

# The Economics of Hedge Funds\*

Yingcong Lan<sup>†</sup>      Neng Wang<sup>‡</sup>      Jinqiang Yang<sup>§</sup>

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## Abstract

We develop an analytically tractable model of hedge fund leverage and valuation where the manager maximizes the present value (PV) of management and incentive fees from current and future managed funds. By leveraging on alpha strategies, skilled managers create value. However, leverage also increases fund volatility and hence the likelihood of poor performance, which may trigger money outflow, drawdown/redemption, and involuntary fund liquidation, causing the manager to lose future fees. The ratio between assets under management (AUM) and high-water mark (HWM),  $w$ , measures the manager's moneyness and is a critical determinant of leverage and valuation.

Our main results are: (1) despite high-powered incentive fees, the risk-neutral manager often behaves in a risk-averse manner and chooses prudent leverage because downside liquidation risk is quite costly, in contrast to the standard risk-seeking intuition; (2) despite time-invariant alpha, leverage tends to increase following good performance and to decrease as the fund loses money; (3) both incentive and management fees contribute importantly to the manager's total value; (4) performance-linked new money inflow encourages leverage and has large effects on the manager's value, particularly on the value of incentive fees; (5) the manager values restart options; (6) managerial ownership has incentive alignment effects; (7) the manager becomes risk loving and the leverage constraint binds as liquidation risk decreases; and (8) investors' payoffs are sensitive to the manager's skill, alpha.

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<sup>†</sup>Cornerstone Research. Email: ylan@cornerstone.com.

<sup>‡</sup>Columbia Business School and NBER. Email: neng.wang@columbia.edu.

<sup>§</sup>Columbia Business School and Shanghai University of Finance & Economics (SUFE). Email: yang.jinqiang@mail.sufe.edu.cn.

# 1 Introduction

Hedge fund assets under management (AUM) reached \$2.13 trillion, a new record, in the first quarter of 2012, according to Hedge Fund Research. One salient feature of this multi-trillion dollar industry is its complicated management compensation structure. Hedge fund management compensation contracts typically feature both management fees and performance-based incentive fees. The management fee is charged as a fraction, e.g. 2%, of AUM. The incentive fee, a key characteristic that differentiates hedge funds from mutual funds, is calculated as a fraction, e.g. 20%, of the fund's profits. The cost base for the profit calculation is often investors' high-water mark (HWM), which effectively keeps track of the maximum value of the invested capital and thus depends on history and the manager's dynamic investment strategies. While "two-twenty" is often observed and viewed as the industry norm, compensation contracts vary with fund managers' track records. For example, Jim Simons' Renaissance Technologies Medallion Fund, one of the most successful hedge funds, charges 5% of the AUM via management fees and a 44% incentive fee.<sup>1</sup>

For investors to pay these fees and break even (net of fees) in present value (PV), managers need to generate risk-adjusted excess returns, known as alpha. Because of returns to scale, managers have incentives to leverage their alpha generating strategies. Indeed, an important feature of the hedge fund industry is the sophisticated and prevalent use of leverage. Hedge funds may borrow through the repo markets or from prime brokers, as well as use various implicit leverage, often via options and other derivatives. However, leverage also increases fund volatility and hence the likelihood of poor performance. In practice, a fund that performs poorly often faces money outflow, withdraw/redemption, or liquidation.

We develop an analytically tractable dynamic model to analyze hedge fund leverage policy and to value hedge fund management compensation contracts. The manager dynamically allocates the fund's AUM between the alpha-generating strategy and the risk-free asset. By leveraging the alpha strategy, the manager creates value for investors (in expectation) and hence benefits via performance-linked compensation. However, leveraging also increases the fund's volatility and hence the likelihood of liquidation, resulting in the loss of fees in the future. The manager dynamically trades off the benefit and the downside (e.g. liquidation and money outflow) risk of leverage to maximize the present value (PV) of fees not only

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<sup>1</sup>See <http://www.insidermonkey.com/hedge-fund/renaissance+technologies/5/>.

from current but also future managed funds. Outside investors rationally participate in the fund given their beliefs about the managerial skills and leverage strategies.

Specifically, our analytically tractable model contains the following important features: (1) an alpha generating strategy; (2) poor performance-triggered drawdown and liquidation; (3) management fees as a fraction of the AUM; (4) incentive/performance fees linked to the HWM; (5) leverage constraint and margin requirement; (6) managerial ownership, which is often motivated as an incentive alignment mechanism; (7) performance-induced new money inflow; (8) the manager's option to restart a fund (endogenous managerial outside option) at a cost. To simplify exposition, in our baseline model, we incorporate the first five features and focus on the manager's key tradeoff between the value creation benefit and the liquidation risk induced by leverage. We then introduce additional features (6) to (8) individually into our baseline model and analyze the economic and quantitative implications.

We exploit our model's homogeneity property and show that the ratio between the fund's AUM and its HWM, denoted by  $w$ , determines leverage choice. We analytically characterize the solution for the manager's value and the optimal leverage policy via an ordinary differential equation (ODE) in  $w$  with the right boundary condition at  $w = 1$  reflecting the manager's incentive fee collection and the left boundary condition reflecting the consequence due to the fund's liquidation.

In a dynamic framework with downside (drawdown/liquidation) risks, the risk-neutral manager has incentives to preserve the fund's going-concern value so as to collect fees in the future. The risk-neutral manager is averse to liquidation and this precautionary motive induces risk-averse managerial behavior. Intuitively, optimal leverage increases with alpha and decreases with volatility. More interestingly, optimal leverage decreases with the manager's endogenously determined risk aversion. Unlike the Merton-type investor, both the manager's effective risk attitude and optimal leverage depend on  $w$ , as the manager's moneyness (the long position in incentive fees and the short position in investors' liquidation option) varies. The higher the value of  $w$ , the more distant the fund is from liquidation and the closer the manager is to collecting the incentive fees, the less risk averse the manager behaves and the higher the leverage. When the downside liquidation likelihood is very low, the risk-neutral manager may even behave in a risk-seeking way. In this case, the leverage constraint becomes binding and the manager's value is convex in  $w$ .

Our baseline model parsimoniously captures the key tradeoff between value creation via leverage on an alpha strategy and the costly liquidation triggered by sufficiently poor performance. Additional key institutional features of hedge funds, such as (6)-(8) listed above, may have first-order effects on the manager’s leverage choice and valuation of fees. We introduce each new feature, one at a time into the baseline model to study their implications. First, when liquidation risk is low, the manager can be risk seeking and the margin requirement becomes binding. Second, managerial ownership within the fund mitigates agency conflicts. Third, we incorporate money flow-performance relation and find that it has significant implications on the manager’s leverage choices and the PVs of management fees and of incentive fees. Finally, we integrate the manager’s options to close the current fund and start up new funds and find that these options are quantitatively valuable.<sup>2</sup>

We also conduct quantitative analysis by calibrating our model to empirical leverage moments reported in Ang, Gorovyy, and van Inwegen (2011). Our calibration implies that investors will liquidate the fund if the manager loses 31.5% of the fund’s AUM from its HWM. Interestingly, our calibrated drawdown limit of 31.5% is comparable to the drawdown level of 25% quoted by Grossman and Zhou (1993) in their study of investment strategy for institutional (hedge fund) investors. We find that the manager creates about 20% value on the fund’s AUM in PV and captures all the surplus via their compensation. Out of the manager’s total value creation of 20 cents on a dollar, 75% is attributed to management fees (15 cents) and the remaining 25% goes to incentive fees (5 cents).

By incorporating managerial risk-seeking incentives/leverage constraints, managerial ownership, new money inflow, and fund restart options, the manager has additional incentives to leverage on the alpha strategy, which in turn increases the value of incentive fees, *ceteris paribus*. Overall, we find that quantitatively both management and incentive fees are important contributors to the manager’s total value. Metrick and Yasuda (2010) report similar quantitative results for private equity (PE) funds whose managers also charge management and incentive fees via two-twenty-type compensation contracts.<sup>3</sup> We also show that investors’ net payoffs (from their investments in the fund) critically depend on their abilities to cor-

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<sup>2</sup>The closure/restart option is similar to the manager’s option to negotiate with investors to reset the fund’s HWM once the incentive fee is sufficiently out of the money. The cost of negotiating with investors to reset the HWM is that some investors may leave the fund and hence the AUM may decrease.

<sup>3</sup>While compensation structures are similar for hedge funds and private equity funds, institutional details such as how management fees and performance fees are calculated differ significantly.

rectly assess the manager's alpha. For example, in our baseline calculation, compensating an unskilled manager with a two-twenty-type compensation is very expensive, as investors lose about 15% of their invested capital in PV.

**Related literature.** There are only a few theoretical papers on hedge funds' valuation and leverage decisions. Goetzmann, Ingersoll, and Ross (2003), henceforth GIR, provide the first quantitative intertemporal valuation framework for management and incentive fees in the presence of the HWM. They derive closed-form solutions for both investors' and the manager's values for a fund with a constant alpha. GIR focuses solely on valuation and does not allow for endogenous leverage or any other decisions such as fund closure/restart.

Panageas and Westerfield (2009), henceforth PW, obtain explicit leverage and the manager's value in a setting with only HWM-indexed incentive fees and no liquidation boundary for  $w$ . The main predictions of PW are (1) leverage is constant at all times, (2) managers are *worse* off if incentive fees *increase* (e.g. from 20% to 30%), and (3) managers are *worse* off if the HWM *decreases*, *ceteris paribus*. Our calibrated model predicts that (1) leverage is stochastic and tends to increase with  $w$ , (2) managers are better off if incentive fees increase, and (3) managers are better off if the HWM decreases, *ceteris paribus*. Our results are the *opposite* of the PW's because the manager in our model is averse to *downside* liquidation risk and tries to stay away from the liquidation boundary for survival, while the manager in PW is averse to crossing the HWM too soon, because incentive fees leave the fund and do not earn excess returns. Moreover, our model allow for leverage constraints and hence can generate risk loving behavior while PW do not. We also incorporate realistic features including management fees, managerial ownership, managerial restart options, and money inflows and outflows, in addition to performance-linked downside liquidation risk.

Hodder and Jackwerth (2007) numerically solve a risk-averse hedge fund manager's investment strategy in a discrete-time finite-horizon model. They argue the importance of endogenous fund restart options for the manager. Dai and Sundaresan (2010) show that the hedge fund's short positions in investors' redemption options and funding options (from prime brokers and short-term debt markets) influence the fund's risk management policies, but do not study the effects of management compensation on leverage and valuation.

More broadly, our paper relates to the literature on how compensation contracts influence

fund investment strategies.<sup>4</sup> Carpenter (2000) and Ross (2004) show that convex compensation contracts may not increase risk seeking for risk-averse managers. Basak, Pavlova, and Shapiro (2007) and Hugonnier and Kaniel (2010) study mutual fund managers' risk-taking induced by an increasing and convex relationship of fund flows to relative performance.<sup>5</sup>

Few papers have attempted to quantify the effects of management compensation on leverage and valuation of fees. We provide a simple calibration and take a first-step to quantitatively value both management and incentive fees in a model with endogenous leverage choice. Additionally, we show that managerial ownership, performance-dependent new money flows, and manager's voluntary closure/restart options are also important for hedge fund leverage and valuation.

There has been much recent and continuing interest in empirical research on hedge funds. Fung and Hsieh (1997), Ackermann, McEnally, and Ravenscraft (1999), Agarwal and Naik (2004), and Getmansky, Lo, and Makarov (2004) among others, study the nonlinear feature of hedge fund risk and return.<sup>6</sup> Lo (2008) provides a treatment of hedge funds for their potential contribution to systemic risk in the economy.

## 2 Baseline model

We now develop a parsimonious model of dynamic leverage with the following essential building blocks.

**The fund's investment opportunity.** The manager can always invest in the risk-free asset which pays interest at a constant rate  $r$ . Additionally, the manager can also invest in an alpha-generating strategy which earns a risk-adjusted expected excess return. Without leverage, the incremental return for the skilled manager's alpha strategy,  $dR_t$ , is given by

$$dR_t = (r + \alpha) dt + \sigma dB_t, \quad (1)$$

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<sup>4</sup>The other widely used approach studies the design of optimal compensation contracts given agency and/or informational frictions between investors and fund managers. The two approaches are complementary.

<sup>5</sup>Cucuo and Kaniel (2011) analyze equilibrium asset pricing with delegated portfolio management.

<sup>6</sup>For the presence of survivorship bias, selection bias, and back-filling bias in hedge funds databases, see Brown, Goetzmann, Ibbotson, and Ross (1992), among others. For careers and survival, see Brown, Goetzmann, and Park (2001).

where  $B$  is a standard Brownian motion,  $\alpha$  denotes the expected return in excess of the risk-free rate  $r$ , and  $\sigma$  is the return volatility. Managers often conceal the details of their trading strategies, and make it hard for investors and competitors to infer and mimic their strategies.<sup>7</sup> Alpha measures scarce managerial talents, which earn rents in equilibrium (Berk and Green, 2004). As we will show later, even with time-invariant investment opportunity, the optimal leverage will change over time due to managerial incentives.

Let  $W$  be the fund's AUM and  $D$  denote the amount invested in the risk-free asset. The investment amount in the alpha strategy (1) is then  $W - D$ . Let  $\pi$  denote the (risky) asset-capital ratio,  $\pi = (W - D)/W$ . Hedge funds often borrow via short-term debt and obtain leverage from the fund's prime brokers, repo markets, and the use of derivatives.<sup>8</sup> For a levered fund,  $D \leq 0$  and  $\pi \geq 1$ . For a fund hoarding cash,  $D > 0$  and  $0 < \pi < 1$ .

**Management compensation contracts.** Managers are paid via both management and incentive fees. The management fee is specified as a constant fraction  $c$  of the AUM  $W$ ,  $\{cW_t : t \geq 0\}$ . The incentive fee often directly links compensation to the fund's performance via the so-called high-water mark (HWM). In this paper, we take compensation contracts (both management and incentive fees) as given and then analyze optimal leverage and value management compensation.<sup>9</sup>

In the region when  $W < H$ , the HWM  $H$  evolves deterministically. Let  $g$  denote the growth rate of  $H$  without money outflow. This growth rate  $g$  may be zero, the risk-free rate  $r$ , or other values. The contractual growth rate  $g$  may reflect investors' opportunity costs of not investing elsewhere (e.g. earning the risk-free rate of return). Additionally, the fund's HWM should be adjusted downward when money flows out of the fund. As in GIR, we also allow investors to continuously redeem capital  $\delta W_t$  per unit of time, where  $\delta \geq 0$  is a constant. To sum up, when  $W < H$ , the HWM  $H$  grows exponentially at the rate  $(g - \delta)$ ,

$$dH_t = (g - \delta)H_t dt, \quad \text{if } W_t < H_t. \quad (2)$$

When  $g = \delta$ , (2) implies that the HWM  $H$  is the running maximum of  $W$ ,  $H_t = \max_{s \leq t} W_s$ ,

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<sup>7</sup>Additionally, for key employees (portfolio managers) who are informed about details of the strategies, the fund manager often pays them with long-term contracts (e.g. inside equity) with various provisions discouraging them from exiting (e.g. non-competition clause) partly to keep strategies secretive.

