

Home Equity, Foreclosures, and Bail-outs*

Carlos Garriga
Federal Reserve Bank of St. Louis

Don E. Schlagenhauf
Florida State University

October 24, 2008

Abstract

This paper examines the increase in housing foreclosures in the United States in the aftermath of the recent housing boom. Foreclosure rates are at levels that are high by historical standards. We argue that a key element in understanding the increase in foreclosures rate is the leverage. An increase in leverage exposes homeowners to additional risk in the event of a decline in the house price. We develop an equilibrium model of housing to aid in understanding these patterns. In the model, homeowners purchase different size homes, have access to a menu of long-term mortgage loans, and have a default option on these loans. We find that the decline in house prices can account for most of the observed increase in the aggregate foreclosure rate in the United States. The model makes consistent predictions about the default rates across different loan types and the decline in homeownership.

Keywords: Housing default, mortgage contracts, homeowners

J.E.L.:E2, E6

*We acknowledge the useful comments from Gaetano Antinolfi, Dirk Krueger, and José Victor Ríos-Rull. Carlos Garriga and Don Schlagenhauf are grateful to the financial support of the National Science Foundation for Grant SES-0649374. Carlos Garriga also acknowledges support from the Spanish Ministerio de Ciencia y Tecnología through grant SEJ2006-02879. The views expressed herein do not necessarily reflect those of the Federal Reserve Bank of St. Louis nor those of the Federal Reserve System. Corresponding author: Don Schlagenhauf, Department of Economics, Florida State University, 246 Bellamy Building, Tallahassee, FL 32306-2180. E-mail: dschlage@mailier.fsu.edu. Tel.: 850-644-3817. Fax: 850-644-4535.

1 Introduction

Since the early 1990s, the American housing market has experienced an initial period in which homeownership and housing prices rose. During this initial period, there were substantial innovations in housing finance that modified the term structure and the downpayment requirements of mortgage loans. These innovations in conjunction with historically low mortgage rates were partially responsible for the increase in house prices. More recently, this market has experienced a decline in the homeownership rate, a fall in prices, and an increase in foreclosures. In fact, foreclosure rates have reached levels not seen since the Great Depression. Understanding the determinants that account for the increase in foreclosures is critical if policy responses are to be appropriately formulated.

We argue that an important mechanism for understanding changes in the foreclosure rate is the speed at which equity is reduced when house prices decline. In an economy where all homes are free of mortgage debt, a 10 percent decline in house prices results in a 10 percent decline in homeowner's equity. In the United States, only 25 percent of homes are clear of mortgage debt. For the remaining households, the average equity in a house is approximately one third of the value of the property. For individuals with an outstanding mortgage, a 10 percent decline in home prices wipes out 30 percent of their equity. While this "home equity" multiplier effect increases the homeowner's equity when house prices increase, sizeable negative effects occur when house prices decline. The size of this multiplier effect depends on the leverage position of homeowners as measured by the loan-to-value (LTV) ratio. In the last few years, individuals have increased their exposure to house price risk by taking on highly popular, highly leverage loans. In addition, the fraction of properties that are owned free and clear has declined by 20 percent. These two conditions in conjunction with other developments in the economy provide an environment favorable for foreclosures.

The objective of this paper is to construct a model that helps us understand the main determinants of foreclosure and thus accounts for the observed spike in housing defaults. The model provides a tool to measure the distributional impact of the decline in house prices for different individuals. Such a framework can be used to help in understanding an environment with higher levels of risky lending, as well as evaluating the effectiveness of different government policy interventions.

A model designed to understand the determinants of foreclosure should also capture essential features of the housing market. An important element of housing finance is the availability of an array of long-term mortgage loans with different leverage positions. Most existing foreclosure studies are mainly empirical and restricted to aggregates. A limitation of these studies is the lack of individual information on the loan performance as well as borrower characteristics.¹ Access to disaggregated data would provide useful information for understanding the determinants of foreclosure. However, insights can be gained by foreclosure data over various mortgage products. The empirical analysis we carry out indicates that higher foreclosure rates are occurring with loans in the subprime market. These types of loans are characterized by high loan-to-value ratios (LTV) and low initial mortgage payments. Loans with similar characteristics in the prime market have also been subject to an increase in defaults. These products are held mainly by first-time buyers (usually young and low-income households) as well as repeated buyers who choose this type of loan in order to consume a portion of the home equity. The low levels of equity associated with these

¹A notable exception is Gerardi, Shapiro, and Willen (2008) who have a unique dataset which is limited to the state of Massachusetts.

particular types of mortgage holders increases the homeowners' exposure to the widespread decline in house prices. By contrast, the foreclosure rate for borrowers that use fixed-rate mortgage loans with relatively large downpayment levels (i.e., a 15 percent or higher downpayment) has remained consistent with historically low levels. We argue that an essential issue in understanding the sharp increase in the level of foreclosure rates is to understand the determinants of mortgage choice in conjunction with the evolution of house prices, since they determine the levels of home equity.

With this purpose we develop an equilibrium-based model of housing default. We parameterize this model so that it matches relevant features of the U.S. economy and housing market prior to the decline in house prices. A key feature of the model is that housing investment is part of the household's portfolio decision and differs from capital investment along several dimensions. Housing investment is lumpy and indivisible, is subject to idiosyncratic capital gains shocks, and requires a downpayment and long-term mortgage financing. However, at any point in time homeowners can default on their obligations, and lose their property. Households have the option to purchase housing services in the rental market. Mortgage loans are available from a financial sector that receives deposits from households and also loans capital to private firms. We show that the parameterized model is consistent with the relevant housing and foreclosure aggregates observed prior to 1997 and also captures distributional patterns of ownership, housing consumption, mortgage holdings, and foreclosure by loan type.

Our preliminary findings suggest that an unanticipated decline in house price can account for the spike in foreclosure rates. The model predicts sizeable foreclosure rates for prime and subprime lending. Moreover, the dynamic path under a government bailout of the mortgage industry is consistent with a short-term decline in homeownership. Despite the decline in house prices, the increase in supply of tenant-occupied housing reduces the rental price. Cheaper renting combined with higher taxes reduces the fraction of individuals who purchase a home in the short-run. Since the bailout is transitory, the new lending that emerges in the economy provides new loans based on the corrected collateral value and it helps the economy to increase the ownership away from post-house price decline. We argue that the response of the rental market is very important for understanding the response of foreclosure rates to declines in house prices. Models where rental rates are based on an arbitrage pricing relationship do not seem to be able to capture these facts.

An outline of the paper follows. Section II presents the empirical evidence. Section III presents a model of housing default and calibrates it to match the evolution the relevant aggregates before the collapse of housing markets. Section IV uses the calibrated model to assess the importance of the default option for house prices, while Section V uses the model to account for the increase in the level of foreclosure rates in the aftermath of the collapse of housing markets. Section VI presents conclusions.

2 Evolution of Foreclosures and Home Equity in the United States

The level of foreclosures have increased rapidly around 2005 after being at low and relatively constant level for roughly two decades. This section looks beyond aggregate foreclosure patterns shows that foreclosures have distinct patterns across different loan products. We argue that the key to understanding the soaring of default rates in mortgages markets is to

understand the expansion in housing finance options that allow homeowners to purchase a house with a high loan to value ratio (i.e. more than the standard 80 percent), low initial payments (i.e. hybrid loans with teaser rates), or both. The relatively low levels of equity associated to these type of loans increase the home owners' exposure to the decline in the house prices. A change in the value of the collateral increases the default probability of households with negative equity.

2.1 Aggregate Foreclosures

We start displaying the evolution of level of foreclosures in the United States. Aggregate foreclosures measures the percentage rate of loans for which a foreclosure has been initiated, that is, the number of loans sent to the foreclosure process as a percentage of the total number of mortgages in the pool.² Figure 1 illustrates the evolution of foreclosure rates starting in 1990 for the U.S. economy.

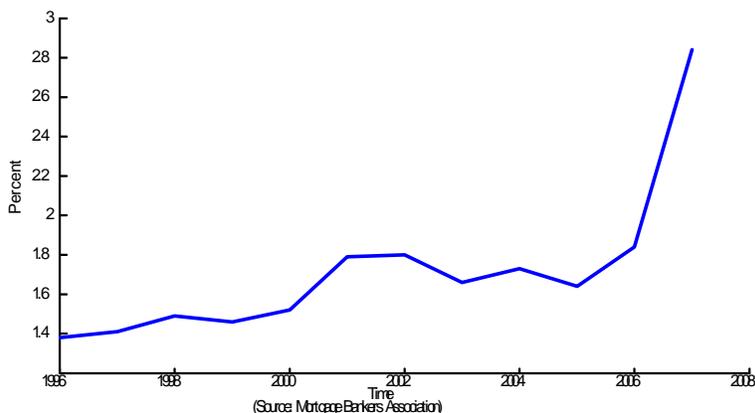


Figure 1: Evolution of Foreclosures in the United States

This picture shows that the aggregate level of foreclosures between 1990 and 2004 has been relatively stable at 1.4 percent. A small exception occurred during the 2001 recession. This relatively stable period ended in 2006 as the foreclosure rate for the total pool of mortgages doubled as this rate increased to three percent.

2.2 Foreclosures by Loan Type in the United States

Focusing on the aggregate rate masks the differences in foreclosure rates that may occur by mortgage loan type. Figure 2 displays the evolution of foreclosure rates by loan type. We group various loans products into a fixed rate mortgage (FRM) group and an adjustable rate mortgage contract (ARM). This means the fixed rate mortgages group includes the prime and subprime market. The fixed rate loan market exhibits a very low foreclosure rate, even over the last four years. Most of the foreclosures are concentrated in the adjustable rate market and in particular in the subprime market. These pictures suggest that understanding the increase in the level of foreclosures observed between 2007 and 2008, requires an examination

²The Mortgage Bankers Association conducts the *National Delinquency Survey* (NDS) since 1953. The survey covers 46 million loans on one-to-four-unit residential properties, representing over 80 percent of all "first-lien" residential mortgage loans outstanding in the United States. Loans surveyed were reported by approximately 120 lenders, including mortgage bankers, commercial banks, and thrifts.

of loan products in the adjustable rate market.

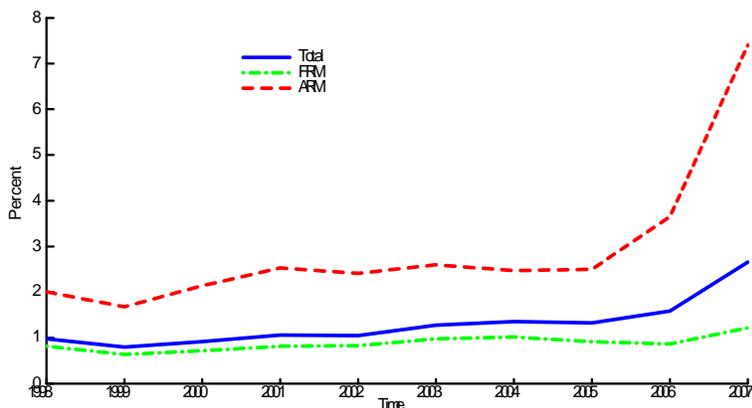


Figure 2: Foreclosures by Loan Type in the United States

In general we observed that the default rates have been relatively stable across loans during the period prior to the decline of house prices. The data suggests that the level of foreclosures are higher in adjustable rate and term loans than with fixed rate loans. Since the market share of fixed rate mortgages is higher, the evolution of the aggregates resembles the evolution for the FRM market. At a more disaggregated level, we find that default rates are substantially larger for subprime loans and loans provided by the Federal Housing Administration (FHA).³ In contrast, loans funded in the conventional prime market have a lower default rate, even in period of declining house prices. The aggregate default rate seem to be driven by the conventional subprime market and the FHA loans. The expansion of subprime lending is a relatively new phenomena. In about three years, this market’s share went from 3 percent in 2001 to 13 percent in 2005. In general, these lenders offer relatively new loan products (i.e. interest-only loans, hybrid loans, combo or piggyback loans, the no- and low documentation mortgages, and specially the option ARMs) that differ in the downpayment requirement, repayment schedule, and interest payments schedule from more traditional loan contracts.⁴ Unfortunately, from an empirical perspective these contracts are often categorized as fixed rate mortgage loan (FRM) and adjustable rate mortgage (ARM); hence it is very difficult to identify the specific nature and characteristics of the individuals

³The importance of the government agencies in origination is relatively large. The share of primary mortgages originated by the Federal Housing Administration, the Veteran Administration (VA), and the Farmer’s Home Administration (FRHA) ranges between 20 and 24 percent in the period 1993 to 2005. The remaining loans are originated by private lenders or mortgage brokers and then sold in the open market by the GSE.

