals pay a fee for education to the rationals. However, this approach presents problems because it requires the specification of irrational beliefs about their irrationality and consideration of the difference between their perceived certainty equivalent as opposed to their actual one. As such, we do not address this aspect in our setting but leave it for future research.

[11] The intuitive notion here is that the more useful the advice, the more the profitability of the information source.

which is equivalent to maximizing

$$\frac{1}{2}\ln\left[|2A_S\Sigma + I|\right] + R\pi(v_c).$$

The optimal v_c is obtained by setting the derivative of expression above to zero and ascertaining that the resulting value of v_c is a maximum. The determinant of $2A_S\Sigma+I$ is quite complex, and the full expression is provided in the appendix. We consider a parametric special case. Thus, for example when all parameters except R and v_c are unity, we have that

$$|2A_S\Sigma + I| = -\frac{2R^6(v_c^2+4v_c+3)+R^4(3v_c^2+8v_c+3)-2R^2(v_c+1)-(v_c+1)^2}{[R^2(v_c+2)+v_c+1]^2}.$$

Since the expected utility involves taking the square root of the determinant, it does not exist for all parameters. As can be seen from the above expression, however, the determinant has a stronger tendency to be positive when R is small. Also note that the second derivative of $\frac{1}{2}\ln\left[|2A_S\Sigma + I|\right]$ with respect to v_c is positive. Thus, for any concave function $\pi(v_c)$ (i.e., one with a negative second derivative) there exists at most one optimum for v_c.[12]

It can be shown that there exists a non-empty set of parameter values where the rational agent leaves the irrational agent optimally unbiased. Consider the parametric case where $\pi(v_c) = Dv_c^n$, where $D > 0$. When all parameters except v_c, R, and n are unity, with $R = 0.2$, and $n = 0.001$, we have that the optimal $v_c = 0.241$. At this level

[12]This follows from the fact that the optimal v_c is given by that point where the first derivative of $\frac{1}{2}\ln\left[|2A_S\Sigma + I|\right]$ with respect to m intersects the first derivative of the profit function. These first derivative functions can intersect at more than one point only if the difference in their slopes reverses in sign between intersection points.

of v_c the utility of the uninformed is worse than that when they are rational (from Proposition 2). Hence the educator does not fully educate the irrationals and they are worse off as a result. This discussion can be summarized by the following proposition:

Proposition 3 *If the risk aversion of the agents is sufficiently low, there exists a non-empty set of exogenous parameter values such that the equilibrium level of financial education preserves the overconfidence of individual investors.*

3 Educating Agents Not to Trade on Noise

The previous section allowed for financial education about reduction in overconfidence to all of a fixed set of agents. In this section, we consider a variant of irrationality that accounts for the notion that agents may believe useless sources of information to be valuable. Such information may take the form of internet bulletin boards, television shows, and the like. This scenario can be considered as an extreme form of overconfidence, so that even though observe a signal that is noise, they believe that it is a signal that is linked to true value. We also parameterize the degree of education provided by the dissemination of information about the uselessness of a signal to some, but not all, agents who could potentially benefit from the education.

More specifically, suppose individual investors believe the risky asset's payoff is $\eta + \epsilon$, but it is actually $\theta + \epsilon$. The variable η is normally distributed with mean zero and is independent of all other random variables in the model. Rational agents, as before, observe $\theta + \epsilon$. Their total masses of individual investors and the rational agents are each normalized to unity. Rational agents control how many other individuals are educated that the signal they are using is actually complete noise. The fraction of individuals that is educated is denoted by m. Thus, a fraction m of individuals trade on noise and a fraction $1-m$ are educated and trade on θ. The total mass of agents who trade on θ is then $2-m$. Later, we will consider the equilibrium mass m of rational agents.

In this section, for tractability, we will assume that w represents the noisy supply provided by price-inelastic liquidity traders.[13] Taking m as exogenous for the moment, the market clearing condition becomes:

$$\frac{(2-m)(\theta-P)}{Rv_\epsilon}+m\frac{\eta-P}{Rv_\epsilon}=w,$$

from which we obtain P. Note that the wealth of the informed is given by $W_S = [(\theta-P)/(Rv_\epsilon)][\theta+\epsilon-P]$. Substituting for the equilibrium P, and from Lemma 1, we have that the expected utility of the informed is given by $-Det^{-0.5}$, where Det is given by

$$\frac{m^2(v_\eta+v_\theta)+v_\epsilon(R^2v_\epsilon v_w+4)}{4v_\epsilon}.$$

It can be seen that the expected utility of the rational agents is increasing in the mass of irrational agents, m, and in the variance of the irrational signal η.