<sup>8</sup>Few hedge funds are able to directly issue long-term debt or secure long-term borrowing.

<sup>9</sup>The optimality of hedge fund management compensation contracts is not the focus of this paper but is an important topic for future research.

in that the HWM is the highest level that the AUM has attained.

When  $W \geq H$ , the fund's profit is  $dH_t - (g - \delta)H_t dt > 0$ . The manager collects a fraction  $k$  of that profit, given by  $k[dH_t - (g - \delta)H_t dt]$ , and then HWM  $H$  is re-set.

**Fund liquidation.** As in GIR, the fund can be exogenously liquidated with probability  $\lambda$  per unit of time. By assumption, the manager can do nothing to influence this liquidation likelihood. Let  $\tau_1$  denote the exogenous stochastic liquidation time.

Alternatively, if the fund's performance is sufficiently poor, investors may liquidate the fund. For example, large losses may cause investors to lose confidence in the manager, triggering liquidation. Specifically, we assume that when the AUM  $W$  falls to a fraction  $b$  of its HWM  $H$ , the fund is liquidated. GIR make a similar liquidation assumption in their valuation model. Unlike GIR, the AUM dynamics in our model are endogenous and depend on leverage. Let  $\tau_2$  denote this endogenous performance-triggered stochastic liquidation time,

$$W_{\tau_2} = bH_{\tau_2} . \tag{3}$$

The above liquidation condition has been used by Grossman and Zhou (1993) in their study of investment strategies by institutional investors facing what they refer to as “draw-down” constraints. In their terminology,  $1 - b$  is the maximum “drawdown” that investors allow the fund manager before liquidating the fund in our model. Grossman and Zhou (1993) state that “it is not unusual for managers to be fired subsequent to achieving a large draw-down, nor is it unusual for the managers to be told to avoid drawdowns larger than 25%.” The drawdown limit of 25% in the above quote maps to  $b = 0.75$  in our model. The fund will be liquidated if the manager loses 25% of the AUM from its HWM.

In reality, investors may increase the withdrawal of capital as the manager's performance deteriorates. We may model this performance-dependent withdrawal by allowing  $\delta$ , the rate at which investors withdraw, to depend on  $w$ , a measure of the fund's performance. By either specifying  $\delta$  as a decreasing function of  $w$  or using the lower liquidation boundary as in (3), we incorporate the downside risk into the model, which induces the manager to behave in an effectively risk-averse manner, the key mechanism of our model as we will show.<sup>10</sup>

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<sup>10</sup>For space considerations, we do not include the details for this extension with continuous performance-triggered withdrawal in the paper. The details are available upon request.

The fund is liquidated either exogenously at stochastic time  $\tau_1$  or endogenously at  $\tau_2$ . At liquidation time  $\tau = \min\{\tau_1, \tau_2\}$ , the manager receives nothing and investors collect the fund's AUM  $W_\tau$ . While leveraging on an alpha strategy creates value, the manager is averse to losing future fees upon liquidation. In an effectively (stationary) infinite horizon framework such as ours, liquidation can be quite costly and hence the manager faces significant downside risk (e.g. losses of all future compensation), unlike *limited* downside risk in typical finite-horizon option-based compensation models. It is thus often optimal for the manager to choose prudent time-varying leverage.

**Leverage constraint.** The fund may also face institutional and contractual restrictions on leverage. We impose the following leverage constraint at all times  $t$ :

$$\pi_t \leq \bar{\pi}, \quad (4)$$

where  $\bar{\pi} \geq 1$  is the exogenously specified maximally allowed leverage. Grossman and Vila (1992) study the effects of leverage constraints on portfolio allocations. For assets with different liquidity and risk profiles,  $\bar{\pi}$  may differ. For example, individual stocks have lower margins than Treasury securities do. See Ang et al. (2011) for a summary of various margin requirements for different assets. Investors also contractually impose bounds on leverage for the fund. With a sufficiently tight leverage constraint, the manager's value and leverage will be finite and the optimization problem is well defined even for a risk-neutral manager.

**Dynamics of AUM.** Prior to liquidation ( $t < \tau$ ), the fund's AUM  $W_t$  evolves as follows

$$dW_t = rW_t dt + \pi_t W_t (\alpha dt + \sigma dB_t) - \delta W_t dt - cW_t dt - k [dH_t - (g - \delta)H_t dt] - W_t dJ_t. \quad (5)$$

The first and second terms in (5) describe the change of AUM  $W$  given the manager's leverage strategy  $\pi$ . The third term gives the continuous payout to investors, i.e. money outflow. The fourth term gives the flow of management fees (e.g.  $c = 2\%$ ), and the fifth term gives the incentive/performance fees which are paid if and only if the AUM exceeds the HWM (e.g.  $k = 20\%$ ).<sup>11</sup> Here,  $J$  is a jump process with a mean arrival rate  $\lambda$ . If the jump occurs, the fund

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<sup>11</sup>The manager collects the incentive fees if and only if  $dH_t > (g - \delta)H_t dt$ , which can only possibly happen at the boundary  $W_t = H_t$ . In the interior region ( $W_t < H_t$ ), incentive fees are zero since  $dH_t = (g - \delta)H_t dt$ . Hence,  $dH_t - (g - \delta)H_t dt \geq 0$  and incentive fees are non-negative at all times.

is exogenously liquidated and hence its AUM  $W$  falls to zero. We can further generalize this jump-induced liquidation by specifying the intensity  $\lambda$  as a function of  $w$ , the ratio between the fund's AUM  $W$  and its HWM  $H$ . By specifying a higher jump likelihood  $\lambda$  for a worse performance (a lower  $w$ ), we introduce performance-triggered stochastic liquidation, which causes the risk-neutral manager to behave in a risk-averse manner.<sup>12</sup>

**Various value functions for investors and the manager.** We now introduce various present values (PVs). Let  $\beta$  denote the manager's discount rate. For a given dynamic leverage strategy  $\pi$ , the PV of total fees, denoted by  $F(W, H; \pi)$ , is given by

$$F(W, H; \pi) = M(W, H; \pi) + N(W, H; \pi), \quad (6)$$

where  $M(W, H; \pi)$  and  $N(W, H; \pi)$  are the PVs of management and incentive fees, respectively,

$$M(W, H; \pi) = \mathbb{E}_t \left[ \int_t^\tau e^{-\beta(s-t)} c W_s ds \right], \quad (7)$$

$$N(W, H; \pi) = \mathbb{E}_t \left[ \int_t^\tau e^{-\beta(s-t)} k [dH_s - (g - \delta) H_s ds] \right]. \quad (8)$$

The manager collects neither management nor incentive fees after stochastic liquidation.<sup>13</sup>

Similarly, we define investors' value  $P(W, H)$  as follows

$$P(W, H; \pi) = \mathbb{E}_t \left[ \int_t^\tau e^{-r(s-t)} \delta W_s ds + e^{-r(\tau-t)} W_\tau \right]. \quad (9)$$

In general, investors' value  $P(W, H)$  differs from the AUM  $W$  because of managerial skills and fees. The total fund value  $V(W, H)$  is given by the sum of  $F(W, H)$  and  $P(W, H)$ :

$$V(W, H; \pi) = F(W, H; \pi) + P(W, H; \pi). \quad (10)$$

**The manager's optimization and investors' participation.** Anticipating that the manager behaves in self interest, investors rationally demand that investors' value,  $P(W, H)$ ,

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<sup>12</sup>As we will show later, our model solution only depends on the sum of  $\lambda + \delta$ . It is thus sufficient to specify  $\lambda + \delta$ , as a function of  $w$ . See Sections 3 and 4.

<sup>13</sup>In Section 7, we extend the baseline model to allow the manager to close the fund and start a new one. The manager then maximizes the PV of fees from both current and future funds.

is at least as large as their time-0 investment  $W_0$  in order to break even in PV. At time 0, by definition, we have  $H_0 = W_0$ . Thus, at time 0, we require

$$P(W_0, W_0; \pi) \geq W_0. \quad (11)$$

Note that the above constraint (11) is only required at the inception of the fund.<sup>14</sup> The surplus division between investors and the manager depends on their bargaining powers. In competitive markets, the skilled manager collects all the surplus, and the participation constraint (11) binds. However, in periods such as a financial crisis, investors may earn some rents by providing scarce capital and the constraint (11) holds with slack.

When the manager only faces the time-0 investors' voluntary participation constraint (11), we may write the manager optimization problem as follows,

$$\max_{\pi} F(W, H; \pi), \quad (12)$$

subject to the liquidation boundary (3), leverage constraint (4), investors' voluntary participation (11), and regularity conditions.

### 3 Solution

We solve the manager's optimization problem using dynamic programming. As we show, the homogeneity property proves valuable in simplifying our analysis. That is, if we double the AUM  $W$  and the HWM  $H$ , the PV of total fees  $F(W, H)$  will correspondingly double. The effective state variable is therefore the ratio between the AUM  $W$  and the HWM  $H$ ,  $w = W/H$ . We use the lower case to denote the corresponding variable in the upper case scaled by the contemporaneous HWM  $H$ . For example,  $f(w) = F(W, H)/H$ .

**Summary of main results.** The manager's leverage policy critically depends on the manager's endogenously determined risk attitude. To measure a risk-neutral manager's

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<sup>14</sup>If investors can liquidate the fund at any time, we then need to require investors' voluntary participation constraints to hold at all times, i.e.  $P(W_t, H_t; \pi) \geq W_t$  for all  $t$ . In this case, investors never lose money in PV. Intuitively, investors' participation constraints for the manager become tighter, and the manager's value will in turn be lower than the value for the case where the manager only faces time-0 investors' participation constraint (11). Fortunately, that version of our model is also tractable. For space considerations, we do not include the detailed analysis here and will provide details upon request.

endogenous risk attitude, motivated by the coefficient of relative risk attitude for a consumer, we define a risk-neutral manager's risk attitude as follows,

$$-\frac{WF_{WW}(W, H)}{F_W(W, H)} = -\frac{wf''(w)}{f'(w)} \equiv \psi(w), \quad (13)$$

where the equality follows from the homogeneity property. When  $\psi(w) > 0$ , we refer the risk-neutral manager as being endogenously risk averse. When  $\psi(w) \leq 0$ , we refer the risk-neutral manager as being endogenously risk seeking. Note that risk attitude  $\psi(w)$  is endogenous and stochastic.

In the Appendix, we show that the optimal leverage policy is given by

$$\pi(w) = \begin{cases} \min \left\{ \frac{\alpha}{\sigma^2 \psi(w)}, \bar{\pi} \right\}, & \psi(w) > 0, \\ \bar{\pi}, & \psi(w) \leq 0, \end{cases} \quad (14)$$

where the endogenous risk attitude  $\psi(w)$  is defined by (13). When  $\psi(w) > 0$  and equivalently  $f''(w) < 0$  implying that the second-order condition (SOC) is satisfied, the manager is endogenously risk averse to downside liquidation risk. If  $\psi(w)$  is sufficiently large, the leverage constraint (4) does not bind and the optimal leverage  $\pi(w)$  is then given by the ratio between (1) the excess return  $\alpha$  and (2) the product of variance  $\sigma^2$  and endogenous risk attitude  $\psi(w)$ . Unlike in the standard portfolio model (e.g. Merton, 1971), the manager's motive for fund survival is strong enough to cause the risk-neutral manager's leverage and value function to be well defined and finite despite the option to infinitely leverage on the alpha strategy. When  $\psi(w)$  is low enough, leverage constraint binds and  $\pi(w) = \bar{\pi}$ .

When  $\psi(w) \leq 0$  and equivalently  $f''(w) \geq 0$  implying that the SOC is not satisfied, the manager then behaves in a risk-seeking manner by choosing the maximally allowed leverage, causing the leverage constraint (4) to bind,  $\pi(w) = \bar{\pi}$ . In the risk-seeking region, the standard FOC-based analysis of leverage choices is no longer valid. A sufficiently tight leverage constraint (4) is necessary to ensure that the manager's optimization problem is well defined. We note that the constraint can bind either when  $\psi(w) < 0$  or when  $\psi(w) \geq 0$ . The mechanisms that causes the constraint (4) to bind differ for the two cases.

Importantly, the risk-neutral manager's risk-taking incentives vary significantly with  $w$ . Let  $\bar{w}$  denote the threshold value of  $w$  where the leverage constraint (4) just becomes binding. Because the manager optimally chooses  $\bar{w}$ , the manager's value  $f(w)$  is twice continuously

differentiable at  $w = \bar{w}$ ,

$$f(\bar{w}_-) = f(\bar{w}_+), \quad f'(\bar{w}_-) = f'(\bar{w}_+), \quad f''(\bar{w}_-) = f''(\bar{w}_+). \quad (15)$$

Here,  $\bar{w}_+$  and  $\bar{w}_-$  denote the right and left limits of the endogenously chosen  $\bar{w}$ .

Both endogenous risk attitude  $\psi(w)$  and leverage constraint (4) help to ensure that the risk-neutral manager's optimization problem is well defined.

Using the Ito's formula and the optimal leverage (14), we write the dynamics for  $w$  as,

$$dw_t = [\pi(w_t)\alpha + r - g - c]w_t dt + \sigma\pi(w_t)w_t dB_t - w_t dJ_t, \quad b < w < 1, \quad (16)$$

where  $J$  is the pure jump process leading to liquidation, as we have previously described.

