

Money and Banking in Search Equilibrium*

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January 4, 2005

Abstract

We develop a new theory of money and banking based on the old story in which goldsmiths start accepting deposits for safe keeping, then their liabilities begin circulating as media of exchange, and they begin making loans. We first discuss the history. We then present a model where agents can open bank accounts and write checks. The equilibrium means of payment may be cash, checks, or both. Sometimes multiple equilibria exist. Introducing banks increases the set of parameters for which money is valued – thus, money and banking are complements. We also derive a microfounded version of the usual money multiplier.

*We thank Ken Burdett, V.V. Chari, Joe Haubrich, David Laidler, Ben Lester, Larry Neal, Ed Nosal, Steve Quinn, Peter Rupert, Irina Telyukova, Francois Velde and Warren Weber for their input. We also thank seminar participants at the Federal Reserve Banks of Atlanta and Cleveland, and the Universities of Western Ontario, Quebec at Montreal, Toronto, Paris 2, Pennsylvania, Maryland and Michigan. The National Science Foundation, the Central Bank Institutue at the Federal Reserve Bank of Cleveland, and ERMES at Paris 2 provided research support. Huang acknowledges the financial support of the Hong Kong RGC Competitive Earmarked Research Grant (CityU 1357/04H). The usual disclaimer applies.

The theory of banking relates primarily to the operation of commercial banking. More especially it is chiefly concerned with the activities of banks as holders of deposit accounts against which cheques are drawn for the payment of goods and services. In Anglo-Saxon countries, and in other countries where economic life is highly developed, these cheques constitute the major part of circulating medium. *Encyclopedia Britannica* (1954, vol.3, p.49).

1 Introduction

This paper develops a new theory of banking based on an old story, and uses the model to derive several substantive predictions about the interaction between money and banks (as summarized at the close of this Introduction). The story is so well known that it is described nicely in standard reference books like *Encyclopedia Britannica*: “the direct ancestors of modern banks were, however, neither the merchants nor the scrivenors but the goldsmiths. At first the goldsmiths accepted deposits merely for safe keeping; but early in the 17th century their deposit receipts were circulating in place of money and so became the first English bank notes.” (*EB* 1954, vol. 3, p. 41). “The cheque came in at an early date, the first known to the Institute of Bankers being drawn in 1670, or so.” (*EB* 1941, vol. 3, p. 68).¹

In case one doubts the authority of general reference books on such matters,²

¹To go into more detail: “To secure safety, owners of money began to deposit it with the London goldsmiths. Against these sums the depositor would receive a note, which originally was nothing more than a receipt, and entitled the depositor to withdraw his cash on presentation. Two developments quickly followed, which were the foundation of ‘issue’ and ‘deposit’ banking, respectively. Firstly, these notes became payable to bearer, and so were transformed from a receipt to a bank-note. Secondly, inasmuch as the cash in question was deposited for a fixed period, the goldsmith rapidly found that it was safe to make loans out of his cash resources, provided such loans were repaid within the fixed period.

“The first result was that in place of charging a fee for their services in guarding their client’s gold, they were able to allow him interest. Secondly, business grew to such a pitch that it soon became clear that a goldsmith could always have a certain proportion of his cash out on loan, regardless of the dates at which his notes fell due. It equally became safe for him to make his notes payable at any time, for so long as his credit remained good, he could calculate on the law of averages the exact amount of gold he needed to retain to meet the daily claims of his note-holders and depositors.” (*EB* 1941, vol. 3, p. 68).

²Note however that contributors to these entries include some of the more recognized banking scholars of the day, such as Ralph George Hawtrey, Oliver M. W. Sprague, Charles

it is good to know that more specialized sources echo this view. As Quinn (1997, p. 411-12) puts it, “By the restoration of Charles II in 1660, London’s goldsmiths had emerged as a network of bankers. ... Some were little more than pawn-brokers while others were full service bankers. The story of their system, however, builds on the financial services goldsmiths offered as fractional reserve, note-issuing bankers. In the 17th century, notes, orders, and bills (collectively called demandable debt) acted as media of exchange that spared the costs of moving, protecting and assaying specie.” Similarly, Joslin (1954, p.168) writes “the crucial innovations in English banking history seem to have been mainly the work of the goldsmith bankers in the middle decades of the seventeenth century. They accepted deposits both on current and time accounts from merchants and landowners; they made loans and discounted bills; above all they learnt to issue promissory notes and made their deposits transferrable by ‘drawn note’ or cheque; so credit might be created either by note issue or by the creation of deposits, against which only a proportionate cash reserve was held.”

This is not to suggest that there were no financial institutions or intermediaries of interest around other than or prior to goldsmith banking.³ Particularly well known are the Italian bankers: “To avoid coin for local payments, Renaissance moneychangers had earlier developed deposit banking in Italy, so two merchants could go to a banker and transfer funds from one account to another.” (Quinn 2002). However, they did not seem to provide anything like circulating debt, and transferring funds from one account to another “generally required the presence at the bank of both payer and payee” (Kohn 1999b). Moreover, “In order that bank credit may be used as a means of payment, it is clearly quite

R. Whittlesey, and Edward Victor Morgan.

³Neal (1994) discusses some that were around along with the goldsmith bankers in England, including the scrivenors, merchant banks, country banks, etc. Kohn (1999a,b,c) and Davies (2002) provide discussions of various other institutions in different places and times.

essential that some convenient procedure should be instituted for assigning a banker's debt from one creditor to another. In the infancy of deposit banking in mediaeval Venice, when a depositor wanted to transfer a sum to someone else, both had to attend the bank in person. In modern times the legal doctrine of negotiable instruments has been developed ... The document may take either of two forms: (1) a cheque, or the creditor's order to the bank to pay; (2) a note or the banker's promise to pay." (*EB* 1941, vol. 3, p. 44).

