

What We Talk about When We Talk about Mutual Fund's Reputation

Chong Huang and Fei Li*

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Abstract

The term reputation appears in our everyday discussions about mutual funds. However, few studies have been done to explore how the mutual fund's reputation forms and evolves. In this paper, we model fund's reputation as the market's belief whether the fund has the information to make profitable investment. We then propose an infinite-horizon model, in which the fund's reputation can be invested by costly information acquisition but may depreciate due to exogenous economic shocks. Reputation has significant effects on the fund's information acquisition decision, performance, and flows. Our results provide theoretical explanations to several empirical observations.

Keywords: Mutual Fund, Reputation, Information Acquisition, Outdated Information

JEL Classification Codes: C73, D83, G23

*Chong Huang, assistant professor in Finance, Paul Merage School of Business, University of California, Irvine, chong.h@uci.edu; Fei Li, assistant professor in Economics, University of North Carolina, Chapel Hill, lifei@email.unc.edu. We thank Vincent Glode, David Hirshleifer, Diego García, George Mailath, Moritz Meyer-ter-Vehn, Guillermo Ordonez, Christopher Schwarz, Zheng Sun, Siew Hong Teoh, Yuichi Yamamoto and Lu Zheng for valuable comments. Chong Huang thanks the CORCL Awards for financial supports.

1 Introduction

In the mutual fund industry, one of the central criteria the investors use to make purchase decision is the reputation of the fund. Capon, Fitzsimons, and Prince (1996) find in a survey that the fund manager’s reputation is the second most important selection criterion when retail investors make their purchasing decisions, with the mean of importance 4.00 and standard deviation 0.77.¹ The Investment Company Institute Survey (1997) confirms the importance of reputation in investors’ funds picking decisions, showing that reputation is the third highest consideration, after risk level and total return. Not only retail investors, but financial advisers also emphasize the importance of the fund manager’s reputation (Jones, Lesseig, and Smyth, 2005). Given the fact that financial advisers are the most important information channel for investors to make investment decisions, the fund’s reputation indeed stands out when investors consider the purchase.

More importantly, fund managers are almost compensated by a percentage of the total assets under management. Given the size of the fund, such a compensation scheme is indeed a flat fee, which seems not able to incentivize the fund manager to manage investors’ money well at a cost of her own disutility. However, fund managers in fact work hard for their investors, because they care about their future revenues as a fraction of future sizes (Berk, 2005), which are usually thought to increase in their reputation.

The reputation of a mutual fund sets up a bridge between its past performance and expected performance in the future. However, the reputation effect is so familiar as to be taken for granted; and thus little attention has been paid to understand the nature of the reputation in such an industry. In particular, what do we talk about when we talk about mutual fund’s reputation? Does a fund’s reputation just reflect some persistent characteristics of the fund so that past performance are positively correlated to future performance? Or the fund’s reputation can affect the fund manager’s behaviors, thereby affecting the fund’s performance?²

In this paper, we address these questions in a new discrete time infinite horizon model and provide theoretical explanations to several empirical observations in the mutual fund industry. We model the reputation of a fund as the market belief about whether the fund has relevant information.³ In the mutual fund industry, information is closely related to the best investment

¹The importance has a 1 to 5 scale, with 5 meaning “most important.”

²Warran Buffet once points out: “... If you think about that (your reputation can be ruined easily), you (fund managers) will do things differently.” This famous quote suggests that since fund managers care about their reputation, they may want to do something to promote their reputation, which may deviate from their myopic actions.

³ We do not consider the agency problem between the fund and the fund manager, which is an optimal contract problem. So in this paper, the fund’s reputation and the fund manager’s reputation refer to the same thing. We also treat a fund family’s reputation and its fund’s reputation same.

strategy. Consequently, it has been widely agreed that funds with information are more likely to obtain good investment outcomes. Moreover, information superiority has been shown the reason why mutual funds are endogenously built (García and Vanden, 2009). Therefore, if a fund is known to have information, investors will purchase its shares. However, whether a fund is informed is unobservable to investors. So investors make decisions based only on their belief whether a fund has information: When the market belief that the fund has information is high, the fund has higher “reputation,” and more investors will purchase the fund.

Two special features of information make mutual fund’s reputation differ from borrower’s reputation in Diamond (1989), which is the market belief over the borrower’s persistent types.⁴ First of all, since information can be acquired, an uninformed mutual fund can become informed by acquiring information. So mutual fund’s reputation in every period contains the market belief about the fund’s information acquisition behavior in that period. Second, more importantly, information has short-run persistence but will become outdated in the long-run. Such a feature may stem from macroeconomic shocks or competitions in the whole wealth management industry. As a result, conditional on the fund is informed in the current period, it is possible that the fund is still informed in the next period even without the next period’s information acquisition; but conditional on that an uninformed fund never acquires information, the fund’s reputation will deteriorate over time.

Our model captures these critical properties of mutual fund’s reputation. Given a fund’s *prior reputation*, which is defined as the market belief whether the fund is currently informed, the uninformed manager makes the information acquisition decision. If she acquires information, she becomes informed; otherwise, she remains uninformed. Investors do not observe the manager’s choice but form a belief over her information acquisition behavior. Such a belief, together with the fund’s prior reputation, constitutes the fund’s *interim reputation*. Investors will then based on the interim reputation to make their purchase decisions. The fund’s period payoff depends on the number of investors who purchase. Investors will reevaluate whether the fund is informed in the current period after observing the investment outcomes and form a *posterior reputation*: Because an informed fund manager can generate a good outcome with a higher probability than an uninformed manager does, good outcomes will always promote the fund’s posterior reputation. The world may change over time, so the fund’s posterior reputation in the current period, discounted by the probability of the change of the world, turns into the prior reputation in the next period.

⁴There is a huge literature in Economics modeling an agent’s reputation as the market belief over his/her types. Kreps and Wilson (1982), Milgrom and Roberts (1982), and Fudenberg and Levine (1989) define an agent’s reputation as the market belief whether the agent is of a commitment type; Mailath and Samuelson (2001) regard an agent’s reputation as the market belief that the agent is not an inept person. Mailath and Samuelson (2013) surveys recent works on reputations in repeated games of incomplete information.

Our model has a “work-work-shirk” stationary equilibrium. In the equilibrium, in every period, an uninformed fund manager’s information acquisition decision depends on her prior reputation. An uninformed “star” fund with a high prior reputation will not acquire information, because the fund’s reputation will not be hurt much even with a bad investment outcome, and thus the reputation premium is dominated by the cost of information acquisition. An uninformed medium fund acquires information with a positive probability, because if the fund obtains a bad investment outcome, the fund’s reputation will be downgraded a lot. An uninformed bad fund also acquires information with positive probability, but it will reach a lower interim reputation than a medium fund does. This is due to the market belief that bad funds are more likely to perform bad, which is required in the equilibrium to support medium funds’ behaviors.

Nowadays, several proxies are used to measure mutual fund’s reputation, such as past performance and size (Gerken, Starks, and Yates, 2014).⁵ Our model provides sufficient structures to analyze these variables’ roles. First, past performance of a fund contribute to its prior reputation that determines the fund’s equilibrium information acquisition decisions. As a result, past performance will have causal effects on the fund’s performance. However, past performance cannot predict the fund’s current performance. For one thing, past performance of a fund are not equivalent to its prior reputation: Given the same performance in the last period, two funds with different interim reputations in the last period may have different prior reputations today. For another, the prior reputation does not contain the probability that an uninformed fund acquires information, which is an important factor determining the fund’s ex-ante current performance. Second, a fund’s size is perfectly correlated to the fund’s interim reputation, therefore it can predict the fund’s ex-ante current performance well. However, the size reflects the market belief about the fund’s information acquisition behavior and is endogenously determined in the model, so it does not have the “causal” effect on the fund’s performance, and investors cannot use the information of the current size to make the current purchasing decision.

In the equilibrium, worst performing mutual funds will keep performing bad. This equilibrium property rationalizes the Rules-of-thumb (I) when choosing funds (Carhart, 1997): Avoid funds with persistently bad performance. Berk and Tonks (2007) attribute the current bad performance to a long history of bad performance. Glode (2011) argues that bad funds perform bad but can survive because the active return the manager generates covaries positively with a component of a pricing kernel that the performance measure omits. In our model, the market belief that bad

⁵A fund’s age is also used to measure the fund’s reputation in Gerken, Starks, and Yates (2014). However, we cannot discuss the fund’s age in our model. First, an old fund has a higher reputation because it survives for a long time. To analyze the survival of a fund, we need a model with fixed operating costs and the fund’s decision to exit. Such a model, though potentially interesting, is beyond the scope of this paper. Second, we focus on stationary equilibria, in which age will not play a role.

funds possess information is low, and investors do not think uninformed bad funds will acquire information with high probability, therefore bad funds' interim reputation is low. The low interim reputation discourages uninformed bad funds to acquire information, so on average bad funds have information with low probability. Consequently, bad funds on average are performing bad.