Suppose that the rational agents can educate the irrationals at a technology that earns them a profit. Let the profit function be $D(1-m)^n$, i.e., the profit is increasing

[13]The results do not substantively change when this assumption is relaxed, but the algebraic expressions become more complicated.

in the proportion of agents that are educated. This can be justified by arguing that the greater the audience reached by the public source such as a website, the greater is the profit earned (e.g., due to the sale of advertisements in the education material). Thus the optimal m solves

$$\max_m 0.5 \ln \left[\frac{m^2(v_\eta + v_\theta) + v_\epsilon(R^2 v_\epsilon v_w + 4)}{4v_\epsilon} \right] + RD(1-m)^n. \tag{3}$$

This immediately implies the following proposition.

Proposition 4 *An equilibrium value of m such that $m \in (0,1)$ satisfies the equation*

$$\frac{m(v_\eta + v_\theta)}{m^2(v_\eta + v_\theta) + v_\epsilon(R^2 v_\epsilon v_w + 4)} - DnR(1-m)^{n-1} = 0$$

so long as second derivative of the left-hand side with respect to m is negative in the interval [0,1].

For expositional convenience, henceforth in the paper, reference to an "equilibrium" will imply an interior equilibrium such that $m \in (0,1)$.[14] Such an equilibrium may or may not exist.However, so long as v_ϵ is sufficiently large and the profit function is concave, the second derivative of the first term in (3) with respect to m is positive, while the second derivative of the profit function is negative. As argued in the previous section, this implies at most one equilibrium m.[15] Examining the left-hand side of the above equation and using the implicit function theorem (and noting that the second-order

[14]Comparative statics from this point also refer to m ranging in the interval $(0,1)$.

[15]Similar arguments hold in the other model variants in the paper and therefore the uniqueness arguments are henceforth omitted for brevity.

condition must be satisfied for the equilibrium) immediately confirms the following comparative statics.

Proposition 5 *The optimal proportion of the uneducated agents is increasing in the variance of information* v_θ.

Increasing the variance of information increases the gains from trading with uninformed agents and reduces the relative advantage of education. Hence, the proportion of uneducated agents rises. Consider the parameter values $c = 0.7$, $n = 0.2$, $R = 1$, $v_\epsilon = 1$, $v_\eta = 3$, $v_\theta = 2$ and $v_w = 3$. Then, the optimal $m = 0.71$, so 29% of the population is educated. Raising the value of v_θ to 3 increases the proportion of uneducated people to 76%. Overall, this demonstrates that in equilibrium, a considerable majority of the population may remain uneducated.

4 A Dynamic Setting

We now consider how trading volume can be sustained in our framework, modified to include a dynamic setting. The framework also serves to explain how it may pay for an intermediary to delay the education of agents when it is able to charge trading commissions. Thus, consider a model where irrational agents trade in two periods. We will first present a simple setting and extend the framework to include price-inelastic liquidity traders in the next section.

4.1 The Basic Framework

The security pays off $\theta + \epsilon$ in the final round of trade, labeled period 3, as in the previous section. In period 1, irrational trade based on complete noise, specifically a variable η_1 that they believe will be revealed in period 2. Thus, their expectation is that the security will pay off $\eta_1 + \epsilon_1$ in period 2, where ϵ_1 is a mean zero random variable with variance v_ϵ and is independent of all other random variables in the model. For now, we impose the assumption that no education is provided in period 1. This can be implemented by assuming that the intermediary provides access to financial tools and only for customers that have been associated with the intermediary for a period of time.[16] We will justify this assumption as part of an equilibrium in the next section.