Going back further, Weatherford (1977) suggests that the first group deserving to be called bankers were the Templars, a religious order of knights that achieved great prominence and wealth during the Crusades. Because they were fierce fighters, they specialized in moving money around safely. After this they began providing other financial services, including loans. According to Sanello (2003), "The Templars also became an indispensable part of the 'business' of travel to and from the Holy Land. Their innovative branch banking was not reserved for crowned heads. A pilgrim or crusader was able to deposit money at a Templar monestary/bank in a European city, then redeem these medieval traveler's checks upon arrival in the Holy Land. The Templars became Europe's primary moneychangers. Indeed, the check was invented by the order." However, it is not clear if the Templars issued any liabilities that were used as a means of payment (as opposed to merely moving wealth around safely).⁴

To summarize, we take it from the literature that even though there were other relevant institutions around prior to the London goldsmiths, it is not unreasonable to say that modern banking started with their accepting deposits for safe keeping, issuing debt that circulated, and making loans. In any case, whoever one wants to say was first, it seems clear that safety was one, if not

⁴Sanello (2003) discusses their prowess in battle, their financial success, and their ultimate demise in detail.

the, critical driving force behind the story of early banking. But a story – even a good one – is not a theory. The goal of this paper is to build a *model* that can be used to study banks as institutions whose liabilities may compete with money as a means of payment. For this task one wants a framework where there is a role for media of exchange in the first place, and where the objects that play this role are endogenous. This is provided by the search-theoretic approach to monetary economics. This approach endeavors to make explicit the frictions necessary for a medium of exchange to be essential, and can be used to determine endogenously which objects circulate as a medium of exchange.

Most existing search models, however, accomplish these things in environments with severe assumptions about the way agents interact. Following Kiyotaki and Wright (1989), typically agents trade exclusively in highly decentralized markets characterized by random, anonymous, bilateral matching. This seems to leave little possibility of introducing banks in a sensible way, although there have been a few interesting attempts.⁵ The recent model developed in Lagos and Wright (2002) deviates from previous analyses by assuming that agents interact periodically in both decentralized markets and centralized markets. This was initially used mainly to simplify outcomes in the decentralized market, as under certain assumptions all agents of a given type choose the same money balances to take out of the centralized and into the decentralized markets. But once the centralized markets are up and running it is easy to introduce labor, capital and other markets into this search model in a natural way. We

⁵Previous work that incorporates some notion of banks into search models includes Cavalcanti and Wallace (1999a,b), Cavalcanti, Erosa and Temzildes (1999), Williamson (1999), and Berentsen, Camera and Waller (2004). We will not attempt to review the vast literature more generally, except to mention a few papers that incorporate banks into models with some microfoundation for money. Analyses in overlapping generations models include Williamson (1987), Champ Smith and Williamson (1996), Schreft and Smith (1998), and Bullard and Smith (2003). Aiyagari and Williamson (2000) and Andolfatto and Nosal (2003) provide different approaches. For an extensive survey of more conventional banking research, most of which is surprisingly far from the microfoundations of money, see Gorton and Winton (2002).

will introduce banks.

The fact that agents sometimes interact in highly decentralized markets in the model makes some medium of exchange essential, and we will determine endogenously whether this ends up being cash, bank liabilities, or both. The fact that agents sometimes interact in centralized markets allows us to think about a competitive banking industry where agents make deposits against which they can make payments, take out loans, etc. The reason agents may want to use bank deposits instead of cash as a means of payment in the model is exactly the reason they did in the historical record: *safe keeping*. That is, in the model cash will be subject to theft while assets deposited in banks' vaults will not. There were other problems with money that contributed to the development of deposit banks and bills of exchange to reduce the need for cash – among other things, coins were in short supply, were hard to transport, got clipped or worn, and were not all that easy to recognize or evaluate – but the modeling the problem as one of safety is natural for our purposes, and leads to some interesting results.

Of course, formalizing banks as providing a means of payment that is relatively safe – as opposed to, say, relatively easy to transport or to recognize – means we need to take some care in the way we interpret things. The most obvious thing to say is that bank liabilities here are like modern checking accounts, or maybe even better, travellers' checks, which are nearly as widely accepted as cash and safer for at least two reasons: they are less valuable to thieves because they cannot be passed without a matching signature; and even if they are lost or stolen you can get your money back at essentially no cost. This safety aspect of our assets is consistent with the historical view of services provided by deposit banks, or more generally by bills of exchange which were not payable to

the bearer.⁶ It is not so clear that our assets are much like bank notes, which presumably were about as easy to steal as money, and payable to the bearer. So we will refer to them as demand deposits or checking accounts in what follows.⁷

Modeling cash and checks as alternative means of payment is not only relevant in term of economic history. Although checks may be less important today than in the past, they remain the most common means of payment in the U.S. “There has been a 20% drop in personal and commercial check-writing since the mid 1990s, as credit cards, check cards, debit cards, and online banking services have reduced the need to pay with written checks. [But] Checks are still king. The latest annual figures from the Federal Reserve show 30 billion electronic transactions and 40 billion checks processed in the United States.” Also, “While credit cards reduce the number of checks that need to be written in retail stores, the credit-card balance still has to be paid every month. And, at least for now, that is usually done by mail – with a paper check.” (*Philadelphia Inquirer*, Feb. 14, 2003, p. A1).⁸ So checks are still hugely important in the

⁶Kahn, McAndrews and Robards (2004) take an alternative view, that cash is a safer means of payment than options like checks or credit cards, because it preserves privacy and hence avoids issues like fraud or identity theft. While there is merit to this view, no one can seriously doubt that it is typically safer to carry around your checkbook than a big bundle of cash.

⁷It may be useful to define some terms. A bill of exchange is an order in writing, signed by the person giving it (the drawer), requiring the person to whom it is addressed (the drawee) to pay on demand or at some fixed time a given sum of money either to a named person (the payee) or to the bearer. A cheque is a particular form of bill, where a bank is the drawee and it must be payable on demand. Quinn (2002) also suggests that bills were “similar to a modern traveller’s check.” It is clear that safety was and is a key feature of checks. For one thing, the payee needs to endorse the check, so no one else can cash it without committing forgery; other features include the option to “stop” a check or “cross” it (make it payable only on presentation by a banker). Indeed, the word “check” or “cheque” originally signified the counterfoil or indent of an exchequer bill, on which was registered details of the principal part in order to reduce the risk of alteration or forgery. The check or counterfoil parts remained in the hands of the banker, the portion given to the customer being termed a “drawn note” or “draft.”

⁸Here are some more details concerning payments. According to the 2003 Census Statistical Abstract, in the year 2002, of all consumer transactions, 41.3% were paid with cash, 24.5% with checks (including traveller’s and official checks, but mostly personal checks), 11% with debit cards, and 17.6% with credit cards. This is for the numbers of transactions. For the value of transactions, 19.5% were paid with cash, 39% with checks, 8.4% with debit cards and 24% with credit cards.

payments system, and developing models where cash and checks coexist seems still pretty relevant. Moreover, the type of model presented here may shed light on the use of alternative media of exchange more generally, including modern innovations like debit cards, electronic money and so on.

