

Obfuscation, Learning, and the Evolution of Investor Sophistication*

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Abstract

Investor sophistication has lagged behind the growing complexity of retail financial markets. To explore this, we develop a dynamic model to study the interaction between obfuscation and investor sophistication. Taking into account different learning mechanisms within the investor population, we characterize the optimal timing of obfuscation for a profit-maximizing monopolist. Obfuscation decreases with competition among firms, but increases with higher investor participation in the market. We show that educational initiatives that are directed to facilitate learning by investors may induce producers to increase wasteful obfuscation, further disorienting investors and decreasing overall welfare.

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1 Introduction

The menu of offerings for retail financial products has grown steadily over the last twenty years, and the sheer number of choices is now daunting.¹ While such proliferation may add value in completing markets, it may also adversely affect investor sophistication.² Newcomers in the market have more to learn when they make their initial choices, and incumbent participants bear a higher burden to keep up with developments in the product market. Moreover, it remains unclear whether having access to more options leads to better decisions, as participants often make suboptimal choices in the face of too much information (e.g. Iyengar, Huberman, and Jiang, 2004; Salgado, 2006; Iyengar and Kamenica, 2008).

The interaction between the number/attributes of product offerings and the evolution of consumer sophistication, therefore, introduces an externality that new products and changing product mixes slow learning and may preserve industry rents for producers. This practice can be used strategically by producers and has been termed obfuscation (Ellison and Ellison, 2008; Ellison and Wolitzky, 2008), shrouding (Gabaix and Laibson, 2006), and complexity (Carlin, 2008). Indeed, many new products do not depart much from old ones, and are redundant even within the lines of specific producers. There are straightforward strategic considerations at play: as Christoffersen and Musto (2002) point out, financial institutions often offer several classes of products to price discriminate among investors of varied levels of sophistication. Discrimination through such purposeful distortions in transparency is an important source of firm value.

The purpose of this paper, then, is to explore the dynamic relationship between obfuscation and sophistication in retail financial markets, taking into account that learning mechanisms within the investor population play an important role. We specifically address the following questions: How often do firms optimally practice obfuscation, given that consumers learn over time? How do specific learning processes (e.g. learning from others, learning from periodicals) affect these dynamics? What effect do competition and participation have on obfuscation? What are the policy implications that follow from the analysis?

To address these questions, we begin by analyzing a retail market in continuous time in which a monopolist markets a class of financial products (say, a class of mutual funds) to a heterogeneous group of consumers. Consumers may either be sophisticated, in which case they always choose the

¹For example, as of 2007, there were 8,029 mutual funds to choose from and 21,631 different share classes (Investment Company Institute 2008)

²Many participants in the market have limited sophistication regarding the products in the market (e.g. NASD Literacy Survey 2003). See also Capon, Fitzsimons, and Prince 1996; Alexander, Jones, and Nigro 1998; Barber, Odean, and Zheng 2005; Agnew and Szykman 2005.

optimal product within the offering, or they may be unsophisticated, in which case they may either pay too much or not get the best quality. As such, the rents achieved from either type are different: monopoly rents from those who are sophisticated and even higher profits from those who are not.

Sophistication evolves according to a general learning process, which is a differential equation with commonly known initial conditions. When the monopolist changes his product mix, the population is “refreshed” so that sophistication returns to its initial level and learning begins again. In essence, then, there are three groups of consumers in the market: experts who are always sophisticated, non-experts who become sophisticated transiently, and non-experts who remain unsophisticated. Controlling the balance of sophisticated and unsophisticated non-experts is at the heart of the monopolist’s problem. As such, the monopolist maximizes profits by deciding how frequently to alter his product mix, affecting the learning process.

The problem that the monopolist faces is stationary and therefore, in equilibrium there exists a unique optimal time to change the product mix. The optimal time is strictly decreasing in the extra rents gained from unsophisticated consumers (compared to monopoly rents) and strictly increasing in the cost incurred in doing so. The intuition is that the more the monopolist gains from unsophisticated consumers, the higher is the benefit from refreshing the population and keeping consumers in the dark. On the other hand, the more costly it is for the monopolist to do this, the lower is the benefit from refreshing the population. These comparative statics have important cross-sectional empirical implications. For example, our analysis predicts that we should observe more product changes and redundancy among classes of mutual funds with higher price dispersion.³ To our knowledge, this prediction is novel and has yet to be tested.⁴

The relationship between the frequency of obfuscation and the speed at which consumers become sophisticated is non-monotonic, however. The intuition is as follows. If consumers learn very quickly, then the gains to refreshing the population are short-lived. Given that there is a fixed cost of changing product attributes, it may not be worthwhile to change the product mix as frequently. Moreover, when consumers are very slow to learn and the extra rents gained from unsophistication are long-lived, the monopolist will optimally choose to refresh the population less frequently, again because there is a cost to doing so. This phenomenon has welfare implications for educational initiatives that the government may undertake. If the population learns relatively slowly, improving

³As Hortacsu and Syverson (2004) note, the amount of price dispersion varies among different classes of funds, which likely indicates a difference in the sophistication (i.e. search costs) among consumers within each asset class. See Table 1 in Hortacsu and Syverson (2004).

⁴Testing this prediction might involve correlating price dispersion within groups of homogeneous offerings with number of share classes offered, either cross-sectionally or in a time-series.

the learning process marginally will actually decrease welfare, as it increases the frequency of wasteful obfuscation.⁵ Small educational initiatives that increase the speed of learning are only likely to increase welfare when the sophistication of the population is sufficiently high.

The type of learning that takes place in the market also affects obfuscation and the policies that are set. Based on the information percolation model of Duffie and Manso (2007), we analyze two specific settings: one in which investors learn by themselves (e.g. by reading periodicals) and one in which investors learn from each other. In the former setting, as the initial set of sophisticated consumers rises, the frequency of obfuscation by the monopolist decreases. This occurs because there is less to gain from refreshing the population. Educational initiatives that improve the level of sophistication in the market are always welfare enhancing in this case because increasing sophistication lowers obfuscation. When investors learn from each other, however, we obtain different results. The comparative statics and policy considerations are very similar to the effect of learning speed. This non-monotonic relationship can be appreciated as follows. When no one is sophisticated initially, there is no one to learn from, and therefore the monopolist never changes their product offerings. When everyone is sophisticated initially, then there is no gain to refreshing the population. Therefore, innovation only takes place when a fraction of the population is sophisticated and there is a non-monotonic relationship between initial sophistication and the frequency of obfuscation.

We then extend the model to consider the effect of participation in the market on obfuscation. In this extension, a fraction of the consumer population does not participate in the market. The monopolist again considers the optimal time to refresh sophistication within the market, which is a stationary problem. We show that for products that have more widespread use (more participation), the frequency of obfuscation is higher. The intuition is that the gains to refreshing the population are higher when more people participate, which makes it more attractive for the monopolist to refresh more quickly over time. This finding predicts empirically that there is a positive correlation between changing product attributes and the scope of a product.

So far, in the model, the only cost to society is the cost that the monopolist incurs when he refreshes. We extend the model to consider two other sources of welfare loss. First, we study how expertise arises endogenously in the market. Indeed, if obfuscation takes place, it may be in the interest of consumers to become experts and always get the best deal without relying on a learning

⁵Policies designed to assist investors have the adverse effect to induce the monopolist to refresh more frequently. This causes the investors fall further behind, making the task of keeping up more costly. Absent such marginal initiatives, the monopolist would also save on the costs of wasteful innovation.

mechanism. This represents a significant opportunity cost, and we quantify this welfare loss as a function of the equilibrium actions of the monopolist. Second, we study how participation arises endogenously in the market. Non-expert consumers may choose not to participate if the amount of complexity in the market is too high. We characterize the relationship between obfuscation and participation, as non-participation represents an important welfare loss.

Finally, we consider the effect of competition on obfuscation. We show that increased competition should slow obfuscation. The reason for this is that there is less to gain for each firm when they refresh the population. In essence, the information rents that firms gain by refreshing dissipate with more competition. This improves welfare as more consumers are willing to participate and fewer of them need to pay the opportunity cost of becoming an expert. This analysis also yields several predictions regarding industry concentration and the evolution of offerings with certain classes of products. We discuss these implications in detail within the paper.