In period 2, a fraction m trade again based on another variable η_2, conjecturing that the asset will pay off $\eta_2 + \epsilon$ in period 2. After trading in period 1, the fraction $1 - m$ may be educated about the true information source θ. The two noise sources η_1 and η_2 are independent and normally distributed with mean zero and common variance v_η. Further, η_i, $i = 1, 2$ are independent of all other random variables in the model. The intermediary observes the information signal θ just after trade in period 1 and just before trading in period 2. The mass of the intermediary is normalized to unity.

For tractability in this dynamic setting, prices are set by risk-neutral and competitive market makers who observe the net demand functions of the other agents. The

[16]For example, Wells Fargo Bank provides brokerage advice only to customers with more than a certain total dollar amount across all accounts (https://www.wellsfargo.com/checking/pma/); this can be viewed as akin to educating only experienced agents who presumably have accumulated more financial assets.

demand at date 1 from the irrational agents is

$$\frac{\eta_1 - P_1}{Rv_\epsilon}.$$

Similarly, the total demand of the irrational agents at date 2 is

$$m\frac{\eta_2 - P_2}{Rv_\epsilon},$$

and that from the total mass of the rational agents is

$$(2-m)\frac{\theta - P_2}{Rv_\epsilon}.$$

The market makers observe the total demand of the irrationals at date 1 and the quantity

$$(2-m)\frac{\theta - P_2}{Rv_\epsilon} + m\frac{\eta_2 - P_2}{Rv_\epsilon},$$

at date 2. Note that observing the net demand is equivalent to observing the random variable $\theta + [m/(2-m)]\eta$.

All of the above implies that the date 1 price is simply zero (since the demand is uninformative and all unconditional means are zero), whereas the date 2 price is given by

$$P_2 = E\{\theta | \theta + [m/(2-m)]\eta\}$$

The wealth of the informed in this case is $[(\theta - P_2)/(Rv_\epsilon)](\theta + \epsilon - P_2)$. From Lemma 1, the utility of the rational agents is again given by $-Det^{-0.5}$, where

$$Det = \frac{m^2[v_\epsilon(v_\eta + v_\theta) + v_\eta v_\theta] - 4mv_\epsilon v_\theta + 4v_\epsilon v_\theta}{v_\epsilon[m^2(v_\eta + v_\theta) - 4mv_\theta + 4v_\theta]}. \tag{4}$$

Note that in the range $m \in [0, 1]$, the above expression is increasing in m so that the informed are better off with more irrationals. As in the previous section, one can calculate the optimal number of irrationals who get educated by differentiating the expression $(1/2)\ln(Det) + RD(1-m)^n$ with respect to m and setting the result equal to zero. This implies solving the following for m:

$$\frac{2mv_\eta v_\theta^2(2-m)}{[(2-m)^2v_\theta + m^2v_\eta][(2-m)^2v_\epsilon v_\theta + m^2v_\eta(v_\epsilon + v_\theta)]} - DnR(1-m)^{n-1} = 0. \tag{5}$$

This immediately leads to the following proposition (again using the implicit function theorem):

Proposition 6 *The equilibrium value of m in the dynamic setting is increasing in v_θ and decreasing in v_ϵ.*

Consider the same parameter values as in the previous section (with $v_\theta = 2$). The optimal m in this case is 0.82, so that only 18% of the population is educated. The reason the optimal m is greater in this instance relative to the previous section is that here, the returns to trading information are diminished because market makers partially back out the information of the informed agents. Thus, the proportion of uneducated agents has to rise to account for this phenomenon.

One implication of the preceding proposition is that the greater the profit potential from information (represented by the variance of the information v_θ), the lower is the proportion of educated agents. This suggests the policy goal of reducing the trading advantage of the sophisticated agents so that they may in equilibrium focus more on educating the unsophisticated agents.

Our second proposition, which follows directly from (5), considers conditions under which the populace is either fully educated or not educated at all.

Proposition 7 *The optimal proportion of educated agents approaches unity when the parameter D, which governs the profitability of financial education, is sufficiently large. On the other hand, when the profitability of financial education approaches zero, the proportion of educated agents approaches zero.*

The straightforward notion in the above proposition is that when it is not sufficiently profitable to disseminate financial education, agents are not educated. An implication of this is that the proportion of educated agents will rise when, due to technological innovations such as the rise of the internet, it becomes more profitable to disseminate educational materials.