The manager's value  $f(w)$  solves the following ordinary differential equation (ODE),

$$(\beta - g + \delta + \lambda)f(w) = cw + [\pi(w)\alpha + r - g - c]wf'(w) + \frac{1}{2}\pi(w)^2\sigma^2w^2f''(w), \quad (17)$$

subject to the following boundary conditions,

$$f(b) = 0, \quad (18)$$

$$f(1) = (k + 1)f'(1) - k. \quad (19)$$

Equation (18) states that the manager's value is zero at the liquidation boundary  $b$ . This assumption is the same as the one in GIR. However, unlike GIR, the manager in our model influences the liquidation likelihood via dynamic leverage. Importantly, performance-based liquidation risk, such as the liquidation boundary (18), plays a critical role in our analysis. In reality, In Section 7, we extend our model to allow the manager to start a new fund, enriching the baseline model by providing the manager with flexible exit and restart options. Equation (19) gives the manager's value at the right boundary  $w = 1$ . Our reasoning for this boundary condition follows GIR and PW. Finally, ODE (17) gives the manager's total value  $f(w)$  in the interior region. Note that the withdrawal rate  $\delta$  and the exogenous liquidation intensity  $\lambda$  appear additively in (17), and thus they have the same effects on valuation.

Investors' voluntary participation condition (11) can be simplified to

$$p(1) \geq 1. \quad (20)$$

We obtain the value of  $p(1)$  by solving the investors' scaled value  $p(w)$  using the ODE (A.11) subject to boundary conditions (A.13)-(A.14). If we further assume perfectly competitive

capital markets, managers collect all the rents from their skills at time 0 and investors' value satisfies  $p(1) = 1$ , as in Berk and Green (2004) in the context of mutual funds.<sup>15</sup>

## 4 Results

We now analyze our baseline model's implications on leverage, valuation of fees, and investors' payoff. We first choose the parameter values and calibrate our model.

### 4.1 Parameter choices and calibration

We choose the commonly used two-twenty compensation contract,  $c = 2\%$  and  $k = 20\%$ . We equate the manager's and the investors' discount rates,  $\beta = r$ . All rates are annualized and continuously compounded, when applicable. As in GIR, our model identifies  $\delta + \lambda$ , the sum of payout rate  $\delta$  and the fund's exogenous liquidation intensity  $\lambda$ . We refer to  $\delta + \lambda$  as the total withdrawal rate. Similarly, our model identifies  $r - g$ , which we refer to as the net growth rate of  $w$ . We thus only need to choose the following parameter values: (1) the un-levered  $\alpha$ ; (2) the un-levered volatility  $\sigma$ ; (3) the total withdrawal rate,  $\delta + \lambda$ ; (4) the net growth rate of  $w$ ,  $r - g$ ; and (5) the liquidation boundary  $b$ . We also set the leverage constraint at  $\bar{\pi} = 4$ .

We set the net growth rate of  $w$  to zero,  $r - g = 0$ . Otherwise, even unskilled managers collect incentive fees by simply holding a 100% position in the risk-free asset. We set the exogenous liquidation probability  $\lambda = 10\%$  so that the implied average fund life (with exogenous liquidation risk only) is ten years. Few hedge funds have regular payouts to investors, we thus choose  $\delta = 0$ . The total expected withdrawal rate is thus  $\delta + \lambda = 10\%$ .

Next, we calibrate the remaining three parameters: excess return  $\alpha$ , volatility  $\sigma$ , and the liquidation boundary  $b$ . We use two moments from Ang, Gorovyy, and van Inwegen (2011), which report that the average long-only leverage is 2.13 and the standard deviation for cross-sectional leverage is 0.616 (for a data-set from a fund-of-hedge funds).

Calibrating to the two leverage moments and the equilibrium condition,  $p(1) = 1$ , we identify  $\alpha = 1.22\%$ ,  $\sigma = 4.26\%$ , and  $b = 0.685$ . The implied Sharpe ratio for the alpha strat-

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<sup>15</sup>Here, if we impose (11) at all times as we discussed earlier, i.e.  $p(w) \geq w$  for all  $w$ . We will have a tighter constraint but the model remains tractable. For space constraints, we leave the details out, and results for the case with  $p(w) \geq w$  are available upon request.

egy is  $\alpha/\sigma = 29\%$ . Our calibration-implied maximum drawdown before investors liquidate the fund (or equivalently fire the manager) is  $1 - b = 31.5\%$ . Interestingly, this calibrated value 31.5% is comparable to the drawdown level of 25% that is quoted by Grossman and Zhou (1993) in their study of investment strategy with drawdown constraints. Our calibration implies an levered (annual) alpha to be about 2.6% and levered volatility to be 9.1%, both of which lie within various estimates in the literature. (Naturally, different styles of hedge funds have different targets for alpha and volatility.) Note that empirical estimates of a fund’s alpha and volatility correspond to the levered ones in our model.<sup>16</sup> Table 6 summarizes all the key variables and parameters in the model. Unless otherwise noted, we use these parameter values for our quantitative analysis.

## 4.2 Leverage $\pi$ , the manager’s value $f(w)$ , and risk attitude $\psi(w)$

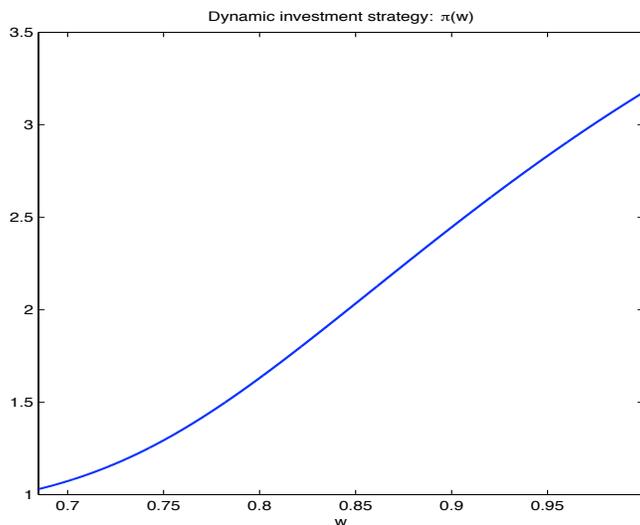


Figure 1: **Dynamic leverage**  $\pi(w)$ .

**Dynamic leverage.** Figure 1 plots leverage  $\pi(w)$ . Despite the fund’s constant investment opportunity, leverage  $\pi_t$  depends on  $w_t$ , the manager’s money in the fund. At the liquidation boundary  $b = 0.685$ , the fund is barely levered,  $\pi(b) = 1.03$ . As  $w$  increases, the manager increases leverage, and reaches  $\pi(1) = 3.18$  at  $w = 1$ . The higher the manager’s

<sup>16</sup>Ibbotson, Chen, and Zhu (2011) report that the estimated annual alpha is about 3%. Similarly, our calibrated levered volatility is also within the range of empirical volatility estimates. See also Fung and Hsieh (1997), Brown, Goetzmann, and Ibbotson (1999), and Kosowski, Naik, and Teo (2007).

moneyness  $w$ , the closer the manager is to collecting incentive fees and the more distant the fund is from liquidation, the higher the leverage  $\pi(w)$ . The implied annual *levered alpha*  $\pi(w)\alpha$  varies from 1.26% at the liquidation boundary  $b = 0.685$  to 3.88% at  $w = 1$  with an average value of 2.60%. The annual volatility of the levered return  $\pi(w)\sigma$  ranges from 4.4% at the liquidation boundary  $b$  to 13.4% at  $w = 1$ .

Common wisdom suggests that managers may gamble for resurrection when their option-based compensation contracts are deep out of the money. In our model, this standard risk-shifting insight does not apply. First, in our infinite-horizon setting, upon liquidation, the manager loses all future management and incentive fees, which is quite costly, as opposed to the limited downside risk assumed in standard finite-horizon settings with option-based convex payoffs. Second, debt is fully collateralized and hence is risk-free. There is thus no risk shifting incentive against creditors.<sup>17</sup> Using their valuation model for management and incentive fees, GIR also show that the manager becomes more prudent as the fund gets closer to the liquidation boundary as opposed to seeking risk, consistent with our findings.<sup>18</sup>

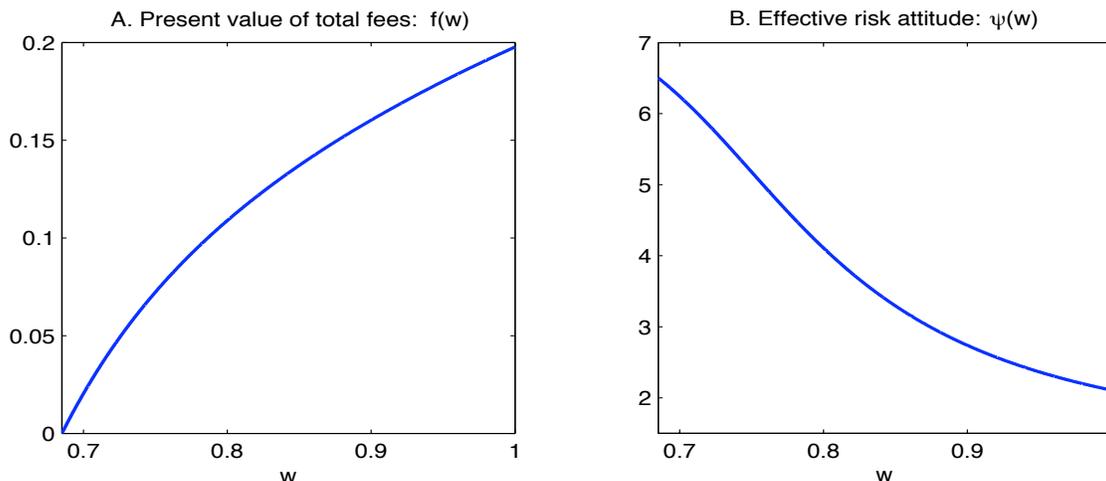


Figure 2: **The manager’s scaled value function  $f(w)$  and the “effective” risk aversion,  $\psi(w) = -w f''(w)/f'(w)$ .**

<sup>17</sup>With risk-free debt, there is no incentive for equityholders to engage in risk shifting since there is no wealth transfer from creditors to equityholders. See Jensen and Meckling (1976). Risk-free debt does not induce under-investment either as shown by Myers (1977).

<sup>18</sup>If the fund incurs a sufficiently large operating cost with a fixed component (independent of fund’s AUM  $W$ ), it is possible that leverage may increase as  $w$  decreases and the fund get close to the liquidation boundary). We leave out the illustration of this result due to space constraints. Notes and results are available upon request.

**The manager’s value  $f(w)$  and “effective” risk attitude  $\psi(w)$ .** Panel A of Figure 2 plots  $f(w)$ . For each unit of AUM, at the inception of the fund, the manager creates 20% surplus in PV,  $f(1) = 0.20$ , and collects all the surplus via management and incentive fees. Panel B of Figure 2 plots the manager’s “effective” risk attitude,  $\psi(w)$ . The risk-neutral manager is averse to costly fund liquidation. As  $w$  increases, liquidation risk decreases and the manager’s endogenous risk attitude  $\psi(w)$  falls from  $\psi(b) = 6.50$  to  $\psi(1) = 2.11$ .

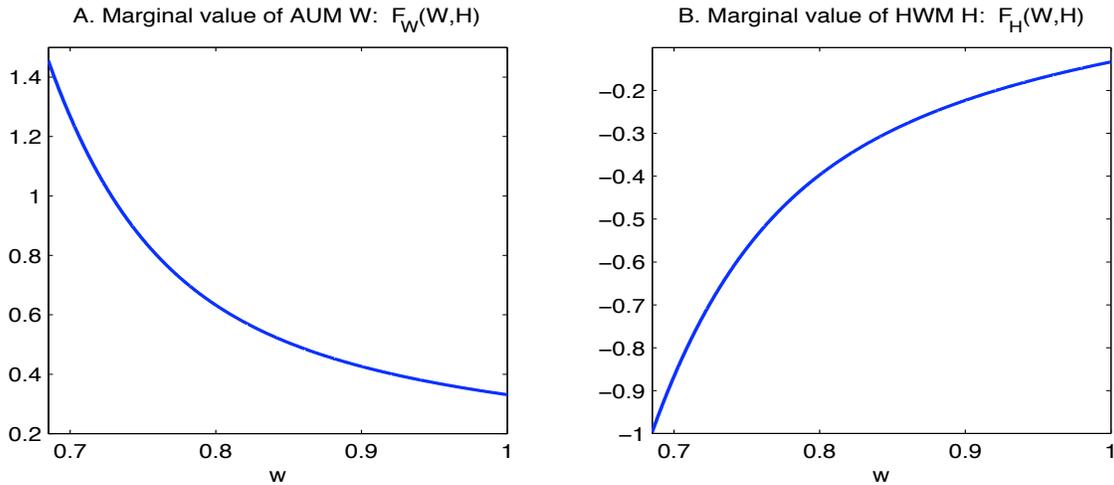


Figure 3: **The sensitivities of the manager’s value function  $F(W, H)$  with respect to the AUM  $W$  and the HWM  $H$ ,  $F_W(W, H)$  and  $F_H(W, H)$ .**

**The marginal value of the AUM  $W$ ,  $F_W(W, H)$ .** Using the homogeneity property, we have  $F_W(W, H) = f'(w)$ . Panel A of Figure 3 plots  $f'(w)$ . As  $w$  increases,  $f'(w)$  decreases from  $f'(b) = 1.46$  at the liquidation boundary  $b = 0.685$  to  $f'(1) = 0.33$ , which is much lower than  $f'(b) = 1.46$ . The higher the value of  $w$ , the lower the liquidation risk and thus the lower the marginal value of AUM  $f'(w)$ . A dollar increase of the AUM near the liquidation boundary  $b$  is much more valuable than a dollar increase near  $w = 1$  because the former decreases the risk of fund liquidation and can potentially save the fund from liquidation.

**The marginal impact of the HWM  $H$ ,  $F_H(W, H)$ .** Again, using the homogeneity property, we have  $F_H(W, H) = f(w) - wf'(w)$ . Panel B of Figure 3 plots  $F_H(W, H)$  as a function of  $w$ . Increasing  $H$  mechanically lowers  $w = W/H$ , which increases the likelihood

of investors' liquidation. Because the manager is averse to fund liquidation, increasing  $H$  lowers  $F(W, H)$ ,  $F_H(W, H) < 0$ . Importantly, downside liquidation risk (a sufficiently high  $b$ ) is critical to generate this result.

Quantitatively, the impact of the HWM  $H$  on  $F(W, H)$  is significant. Even when the manager is very close to collecting incentive fees ( $w = 1$ ), a unit increase of the HWM  $H$  lowers the manager's value  $F(H, H)$  by 0.13, which follows from  $F_H(H, H) = f(1) - f'(1) = -0.13$ . The impact of  $H$  on  $F(W, H)$  is even greater for lower values of  $w$ . At the liquidation boundary  $b = 0.685$ , the impact of HWM on manager's value is about one to one in our calibration,  $F_H(bH, H) = f(b) - bf'(b) = -1.00$ .