The rest of the paper and some of our main results can be summarized as follows. The analysis is organized around a sequence of models, starting extremely simple and then adding additional components. Section 2 presents the basic assumptions on the environment. Section 3 analyzes the simplest case, where theft is exogenous, money and goods are indivisible, and we have 100% reserve requirements. With no banking, this model is a simple extension of the textbook search-based model of monetary exchange. Once banks are added, we show there can exist equilibria where all agents, no agents, or some agents use checks, and sometimes these equilibria coexist. An interesting result is that sometimes monetary equilibria cannot exist without banks, but can exist with banks; hence, money and banking are complementary. We also show how the equilibrium set changes as parameters like the cost of banking or the supply of outside money change.

Section 4 endogenizes theft. Without banks, this provides a slightly more interesting extension of the textbook search model. When we introduce banks we again expand the set of parameters for which monetary equilibria exist, so again money and banking are complementary, but now there are interesting general equilibrium effects. In particular, as in the model with exogenous theft, the safety provided by banks makes money more valuable, but also, when more people put their money in the bank the number of thieves can fall and this can make money even more valuable. A key result in this model is that checks can never completely drive out money when theft is endogenous. The reason is

that if no one carries cash there are no thieves, but then you may as well use cash. Hence, concurrent circulation of money and bank liabilities is a natural outcome.

Section 5 generalizes the model to allow divisible goods and shows the basic results are robust.⁹ Section 6 relaxes the 100% reserve requirement by allowing banks to lend a certain fraction of their deposits. The loan rate is determined by supply and demand in the centralized market. One feature of this model is that the fee charged for checking service will be less than the resource cost of managing the account, since banks profit from making loans, and as with the goldsmiths they may end up charging a negative fee (i.e. paying interest on demand deposits). In this case, checks may possibly drive cash out of circulation. This version of the model also generates a simple money multiplier, as in the undergraduate textbooks, although here the role of both money and banks are explicitly modeled from microeconomic principles. Section 7 concludes.

2 Basic Assumptions

The economy is populated by a $[0, 1]$ continuum of infinitely-lived agents. Time is discrete, and as in Lagos and Wright (2002) we assume each period is divided into two subperiods, say day and night. During the day agents will interact in a centralized (frictionless) market, while at night they will interact in a highly decentralized market characterized by random, anonymous, bilateral matching. Trade is difficult in the decentralized markets because of a standard double coincidence problem: there are many specialized goods traded in this market, and only a fraction $x \in (0, 1)$ of the population can produce a specialized good

⁹Although we allow divisible goods, we retain the assumption of indivisible money and a unit upper bound on asset holdings throughout the paper. It is not difficult to relax this, given the periodic meetings of the centralized and decentralized markets, and we plan to pursue this idea, but we wanted to explore the indivisible asset models first.

that you want. A meeting where someone can produce what you want is called a single coincidence meeting; for simplicity, there are no double coincidence meetings, so we can ignore pure barter (this is easily relaxed). As they are nonstorable, these goods are produced for immediate trade and consumption. Goods are also indivisible for now, but this is relaxed below. Consuming a specialized good that you want conveys utility u . Producing a specialized good for someone else conveys disutility $c < u$. The rate of time preference is $r > 0$.

Due to the frictions in the decentralized market, trade would shut down if not for a medium of exchange. In particular, since agents are anonymous there can be no credit, or at least not without banks. A fraction $M \in [0, 1]$ of the population are each initially endowed with one unit of money – an object that is consumed or produced by no one, but may have potential use as a means of payment. For simplicity, if not historical accuracy, one can think of this money as fiat, although it would be easy to redo things in terms of commodity money (say, as in Velde, Weber and Wright 1999). Here we follow the early literature in this area and assume money is indivisible and agents can store at most one unit at a time. A key feature of the model is that this money is unsafe – it can be stolen. We assume for simplicity that goods cannot be stolen. Because individuals can store at most one unit of money, only individuals without money steal.

In the decentralized market, an agent who meets someone with cash attempts to steal it with probability λ , which is endogenous in some versions of the model and exogenous in others.¹⁰ Given that he tries to steal, with probability γ he succeeds. Theft has a cost $z < u$. Note that theft may be more or less costly than honest trade, depending on $z - c$, and also may be more or less likely to succeed,

¹⁰The role of crime here is related to the role of private information in Williamson and Wright (1994), since stealing is similar to selling low quality merchandise.

depending on $\gamma - x$. Agents in the day subperiod can deposit their money into a bank account, on which they can write checks. Checks are assumed to be relatively safe (harder to steal than cash). Also, all agents believe a bank can be counted on to honor any check signed by the depositor.¹¹ This means that everyone is as willing to accept checks in the decentralized market as cash, since the former can be turned into the latter in the next day's centralized market.

We keep the centralized market as simple as possible here. First, we assume that different goods are produced during the day and night: during the day agents cannot produce the specialized goods traded at night, but rather some general good. Consuming Q units of this general good conveys utility Q and producing Q units conveys disutility $-Q$. The assumption of linear utility can be relaxed, but eases the presentation slightly since it implies agents would never trade general goods for their own sake; their only role will be to settle interest or fees with banks. General goods are perfectly divisible, but they are nonstorable so they cannot be used to trade for special goods in the decentralized market. Claims to future general goods also cannot be used in the decentralized market, unless these are claims drawn on a bank (i.e. personal IOU's cannot be used because agents are anonymous).