It is also worth exploring the issue of how much information is reflected in the price in this model. Note that the informational efficiency of the price system is given by

$$\mathrm{var}^{-1}(\theta|P_1, P_2) = \mathrm{var}^{-1}[\theta|\theta + \{m/(2-m)\}\eta].$$

Then, the value of the above precision is given by

$$\frac{(2-m)^2 v_\theta + v_\eta}{m^2 v_\theta v_\eta}.$$

This directly leads us to the following proposition.

Proposition 8 *The informational efficiency of the price is decreasing in the proportion of uneducated agents.*

Thus, leaving the populace uneducated has a cost that goes beyond an issue of "fairness." Specifically, the price is more noisy due to the presence of unsophisticated agents. This result is in contrast to the model of Kyle (1985) where, under risk neutrality of informed agents and market makers, uninformed agents have no effect on informational efficiency. This is because in that model informed agents scale up their activity in response to an increase in irrational trading, and in equilibrium, prices set by market makers adjust to leave informational efficiency unchanged. In our case, while market makers are risk neutral, the informed agents are risk averse which dampens their response to irrational traders.

4.2 Support of the Equilibrium with Delayed Education

We now consider parameter values that justify the equilibrium with delayed education. Specifically, as we mentioned in the previous section, we suppose that a fixed commission of K per period is paid by the irrational agents to the financial intermediary. To allow for individual investor participation in each of the two rounds, it suffices to have their perceived certainty equivalents exceed $2K$. We assume the irrational agents conjecture that market makers do not consider their trades to be informed. In this case, the irrational agents' perceived wealth is given by

$$[(\eta_1 - P_{c1})/(Rv_\epsilon)](\eta_1 + \epsilon_1) + [(\eta_2 - P_{c2})/(Rv_\epsilon)](\eta_2 + \epsilon_2).$$

where P_{c1} and P_{c2} are their conjectured date 1 and 2 prices, which are zero (under the conjecture that market makers do not consider their trades to be informed). Straightforward calculations show that the certainty equivalent of the above perceived wealth is

$$\frac{1}{2R}\ln\left[\frac{(v_\epsilon+v_\eta)^2}{v_\epsilon^2}\right].$$

So long as the above quantity is greater than $2K$ the irrational agents will enter and trade in both periods.

Now, for the financial intermediary, there is no benefit to educating agents at date 1 as opposed to date 2. This is because no fundamental information is received at date 1 so the intermediary cannot benefit by trading at this date. Note, however, there is a cost of education at date 1 in that it causes irrational agents to realize that η_1 is useless and hence they eschew trading. This causes a loss of commission revenue. This immediately leads us to the following proposition.

Proposition 9 *So long as*

$$\frac{1}{2R}\ln\left[\frac{(v_\epsilon+v_\eta)^2}{v_\epsilon^2}\right] > 2K > 0,$$

irrational agents trade in both periods and financial education is delayed, i.e., is provided to the irrational agents only in period 2.

4.3 Volume

The volume by each agent class is proportional to the standard deviation of the total trade of that class.[17] Thus, the volume at date 1 is simply $\text{std}[\eta/(Rv_\epsilon)]$. At date 2, volume is given by the standard deviation of the trade by each agent class and the standard deviation of the trade between the agent classes. Thus, the date 2 volume is given by

$$(2-m)\text{std}[(\theta - P_2)/(Rv_\epsilon)] + m\ \text{std}(\eta - P_2)/Rv_\epsilon)]$$

$$+\text{std}[(2-m)(\theta - P_2)/(Rv_\epsilon) + m(\eta - P_2)/(Rv_\epsilon)].$$

This volume may be increasing or decreasing in m depending on the variance of η among other things. If the variance of η is large then irrationals contribute a lot of volume so more irrationals imply more volume. Indeed, one can show the following (the appendix contains a proof):

Proposition 10 *If the parameter D is sufficiently low, then, as the variance of the irrational signal $v_\eta \to \infty$, the volume of trading becomes infinitely large.*

Thus, large levels of volume may be sustained in this model because of the actions of the irrational traders. The more the variance of the useless signals used by the irrationals, the greater is expected trading volume induced by these agents.