Intuitively, increasing  $H$  lowers the manager's total value  $F(W, H)$  as the manager's option to collect incentive fees is further out of the money and the manager is closer to the liquidation boundary, both of which make the manager worse off, *ceteris paribus*. In a model with incentive fees only, PW show that the manager's value function increases with the HWM  $H$ , which is the opposite of our result. This is because the first-order tradeoffs in the two models are different. In PW, there is no liquidation boundary ( $b = 0$ ) and the manager is averse to crossing the HWM too soon.<sup>19</sup> In our model, the manager is primarily concerned about the downside liquidation risk.<sup>20</sup>

### 4.3 Understanding leverage and valuation via a sample path

Figure 4 illustrates a sample path for the fund's performance from its inception until its stochastic liquidation. For the simulation, we set the time step  $\Delta = 10^{-3}$ . Panel A plots the fund's AUM  $W_t$  and HWM  $H_t$ . Panel B plots  $w_t = W_t/H_t$  and leverage  $\pi_t$ . Finally, Panels C and D plot the management fee and incentive fee (both in cash flows), respectively.

At the fund's inception,  $W_0 = H_0 = 1$ , and  $w_0 = 1$  by definition. The manager chooses leverage  $\pi_0 = \pi(1) = 3.18$ . At the end of the first time step,  $t = \Delta = 10^{-3}$ , the manager collects the management fee  $c \times W_0 \times \Delta = 2 \times 10^{-5}$ , as expected. Given that the realized

<sup>19</sup>In PW,  $F_H(W, H) = f(w) - wf'(w) > 0$ , which follows from the homogeneity property and no liquidation boundary,  $b = 0$ . In PW,  $F_H > 0$  is implied by a concave  $f(w)$  and  $b = 0$ . Therefore, with  $b = 0$ ,  $F_H > 0$  is necessary and sufficiently to ensure that the manager's optimization problem is well defined in PW.

<sup>20</sup>It is worth noting that the manager also earns a lower rate of return (no alpha) from collected fees in our model as in PW. However, in our model, the lower rate of return that a manager earns on fees only has a second-order effect in deterring the manager from leveraging and does not influence the main tradeoff between leveraging on the alpha strategy to generate excess returns and being prudent to reduce liquidation risk. Our model and PW focus on very different parameter regions.

shock at  $t = \Delta$ ,  $dR_\Delta = -0.001$ , is negative, the AUM  $W_\Delta = 0.9967$  and  $H_\Delta = 1.00005$  implying  $w_\Delta = 0.9966 < 1$  and no incentive fee cash flow. Leverage drops to  $\pi_\Delta = 3.15$ .

At  $t = 2\Delta = 2 \times 10^{-3}$ , the fund draws another shock  $dR_{2\Delta}$  and the process repeats. As long as  $W_t < H_t$ , the HWM  $H_t$  grows exponentially at a deterministic rate  $g - \delta = 5\%$ , as we see from the smooth curve in the region  $W_t < H_t$ .

In contrast, when  $W_t = H_t$ , a positive shock causes the manager to collect incentive fee cash flow. For example, at  $t = 8\Delta = 0.008$ ,  $w_t \approx 1$ , the realized shock at  $t = 9\Delta$ ,  $dR_{9\Delta} = 0.0015$ , is positive, and the manager collects  $0.0023 \times 20\% = 0.00046$  in incentive fees, and then HWM is reset so that  $w_{9\Delta} = 1$ .

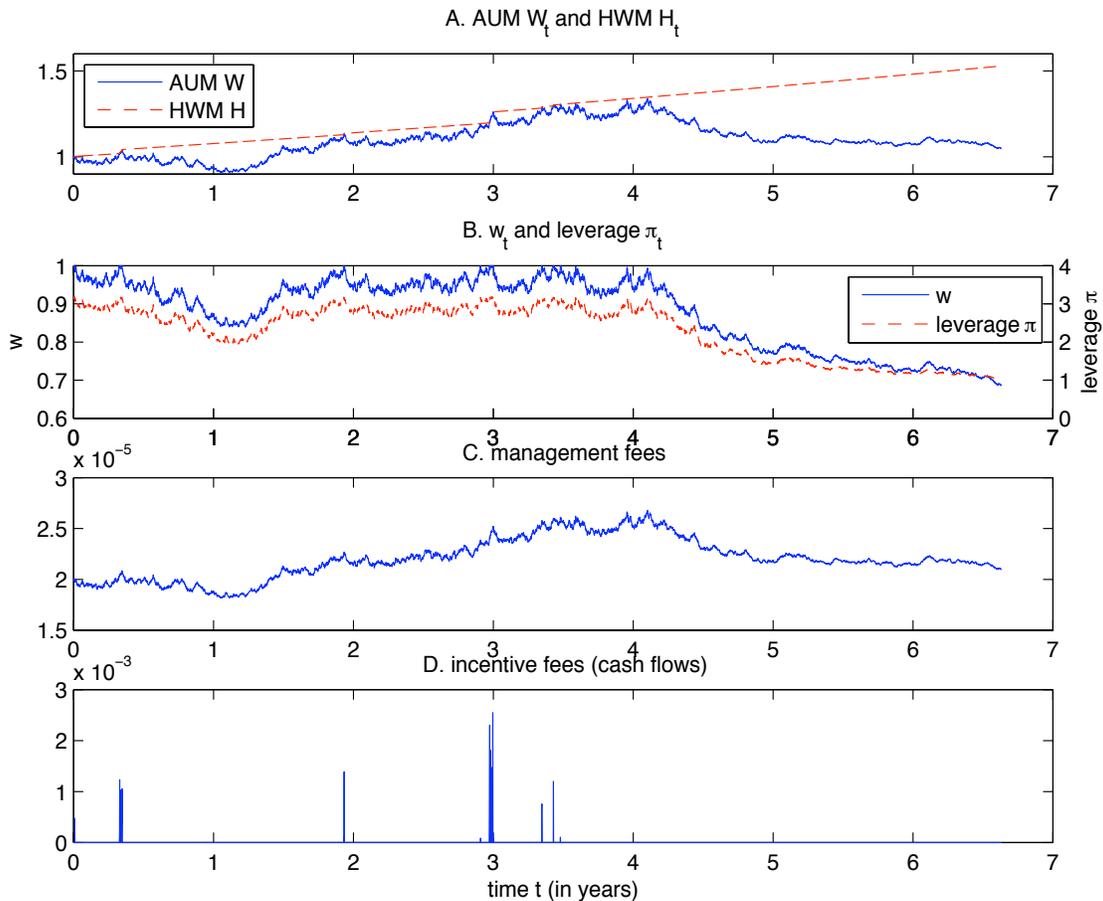


Figure 4: A sample path of AUM  $W_t$ , HWM  $H_t$ ,  $w_t$ , leverage  $\pi_t$ , management fee (cash flows), and incentive fees (cash flows) over time  $t$ .

This sample path shows that as  $w_t$  decreases, leverage  $\pi_t$  decreases in a nonlinear way.

For this simulation, the fund is liquidated at time  $\tau = 6.63$ , the first moment that  $w_\tau$  reaches the lower liquidation boundary  $b = 0.685$ . Panel C shows that the management fee cash flow  $cW_t\Delta$  linearly tracks the evolution of AUM  $W_t$ . Incentive fee cash flows are paid occasionally and lumpy, as the fund's AUM occasionally exceeds its HWM at random moments. Next, we turn to the valuation of management fees and incentive fees.

#### 4.4 Valuing incentive and management fees

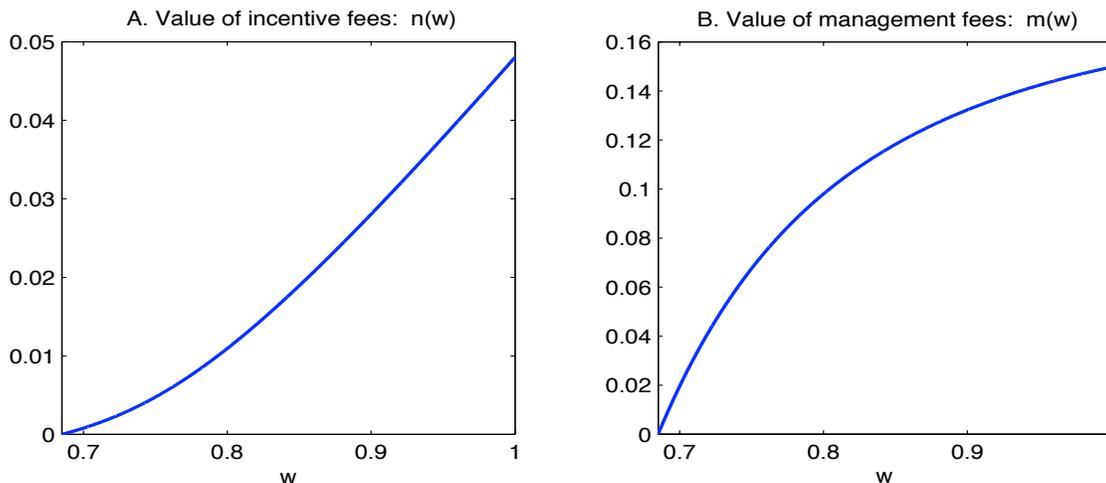


Figure 5: **The PV of incentive fees  $n(w)$  and the PV of management fees  $m(w)$ .**

**The value of incentive fees  $n(w)$ .** Panel A of Figure 5 plots  $n(w)$ . As  $w$  increases, the manager is closer to collecting incentive fees and  $n(w)$  increases. At  $w = 1$ ,  $n(1) = 0.05$ , which is one quarter of the manager's total value  $f(1) = 0.20$ . Incentive fees are a sequence of embedded call options. Intuitively, the delta of  $n(w)$ ,  $n'(w)$ , increases with  $w$ , opposite to the result in PW. Again, the manager is primarily concerned about downside liquidation risk in our model, but is averse to collecting incentive fees too soon in PW.

**The value of management fees  $m(w)$ .** Panel B of Figure 5 plots  $m(w)$  which increases with  $w$  from  $m(b) = 0$  to  $m(1) = 0.15$ , about 75% of the manager's total value  $f(1) = 0.20$ . Quantitatively, management fees contribute significantly to total compensation. Intuitively, the management fee is effectively a wealth tax on the AUM. The PV of the AUM tax at 2% over a fund's lifespan is thus significant as a fraction of its AUM. In our case,  $m(1)$  is

Table 1: **Comparative static effects of the un-levered  $\alpha$  on leverage  $\pi(1)$ , effective risk attitude  $\psi(1)$ , and various value functions.**

$\alpha$	$\pi(1)$	$\psi(1)$	$m(1)$	$n(1)$	$f(1)$	$p(1)$	$v(1)$
0	0	2.0425	0.1495	0	0.1495	0.8505	1
0.61%	2.5038	1.3381	0.1413	0.0213	0.1627	0.8934	1.0560
1.22%	<b>3.1753</b>	<b>2.1105</b>	<b>0.1497</b>	<b>0.0480</b>	<b>0.1977</b>	<b>1</b>	<b>1.1977</b>
2.44%	4.0000	2.6598	0.2248	0.1653	0.3901	1.4899	1.8800

15% of the AUM. For the private equity industry, Metrick and Yasuda (2010) also find that management fees contribute to the majority of total management compensation.

Unlike the value of incentive fees  $n(w)$ ,  $m(w)$  is concave in  $w$ . The manager collects management fees as long as the fund survives, but only receives incentive fees when the AUM exceeds the HWM. Management fees, as a fraction  $c$  of the fund’s AUM, effectively give the manager an *un-levered* equity-type cash flow claim from the fund while the fund is alive. Upon the fund’s liquidation, the manager receives nothing and loses all future fees. Therefore, fund liquidation is quite costly for the manager. The manager thus optimally chooses a prudent level of leverage for survival so as to collect fees in the future.

#### 4.5 How important is the manager’s alpha?

Table 1 reports the comparative static effects of changing the un-levered  $\alpha$  on leverage  $\pi(1)$ , effective risk attitude  $\psi(1)$ , and various values. Recall that in the baseline case (the highlighted row), investors break even,  $p(1) = 1$ , and managers collect  $f(1) = 0.2$  in PV via fees for their skills. Other than the baseline case, investors either make losses ( $p(1) < 1$ ) or collect surplus ( $p(1) > 1$ ) in our comparative static exercises, as we expect. The bottom-line of this comparative static exercise is that it is important for both investors and the manager to correctly assess the manager’s skill, alpha, from a quantitative perspective.

As we decrease  $\alpha$  from 1.22% to 0.61%, investors lose 10.7% by investing with a manager whose un-levered alpha is only half of the the baseline value 1.22%. The manager still collects about 16.3% in total fees where the vast majority of the fees, 14.1% out of the total 16.3%, comes from management fees, as we expect. Intuitively, a less skillful manager milks

investors by over-charging fees, primarily the management fees. This calculation shows that a modern mis-assessment of the manager’s skill leads to significant value losses to investors.

By doubling the manager’s skill (increasing the un-levered  $\alpha$  from 1.22% to 2.44%), investors’ value  $p(1)$  increases about by 50% from one to 1.49. And the manager’s total value  $f(1)$  almost doubles from 19.8% to 39.0%, with the majority of the increase coming from incentive fees  $n(1)$ , as we expect. This calculation suggests that if investors can hire a manager who is more skilled than the market expectation, investors can benefit significantly. Correctly assessing the manager’s skill is thus essential for investors, which is consistent with the commonly held belief in the hedge fund industry.

Next, we consider the special case where the manager has no skills,  $\alpha = 0$ . With no skills, the manager behaves quite prudently by optimally choosing no leverage. Since the growth rate of HWM is indexed to  $r$ , the manager thus collects no incentive fees,  $n(1) = 0$ , but still collects management fees. Table 1 reports that  $m(1)$  is effectively unchanged, remaining at 15%. Investors are thus worse off in that  $p(1)$  is lowered by 15% from 1 to 0.85. Management fees transfer wealth from investors to the manager. Hiring an unskilled manager with a 2-20 compensation contract costs investors about 15% of their AUM in PV, which is very costly. Our analysis suggests that it is critically important for investors to choose the right manager (with high enough alpha) for a given compensation contract (e.g. two-twenty).