We show that performance of star funds and medium funds are not persistent, which is one of the most important observations in the mutual fund industry (Carhart, 1997). In a seminal paper, Berk and Green (2004) argue that the mutual fund performance non-persistence is due to the competition among investors and the decreasing return to scale of mutual funds' production functions. In our model, the total capitals available in the market are limited, and mutual funds' production functions have a constant return to scale. Mutual funds' performance are not persistent in the equilibrium, precisely because the probability of information acquisition is insufficient to compensate the information depreciation. Consider a star fund who obtains a good investment outcome today. The fund's posterior reputation is higher than its interim reputation, because investors think the fund is likely to be informed today. However, the best investment strategy today may not work well tomorrow, because the economic fundamentals may change. Since investors cannot observe exogenous shocks to the fund, they just discount the posterior reputation today by the probability of the change of the economic fundamentals to form tomorrow's prior reputation. This discounting effect is so significant for a star fund that the prior reputation tomorrow is lower than the interim reputation today. In the equilibrium, the consistent market belief that the fund will acquire information tomorrow cannot be too high; otherwise, the fund will not acquire information if uninformed. Therefore, on average, the interim probability a fund being informed tomorrow is lower than today. This leads to the performance non-persistence in the mutual fund industry.

Berk and Tonks (2007) also document that after a bad investment outcome, funds with a long history of bad performance experience insignificant cash outflows; in contrast, funds that only perform bad recently have much larger cash outflows. Berk and Tonks argue that these phenomena are due to the heterogeneity of investors' sensitivity to bad investment outcomes. These empirical facts are present in the equilibrium of our model for different reasons. For funds with a long history of bad performance, their prior reputation is low; that is, the majority of such funds are uninformed at the beginning of the current period. However, in the equilibrium, investors believe some uninformed funds with low prior reputation will acquire information. As a result, the interim probability that a fund with low prior reputation is not low, and we should not expect significant outflows. Conversely, a fund that performs well in the past gains a high prior reputation. But a bad investment outcome in the current period makes investors pessimistic, so that the fund's prior reputation in the next period is downgraded. Investors then believe such a fund acquires

information with low probability, if it is uninformed. Therefore, the interim probability the fund is informed tomorrow is much lower than today, which implies significant flows out of the fund.

This paper enriches the reputation literature by studying a discrete time infinite horizon model, in which reputation is defined as the market belief over an endogenous variable. In a recent paper, Board and Meyer-ter veht (2013) study a continuous time model in which a firm’s reputation is the consumers’ belief about the quality of its product.⁶ In their setting, quality is periodically replaced via a Poisson arrival process; when a replacement occurs, the firm’s current effort determines the new quality. Consumers learn about quality through noisy signals according to a quality-dependent Poisson process. In the time period that no signal arrives, the firm’s reputation smoothly changes, while once a signal arrives, the firm’s reputation jumps. Hence, the realized reputation is discontinuous, which plays a critical role in determining intertemporal incentives. They consider models with perfect good/bad Poisson signals and imperfect Poisson signals, and show that reputation incentives critically depend on the specification of market learning.⁷ In our model, the investors learn whether the manager has the relevant information via her past investment performance, which is a non-Poisson process. The manager’s reputation either goes up or down smoothly in each period. Hence, we view our model as a complementary of Board and Meyer-ter veht (2013).

The remainder of this paper is organized as follows. In Section 2, we introduce a static reputation game, in which a fund cares about its current size and its posterior reputation. In Section 3, we present an infinite-horizon reputation model and solve the “work-work-shirk” equilibrium. Section 4 describes the equilibrium properties, which provide theoretical explanations to salient empirical facts. Section 5 concludes.

2 A Static Model

In this section, we introduce a static model, which could be viewed as the reduced form of the dynamic model analyzed in Section 3. The analysis illustrates how the reputation concern provides incentive to the manager to acquire information.

There is an active mutual fund, to whom a continuum of investors with measure one may

⁶In this literature, one typically employs a continuous time framework since the monotonicity of the firm’s value function is critical to obtain their results, but it has not been proved in a discrete time model with a Poisson learning process.

⁷Bohren(2013) also considers a continuous time reputation model where the reputation of a firm is the persistent quality of its product. However, in her model, there is no asymmetric information. A firm’s reputation is an observable quality stock, which can be enhanced through the firm’s costly investment. Dilme (2012) proposed an alternative model of firm reputation where actions have lasting effects due to switching costs.

delegate their wealth management. The best investment strategy is described by $\theta \sim U[0, 1]$. If the fund manager chooses the right investment strategy θ , she will obtain a good return G (relative to the passive money management strategy) with probability $q > 1/2$; with the complementary probability $1 - q$, she gets a bad return B due to a bad luck. However, if the mutual fund manager employs any other investment strategy $\theta' \neq \theta$, she will generate a good return with probability $1 - q$ and a bad return with probability q .

At the beginning of the game, investors believe that the manager knows θ with probability $x \in (0, 1)$. The prior is $x \sim U(0, 1)$. This is also the fund manager's "prior reputation." The fund manager knows whether she is informed or not. If the manager is informed, she will implement θ . If the manager does not know θ , the manager randomly chooses one investment strategy $\theta' \in [0, 1]$.

Before investing, an uninformed manager needs to choose whether to acquire information. Information acquisition is costly: The manager incurs an effort cost $c > 0$ when acquiring information. But the information acquisition is effective: The manager knows the best investment strategy θ for sure by acquiring information. The manager's information acquisition choice is unobservable to investors, but investors form a belief about the manager's information acquisition activity, given the manager's prior reputation x . Denote such a belief by $\hat{\sigma}$. Then investors' belief that the manager knows θ is

$$z = x + (1 - x)\hat{\sigma}.$$

Such a belief is defined as the fund's "interim reputation." Assume that measure z investors will purchase the fund, so the fund's size is just the fund's interim reputation.

At the end of the game, investors will assess the fund, using the observable investment outcome. Denote by $z^g = \frac{zq}{zq+(1-z)(1-q)}$ and $z^b = \frac{(1-z)(1-q)}{zq+(1-z)(1-q)}$ the manager's "posterior reputation" when the investment return is good and bad, respectively. The fund manager cares both the amount of money under management (because of the fee structure of the mutual fund) and the fund's posterior reputation (because of investors' future purchase decisions). Given the interim reputation z , after the information acquisition decision, the manager's payoff depends on whether she knows θ or not:

$$\begin{cases} V_I(z) = z + [q\delta z^g + (1 - q)\delta z^b], & \text{if informed;} \\ V_U(z) = z + [(1 - q)\delta z^g + q\delta z^b], & \text{if uninformed.} \end{cases}$$

Then an uninformed manager's payoff is (given her prior reputation x and investors' belief that she will acquire information with probability $\hat{\sigma}$)

$$\begin{cases} V_I(x) - c, & \text{if acquiring information;} \\ V_U(x), & \text{if not acquiring information.} \end{cases}$$

The fund manager's information acquisition strategy $\sigma(x)$ and the market belief $\hat{\sigma}$ constitute an equilibrium of the static model, if (i) $\sigma(x)$ maximizes an uninformed manager's payoff given $\hat{\sigma}$; and (ii) investors have rational expectation: $\hat{\sigma} = \sigma(x)$.

We call $V_I(x) - V_U(x)$ the reputation premium at prior reputation x . Then, the manager's optimal information acquisition rule is

$$\sigma(x) \begin{cases} = 0, & \text{if } V_I(x) - V_U(x) < c; \\ \in [0, 1], & \text{if } V_I(x) - V_U(x) = c; \\ = 1, & \text{if } V_I(x) - V_U(x) > c. \end{cases}$$

That is, an uninformed manager acquires information if and only if the reputation premium is greater than the information acquisition cost. Notice that the reputation premium depends on the market belief $\hat{\sigma}$ via the interim reputation z . In other words, the manager's incentive to acquire information depends on whether the market believes she will do so.

Given z , the equation $V_I(z) - V_U(z) \geq c$ is equivalent to

$$z^g - z^b \geq \frac{c}{\delta(2q-1)}. \quad (1)$$

One can show that $z^g - z^b$ is concave in z , and when $z = 0$ or $z = 1$, $z^g - z^b = 0$. Then Figure 1 shows how the interim reputation determines the manager's decision. In the figure, $\bar{\omega}$ and $\underline{\omega}$ are the two solutions to $z^g - z^b = c/\delta(2q-1)$. Then, if $z > \bar{\omega}$ or $z < \underline{\omega}$, the reputation premium is strictly less than the information acquisition cost. As a result, the manager will choose not to acquire information, when her interim reputation z is strictly greater than $\bar{\omega}$ or strictly less than $\underline{\omega}$. If $z \in (\underline{\omega}, \bar{\omega})$, the reputation premium is strictly larger than the information acquisition cost. Consequently, the manager will choose to acquire information for sure. Finally, when $z = \underline{\omega}$ or $z = \bar{\omega}$, the manager is indifferent between acquiring information or not, so the manager may randomize.