[17] This follows from the observation that the expectation of the absolute value of a normal random variable is proportional to the standard deviation of the variable (the constant of proportionality is $\sqrt{2\pi^{-1}}$).

4.4 A Competitive Market for Financial Education

In the previous analysis, the optimal amount of financial education was obtained by maximizing an objective that included the certainty equivalent of trading with unsophisticated agents and the profits from making unsophisticated agents sophisticated. This implied monopolistic access to the education technology on the part of the sophisticated intermediary. An alternative would be to model the market for education as competitive. In this instance, to close the equilibrium, one may say that the intermediary has to cover a fixed set up cost G, and, in equilibrium, the gains from education and trading equal this setup cost. Thus, in equilibrium, from (4) we have

$$(1/2)\ln(Det) + RD(1-m)^n = G. \tag{6}$$

where

$$Det = \frac{m^2[v_\epsilon(v_\eta + v_\theta) + v_\eta v_\theta] - 4mv_\epsilon v_\theta + 4v_\epsilon v_\theta}{v_\epsilon[m^2(v_\eta + v_\theta) - 4mv_\theta + 4v_\theta]}.$$

Interestingly, the comparative statics relating to the competive market for education are different from thise in Subsection 4.1. Specifically, the following proposition (proved in the appendix) can be obtained:

Proposition 11 *In the competitive model, so long as D is sufficiently small, an increase in the variance of information v_θ, reduces the equilibrium proportion of uneducated agents in equilibrium.*

The intuition for the above proposition is that increasing v_θ increases the gains from trading against the unsophisticated agents. In a competitive model this absolute increase simply has the effect of reducing the equilibrium proportion of unsophisticated agents as the rents get competed back to zero. In contrast, the monopolist trades off the marginal gain from trading against the marginal loss of profit from educating the unsophisticated, and this leads to the opposite comparative static.

5 Price-Inelastic Liquidity Trading and Survival

It can be shown that the irrationals cannot earn positive expected profits in the model of the previous section, because they lose money on average to the informed agents. However, consider an extension of the previous setting where price-inelastic liquidity traders with a total trade of w trade in the market in the second period. Their trades are independent of all other random variables in the model. In this case, the price at date 2 is modified to

$$P_2 = E\{\theta | \theta + [m/(2-m)]\eta - w/(2-m)\},$$

and the wealth retains the same form as a function of price as that in Subsection 4.1. It can easily be shown in this case that the utility of the rationals is given by $-Det^{-0.5}$, where

$$\frac{m^2[v_\epsilon(v_\eta + v_\theta) + v_\eta v_\theta] - 4mv_\epsilon v_\theta + 4v_\epsilon v_\theta + v_w(v_\epsilon + v_\theta)}{v_\epsilon[m^2(v_\eta + v_\theta) - 4mv_\theta + 4v_\theta + v_w]}.$$

As before, the intermediary maximizes the objective

$$(1/2)\ln(Det) + RD(1-m)^n$$

so that the equilibrium m satisfies the equation

$$\frac{v_\theta^2[2v_w - 2m^2v_\eta + m(4v_\eta - v_w) - 2v_w]}{[m^2(v_\eta + v_\theta) - 4mv_\theta + 4v_\theta + v_w][m^2\{v_\epsilon(v_\eta + v_\theta) + v_\eta v_\theta\} - 4mv_\epsilon v_\theta + v_\epsilon(4v_\theta + v_w) + v_\theta v_w]}$$

$$-DnR(1-m)^{n-1} = 0.$$

The equilibrium m in this case reduces to 58% under the parameter values of the previous section, indicating that the presence of the liquidity traders reduces the proportion of people that stay uneducated. Generally, the equilibrium m tends to decrease in v_w. This is because the presence of the liquidity traders reduces the sensitivity of the gains to trade to the mass of uneducated agents. This is because the signal-to-noise ratio of the price decreases, which, in turn, increases price volatility for the risk-averse rational agents. Since the profit from education does not depend on v_w, the proportion m of uneducated investors falls.