⁴Interest-only loans allow borrowers to delay principal payments for some period before amortization starts. Hybrid ARMs allow borrowers to pay low interest rate for a specific amount of time, between 1 and 5 years, and then it floats according to some reference rate. Combo or piggyback loans allow to take a secondary mortgage to cover the downpayment amount. In some cases, the lender allows to borrow the full downpayment so the loan-to-value ratio is 100 percent. These loans are very attractive to borrowers since they allow to avoid the private mortgage insurance (PMI) required in traditional loans with a high LTV ratio. No- and low-documentation loans allow for less detailed proof of income than traditional lenders would require. The payment-option adjustable differ from the common ARMs since gives borrowers a choice of several payment alternatives each month, ranging from full amortization of principal and interest to minimum payments. There are other adjustable rate mortgages that do give the option to choose the payment structure, but the payments and the amortization schedule increase over the life of the loan at a predetermined rate. This product is very attractive for borrowers because of the initial lower cost of the loan.

with nonperforming loans. Given this data limitation, we have to restrict the analysis to these two general class of mortgage loans.

If we condition delinquency rates by loan type, we observe that most defaults are associated to mortgage loans that adjust payments over the length of the contract. The terms of these loans usually differ from the traditional FRM contract as they are characterized by higher loan-to-value ratio and time varying repayment structure. A changing repayment schedule allows the lender to offer introductory teaser rates that reduce the initial cost of purchasing a house. Ideally, it would be nice to have detailed data with the share of these loan contracts by different characteristics. Unfortunately, this type of data is not available so we have to rely on less direct information to argue that the market share of these loan products grew after 2003. Using indirect evidence, we present two sets of facts. Figure 4 shows that the share of FRM fell by 14 percent between 2003 and 2006. After the subprime crises in 2006, the share of FRM recovered one third of the original market share. The decline in market share of FRM is consistent with the expansion of subprime lending and the increasing demand of non-traditional loan products.

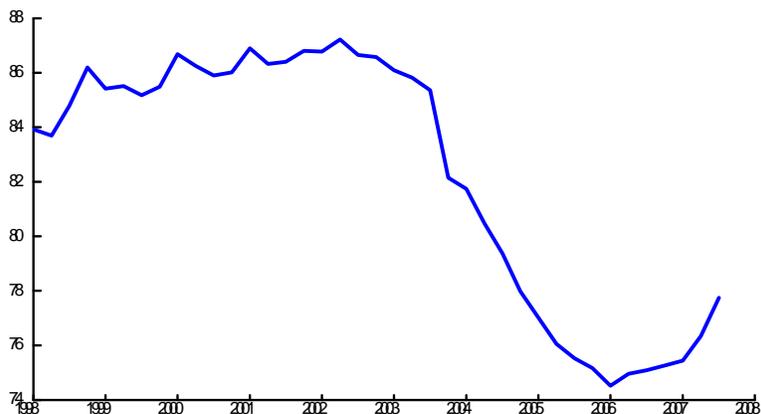


Figure 3: Market Share of FRM in the United States

The second source of evidence is presented in Table 1 that reports the relative importance of non-traditional loans in the subprime market. Two interesting facts stand out. First, the demand for nontraditional products increase 76 percent between 2002 and 2005. Second, nontraditional loans have become increasingly significant in the market. For example, we observe a relative decline in the importance of traditional ARM contracts, and an increase in other products. It is important to remark that since the market grew, the number of individuals holding each type of contracts increased but the relative distribution changed.

Table 1: Relative Importance of Nontraditional Loans

Loan Type	2002	2005	
	Total=Share	Total	Share
Interest only	0%	29%	16%
Combo or Piggyback	30%	41%	23%
No-or low documentation	2%	33%	19%
ARMs	68%	73%	41%
TOTAL	100%	176%	100%

Data: Loan Performance

This national level evidence is consistent with the conclusions in Gerardi, Willen, and Rosen (2008). They study in the Massachusetts loan market using a panel of subprime

borrowers between 1989 and 2007 to estimate how often these borrowers end up in foreclosure. They find that a subprime borrower ends up 6 times as often in foreclosure in comparison to a prime borrower. In addition, they find that the dramatic increase in Massachusetts foreclosures during 2006 and 2007 can be attributed to the decline in house prices that began in the summer of 2005.

2.3 Decomposition of Aggregate Foreclosures

The aggregate foreclosure rate can also be viewed as the weighted average of the foreclosure rates across mortgage loan types. The previous section illustrates that the default rates and relative importance of various contracts has substantially changed in the last decade. The aggregate level of foreclosures, D , is simply the sum of foreclosures over all mortgage product types. That is,

$$D = \sum_i D_i$$

where D_i represents the number of foreclosures of mortgage type i . The aggregate number of mortgages, M , is the sum over all mortgage types, or

$$M = \sum_i M_i$$

Now, we define $d = D/M$ and $d_i = D_i/M_i$ to be the aggregate default rate and the default rate of a mortgage of type i . By using these definitions, we can derive an expression for the aggregate foreclosure rate as a function of the default rate for a particular mortgage product and the relative share of that product in the mortgage market. That is,

$$\frac{D}{M} = \frac{\sum_i D_i}{M_i} \frac{M_i}{M}$$

or

$$d = \sum_i m_i \cdot d_i$$

where the term $m = M_i/M$ captures the relative size of each market. This expression suggests that an increase in aggregate foreclosures has to result from either an increase in default by loan type, a change in the market share towards loans with high default rates, or both. The prior discussion suggest that the increase in foreclosures is due to both factors.

Consequently, the contribution of each factor can be easily calculated as

$$\Delta d = \sum_i m_i \cdot \Delta d_i + \sum_i \Delta m_i \cdot d_i + \sum_i \Delta m_i \cdot \Delta d_i.$$

The first term represents the intensive margin, the second term represents the extensive margin, and the last term represents the covariance term. To compute the contribution of each factor we use data from the *Mortgage Banker's Association*. Given the market share of each loan product and the respective foreclosure rate we can compute the contribution in accounting for the increase in aggregate foreclosures.

The measures of foreclosed properties reported in the paper are constructed using the above definitions and letting $i = FRM, ARM$. In particular, the share of FRM captures the total number of outstanding fixed rate mortgage loans in the prime and subprime market whereas the share of ARM is constructed to represent the total number of outstanding adjustable rate mortgage loans and nontraditional (non FRM) products. Table 2 we construct

a decomposition for 1998, which is prior to house price declines, and 2007 when house prices were falling.

Table 2: United States: Actual and Hypothetical Foreclosure Rates

	Expression	Foreclosures	Contribution	
			% Change	Total
Share (1998) and Foreclosures (1998)	$\sum_i m_i^{98} \cdot d_i^{98}$	0.97%		
Share (2007) and Foreclosures (2007)	$\sum_i m_i^{07} \cdot d_i^{07}$	2.63%	171.3	
Share (1998) and Foreclosures (2007)	$\sum_i m_i^{98} \cdot d_i^{07}$	2.07%	113.6	66.4%
Share (2007) and Foreclosures (1998)	$\sum_i m_i^{07} \cdot d_i^{98}$	1.08%	11.2	6.5%
Covariance(m,d)	$\sum_i \Delta m_i^{07} \cdot \Delta d_i^{98}$	1.42%	46.5	27.4

Data: Loan Performance

The decomposition exercise s that the increase in foreclosure rates in each market accounts for two thirds of the total increase in the aggregate level. This factor alone would have taken the aggregate default rate from 0.97 to 2.07 percent. The importance of the loan share appears to be very small and it only accounts for 7 percent of the total change, and the implied aggregate foreclosure level is 1.08 percent. These two factors represent 75 percent of the total contribution. The remaining 25 percent is due to the covariance terms that captures the joint effects associated to a change in the market share of each loan and the foreclosure rates in each market. The implication of this analysis is that the answer for the increase in the aggregate foreclosure rate lies the default rates by loan contract type.

2.4 House Prices, Home Equity, and the Equity Multiplier

The objective is to use the model to address the impact of a decline in house prices for the aggregate level of foreclosures. Between the peak in 2005 and Q4 2007 national house prices fell over 5 percent in annual terms. Figure 4 displays the evolution of house prices between 1998 and 2007 using the Case-Shiller, OFHEO, and Conventional Mortgage price series. The figure clearly suggests that the recent adjustment in house prices has been very dramatic. This Figure understate the decline in prices observed in certain local markets. The housing markets in Arizona, California, Florida and Nevada, the price declines have been over 10 percent.

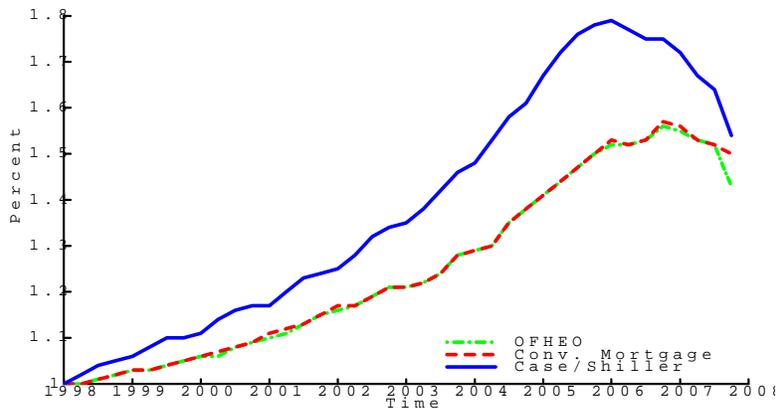


Figure 4: Evolution Houses Prices in the United States

Figure 4 suggests a connection between house price depreciation and foreclosures. However, a decline in house prices is not necessarily have to result in an increase in foreclosures.

Between 1979 and 1982, home prices fell 11 percent in real terms, according to the Case-Shiller index, but foreclosure rates remained around 0.7 percent. Between 1990 and 1993 the decline in house prices was a bit more modest, around 7 percent, but the foreclosure rate remained at 1.3 percent. These figures are summarized in Table 3.

Table 3: Home Price Appreciation and Foreclosure Rates

Variable	1979-82	1990-93	2005-07
Real Home Price Appreciation (HPA)			
Case-Shiller Index	-11.0%	-7.0%	-14.0%
OFHEO Index	-2.7%	-1.6%	-4.0%
Foreclosures Rates			
	0.7%	1.3%	3.0%

Data: Loan Performance

The difference between these two episodes (1979-82 and 1990-93) and the more recent events (2005-07) is a change in the type of mortgage product being used which has changed the leverage position of the homeowner. Since most properties are mortgaged, the decline in house prices will have a larger effect in homeowners' equity the greater the amount of leveraged. To measure the homeowners' exposure to a change in the value of the house we use the concept of a "home equity multiplier" and show how the size of the multiplier depends on the economy's total leverage.

Let V_0 represent the property value at $t = 0$. This value can be further decompose into outstanding mortgage debt D_0 and the equity in the house E_0 . Formally,

$$V_0 = D_0 + E_0$$

A percent change in the house value can be written as

$$e = \frac{1}{1 - LTV}v.$$

where $v = (V - V_0)/V$ represents a percent change in house value, $e = (E - E_0)/E_0$ a percent change in equity value, and $LTV = D_0/V_0$. The home equity multiplier implies that a percent changes in equity value is amplified by the size of leverage. When the $LTV = 0$, the percentage change in house value and equity are equal, but otherwise the effects are larger.

To compute the equity multiplier we have to take a snap shot look of the US residential real estate market for 2007. According to the *Flow of Funds Accounts* and the *Survey of Consumer Finances* show that the value of all houses is \$20 trillion. Roughly 25 percent of all houses are owned free and clear. We can approximate the value of these homes at \$5 trillion. The remaining \$15 trillion are is own by households who have an outstanding mortgage(s) on the property. Data indicates that approximately 1/3 of the \$15 is homeowners' equity with the remaining an estimate of outstanding mortgage debt. The implied aggregate LTV ratio of 67 percent. As a result, a widespread decline in home prices of 10 percent reduces the value of the mortgaged houses to \$13.5 trillion. However, the total level of mortgage debt remains unchanged to \$10 trillions and homeowner's equity is reduced to \$3.5 trillion. This mean the equity position has declined by 30 percent.