By considering possible interim reputation for any given prior reputation, we have the following Lemma 1.

Lemma 1. *In any equilibrium of the static model,*

$$\sigma(x) = \begin{cases} 0, & \text{if } x > \bar{\omega}; \\ \frac{\bar{\omega}-x}{1-x}, & \text{if } x \in (\underline{\omega}, \bar{\omega}]. \end{cases}$$

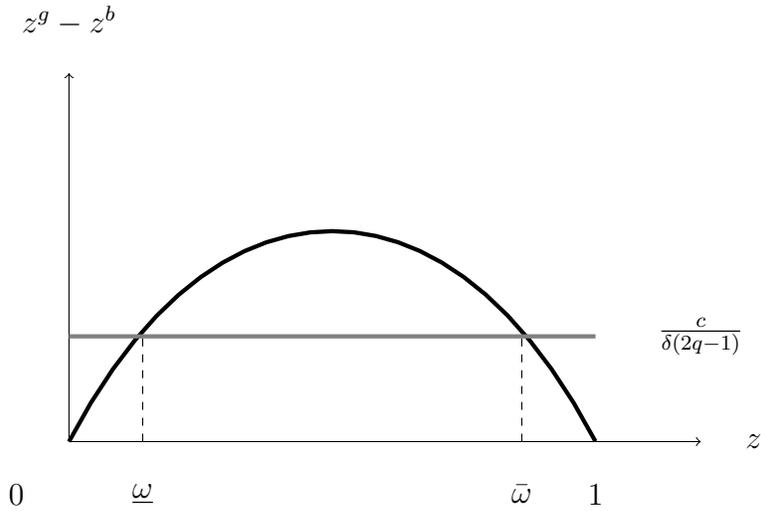


Figure 1: Equilibrium Construction in the Static Example

The equilibrium description of the static model is then complete after specifying the manager’s behavior when the fund’s prior reputation $x \leq \underline{\omega}$. For each prior reputation in this case, there are three possible interim reputation consistent with an equilibrium information acquisition decision. The market belief $\hat{\sigma}(x)$ could be 0, so that the interim reputation $z = x$; or $\hat{\sigma}(x)$ could be $(\underline{\omega} - x)/(1 - x)$, so that the interim reputation $z = \underline{\omega}$; or $\hat{\sigma}(x)$ could be $(\bar{\omega} - x)/(1 - x)$, so that the interim reputation $z = \bar{\omega}$. Therefore, the static model has three kinds of equilibria, differing in the behavior of the uninformed manager, when the fund’s reputation is lower than $\underline{\omega}$. We are interested in the “work-work-shirk” equilibrium, described in Proposition 1 below, because there is a similar equilibrium in the dynamic model, analyzed in Section 3.

Proposition 1. *The static model has a “work-work-shirk” equilibrium. In particular, the manager’s equilibrium strategy is*

$$\sigma^*(x) = \begin{cases} \frac{\underline{\omega} - x}{1 - x}, & \text{if } x \in (0, \underline{\omega}]; \\ \frac{\bar{\omega} - x}{1 - x}, & \text{if } x \in (\underline{\omega}, \bar{\omega}]; \\ 0, & \text{if } x \in (\bar{\omega}, 1). \end{cases}$$

Due to the self-fulfilling nature of the model, there exists other equilibrium. For example, there exists a “work-shirk” equilibrium in which $\sigma(x) = \frac{\bar{\omega} - x}{1 - x}$ when $x \in (0, \bar{\omega})$.

Although the simple static model can illustrate the role of reputation as an incentive mechanism, it fails to explain how the reputation forms, and evolves. More importantly, a static model cannot capture important features of information. First, any information may become outdated.

That is, old profitable investment strategy may not work well in the future. This may be attributed to the change of the macroeconomic environments, such as new policies, new emerging markets, and new technologies. Another reason why information may become outdated is the competition of the fund industry. For example, when a profitable investment strategy is employed by too many funds, such an investment strategy will never be profitable. Since the information will become useless in the long-run, without acquiring new information, the fund's reputation cannot persist in the long-run. Second, useful information has short-term persistence. Therefore, the market believes that if the fund has information today, with positive probability, the fund is still informed tomorrow. That is, today's information acquisition can promote the fund's prior reputation in the next period. Such information persistence provides short-term persistence of the fund's reputation. Therefore, in order to understand the fund's reputation and its effects, we need an infinite-horizon model, which can capture these two features of information.

3 A Dynamic Model of Mutual Fund's Reputation

In this section, we analyze mutual fund's reputation in an infinite-horizon model. The fund's payoff is the average discounted sum of management fees it collects in every period.

3.1 The Model

State of the economy. Time is indexed by $t = 1, 2, 3, \dots$. In each period, the manager chooses an investment portfolio $a_t \in [0, 1]$. The return of each portfolio depends on the fundamental state of the economy, which follows a stochastic process. In particular, denote by $\theta_t \in [0, 1]$ the state of the economy in period t . In each period t , the state of the economy remains, $\theta_{t+1} = \theta_t$ with probability $1 - \lambda$. With probability λ , the state changes and the new state $\theta_{t+1} \sim U[0, 1]$.⁸

Asset Return. The return of the portfolio $a_t = \theta_t$ is 1 with probability q and 0 with probability $1 - q$. The return of the portfolio $a_t \neq \theta_t$ is 1 with probability $1 - q$ and 0 with probability q . Here, $q \in (1/2, 1)$. Investors lack of information of the state of the economy, so they want to delegate their wealth management to a fund, which may possess the information of the state of the economy.

Fund Manager. A manager may or may not know the state of the economy. The investors form public belief about whether the manager is informed or uninformed. We interpret the public belief that the manager being informed as the manager's *reputation*. The law of motion of the

⁸Notice that we rule out the possibility that an uninformed manager can learn θ_t from "trial and error" investment.

reputation is described as follows. At the beginning of each period t , the manager's prior reputation is $x_t \in (0, 1)$, and x_1 is exogenously given. In each period, the uninformed manager will decide whether to acquire information about the true state of the world in period t . Denote by σ_t the probability that the uninformed manager acquires information. We will focus on stationary equilibrium in which the uninformed manager's information acquisition decision is a function of her prior reputation only: $\sigma_t = \sigma(x_t)$. If the uninformed manager chooses not to acquire information, the manager remains uninformed; if the manager acquires information, the manager will become informed in period t , but she needs to pay an effort cost $c > 0$. We assume that the manager knows whether she knows, but this is her private information. Notice that the manager's information acquisition choice is not unobserved, but the market believes that she acquires information with probability $\hat{\sigma}_t$. Then the manager's interim reputation in period t is $z_t = x_t + (1 - x_t)\hat{\sigma}_t$, where $\hat{\sigma}_t$ is the market belief that the manager acquires information in period t .

Potentially Outdated Information. An informed manager in period t , however, will become uninformed in period $t + 1$ with probability $\lambda \in (0, 1)$, since the state of the world of the economy may change. When the economic fundamentals change, old investment portfolio will not work well anymore. Once the state of the world changes, the manager knows his previous information is outdated, but he does not know what is the new state without acquiring information. Since we assume that the new state θ_{t+1} is uniformly drawn from a continuum, we rule out the possibility that the manager can learn the state from trial-and-error over time.

Given an interim reputation z_t in period t , the prior reputation in period $t + 1$ is:

$$x_{t+1} = \begin{cases} (1 - \lambda)z_t^g = (1 - \lambda)\frac{qz_t}{qz_t + (1-q)(1-z_t)}, & \text{if the investment outcome is good;} \\ (1 - \lambda)z_t^b = (1 - \lambda)\frac{(1-q)z_t}{(1-q)z_t + q(1-z_t)}, & \text{if the investment outcome is bad.} \end{cases} \quad (2)$$

For simplicity, we assume that the manager's flow payoff in period t is z_t . We are interested in a Markov perfect equilibrium, in which

1. the manager's strategy depends only on the history only through her prior reputation x ;
2. the manager's strategy is consistent with the market belief: $\hat{\sigma}_t = \sigma(x_t)$, and
3. over time, the reputation evolves according to Bayes' rule: $z_t = x_t + (1 - x_t)\sigma(x_t)$ and equation (2).

Because of Markov strategies, we can ignore the subscript t .

3.2 Discussion of Assumptions

Before analyzing the model, we discuss some of the assumptions. First of all, in each period, the fund's size is assumed to be its interim reputation. This reduced form assumption is just for simplicity. It is sufficient to capture the fact that the number of investors is increasing in the fund's interim reputation, because by definition, investors think the fund is more likely to possess the information. However, this assumption can be justified in a number of ways. For example, we can imagine that each individual investor can access a private signal from private research about whether the fund manager is informed. Then given the interim reputation, there exist private signal structures that lead to the equation of the fund's interim reputation and the number of investors purchasing the fund. In fact, our result is robust if we assume that in each period, the size of the fund is an increasing function of the fund's interim reputation.