Lack of sophistication regarding financial decisions degrades personal welfare as well as economic growth. Remedies for this problem have recently received much attention, though the optimal solution remains hotly debated. The analysis in this paper tends to support the view that education may not be an effective solution in retail financial markets. Indeed, as Choi, Laibson, and Madrian (2008) show, retail investors do not make improved investment choices when they have better information about the market.⁶ Based on this, there is now growing support for the use of default options to assist retail investors and improve welfare (e.g. Choi, Madrian, Laibson, and Metrick 2004). While not specifically modeled in our paper, default options would in essence make more consumers experts (by proxy) and may decrease obfuscation, especially when used on a grand scale or in markets in which people learn on their own. Libertarian paternalism as posed by Thaler and Sunstein (2003) makes sense in our model, as it would slow obfuscation and encourage participation. Investigating this debate is the subject of future research.

Our paper also adds a new and important dimension to an already extensive literature on oligopoly competition with consumer search (e.g. Diamond (1971), Salop and Stiglitz (1977), Varian (1980), Stahl (1989)). Indeed, in many existing models, consumers search for the best alternative, but the firms are unable to affect the search environment except through the prices they choose. Few notable exceptions are papers by Robert and Stahl (1993), Carlin (2008), and Ellison and Wolitzky (2008). Robert and Stahl (1993) analyze a model of sequential search in which firms may advertise to consumers in the population. Carlin (2008) and Ellison and Wolitzky (2008)

⁶Investors appear to ignore fees when making investment decisions, even when they are given salient information about the importance of taking fees into consideration.

analyze a static model in which firms simultaneously choose whether to add complexity to their pricing schedules. The analysis that we consider here departs from the above papers in several ways. First, our model is fully dynamic so that we can characterize how obfuscation evolves over time. Second, our model is general in that it can account for innovation in both prices and product characteristics. Finally, in our analysis we are able to characterize how different types of learning affects both obfuscation and policies that govern these markets.

The rest of the paper is organized as follows. In Section 2, we pose a dynamic model of obfuscation and investor sophistication, given that the producer in the market is a monopolist. In Section 3, we characterize optimal obfuscation by the monopolist, and evaluate the effect that different learning models have on welfare and policy considerations. In Section 4, we analyze how expertise arises endogenously. In Section 5, we consider the effect of participation on obfuscation. In Section 6, we characterize the effect that competition has on obfuscation. Section 7 concludes. All of the proofs are contained in the Appendix.

2 The Model

Consider a continuous time economy in which a monopolist sells a class of retail financial products to a unit mass of household consumers I . Time evolves continuously, and future cash flows are discounted at an interest rate r . The products could be a group of instruments used to finance the purchase of consumption goods (for example, a line of credit cards) or a set of investment vehicles that are available to maximize lifetime utility (for example, a family of mutual funds). For our purposes here, we describe our model using the mutual fund industry as an example, while keeping in mind that the analysis applies to a broader set of financial services.

Consumers in the market have unit demand for the product. Based on the information that they have, each consumer chooses within the class of offerings to maximize their expected payoff. At any time t , consumers are divided into two groups: sophisticated x_t and unsophisticated $y_t = 1 - x_t$. As such, sophisticated consumers always make the best choice, whereas the unsophisticated may not. For example, within a group of homogenous funds (e.g. money funds), sophisticated consumers will always choose the one with the lowest fees, whereas unsophisticated consumers may sometimes invest in funds with higher costs, and therefore not get the best deal.⁷

⁷This holds generally in our model on many product dimensions. For example, within a group of funds that are heterogeneous across multiple dimensions, sophisticated consumers will choose the one that offers the most value, whereas unsophisticated consumers may not.

Sophistication evolves according to the differential equation

$$dx_t = f(\lambda, x_t)dt, \tag{1}$$

with initial values x_0 and y_0 . By construction, $dy_t = -dx_t$. We assume that $f(\lambda, x_t)$ is strictly positive and increasing in λ so that, without a change in the product offerings, x_t increases over time while y_t decreases. Such learning might evolve individually (e.g. access to periodicals) or through contact with other consumers in the market (e.g. through information percolation).⁸ Given any particular learning process, λ parameterizes how easy it is for consumers to access public information or learn from each other and thus measures the speed of learning. For now, we consider a general learning process and characterize the optimal behavior by the monopolist. Later, in Section 3.2, we consider particular processes and contrast the effects that they have on the monopolist's behavior and economic welfare.

The monopolist earns profits $\pi(x_t, y_t)$ from its consumers at each instant in time. The rents that they capture are larger for unsophisticated consumers, since these consumers are less able to choose the most advantageous product. To capture this, we consider that at any instant, the firm's profit is

$$\pi(x_t, y_t) = ax_t + by_t,$$

where $b > a$. The profit a represents the monopoly rent that is gained from selling the product to informed consumers, whereas $b - a$ is the added gain from selling to unsophisticated consumers.⁹ As sophistication increases, the added gains to the monopolist diminish.

The monopolist may alter learning, however, by changing their product attributes (i.e. obfuscation). Specifically, we assume that, at any time t , the monopolist can refresh population learning by returning the fraction of sophisticated and unsophisticated consumers to their initial levels x_0 and y_0 . The monopolist pays a cost c to do so, and chooses their timing optimally to maximize lifetime profits. As such, the times in which the monopolist changes their product mix or their prices are given by the vector $T = (t_1, t_2, t_3, \dots)$, and for any $t \in [t_i, t_{i+1}]$, the dynamics of sophistication evolves according to (1). In the sequel, we will refer to x_0 as the fraction of expert consumers since they always catch on quickly when the monopolist makes changes. This is untrue for the fraction y_0 : whereas they may become sophisticated for a while, they do not routinely keep up with changes

⁸See Duffie and Manso (2007) for a general model of information percolation in consumer markets.

⁹The microfoundations of the assumption that the monopolist gains higher rents from unsophisticated consumers is left unmodeled, but is consistent with the search literature (e.g. Varian, 1980; Stahl, 1989) and the literature in which the timing of sales is used to discriminate between well-informed and ill-informed consumers (Salop, 1977; Conlisk, Gerstner, and Sobel, 1984; Rosenthal, 1982; Sobel, 1984).

in the market. For now, we take the values of x_0 and y_0 as given exogenously, but we consider their evolution endogenously in Section 4.

Before posing the monopolist’s problem, it is instructive to consider an example of obfuscation. Consider a firm that offers two homogeneous funds with different fee structures. Sophisticated consumers invest in the one with lower fees, whereas unsophisticated consumers choose one of the them randomly. As time progresses and unsophisticated investors learn about the fund with lower fees, fewer investors demand the fund with higher fees. The firm may then add a third fund of the same type (or a new class of shares), that have low fees and progressively raise the fees of the “old” low-fee fund. Such pricing behavior has been documented previously in money funds by Christoffersen and Musto (2002). The result is that expert investors will catch on quickly to this strategy and switch funds, while a new breed of unsophisticated customers evolves. What our model captures, therefore, is the idea that consumers need to keep up with innovation in prices and quality in order to continue to get the best deal. Other examples might include different funds (or share classes) that sell for the same price, but have quality differences that require different investment on the part of the firm. The model as posed is general to consider heterogeneity on multiple dimensions.

The problem that the monopolist faces may be posed as

$$\sup_{T \in \mathcal{T}} \int_{t=0}^{\infty} e^{-rt} \pi(x_t, y_t) - C(r, T) \tag{2}$$

where $C(r, T)$ sums the discounted lifetime costs of obfuscation according to the plan T . We characterize the solution to this problem in the next section.

3 Obfuscation and Sophistication

We begin by solving the monopolist’s problem and characterize its solution in generality, given that learning proceeds according to (1). Then, we consider specific examples of learning processes and contrast the obfuscation that takes place in each case. Following this, we discuss several welfare and policy considerations that arise based on the type of learning that takes place in the market.

3.1 Optimal Obfuscation

The monopolist’s problem is stationary. That is, after the monopolist obfuscates, he faces a problem that is isomorphic to the one he faced at $t = 0$. The following proposition relies on dynamic programming techniques to simplify the monopolist’s problem.