In Figure 5, we present aggregate foreclosures between 1970 and 2007 as well as an estimate of the home equity multiplier for each year.

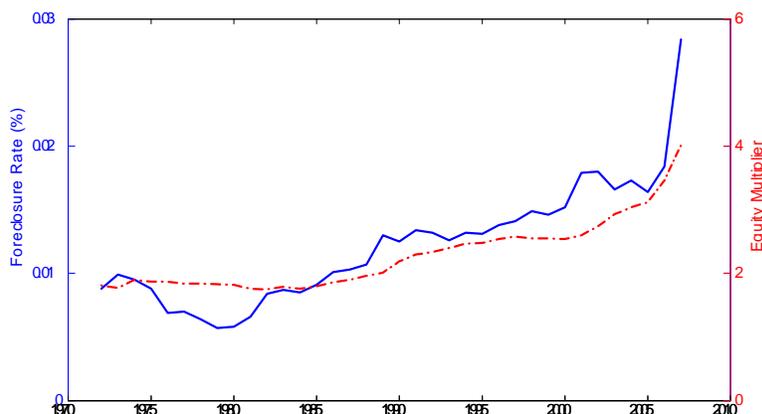


Figure 5: Equity multiplier and Foreclosures in the United States

This figure suggests that the increase in leverage in the housing markets has increased the equity multiplier and that this increase seems to be correlated with the increase in foreclosures. The pattern in the home equity multiplier not only captures the steady increase in the foreclosure rate between 1970 and 2000, but also captures part of the recent spike in defaults. It is interesting to examine the home equity multiplier for the period 1979-1982 and 1990-1993. These were periods of house price declines, but relatively no change in the foreclosure rate. Interestingly, these were also periods where the home-equity multiplier remained low.

2.5 Mortgage Choice and Foreclosures

The previous subsections analyzed the evolution of foreclosure rates at the aggregate level and across different loan types. The decision to foreclose a property requires the solution of a complex problem. The individual has to take into consideration current and future benefits and losses. In addition, when an individual chooses a particular mortgage loan, consideration must be given to the spread in mortgage rates that are due to the associated default risk. As a result, the choice of purchase a house and the type of mortgage loan are not independent of the decision to foreclose a property. The introduction of a default option limits the home owners losses in the event of foreclosure and as a result should increase the incentives to participate in the owner-occupied housing market and to purchase large units. Understanding of the foreclosure decision requires bridging the housing literature with the literature that examines default on unsecured lending. The housing literature provides the foundation for the modelling of housing decisions whereas the default literature provides a framework to formalize the pricing of short-term loan contracts with default option. Some of the relevant papers in the housing literature in the context of general equilibrium models include Díaz and Luengo-Prado (2005), Davis and Heathcote (2006), Fernández-Villaverde and Krueger (2002), Gervais (2002), Kiyotaki, Michaelides, and Nikolov (2007), Li and Yao (2007), Nakajima (2004), Ortalo-Magne and Rady (2006), Sánchez-Marcos and Ríos-Rull (2008), Chambers, Garriga and Schlagenhaut (2007). However, one of the important limitations in this literature, with the exception of Chambers, Garriga, and Schlagenhaut, is that houses are financed with one-period collateralized loans, thus abstracting from longer-term mortgage choice. As a result, the option of foreclosure is considered.

There has been a growing literature on default in unsecured credit market using an equilibrium model approach. Some of the papers in this literature include Athreya (2002),

Li and Sarte (2006), Livshits, MacGee, and Tertilt (2007), Chatterjee, Corbae, Nakajima, and Ríos-Rull (2005), Chatterjee, Corbae, and Ríos-Rull (2006), Athreya, Tam, and Young (2008), Sánchez (2008), Nakajima (2008).⁵ The main limitations of this literature is that deals with unsecured lending and short-term relationships. In addition, all these models only deal with one-asset economy that results in relatively low equilibrium default rates.

The paper by Jeske and Krueger (2005) is the one paper that introduces housing default option into an equilibrium model that includes housing and one-period mortgage contracts. The focus of their paper is to study the macroeconomic effects of the interest rate subsidy provided by government-sponsored enterprises (GSEs). They set up a infinitely lived model in which households face an idiosyncratic house depreciation shock that can result in negative equity on their home and consequently default. In addition, they allow households to choose any downpayment ratio but the interest rate charged in the loan depends on the leverage ratio. Two limitations of their modelling approach are that the loan structure is irrelevant and housing is not subject to adjustment costs. As a result, households that face negative income shocks can downsize at no cost. In addition, the infinitely lived structure implies that mortgage loans are never repaid since the homeowner keeps refinancing the house purchase to buffer income shocks. In this paper we use a housing model that allows households to choose to finance their home purchase from a finite menu of loan products. These mortgage loans represent a long-term commitment to repay the property under certain conditions, otherwise in the event of foreclosure the property is repossessed and the owner loses the collateral.

2.6 A Primer on Housing Foreclosure

Prior to the construction of a model to study housing foreclosure, it is important to study the legal environment as it pertains to foreclosure. This allows the essential features of the legal environment to be embedded into the economic environment faced by households and mortgage investment banks. In this section, we briefly discuss the essential elements of the legal environment.

Foreclosure is a legal proceeding in which a lender obtains a court ordered termination of the borrower's, or mortgagor's equitable right of redemption. The redemption is in the form of the asset used to secure the loan. Foreclosure allows the lender to foreclose the right of redemption which allows the borrower to repay the debt and redeem the property. Mortgage documents specify the period of time after which default occurs and thus foreclosure can be initiated. Foreclosure can be supervised by a court in which case is known as Judicial Foreclosure. If the courts do not supervise, then the sale of the property determines foreclosure. This is known as Foreclosure by Power of Sale.

Another important concept in foreclosure law is *acceleration*. This concept allows the lender to declare the entire debt of a defaulted mortgage due and payable. The question from a modelling perspective is what is the amount due and payable. The answer to the question depends on the state a homeowner lives in. In most states, the mortgage is recourse debt. This means the lender can get a deficiency judgement to place a lien on the borrower's other property that obligates the original borrower to repay the difference from these other assets, if any. However, there are States where mortgages are treated as non-recourse debt. In

⁵Drozd and Nosal (2008) and Mateos-Planas and Ríos-Rull (2008) provide a notable exception that deals with default and long-term relationships.

this case, the lender can not go after the borrower's other assets to recoup losses.⁶. It should be pointed out that only the original mortgage is treated as non recourse debt. Refinanced loans and home equity lines of credit are still treated as recourse debt. Our initial model will assume mortgages are non-recourse debt.

If a lender chooses not to pursue a *deficiency judgement* because the borrower has insufficient assets or the mortgage is legally treated as a non-recourse debt, the debt write-off the borrower may have an income tax obligation on the remaining unpaid principal if it can be considered as "forgiven debt." Recently, the tax law has been changed on forgiven debt as it pertains to foreclosed property. For the period January 1, 2007 through December 31, 2009, homeowners are not obligated to pay tax on any debt on a primary residence that is cancelled.

3 Equilibrium Model of Mortgage Choice with Foreclosure

In this section we modify the housing framework used by Chambers, Garriga, and Schlagenhaut (2007) to include foreclosure and calibrate it to match the relevant empirical evidence in housing default. The model emphasizes two relevant factors that determine the evolution of foreclosure rates: the availability of mortgage choice with different levels of leverage and the riskiness of housing investment. In addition, it is important for the model to capture relevant features on the U.S. housing markets. These include the amount of homeowners, house size, and types of mortgage financing. Section V will use the baseline model to assess the effect of changes in house prices in the level and the distribution of foreclosure rates. To keep matters simple the decline in house prices will be modelled as improvements in the productivity of the construction sector that reduces the unit price of housing investment goods. However, the market for tenant-occupied housing responds to changes in house prices adjusting the rental price accordingly.

3.1 Households

A key criterion for the model is that it be able to replicate the observed foreclosure rates across different mortgage contracts. That requires a set up with heterogeneous consumers and incomplete markets. In this arrangement, the decision to purchase a house is not determined by the household's permanent income, but rather the current income, and resources to meet a certain downpayment, and the menu of mortgage loans available. To ease the notation, we have suppress time subscripts and focus on the problem for a particular time period. In addition, some of the modelling choices have been made to capture an empirical counter part, while others have been made to simplify the problem while maintaining essential features of the problem.

Demographic structure and preferences: We consider an overlapping generation structure where a newborn cohort is born at every period and lives a maximum of J periods. Survival each period is subject to mortality risk. The probability of surviving from age j to age $j + 1$ is denoted by $\pi_{j+1} \in (0, 1)$, with $\pi_1 = 1$. The demographic structure is given by $\mu_j = \pi_j \mu_{j-1} / (1 + \rho)$ for $j = 2, 3, \dots, J$ and $\sum_{j=1}^J \mu_j = 1$, where ρ denotes the rate of growth

⁶In the United States, eight states treat mortgage debt as non recourse debt. The States with anti-deficiency laws are: Alaska, Arizona, California, Minnesota, Montana, North Dakota, Oregon, Washington.

of population.⁷ Each newborn cohort has preferences defined over the expected discounted sum of momentary utility functions,

$$E\sum_{j=1}^J\beta^{j-1}\pi_j u(c_j, d_j),$$

where $0 < \beta < 1$ is the discount factor. The momentary utility functions are defined over consumption of goods, c_j , and housings services or dwelling size, d_j . The period utility function is neoclassical and satisfies the standard properties of continuity and differentiability $u : \mathfrak{R}_2 \rightarrow \mathfrak{R}$, and $u' > 0$ and $u'' < 0$. An important issue in the demand for housing over the life-cycle is the income elasticity. In our formulation we emphasize the non-homotheticity in preferences to generate an income elasticity that is not unitary. Under this assumption, as income increases over the life-cycle the fraction of resources devoted to housing consumption increase relative to goods consumption.⁸ A unitary elasticity generates distribution of housing consumption over the life-cycle that is not consistent with the evidence. A noteworthy feature is that it is possible to generate a cross section distribution consistent with the data using homothetic preferences, but that would require an assumption on age-specific consumption shares.

Housing: The characteristics of housing are very different from other consumption goods and assets. It is important to be very precise the nature of houses in our model since it differs from the more common specifications that housing as a standard durable good.

- **Consumption/investment good:** We model a house as an asset tree where the tree represents the investment component of the house, h , and the fruit produced at every period represent the flow of housing services the owner is entitled, d . To map the housing investment into services we assume a constant returns to scale technology, $d = g(h) = h$.
- **Lumpy investment good:** Houses come in different sizes (lumpy and indivisible investment) restricted by the set \mathcal{H} where $\mathcal{H} \equiv \{0\} \cup \{\underline{h}, \dots, \bar{h}\}$ where \underline{h} is the smallest housing investment and \bar{h} represents the largest. The cost economy cost of purchasing a house of a given size is ph where p represents the price per unit size (i.e. price per square feet or square meter). The indivisibility of housing $\underline{h} > 0$ forces some individuals to rent property since the cost of purchasing the smallest house size $p\underline{h}$ might be too expensive.
- **Housing investment is risky:** An important element to generate foreclosure is to have housing being a risky asset. The nature and the timing of riskiness is the key element to determine foreclosures in the model. We assume that the house purchase ph and the consumption of housing services d are not subject to any source of risk. However, the decision to sell property is subject to an idiosyncratic capital gains (or amenity) shock, $\xi \in \Xi \equiv \{\xi_1, \dots, \xi_z\}$ that affects the final sale value $p\xi h$ received by the homeowner. This approach is similar to Jeske and Krueger (2005) that use an exogenous maintenance shock affecting the net value of the property to generate

⁷In contrast with an infinitely lived or permanent youth model structure, the the choice of a life-cycle ensures that mortgages are paid-off. With the alternative formulations households choose an optimal level of mortgage debt to assets since they have no incentives to repay the house and keep refinancing.