Second, again just for simplicity, we assume the fund's period revenue equals to the size of the fund. By this assumption, we capture the fact that the management fee is a fixed percentage of the total money under management. Moreover, for any exogenous management fee structure depending on the fund's prior reputation, our result is robust.

Finally, we assume the potential capitals for a fund is limited, and the fund's productivity has a constant return to scale. In most cases, especially in the recession of the economy, the total capitals in the market is finite. Under such circumstances, we normalize the total available capitals in the market to be 1, as assumed in the model. When the total available capitals are finite, the fund's revenue is well defined with a constant return to scale production function. If a fund is a price taker in the financial market, the fund's short-run excess rate of return from investment, the α , is independent of the volume it trades. Therefore, if the fund has information, any dollar the fund invests can generate the same return, which means the fund's productivity has a constant return to scale in the short run.

3.3 Preliminary Analysis

One of the most important features of our model is that the information has some persistence. That is, if the manager is informed today, she will remain informed with positive probability, so $\lambda \in (0, 1)$. This is important for a fund manager to have incentives to acquire information. Consider the case of $\lambda = 1$. Because the manager's information acquisition is unobservable to the market, for any given interim reputation in the current period, the fund's prior reputation in the next period is 0. This is so because the information has no persistence, then today's best investment strategy surely does not work in the next period. Since the fund will be uninformed at the beginning of the next period, the fund's continuation value is independent of whether the

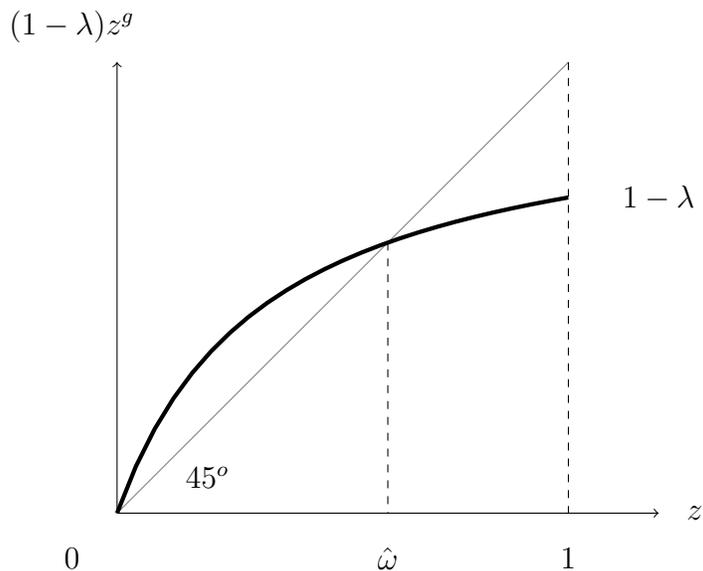


Figure 2: The determination of $\hat{\omega}$. The thick curve represents the posterior reputation following a good performance by considering a possible change of the state. $\hat{\omega}$ is defined as the interim reputation, such that $\hat{\omega} = (1 - \lambda)\hat{\omega}^g = (1 - \lambda)\frac{q\hat{\omega}}{q\hat{\omega} + (1-q)(1-\hat{\omega})}$.

fund manager is informed or not in the current period. This argument, together with the fact that the interim reputation is also independent of the fund manager's information, implies that the reputation premium is 0. Since the effort cost is positive, the unique equilibrium when $\lambda = 1$ is that the fund never acquires information. Therefore, to analyze the reputation effects, we should assume that λ is sufficiently small.

Similarly, for an uninformed manager to have incentives to acquire information, a good investment outcome must be significantly more possible for an informed manager. Therefore, we maintain the assumption below in the rest of the paper.

Assumption 1. $(2 - \lambda)q > 1$.

Even with Assumption 1, when the fund's interim reputation is too large, the effect of a good investment outcome on the prior reputation in the next period is dominated by that of the fund manager's information depreciation. The reason is that the economic fundamentals change linearly with probability $1 - \lambda$, while the effect of a good outcome is concave. This intuition can be seen from figure 2.

Simple algebra implies that $\hat{\omega} = 1 - \frac{\lambda q}{2q-1} \in (0, 1)$ by Assumption 1. Therefore, conditional on a good investment outcome, when the manager's interim reputation is smaller than $\hat{\omega}$, the

manager’s prior reputation in the next period will be higher than the current interim reputation. Conversely, the fund’s prior reputation in the next period will be smaller than its current interim reputation, when its current interim reputation is greater than $\hat{\omega}$.

In the model, whether investors want to delegate the wealth management to the fund manager is determined only by their belief that the manager has the information of the true state of the world in the current period. This belief is not only affected by the manager’s prior reputation, but also affected by the probability that the uninformed manager acquires information. Because the manager’s information acquisition activity cannot be observed by investors, the manager’s interim reputation is independent of the manager’s working decision in the current period.

By acquiring information, the manager will become informed in the current period. There are two benefits from information acquisition. First, the manager can generate a good outcome with a higher probability, so the manager can increase her prior reputation in the next period. Second, if the manager is informed in the current period, with probability λ , the manager is still informed in the next period. The cost of information acquisition is the current cost of efforts c . Lemma 2 below shows that in an equilibrium, the manager cannot acquire information for sure.

Lemma 2. *In any equilibrium, $\sigma(x) < 1$ for each $x \in (0, 1)$.*

Therefore, in the rest of the paper, the statement that “the manager acquires information” means the manager acquires information with positive probability.

3.4 A “Work-Work-Shirk” Equilibrium

Consider the following “work-work-shirk” strategy.

$$\sigma(x) = \begin{cases} \frac{\omega_1 - x}{1 - x}, & \text{if } x \in [0, \omega_1]; \\ \frac{\omega_0 - x}{1 - x}, & \text{if } x \in (\omega_1, \omega_0]; \\ 0, & \text{if } x \in (\omega_0, 1]. \end{cases} \quad (3)$$

Here, $\omega_0 \geq 1 - \lambda$, and

$$\omega_1 = (1 - \lambda)\omega_0^b = (1 - \lambda) \frac{(1 - q)\omega_0}{(1 - q)\omega_0 + q(1 - \omega_0)} \leq \hat{\omega};$$

that is, fix the interim reputation ω_0 in the current period, ω_1 is the manager’s next period prior reputation, if she generates a bad outcome in the current period. This proposed strategy could be represented by the automaton in Figure 3. In the strategy, when the fund’s prior reputation x is smaller than the lower threshold ω_1 , the fund will acquire information with probability $(\omega_1 -$

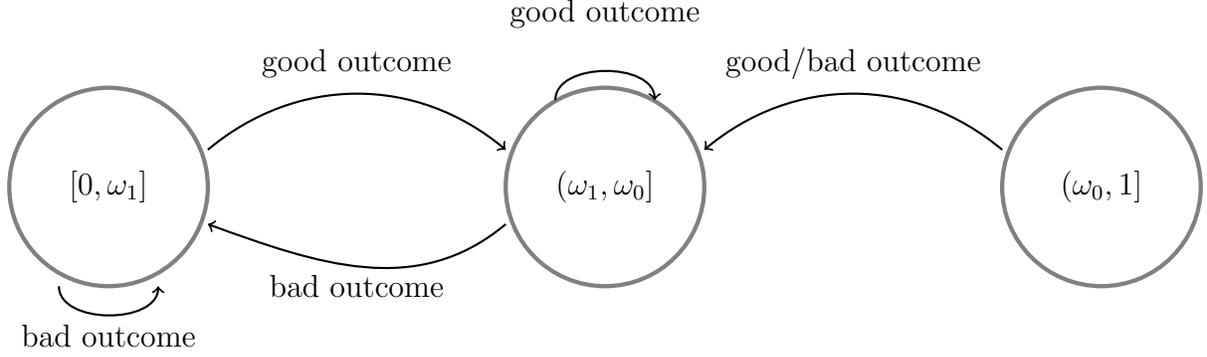


Figure 3: The automaton representation of the “work-work-shirk” equilibrium. When the prior reputation $x \leq \omega_1$, $\sigma(x) = \frac{\omega_1 - x}{1 - x}$ and the interim reputation $z = x + (1 - x)\sigma(x) = \omega_1$, and the posterior reputation following a good (bad) outcome is $z_g \in (\omega_1, \omega_0]$ ($z_b < \omega_1$); when $x \in (\omega_1, \omega_0]$, $\sigma(x) = \frac{\omega_0 - x}{1 - x}$ and $z = \omega_0$, and $z_g \in (\omega_1, \omega_0]$ ($z_b \in [0, \omega_1]$); when $x > \omega_0$, $\sigma(x) = 0$ and $z = x$, and $z_g, z_b \in (\omega_1, \omega_0]$.