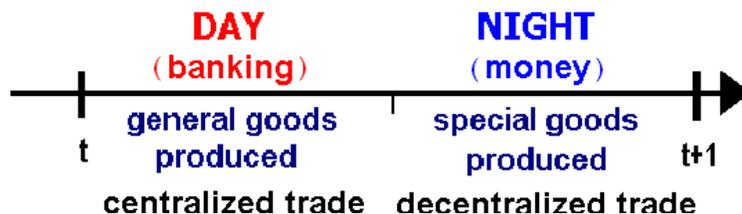


Figure 1: Timing

The timing is shown in Figure 1. Agents do their banking (e.g. making

¹¹This belief is assumed exogenously here, say because there is a legal enforcement mechanism at work, but it should not be hard to use reputation to get this endogeneously.

deposits or withdrawals and cashing checks) during the day, and then go out at night to the decentralized market. They can use either cash or checks to buy specialized goods at night, but carrying cash is risky. Checks are safe but you must pay a fee ϕ for checking services; if $\phi < 0$ you earn interest on your checking account. Banks have a resource cost $a > 0$, in terms of general goods, per unit of money deposited. Also, they are required (legally) to keep a fraction α of their deposits on reserve, while the rest can be loaned out. Loans may be demanded by some of the $1 - M$ agents who begin the day without purchasing power. We assume competitive banking, and the cost of a loan ρ will equate supply and demand. If $\alpha = 1$, e.g., then banks must keep all cash deposits in the vault, in which case competition implies $\phi = a$.

3 Exogenous Theft

In this section we study the model where stealing is exogenous: if an agent with money meets one without, the latter will try to rob the former with some fixed probability λ . With probability γ he succeeds, while with probability $1 - \gamma$ he fails and walks away empty handed. We first study the case where the only asset is money and then we introduce banking. We also assume 100% reserve requirements for now, $\alpha = 1$, but we will relax this in Section 6 below, when we allow banks to make loans.

3.1 Money

Throughout the paper we use V_1 to represent the value function of an agent with 1 unit of money, called a buyer, and V_0 the value function of an agent with no money, called a seller. When there are no banks, Figure 2 shows the event tree for a buyer in the night market. With probability M he meets another buyer, and he leaves without trading; with probability $1 - M$ he meets a seller,

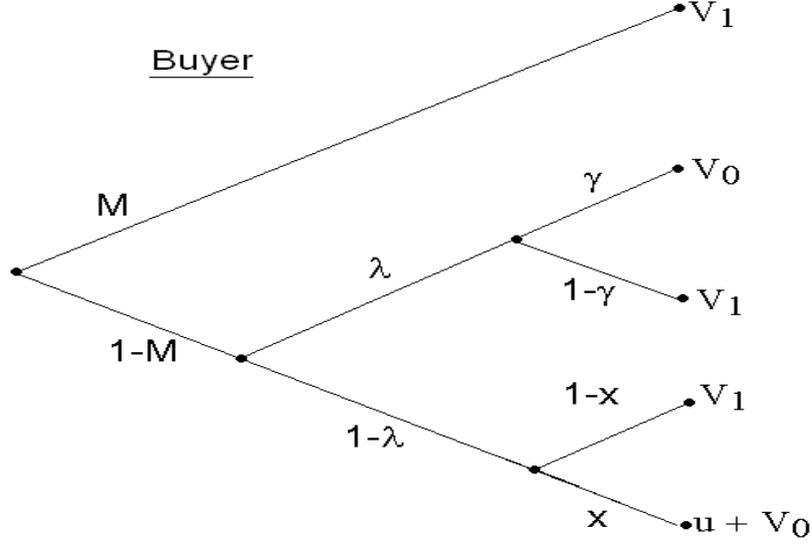


Figure 2: Event trees for a buyer

and in this case with probability λ the seller tries to rob him and succeeds with probability γ , while with probability $1 - \lambda$ he tries to trade and succeeds with probability x . There is a similar event tree for a seller. The flow Bellman equations corresponding to these trees are

$$\begin{aligned}
 rV_1 &= (1 - M)(1 - \lambda)x(u + V_0 - V_1) + (1 - M)\lambda\gamma(V_0 - V_1) \\
 rV_0 &= M(1 - \lambda)x(V_1 - V_0 - c) + M\lambda\gamma(V_1 - V_0 - z).
 \end{aligned}$$

We are interested in monetary equilibria (obviously there always exists a nonmonetary equilibrium as well). The incentive condition for agents to produce in order to acquire money is $V_1 - V_0 - c \geq 0$. Note that in this section we do not impose a symmetric condition for stealing, $V_1 - V_0 - z \geq 0$ since, as we said, theft is exogenous for now. However, we do impose participation constraints $V_0 \geq 0$ and $V_1 \geq 0$, since agents are free to not go out at night. It is clear from the incentive condition $V_1 - V_0 - c \geq 0$ that the binding participation constraint

is $V_0 \geq 0$. Hence, a monetary equilibrium here simply requires that that $V_0 \geq 0$ and $V_1 - V_0 - c \geq 0$ both hold.

In order to describe the regions of parameter space where these conditions are satisfied, and hence where a monetary equilibrium exists, define

$$C_M = \frac{(1-M)(1-\lambda)xu + M\lambda\gamma z}{r + (1-M)(1-\lambda)x + \lambda\gamma}$$

$$C_A = \frac{(1-M)[\lambda\gamma + (1-\lambda)x]u}{r + (1-M)[\lambda\gamma + (1-\lambda)x]} - \frac{\lambda\gamma z}{(1-\lambda)x}.$$

Figure 3 depicts C_M and C_A in (x, c) space using properties in the following easily verified Lemma.

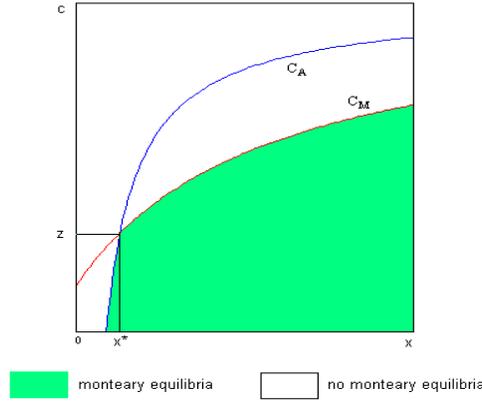


Figure 3: Existence region for monetary equilibrium

Lemma 1 (a) $x = 0 \Rightarrow C_M = \frac{M\lambda\gamma z}{r + \lambda\gamma}$, $C_A = -\infty$. (b) $C'_M > 0$, $C'_A > 0$. (c) $C_M = C_A$ iff $(x, c) = (x^*, z)$, where $x^* = \frac{[r + (1-M)\lambda\gamma]z}{(1-M)(1-\lambda)(u-z)}$.

We can now verify the following.