We now show that in the model of the previous subsection, the irrationals can earn positive expected profits and survive. Note that the expected profits of the irrationals are given by

$$E[(\eta_1 - P_1)/(Rv_\epsilon)] + m(\eta_2 - P_2)/(Rv_\epsilon) + (1-m)(\theta - P_2)/(Rv_\epsilon).$$

Specifically, straightforward calculations show that the average profits across the outcomes of being educated and not being educated are given by

$$\frac{v_\theta[v_w(1-m) - m^2v_\eta]}{Rv_\epsilon[(m-2)^2v_\theta + m^2v_\eta + v_z]}.$$

This immediately leads to the following proposition:

Proposition 12 *The expected profits of the irrational investors are positive if and only if*

$$(1-m)v_w > m^2 v_\eta + \frac{Rv_\epsilon[(m-2)^2 v_\theta + m^2 v_\eta + v_z]}{v_\theta}.$$

Since v_w is positive, the above inequality is satisfied across a wide range of parameter values. In the parametric example considered in Subsection 5 and earlier the inequality is indeed satisfied. The proposition demonstrates that in this specification of the model, even though financial education may be limited, the overall expected profits of the irrationals may still sustain the presence of such agents in financial markets.

6 Conclusion

When agents first become active investors in financial markets, they are relatively inexperienced. However, experienced agents profit off the inexperienced, which creates a barrier to the transfer of experience. We show that the optimal amount of financial education does not fully correct the biases of the inexperienced agents. This result complements hindrances to learning due to the self-attribution bias. We also consider a setting where individuals trade on complete noise. In this setting we show that a sophisticated financial intermediary may leave a proportion unsophisticated, i.e., prone to trade on noise. Among other results, we show that the activity of becoming educated creates volume and that unsophisticated irrationals may earn positive expected profits across the outcomes of being educated and not being educated. Thus, they may survive as part of a long-run equilibrium.

We also find that the equilibrium proportion of educated agents tends to decrease with the profit potential of the information possessed by sophisticated agents. On the other hand, in a competitive setting, increasing the variance of information tends to increase the rents from trading and thus decrease the proportion of uneducated agents. This is because in the latter case, rents have to be competed away and any increase in rent at the expense of the uneducated simply increases the equilibrium proportion of the educated agents. Thus, the implications of endowing the informed agents with more valuable private information depends on whether they also have privileged access to the technology for effective financial education.

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Appendix

Proofs of Propositions 1 and 2: The A matrices in Section 2 are as follows:

$$A_S = R \times \begin{bmatrix} \frac{(1-\alpha_1)^2}{v_\epsilon} & \frac{1-\alpha_1}{2v_\epsilon} & \frac{-\alpha_2(1-\alpha_1)}{v_\epsilon} + \frac{R\alpha_1}{2} \\ \frac{1-\alpha_1}{2v_\epsilon} & 0 & \frac{-\alpha_2}{2v_\epsilon} \\ \frac{-\alpha_2(1-\alpha_1)}{v_\epsilon} + \frac{R\alpha_1}{2} & \frac{-\alpha_2}{2v_\epsilon} & \frac{\alpha_2^2}{v_\epsilon} + R\alpha_2 \end{bmatrix},$$

and

$$A_O = R \times \begin{bmatrix} \frac{(k_1-\alpha_1)(1-\alpha_1)}{v_c+k_2} & \frac{k_1-\alpha_1}{2(v_c+k_2)} & \frac{(1-\alpha_1)(-\alpha_2-k_1Rv_\epsilon)-\alpha_2(k_1-\alpha_1)}{2(v_c+k_2)} \\ \frac{k_1-\alpha_1}{2(v_c+k_2)} & 0 & \frac{-\alpha_2-k_1Rv_\epsilon}{2(v_c+k_2)} \\ \frac{(1-\alpha_1)(-\alpha_2-k_1Rv_\epsilon)-\alpha_2(k_1-\alpha_1)}{2(v_c+k_2)} & \frac{-\alpha_2-k_1Rv_\epsilon}{2(v_c+k_2)} & \frac{\alpha_2(\alpha_2+k_1Rv_\epsilon)}{v_c+k_2} \end{bmatrix}.$$