## 4.6 Leverage constraint

Downside liquidation risk plays a critical role by giving the risk-neutral manager incentives to potentially behave in a risk-averse manner and hence delivers sensible economic predictions in our model. We now conduct the comparative static analysis with respect to the downside risk. As we decrease the liquidation boundary  $b$ , the downside risk decreases, leverage increases and hence the constraint  $\pi(w) = \bar{\pi}$  may bind.

**Dynamic leverage  $\pi(w)$  and “effective” risk attitude  $\psi(w)$ .** Figure 6 and Table 2 report the comparative static analysis of leverage policy  $\pi(w)$  and “effective” risk attitude  $\psi(w)$  with respect to the liquidation boundary  $b$ . We choose  $b = 0.2, 0.5$ , and  $0.685$  and set the leverage constraint  $\bar{\pi} = 4$ .

First, we restate results for our baseline case. With sufficiently large liquidation risks

( $b = 0.685$ ), the leverage constraint  $\pi \leq 4$  does not bind (the maximal leverage is  $\pi(1) = 3.18$ ), and hence the value of relaxing this leverage constraint has no value for the manager.

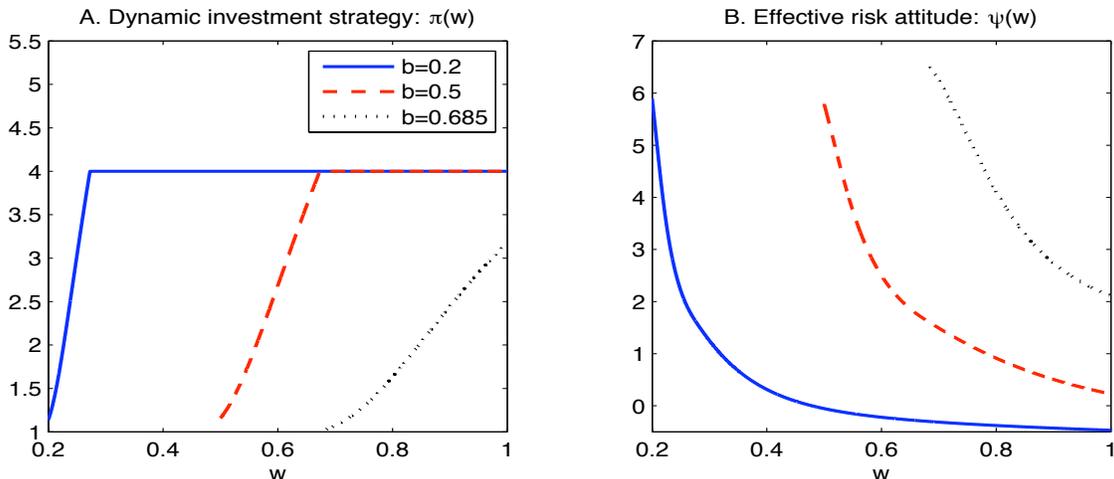


Figure 6: **Dynamic leverage  $\pi(w)$  and the manager's effective risk attitude  $\psi(w)$  for various levels of liquidation boundary  $b = 0.2, 0.5, 0.685$ .**

Second, as we decrease  $b$  from 0.685 to 0.5, with more flexibility in managing downside liquidation risk, the manager rationally increases leverage  $\pi(w)$ . The leverage constraint (4) then binds for  $w \geq 0.67$  but remains non-binding otherwise. Importantly, the manager is effectively risk averse for all  $w$  when  $b = 0.5$ ,  $\psi(w) > 0$ , as seen from Panel B of Figure 6.

Finally, the manager may be *risk seeking* when liquidation risk is low. When  $b = 0.2$ , in the region  $0.48 \leq w \leq 1$ , the leverage constraint binds,  $\pi(w) = \bar{\pi} = 4$ , as the manager is effectively risk seeking ( $\psi(w) < 0$ ). Table 2 reports  $\psi(1) = -0.4702$ . In the region  $0.27 \leq w \leq 0.48$ , the leverage constraint also binds  $\pi(w) = \bar{\pi} = 4$  but the manager behaves in a risk-averse manner,  $\psi(w) > 0$ . When  $w < 0.27$ , the manager is sufficiently risk averse and hence the optimal leverage  $\pi(w) < \bar{\pi}$  does not bind.

Importantly, we note that the manager may engage in risk seeking for high values of  $w$  but still behaves prudently for low  $w$ . This is to the opposite of the typical risk-seeking intuition in option-based models. We often hear that managers become risk seeking as the firm gets close to liquidation because the manager effectively holds an option with convex payoffs. This intuition does not apply here because debt in our model is risk-free and the manager's loss upon liquidation is quite costly in contrast to limited downside losses. Managerial survival

Table 2: **Comparative static effects of the liquidation boundary  $b$  on leverage  $\pi(1)$ , “effective” risk attitude  $\psi(1)$ , and various values.**

$b$	$\pi(1)$	$\psi(1)$	$m(1)$	$n(1)$	$f(1)$	$p(1)$	$v(1)$
0.2	4	-0.4702	0.2445	0.1235	0.3680	1.2259	1.5939
0.5	4	0.2269	0.1981	0.1091	0.3072	1.1467	1.4539
0.685	<b>3.1753</b>	<b>2.1105</b>	<b>0.1497</b>	<b>0.0480</b>	<b>0.1977</b>	<b>1</b>	<b>1.1977</b>

is the dominant consideration in our model when the fund performs poorly.

For simplicity, we have so far intentionally chosen a parsimonious baseline model. In the next three sections, we extend our model along three important dimensions: managerial ownership, new money flows, and managerial voluntary fund closure/restart options.

## 5 Managerial ownership

Hedge fund managers often have equity positions in funds that they run, which potentially mitigates managerial conflicts with investors. We next incorporate inside ownership.

Let  $\phi$  denote managerial ownership in the fund. For simplicity, we assume that  $\phi$  remains constant over time. Let  $Q(W, H)$  denote the manager’s total value including both the value of total fees  $F(W, H)$  and the manager’s *pro rata* share of investors’ value  $\phi P(W, H)$ ,

$$Q(W, H) = F(W, H) + \phi P(W, H). \quad (21)$$

The manager dynamically chooses the investment policy  $\pi$  to maximize (21). Using the homogeneity property, we write  $Q(W, H) = q(w)H$  where  $q(w)$  is the manager’s scaled total value. In the Appendix, we show that the optimal investment strategy  $\pi(w)$  is given by

$$\pi(w) = \begin{cases} \min \left\{ \frac{\alpha}{\sigma^2 \psi_q(w)}, \bar{\pi} \right\}, & \psi_q(w) > 0, \\ \bar{\pi}, & \psi_q(w) \leq 0. \end{cases} \quad (22)$$

where  $\psi_q(w)$  is the manager’s effective risk attitude defined by

$$\psi_q(w) = -\frac{wq''(w)}{q'(w)}. \quad (23)$$

Table 3: **Comparative static effects of managerial ownership  $\phi$  on leverage  $\pi(1)$ , effective risk attitude  $\psi_q(1)$ , and various values.**

$\phi$	$\pi(1)$	$\psi_q(1)$	$m(1)$	$n(1)$	$f(1)$	$p(1)$	$v(1)$
<b>0</b>	<b>3.1753</b>	<b>2.1105</b>	<b>0.1497</b>	<b>0.0480</b>	<b>0.1977</b>	<b>1</b>	<b>1.1977</b>
10%	3.2300	2.0749	0.1442	0.0528	0.1970	1.0142	1.2112
20%	3.3038	2.0282	0.1388	0.0568	0.1956	1.0242	1.2197
50%	3.5188	1.9042	0.1255	0.0646	0.1901	1.0408	1.2309

With managerial ownership  $\phi$ , the manager’s effective risk attitude  $\psi_q(w)$  and hence leverage  $\pi(w)$  naturally depend on  $q(w)$ , the sum of the value of fees  $f(w)$  and the value of the fund’s ownership  $\phi p(w)$ . The Appendix provides the ODE and boundary conditions for  $q(w)$ .

Table 3 shows the effects of managerial ownership  $\phi$  on leverage and values, and the parameter values are the same as in the baseline calibration. As we increase  $\phi$  from 0 to 20%, leverage  $\pi(1)$  increases from 3.18 to 3.30, and effective risk attitude  $\psi_q(1)$  changes from 2.11 to 2.03, as managerial ownership improves incentive alignments between the manager and investors by making the manager less concerned about liquidation risk. Managerial ownership increases the value of incentive fees  $n(1)$  but lowers the value of management fees  $m(1)$ , as higher leverage makes liquidation more likely.

## 6 New money flow

Money chases performances in hedge funds. We next incorporate this feature into the baseline model. We show that the manager benefits significantly from new money flows. We can also incorporate a richer specification for money outflow due to poor performances.<sup>21</sup>

### 6.1 Model setup

We model performance-triggered money inflows as follows. Whenever the fund’s AUM exceeds its HWM, new money flows into the fund.<sup>22</sup> Let  $dI_t$  denote the new money inflows

<sup>21</sup>Due to space considerations, we leave out this extension.

<sup>22</sup>Chevalier and Ellison (1997) empirically study flows-induced risk taking by mutual fund managers. Basak, Pavlova, and Shapiro (2007) study mutual fund managers’ risk-taking induced by an increasing and convex relationship of fund flows to relative performances.

over time increment  $(t, t + \Delta t)$ . We assume that

$$dI_t = i [dH_t - (g - \delta)H_t dt] , \quad (24)$$

where the constant parameter  $i > 0$  measures the sensitivity of  $dI_t$  with respect to the fund's profits measured by  $[dH_t - (g - \delta)H_t dt] \geq 0$ . Because the HWM  $H$  grows deterministically at the rate of  $(g - \delta)$  in the interior region  $W < H$ , the fund's AUM only exceeds its HWM and new money subsequently flows in, when  $W_t = H_t$  and  $dH_t - (g - \delta)H_t dt > 0$ . For example, suppose  $W_t = H_t = 100$  and the next year's realized AUM is  $W_{t+1} = 115$ . Then, the manager collects  $3 = 20\% \times 15$  in incentive fees. With  $i = 0.8$ , the new money inflow is  $dI_t = 12 = 0.8 \times 15$ , which is 12% of the fund's AUM  $W_t = 100$ .

New money flow increases the fund's AUM which in turn rewards the manager with more future fees. Let  $X(W, H)$  denote the present discounted amount of all future money inflows,

$$X(W, H) = \mathbb{E}_t \left[ \int_t^\tau e^{-r(s-t)} i [dH_s - (g - \delta)H_s ds] \right] , \quad (25)$$

where  $\tau$  is stochastic liquidation time. Let  $x(w) = X(W, H)/H$ .

Current and future investors may attach different values to the fund. Let  $P_1(W, H) = p_1(w)H$  and  $P_2(W, H) = p_2(w)H$  denote the PV of current investors' value and the PV of future investors' contributed capital, respectively. The sum  $p_1(w) + p_2(w)$  gives the scaled value of all investors' value,  $p(w)$ . In general, for future investors, the PV  $p_2(w)$  differs from the discounted amount  $x(w)$ . Because new money only possibly flows into the fund at  $w = 1$ , current and future investors share the same HWM. This property substantially simplifies our analysis because we only need to track a single fund-wide HWM for all investors. See the Appendix for technical details.

## 6.2 Results

Figure 7 plots leverage  $\pi(w)$  and the manager's value  $f(w)$  for the case with new money flow,  $i = 0.8$ , and then compares to the baseline case with  $i = 0$ . Panel A of Figure 7 plots leverage  $\pi(w)$ . Leverage is substantially higher with new money inflow,  $i = 0.8$ , than with  $i = 0$ . The leverage constraint (4) binds for  $w \geq 0.909$  with  $i = 0.8$ . Panel B of Figure 7 plots  $f(w)$ . Intuitively,  $f(w)$  is higher with  $i = 0.8$  than with  $i = 0$ . The impact of new money flow on  $f(w)$  is much bigger for larger values of  $w$ . For example, at  $w = 1$ ,  $f(1)$

increases significantly from 0.198 to 0.261. The new money inflow rewards the manager by increasing the AUM base upon which the manager collects future management and incentive fees. With more rewards at the upside, the manager behaves in a less risk-averse manner.

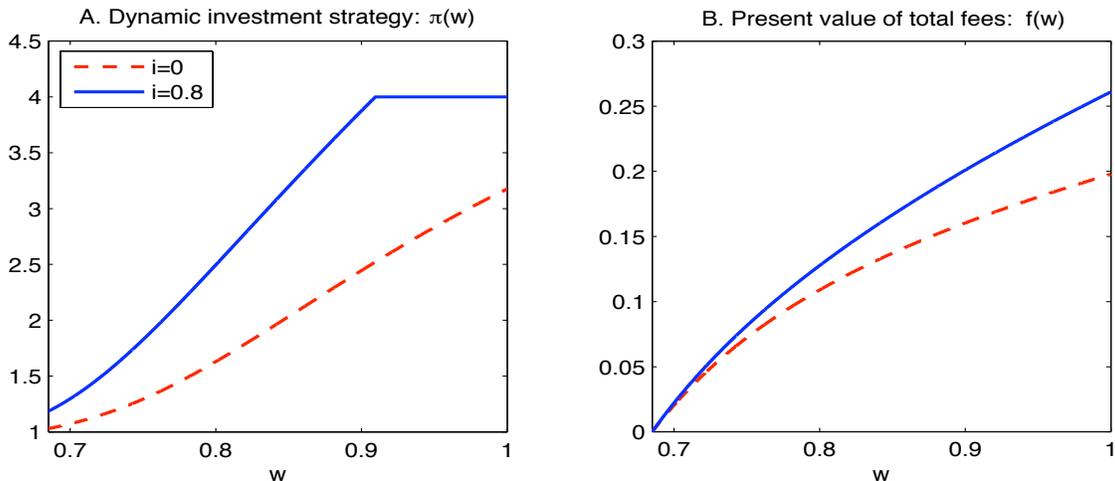


Figure 7: **Leverage policy  $\pi(w)$  and  $f(w)$  with new money flows.**

Table 4 reports the comparative static effect of new money flow  $i$ . Quantitatively, the effect of new money flow is significant. As we increase  $i$  from 0 to 1, leverage  $\pi(1)$  increases from 3.18 to 4.00 (where the leverage constraint binds), and the manager's total value  $f(1)$  increases by 46% from 0.2 to 0.29. Mostly the new money flow effect operates through the value of incentive fees  $n(1)$ , which increases by 2.3 times from 4.8% to 11.2%. New money flow rewards the manager when the fund is doing well and thus strongly influences the value of incentive fees  $n(1)$ .