Proposition 1 *Monetary equilibrium exists iff $c \leq \min\{C_M, C_A\}$.*

Proof: Subtracting the Bellman equations and rearranging implies

$$V_1 - V_0 = \frac{(1-\lambda)x[(1-M)u + Mc] + \lambda\gamma Mz}{r + (1-\lambda)x + \lambda\gamma}.$$

Algebra implies $V_1 - V_0 - c \geq 0$ iff $c \leq C_M$ and $V_0 \geq 0$ iff $c \leq C_A$. ■

Naturally, monetary equilibrium is more likely to exist when c is lower or x bigger.¹² Also notice that either of the two constraints $c \leq C_M$ and $c \leq C_A$ may bind (neither is redundant). Average utility

$$W = MV_1 + (1 - M)V_0 = \frac{M(1 - M)}{r} [(1 - \lambda)x(u - c) - \lambda\gamma z],$$

is decreasing in λ and γ ; this is due to the resource cost z and the opportunity cost of thieves not producing, as stealing per se is a transfer and not inefficient. In any case, when $\lambda = 0$, so that $C_M = C_A = \frac{(1-M)xu}{r+(1-M)x}$, things reduce to the most basic model of monetary exchange (e.g. Kiyotaki and Wright 1993). Allowing money to be subject to theft provides a simple but not unreasonable extension of this framework. We next show how it can be used to think about banking in the context of the economic history discussed above.

3.2 Banking

We now allow agents with money to deposit it in checking accounts. Since $\alpha = 1$ in this section, banks like the early goldsmiths simply keep the money in the vault and earn revenue by charging a fee ϕ for this service, paid here in general (day) goods. Let θ be the probability an agent with money decides each day to put it in the bank – or, if he already has an account, to not withdraw it. Then $M_0 = M(1 - \theta)$ is the amount of cash in circulation, and $M_1 = M_0 + M\theta = M$ is cash plus demand deposits. Let V_m be the value function of an agent in the night market with cash, and V_d the value function of an agent at night with

¹²One might also expect $\partial C_M / \partial \lambda < 0$, but actually this is true iff $z < \tilde{z} = \frac{(1-M)(r+\gamma)xu}{M[r+(1-M)x]^\gamma}$; thus, for large z , when λ increases agents are more willing to accept money. This is because λ measures not only the probability of being robbed but also the probability of trying to rob someone; when the cost z is big, if λ goes up agents are more willing to work for money just to keep themselves from crime. This effect may seem strange, but in any case it goes away once λ is endogenized.

money in the bank, exclusive of the fee, which will be $\phi = a$ in equilibrium.

Hence, $V_1 = \max\{V_m, V_d - a\}$.

Although checks will be perfectly safe in most of what follows, it facilitates the discussion to proceed more generally and let γ_m and γ_d be the probabilities you can successfully steal from someone with money and from someone with a bank account. Bellman's equation for an agent with asset $j \in \{m, d\}$ is then¹³

$$rV_j = (1 - M)(1 - \lambda)x(u + V_0 - V_1) + (1 - M)\lambda\gamma_j(V_0 - V_1) + V_1 - V_j.$$

For an agent with no asset,

$$rV_0 = (1 - \lambda)Mx(V_1 - V_0 - c) + \lambda[M_0\gamma_m + (M - M_0)\gamma_d](V_1 - V_0 - z).$$

If we set $\gamma_m = \gamma$ and $\gamma_d = 0$, then

$$rV_m = (1 - M)(1 - \lambda)x(u + V_0 - V_1) + (1 - M)\lambda\gamma(V_0 - V_1) + V_1 - V_m$$

$$rV_d = (1 - M)(1 - \lambda)x(u + V_0 - V_1) + V_1 - V_d$$

$$rV_0 = (1 - \lambda)Mx(V_1 - V_0 - c) + \lambda M_0\gamma(V_1 - V_0 - z).$$

From $V_1 = \max\{V_m, V_d - a\} = \theta(V_d - a) + (1 - \theta)V_m$, it is clear that

$$\theta = 1 \Rightarrow V_d - a \geq V_m; \theta = 0 \Rightarrow V_d - a \leq V_m; \text{ and } \theta \in (0, 1) \Rightarrow V_d - a = V_m.$$

Equilibrium must satisfy this condition, plus the incentive condition for money to be accepted, $V_1 - V_0 - c \geq 0$, and the participation condition, $V_0 \geq 0$. To

¹³The value of entering the decentralized market with asset j is

$$V_j = \frac{1}{1 + r}[(1 - M)(1 - \lambda)x(u + V_0) + (1 - M)\lambda\gamma_j V_0 + \zeta V_1]$$

where $\zeta = 1 - (1 - M)(1 - \lambda)x - (1 - M)\lambda\gamma_j$. Multiplying by $1 + r$ and subtracting V_j from both sides yields the equation in the text.

characterize the parameters for which different types of equilibria exist we define

$$\begin{aligned} C_1 &= -\frac{(1-M)u}{M} - \frac{\lambda\gamma z}{(1-\lambda)x} + \frac{[r + (1-\lambda)x + \lambda\gamma]\hat{a}}{M(1-M)\lambda(1-\lambda)\gamma x} \\ C_2 &= \frac{(1-M)(1-\lambda)xu - \hat{a}}{r + (1-M)(1-\lambda)x} \\ C_3 &= -\frac{(1-M)u}{M} + \frac{[r + (1-\lambda)x + (1-M)\lambda\gamma]\hat{a}}{M(1-M)\lambda(1-\lambda)\gamma x} \end{aligned}$$

where $\hat{a} = (1+r)a$.

Figure 4 show the situation in (x, c) space for the two possible cases, $z < C_4$ and $z > C_4$, where

$$C_4 = \frac{\hat{a}}{(1-M)\lambda\gamma}.$$

We assume $C_4 < u$, or $\hat{a} < (1-M)\lambda\gamma u$, to make things interesting. The following Lemma establishes that the Figures are drawn correctly by describing the relevant properties of C_j , and relating them to C_M and C_A from the case with no banks; again the proof is omitted.