Proposition 1. *If the vector $T = (t_1, t_2, t_3, \dots)$ solves (2), then $t_{i+1} - t_i = t_i - t_{i-1}$ for any i . Therefore, the monopolist's problem reduces to choosing the duration t of each cycle:*

$$\max_t \frac{\int_0^t e^{-rs} \{ax_s + b(1 - x_s)\} ds - e^{-rt}c}{1 - e^{-rt}}. \quad (3)$$

According to Proposition 1, we may focus on the solution to the problem in (3) to derive the optimal plan T^* , which is stationary. Moreover, as we show in the proof of Proposition 1, the stationary plan is strictly superior to any arbitrary control sequence that changes over time (non-stationary). The next proposition characterizes the solution to (3).

Proposition 2. *(Optimal Obfuscation) There exists a unique optimal stopping time $t^* > 0$ that solves the monopolist problem. If*

$$c < \bar{c}(\lambda, x_0) \equiv \lim_{t \rightarrow \infty} \frac{r \int_0^t e^{-rs} \{ax_s + b(1 - x_s)\} ds - \{ax_t + b(1 - x_t)\}}{r} \quad (4)$$

then the optimal stopping time t^* is finite and solves

$$(1 - e^{-rt^*})\{ax_{t^*} + b(1 - x_{t^*})\} - r \int_0^{t^*} e^{-rs} \{ax_s + b(1 - x_s)\} ds + rc = 0. \quad (5)$$

Otherwise, $t^* = \infty$. Moreover, for $t^* < \infty$,

$$(i) \quad \frac{\partial t^*}{\partial b} < 0$$

$$(ii) \quad \frac{\partial t^*}{\partial a} > 0$$

$$(iii) \quad \frac{\partial t^*}{\partial c} > 0$$

According to Proposition 2, obfuscation takes place more frequently when the additional rents that are gained from unsophisticated consumers are higher ($b - a$ high) and when the cost is lower. That is, if resetting the learning process is more valuable because unsophisticated consumers forfeit significant surplus, then the monopolist will wish to capture these rents as frequently as possible. Since $c > 0$, though, they will not optimally do this continuously, that is, $t_{i+1} - t_i > 0$ for all i . When the cost is higher, the monopolist will wait longer before he incurs this cost, *ceteris paribus*. If the cost is sufficiently high, that is, if $c > \bar{c}$, the monopolist will never change their prices or their product mix.

The relationship between t^* and λ is a bit trickier. Intuitively, if learning occurs very slowly, the monopolist will make changes to their product line infrequently. In fact, if $\lambda \rightarrow 0$, then the monopolist will never make any changes because there is no benefit to paying the cost c . As learning

occurs more frequently (higher λ), then the monopolist might want to make changes more quickly to keep resetting the process and stay ahead of consumer sophistication. However, as learning becomes sufficiently fast, there is a diminishing benefit to obfuscation. To see this, consider a limiting case when $\lambda \rightarrow \infty$, that is when sophistication evolves instantaneously. In this case, the rents that are gained compared to the monopoly rent a are negligible because they are short-lived. Since, $c > 0$, the optimal strategy for the monopolist is never to make changes. Therefore, when $\lambda \rightarrow 0$ and when $\lambda \rightarrow \infty$, we expect $t^* = \infty$, whereas for values of λ in between we may observe a finite t^* . Thus, there exists a non-monotonic relationship between λ and t^* .

The following proposition formalizes this result.

Proposition 3. *Suppose that $c < \sup_{\lambda} \bar{c}(\lambda, x_0)$. If*

$$\lim_{\lambda \rightarrow 0} x_t = x_0 \quad \text{and} \quad \lim_{\lambda \rightarrow \infty} x_t = 1 \quad \forall t > 0, \quad (6)$$

then

$$\lim_{\lambda \rightarrow 0} t^*(\lambda) = \infty \quad \text{and} \quad \lim_{\lambda \rightarrow \infty} t^*(\lambda) = \infty, \quad (7)$$

and the function $t^*(\lambda)$ is non-monotone in λ .

Proposition 3 can be appreciated as follows. The conditions about a particular learning process in (6) are sufficient for $t^*(\lambda)$ to be non-monotone in λ . The first condition says that as the rate of learning converges to zero, then the fraction of sophisticated consumers remains the same (the initial level) for any fixed time. The second condition says that as the rate of learning converges to infinity, the entire population becomes sophisticated for any arbitrarily small time. If these two conditions hold for a particular learning process, then $t^*(\lambda)$ is non-monotone in λ .

Now, we consider the relationship between t^* and x_0 . We first state the proposition that characterizes this relationship and then describe it intuitively.

Proposition 4. *Suppose that $c < \sup_{x_0} \bar{c}(\lambda, x_0)$. If*

$$\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0, \quad (8)$$

then $\frac{\partial t^*}{\partial x_0}$ is monotone and positive. If

$$\lim_{x_0 \rightarrow 0} x_t = 0 \quad \forall t > 0 \quad (9)$$

then

$$\lim_{x_0 \rightarrow 0} t^*(x_0) = \infty \quad \text{and} \quad \lim_{x_0 \rightarrow 1} t^*(x_0) = \infty, \quad (10)$$

and the function $t^*(x_0)$ is non-monotone in x_0 .

According to Proposition 4, if the learning process has the property in (8), then as x_0 increases, the monopolist obfuscates less frequently. This means that for higher x_0 , the rents that are collected from unsophisticated consumers are strictly less, and therefore the benefits to paying the cost c are decreased. The higher x_0 is, the more time passes between cycles. As we will soon illustrate by example, the condition in (8) may exist when consumers learn independently from each other, but does not hold generally for group learning processes.

If the condition in (9) holds, however, then the relationship between t^* and x_0 is non-monotonic. Intuitively, the limit in (9) says that if the fraction of sophisticated consumers tends to zero, that no matter how much time passes, all consumers remain unsophisticated. This condition will hold generally when consumers learn from each other. If no one is sophisticated in the first place, then there is no one to learn from. In contrast, if consumers learn on their own from accessing information from outside sources, it may be that x_t will increase over time, despite the fact that $x_0 = 0$ initially.

To better appreciate how the conditions in Propositions 3 and 4 affect obfuscation, it is instructive to consider a few examples.

3.2 Learning and Information Percolation

Based on the information percolation model of Duffie and Manso (2007), we analyze two specific settings: one in which investors learn by themselves and one in which investors learn from each others.

Let us first consider a learning process in which investors learn from their own research. This may occur by reading periodicals, accessing news through the media, or reading a prospectus. Formally, let us suppose that (1) takes the form

$$dx_t = \lambda(1 - x_t)dt, \quad (11)$$

which is an exponential learning model in which a fixed proportion λ of unsophisticated consumers become sophisticated at each point in time. Integrating and using the initial condition x_0 yields the solution