$x)/(1 - x)$; when the prior reputation is between ω_1 and ω_0 , the fund will acquire information with probability $(\omega_0 - x)/(1 - x)$; when the prior reputation is high such that $x > \omega_0$, the fund will certainly not acquire information.

Given the proposed strategy, there are only two relevant interim reputations: ω_0 and ω_1 . First of all, a manager’s prior reputation in the current period cannot be larger than $1 - \lambda$, no matter how likely she was informed in the previous period. This is due to the assumption that the world will change with probability $1 - \lambda$, in which case the old investment strategy does not work. Since ω_0 by construction is greater than $1 - \lambda$, the manager’s prior reputation will be less than ω_0 in the next period. Second, when the manager’s prior reputation is in $(\omega_1, \omega_0]$, according to the strategy (and the equilibrium belief consistency condition), the manager’s interim reputation will jump to ω_0 . Finally, when the manager’s prior reputation is in $[0, \omega_1]$, the manager’s interim reputation will jump to ω_1 .

To save notations, we denote by $V_K^i = V_K(\omega_i)$ the manager’s value function, when the manager’s interim reputation is ω_i and she is in the status $K = I$ (informed) or $K = U$ (uninformed). Let’s first consider the manager’s value function, when her prior reputation $x > \omega_0$. Since the investors believe the manager will not acquire information, the manager’s interim reputation $z = x$.

$$\begin{aligned} V_I(x) &= x + \delta [(1 - \lambda)V_I^0 + \lambda V_U^0], \\ V_U(x) &= x + \delta V_U^0. \end{aligned}$$

No matter whether the manager is informed or not, her flow payoff will be her interim reputation

$z = x$. Because $z > \omega_0$, when the manager gets a good investment outcome, the manager's prior reputation in the next period will be less than $1 - \lambda$, so less than ω_0 ; when the manager gets a bad investment outcome, the manager's prior reputation in the next period will be less than $1 - \lambda$ but larger than ω_1 . Therefore, when the manager's interim reputation $z > \omega_0$, the manager's interim reputation in the next period is ω_0 , which is independent of the manager's performance in the current period. Then, when the manager is informed, the manager will still be informed in the next period with probability $1 - \lambda$; but with probability λ , an informed manager will become uninformed. For an uninformed manager, she will still be uninformed at the beginning of the next period. By Lemma 2, in an equilibrium, keeping uninformed is at least as good as acquiring information to the uninformed manager, so her continuation value is just V_U^0 .

By the strategy, when the manager's prior reputation $x \in (\omega_1, \omega_0]$, the manager's interim reputation will jump to ω_0 . So the manager, with the prior reputation $x \in (\omega_1, \omega_0]$, has value functions:

$$\begin{aligned} V_I(x) &= \omega_0 + \delta [q(1 - \lambda)V_I^0 + q\lambda V_U^0 + (1 - q)(1 - \lambda)V_I^1 + (1 - q)\lambda V_U^1], \\ V_U(x) &= \omega_0 + \delta [(1 - q)V_U^0 + qV_U^1]. \end{aligned}$$

When the manager's prior reputation $x \in [0, \omega_1]$, the manager's interim reputation will jump to ω_1 . In this case, the manager's value function is:

$$\begin{aligned} V_I(x) &= \omega_1 + \delta [q(1 - \lambda)V_I^0 + q\lambda V_U^0 + (1 - q)(1 - \lambda)V_I^1 + (1 - q)\lambda V_U^1], \\ V_U(x) &= \omega_1 + \delta [(1 - q)V_U^0 + qV_U^1]. \end{aligned}$$

If the strategy described in Equation (3) is an equilibrium, we must have $\omega_1 < \hat{\omega}$. Suppose $\omega_1 \geq \hat{\omega}$, then when the fund's interim reputation is ω_1 , the fund's next period prior reputation will be lower than ω_1 , no matter the investment outcome is good or bad, by the definition of $\hat{\omega}$. Then, according to the proposed strategy, the fund's next period interim reputation is ω_1 , independent of the current period investment outcome. This will break the equilibrium, because the fund with prior reputation less than ω_1 will certainly not to acquire information. Therefore, to support the existence of ω_0 and ω_1 and thus the equilibrium, we require that the distance between $1 - \lambda$ and $\hat{\omega}$ is small enough, such that when the manager with the current interim reputation $1 - \lambda$ gets a bad outcome, the manager's next period prior reputation is smaller than $\hat{\omega}$. This requirement can be rewritten as $1 < \lambda + q(1 + q)$. By Assumption 1, $q > 1/(2 - \lambda)$. So a sufficient condition is $\frac{1}{2 - \lambda} \frac{3 - \lambda}{2 - \lambda} \geq 1 - \lambda$, which is equivalent to $f(\lambda) = 1 - 7\lambda + 5\lambda^2 - \lambda^3 \leq 0$. We can show that $f(\lambda)$ is strictly decreasing for all $\lambda \in (0, 1)$, $f(0) > 0$ and $f(1) < 0$, so there is a unique $\hat{\lambda} \simeq 0.16$, such that $f(\hat{\lambda}) = 0$. Therefore, we will maintain the Assumption 2 below.

Assumption 2. $\lambda \geq \hat{\lambda}$.

The requirement that $\omega_1 < \hat{\omega}$ also sets an upper bound for us to choose ω_0 . That is, the supremum of ω_0 to support the equilibrium will be the interim reputation, with which the manager reaches the next period prior reputation $\hat{\omega}$ after getting a bad outcome. This supremum is the solution to the equation $(1 - \lambda) \frac{(1-q)x}{(1-q)x + q(1-x)} = \hat{\omega}$, which is denoted by

$$\bar{\omega} = \frac{q(2 - \lambda) - 1}{(1 - \lambda(1 - q)(2q - 1) + 2q[q(2 - \lambda) - 1]}.$$

Therefore, we can only choose $\omega_0 \in [1 - \lambda, \bar{\omega}]$.

In the strategy profile described in Equation (3), an uninformed fund manager will be indifferent between acquiring information and remaining uninformed, when her interim reputation is ω_0 or ω_1 . Hence, if this strategy profile is an equilibrium, we must have

$$V_I^0 - V_U^0 = V_I^1 - V_U^1 = c. \quad (4)$$

In addition, since the fund with the prior reputation greater than ω_0 will certainly not acquire information,

$$V_I(x) - V_U(x) < c. \quad (5)$$

Figure 4 illustrates the fund's value function for any prior reputation.

Equation (4) will help to pin down ω_0 , which will determine the whole equilibrium construction. Specifically, Equation (4) implies that

$$\frac{\delta(2q - 1)}{1 - \delta(1 - \lambda)} \left[\omega_0 - (1 - \lambda) \frac{(1 - q)\omega_0}{(1 - q)\omega_0 + q(1 - \omega_0)} \right] = c. \quad (6)$$

The left-hand side of Equation (6) is a continuous function over $[1 - \lambda, \bar{\omega}]$, therefore, the left-hand side of Equation (6) has its maximum and minimum over $[1 - \lambda, \bar{\omega}]$, which are denoted by \bar{y} and \underline{y} , respectively. The idea can be illustrate via Figure 5. We characterize the “work-work-shirk” equilibrium in the proposition below.

Proposition 2. *Suppose the parameter vector (δ, q, λ) satisfies the Assumption 1 and Assumption 2. There exists a pair (\underline{y}, \bar{y}) with $0 < \underline{y} < \bar{y}$, such that for each $c \in (\underline{y}, \bar{y})$, there exists a “work-work-shirk” stationary equilibrium described in Equation (3), where ω_0 is pinned down by Equation (6).*

Some remarks of Proposition 2 are in order. First of all, the key equilibrium condition is Equation (4). Because the flow payoffs to the informed manager and the uninformed manager are same, the difference of value at any prior reputation depends only on continuation values. More importantly, from the proposed strategy, no matter the prior reputation is ω_0 or ω_1 , if the manager

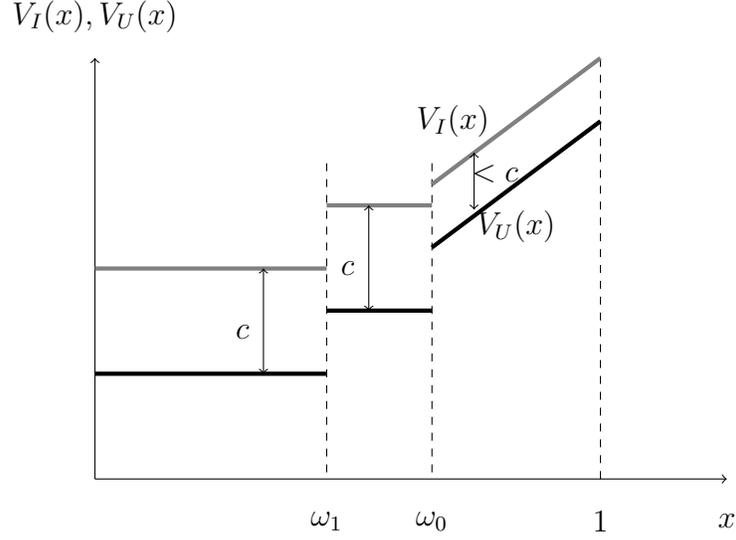


Figure 4: The Value Functions in the Equilibrium. An informed (uninformed) manager's payoff depends on her interim reputation z instead of her prior reputation x . For an informed (uninformed) manager whose prior reputation is $x \in (0, \omega_1]$, her interim reputation is ω_1 regardless of her prior reputation, so her value function is constant in such an interval. The same logic applies in the interval $(\omega_1, \omega_0]$. For a manager whose prior reputation is $x \in (\omega_0, 1)$, the market beliefs is $\hat{\sigma} = 0$, so her interim reputation is her prior reputation, and her value function is a linear function of her prior reputation.