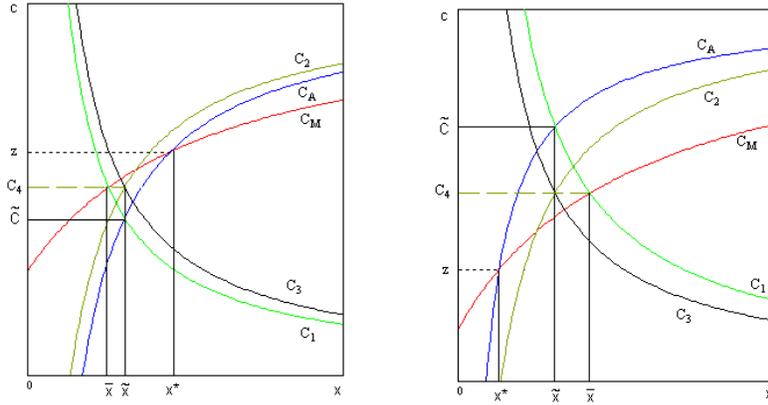


Figure 4: Conditions in Lemma 2

Lemma 2 (a) $x = 0 \Rightarrow C_1 = \infty$ or $-\infty$, $C_2 = -\hat{a}/r < 0$, $C_3 = \infty$. (b) $C_2' > 0$, $C_3' < 0$, and C_1 is monotone but can be increasing or decreasing. (c) $C_1 = C_M$

iff $(x, c) = (\bar{x}, C_4)$, $C_2 = C_3$ iff $(x, c) = (\tilde{x}, C_4)$, and $C_1 = C_A$ iff $(x, c) = (\tilde{x}, \tilde{C})$,

where

$$\begin{aligned}\bar{x} &= \frac{\hat{a}(r + \lambda\gamma) - M(1 - M)\lambda^2\gamma^2z}{(1 - M)(1 - \lambda)[(1 - M)\lambda\gamma u - \hat{a}]} \\ \tilde{x} &= \frac{\hat{a}[r + (1 - M)\lambda\gamma]}{(1 - M)(1 - \lambda)[(1 - M)\lambda\gamma u - \hat{a}]}.\end{aligned}$$

(d) $\bar{x} > \tilde{x}$ iff $z < C_4 < \tilde{C}$ and $\bar{x} < \tilde{x}$ iff $z > C_4 > \tilde{C}$.

We can now prove the following:

Proposition 2 (a) $\theta = 0$ is an equilibrium iff $c \leq \min(C_M, C_A, C_1)$. (b) $\theta = 1$ is an equilibrium iff $C_3 \leq c \leq C_2$. (c) $\theta \in (0, 1)$ is an equilibria iff either: $z < C_4$, $c \in [C_3, C_1]$ and $c \leq C_4$; or $z > C_4$, $c \in [C_1, C_3]$ and $x \geq \tilde{x}$.

Proof: Consider $\theta = 0$, which implies $M_1 = M_0 = M$ and $V_1 = V_m$. For this to be an equilibrium we require $V_m - V_0 - c \geq 0$ and $V_0 \geq 0$, which is true under exactly the same conditions as in the model with no banks, $c \leq \min(C_M, C_A)$. However, now we also need to check $V_m \geq V_d - a$ so that not going to the bank is an equilibrium strategy. This holds iff $c \leq C_1$. Now consider $\theta = 1$, which implies $M_0 = 0$ and $V_1 = V_d - a$. In this case, $V_1 - V_0 - c \geq 0$ holds iff $c \leq C_2$. We also need $V_0 \geq 0$, but this never binds. Finally, we need to check $V_d - a \geq V_m$, which is true iff $c \geq C_3$.

Finally consider $\theta \in (0, 1)$, which implies $M_0 = M(1 - \theta) \in (0, M)$ is endogenous and $V_1 = V_d - a = V_m$. The Bellman equations imply

$$(1 + r)(V_d - V_m) = (1 - M)\lambda\gamma(V_1 - V_0).$$

Inserting $V_d - V_m = a$ and

$$V_1 - V_0 = \frac{(1 - \lambda)x[(1 - M)u + Mc] + M_0\lambda\gamma z}{r + (1 - \lambda)x + (1 - M + M_0)\lambda\gamma},$$

we can solve for

$$M_0 = \frac{(1-M)\lambda(1-\lambda)\gamma x[(1-M)u+Mc] - [r+(1-\lambda)x+(1-M)\lambda\gamma]\hat{a}}{(C_4-z)(1-M)\lambda^2\gamma^2}.$$

We need to check $M_0 \in (0, M)$, which is equivalent to $\theta \in (0, 1)$. There are two cases, depending on the sign of the denominator: if $z < C_4$ then $M_0 \in (0, M)$ iff $c \in (C_3, C_1)$, and if $z > C_4$ then $M_0 \in (0, M)$ iff $c \in (C_1, C_3)$. We also need to check $V_1 - V_0 - c \geq 0$, which holds iff $c \leq C_4$, and $V_0 \geq 0$, which holds iff $x \geq \tilde{x}$. When $\bar{x} > \tilde{x}$ the binding constraint is $c \leq C_4$, and when $\bar{x} < \tilde{x}$ the binding constraint is $x \geq \tilde{x}$. ■

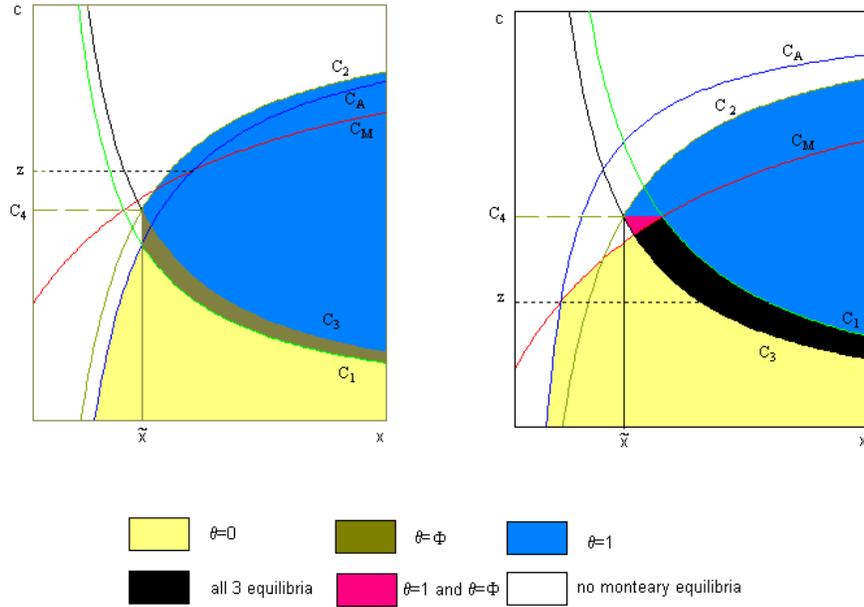


Figure 5: Existence regions with banking

Figure 5 shows the situation when \bar{x} and \tilde{x} from Lemma 2 are in $(0, 1)$.¹⁴