$$x_t = 1 - (1 - x_0)e^{-\lambda t}. \quad (12)$$

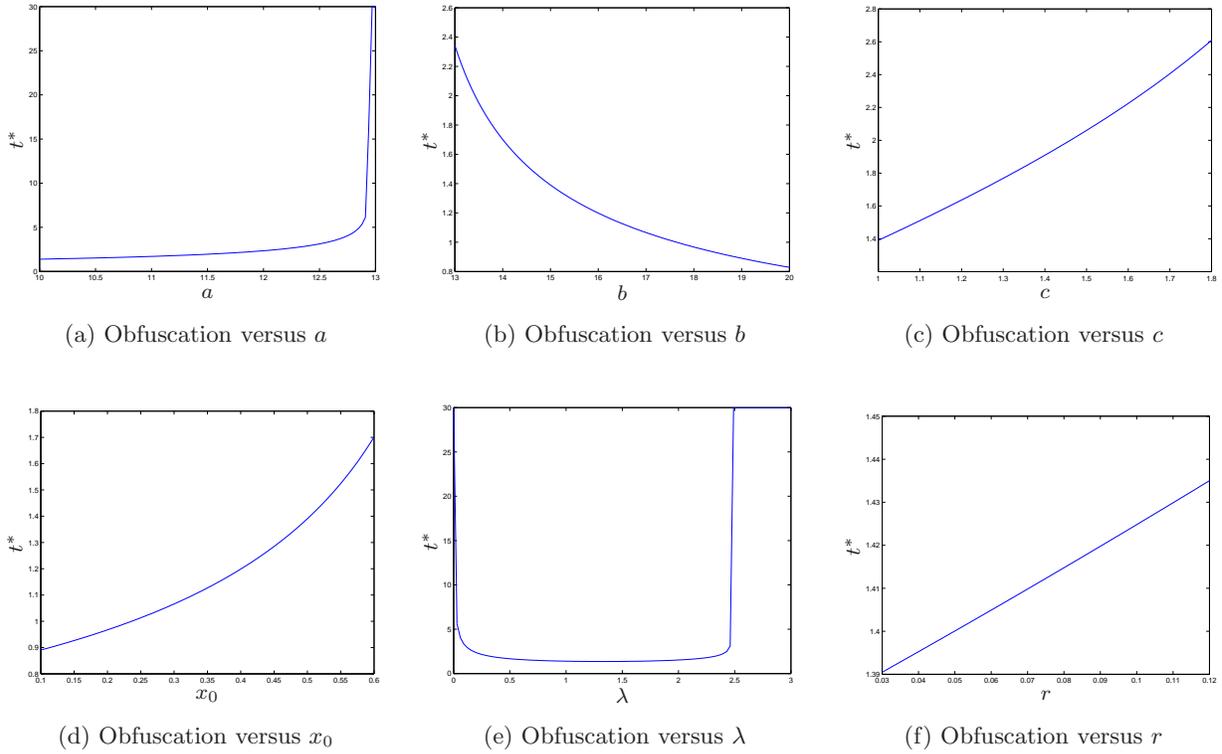


Figure 1: Learning On Your Own: The series of figures (a)-(f) plot t^* versus the fundamental parameters in the model, when the learning process involves learning about products through the use of periodicals or media. The time t^* is monotonically decreasing in b and increasing in a , c , x_0 , and r . The relationship between t^* and λ is U-shaped. Parameters, when held fixed, are $r = 0.03$, $\lambda = 1$, $x_0 = 0.5$, $a = 10$, $b = 15$, and $c = 1$.

Now, we can consider (12) in terms of the conditions in Propositions 3 and 4. For this process,

$$\frac{\partial^2 x_t}{\partial x_0 \partial t} = -e^{-\lambda t} \lambda, \quad (13)$$

which is always negative. From Proposition 4, we have that the optimal time to obfuscate is increasing in x_0 . This makes intuitive sense, since in this example the ability to learn is independent of other consumers' sophistication, so that as x_0 rises the rents available to the monopolist decrease. Obfuscation is less attractive and occurs with decreased frequency.

To show that the relationship between t^* and λ is non-monotone, we compute that

$$\lim_{\lambda \rightarrow 0} x_t = x_0$$

and

$$\lim_{\lambda \rightarrow \infty} x_t = 1$$

Consequently, $t^* = \infty$ when λ approaches 0 or ∞ . Intuitively, this implies that if there are no sources of information and consumers do not learn, then there is no incentive for the monopolist to refresh the consumer population. In contrast, if access to media or periodicals allows consumers to educate themselves quickly, then the monopolist will avoid obfuscation because refreshing the consumer population is a futile effort.

Consider the example in Figure 1, in which consumers learn without the help of others. The series of subfigures plot t^* versus the underlying parameters, while holding all else fixed. As predicted by Proposition 2, t^* is strictly decreasing in b and increasing in a and c . As discussed, t^* is strictly increasing in x_0 and is non-monotonic (U-shaped) in λ .

Now, let us consider an alternative learning process in which investors learn from each other. Indeed, as unsophisticated consumers meet those who are informed, sophistication within the population rises. Such meetings may occur via friends, relatives, co-workers, or advisors. The key factor in these types of learning processes is that the chance that an unsophisticated consumer becomes sophisticated depends directly on the fraction of consumers who are already knowledgeable. Let us consider a particular example in which investors meet each other in bilateral meetings and (1) takes the form

$$dx_t = \lambda x_t(1 - x_t)dt. \quad (14)$$

This process differs from (11) in that the rate at which market participants become sophisticated depends on the proportion of sophisticated consumers in the market. Integrating and using the initial condition x_0 yields the solution

$$x_t = \frac{x_0 e^{\lambda t}}{(1 - x_0) + x_0 e^{\lambda t}}. \quad (15)$$

Again, we can consider (15) in terms of the conditions in Propositions 3 and 4. With this process,

$$\frac{\partial^2 x_t}{\partial x_0 \partial t} = \frac{e^{\lambda t} \lambda (1 - x_0 + e^{\lambda t} x_0)}{(1 - (1 - e^{\lambda t}) x_0)^3}, \quad (16)$$

which can be positive or negative depending on t . Therefore, based on Proposition 4, we are not guaranteed that the optimal time to obfuscate is monotonically increasing in x_0 . In fact, we can use the condition in (9) to show that the relation between t^* and x_0 is non-monotone. Specifically,

$$\lim_{x_0 \rightarrow 0} x_t = 0. \quad (17)$$

This follows from the fact that if no one is sophisticated ($x_0 = 0$), there is no one to learn from. It then follows from Proposition 4 that the relationship between x_0 and t^* is non-monotone.

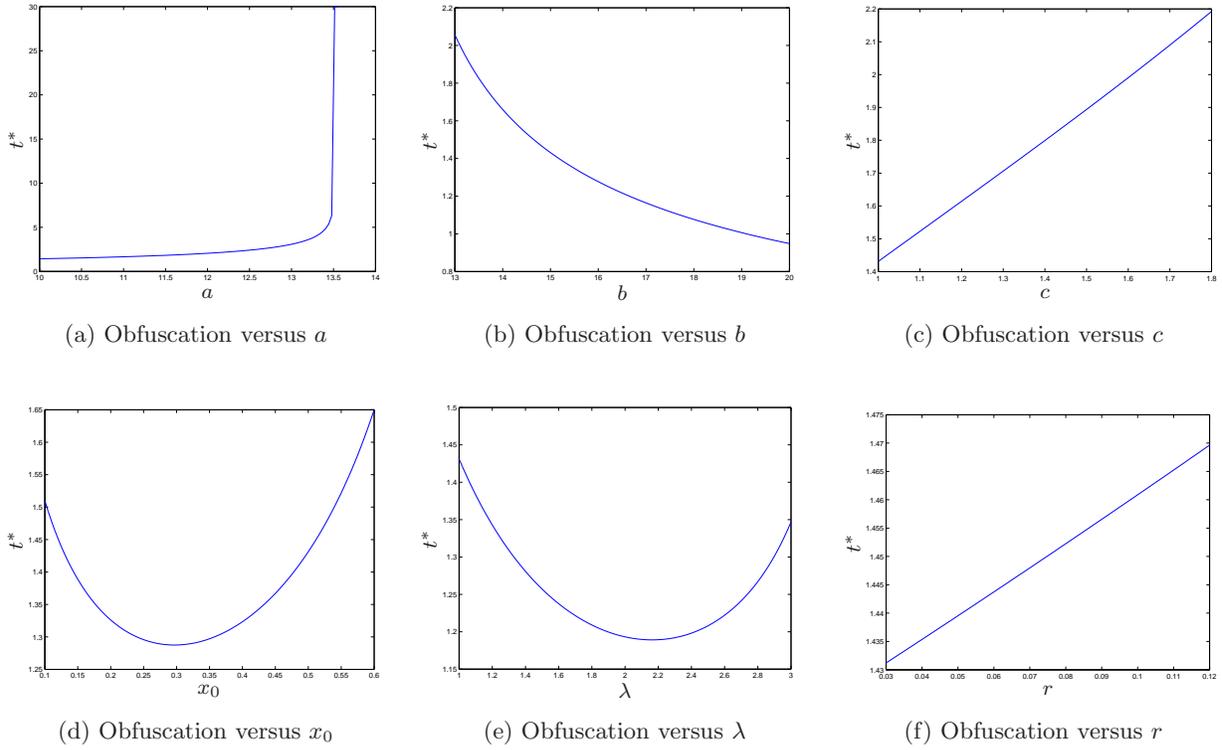


Figure 2: Learning From Others: The series of figures (a)-(f) plot t^* versus the fundamental parameters in the model, when the learning process involves learning about products through bilateral meetings between consumers. The time t^* is monotonically decreasing in b and increasing in a , c , and r . The relationship between t^* and λ is U-shaped, as is the relationship between t^* and x_0 . Parameters, when held fixed, are $r = 0.03$, $\lambda = 1$, $x_0 = 0.5$, $a = 10$, $b = 15$, and $c = 1$.