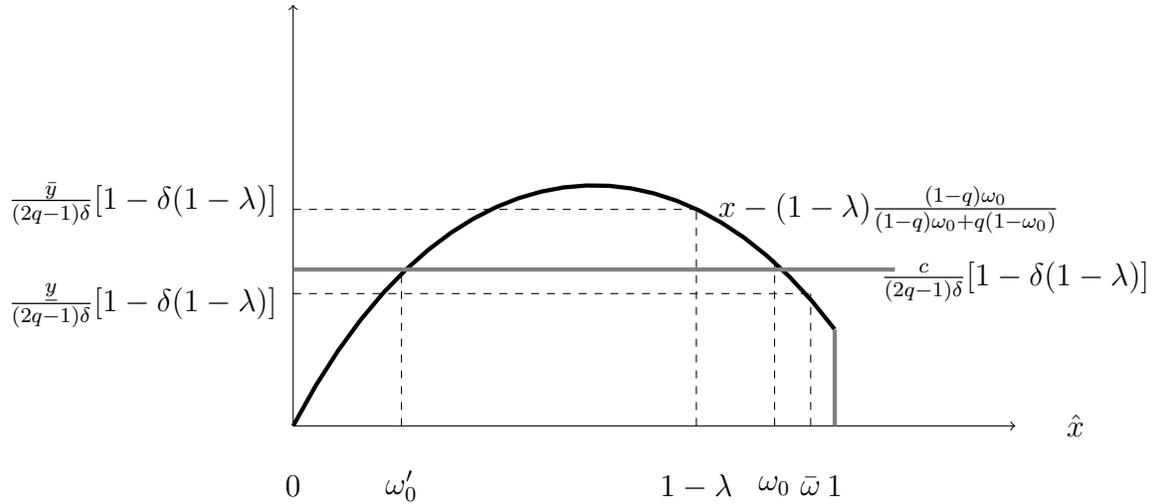


Figure 5: Equilibrium Construction in the Dynamic Model

receives a good outcome, her interim reputation in the next period is ω_0 ; if the manager receives a bad outcome, her interim reputation in the next period is ω_1 . Since the conditional probability of a good outcome is independent of the manager’s reputation, the difference between an informed manager’s continuation value and an uninformed manager’s continuation value is independent of her reputation.

Second, the “work-work-shirk” equilibrium requires medium levels of the effort cost; that is, $c \in (\underline{y}, \bar{y})$. When the effort cost is too large, we cannot have an equilibrium in which the fund acquires information. Because the reputation premium is bounded, the reputation premium will be dominated by the effort cost, when the effort cost is too large. On the other hand, when the effort cost is too small, the “work-work-shirk” strategy is not an equilibrium. Because $\omega_0 \geq 1 - \lambda$ and $\omega_1 < \hat{\omega}$, their difference $\omega_0 - \omega_1$ is bounded below by $(1 - \lambda) - \hat{\omega}$. From Equation (4), the reputation premium at interim reputations ω_0 and ω_1 is determined by $\omega_0 - \omega_1$, so if the effort cost is too small, the fund will acquire information for sure at interim reputations ω_0 and ω_1 , which contradicts Lemma 2.

Finally, the “work-work-shirk” equilibrium described in Equation (3) looks similar to the one in Proposition 1, but their differences are substantial. In the equilibrium of the static model, the behavior of a fund with a very low prior reputation ($x < \underline{\omega}$) is independent of the behavior of a fund with a prior reputation $x \in (\underline{\omega}, \bar{\omega})$; but in the “work-work-shirk” equilibrium in the dynamic model, equilibrium behaviors at different prior reputations are closely connected. This is because in a dynamic environment, the behavior at any prior reputation determines the fund’s value at that reputation, which is part of the continuation value at the current prior reputation. In particular, in the dynamic model, ω_1 is the next period prior reputation when a fund with the interim reputation ω_0 receives a bad investment outcome, and ω_0 and ω_1 must satisfy Equation (4).

4 Reputation Effects

The “work-work-shirk” equilibrium characterized in Proposition 2 shows that a fund’s initial reputation has great effects on an uninformed fund’s incentives to acquire information. Because a fund’s information acquisition decision determines its ex-ante investment outcome, we should expect that the fund’s reputation has significant causal effects on the fund’s performance, sizes, and cash flows. In this section, we first demonstrate that our model provides sufficient structures to understand measures of funds’ reputations in existing studies. Then we analyze the reputation effects in the “work-work-shirk” equilibrium and provide potential explanations to some empirical observations.

4.1 Reconcile Existing Measures of Fund’s Reputation

A fund’s past performance are usually employed to measure the fund’s reputation. Ippolito (1992) and Gerken, Starks, and Yates (2014), among others, treat past performance as one proxy of funds’ reputations, when empirically analyzing investors’ fund purchasing choices. Berk and Green (2004) assume that more investors will purchase the fund when the fund has a good investment performance. All these studies implicitly assume that past investment performance reflect the quality of the fund’s money management, and that such a quality is persistent. As a result, if a fund performed well before, it should perform well in the future. These studies, however, overlook the fund’s incentives. Hence, the fund’s reputation will not have causal effects on the fund’s future performance in these frameworks.

Past performance are important factors determining the fund’s prior reputation in our model. This is due to the persistence (though imperfect) of information. In our model, if a fund’s investment outcome is good today, investors will believe that the fund is more likely to know today’s best investment strategy. If the fund is informed today, the fund will know the best investment strategy tomorrow with probability $1 - \lambda$, without considering the probability that an uninformed fund manager acquires information.

In the “work-work-shirk” equilibrium, the fund’s prior reputation determines the fund’s information acquisition decision, so the fund’s past performance have causal effects on the fund’s investment performance. However, a fund’s past performance are not best predictors of the fund’s future performance, because we should take into account the probability that an uninformed fund acquires information. Let’s consider a fund with a prior reputation $x \in (\omega_1, \omega_0)$ and assume that the prior reputation is uniquely determined by past performance. Just from the fund’s prior reputation, the expected probability that the fund will generate a good investment outcome is $xq + (1 - x)(1 - q)$. However, in the equilibrium, an uninformed fund with a prior reputation $x \in (\omega_1, \omega_0)$ acquires information with probability $(\omega_0 - x)/(1 - x)$ so that the interim reputation is ω_0 . That is, in the equilibrium, the fund is informed with probability ω_0 , implying that the expected probability that the fund will generate a good investment outcome is $\omega_0q + (1 - \omega_0)(1 - q)$. Because $\omega_0 > x$ and $q > 1/2$, the fund’s performance will be underestimated, if we use only the past performance as the predictors.

Another common proxy of a fund’s reputation is the fund’s size (For example, see Gerken, Starks, and Yates, 2014). In the “work-work-shirk” equilibrium, the fund’s interim reputation is the best predictor of the fund’s performance. Since the fund’s interim reputation equals the fund’s size, it follows that the fund’s current size is the best predictor of its performance in the current period. That is, a fund with a larger size is expected to perform better. This is consistent with empirical findings in Elton, Gruber, and Blake (2011). However, the fund’s size, as the interim

reputation, has no causal effect on the fund’s performance, because the fund size is endogenously determined in the equilibrium. Also note that investors cannot use the information of a fund’s size to smartly make purchasing decisions, because when an investor receives information about the fund’s size, the investor has lost the current period’s purchasing opportunity. This is similar to the argument in Zheng (1999) that investors cannot use the Gruber’s “smart money” (1996) information to select funds.

4.2 Reputation Effects on performance

To better explain documented empirical observations, we sort all funds into three groups according to their prior reputations: Bad funds, with prior reputations smaller than or equal to ω_1 ; medium funds, with prior reputations between ω_1 and ω_0 ; and star funds, with prior reputations strictly greater than ω_0 . To provide cross-sectional prediction, we also assume that there is a continuum of funds with measure 1, who are identical except their past performance. Assume funds’ past performance are represented by their prior reputations, which are uniformly distributed over $[0, 1]$.