What is the fund's expected discounted amount of new money flow (scaled by  $H$ ),  $x(1)$ ? For  $i = 1$ ,  $x(1) = 0.557$ , which implies that the new money flow increases the effective AUM by 56% in the PV sense. While most benefits of the new money flow accrue to the manager, investors are also better off in our comparative analysis as the manager becomes less risk averse. The current investors' value  $p_1(1)$  increases by 4% from 1 to 1.044 as the new money flow  $i$  increases from 0 to 1. Interestingly, future investors are also better off by 4% per unit of AUM. This is due to the property that all investors, current and future, in our model share the same HWM, which substantially simplifies our analysis.

Table 4: **Comparative static effects of new money flow  $\phi$  on leverage  $\pi(1)$ , effective risk attitude  $\psi(1)$ , and various values.**

$i$	$\pi(1)$	$\psi(1)$	$m(1)$	$n(1)$	$f(1)$	$p_1(1)$	$x(1)$
<b>0</b>	<b>3.1753</b>	<b>2.1105</b>	<b>0.1497</b>	<b>0.0480</b>	<b>0.1977</b>	<b>1</b>	0
0.2	3.6565	1.8325	0.1511	0.0574	0.2085	1.0107	0.0573
0.5	4.0000	1.5189	0.1556	0.0744	0.2300	1.0248	0.1859
0.8	4.0000	1.2212	0.1661	0.0948	0.2609	1.0372	0.3790
1.0	4.0000	1.0260	0.1777	0.1115	0.2892	1.0439	0.5572

## 7 Restart options

In reality, hedge fund managers often have outside options. It may sometimes be optimal for managers to voluntarily close the current fund as its AUM is sufficiently down from its HWM and incentive fees become sufficiently out of the money. A recent *Economist* magazine article titled “Hedge-fund closures: Quitting while they’re behind” provides one such topical discussion.<sup>23</sup> We next model the manager’s optimal fund closure and restart options.

### 7.1 Model setup and solution

We analyze a stationary framework with infinite restart options. In reality, the manager has finite restart options. We later provide a sensitivity analysis with respect to the number of restart options.

**A stationary model with infinite restart options.** At any moment when the current fund’s AUM is  $W$  and its HWM is  $H$ , the manager has an option to start a new fund with a new AUM, which we denote as  $S(W, H)$ . Let  $\nu$  denote the ratio between the new fund’s AUM  $S(W, H)$  and the previous fund’s closing AUM  $W$ , i.e.  $\nu = S(W, H)/W$ . To illustrate the effects of restart options, we assume that the ratio  $\nu$  satisfies

$$\nu(w) = \theta_0 + \theta_1 w + \frac{\theta_2}{2} w^2, \quad (26)$$

where  $\theta_0, \theta_1 > 0$ , and  $\theta_2 < 0$  are all constant. Intuitively, the better the fund’s performance, the larger  $\nu$ . Additionally, the impact of  $w$  on  $\nu$  diminishes as we increase  $w$ . By closing the

<sup>23</sup>See <http://www.economist.com/node/21547807>.

existing fund and starting a new one, the manager benefits by resetting the fund's HWM and hence being much closer to collecting incentive fees but forgoes the fees on the closed fund. Additionally, the new fund's AUM  $S(W, H)$  may be smaller than the closed fund's AUM  $W$ . Finally, it is costly to close the existing fund and start a new fund. The manager optimally chooses the closure/restart timing as well as leverage policies.

Let  $f_\infty(w)$  denote the manager's scaled value with infinite restart options. Let  $\bar{w}_\infty$  denote the optimal threshold for the restart option. The manager chooses  $\bar{w}_\infty$  so that

$$f_\infty(\bar{w}_\infty) = \bar{w}_\infty \nu(\bar{w}_\infty) f_\infty(1), \quad (27)$$

$$f'_\infty(\bar{w}_\infty) = (\nu(\bar{w}_\infty) + \bar{w}_\infty \nu'(\bar{w}_\infty)) f_\infty(1). \quad (28)$$

The value-matching condition (27) requires that the manager's value  $f_\infty(w)$  is continuous at the moment of abandoning the existing fund and starting a new fund. The smooth pasting condition (28) ensures that  $\bar{w}_\infty$  is optimally chosen. Finally, the ODE for  $f_\infty(w)$  is the same as (17), the one for the baseline case.

We may equivalently interpret restart options in our model as options to reset the fund's HWM. Resetting the HWM allows the manager to collect incentive fees much sooner, but also causes some investors to withdraw their capital or leave the fund. We see that both fund restart and HWM reset interpretations are consistent with our model.

## 7.2 Model results

**Parameter choice and calibration.** We now calibrate the three new parameters,  $\theta_0, \theta_1$ , and  $\theta_2$ , in (26). We target (1) the restart boundary  $W$  to be 80% of the fund's AUM  $H$ ,  $\bar{w}_\infty = 0.8$ ; (2) the subsequent fund's AUM to be 75% of the previous fund's AUM,  $\nu(\bar{w}_\infty) = 0.75$ ; and (3) the new fund's size is zero when the manager is forced to liquidate at  $w = b$ ,  $\nu(b) = 0$ . Using these three conditions, we obtain  $\theta_0 = -24.75$ ,  $\theta_1 = 61.47$ , and  $\theta_2 = -74$ . Other parameter values are the same as in the baseline calibration.

**Leverage  $\pi(w)$  and the PV of total fees  $f(w)$ .** Panel A of Figure 8 plots the optimal leverage  $\pi(w)$ . Restart options make the manager less risk averse, which causes the leverage constraint to bind,  $\pi(1) = 4.00$ . In our calibration, at the moment of starting up the new fund, the fund is 20% down from its HWM, i.e.  $\bar{w}_\infty = 0.8$ . The corresponding leverage

$\pi(0.8) = 2.20$ , which is larger than  $\pi(0.8) = 1.63$ , the leverage in the baseline case with no restart option. Intuitively, restart options cause the manager to be less risk averse and hence leverage increases. Panel B of Figure 8 plots  $f(w)$ . At the optimally chosen restart option boundary  $\bar{w}_\infty = 0.8$ ,  $f_\infty(0.8) = 0.130$ , which is about 20% higher than the manager's value  $f(0.8) = 0.109$  when the fund's AUM is 20% down from its HWM in the baseline case with no restart options. Even at  $w = 1$ , when the restart option becomes least valuable,  $f_\infty(1) = 0.217$ , which is 10% higher than  $f(1) = 0.198$  in the baseline case.

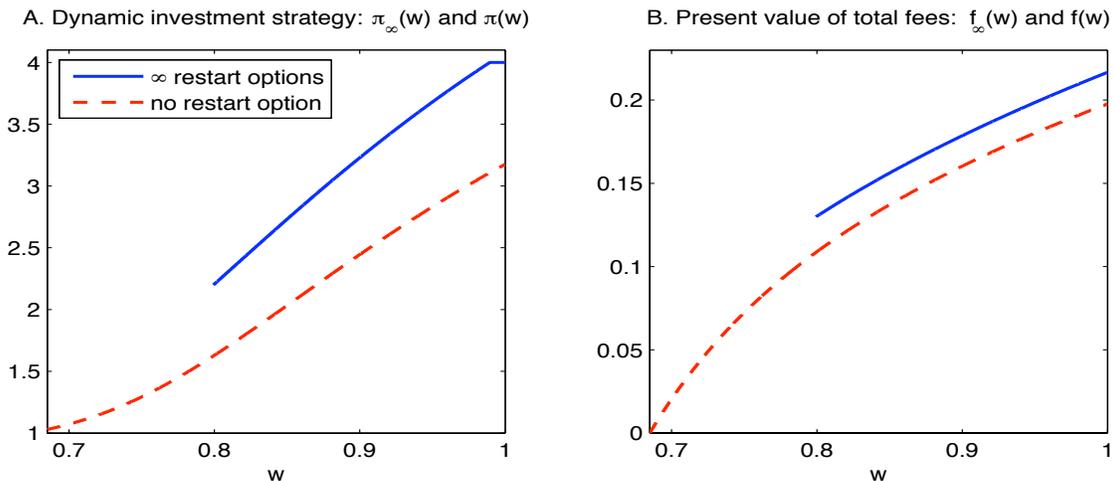


Figure 8: **Investment strategy**  $\pi(w)$  and the **PV of total fees**  $f(w)$ .

**The value of first, second, and remaining restart options.** Table 5 reports the results for the sensitivity analysis with respect to the number of restart options.<sup>24</sup> As we increase the number of restart options, the manager exits the fund sooner (higher  $\bar{w}$ ), chooses a more aggressive investment strategy (higher  $\pi(w)$ ), becomes more risk tolerant (lower  $\psi(w)$ ), values incentive fees more, but interestingly, the value of management fees  $m(1)$  decreases. Intuitively, more aggressive investment and earlier exits for each fund lower  $m(1)$ . But the loss in  $m(w)$  is outweighed by the gains in  $n(w)$ . For example, the value of incentive fees  $n(1)$  increase from 4.8% for the case with no restart option to 10.4% with infinite restart options, while  $m(1)$  decreases from 15% to 11.3%, resulting in an increase of total fees  $f(1)$  from 20% to 21.7%. Quantitatively, restart options have much stronger effects on the value of incentive

<sup>24</sup>See the Appendix for the case with one restart option. For cases with multiple restart options, we have more complicated notations, but the analysis is essentially the same and is available upon request.

Table 5: Comparative static effects of increasing the number of restart options on fund’s restart boundary  $\bar{w}$ , leverage  $\pi(1)$ , effective risk attitude  $\psi(1)$ , and various values.

Option #	$\bar{w}$	$\pi(1)$	$\psi(1)$	$m(1)$	$n(1)$	$f(1)$	$p(1)$	$v(1)$
0	<b>0.685</b>	<b>3.1753</b>	<b>2.1105</b>	<b>0.1497</b>	<b>0.0480</b>	<b>0.1977</b>	<b>1</b>	<b>1.1977</b>
1	0.788	3.6051	1.8587	0.1390	0.0675	0.2065	1.0070	1.2135
2	0.794	3.8204	1.7539	0.1308	0.0802	0.2110	1.0072	1.2182
$\infty$	<b>0.800</b>	<b>4.0000</b>	<b>1.6420</b>	<b>0.1125</b>	<b>0.1041</b>	<b>0.2166</b>	<b>1.0067</b>	<b>1.2233</b>

fees than on the value of management fees. Our calibrated exercise also suggests that the first few restart options carry the most value for the manager. If the manager restarts twice, that probably indicates the end of the manager’s career. Therefore, the manager may still deploy leverage prudently even with restart options as our analysis indicates.

## 8 Two important special cases

To highlight our model’s mechanism and to better relate to the literature, we next summarize Goetzmann, Ingersoll, and Ross (2003) and Panageas and Westerfield (2009), two most closely related papers to ours, and compare our main results with these two models.

### 8.1 Goetzmann, Ingersoll, and Ross, 2003 (GIR)

GIR value hedge fund compensation contracts when managers are paid via both management fees and HWM-indexed incentive fees. GIR derive closed-form solutions for the values of management fees, incentive fees, and investors’ value but do not model the manager’s decisions such as leverage and fund closure/restart. We summarize their results below.

**Proposition 1** *By setting  $\pi(w) = 1$  at all times and  $\beta = r$ , we obtain GIR results via the*

following explicitly solved value functions:

$$n(w) = \frac{k(w^\gamma - b^{\gamma-\zeta}w^\zeta)}{\gamma(k+1) - 1 - b^{\gamma-\zeta}(\zeta(1+k) - 1)}, \quad (29)$$

$$f(w) = \frac{c}{c + \delta + \lambda - \alpha}w + \frac{(\delta + \lambda - \alpha)k + (\zeta(1+k) - 1)cb^{1-\zeta}}{(c + \delta + \lambda - \alpha)(\gamma(k+1) - 1 - b^{\gamma-\zeta}(\zeta(1+k) - 1))}w^\gamma - \frac{b^{\gamma-\zeta}(\delta + \lambda - \alpha)k + (\gamma(1+k) - 1)cb^{1-\zeta}}{(c + \delta + \lambda - \alpha)(\gamma(k+1) - 1 - b^{\gamma-\zeta}(\zeta(1+k) - 1))}w^\zeta, \quad (30)$$

$$p(w) = \frac{\delta + \lambda}{c + \delta + \lambda - \alpha}w - \frac{(\delta + \lambda)k + (\zeta(1+k) - 1)(c - \alpha)b^{1-\zeta}}{(c + \delta + \lambda - \alpha)(\gamma(k+1) - 1 - b^{\gamma-\zeta}(\zeta(1+k) - 1))}w^\gamma + \frac{b^{\gamma-\zeta}(\delta + \lambda)k + (\gamma(1+k) - 1)(c - \alpha)b^{1-\zeta}}{(c + \delta + \lambda - \alpha)(\gamma(k+1) - 1 - b^{\gamma-\zeta}(\zeta(1+k) - 1))}w^\zeta, \quad (31)$$

where  $\gamma$  and  $\zeta$  are two constants given by

$$\gamma = \frac{1}{2} - \frac{\alpha + r - g - c}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha + r - g - c}{\sigma^2}\right)^2 + \frac{2(r + \lambda - g + \delta)}{\sigma^2}}, \quad (32)$$

and

$$\zeta = \frac{1}{2} - \frac{\alpha + r - g - c}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\alpha + r - g - c}{\sigma^2}\right)^2 + \frac{2(r + \lambda - g + \delta)}{\sigma^2}}. \quad (33)$$

Finally,  $m(w) = f(w) - n(w)$  and the total fund's value is  $v(w) = p(w) + f(w)$ .

## 8.2 Panageas and Westerfield, 2009 (PW)

Building on the valuation framework of GIR, PW derive explicit formulas for leverage and the manager's value, by making two key simplifying assumptions: (1) no liquidation boundary,  $b = 0$ , and (2) no management fees,  $c = 0$ . We next summarize the main results in PW.