As before, using conditions in (6), we can show that the relation between t^* and λ is also non-monotone. For that, it is enough to note that

$$\lim_{\lambda \rightarrow 0} x_t = x_0 \quad (18)$$

and

$$\lim_{\lambda \rightarrow \infty} x_t = 1. \quad (19)$$

Consequently, t^* approaches infinity when λ approaches 0 or ∞ .

Consider the example in Figure 2, in which consumers learn from each other. The series of subfigures plot t^* versus the underlying parameters, while holding all else fixed. Again, as predicted by Propositions 2-4, t^* is strictly decreasing in b , increasing in a and c , and non-monotonic (U-shaped) in λ and x_0 .

Now, we consider how these different learning process may affect welfare in the market and the efficacy of educational initiatives.

3.3 Welfare and Policy Implications

So far in the model, the only source of welfare loss is the cost that the monopolist incurs when he changes his product line. The prices that consumers pay and the extra rents gained from unsophisticated consumers are transfers between the parties to the transaction. A social planner who wishes to maximize welfare in this market, therefore, seeks to minimize the quantity

$$\mathcal{L} = C(r, T),$$

where we recall that $C(r, T)$ is the lifetime costs of obfuscation, given the plan T .

The social planner may consider undertaking initiatives to raise the rate of learning λ or alter the initial number of sophisticated investors in the market x_0 . For example, subsidizing websites to enhance consumer education or legislating initiatives to enhance disclosure might increase the ability for people to learn about the market (i.e. increase λ). Requiring that financial education be an integral part of secondary education would be likely to make more people sophisticated in the first place (i.e. raise x_0).

The discussion in Section 3.2, though, implies that optimal intervention through policies needs to take into account the way in which people learn. Likewise, the magnitude of intervention is equally important as small scale programs might actually decrease welfare. For example, consider in the two examples discussed that the relationship between t^* and λ is non-monotonic. For low λ , small increases in the speed of learning will decrease the time to obfuscation and will thus decrease welfare.

The key here is that when a social planner considers an initiative to improve investor sophistication, they need to consider the natural response on the part of the monopolist to maximize rents, given the initiative that is undertaken. In this way, for any λ on the decreasing portion of the curve in Figure 1e or 2e, a small supplement to λ will lead to more obfuscation by the monopolist, which destroys value. Only if the magnitude of intervention is large enough will the market reach the upward sloping portion in which increased access to information leads to lower obfuscation. For low values of λ it may be more effective for the social planner to make learning more difficult, decreasing λ .

The differences in the relationship between t^* and x_0 among the two examples of learning also highlights that the social planner needs to take into account the mechanism by which people

learn when they set policy. If people learn from periodicals, increasing education is always welfare enhancing (see Figure 1d) no matter how unsophisticated the population is. This does not hold for learning processes in which people learn from each other (see Figure 2e). If x_0 is low, small positive increments will induce the monopolist to obfuscate more frequently and destroy welfare. Only larger scale initiatives are able to overcome this loss in value.

So far, in the model, the only loss from obfuscation arises from the cost the monopolist pays to “refresh” the product line. Realistically, though, there are other costs of obfuscation in the market that a social planner needs to consider. First, consumers incur costs to keep up with changes in the market (gain expertise). The more that firms invest in obfuscation, the more consumers will have to pay an opportunity cost (of time and resources) to make sure they get a good deal. Second, obfuscation has an adverse effect on the willingness of consumers to participate in the market. If the market is too confusing, some consumers may choose to just drop out, which represents an important deadweight loss. We turn to these issues in the next two sections.

4 Endogenous Expertise

Now, we consider that the fraction of expert consumers x_0 arises endogenously. The timing of the game is as follows. First, each consumer $i \in I$ chooses whether to pay a cost k_i to become an expert and join the x_0 population. Following this, the rest of the game follows in the same fashion as in Section 2. The cost k_i is a one-time cost, and could be considered to be the decision whether to obtain a financial education. Alternatively, it could represent the decision whether to become familiar with a particular sector of the financial system.¹⁰ Consumers in the x_0 pool are experts and know to keep up with developments as time goes on. Those in the y_0 pool may learn about particular products over time, but do not have higher levels of sophistication that are required to make sure that they always get the best deal. Learning takes place as before according to the differential equation in (1).

Suppose that consumers are heterogeneous and the costs to become an expert are distributed over the support $[0, \bar{k}]$ according to a twice continuously differentiable function $G(\cdot)$. Define B as the expected benefit of becoming an expert given the actions of other consumers and the expected actions of the monopolist. Therefore, if $k_i \leq B$, consumer i becomes sophisticated. It follows then

¹⁰An alternative specification of the model would be to allow consumers to make this choice each time the monopolist refreshes their product line. Focusing on Markov Perfect Equilibria, the results would be qualitatively similar to what we derive here. Of course, in such a model, other Nash Equilibria that are not stationary might arise, but this is beyond the scope of our analysis.

that

$$x_0 = G(B).$$

The value of B will depend on x_0 and on the t^* that is chosen by the monopolist based on x_0 and the underlying parameters. We denote this dependence as $B(t^*, \vec{v})$, where \vec{v} represents the underlying parameters. The function B is decreasing in t^* since the rents that the unsophisticated pay decrease when less obfuscation takes place. Also, $\frac{\partial B}{\partial b} > 0$ and $\frac{\partial B}{\partial a} < 0$. Define \underline{B} as the lower limit of B , that is, the benefit to becoming an expert when $t^* = \infty$. We assume that $\underline{B} < \bar{k}$; otherwise, all consumers would become experts leading to an uninteresting interaction.

In any equilibrium of this game, the fraction of expert consumers, denoted x_0^* , is implicitly defined by

$$x_0^* = G\left(B\left(t^*, \vec{v}\right)\right) \equiv H(x_0^*). \quad (20)$$

Going forward, we follow the standard approach of Debreu (1970) and Mas-Colell (1985) and focus on the “regular” equilibria of the game. Such equilibria are robust to small perturbations of the set of parameters, and are therefore locally unique, which allows for meaningful comparative statics.¹¹ By straightforward application of Sard’s Theorem and Mas-Colell (1985), other pathologic equilibria may be ruled-out as non-generic. Specifically, it is straightforward to show that, except for a set of parameters having zero measure in the general parameter space, the equilibria that will arise will be regular.¹²

The following proposition proves existence of a fixed point and characterizes the regular equilibria of the game. In doing so, we distinguish equilibria based on their stability in the sense of Vives (1990, 1999, 2005). Specifically, an equilibrium is said to be locally stable at a point x_0^* if there exists a neighborhood around it such that for any initial position x_0 within that neighborhood, the system converges to the point x_0^* according to the function $H(x_0)$.

Proposition 5. *A solution $x_0^* \in (0, 1)$ exists for the expression in (20) for any $G(\cdot)$. If $\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0$, there exists a unique $x_0^* > 0$ with the following properties:*

$$(i) \quad \frac{\partial x_0^*}{\partial b} > 0$$

¹¹Originally, Debreu (1970) and Mas-Colell (1985) focused on “regular” equilibria to characterize general equilibria in exchange economies. Indeed, there are pathologic situations in which the excess demand function $z(p)$ might lead to an infinite number of equilibria, preventing comparative statics exercises. By limiting the focus to regular equilibria and proving that such pathologic cases are non-generic, local uniqueness and differentiability of the equilibria is guaranteed, thereby allowing for comparative statics to be generated.

¹²See Chapter 8 in Mas-Colell (1985) for a thorough discussion of genericity analysis and Carlin, Dorobantu, and Viswanathan (2008) for an application of this in finance. A proof of this statement would follow from the same arguments in the proof of Proposition 2 in Carlin, Dorobantu, and Viswanathan (2008).

$$(ii) \frac{\partial x_0^*}{\partial a} < 0$$

$$(iii) \frac{\partial x_0^*}{\partial c} < 0$$

If $\frac{\partial^2 x_t}{\partial x_0 \partial t} \geq 0$, there may exist multiple regular equilibria. However, in any stable equilibrium, the same properties hold.