Let’s first analyze bad funds’ performance. In the “work-work-shirk” equilibrium, an uninformed fund with prior reputation $x \leq \omega_1$ will acquire information with probability $(\omega_1 - x)/(1 - x)$, so its interim reputation is ω_1 . Therefore, a bad fund’s expected investment outcome in the current period is $\omega_1 q + (1 - \omega_1)(1 - q)$, which is less than ω_1 . That is, a bad fund will perform bad. Two reasons lead to this prediction. First, a bad fund with bad past performance is less likely to have information at the beginning of the current period. More importantly, a bad fund acquires information with a low probability because investors do not believe it is likely to acquire information. In the “work-work-shirk” equilibrium, bad funds’ information acquisition decisions are necessary to support behaviors of medium funds and star funds. This rationalizes Carhart’s rules-of-thumb I (1997): Investors should avoid funds with persistent bad performance.

However, bad funds can still live because some investors are still purchasing them. Such investors are rational because an uninformed bad fund will acquire information with a positive probability. Therefore, the ex-ante average performance of bad funds in the current period, $\omega_1 q + (1 - \omega_1)(1 - q)$, is greater than the average bad funds’ past performance, $\omega_1/2$.

Now, let’s compare the performance of medium funds and star funds. A medium fund with a prior reputation $x \in (\omega_1, \omega_0]$ acquires information with a probability $(\omega_0 - x)/(1 - x)$. Therefore, all medium funds will have the same interim reputation in the current period. Conversely, a star fund with prior reputation $x > \omega_0$ will not acquire information, so its interim reputation is same as its prior reputation. Denote by D_0 the difference between average past performance of star

funds and those of medium funds, then

$$D_0 = \frac{\omega_0 + 1}{2} - \frac{\omega_1 + \omega_0}{2} = \frac{1 - \omega_1}{2}.$$

Also denote by D_1 the difference between average expected current performance of star funds and those of medium funds, then

$$D_1 = \left[\frac{1 + \omega_0}{2}q + \left(1 - \frac{1 + \omega_0}{2}\right)(1 - q) \right] - [\omega_0q + (1 - \omega_0)(1 - q)] = (2q - 1)\frac{1 - \omega_0}{2}.$$

Because $\omega_0 > \omega_1$ and $q \in (1/2, 1)$, we have Proposition 3 below, which shows that star funds are performing better than medium funds, but the difference between average star funds' performance and average medium funds' performance shrinks.

Proposition 3. *In the “work-work-shirk” equilibrium,*

$$D_1 > 0 \quad \text{and} \quad D_1 - D_0 < 0.$$

The mechanism leading to the unpredictability of funds' past performance on their current performance in our model differs substantially from that in Berk and Green (2004). In Berk and Green (2004), star funds receive a huge amount of cash inflows, which drive down star funds' performance because of the decreasing return to scale production function. This will equalize the ex-ante current performance of star funds and those of medium funds instantly. In the “work-work-shirk” equilibrium of our model, the difference between star funds' performance and medium funds' performance shrinks, because an uninformed star fund will not acquire information while an uninformed medium fund will acquire information with a positive probability. Star funds still perform better than medium funds in the short-run, because star funds are more likely to be informed at the beginning of the current period: Since the information has some persistence, star funds, which perform well in the past, are more likely to be informed.

4.3 Reputation Effects on Sizes and Cash flows

In our model, the current size of a fund is its interim reputation. Therefore, the effect of a fund's prior reputation on its size is equivalent to that on its interim reputation. In the “work-work-shirk” equilibrium, the reputation effect on a fund's size is illustrated in Figure 6.

From Figure 6, we can see that the fund's size is increasing in its prior reputation, thereby increasing in its past performance, since past performance determine the fund's prior reputation. However, only in the star funds group, the fund's size is strictly increasing in its past performance: Within either medium funds group or bad funds group, funds have a similar size, even though

they have different past performance. This is due to the market belief that within one group, funds with worse past performance will acquire information with higher probabilities.

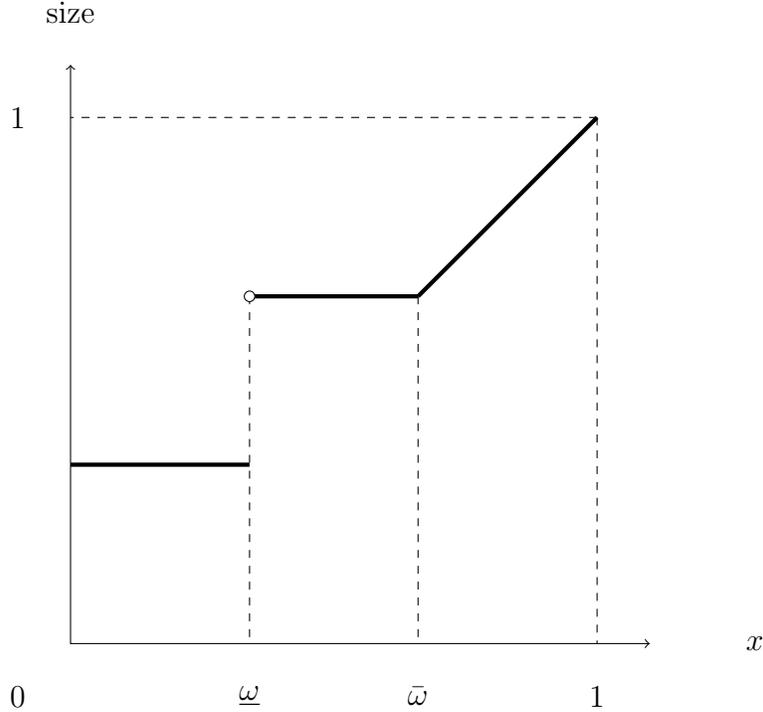


Figure 6: Reputation Effects on Sizes

Our model can also help us understand how a recent bad investment performance affects the fund’s cash flow. Consider that a fund with a size z_t receives a bad investment outcome in period t , and its size changes to z_{t+1} in period $t + 1$. In the “work-work-shirk” equilibrium, the cash flow $z_{t+1} - z_t$ following the bad investment outcome depends on the fund’s prior reputation x_t in period t . If the fund is bad (so $x \leq \omega_1$), the fund’s current size will be $z_t = \omega_1$. A bad investment outcome makes investors believe the fund is unlikely to have information in period t . Therefore, the fund’s posterior reputation drops below ω_1 . However, because the fund’s prior reputation in period $t + 1$ is low, investors believe that the fund will acquire information with a probability such that the fund’s interim reputation will jump back to ω_1 . Therefore, a bad investment outcome will not have a significant effect on the cash flow of a bad fund, which has a long bad investment history. Now consider a medium fund. In the equilibrium, its size in period t is ω_0 . Then a bad investment outcome in period t will pull its prior reputation down to ω_1 in period $t + 1$. Then the cash flow of such a medium fund is $\omega_1 - \omega_0 < 0$; that is, there will be significant cash outflows from a medium fund, if it receives a bad investment outcome. These prediction have been documented

by Berk and Tonks (2007).

Similarly, how a recent good investment outcome affects a fund's cash flow also depends on the fund's prior reputation: For a bad fund, a good investment outcome will lead to cash inflows $\omega_0 - \omega_1$; for a medium fund, a good investment outcome will not lead to significant cash flows. While it is intuitive that a bad fund receives significant cash inflows after a good investment outcome, the reason why a good investment outcome cannot bring significant cash inflows to a medium fund is that information may become outdated. With a good investment outcome, the investors are almost sure that a medium fund is informed in the current period. However, the world may change, so the fund's prior reputation in the next period is still below ω_0 .

Surprisingly, for a star fund, there is no significant difference between cash flows following a good investment outcome and those following a bad investment outcome. On one hand, after a good investment outcome, investors are more confident that a star fund is informed in the current period. However, such a positive effect of the good investment outcome is relatively small, because investors believe that the fund is informed *ex ante*. Then the information depreciation effect dominates, causing the fund's prior reputation in the next period lower than ω_0 and thus its size in the next period equal to ω_0 . On the other hand, when a star fund receives a bad investment outcome, investors think it is more likely that the fund gets a bad luck. Such a negative effect of a bad investment outcome is so small that even with the information depreciation effect, the fund's prior reputation in the next period is still greater than ω_1 . Then in the equilibrium, the fund's size in the next period is also ω_0 .

5 Conclusion

In this paper, we define a mutual fund's reputation as the market belief whether the fund is informed about the current best investment strategy. Information has two important features: First, it has short-run persistence but will become outdated in the long run; second, an uninformed mutual fund can become informed by acquiring information. Therefore, our definition of a mutual fund reputation is the market belief over an endogenous variable. Therefore, we propose a new discrete time infinite-horizon model to analyze the reputation effects on the fund's performance, size, and cash flows.