⁸This is consistent with the evidence suggested by Jeske (2005) and Yang (2005).

foreclosures.⁹ We assume that the capital gain shock ξ is i.i.d. with an expected value $E(\xi) = 1$ and variance σ_ξ . The timing of uncertainty has to be consistent with no capital gains, that requires the shock to not be observed until the house is put in the house is sold.¹⁰ In addition, this shock is not observed until the house is sold. Households know the unconditional probability of this event which is represented by π_ξ . We discuss this assumption and its implications in more detail in the consumer problem.

- **Transaction costs of moving:** Purchasing or changing the existing housing investment is subject to non-convex transaction costs. The homeowner pays a proportional cost to the purchase $\phi_b p h$ and/or selling price $\phi_s p \xi h$. Since housing is indivisible, if a homeowners desires to consume more housing $d > h$ it has to sell the existing investment and purchase a larger unit. However, the decision to consumer a different amount of housing services is not subject to transaction costs.¹¹
- **Owner-occupied vs. tenant-occupied housing:** The separation between housing investment and housing consumption allows us to formalize rental markets. While housing investment in indivisible, the flow of services provided by the house is perfectly divisible, i.e. one way to rationalize that is to think about a home as being a full building with different floors. The investment cannot be divided and must be purchased by a single household, however, the individual does not need to utilize all the housing space in the building since part of it can be rented out. Therefore, housing services can be enjoy by either purchasing directly a house of a given size or rented in the market for one period. Under this formulation owner-occupied housing is “perfectly convertible” into tenant or rental-occupied housing and viceversa. This approach keeps us from the need to formalize these two type of homes either using different stocks or tightening rental property as a by product of capital investment.¹² This property is particularly important in an environment with foreclosures since the homeowner always has the option to rent a fraction of the housing investment instead of turning it to the mortgage bank. In the model, ownership is view as a particular choice to consume housing services and not as consuming a good with a superior quality or size.
- **Rental market for housing services:** Individuals that supply tenant-occupied prop-

⁹In Jeske and Krueger (2005), homeowners are subject to an exogenous depreciation shock that changes the value of the house for next period $p(1 - \delta)h$. Those individuals that have negative equity automatically choose to default. In our formulation, the capital gain shock is only realized upon the transaction of the property. We can also think of the shock being drawn ex-post by all individuals. Since most of them choose not to sell, the realization of the shock is irrelevant even if they have negative equity.

¹⁰The idiosyncratic capital gains or amenity shock allows a risk to be associated with housing without introducing an aggregate shock that determines capital gains. Adding aggregate uncertainty is not computationally feasible in this model at this time. The amenity shock can be thought of as what happens to a property if the surrounding neighborhood deteriorates (or improves). This change would be reflected in the house value at the time of sale. An additional advantage of the formulation is that the necessity of matching buyers and sellers is avoided. Since any buyer can always purchase a home independent of the shock received by the seller.

¹¹This assumption differs from the standard durable good model where individuals can expand the set of durables every period until they attain their desired level. Households can purchase homes of different sizes, but they are forced to sell if they desire to buy a different unit. Since housing investment requires the use of a long-term mortgage contract, it becomes computationally infeasible to have households holding a housing portfolio with different mortgage balances.

¹²**NOTE: Explain rental firm formulation.**

erty (i.e. floor space) receive a rental income $R(h - d)$ where R represents the rental price per unit of housing services. However, these activity has two pecuniary costs. First, requires landlords to pay a monetary fixed cost $\varpi > 0$ anytime property is rented.¹³ Second, maintenance expense associated to the housing investment $\varphi(h', d)$ depends on whether housing is owner-occupied or rental-occupied. Rental-occupied housing depreciates at a higher rate than owner-occupied housing $\Delta\delta = \delta_r - \delta_o > 0$. The different depreciation rates are a result of a moral hazard problem that occurs in rental markets as renters decide how intensively to utilize the dwelling. The maintenance cost depends on the fraction of services the household consumes and the fraction rented out, $\varphi(h', d) = \delta_o p d + \delta_r p (h' - d)$. This approach allows to have individuals consuming the same good but paying different prices. For renters, the cost of one unit of tenant-occupied housing is R , whereas for homeowners is given by $R - p\Delta\delta$. The opportunity cost of owner-occupied housing is given by the market value of rental property net of the excess maintenance cost. Holding constant R , the higher the spread in depreciations the lower the cost associated to owner-occupied housing. That increases the incentives to consume large homes reducing the supply of rental property. The theory and the empirics of the supply of tenant-occupied housing are studied in more detail in Chamber, Garriga, and Schlagenhauf (2007).

Housing finance and the default option: House are purchased using long-term mortgage contracts provided by a competitive lending sector. We assume that lenders offer a finite number of exogenous type loans $z \in \mathcal{Z} = \{1, \dots, Z\}$. These contracts can potentially differ along a number of dimensions such as downpayment, length of contract, and repayment structure. All these different characteristics can be easily be accommodated in a general formulation that specifies the long-term contract for a given loan amount. In general, the purchase of a house of value ph requires a downpayment requirement $\chi(z) \in [0, 1]$ that can vary by loan type z . The size of the mortgage loan is given by $D(N(z)) = (1 - \chi(z))ph'$ where $N(z)$ is the length of the mortgage contract z . The choice of a particular loan product commits the borrower to certain obligations. The first one is to make mortgage payments every period to repay the loan. The magnitude of the mortgage payment $m(x, n, z)$ in a given period n in the contract depends on the loan amount $D(N(z))$, the mortgage interest rate $r^m(z)$, and the repayment structure associated to the loan type z . The term $x \in (p, h', \chi(z), N(z), r^m(z))$ summarizes the set of relevant information necessary to keep track of the loan for any given period n . Second, borrowers are committed to repay the loan as long as they stay in the property. Selling the house immediately terminates the contract. Early prepayments without transacting the property are not allowed.

The mortgage payment can be decomposed into an amortization term, $A(x, n, z)$, and an interest rate payment term $I(x, n, z)$,

$$m(x, n, z) = A(x, n, z) + I(x, n, z),$$

where the interest payments are calculated by $I(x, n, z) = r^m(z)D(x, n, z)$. The law of motion for the level of housing debt $D(x, z)$ is given by

$$D(x, n - 1, z) = D(x, n, z) - A(x, n, z),$$

¹³The introduction of the fixed cost prevents homeowners from freely using the rental market to buffer negative income shocks. This cost should be viewed as either a time opportunity cost, or as a management fee. These costs are paid every period and are independent of the size of the property.

The law of motion for home equity increases with every payments. That is

$$e(x, n - 1, z) = e(x, n, z) + [m(x, n, z) - r^m(z)D(x, n, z)],$$

where $e(x, N, z) = \chi(z)ph'$ denotes the home equity in the initial period. This general specification covers a large number of loans offered by the mortgage industry. For example, the standard fixed rate mortgage has a constant payment schedule that satisfies $m(x, n, z) = \lambda D(N(z))$ where $\lambda = r^m(z)[1 - (1 + r^m(z))^{-N(z)}]^{-1}$. A cash purchase implies $\chi(z) = 1$ that immediately implies $D(N(z)) = m(x, n, z) = 0$. In this context, a 30 year fixed rate mortgage with a 20 percent downpayment is view as a different loan product than a 30 year fixed rate mortgage with a 10 percent downpayment. Since these two loans have different downpayment levels, the implied mortgage payments will be different even though the repayment structure is constant over time for these two loans contracts. In our model homeowners choose among exogenously given contracts that differ in some of the aforementioned characteristics, they do not choose the characteristics of the contract individually.

The long-term mortgage loan has incorporated a default option that can only be executed upon selling the property and serves to limit the homeowner's losses. The precise procedure works as follows. First, the homeowners chooses to sell the current housing investment h . Once the house is on the market, the idiosyncratic capital gain shock ξ is realized. Given the observed realization, the households chooses to default. If the option value of defaulting is higher than the one associated with selling the house and clearing any outstanding balance with the financial intermediary.¹⁴ Since this is a collateralized loan with default option, the borrower is forced to repay the loan when the net revenue exceeds the outstanding remaining principal with the bank, $\Pi_\xi = (1 - \phi_s)p\xi h - D(x, n, z)$. In this situation the homeowner has positive capital gains, $\Pi_\xi > 0$, so is always beneficial to sell the property. We implicitly assume that if the homeowners chooses not to repay the bank loan the lender could immediately go to court to enforce the contract, reposes the house, and sell it in the market for a profit. When the revenue from selling the house is negative, $\Pi_\xi < 0$, that is, the market value of the property is lower than the current outstanding principal, the homeowner let's the bank reposes the property and absorbs the loss. Consequently, the foreclose option built in the mortgage contract implies that the homeowners profits from transacting the property satisfy $\max(\Pi_\xi, 0)$. There are two essential elements that trigger the default decision. The first one is the size of the capital gain shock, ξ . If the capital gain shock has a low variance σ_ξ homeowners are not likely to foreclose the property. Changes in the riskiness of housing could certainly be a relevant factor for understanding the increase in foreclosures. The second element is leverage. Mortgage loan that allow high levels of leverage imply $D(x, n, z) \approx ph$ (i.e. with contracts that allow zero downpayment $\chi(z) = 0$ depending on the repayment structure we could have negative amortization, $D(x, n, z) > ph$). In this situation the size of the capital income shock can be smaller to induce a homeowner to foreclose the property. The model can be use to determine the leverage levels that trigger

¹⁴The advantage of this approach is computational, since it does not require to introduce an additional state variable. There are alternative timing conventions that could have been used. One could consider a one time capital gain shock. After purchaing the house, the individual observes a one time idiosyncratic shock, ξ . The cost of this approach is to include the shock as an additional state variable. An extension of this timing could allow for an idiosyncratic capital gain with early revelation of uncertainty. The approach is similar to the previous one, but we allow the shocks to change every period according to an *iid* shock with a probability distribution, π_s . The individuals observe the house price shock, ξ , and then they decide to sell or not.

volumes of foreclosure similar to the observed in the data. We defer the discussion about the cost/punishment associated to foreclose to the next section.

Household's Income: In this economy households have four different sources of income that include labor earnings, the return from deposits in the bank, net revenue from leasing tenant-occupied housing, and accidental bequest. During working life $j < j^*$, each household is endowed with one unit of time that is inelastically supplied to the labor market. The market value of time across households differs due to two exogenous factors: an age component and a period specific productivity shocks. The age component is defined by v_j and evolves over time in a deterministic pattern $\{\nu_j\}_{j=1}^{j^*}$. The stochastic component, $\epsilon \in \mathcal{E}$, is drawn from a probability space where the realization of the current period productivity component evolves according to the transition law $\Pi_{\epsilon, \epsilon'}$. Each period labor earnings are determined by $w\epsilon v_j$ where w is the market wage rate. The return from bank deposits is given by ra where r represents the interest rate and a denotes the level of deposits. Formally, we define the household's disposable income as

$$y = \begin{cases} w\epsilon v_j + (1+r)a + tr + y_r & \text{if } j < j^*, \\ w_{ss} + (1+r)a + tr + y_r & \text{if } j \geq j^*. \end{cases} \quad (1)$$

where w_{ss} is retirement benefit, tr represents a lump-sum transfer from accidental bequests, and y_r represents net rental income. Households earn income in the labor market if they are under the age j^* , or from retirement benefits if they are of age j^* or older. Net rental income earned from the housing investment y_r is defined as

$$y_r = \begin{cases} R(h' - d) - \varpi - \varphi(h', d) & \text{if } d < h' \text{ and } h' > 0 \\ -\varphi(h', d) & \text{if } d = h' \text{ and } h' > 0 \\ 0 & \text{if } h' = 0 \end{cases}$$

Note that rental income is determined by the revenue from leasing tenant-occupied property (selling housing services from the housing investment) net of fixed costs and maintenance costs. For renters ($h' = 0$), the implied rental income is zero.