According to Proposition 5, if $\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0$ then there exists only one equilibrium for the game. This will generally be the case when people learn on their own from periodicals, which was the case when $f(\lambda, x_t)$ took the form in (11). In other cases, where this condition does not hold (e.g. the process in (14)), we are not guaranteed to have a unique equilibrium. Indeed, two classes of regular equilibria may form: a stable one with the properties specified in Proposition 5 and an unstable variant that has the opposite comparative statics. The proof of Proposition 5 details this distinction.

The comparative statics in Proposition 5 imply that as the rents to the monopolist $b - a$ increase, more consumers will become experts in the first place. This occurs through two channels. First, as $b - a$ increases, unsophisticated consumers will pay higher rents over time, so they are more willing to invest in education. Second, and equally important, rising $b - a$ induces the monopolist to increase the frequency with which they change their product mix, which drives more consumers to gain expertise in the first place. The same relationship holds with regard to the cost c .

Based on this analysis, there is another cost of obfuscation to society. Specifically, a proportion of consumers will have to allocate resources to gain expertise, which represents an opportunity cost. This cost may then be computed as

$$\mathcal{K} = \int_0^B kdG(k).$$

As in Section 3.3, this has important policy implications when a social planner considers initiatives to change learning through λ . As shown there, a change in λ may induce the monopolist to change their product mix more frequently. Looking forward, more consumers will expend resources to gain expertise, which destroys value. If the magnitude of intervention is large enough though increased access to information will lead to lower obfuscation and higher welfare.

5 Participation and Obfuscation

So far, we have assumed that all consumers participate in the market and make a purchase. Now, we relax that assumption and consider an alternative specification for the consumer population.

In addition to sophisticated and unsophisticated consumers (fraction ϕ), there is a group $1 - \phi$ of consumers who are either unaware of the product, or who have decided that this product is not appropriate for their needs. As such, the parameter ϕ may proxy for the scope of the product: low ϕ is a specialized product whereas products with high ϕ have more widespread use. The monopolist receives no demand from consumers who do not participate. At any given point in time, then, there is fraction ϕx_t of sophisticated consumers and ϕy_t of unsophisticated consumers.

We begin by taking ϕ as given exogenously. The monopolist's instantaneous profit is

$$\pi(x_t, y_t, \phi) = \phi\{ax_t + by_t\},$$

where $b > a$. As before, the firm may choose to refresh the population of market participants at any time t . The times in which the monopolist does this are given by the vector $T = (t_1, t_2, t_3, \dots)$.

The monopolist, therefore, solves

$$\sup_{T \in \mathcal{T}} \int_{t=0}^{\infty} e^{-rt} \pi(x_t, y_t, \phi) - C(r, T). \quad (21)$$

As in Section 3, the monopolist's problem is stationary and can be reposed as

$$\max_t \frac{\int_0^t e^{-rs} \phi\{ax_s + b(1 - x_s)\} ds - e^{-rt} c}{1 - e^{-rt}}. \quad (22)$$

The following proposition characterizes the effect that participation has on the solution to the problem.

Proposition 6. (*Obfuscation and Participation*) *There exists a unique optimal stopping time $t^* > 0$ that solves the monopolist problem. For $t^* < \infty$, $\frac{\partial t^*}{\partial \phi} < 0$.*

According to Proposition 6, products that are more specialized (low ϕ) will have lower rates of obfuscation than products that have more widespread use (high ϕ). The result is a straightforward application of the analysis in Section 3. As ϕ rises, the aggregate rents to be gained from unsophisticated consumers rises, which induces the monopolist to refresh the population more frequently. The result has important and interesting empirical implications. Comparing two classes of mutual funds with different specialization (e.g. S&P 500 Index funds versus Precious Metals funds), we would expect the product mix of funds to change more frequently with the funds with less specialization. Empirically, one could proxy for ϕ by considering the fraction of total assets in the marketplace invested in a particular class of funds, or by considering the number of investors who invest in that class of funds. Using this as a proxy, one could test the implication by correlating it

with the number of share classes or offerings per fund family in that market. The analysis might be performed either cross-sectionally, or with a time-series.

From a welfare perspective, it is reasonable to assume that there is a cost associated with having unaware consumers that do not participate in the market. In this case, a social planner would like to increase awareness and participation to reduce those costs. However, as Proposition 6 shows, increasing participation leads to more frequent wasteful obfuscation. From the social planner perspective, there should exist an optimal level of participation that balances the losses from leaving consumers out of the market with the losses from the more frequent wasteful obfuscation.

Let us now suppose that all x_0 -types participate, but that y_0 -types decide ex ante whether to participate in this market. Let $V(t^*, \vec{v})$ be the value that a y_0 -type consumer obtains when they purchase the product. This is composed of a positive surplus S from owning the product and rents that are forfeited to the monopolist. The rents are a function of the underlying parameters \vec{v} and the choice of t^* by the monopolist. By construction, the value $V(t^*, \vec{v})$ is strictly less than that gained by experts in the market.

Let us suppose that y_0 -types are averse to complexity in the market. This may arise because they are ambiguity averse or because complexity makes learning more challenging. Consumer i suffers a disutility $d_i(t^*)$ that is increasing in obfuscation (decreasing in t^*). Consumers are heterogeneous on this dimension, and disutility is distributed in the y_0 population according to a twice continuously differentiable distribution function M over the support $[0, \bar{d}]$, which depends on t^* . We denote this dependence as M_{t^*} . For example, if $t_1^* < t_2^*$, then $M_1(d) \leq M_2(d)$ for all $d \in [0, \bar{d}]$.

As such, a consumer participates in the market iff

$$d_i \leq V(t^*, \vec{v}). \tag{23}$$

Participation in the market is a fixed point implicitly defined by

$$\phi(t^*) = M_{t^*}(V(t^*, \vec{v}), \phi). \tag{24}$$

Proving that a fixed point in this problem exists is straightforward and follows the same logic as in the proof of Proposition 5. Lack of participation adds another deadweight loss to the analysis in Sections 3.3 and 4. Specifically, the loss from non-participation can be computed as

$$\mathcal{NP} = (1 - \phi)y_0S. \tag{25}$$

Since ϕ is a function of t^* , by inspection of (25), it is clear that more frequent obfuscation causes a deadweight loss through non-participation.

The effect that rents have on obfuscation operate through several channels. First, as before, holding all else constant, as b increases fewer y_0 types are willing to participate in the market. Second, as b increases and the monopolist has a greater incentive to obfuscate more frequently, the rents that are lost when participating are higher. Third, as there is more complexity in the market, the disutility that consumers experience is higher.

The effect on λ remains ambiguous, though, and depends on the other underlying parameters in the model. This is because changes in λ may have direct and indirect effects on $V(t^*, \vec{v})$. Increasing λ allows non-experts to learn faster about the market and makes it more attractive to participate (increases $V(t^*, \vec{v})$ directly). However, as noted in Section 3.3, increasing λ may induce the monopolist to lower t^* , which has the effects of decreasing $V(t^*, \vec{v})$ and increasing disutility (indirect effects). Which effects, direct or indirect, dominate is a function of the underlying parameters. Therefore, while educational initiatives may sometimes destroy value by inducing non-participation, this does not hold under all parameter specifications.

Of course, throughout the paper so far, we have considered the effects of obfuscation when a monopolist produces in the market. Now, we turn our attention to the effects of competition on obfuscation and welfare.

6 Competition and Obfuscation

Consider now that n homogeneous firms, indexed by $j \in N = \{1, \dots, n\}$, sell a line of retail financial products. The firms have no ex ante advantage over competitors, have no capacity constraints, and produce the same number of choices within the class of products that are marketed to consumers.