**Proposition 2** *With no liquidation boundary ( $b = 0$ ), no management fees ( $c = 0$ ), a non-binding leverage constraint, and  $g = \delta = 0$ , PW derive the following optimal leverage:*

$$\pi = \frac{\alpha}{(1 - \eta)\sigma^2}. \quad (34)$$

The manager's scaled value  $f(w)$  is given by

$$f(w) = n(w) = \frac{k}{\eta(k+1) - 1}w^\eta, \quad (35)$$

where  $\eta$  is a constant and given by

$$\eta = \frac{r + \beta + \lambda + \frac{\alpha^2}{2\sigma^2} - \sqrt{\left(r + \beta + \lambda + \frac{\alpha^2}{2\sigma^2}\right)^2 - 4r(\beta + \lambda)}}{2r}. \quad (36)$$

To ensure that  $f(w)$  is increasing, concave, and finite, PW impose the following condition:

$$\eta(1 + k) > 1. \tag{37}$$

The main results of PW are as follows: (1) leverage  $\pi(w)$  is constant at all times; (2) the higher the incentive fee  $k$ , the *lower* the manager’s value  $F(W, H)$ ; (3) the higher the HWM  $H$ , the *higher* the manager’s value  $F(W, H)$ ; and (4) the value of incentive fees  $N(W, H)$  is concave in  $W$ . In PW, the risk-neutral manager is averse to collecting incentive fees too soon, because incentive fees are consumed by the manager rather than being reinvested in the fund to earn a (levered) excess return. The manager may delay the timing of collecting incentive fees by reducing leverage because of aversion to the *upside* risk caused by the incentive payout at  $w = 1$ . The condition (37), which is equation (15) in PW, is critical and ensures that the mechanism discussed above works in PW. However, PW’s predictions (2)-(3)-(4) listed above are hard to reconcile with commonly held belief in the industry.

In contrast, we find that (1) leverage  $\pi(w)$  is time varying and tends to increase following good performances; (2) the higher the incentive fee  $k$ , the *higher* the manager’s value  $F(W, H)$ ; (3) the higher the HWM, the *lower* the manager’s value  $F(W, H)$ ; and (4) the value of incentive fees is convex in  $W$ . The two models generate opposite predictions as the mechanisms are different. The manager in our model is averse to the *downside* liquidation risk, while the manager in PW is averse to collecting incentive fees too soon.<sup>25</sup> By incorporating management fees as we do, the manager has more to lose upon liquidation, which further makes the manager behave prudently.

## 9 Conclusions

We develop a dynamic model of hedge fund leverage and compensation valuation where the manager dynamically chooses leverage to maximize the PV of the sum of AUM-based management and HWM-indexed incentive fees from the current and future managed funds (with money inflow/outflow, fund closure/restart options, and investors liquidation options). The risk-neutral manager trades off the value creation by leveraging on the alpha strategy

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<sup>25</sup>In our calibration, the “upside” risk of crossing the HWM too soon (the key mechanism in PW) still exists but plays a minor role. This should be expected since our model includes theirs as a special case. However, the key tradeoff and intuition in our model are different from those in PW.

and the cost of inefficient liquidation and redemption/drawdown. The manager has incentives to preserve the fund's going-concern value so as to collect fees in the future. This survival/precautionary motive causes the manager to behave in an effectively risk-averse manner. The greater the liquidation risk/costs, the more prudently the manager behaves. In our calibrated setting, leverage increases with alpha and decreases with both volatility and the manager's endogenously determined effective risk attitude. The ratio between the fund's AUM and its HWM,  $w$ , measures the manager's moneyness in the fund. The higher the value of  $w$ , the less likely the fund is liquidated and the more likely the manager collects the incentive fees, and, consequently, the higher the manager's leverage.

Quantitatively, our calibration suggests that the manager needs to create significant value to justify their compensation contracts. Both management fees and incentive fees are quantitatively important contributors. Our baseline calibration suggests that the manager needs to create 20% surplus in PV on the AUM to justify a two-twenty contract. Out of the manager's total surplus, 75% is attributed to management fees (15 cents on a dollar) and the remaining 25% is due to incentive fees (5 cents on a dollar). By incorporating features such as managerial ownership, new money flows, and fund closure/restart options, the relative contributions of management fees and incentive fees to the manager's total value become more balanced. In private equity, Metrick and Yasuda (2010) also find that management and incentive fees are significant contributors to the manager's value.

In reality, managerial skills may be unknown and time-varying. Learning about managerial skills by investors is a topic for future research. Moreover, managers with no skills may pretend to be skilled, which further complicates investors' inference and learning about the manager's skill. To highlight the effect of managerial incentives on leverage and valuation, we have intentionally assumed that the managerial skill features constant alpha and Sharpe ratio. Empirically, it is likely that the manager's skill is time varying. Additionally, the marginal skill (alpha) decreases with fund size. This decreasing returns to scale feature of managerial skill may explain why some funds with strong performance records voluntarily close to new investors. Finally, it is important to incorporate managerial risk aversion (Hodder and Jackwerth (2007)) because managers are often poorly diversified and incomes from fund management compensation thus carry additional idiosyncratic risk premium. We leave these topics for future research.

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Table 6: **Summary of Key Variables and Parameters.** This table summarizes the symbols for the key variables used in the model and the parameter values.

Variable	Symbol	Parameters	Symbol	Value
Risk-free asset	$D$	Risk-free rate	$r$	5%
Assets under management	$W$	Un-levered alpha	$\alpha$	1.22%
Cumulative returns	$R$	Volatility	$\sigma$	4.26%
High-water mark	$H$	Growth rate of $H$	$g$	5%
Leverage	$\pi$	Manager's discount rate	$\beta$	5%
Outside liquidation risk	$J$	Investors' withdrawal	$\delta$	0
Present value of management fees	$M$	Probability of liquidation	$\lambda$	10%
Present value of incentive fees	$N$	Management fee	$c$	2%
Present value of investors' payoff	$P$	Incentive fee	$k$	20%
Present value of total fees	$F$	Lower liquidation boundary	$b$	0.685
Total value of the fund	$V$	Ownership of equity	$\phi$	0.2
Manager's total value with ownership	$Q$	New money inflow rate	$i$	0.8
Effective risk attitude	$\psi$	Restart option parameter	$\theta_0$	-24.75
Point of leverage constraint binding	$\overline{W}$	Restart option parameter	$\theta_1$	61.47
New fund's size	$S$	Restart option parameter	$\theta_2$	-74
		Leverage constraint	$\overline{\pi}$	4

# Appendix

## A Technical details

### A.1 For Section 3.

**The interior region ( $W < H$ ).** In this region, we have the following Hamilton-Jacobi-Bellman (HJB) equation,

$$(\beta + \lambda)F(W, H) = \max_{\pi \leq \bar{\pi}} cW + [\pi\alpha + (r - \delta - c)]WF_W(W, H) + \frac{1}{2}\pi^2\sigma^2W^2F_{WW}(W, H) + (g - \delta)HF_H(W, H). \quad (\text{A.1})$$

subject to the leverage constraint (4). The discount rate on the left side changes from  $\beta$  to  $(\beta + \lambda)$  to reflect the exogenous stochastic liquidation likelihood.

**The upper boundary ( $W = H$ ).** Our reasoning for the boundary behavior essentially follows GIR and PW. A positive return shock increases the AUM from  $W = H$  to  $H + \Delta H$ . The PV of the manager's total fees is then given by  $F(H + \Delta H, H)$  before the HWM adjusts. Immediately after the positive shock, the HWM adjusts to  $H + \Delta H$ . The manager collects an incentive fee  $k\Delta H$ , and thus the AUM is lowered from  $H + \Delta H$  to  $H + \Delta H - k\Delta H$ . The PV of total fees is  $F(H + \Delta H - k\Delta H, H + \Delta H)$ . Using the continuity of  $F(W, H)$  before and after the adjustment of the HWM, we have

$$F(H + \Delta H, H) = k\Delta H + F(H + \Delta H - k\Delta H, H + \Delta H). \quad (\text{A.2})$$

By taking the limit as  $\Delta H$  approaches zero and using Taylor's expansion rule, we obtain

$$kF_W(H, H) = k + F_H(H, H). \quad (\text{A.3})$$

The above is the value-matching condition for the manager on the boundary  $W = H$ .

**The lower liquidation boundary ( $W = bH$ ).** At the liquidation boundary  $W = bH$ , the manager loses all future fees in our baseline model, in that

$$F(bH, H) = 0. \quad (\text{A.4})$$

This assumption is the same as the one in GIR. Unlike GIR, the manager in our model influences the liquidation likelihood via dynamic leverage.

**Homogeneous value functions.** We conjecture that the value function  $F(W, H)$  takes the following homogeneous form in  $W$  and  $H$ :

$$F(W, H) = f(w)H, \quad (\text{A.5})$$

where  $w = W/H$ . When the manager's value is given by (A.5), we have

$$F_W(W, H) = f'(w), \quad F_{WW}(W, H) = f''(w)/H, \quad F_H(W, H) = f(w) - wf'(w). \quad (\text{A.6})$$

Substituting them into the HJB equation (A.1) and the boundary conditions (A.3)-(A.4), we obtain ODE (17) with boundary conditions (18)-(19).

**Leverage policies.** We solve the optimal leverage policy as follows.

- First, we use the following first-order condition (FOC) for leverage  $\pi$ :

$$\alpha W F_W(W, H) + \pi \sigma^2 W^2 F_{WW}(W, H) = 0. \quad (\text{A.7})$$

With the homogeneity property, we further simplify the leverage policy as,

$$\pi = \frac{\alpha F_W(W, H)}{-\sigma^2 W F_{WW}(W, H)} = \frac{\alpha}{\sigma^2 \psi(w)}, \quad (\text{A.8})$$

where  $\psi(w)$  is the manager's effective risk attitude defined in (13).

- However, the FOC is not sufficient for the optimality. We also need to check the second-order condition (SOC) for leverage, which is given by  $\sigma^2 W^2 F_{WW} < 0$ . The homogeneity property of our model simplifies the SOC as,

$$f''(w) < 0, \quad \text{which can be equivalently written as } \psi(w) > 0. \quad (\text{A.9})$$

- Third, we incorporate the leverage constraint (4) which may sometimes bind by writing the optimal leverage policy as,

$$\pi(w) = \min \left\{ \frac{\alpha}{\sigma^2 \psi(w)}, \bar{\pi} \right\}, \quad (\text{A.10})$$

in the region where the risk-neutral manager behaves in a risk-averse manner,  $\psi(w) > 0$ .

- Now, we turn to the region where the risk-neutral manager behaves in a risk-seeking manner,  $\psi(w) \leq 0$ . In this case, the FOC (A.7) no longer characterizes the manager's optimality. Indeed, the SOC is violated. The manager chooses the maximally allowed leverage  $\bar{\pi}$ , and hence the leverage constraint (4) binds.
- Combining the optimal leverage policy solutions in both the risk-averse and the risk-seeking regions. we obtain (14) for the optimal leverage policy.

**Scaled investors' value  $p(w)$  and the total fund value  $v(w)$ .** Applying the standard pricing method to  $P(W, H)$  defined in (9), we have the following ODE for  $p(w)$ ,

$$(r - g + \delta + \lambda)p(w) = (\delta + \lambda)w + [\pi(w)\alpha + r - g - c]p'(w) + \frac{1}{2}\pi(w)^2\sigma^2w^2p''(w), \quad (\text{A.11})$$

where  $\pi(w)$  is given in (14). If a positive return shock increases the AUM from  $W = H$  to  $H + \Delta H$ , we could use the same argument as the one for  $F(W, H)$  and obtain the continuity of value function before and after the adjustment of the HWM,

$$P(H + \Delta H, H) = P(H + \Delta H - k\Delta H, H + \Delta H). \quad (\text{A.12})$$

By taking the limit as  $\Delta H$  approaches zero and using Taylor's expansion rule, we have  $kP_W = P_H$ , which implies

$$p(1) = (k + 1)p'(1), \quad (\text{A.13})$$

Recall that the investors collect the AUM at the liquidation boundary ( $W = bH$ ), and hence the following lower condition is held:

$$p(b) = b. \quad (\text{A.14})$$

The scaled total PV of the fund  $v(w)$  is given by  $v(w) = f(w) + p(w)$ .

## A.2 For Section 4

Applying the standard differential equation pricing method to  $M(W, H)$  defined in (7) and  $N(W, H)$  defined in (8), we obtain the following ODEs for  $m(w)$  and  $n(w)$ , respectively,

$$(\beta - g + \delta + \lambda)m(w) = cw + [\pi(w)\alpha + r - g - c]m'(w) + \frac{1}{2}\pi(w)^2\sigma^2w^2m''(w) \quad (\text{A.15})$$

$$(\beta - g + \delta + \lambda)n(w) = [\pi(w)\alpha + r - g - c]n'(w) + \frac{1}{2}\pi(w)^2\sigma^2w^2n''(w). \quad (\text{A.16})$$

For the boundary behavior at  $W_t = H_t$ , we use the same argument as the one for  $F(W, H)$ . Consider a positive return shock increases the AUM from  $W = H$  to  $H + \Delta H$ . Using the continuity of value function before and after the adjustment of the HWM,

$$M(H + \Delta H, H) = M(H + \Delta H - k\Delta H, H + \Delta H), \quad (\text{A.17})$$

$$N(H + \Delta H, H) = k\Delta H + N(H + \Delta H - k\Delta H, H + \Delta H). \quad (\text{A.18})$$

By taking the limit as  $\Delta H$  approaches zero and using Taylor's expansion rule, we obtain

$$kM_W = M_H, \quad kN_W = k + N_H. \quad (\text{A.19})$$

And then using the homogeneity property, we obtain the following boundary conditions

$$m(1) = (k + 1)m'(1), \quad (\text{A.20})$$

$$n(1) = (k + 1)n'(1) - k. \quad (\text{A.21})$$

Notice that the manager loses both of management fees and incentive fees forever at the liquidation boundary ( $W = bH$ ), and hence the following lower conditions are held

$$m(b) = n(b) = 0. \quad (\text{A.22})$$

Panels A and B of Figure 9 plot scaled investors' value  $p(w)$  and its sensitivity  $p'(w)$  with respect to AUM. Note that  $p(w)$  is convex for  $w \leq 0.80$  and concave for  $w \geq 0.80$ . Panels C and D of Figure 9 plot the scaled total fund value  $v(w)$ , and the sensitivity  $v'(w)$  with respect to AUM. Total fund value  $V(W, H)$  is increasing and concave in AUM  $W$ .