In the case $z > C_4$, shown in the left panel, there is a unique equilibrium,

¹⁴It is easy to provide conditions that guarantee \bar{x} and \tilde{x} are in $(0, 1)$, but they are not especially interesting. If \bar{x} and \tilde{x} are not in $(0, 1)$, some equilibria would not appear in the figure.

and it may entail $\theta = 0$, $\theta = 1$, or $\theta = \Phi$, where we use the notation Φ for a number in $(0, 1)$ (i.e. for a mixed strategy). In the case $z < C_4$, shown in the right panel, we can have uniqueness or multiple equilibria (sometimes $\theta = 1$ and $\theta = \Phi$; sometimes all three equilibria). Recall that without banks monetary equilibrium exists iff $c \leq \min\{C_M, C_A\}$. Hence, if it exists without banks, monetary equilibrium still exists with banks. However there are parameters such that there are no monetary equilibria without banks while there are with banks. In these equilibria we must have $\theta > 0$, although not necessarily $\theta = 1$. The important point is that for some parameters, and in particular for large x or c , money cannot work without banking but it can work with it. This says that money and banking are complements.

Figure 6 shows how the set of equilibria evolves as a falls. For very large a banking is not viable, so the only equilibrium is $\theta = 0$. As we reduce a two things happen: in the some regions where there was a monetary equilibrium without banks, agents may start using banks; and in some regions where there were no monetary equilibria without banks, a monetary equilibrium emerges. As a falls further the region where $\theta = 0$ shrinks. As a falls still further we switch from the case $z < C_4$ to the case $z > C_4$; so for relatively small a equilibrium is unique, but could entail $\theta = 1$, $\theta = 0$, or $\theta = \Phi$. As a falls even further, we lose equilibrium with $\theta = 0$ as checks eventually drive currency from circulation (although note that when we endogenize λ below, banking can never completely drive out money). Equilibria with $\theta \in (0, 1)$ seem particularly interesting, as they entail the concurrent circulation of cash and checks.

A similar picture emerges if we let M fall, since reducing M also raises the demand for checking services. One arguably strange reason for this is that, in the model, lower M implies a greater number of criminals, $(1 - M)\lambda$. We can

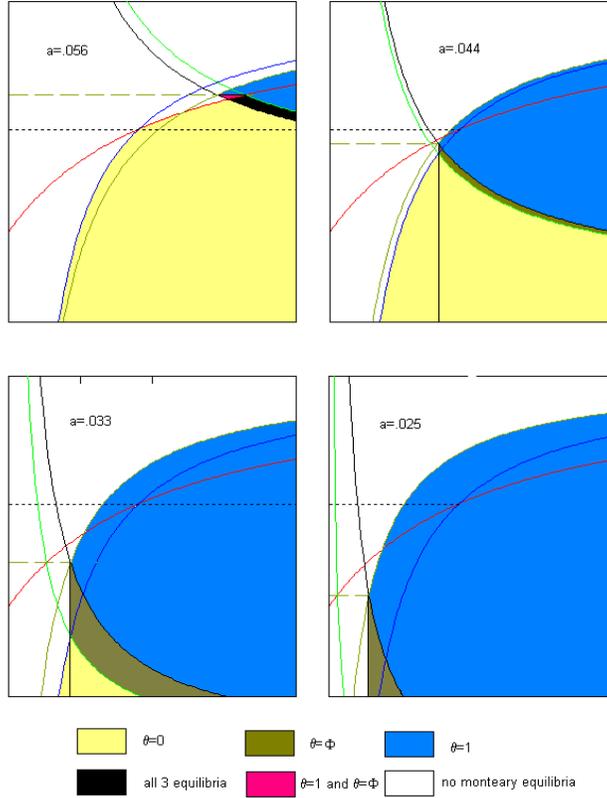


Figure 6: Equilibrium as a changes

circumvent this by considering what happens as we vary M and adjust λ to keep $(1 - M)\lambda$ constant. Now lower M still makes $\theta > 0$ more likely, but the reason is that it makes money more valuable, and so you are more willing to pay to keep it safe. See Figure 7, where the two rows are for $z < C_4$ and $z > C_4$, and in either case M falls as we move from left to right. The result that lower M makes it more likely that agents use banking is consistent with the historical evidence that people were more likely to use demandable debt as a means of payment when cash was scarce (Ashton 1945, Cuadras-Morato and Rosés 1998). In any case, although the model is very simple, it captures something interesting.

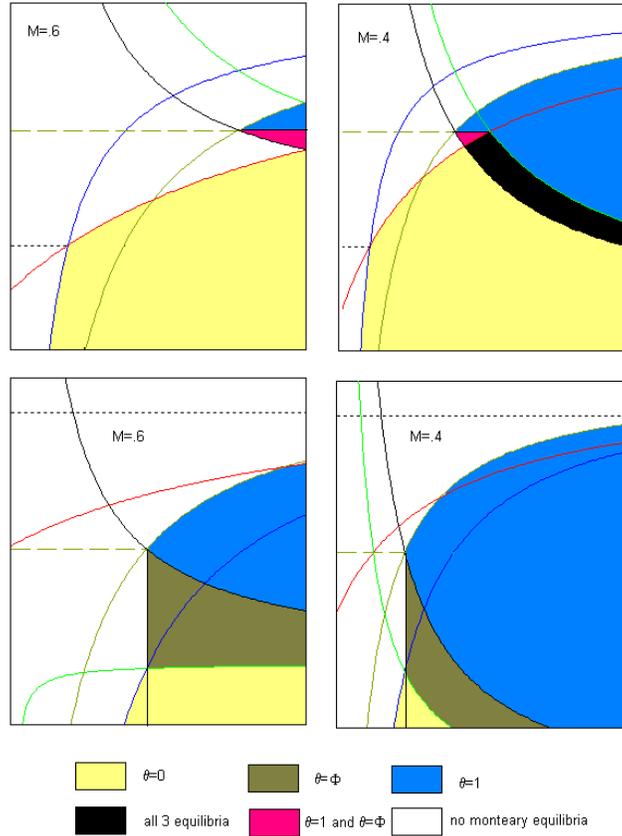


Figure 7: Equilibrium as M changes

4 Endogenous Theft

Here we endogenize the decision to be a thief. This is useful not only for the sake of generality, but because the model with λ exogenous does have some features one may wish to avoid (e.g. when M goes down the crime rate mechanically increases). Moreover, an interesting implication of the model with λ endogenous is that checks can never completely drive currency from circulation: if no one uses cash, no one will choose crime, but then cash is safe and no one would use

checks. Hence concurrent circulation is a natural outcome. As in the model with exogenous λ , we start with the case without banks.