The composition of the consumer population remains unchanged from Section 2 except that we make the assumption that unsophisticated consumers purchase their goods randomly from any of the n firms. Specifically, at any time t , the demand that any firm receives from unsophisticated consumers is $\frac{y_t}{n}$. As such, we adopt the standard approach that is taken in both the literature on consumer search theory (e.g. Salop and Stiglitz (1977) and Varian 1980) and household finance (e.g. Carlin 2008).¹³

Each firm, therefore, faces a symmetric problem in which the fraction of uninformed consumers is identical for all of the firms:

$$\sup_{T \in \mathcal{T}} \int_{t=0}^{\infty} e^{-rt} \pi(x_t, y_t, n) - C(r, T), \quad (26)$$

¹³See either Stahl (1989) or Baye, Morgan, and Scholten (2006) for a complete review of consumer search theory.

given that all of the other firms do likewise. Since this is a symmetric and stationary problem, (26) may then be expressed as

$$\max_t \frac{\int_0^t e^{-rs} b \frac{y_s}{n} ds - e^{-rt} c}{1 - e^{-rt}}. \quad (27)$$

Note that, as compared to the problem in (3), the rents from unsophisticated consumers are shared equally and $a = 0$ since sophisticated consumers will always get the best deal. That is, for the most attractive product in each line, a Bertrand-type competition ensues which makes the rents from sophisticated consumers equal zero.

The following proposition characterizes optimal firm behavior when there is competition in the product market.

Proposition 7. (*Competition and Obfuscation*) *For any n , there exists a unique symmetric equilibrium in which all firms use an optimal stopping time $t^*(n) > 0$ such that*

$$(i) \quad \frac{\partial t^*(n)}{\partial b} < 0$$

$$(ii) \quad \frac{\partial t^*(n)}{\partial c} > 0$$

$$(iii) \quad \frac{\partial t^*(n)}{\partial n} > 0$$

As $n \rightarrow \infty$, $t^*(n) \rightarrow \infty$.

According to Proposition 7, each firm's optimal choice is qualitatively similar to that when they are a monopolist. The primary difference, though, is that the frequency of obfuscation decreases when there is more competition. In fact, under perfect competition, obfuscation disappears altogether and all consumers eventually become sophisticated. As such, the information rents from obfuscation dissipate as competition becomes fierce.

One concern that may arise is that the results of Proposition 7 depend on the assumption that obfuscation involves only fixed costs c to each firm, while the rents earned by each firm are decreasing in the number n of firms. However, the results of Proposition 7 are robust to an alternative formulation in which the obfuscation cost incurred by firms each time they refresh their product line is $c(\alpha) = c_1 + c_2\alpha$, where α is the market share of the firm. In this case, as long as $c_1 > 0$, obfuscation is decreasing in n and disappears altogether when all consumers become sophisticated as in the benchmark case studied above.

The decrease in obfuscation associated with competition has straightforward welfare implications based on the discussion in previous sections of the paper. Clearly, as obfuscation decreases the

incentive to become an expert (an x_0 -type) decreases, so that consumers in aggregate incur lower costs when they participate in the market. Additionally, more y_0 -types may participate in the first place, as the value to doing so increases with lower obfuscation. It remains ambiguous, however, the effect that lower obfuscation has on the aggregate costs that firms incur. On an individual basis, $C(r, T)$ is lower when there is more competition, but it is unclear whether $nC(r, T)$ is lower in aggregate.

Proposition 7 has several empirical implications. First, the model predicts that the number of redundant products within each retail line should decrease with higher competition. This might be tested cross-sectionally in the mutual fund sector by correlating the Herfindahl index with the number of share classes within a fund line. Based on theory, there should be a negative correlation between the number of firms competing and the number of share classes that are offered by each firm. The second prediction is that changes within classes of products should occur less frequently with more competition. This might be tested in the same fashion using panel data from the mutual fund industry.

7 Concluding Remarks

Many retail investors lack sophistication regarding financial products, but choose to participate in the market. Over time, investors learn but are required to keep abreast of developments in the market as they occur. Such changes are endogenously induced by producers of financial market, and must be taken into account when government-sponsored educational initiatives are implemented.

In this paper, we study the interaction between obfuscation and investor sophistication in a dynamic setting. We characterize optimal cycles of obfuscation and demonstrate how they change based on primitives in the market: the extra rents available from unsophisticated investors, the baseline financial education that investors possess, the speed at which learning takes place, and the underlying mechanism in which sophistication evolves. Strikingly, we show that small educational initiatives may induce further obfuscation by producers, which destroys economic surplus. Such wasteful obfuscation is enhanced as more consumers participate in the market. On the other hand, major educational initiatives are effective in protecting investors and reducing wasteful obfuscation, but may entail high implementation costs. Our results suggest that an alternative way to reduce obfuscation and increase welfare is to increase competition among producers.

The analysis in this paper supports the view that education may not be an effective solution in retail financial markets. As Choi, Laibson, and Madrian (2008) show, retail investors do not

make improved investment choices when they have better information about the market. There is now growing support for the use of default options to assist retail investors and improve welfare (e.g. Choi, Madrian, Laibson, and Metrick 2004). While not specifically modeled in our paper, default options would in essence make more consumers experts (by proxy) and may decrease obfuscation, especially when used on a grand scale or in markets in which people learn on their own. Such libertarian paternalism makes sense in our model, as it would slow obfuscation and encourage participation. Given the welfare impact of such policies, continued exploration appears warranted.

Appendix

Proof of Proposition 1

Let t^* solve (3). Then the discounted profits achieved by the monopolist under the optimal policy are given by:

$$V \equiv \frac{\int_0^{t^*} e^{-rs} \{ax_s + b(1 - x_s)\} ds - e^{-rt} c}{1 - e^{-rt^*}}. \quad (28)$$

We need to show that there does not exist another policy that achieves a higher discounted profits. Let $T = (t_1, t_2, \dots)$ be an arbitrary policy. Then, by the above definition of t^* ,

$$V \geq \int_{t_i}^{t_{i+1}} e^{-rs} \{ax_s + b(1 - x_s)\} ds - e^{-r(t_{i+1} - t_i)} c + e^{-r(t_{i+1} - t_i)} V, \quad (29)$$

for $i = 0, 1, \dots$ and $t_0 = 0$. Multiplying both sides of the above inequalities by e^{-rt_i} we have

$$e^{-rt_i} V \geq e^{-rt_i} \int_{t_i}^{t_{i+1}} e^{-rs} \{ax_s + b(1 - x_s)\} ds - e^{-rt_{i+1}} c + e^{-rt_{i+1}} V \quad (30)$$

Summing the inequalities from $i = 0$ to $i = I$ causes telescopic cancellation on the left-hand side, leaving only

$$V - e^{-rt_{I+1}} V \geq \sum_{i=0}^I \int_{t_i}^{t_{i+1}} e^{-rs} \{ax_s + b(1 - x_s)\} ds - e^{-rt_{i+1}} c. \quad (31)$$

Taking the limit as I goes to infinity yields the result. \blacksquare

Proof of Proposition 2

The derivative of the objective function in (3) with respect to t is:

$$\frac{e^{-rt} \{ax(t) + b(1 - x(t))\} + re^{-rt} c}{1 - e^{-rt}} - \frac{re^{-rt} (\int_0^t e^{-rs} \{ax(s) + b(1 - x(s))\} ds - e^{-rt} c)}{(1 - e^{-rt})^2} \quad (32)$$

The optimal time t^* at which the monopolist decide to innovate solves thus the following first order condition:

$$rc + (1 - e^{-rt}) \{ax(t) + b(1 - x(t))\} - r \int_0^t e^{-rs} \{ax(s) + b(1 - x(s))\} ds = 0. \quad (33)$$

Because $x(t)$ is increasing in t , the derivative left-hand side of (33) is positive for $t < t^*$ and negative for $t > t^*$ showing that t^* , when it exists, is indeed a global maximum. If there does not exist a t^* that solves (33), then the derivative of the objective function with respect to t is always positive and the maximum is at infinity, meaning that it is optimal for the monopolist never to innovate.

Comparative statics are generated by using the implicit function theorem. First, we take the cross-derivative of the objective function with respect to b and t :

$$(1 - e^{-rt^*})e^{-rt^*}(1 - x(t^*)) - re^{-rt^*} \left(\int_0^{t^*} e^{-rs}(1 - x(s))ds \right) \quad (34)$$

which is negative if x_t is increasing in t . Therefore, the optimal time to innovate is decreasing in b . A similar calculation for a shows that the optimal time to innovate is increasing in a .