3.2 Households' Problem

To solve the consumer decision problem we use dynamic programming techniques. The individual state of a household is indexed by their asset holding, a , investment position in housing, h , mortgage choice, z , remaining periods on the mortgage, n , the idiosyncratic income shock, ϵ , and age, j . To keep the notation compact, we summarize the household state by $s = (a, h, n, z, \epsilon, j)$. It is important to notice that this formulation does not consolidate the household balance sheet into a single account. This approach contrasts with the formulation used by Jekse and Krueger (2007) where bank deposits and the net housing are consolidated in a single account using cash-on-hand. In addition, the separation of balance sheets allows to break the link between household wealth and home equity.¹⁵ The formulation effectively separated the default decision is separated from income and wealth. That allows either poor or wealthy individuals to foreclose. The key difference would be that wealthy individuals might choose mortgage contracts with low leverage decreasing their likelihood of default. Based on the households housing status at the start of the period, the decision tree is summarized in Table 2.

¹⁵The default literature usually assumes asset consolidation. In this set up households that choose to default are poor (low income and wealth).

Table 2: Basic Structure of the Model

Renter: $h = 0$	[Continues renting $h' = 0$]
	[Purchases a house $h' > 0$]
Owner: $h > 0$	[Stay house: $h' = h$]
	[Change size: $h' \neq h$]
		[Repay loan $\Pi_\xi > 0$
]	Forecloses property $\Pi_\xi > 0$
	[Sell and rent: $h' = 0$]
		[Repay loan $\Pi_\xi > 0$
]	Forecloses property $\Pi_\xi > 0$

Next, we formally describe the household optimization problem for each case. We start with problem of an individual that starts as a renter, and then we consider the decision problem of the individual that starts as a homeowner.

Renters: A household that begins the period renting in an individual state $s = (a, h, z, n, \epsilon, j) = (a, 0, 0, 0, \epsilon, j)$ has the option of continue renting ($h' = 0$) or purchase a house ($h' > 0$). The discrete non concave problem is given by

$$v(s) = \max\{v^r, v^o\}.$$

- **Continue renting:** The value associated to continue renting is determined by the choice of consumption, c , housing services, d , and asset holdings, a , that solve

$$v^r(s) = \max_{(d, a') \in \mathbb{R}_+} \{u(y - a' - Rd, d) + \beta_{j+1} E_{e'}[v(s')]\}$$

where $s' = (a', 0, 0, 0, \epsilon', j+1)$ represents the future state variable, Rd is the expenditure in tenant-occupied housing, and denotes an age specific discount rate that incorporates the survival probability, $\beta_j = \beta\pi_j$. The first-order condition with respect to housing services and consumption goods is given by

$$\frac{U_d}{U_c} = R,$$

This equation states that the marginal rate of substitution between housing services and consumption is equal to the relative price of tenant-occupied housing and consumption. Note that there is no restriction on the size of house rented other than the non-negativity constraint in consumption.¹⁶ In addition, the restriction in the choice

¹⁶Other housing papers impose some limits in the size of rental-occupied housing. In this paper, renters can consumer any size of housing services. In equilibrium, renters consume smaller housing units that home buyers. This comes as an endogenous outcome in the model as opposed to be assumed.

set indicates that asset markets are incomplete since individuals only have access to an uncontingent asset and borrowing via this asset is precluded.

- **Purchase a house:** When an individual that rents purchases a house solves a different problem with a larger number of choices than the previous problem. In addition to the previous choice, it has to decide the size of the housing investment, h' , the type of mortgage financing used to purchase the house, z' , and the discrete choice of selling housing services in the rental market, $I_r = 1$, or fully consume the dwelling $d = h'$. This decision problem solves

$$v^o(s) = \max_{\substack{(c,d,a',h') \in \mathbb{R}_+ \\ z' \in \mathcal{Z}, I_r \in \{0,1\}}} \{u(c, d) + \beta_{j+1} E_{e'}[v(s')]\},$$

$$s.t. \quad c + a' + [\phi_b + \chi(z')]ph' + m(x, n, z') = y.$$

The purchase of a house has two up front expenditures: a transaction costs (i.e. realtors fees, closing costs, etc.) that are proportional to the value of the house $\phi_b ph'$, and a downpayment to the mortgage bank for a fraction $\chi(z')$ of the value of the house (i.e. 20 percent down of the purchase price excluding transaction costs). The downpayment represents the amount of equity in the house at the time of purchase and varies with the choice of mortgage contract, z' . In addition to the expenditures associated to the home purchase, we assume that the homeowner starts to repay the mortgage loan immediately. The mortgage payments are a function of the variable $x = (p, h', \chi(z), N(z), r^m(z))$, the number of period remaining in the loan $n = N$, and the loan choice z' . The optimal decision with respect to housing satisfies

$$\frac{U_d}{U_c} - R + p(\delta_r - \delta_o) \geq 0, \quad (= 0 \text{ when } d < h' \text{ or } I_r = 1)$$

This equation equates the marginal rate of substitution between housing services and consumption to the opportunity cost of consuming owner-occupied housing. Notice that the implicit cost depends on the magnitude of the depreciation spreads $\Delta\delta$ and the price of housing p determined in equilibrium. For those individuals that choose to supply rental property in the market $I_r = 1$, the first-order condition is satisfied with equality, the optimal amount of housing services consumed satisfies $d^* < h'$, and receive a net rental income equal to $R(h' - d^*) - \varpi - \varphi(h', d^*)$. The homeowners that do not supply housing services in the rental market avoid the fixed cost ($I_r = 0$) and consume $d = h'$. The optimal choices this period imply next period states according, $s' = (a', h', z', N - 1, e', j + 1)$.

The choice of whether to continue renting or purchase a home is determined by the highest value between $v^r(s)$ and $v^o(s)$. When $v^r(s) > v^o(s)$ the individual continues to rent, and otherwise becomes a home owner.

Owners: The decision problem for an individual that starts the period owning a house ($h > 0$) has more choices. The homeowner can choose to stay in the house ($h' = h$), purchase a different house ($h' \neq h$), or become a renter ($h' = 0$). In addition, anytime that the homeowner chooses to sell the property, the transacted price is subject to the capital gains shock, $\xi \in \Xi$, and can choose to foreclose the property. Given the homeowner state variable at the start of the period, $s = (a, h, z, n, \epsilon, j)$, the individual solves

$$v(s) = \max\{v^m, v^e, v^b\}$$

The different value function are calculated by solving three subproblems.

- **Stay same house:** The value function associated to stay in the same house is given by

$$v^m(s) = \max_{\substack{(c,d,a') \in \mathbb{R}_+ \\ I_r \in \{0,1\}}} \{u(c, d) + \beta_{j+1} E_{\epsilon'}[v(s')]\},$$

$$s.t. \quad c = y - (a' + m(x, n, z)).$$

In this case the individual makes mortgage payments when $n > 0$, otherwise $m(x, 0, z) = 0$ and he simple decides the amount of consumption and savings that result from disposable income, and whether to lease tenant-occupied housing (I_r). Given that the individual stays maintain the housing investment, $h' = h$, he is not subject to transaction costs. The future vector of state variables is then determined by $s' = (a', h', z', n', \epsilon', j + 1)$ where $n' = \max\{n - 1, 0\}$. This counter determines whether the mortgage loan is paid off or not.

- **Sell current property and rent or buy:** For the individuals that choose the sell the current property, h , the default option becomes available, $\max(\Pi_s, 0)$. Among those that sell, some individuals prefer to exit the housing market and rent property where v^e represents their value function, and others prefer to buy a different size house $h' \neq h$ (larger or smaller than the previous one) where the term v^b represents their value function. It is important to note that the capital gain shock is realized after the selling decisions has been made. For the individuals that sell and rent we solve

$$v^e(s) = \max_{(c,d,a') \in \mathbb{R}_+} \{E_{s,\epsilon'}[u(c, d) + \beta_{j+1}v(s')]\}$$

$$s.t. \quad c = y + \max(\Pi_s, 0) - (a' + Rd)$$

where $\Pi_s = (1 - \phi_s)p\xi h - D(x, n, z)$ represent the net revenue of selling the property. The foreclosure option is straight forward, individuals with negative equity walk away from their mortgage obligations but loose the property. The key element is the household leverage at the time of sell, $D(x, n, z)$, relative to the proceedings associated to sell the house. This difference determines the level of equity in the house. When the house is pay-off, $D(x, n, z) = 0$, the homeowner does never default even when the realization of the idiosyncratic capital gains is the worse one, ξ . The individual problem is equivalent to the one of a renter with the exception that the level of wealth is affected by the option on capital gains. The future state variable is given by $s' = (a', 0, 0, 0, \epsilon', j + 1)$.

The individual that purchases a different house size, $h' \neq h$, solves

$$v^b(s) = \max_{\substack{(c,s,a',h') \in \mathbb{R}_+ \\ z' \in \mathcal{Z}, I_r \in \{0,1\}}} \{E_{s,\epsilon'}[u(c, d) + \beta_{j+1}v(s')]\},$$

$$s.t. \quad c + [\phi_b + \chi(z')]ph' + m(x, n, z') + a' = y + \max(\Pi_s, 0),$$

This individual sells the property and either receives Π_s or zero. Then, chooses to purchase a new house, h' , pays transaction costs, $\phi_b ph'$, and chooses a mortgage loan, z' , that determines the size of the downpayment, $\chi(z')ph'$. The state variable for tomorrow is $s' = (a', h', N - 1, z', \epsilon', j + 1)$.

3.3 Production of Housing Units

We follow the approach of Jeske and Krueger (2007) to model the real estate construction sector. We assume a competitive sector that manufactures housing units using a linear technology, $I_H = C_H/\theta$, where I_H represents the output of new homes, C_H is the input of the consumption good, and θ is a technology constant used to transform consumption goods into new housing units. To keep matters simple, we assume that the technology is reversible, so homes can be transformed into consumption goods. The optimization problem of the representative firm is given by

$$\begin{aligned} & \max_{H, C_H} pI_H - C_H \\ & s.t. \quad I_H = C_H/\theta \end{aligned}$$

The first-order condition of the housing sector determines the equilibrium house price that satisfies.

$$p = \theta$$

The homes produced are added to the existing housing stock as either new units or as repairs of the existing stock. The aggregate law of motion for housing investment is

$$I_H = (1 + \rho)H' - H + \varkappa(H, \delta_o, \delta_r).$$

The depreciation of the housing stock $\varkappa(H, \delta_o, \delta_r)$ depends on utilization (i.e. owner vs. tenant-occupied housing). The larger the size of the rental market, the larger the investment in housing repairs. When $\delta_o = \delta_r$, the investment function is the standard linear expression, $\varkappa(H, \delta_o, \delta_r) = \delta H$, independent of the distribution of housing consumption. To study the implications of declines in house prices, we assume an exogenous technological change that reduces the marginal cost of manufacturing new housing units, $\Delta\theta = \theta' - \theta < 0$

3.4 Mortgage Brokers or Investment Banks

Mortgage brokers use global capital markets to finance mortgage lending. We assume a competitive lending sector that maximizes expected profits per mortgage contract. The type of contracts transacted is finite, z , and exogenously determined. The objective is to understand the observed mortgage default in the existing contracts and not to provide a positive theory of the type of mortgage contracts offered that is consistent with the evidence. The balance sheet per credit line is given by

Balance Sheet

Assets	Liabilities
New Mortgage Loans (-)	Credit Borrowed (r^*)
Repayment of Principal (+)	(Existing loans \equiv Outstanding Principal)
Foreclose properties (+)	
Mortgage payments	
Interest payments (+)	
Principal payments (-)	

We assume that the lender collects foreclose property with a haircut, γ . Therefore, the individual loss is smaller than the bank loss, $\Pi_s < \Pi_b = (1 - \phi_s)\gamma p \xi h - D(x, n, z)$. To recoup

the loss, the lender has to charge a premium in each credit line. The base interest rate per mortgage contract is given by $r^* + \varrho(z)$, where $\varrho(z)$ is the required mortgage premium in contract z that guarantees zero profits. The profit condition for the line of credit of mortgage z is

$$M(z) - r^* RP'(z) + FL(z) + T(z) = 0, \quad \forall z$$

where $M(z)$ represent mortgage interest payments, $RP'(z)$ represents the beginning of next period outstanding principal, and $FL(z)$ defines the bank proceedings from selling foreclosed property. The mortgage broker borrows in the international capital markets and the premium is used to cover the default rate probability. With the law of large numbers the expected level of profits per line of credit is zero. For every contract, we need to determine $\varrho^*(z)$ such that the mortgage broker makes zero profits per contract. With the equilibrium conditions we need to compute $\{\varrho^*(z)\}_{z=1}^Z$ that guarantee zero profits.