The expected utility of each class of agent EU_i is monotonic in

$$D_i = Det(2A_i\Sigma + I),$$

where $i = S, O$. The determinant D_S can be written as

$$\Gamma_5^{-1}[\Gamma_1 R^6 + \Gamma_2 R^4 + \Gamma_3 R^2 + \Gamma_4],$$

where

$$\Gamma_1 = -v_\epsilon^4 v_w^3 (v_\epsilon + v_\theta)[v_c^2 + 2v_c(v_\epsilon + v_\theta) + v_\theta(2v_\epsilon + v_\theta)],$$

$$\Gamma_2 = v_\epsilon^2 v_w^2 [v_c^2(v_\epsilon^2 - 2v_\epsilon v_\theta - 2v_\theta^2) + 2v_c(v_\epsilon^3 - v_\epsilon^2 v_\theta - 3v_\epsilon v_\theta^2 - v_\theta^3),$$

$$+v_\epsilon(v_\epsilon^3 + v_\epsilon^2 v_\theta - 3v_\epsilon v_\theta^2 - 2v_\theta^3)],$$

$$\Gamma_3 = v_\theta v_w [v_c(2v_\epsilon^2 - v_\epsilon v_\theta - v_\theta^2) + v_\epsilon(2v_\epsilon^2 + v_\epsilon v_\theta - v_\theta^2)][v_c + v_\epsilon],$$

$$\Gamma_4 = v_\theta^2 (v_c + v_\epsilon)^2,$$

and

$$\Gamma_5 = [R^2 v_\epsilon^2 v_w (v_c + v_\epsilon + v_\theta) + v_\theta(v_c + v_\epsilon)]^2.$$

The derivative of D_S with respect to v_c is

$$-\frac{2R^4 v_\epsilon^5 v_w^2 [R^4 v_\epsilon^3 v_w^2 (v_\epsilon + v_\theta) + R^2 v_\epsilon v_\theta v_w (2v_\epsilon + v_\theta) + v_\theta^2]}{[R^2 v_\epsilon^2 v_w (v_c + v_\epsilon + v_\theta) + v_\theta (v_c + v_\epsilon)]^3}$$

and is negative, so that the expected utility of the rationals is higher, the lower is v_c. Similarly,

$$\frac{dD_O}{dv_c} = \frac{2R^4 v_\epsilon^4 v_w^2 (2v_\epsilon - v_c)(R^2 v_\epsilon^2 v_w + v_\theta)^2}{[R^2 v_\epsilon^2 v_w (v_c + v_\epsilon + v_\theta) + v_\theta (v_c + v_\epsilon)]^3]}$$

and is positive.

Proof of Proposition 10: The volume of trade by the irrationals is monotonically related to the variance of their trades. This variance is given by

$$\text{var}\left[m\frac{\eta - P_2}{Rv_\epsilon}\right] = \frac{m^2[m^2(v_\eta^2 + 3v_\eta v_\theta + v_\theta^2) - 4mv_\theta(2v_\eta + v_\theta + 4v_\theta(v_\eta + v_\theta))]}{R^2 v_\epsilon^2 [(2-m)^2 v_\theta + m^2 v_\eta]}.$$

As m approaches the corner solution of unity for sufficiently large D, and v_η becomes unboundedly large, the above expression tends to infinity. This is because the exponent of v_η is greater in the numerator than in denominator. Since total volume is the sum of three positive variables and one of them satisfies the claimed limit, this proves the proposition.

Proof of Proposition 11: Denote the left-hand side of (6) as H. Then we have that

$$\frac{dm}{dv_\theta} = -\frac{\partial H/\partial v_\theta}{\partial H/\partial m}.$$

Now, it follows from simple differentiation that

$$\frac{\partial H}{\partial v_\theta} = \frac{m^4 v_\eta^2}{2[\{(2-m)^2 v_\theta + m^2 v_\eta\}\{(2-m)^2 v_\epsilon v_\theta + m^2 v_\eta (v_\epsilon + v_\theta)\}]} > 0.$$

and that

$$\frac{\partial H}{\partial m} = \frac{2mv_\eta v_\theta^2(2-m)}{2[\{(2-m)^2 v_\theta + m^2 v_\eta\}\{(2-m)^2 v_\epsilon v_\theta + m^2 v_\eta(v_\epsilon + v_\theta)\}]} - DnR(1-m)^{n-1}$$

The latter partial derivative will also be positive for sufficiently small D, and in this case $dm/dv_\theta < 0$, as asserted in the proposition.