Next, we take the cross-derivative of the objective function with respect to c and t :

$$(1 - e^{-rt^*})re^{-rt^*} + re^{-2rt^*} \quad (35)$$

which is always positive. Therefore, the optimal time to innovate is increasing in c . ■

Proof of Proposition 3

If the first limit condition in (6) holds, it is easy to see that there is a λ sufficiently small such that \bar{c} as defined in 4 is lower than c and therefore $t^*(\lambda) = \infty$. Using the same argument and the second limit condition in (6), we have that for λ sufficiently large $t^*(\lambda) = \infty$. ■

Proof of Proposition 4

For the first part of the proposition, we take the cross-derivative of the objective function with respect to x_0 and t to obtain:

$$(1 - e^{-rt^*})e^{-rt^*}(a - b) \frac{dx(t^*; \lambda, x(0))}{dx(0)} - re^{-rt^*} \left(\int_0^{t^*} e^{-rs}(a - b) \frac{dx(s; \lambda, x(0))}{dx(0)} ds \right) \quad (36)$$

which is negative if $\frac{dx(t; \lambda, x(0))}{dx(0)}$ is increasing in t and positive if $\frac{dx(t; \lambda, x(0))}{dx(0)}$ is decreasing in t . Therefore, the optimal time to innovate is decreasing (increasing) in λ if $\frac{dx(t; \lambda, x(0))}{dx(0)dt}$ is positive (negative).

The second part of the proposition is proved with a similar argument as in the previous proposition. ■

Proof of Proposition 5

First, we show that if a solution x_0^* exists, it must be that $x_0^* \in (0, 1)$. Since $\lambda < \infty$ and $b > a$, it must be that $\underline{B} > 0$. There must exist a fraction of consumers $G(\underline{B})$ such that $k_i \leq \underline{B}$, which implies that x_0^* cannot be zero. Now, suppose that $x_0^* = 1$. Then, $t^* = \infty$ and $B = \underline{B}$. Since $\bar{k} > \underline{B}$, there exists an $i \in I$ such that $k_i > \underline{B}$. Specifically, a fraction $1 - G(\underline{B})$ will not pay the cost k_i . Therefore, it cannot be that $x_0^* = 1$.

Now, we can prove existence of an equilibrium. We know that $H(x_0) > 0$ when $x_0 = 0$ and that $H(x_0) < 1$ when $x_0 = 1$. Therefore, the function $H(\cdot)$ must cross the 45-degree line at least once. Given the continuity of H , there must exist at least one point x_0^* at which (20) holds with equality.

According to Proposition 4, if $c < \sup_{x_0} \bar{c}(\lambda, x_0)$ and $\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0$, then we know that $\frac{\partial t^*}{\partial x_0}$ is monotone and positive. Since $H(x_0)$ is decreasing in t^* , this implies that once $H(x_0)$ crosses the 45-degree line from above and never crosses again. Therefore, when $\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0$, the fixed point at x_0^* is unique. Comparative statics follow from using the implicit function theorem.

For convenience, we define the function $\omega(t^*, \vec{v})$ as

$$\omega(t^*(x_0), \vec{v}) = H(x_0) - x_0,$$

so that in any equilibrium $\omega(t^*(x_0^*), \vec{v}) = 0$. If $\frac{\partial^2 x_t}{\partial x_0 \partial t} \geq 0$, then $H(x_0)$ may cross the 45-degree line multiple times, sometimes from above and sometimes from below. In such case, there will not exist a unique x_0^* , but rather two classes of equilibria (Class 1 and Class 2). For those that cross from above (Class 1),

- (i) $\frac{\partial x_0^*}{\partial b} > 0$
- (ii) $\frac{\partial x_0^*}{\partial a} < 0$
- (iii) $\frac{\partial x_0^*}{\partial c} < 0$

This follows from the implicit function theorem and the fact that $\frac{\partial \omega(t^*(x_0^*), \vec{v})}{\partial x_0} < 0$ at the point of equilibrium. For those that cross from below (Class 2),

- (i) $\frac{\partial x_0^*}{\partial b} < 0$
- (ii) $\frac{\partial x_0^*}{\partial a} > 0$
- (iii) $\frac{\partial x_0^*}{\partial c} > 0$

This follows from the implicit function theorem and the fact that $\frac{\partial \omega(t^*(x_0^*), \vec{v})}{\partial x_0} > 0$ at the point of equilibrium.

Now, we follow the discussion of Vives (2005, pages 440-445) and show that Class 1 equilibria are stable and Class 2 equilibria are unstable. The function $H(x_0) = G(B(x_0))$ is an aggregate best response function. Note that the aggregate best-response function as defined by Vives (2005) is $r(\tilde{a}) = F(g(\tilde{a}))$, where F is our function G , g is our benefit function B , and \tilde{a} is the fraction of players who take a particular binary action.

Consider a particular Class 1 equilibrium x_0^* and the neighborhood $\mathcal{N}_1 = [x_0^* - \epsilon, x_0^* + \epsilon]$. By the definition of this class of equilibrium, $H'(x_0) < 1$ at $x_0 = x_0^*$. First, choose an arbitrary $x_0 \in \mathcal{N}_1$ such that $x_0 - x_0^* = \delta > 0$. By the definition of a Class 1 equilibrium, it follows that $H(x_0) - H(x_0^*) < \delta$. Since $x_0^* = H(x_0^*)$, we know that $x_0^* + \delta > H(x_0^* + \delta)$ or $x_0 > H(x_0)$, which implies that the cost of becoming an expert exceeds the benefit of doing so for the marginal consumer. Converging toward equilibrium implies that $x_0 \rightarrow x_0^*$. In words, since the benefit to becoming an expert is lower than x_0 , fewer consumers will become experts. The same can be shown for an arbitrary $x_0 \in \mathcal{N}_1$ such that $x_0 - x_0^* = -\delta < 0$: the system will again converge toward the equilibrium point x_0^* . These two observations together assure that Class 1 equilibria are locally stable (Vives 1999).

Now, we show the opposite for a Class 2 equilibrium. Consider a particular Class 2 equilibrium x_0^* and the neighborhood $\mathcal{N}_2 = [x_0^* - \epsilon, x_0^* + \epsilon]$. By the definition of this class of equilibrium, $H'(x_0) > 1$ at $x_0 = x_0^*$. First, choose an arbitrary $x_0 \in \mathcal{N}_2$ such that $x_0 - x_0^* = \delta > 0$. By the definition of a Class 2 equilibrium, it follows that $H(x_0) - H(x_0^*) > \delta$. Since $x_0^* = H(x_0^*)$, we know that $x_0^* + \delta < H(x_0^* + \delta)$ or $x_0 < H(x_0)$, which implies that the cost of becoming an expert is less than the benefit of doing so for the marginal consumer. In words, more consumers will have an incentive to become an expert. Therefore, the system does not converge back to x_0^* . The same can be shown for an arbitrary $x_0 \in \mathcal{N}_2$ such that $x_0 - x_0^* = -\delta < 0$: the system will again fail to converge toward the equilibrium point x_0^* . Either of these two observations assure us that Class 2 equilibria are not locally stable (Vives 1999). ■

Proof of Proposition 6

We take the cross-derivative of the objective function with respect to ϕ and t :

$$(1 - e^{-rt^*})e^{-rt^*}(ax(t^*) + b(1 - x(t^*))) - re^{-rt^*} \left(\int_0^{t^*} e^{-rs}(ax(s) + b(1 - x(s)))ds \right) \quad (37)$$

which is negative if x_t is increasing in t . Therefore, the optimal time to innovate is decreasing in ϕ . ■

Proof of Proposition 7

The proof of the first two parts is similar to the proof of Proposition 2. For the third part, we take the cross-derivative of the objective function with respect to n and t :

$$\left(-(1 - e^{-rt^*})e^{-rt^*}(1 - x(t^*)) + re^{-rt^*} \left(\int_0^{t^*} e^{-rs}(1 - x(s))ds \right) \right) \times \left(\frac{1}{n^2} \right) \quad (38)$$

which is positive if x_t is increasing in t . Therefore, the optimal time to innovate is increasing in n . ■

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