3.5 Firms

In this economy, a representative firm produces a good in a competitive environment that can be used either for consumption, government, capital purposes, or housing purposes. The representative firm produces goods using a constant returns to scale technology $F(K, L)$, where K and L denote the amount of capital and labor utilized. In the economy with global capital markets the interest rate is fixed, r^* . Given the competitive nature of financial and labor markets, the optimal firm chooses $\{K^*, L^*\}$ such that:

$$\begin{aligned} r^* &= F_1(K, L) - \delta, \\ w &= F_2(K, L). \end{aligned}$$

Given the global interest rate r^* , the first-order conditions of the firm's problem determine the amount of capital (K^*) used by domestic firms and the equilibrium wage rate (w). Since households supply labor inelastically, the aggregate level of output can be easily computed $Y^* = F(K^*, L^*)$

3.6 Government

In this economy, the government engages in a number of activities: provides retirement benefits through a social security program; and redistributes the wealth of those individuals who die unexpectedly. We assume that the financing of government expenditure and social security are run under different budgets.

The government provides social security benefits to retired households. The benefit, w_{ss} , is based on a fraction, \bar{w} , of the average income of workers. These payments are financed by taxing the wage income of employed households at the tax rate τ_p . Since this policy is self-financing, the tax rate depends on the replacement ratio \bar{w} . The social security benefit can be defined as:

$$w_{ss} \equiv \bar{w} \sum_{j=1}^{j^*-1} \sum_i \mu_j w v_j \epsilon / \sum_{j=1}^{j^*-1} \mu_j$$

where μ_j is the size of the age j cohorts. The social security budget constraint is:

$$\tau_p \sum_{j=1}^{j^*-1} \sum_i (\mu_j w v_j \epsilon) = w_{ss} \sum_{j=j^*}^J \mu_j. \quad (2)$$

The government also has the responsibility to collect the physical and housing assets of those individual who unexpectedly die. Both of these assets are sold and any outstanding debt on housing is paid off. The remaining value of these assets is distributed to the surviving households as a lump sum payment, tr . This transfer can be defined as

$$tr = \int \mu_j(1 - \psi_j)a(\Lambda)\Phi(d\Lambda) + \sum_{\xi \in \Xi} \pi_\xi \int \mu_j(1 - \psi_j)[(1 - \phi_s)p\xi h(\Lambda) - D(\Lambda)]\Phi(d\Lambda). \quad (3)$$

where $\Phi(d\Lambda) \equiv \Phi(da \times dh \times dn \times d\epsilon \times dj)$.

3.7 Market Equilibrium

This economy has four competitive markets: the goods market, labor market, the rental of housing services market, and the housing market.

- **Housing market:** We assume that the aggregate supply of housing is fixed \bar{H} . The market clearing condition is then given by

$$\int_{I_s(\Lambda)=0} \mu_j h'(\Lambda)\Phi(d\Lambda) + \int_{I_s(\Lambda)=1} \sum_{\xi \in \Xi} \pi_\xi \mu_j h'_\xi(\Lambda)\Phi(d\Lambda) = \bar{H},$$

or in compact notation

$$\int \mu_j h'(\Lambda)\Phi(d\Lambda) = \bar{H},$$

- **Rental market:** The equilibrium in this market is determined by the aggregate amount of housings services made available by landlords and the total demand of rental housing services. That is

$$\begin{aligned} \int_{I_s(\Lambda)=0} \mu_j [h'(\Lambda) - s(\Lambda)]\Phi(d\Lambda) + \int_{I_s(\Lambda)=1} \sum_{\xi \in \Xi} \pi_\xi \mu_j [h'_\xi(\Lambda) - s_\xi(\Lambda)]\Phi(d\Lambda) = \\ \int_{I_s(\Lambda)=0} \mu_j s(\Lambda)\Phi(\Lambda) + \int_{I_s(\Lambda)=1} \sum_{\xi \in \Xi} \pi_\xi \mu_j s_\xi(\Lambda)\Phi(\Lambda) \end{aligned} \quad (4)$$

This definition accounts for the effect of the idiosyncratic capital gains shock for both the landlord and the renter that just sold a property.

- **Goods market:** The aggregate resource constraint is given by

$$C + I_K + pI_H + \Upsilon = F(K, L), \quad (5)$$

where C , K , I_K , I_H and Υ represent aggregate consumption, the aggregate capital stock at the beginning of the next period, capital investment, housing investment to maintain properties, and various transactions costs, respectively.¹⁷ Investment in capital goods is defined as $I_K = (1 + \rho)K' - (1 - \delta_K)K$ where the parameter δ_K denotes the depreciation rate for physical capital. The additional term pI_H represents the expenditure in consumption goods necessary to produce the new housing investment.

- **Labor market:** In the labor market, labor demand is determined by the marginal product of labor, $F_2(K, L)$. Labor is inelastically supplied and determined by $L = \sum_{j=1}^{j^*-1} \mu_j v_j \epsilon$.

¹⁷The definition of aggregate housing investment and total transactions cost are define in the appendix.

4 Model Parameterization

In order to evaluate the model it is necessary to specify parameters and functional forms. Some of the parameters of the model can be set independently. The equilibrium objects (allocations and prices) are functions of the underlying parameter values and our objective is to set these parameter values so the model matches the desired counterparts in the data. We use a minimum distance approach to ensure that the match is carefully done. However, we cannot guarantee that there exists a unique constellation of parameters consistent with the data. We first describe baseline parameters and functional forms. Then we discuss the choice of targets and model performance.

4.1 Description of parameters

There is a number of parameters that can be set independently of the model solution. We first discuss the determination of these parameters, then we discuss the choice of functional forms and the set of structural parameters that need to be estimated.

Population structure: A model period is taken to be 3 years where an individual enters the labor market at the age of 20 (model period 1) and lives until age 83 (model period $J = 23$). The mandatory retirement age is age 65 (model period $j^* = 16$). The survival probabilities, π_{j+1} , are determined using data from the National Center for Health Statistics, *United States Life Tables* (1994). The population growth rate is set to 1.2 percent in annual terms.

Transaction costs and mortgage contracts: The housing market introduces a number of parameters that need to be determined. The purchase of a house is subject to transaction costs. We assume that all these costs are incurred at the purchase time and paid by the buyer only, $\phi_s = 0$ and $\phi_b = 0.06$. We explore the sensitivity of the default decision to changes in this assumption. The purchase of the house requires long-term mortgage finance. For computational reasons, we limit the number of mortgage loans to two distinct types, $Z = 2$. The first one is the standard 30 year, $N(1) = 10$, fixed rate mortgage with a 20 percent downpayment $\chi(1) = 0.20$.¹⁸ The second mortgage loan is modelled to allow more leverage, low initial loan costs, and changing payments like most subprime market loans. One contract that satisfies this criterion in a stationary environment with no interest rate risk is a graduated payment loan. We set the length of this mortgage to be 30 years, i.e., $N(2) = 10$. Under this contract the mortgage payments follow $m_{n+1} = (1 + g)m_n$ where $g \geq 0$ represents the growth rate. The value g is solved in the estimation process to match the observed level of foreclosures in the subprime market and downpayment requirement $\chi(2)$ to match the share of adjustable mortgages in the market.

Preferences: Preferences are time separable with an exogenous discount rate β and CRRA period utility function defined over each of the two goods. The curvature of consumption σ_c differs from the curvature of housing services σ_d .

$$U(c, d) = \gamma \frac{c^{1-\sigma_c}}{1-\sigma_c} + (1-\gamma) \frac{d^{1-\sigma_d}}{1-\sigma_d}$$

This preference structure implies a non-homothetic relation between consumption of goods and housing services that is consistent with an increasing ratio of housing services expenditures to consumption expenditures by age observed in the data, [see Jeske (2005) for a

¹⁸The American Housing Survey in 1993-99 presents data that shows that the average downpayment is approximately twenty percent.

detailed discussion].¹⁹ The relative curvature between σ_c and σ_d determines the growth rate of the housing services to consumption.

$$\frac{Rd^{\sigma_d}}{c^{\sigma_c}} = \frac{1 - \gamma}{\gamma}$$

When $\sigma_c > \sigma_d$ implies that the marginal utility of consumption declines faster than the marginal utility of housing services. Therefore, when income increases over the life-cycle households choose to consumer large homes. We set $\sigma_d = 1$ and determine σ_c to pin down the ratio of expenditures Rd/c . The relative share parameter γ is determined to capture at aggregate amount of housing relative to goods consumption.

Technology: Aggregate output is produced with a constant returns to scale Cobb-Douglas technology, $F(K, L) = K^\alpha L^{1-\alpha}$. The parameter α is estimated.

Housing: There are some parameters of housing that are endogenously determined. Given the lumpy nature of housing, the specification of the smallest house size, \underline{h} , has important implication for housing tenure and portfolio allocations. This value is determined to replicate the aggregate homeownership. There are two relevant set of parameters for the supply of rental property. The depreciation rates for owner and tenant-occupied housing (δ_o and δ_r), the fixed cost that a household has to pay to become a landlord, ϖ , and the technology parameter for the construction of new homes, θ .²⁰ These four parameters are determined in the estimation.

Idiosyncratic capital gains shocks: To determine the distribution of idiosyncratic capital gains shocks we use data from the *American Housing Survey*. To calculate the probability distribution for this shock we measure capital gains based on the purchase price of the property and their reported estimate of the current market value. The implied rate of return is adjusted by the maturity of the investment and is express the appreciation in annualized terms. We estimate a kernel density and then discretize the density in seven uneven partitions. The values of the capital gain shock can be easily computed as deviation of the mean value $\xi - E(\xi)$ and are given by

$$\xi - E(\xi) = [-0.20, -0.097, -0.013, 0.059, 0.122, 0.179, 0.230]$$

and the implied probability distribution is

$$\pi_\xi = [0.0388, 0.2046, 0.4917, 0.1437, 0.0670, 0.0347, 0.0195]$$

The values used in the model are adjusted to be consistent with a period being defined as three years.²¹

¹⁹We also find that such a momentary utility function generates insufficient movements in housing position as well as introducing some counterfactual implications for the rental market.

²⁰The parameter ϖ affects the number of households that choose to become landlords. Determination of the this parameters is difficult as we have little direct evidence on the number of households who own rental property. An indirect measure is to calculate the number of homeowners that report rental income.

²¹In order to test the robustness of the data from the *American Housing Survey*, we employed a similar approach using 1995 Tax Roll Data for Duval County in Florida. Jacksonville is the major city in Duval County. This data follows real estate properties as opposed to individuals. As a result, we can calculate annualized capital gains based in actual sales. We find very similar estimates for the idiosyncratic capital gains shock using this data source.

Endowments: Workers are assumed to have an inelastic labor supply, but the effective quality of their supplied labor depends on two components. One component is an age-specific, v_j , and is designed to capture the “humped shaped” profile of earnings over the life cycle. We use data from U.S. Bureau of the Census, “Money, Income of Households, Families, and Persons in the United States, 1994,” *Current Population Reports*, Series P-60 to construct this variable. The second component captures the stochastic element of earnings, ϵ , and is constructed using the approach used by Storesletten, Telmer and Yaron (2004) that estimate a continuous income process with a permanent and a transitory component for the U.S. economy. We discretize this income process into a five state Markov chain using the methodology presented in Tauchen (1986). The values we report reflect the three year horizon employed in the model. As a result, the efficiency values associated with each possible productivity value ϵ are

$$\epsilon \in \mathcal{E} = \{1.89, 2.37, 2.88, 3.51, 4.41\}$$

and the transition matrix is:

$$\Pi(\epsilon, \epsilon') = \begin{bmatrix} 0.47 & 0.33 & 0.14 & 0.05 & 0.01 \\ 0.29 & 0.33 & 0.23 & 0.11 & 0.03 \\ 0.12 & 0.24 & 0.29 & 0.23 & 0.12 \\ 0.03 & 0.11 & 0.23 & 0.33 & 0.29 \\ 0.01 & 0.05 & 0.14 & 0.33 & 0.47 \end{bmatrix}.$$

Government: The government role is to fund retired households consumption through a social security program. The retirement program is self-financed via a payroll tax on the earnings of workers. After retirement, households receive a transfer based on some fraction of the average labor income. The replacement ratio is set at thirty percent which results in a payroll tax on the worker of 5.25 percent.