The model prediction provide potential explanations to several empirical observations in the mutual fund industry. First, bad funds will perform bad, while the difference between medium funds' performance and star funds' performance shrinks. Second, the fund's size is increasing in the fund's prior reputation, but there is no significant variation of sizes within the bad funds group and the medium funds group. Third, after a bad recent investment outcome, there is no significant

cash outflows from a bad fund, but a medium fund will suffer huge cash outflows.

Our model provides sufficient structures to understand the empirical measures of a mutual fund's reputation. In particular, a fund's past performance contribute to the fund's prior reputation, thereby causally affecting its current performance, size and cash flows. However, a fund's past performance neither equals the fund's prior reputation nor contains the probability an uninformed fund acquires information; therefore, past performance cannot provide good prediction of the fund's current performance. A fund's size can best predict the fund's current performance. However, the fund's size is endogenously determined, so it does not have causal effects. In addition, investors cannot use the information of the fund's size to select funds.

This paper has both applied contributions and theoretical contributions. From applied aspects, our model provides a framework for future studies of reputations in industries, where information or knowledge plays the most important role, for example, reputations of financial analysts, consultants, and investment banks. From theoretical aspects, this paper enriches the reputation literature by studying an infinite-horizon discrete time model, in which reputation is defined as the market belief over an endogenous variable.

Appendix A Omitted Proofs

Proof of Lemma 1. Recall that $z = x + (1-x)\hat{\sigma}$, so $z \geq x$. Then when $x > \bar{\omega}$, $z > \bar{\omega}$, which implies that when the fund's prior reputation is greater than $\bar{\omega}$, the manager will choose not to acquire information. Therefore, the equilibrium belief consistence condition requires that $\hat{\sigma}(x) = \sigma(x) = 0$, when $x > \bar{\omega}$.

For $x \in (\underline{\omega}, \bar{\omega})$, if $\hat{\sigma}(x)$ leads to $z > \bar{\omega}$, $\hat{\sigma}(x) > 0$. But $z > \bar{\omega}$ implies $\sigma(x) = 0$, which leads to $\hat{\sigma}(x) > \sigma(x)$, a contradiction to the belief consistency requirement in an equilibrium. If $\hat{\sigma}(x)$ leads to $z < \bar{\omega}$, the manager will certainly acquire information, so $\sigma(x) = 1$. Then, because $z < \bar{\omega}$, $\hat{\sigma}(x) < 1 = \sigma(x)$, which also contradicts the belief consistency requirement. Therefore, when the manager's prior reputation $x \in (\underline{\omega}, \bar{\omega})$, the only possible interim reputation z is $\bar{\omega}$. □

Proof of Lemma 2. Suppose $\sigma(x) = 1$ for some x . Then the manager with the prior reputation x will have the interim reputation

$$z = x + (1-x)\sigma(x) = 1.$$

Then, the manager's reputation in the next period will be $1 - \lambda$, independent of the investment outcome. To support the equilibrium, we must have $V_I(x) - V_U(x) \geq c$, which implies that

$$V_I(1 - \lambda) - V_U(1 - \lambda) \geq \frac{c}{\delta(1 - \lambda)}$$

Since $\delta, 1 - \lambda \in (0, 1)$, we must have $V_I(1 - \lambda) - V_U(1 - \lambda) > c$ and $\sigma(1 - \lambda) = 1$ as well. As a result, we have

$$V_I(1 - \lambda) - V_U(1 - \lambda) = \delta(1 - \lambda)[V_I(1 - \lambda) - V_U(1 - \lambda)],$$

which is a contradiction! □

Proof of Proposition 2. From the description of the “work-work-shirk” strategy, we can see that only values of V_I^0, V_U^0, V_I^1 and V_U^1 matter. That is, in order to verify whether the proposed strategy is an equilibrium, we first need to solve these values. Consider the manager with prior reputation ω_0 first. Since the uninformed manager is believed not to acquire information at this reputation, her interim reputation is the same as the prior reputation. Then her value function is

$$\begin{aligned} V_I(\omega_0) &= \omega_0 + \delta [q(1 - \lambda)V_I^0 + q\lambda V_U^0 + (1 - q)(1 - \lambda)V_I^1 + (1 - q)\lambda V_U^1], \\ V_U(\omega_0) &= \omega_0 + \delta [(1 - q)V_U^0 + qV_U^1]. \end{aligned}$$

Take difference, we have

$$V_I^0 - V_U^0 = \delta [q(1 - \lambda) (V_I^0 - V_U^0) + (1 - q)(1 - \lambda) (V_I^1 - V_U^1) + (2q - 1) (V_U^0 - V_U^1)] \quad (7)$$

Similarly, if we calculate the difference between the informed manager's value and the uninformed manager's value at the prior reputation ω_1 , we have

$$V_I^1 - V_U^1 = \delta [q(1 - \lambda) (V_I^0 - V_U^0) + (1 - q)(1 - \lambda) (V_I^1 - V_U^1) + (2q - 1) (V_U^0 - V_U^1)] \quad (8)$$

We can see that

$$V_I^0 - V_U^0 = V_I^1 - V_U^1.$$

Set

$$V_I^0 - V_U^0 = V_I^1 - V_U^1 = c,$$

then the equilibrium requirement Equation (4) is satisfied, because the manager will randomize when the prior reputation $x \in [0, \omega_1) \cup (\omega_1, \omega_0)$. Given Equation (4), when the manager's prior reputation $x > \omega_0$, the difference between values of an informed manager and an uninformed manager is

$$V_I(x) - V_U(x) = \delta(1 - \lambda)(V_I^0 - V_U^0) < c,$$

because $\delta(1 - \lambda) < 1$. That is, the equilibrium condition Equation (5) is satisfied. As a result, the uninformed manager with the prior reputation $x > \omega_0$ will choose not to acquire information.

With the equilibrium condition (4), Equation (7) implies

$$\begin{aligned} c &= V_I^0 - V_U^0 \\ &= \delta [q(1 - \lambda) (V_I^0 - V_U^0) + (1 - q)(1 - \lambda) (V_I^1 - V_U^1) + (2q - 1) (V_U^0 - V_U^1)] \\ &= \delta [(1 - \lambda)c + (2q - 1) (V_U^0 - V_U^1)], \end{aligned}$$

which implies that

$$V_U^0 - V_U^1 = \omega_0 - \omega_1 = \frac{1 - \delta(1 - \lambda)}{\delta(2q - 1)}c.$$

By definition,

$$\omega_1 = (1 - \lambda) \frac{(1 - q)\omega_0}{(1 - q)\omega_0 + q(1 - \omega_0)},$$

so we have Equation (6)

$$\omega_0 - (1 - \lambda) \frac{(1 - q)\omega_0}{(1 - q)\omega_0 + q(1 - \omega_0)} = \frac{1 - \delta(1 - \lambda)}{\delta(2q - 1)}c.$$

That is, for a given set of parameters, we can identify ω_0 , which is the starting point of the equilibrium construction.

Given the constructed ω_0 and ω_1 , we know

$$\begin{aligned} V_U^0 &= \omega_0 + \delta [(1 - q)V_U^0 + qV_U^1], \\ V_U^1 &= \omega_1 + \delta [(1 - q)V_U^0 + qV_U^1]. \end{aligned}$$

Therefore,

$$V_U^0 - V_U^1 = \omega_0 - \omega_1.$$

By arranging terms, we can solve

$$\begin{aligned} V_U^0 &= \frac{1}{1 - \delta} [\omega_0 - \delta q (V_U^0 - V_U^1)] \\ &= \frac{1}{1 - \delta} [\omega_0 - \delta q (\omega_0 - \omega_1)] \\ &= \frac{(1 - \delta q)\omega_0 + \delta q\omega_1}{1 - \delta}. \end{aligned}$$

and

$$\begin{aligned} V_U^1 &= V_U^0 - (\omega_0 - \omega_1) \\ &= \frac{\delta(1 - q)\omega_0 + (1 - \delta(1 - q))\omega_1}{1 - \delta}. \end{aligned}$$

Then we can calculate that

$$\begin{aligned} V_I^0 &= c + \frac{(1 - \delta q)\omega_0 + \delta q\omega_1}{1 - \delta}, \\ V_I^1 &= c + \frac{\delta(1 - q)\omega_0 + (1 - \delta(1 - q))\omega_1}{1 - \delta}. \end{aligned}$$

Given the derived value functions, an uninformed fund will be indifferent between acquiring information and remaining uninformed when its prior reputation is below ω_0 ; the fund will certainly not acquire information if its prior reputation is greater than ω_0 . By construction, the fund's strategy is consistent with the market belief. Therefore, the proposed strategy profile is a stationary equilibrium. □

Proof of Proposition 3. $D_1 > 0$ directly follows from the assumptions $q \in (1/2, 1)$ and $\omega_0 < 1$. We also have

$$\frac{1 - \omega_1}{2} > \frac{1 - \omega_0}{2},$$

because $\omega_0 > \omega_1$. Then $2q - 1 < 1$ implies that $D_1 < D_0$. □

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