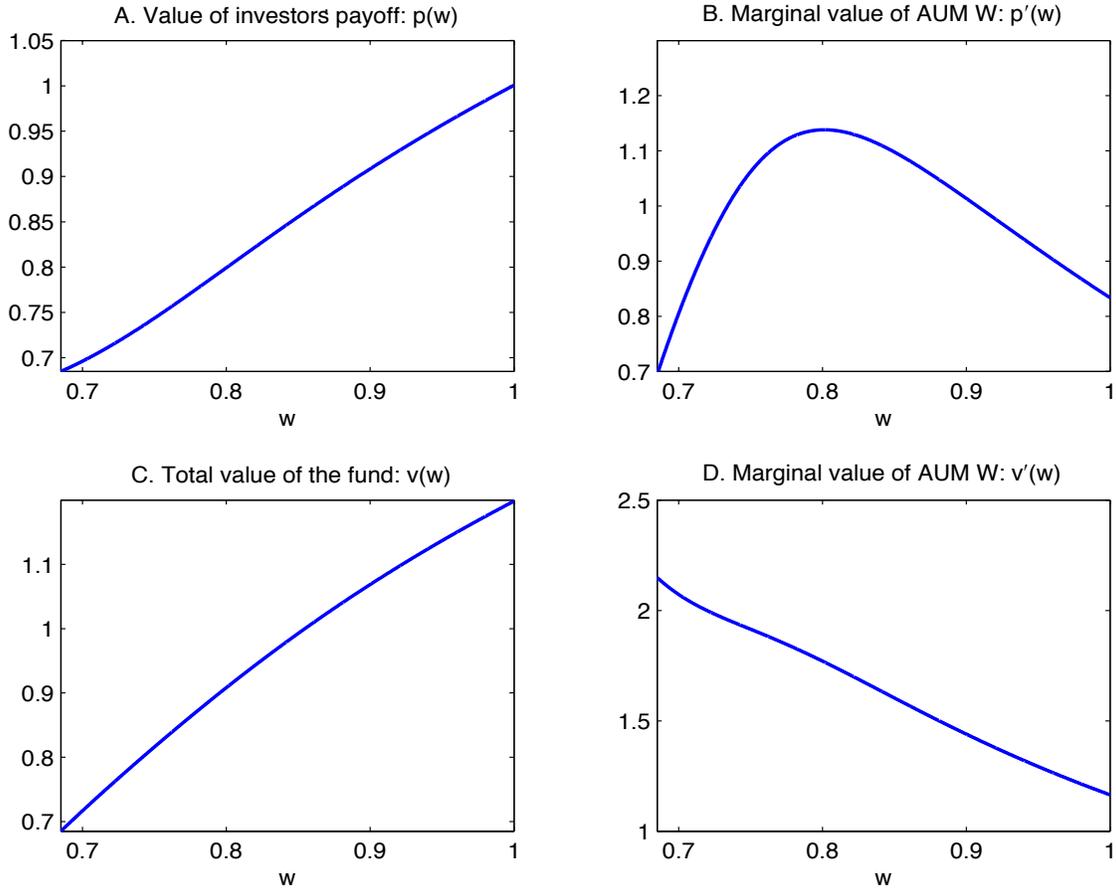


Figure 9: The scaled investors' value  $p(w)$ , the scaled total fund value  $v(w)$ , and their sensitivities,  $p'(w)$  and  $v'(w)$ , with respect to AUM  $W$ .

### A.3 For Section 5

With managerial ownership, in the region  $W < H$ , the manager's total value  $Q(W, H)$  solves

$$(\beta + \lambda)Q(W, H) = \max_{\pi \leq \bar{\pi}} [c + \phi(\delta + \lambda)]W + [\pi\alpha + (r - \delta - c)]WQ_W(W, H) + \frac{1}{2}\pi^2\sigma^2W^2Q_{WW}(W, H) + (g - \delta)HQ_H(W, H). \quad (\text{A.23})$$

When  $\pi < \bar{\pi}$ , the FOC for leverage  $\pi$  is

$$\alpha WQ_W(W, H) + \pi\sigma^2W^2Q_{WW}(W, H) = 0. \quad (\text{A.24})$$

Using the homogeneity property,  $Q(W, H) = q(w)H$ , we simplify (A.23) and obtain the following ODE for  $q(w)$ :

$$(\beta - g + \delta + \lambda)q(w) = [c + \phi(\delta + \lambda)]w + [\pi(w)\alpha + r - g - c]wq'(w) + \frac{1}{2}\pi(w)^2\sigma^2w^2q''(w). \quad (\text{A.25})$$

The optimal leverage  $\pi(w)$  is given by (22)-(23). Because  $q(w) = f(w) + \phi p(w)$ , the lower boundary condition becomes  $q(b) = \phi b$ . Using the same analysis as the one for  $F(W, H)$ , we obtain the upper boundary condition  $q(1) = (k + 1)q'(1) - k$ .

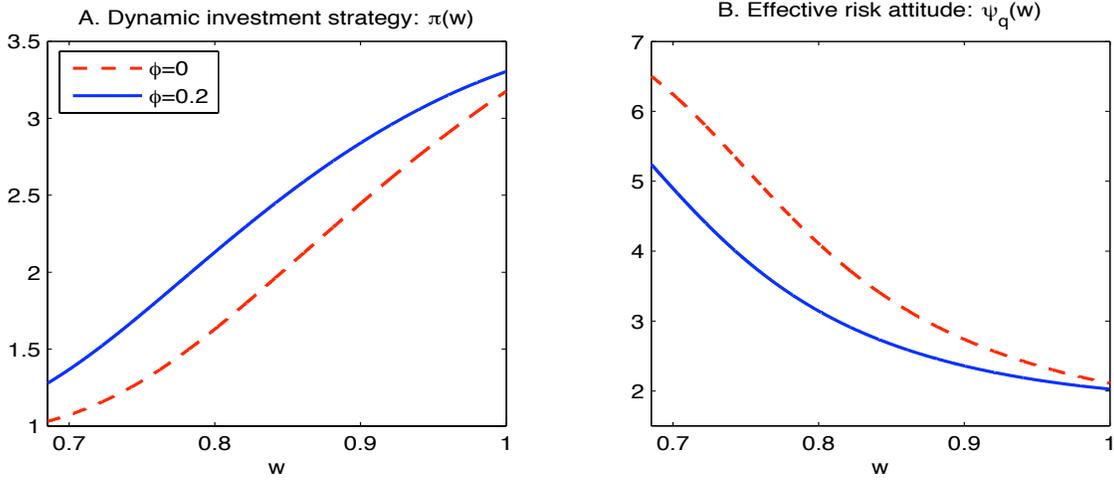


Figure 10: **Dynamic leverage strategy  $\pi(w)$  and effective risk attitude  $\psi_q(w)$  with managerial ownership ( $\phi = 0.2$ ).**

Figure 10 plots the manager's optimal leverage  $\pi(w)$  and the risk attitude measure  $\psi_q(w)$  for  $\phi = 0.2$  and  $\phi = 0$  (the baseline case). All other parameter values are the same for the two cases. Leverage  $\pi(w)$  is higher and correspondingly the effective risk attitude  $\psi_q(w)$  is lower, with managerial ownership  $\phi = 0.2$  than without ownership,  $\phi = 0$ , *ceteris paribus*. The larger the inside equity position  $\phi$ , the more the manager cares about investors' value  $p(1)$ , encouraging the manager to choose a higher leverage.

## A.4 For Section 6

We solve various value functions in four steps: (1)  $f(w)$ ,  $m(w)$ , and  $n(w)$ ; (2) the current investors' value  $p_1(w)$ ; (3) the total expected discounted amount of new capital  $x(w)$ ; and (4) the PV of future investors' contributed capital  $p_2(w)$ .

**The PVs of total fees, management, and incentive fees:  $f(w)$ ,  $m(w)$ , and  $n(w)$ .** The continuity of the manager's value before and after hitting the HWM implies  $F(H + \Delta H, H) = k\Delta H + F(H + \Delta H - k\Delta H + i\Delta H, H + \Delta H + i\Delta H)$ . By taking the limit as  $\Delta H$  approaches zero and using Taylor's expansion rule, we obtain

$$(k - i)F_W(H, H) = k + (1 + i)F_H(H, H). \quad (\text{A.26})$$

Using the homogeneity property, we simplify the boundary condition (A.26) as

$$f(1) = \frac{(k + 1)f'(1) - k}{1 + i}. \quad (\text{A.27})$$

Similarly, we may also obtain the following boundary conditions for  $m(w)$  and  $n(w)$ :

$$m(1) = \frac{(k + 1)m'(1)}{1 + i}, \quad (\text{A.28})$$

$$n(1) = \frac{(k + 1)n'(1) - k}{1 + i}. \quad (\text{A.29})$$

At the liquidation boundary  $w = b$ , the manager collects no fees, which implies

$$f(b) = m(b) = n(b) = 0. \quad (\text{A.30})$$

**Current investors' value  $p_1(w)$ .** The amount of the new money inflow upon the adjustment of the HWM is  $i\Delta H$ , which has a PV of  $i\Delta H P_1(H + \Delta H, H)$ . Before the new money inflow, the existing current investors' value (i.e. before HWM adjustment) equals the existing current investors' value after the HWM adjustment minus  $i\Delta H P_1(H + \Delta H, H)$ , the PV of the new money inflow. The continuity of the value function  $P_1$  implies

$$P_1(H + \Delta H, H) = -i\Delta H P_1(H + \Delta H, H) + P_1(H + \Delta H(1 - k + i), H + \Delta H + i\Delta H). \quad (\text{A.31})$$

By taking the limit as  $\Delta H$  approaches zero and using Taylor's expansion rule, we obtain

$$(k - i)\frac{\partial P_1(H, H)}{\partial W} = -iP_1(H, H) + (1 + i)\frac{\partial P_1(H, H)}{\partial H}. \quad (\text{A.32})$$

Simplifying the above condition yields

$$p_1(1) = (k + 1)p_1'(1). \quad (\text{A.33})$$

Using the standard pricing method, the current investors' value  $p_1(w)$  solves

$$(r - g + \delta + \lambda)p_1(w) = (\delta + \lambda)w + [\pi(w)\alpha + r - g - c]wp_1'(w) + \frac{1}{2}\pi(w)^2\sigma^2w^2p_1''(w), \quad (\text{A.34})$$

with the boundary conditions (A.33) and

$$p_1(b) = b. \quad (\text{A.35})$$

The current investors' voluntary participation condition is

$$p_1(1) \geq 1. \quad (\text{A.36})$$

**The total expected discounted amount of new money flow  $x(w)$ .** At the moment when new money flows in,  $X(W, H)$  satisfies the value matching condition,

$$X(H + \Delta H, H) = i\Delta H + X(H + \Delta H - k\Delta H + i\Delta H, H + \Delta H + i\Delta H). \quad (\text{A.37})$$

Intuitively, the first term on the right side gives the instant new money flow and the second term gives the total expected discounted amount of future new money inflows. By taking the limit  $\Delta H \rightarrow 0$  and using Taylor's expansion rule, we obtain

$$(k - i)X_W(H, H) = i + (1 + i)X_H(H, H). \quad (\text{A.38})$$

Using the homogeneity property, we simplify (A.38) as follows,

$$x(1) = \frac{(k + 1)x'(1) - i}{1 + i}. \quad (\text{A.39})$$

The homogeneity property also implies that  $x(w)$  satisfies the following ODE,

$$(r - g + \delta + \lambda)x(w) = [\pi(w)\alpha + r - g - c]wx'(w) + \frac{1}{2}\pi(w)^2\sigma^2w^2x''(w). \quad (\text{A.40})$$

From ODE (A.40), there is no money inflow when  $w < 1$ . In sum,  $x(w)$  solves (A.40) subject to boundary condition (A.39) and  $x(b) = 0$ , the condition at the liquidation boundary  $b$ .

**The PV of future investors' contributed capital  $p_2(w)$ .** Following the same argument as the one for our analysis of  $p_1(w)$ , the continuity of value function  $P_2$  implies

$$P_2(H + \Delta H, H) = i\Delta HP_1(H + \Delta H, H) + P_2(H + \Delta H - k\Delta H + i\Delta H, H + \Delta H + i\Delta H). \quad (\text{A.41})$$

By taking the limit as  $\Delta H$  approaches zero and using Taylor's expansion rule, we obtain

$$(k - i)\frac{\partial P_2(H, H)}{\partial W} = iP_1(H, H) + (1 + i)\frac{\partial P_2(H, H)}{\partial H}. \quad (\text{A.42})$$

Simplifying the above condition yields

$$p_2(1) = \frac{(k+1)p_2'(1) - ip_1(1)}{1+i}. \quad (\text{A.43})$$

The future investors' scaled value  $p_2(w)$  satisfies the following ODE

$$(r-g+\delta+\lambda)p_2(w) = [\pi(w)\alpha + r-g-c]wp_2'(w) + \frac{1}{2}\pi(w)^2\sigma^2w^2p_2''(w), \quad (\text{A.44})$$

with the boundary conditions (A.43) and

$$p_2(b) = 0. \quad (\text{A.45})$$

**The fund's total net surplus  $z(w)$ .** The scaled fund's value  $v(w)$  equals the sum of all investors' value  $p(w)$  and the manager's value  $f(w)$ . Summing the existing capital  $w$  and discounted amount of future money inflows  $x(w)$  gives the total capital,  $w+x(w)$ . Let  $z(w)$  denote the scaled total net surplus, which equals the difference between  $v(w)$  and  $w+x(w)$ ,

$$z(w) = v(w) - (w+x(w)) = p_1(w) + p_2(w) + f(w) - (w+x(w)). \quad (\text{A.46})$$

## A.5 For Section 7

Consider the case where the manager has one restart option. Let  $W^1$  and  $H^1$  denote the first fund's AUM and its HWM at the moment of closure. And the manager has an option to start up a new fund whose size is denoted as  $W^2$ , equals  $S(W^1, H^1) = \nu(w^1)W^1$ , where  $w^1 = W^1/H^1$  and the function  $\nu(w)$  is given in (26).

At the moment of closing the current fund and starting a new one, value is continuous,

$$F^1(W^1, H^1) = F^2(\nu(w)W^1, \nu(w)W^1). \quad (\text{A.47})$$

Note that the HWM is re-set when the manager restarts the fund as seen from the right side of (A.47). Let  $\bar{w}_1$  be the optimal boundary to restart the new fund. We thus have

$$f_1(\bar{w}_1) = \bar{w}_1\nu(\bar{w}_1)f_2(1), \quad (\text{A.48})$$

$$f_1'(\bar{w}_1) = (\nu(\bar{w}_1) + \bar{w}_1\nu'(\bar{w}_1))f_2(1), \quad (\text{A.49})$$

where (A.48) is the value matching condition and the smooth-pasting condition (A.49) describes the manager's optimal exercising of the exit/restart option.