4.2 Description of Targets and Model Performance

The parameters that need to be determined are the share of capital income, α , the depreciation rate of the capital stock, δ_k , the depreciation rate for rental units, δ_r , the depreciation rate for ownership units, δ_o , the growth rate of housing expenditures to consumption, σ_c , the relative importance of consumption goods to housing services, γ , the individual discount rate, β , the minimum house size, \underline{h} , and the growth rate of payments, g , in the GPM contract. It is worth pointing out that the determination of the structural parameters is not separated from the computation of equilibrium. The specified targets are given by nine moments observed in the U.S. economy.

1. The share of capital income of 0.29. This value is calculated by dividing a measure of capital that includes private fixed assets plus the stock of consumer durables less the stock of residential structures and the measure of output that includes the service flows from consumer durables less the service flow from housing.
2. The ratio of investment in capital goods to output: 0.14.
3. The ratio of capital to gross domestic product: 2.54.

4. The ratio of the housing capital stock to the nonhousing capital stock: 0.48. The housing capital stock is defined as the value of fixed assets in owner and tenant residential property.
5. The ratio of the investment in residential structures to housing capital stock: 0.121.
6. Housing consumption relative to nonhousing consumption: 0.24. Housing services are defined as personal consumption expenditure for housing and non housing consumption is defined as nondurable and services consumption expenditures net of housing expenditures.
7. The growth rate of housing expenditure relative to consumption expenditure.
8. The homeownership rate in the period 1998 is 0.657 percent.
9. The default rate of adjustable rate loans of 2 percent.

The model performs quite well matching all the targeted moments. The implied targets generated by the model solution are within one percent error for all the moments. The estimated parameters expressed in annual terms are presented in Table 2.

Table 2: Estimation of Model (Annualized Values)

	Parameter	Value
Individual discount rate	β	0.9749
Share of capital	α	0.29
Depreciation rate of capital stock	δ_k	0.0428
Depreciation rate of rental housing	δ_r	0.0749
Depreciation rate of owner occupied housing	δ_o	0.0340
Curvature utility with respect consumption	σ_c	3.0
Share of consumption goods in the utility function	γ	0.9541
Minimum house size	\underline{h}	1.4726
Growth rate payments	g	0.015

In addition to the main targets, the model can be evaluated along a number of dimensions. Table 3 shows some selected housing statistics for homeownership and housing consumption by age groups. We view that the model fit is close enough given the limited amount of heterogeneity we impose on individual preferences. The model captures the hump-shaped profile of homeownership by age and also captures the housing downsize observed in the data. The fit is specially good considering that the model does not consider additional shocks that

can affect the pattern of homeownership (i.e. shocks to family structure, or health shocks).

Table 3: Housing Distributions: Model and Data (1998 AHS)

Variable	Homeownership Rate						
	by Age Cohorts	Total	20-34	35-49	50-64	65-74	75-89
Data 1998		66.3	39.3	75.8	80.1	79.1	77.4
Baseline		66.5	46.2	79.6	81.9	84.1	76.9

Variable	Sqft. Owners ¹						
	by Age Cohorts	Total	20-34	35-49	50-64	65-74	75-89
Data 1998		2,137	1,854	2,220	2,301	2,088	2,045
Baseline		2,228	1,957	2,185	2,392	2,463	2,377

¹ Owner occupied house size is measured in terms of square feet.

An important test for the model is to check whether individuals purchase a distribution of home sizes consistent with the empirical evidence. Some papers measure housing consumption using expenditure to measure housing services whereas others report the ratio with respect to goods consumption (defined in a broad sense). We choose to report the consumption in housing services using square feet - the measure most frequently used to measure house size. This is done by renormalizing the average house size in the model to the average value reported in the *American Housing Survey* that is roughly 1,700 square feet (or 156 square meters). This measure is not conditioned by the type of ownership. If we condition, the data suggests that the average owners-occupied house (2,100 sqft) is roughly twice the size of tenant-occupied housing (1,100 sqft). The model captures two important features observed in the data. First, the level of the average owner-occupied house, and second the hump-shaped distribution of houses over the life-cycle. The pattern suggest that young households purchase a small house, and the house is upgraded to a larger one as income grows over the life-cycle. Upon retirement, individuals move to again to smaller units. The model replicates the hump-shaped profile of house sizes over the life-cycle. However, the peak house size seems to be later in the cycle when compared to the data, and the average house. Although it is not reported, the model also captures the increasing pattern of housing consumption by income levels.

The model also makes predictions about total foreclosures and the distribution of foreclosures. The evolution of total foreclosures and foreclosures by contract is summarized in Table 4.

Table 4: Foreclosures by Loan Type: Model and Data (1998)

	Data		Model	
	Foreclosed	Share	Foreclosed	Share
Total	1.0-15	-	1.8	-
FRM	0.8	0.86	1.7	0.61
GPM	2.0	0.14	2.0	0.39

The model predicts an aggregate foreclosure rate of 1.8 percent which is higher than the observed in the data ranges between 1.0 and 1.5 percent. The difference depends on the weights assigned to each type of mortgage contract and to the exclusion of some type of

lending. The model overprediction is mainly driven by foreclosures in the FRM loans where 1.7 percent of the loans are non-performing instead of 0.8 percent observed in the data. However, the model replicates the 2.0 percent of the ARM do not perform. The market share of GPM is slightly higher than the observed in the data for 1999 and more consistent with the levels observed in 2004. The model also predicts a smaller aggregate number of mortgages that are owe free and clear, around 10 percent that contrast with the 25 percent observed in the data. However, it is important to remark the fact that mortgage loans get fully repaid in the model, and there could other motives that explain why a quarter of the properties are clear of debt.

The distributional implications of the model are summarized in Tables 5 and 6. We consider the distribution of foreclosures by age, income, and loan type.

Table 5: Foreclosure Rates by Age

by Age Cohorts	20-34	35-49	50-64	65-74	75-89
Total	1.6	1.5	1.9	2.5	2.1
Share	16.7	15.6	19.8	26.0	21.9

In this tables, the model distributions are not compared with the data since foreclosures rates by age or income levels do not exist at the national level yet. These age and income specific default rates are computed as the fraction of foreclosures by individuals in group x (i.e. age group or income quintiles) over the total number of outstanding loans in the groups. The model predicts low default rates by age and income, but interesting distributional implications. The default rates by age is consistent with the pattern of housing mobility over the life-cycle. A fraction of the first-time buyers cannot afford the mortgage payments and choose to exit the market. That explains why the foreclosure rates falls for borrowers between age 35 and 49. Around the peak of earnings individuals either choose to upsize or downsize before retirement, with the housing trade some households realize negative capital gains and that increases the default rate 25 percent when compared to the previous age group. Finally, at retirement age households run-down the asset and sell property some of which has low levels of equity. The result is a relatively high level of defaults for this age group. The life-cycle pattern of default become more clear if we look at the distributions by loan type as in Figure 6.

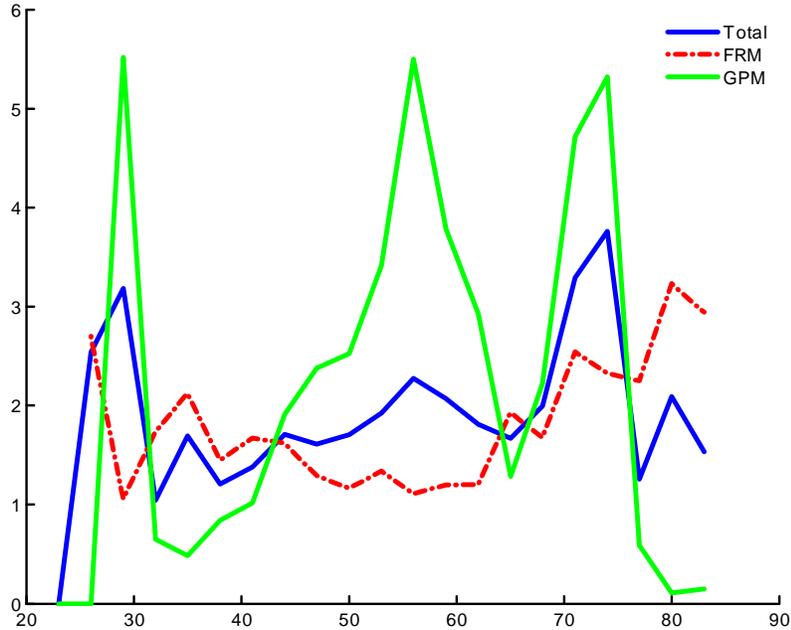


Figure 4: Model Distribution of Foreclosures by Loan Type

In general, the model predicts a relatively stable level of default for the FRM loans with the exception of very young first-time buyers and retired individuals. Most individuals that choose a FRM do not move during the middle age and that accounts for the relatively flat profile. In general, individuals that are more prone to move choose a GPM loan since it has a lower downpayment and an increasing repayment structure. Despite a relatively high interest premium, is a cost effective loan for those that plan to move in a short period of time. The flexibility provided by this contract explains the spike in default rates in those periods where individuals are likely to move. That includes first-time buyers that enter in the housing market to find out that cannot afford the house, and middle households that have not accrued enough equity in the house and are exposed to capital gains risk at the moment of either upsize or downsize, and retired individuals that choose to downsize. It is important to mention that the default decision is entirely driven by the level of equity in the house and the current market value. Since both contracts accrue equity very slowly, most homeowners do not have a lot of equity in the house. That is the case even with the FRM loan.

To provide addition insight, in Table 6, we look at the mortgage holdings distribution and the foreclosure rates by income levels.

Table 6: Mortgage Holdings Distribution and Foreclosure Rates by Income

	Income				
Fraction Own	1st	2nd	3rd	4th	5th
	28.7	47.8	61.1	81.1	90.5
Loan Distribution by Type	1st	2nd	3rd	4th	5th
FRM	4.7	11.9	16.3	30.7	36.2
GPM	6.2	18.8	29.4	23.5	22.1
Foreclosure Rates	1st	2nd	3rd	4th	5th
Total ($d_i = D_i/M_i$)	4.1	2.2	1.7	1.3	1.7
FRM	6.7	3.6	2.4	1.0	0.6
GPM	1.0	0.9	1.1	1.9	4.4

The model predicts different patterns for the distribution of mortgage holdings by income. The distribution of FRM is strictly increasing and skew towards the highest income groups that account for two thirds of this mortgage holdings. By contrary, the distribution of GPM is hump-shaped with 55 percent of the holdings accounted by the lowest three income quintiles. Even though GPM loans represent over one third of the market, its relative importance for the lowest income groups is clear.

The distribution of foreclosure rates by income levels exhibit some interesting features. The default rates across all income levels are relatively small with the exception of the lowest income group. Given that the fraction of homeowners within this income group is around 30 percent, a 4.1 percent of non-performing loans is relatively high. Most of this group is comprised of first-time buyers with a high default rate and retired individuals. High income individuals move more often, hence, they are more exposed to negative capital gains shocks than individuals that never move. When we condition the default rate by loan type we find that with contracts with low leverage, such as the FRM, the default rate declines with income. That comes from the fact the number of homeowners in these income groups increases in addition to the market share of FRM. The pattern of foreclosure in the GPM surprisingly shows a high default rate of the highest income quintile. This pattern is consistent with Figure 6 that shows a spike in defaults between age 50 and 60. Some of these individuals are selling their first property purchased with a GPM a few years back. It is worth remark, the table captures the income status at the moment of default and not at the time of purchase. A subset of the individuals that initially purchased a house using a leveraged loan receive a sequence of positive income incomes shocks. Since the initial payments on the GPM loan are relatively low in the early years of the contract, most of these individuals do not have any equity in the house when they choose to sell. As a result they default on the loan.²² Overall, the number of individuals in the highest income group with defaulted GPM is very small.

²²One way to prevent this type of default is to incorporate additional penalties. In the model we have assumed an anti-deficiency foreclosure law. With a deficiency law, the lender could seize part of the borrowers assets (specially for high income individuals) to recover part/full amount of the losses.

5 Home Equity, Foreclosures, and Bail-outs

We use the model to address the impact of a decline in house prices in the aggregate level of foreclosures. From our previous work in Garriga, Jeske, and Schlagenhauf (2008) we have concluded that in a steady state environment the introduction of housing default has a small impact in house prices. However, at the aggregate level a widespread decline in house prices is likely to have a large impact in foreclosures. To generate a decline in house prices, we consider a one time unanticipated increase in the technology parameter of the construction sector $\Delta\theta$. This increase reduces the marginal cost of producing homes and generates a decline in the current house price. To identify the impact of the decline, we assume that the uncertainty structure of the capital gains, σ_ξ , remains unchanged. With the decline in house prices, existing homeowners face an equity loss, $p_0 > p_1$, and increases their leverage. For some of the households the drop in house value wipes out all their equity and they choose to foreclose the property.

To understand the impact on the home equity at the individual level it is import to discuss its operating mechanism. Consider a homeowners that recently purchased a house and paid a price p_0 borrowing $D(x, N, z; p_0) = (1 - \chi(z))p_0h$ where the amount of borrowing is indexed by the purchase price. A decline in house prices ($p_1 < p_0$) reduces the current selling price of the property p_1h , but does not affect the outstanding principle, $D(x, n, z; p_0)$, and the mortgage payment obligations, $m(x, n, z; p_0)$, with the bank. The homeowner can be in two distinct situations based on the size of equity lost.

1. **Home equity loss:** In this case the homeowner has sufficient equity in the house

$$\tilde{e}(x, n, z; p_1) = e(x, n, z; p_0) - (p_0 - p_1)h \geq 0,$$

where $\tilde{e}(x, n, z; p_1)$ represents the current market value of the homeowner's equity before the realization of the capital gain shock, $e(x, n, z; p_0)$ represents the equity at the end of the period based on p_0 house prices, and $(p_0 - p_1)h$ represents the loss in house value. In this case, the current market value of equity is higher than the loss of property value. This situation happen when the homeowner chooses a mortgage loan with a low LTV ratio, $e(x, N, z; p_0) = \chi(z)p_0h'$, or has been making mortgage payments many periods decreasing the loan outstanding principle. In this cases the decline in house prices need to be quite large to induce negative equity. The homeowner still has a positive amount of equity in the house, so ignoring the model capital gain shocks, there are no incentives to foreclose the property since that would imply a full loss of equity, $\tilde{e}(x, n, z; p_1)$.

2. **Negative equity:** In this case the homeowner's equity has been completely wipe out by the decline in house prices

$$\tilde{e}(x, n, z; p_1) = e(x, n, z; p_0) - (p_0 - p_1)h < 0.$$

That happens when homeowners are highly leverage or they have not lived in the property for too long. The negative equity in the house increases the incentives to foreclose the property. At this point is important to clarify that negative equity does not necessarily imply default in the model. The purchase of a house requires transaction costs, therefore, the decision of foreclosing has to weigh the benefits and losses.

A widespread decline in house price also has important implication in the bank balance sheet. First, it increases the riskiness of the loans in the bank's portfolio since the market value of repossess properties, $p_0 - p_0$, is lower. That directly implies that the mortgage premiums across loan products based on the initial house prices, p_1 , are not sufficient to cover the loss in principal value. In addition, the decline in house prices increases the amount of non performing loans. The decline in the value of the collateral and the aggregate increase in the rate of foreclosures makes the lender insolvent in the short-run. These lenders could charge a high premium to new borrowers to recover from the loss, however, this practice would not be consistent with free entry in the mortgage industry. A new bank could enter and take over the lending market. There are several ways of dealing with the bank losses. One way to absorb these losses is to assume that banks hold capital. That capital can be used in the absorb some of the short-run losses. While this option is attractive, it requires to formalize bank decisions with respect to capital requirements and shareholders. The optimal level of capital has to be determined, and the compensation for shareholders or profits need to be distributed thus affecting the consumers budget constraints Another alternative it to assume to that rate of return of deposits paid domestically is adjusted accordingly. Depositors made investment decisions expecting a rate of return r_0 , but the realized deposit payment is $r_1 = r_0 - \Delta$ where Δ represents the decline in returns necessary to ensure zero profits. The implicit assumption is that the cost of default is bear by domestic households and not by foreign investors, otherwise there is no social cost associated to the increase in aggregate foreclosures. Since the computational complexity of the model is very large, we choose an different alternative by allowing the government to bail-out mortgage banks. We assume that the government uses the lump-sum taxes to raise enough resources to fund the losses in the lending industry. The new loans are going to be priced based on the assessed risk on that pool, and need to be consistent with the firm making zero profits at the time of origination. This approach allows to separate the losses from existing loans with new originations priced accordingly.

The negative income effects in conjunction with the adjustment in the rental market are two important channels to prevent the homeownership to increase. With our preference specification, a small decrease in income implies a larger decline in housing services than consumption. In addition, the lower rental price that results from the increase in rental supply makes tenant-occupied housing more attractive. Since we model the house price decline as a one time shock, homeowners expect to live in a world with permanently lower house prices. This lower price level reduces the opportunity cost of owner-occupied housing ($R - p_1\Delta\delta$) and makes it more attractive (notice that the riskiness level is maintained constant). At the aggregate level, that should increase the homeownership rate, feature that is not consistent with the empirical evidence that suggest the opposite. The previous analysis assume that the rental price did not adjust.²³ The adjustment in the rental market generates a non trivial impact in the effective opportunity cost of owner-occupied housing, $R_1 - p_1\Delta\delta$ where $R_1 \neq R_0$. A relative decline in the rental price can make owner-occupied housing less attractive. If the downward adjustment is sufficient, the implied homeownership rate could decrease instead of increase.

²³That would be the case in a model where a the supply of rental property ties the price of tenant-occupied housing to the interest rate and the depreciation rate, $R = r - \delta_o$. Since we consider a global capital market and r remains constant, the implied R would be fixed too.

5.1 Spike Foreclosures, Mortgage Collapse, and Government Bail-outs

The model immediate response to a 15 percent decline in house prices is an increase in the aggregate foreclosure rate from 1.8 to 2.7 percent. This increase is consistent with the spike in foreclosures observed in the data, that went from 1.0-1.5 percent to 2.8 percent. The spike in foreclosures was also consistent with an decline in homeownership rate. The data suggest that prior to the housing collapse, this rate was 0.9 percent higher but the model only predicts one third of the total decline. The model suggests that a 8.6 percent decline in rental prices in conjunction with the negative income effects are sufficient to compensate for the 15 percent decline in house prices.

Table 7: Short-Run Response to a Decline in House Prices

	Default Rate	Total	Ownership (%Δ)
Model (t=0)	1.8	66.5	
Model (t=3)	2.7	66.3	-0.3%
Data 1998	1.0-1.5	66.3	
Data 2006	1.6	68.8	2.9%
Data 2007	2.8	68.2	-0.9%

To understand the nature of the spike in foreclosures, we report the foreclosure rates by loan type in Figure 8. The model predicts that foreclosures in FRM increased by roughly 30 percent whereas in the GPM by 100 percent. When we compare the model with the data during the same time period, we find that with respect to 1998 levels, the level of foreclosures increased by 50 percent in the FRM market and 270 percent in the ARM. However, since there was an important composition effect in the relative weight of FRM with respect to ARM's (the relative share of ARM's over FRM decline 14 percent) not captured in the model before the decline in house prices, we report the data change between 2006 and 2007. In this case we find that the increase in foreclosures in the FRM market was 33 percent and 105 percent for the ARM case.

Table 8: Foreclosures by Loan Type (at t=3)

	Data			Model	
	1998	2006	2007	Baseline	$\nabla 15\%$
Aggregate	1.0-1.5	1.6	2.8	1.8	2.7
by loan type					
FRM	0.8	0.9	1.2	1.7	2.2
GPM	2.0	3.6	7.4	2.0	4.0
market shares					
FRM	0.85	0.74	0.77	0.61	0.72
GPM	0.15	0.26	0.23	0.39	0.28

Although the aggregate levels and the percent increases in foreclosures are consistent

with the evidence, the specific levels in each market are a bit off. The model predicts an foreclosure rate of 2.2 percent in the FRM market (1.2 percent in the data), and 4.0 percent in the GPM market (7.4 percent in the data). When we compare the market shares in the baseline year, the data suggest a decline in the share of FRM between 1998 and 2007. However, this apparent decline disappears when we compare it between 2006 and 2007. The increase in foreclosures reduces the fraction of individuals holding GPM loans, since they have a higher premium, and the demand for FRM increases given that the entry cost have declined given the lower house prices. The model captures this flight to FRM observed in the data.

6 Conclusions

The empirical evidence from the last decade suggest than sizeable increases in housing defaults. We argue that an important mechanism to understand the evolution of foreclosure rate is the leverage in the economy. An increase in the leverage exposes homeowners to more risk in the event of a decline in house prices. For example, a 10 percent decline in home prices wipes out 30 percent of the equity. The objective of this paper is to a construct model that aids in understanding the main determinants of foreclosure and thus account for the observed spike in housing defaults. The model allows the distributional impact of a decline in house prices for different individuals to be identified. Such a framework can be used to help in understanding an environment with higher levels of risky lending, as well as evaluating the effectiveness of different government policy interventions.

Our preliminary findings suggest that an unanticipated decline in house prices can generate sizeable default rates at the aggregate level and across mortgage types. The model predicts that a decline in house prices can partially rationalize the spike in foreclosure rates, but the composition of default across loan products is harder to pin down. That suggests that mortgage rates probably include additional premiums not formalized in the model. We argue that the aggregate leverage level makes the economy more vulnerable to declines in house price. Moreover, the dynamic path under a government bailout of the mortgage industry is consistent with a short-term decline in homeownership. Despite the decline in house prices, the increase in supply of tenant-occupied housing reduces the rental price. Cheaper renting combined with higher taxes reduces the fraction of individuals purchase home in the short-run. Since the bailout is transitory, the new lending that emerges in the economy provides new loans based on the corrected collateral value and it helps the economy to increase the ownership away from post-collapse level. We argue that the response of the rental market is very important to understand the response of foreclosure rates to declines in house prices that models based on arbitrage pricing are incapable of replicate.

7 References

Chambers, M. , C. Garriga, and D. Schlagenhauf, "Accounting for Changes in the Homeownership Rate," forthcoming *International Economic Review*.

Davis, M., and J. Heathcote, "Housing and the Business Cycle," *International Economic Review*, 46 (August, 2005), 751-784.

Díaz, A. and M.J. Luengo Prado, "Durable Goods and the Wealth Distribution". Unpublished Manuscript, (December, 2006).

Fernández-Villaverde, J. and D. Krueger, "Consumption and Saving over the Life-Cycle: How Important are Consumer Durables?" Working paper, University of Pennsylvania, (December, 2005).

Fisher, J. D. M., and S. Quayyum, "The Great Turn-of the Century Housing Boom," *Economic Perspectives*, Federal Reserve Bank of Chicago, (3rd Quarter, 2006), 29-44.

Jeske, K. "Macroeconomic Models with Heterogenous Agents and Housing," *Federal Reserve Bank of Atlanta Economic Review*, (Fourth Quarter 2005), 39-56.

Jeske, K. and D. Krueger, "Housing and the Macroeconomy: The Role of Implicit Guarantees for Government-Sponsored Enterprises," Federal Reserve Bank of Atlanta working paper, (August, 2005).

Kiyoyaki, N., Michaelides, A., and K. Nikolov, "Winners and Losers in the Housing Market," Working Paper, Princeton University, (June 2007).

Li, W. "Moving Up: Trends in Homeownership and Mortgage Indebtedness," *Federal Reserve Bank of Philadelphia Business Review*, (First Quarter 2005), 26-34.

Nakajima. M., "Rising Prices of Housing and Non-Housing Capital and Rising Earnings Instability: The Role of Illiquidity of Housing," Working Paper, University of Pennsylvania, (2003).

Ortalo-Magne, F., and S. Rady, "Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints," *Review of Economic Studies*, " 78 (April, 2006), 459-485.

Ríos-Rull, J.V., and V. Sanchez-Marcos, "An Aggregate Economy with Different House Sizes," Working paper, University of Pennsylvania, (September 2006).

Storesletten, K., C. I. Telmer, and A. Yaron, Consumption and Risk Sharing over the Life Cycle, *Journal of Monetary Economics* 51 (April, 2004), 609-633.