4.1 Money

Let λ now be the probability an individual without money chooses to be a thief before going out at night. Bellman's equation for V_1 is the same as in Section 3.1. Bellman's equations for producers and thieves are now

$$\begin{aligned} rV_p &= Mx(V_1 - V_p - c) + (1 - Mx)(V_0 - V_p) \\ rV_t &= M\gamma(V_1 - V_t - z) + (1 - M\gamma)(V_0 - V_t), \end{aligned}$$

where $V_0 = \max\{V_t, V_p\} = \lambda V_t + (1 - \lambda)V_p$. The equilibrium conditions are

$$\lambda = 1 \Rightarrow V_t \geq V_p, \lambda = 0 \Rightarrow V_t \leq V_p, \text{ and } \lambda \in (0, 1) \Rightarrow V_t = V_p,$$

plus the incentive condition $V_1 - V_0 - c \geq 0$. With λ endogenous we do not have to check the participation constraint $V_0 \geq 0$, as it is implied by $V_1 - V_0 - c \geq 0$.¹⁵

The next observation is that we can never have equilibrium with $\lambda = 1$, since obviously no agent will accept money if everyone else is a thief. To say more, define the following thresholds

$$\begin{aligned} c_0 &= \frac{(1 - M)xu}{r + (1 - M)x} \\ c_1 &= \frac{(x - \gamma)(1 - M)xu + \gamma(r + x)z}{x[r + (1 - M)x + M\gamma]} \\ c_2 &= \frac{[r + Mx + (1 - M)\gamma]\gamma z}{(r + \gamma)x}. \end{aligned}$$

The next Lemma establishes properties of these thresholds illustrated in Figure 8, shown for the case $\gamma > x^* = \frac{rz}{(1 - M)(u - z)}$ (in the other case the region labeled $\lambda = \Phi$ disappears). Again we omit the routine proof.

¹⁵Since one can always set $\lambda = 0$, $rV_0 \geq Mx(V_1 - V_0 - c) \geq 0$ as long as the incentive condition holds.

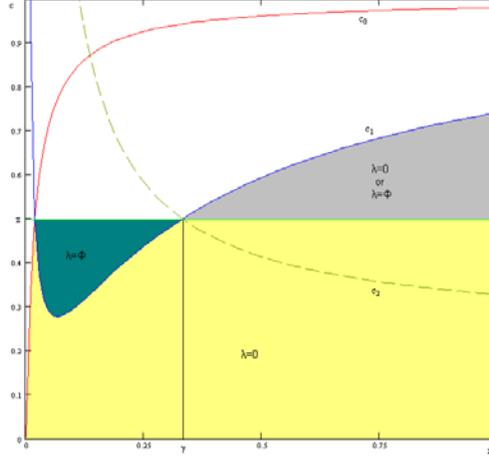


Figure 8: Equilibrium with endogenous λ , no banks

Lemma 3 (a) $x = 0 \Rightarrow c_0 = 0, c_1 = c_2 = \infty$. (b) $c'_0 > 0, c'_2 < 0$. (c) $c_0 = c_1$ iff $(x, c) = (x^*, z)$ and $c_1 = c_2$ iff $(x, c) = (\gamma, z)$. (d) $c_0, c_1 \rightarrow u$ as $x \rightarrow \infty$.

We can now establish the following:

Proposition 3 (a) $\lambda = 0$ is an equilibrium iff either: $x < x^*$ and $c \in [0, c_0]$; or $x > x^*$ and $c \in [0, c_1]$. (b) $\lambda \in (0, 1)$ is an equilibrium iff $x > \gamma$ and $c \in [z, c_1]$, or $x < \gamma$ and $c \in (c_1, z]$.

Proof: Equilibrium with $\lambda = 0$ requires $V_t \leq V_p$ and $V_1 - V_p - c \geq 0$. Inserting the value functions and simplifying, the former reduces to $c \leq c_1$ and the latter to $c \leq c_0$. Hence, $\lambda = 0$ is an equilibrium iff $c \leq \min\{c_0, c_1\}$, and the binding constraint will depend on whether x is below or above x^* . Equilibrium with $\lambda \in (0, 1)$ requires $V_t = V_p$ and $V_1 - V_0 - c \geq 0$. We can solve $V_t = V_p$ for $\lambda = \lambda^*$, where

$$\lambda^* = \frac{(\gamma - x)x[(1 - M)u + Mc] - (r + x)(\gamma z - xc)}{(\gamma - x)(1 - M)(xu - xc + \gamma z)}.$$

It can be checked that $\lambda^* \in (0, 1)$ iff $c \in (c_1, c_2)$ when $x < \gamma$ and $\lambda^* \in (0, 1)$ iff $c \in (c_2, c_1)$ when $x > \gamma$. The condition $V_1 - V_0 - c \geq 0$ can be seen to hold iff $c \leq z$ when $x < \gamma$ and iff $c \geq z$ when $x > \gamma$. Hence, $\lambda = \lambda^* \in (0, 1)$ is an equilibrium iff $c \in (c_1, z]$ when $x < \gamma$, and iff $c \in [z, c_1)$ when $x > \gamma$. ■

As seen in the figure, a monetary equilibrium is again more likely to exist when c is low or x is high. Given a monetary equilibrium exists, it is more likely that $\lambda = 0$ when c is low or x is high, since both of these make honest production relatively attractive. Given x , it is more likely that $\lambda \in (0, 1)$ when c is bigger. When $x^* < \gamma$, there is a region of (x, c) space with $x < \gamma$ where an equilibrium with $\lambda \in (0, 1)$ exists and is unique. Regardless of x^* , as long as $c_0 > z$ at $x = 1$, there exists a region where equilibrium with $\lambda \in (0, 1)$ and $\lambda = 0$ coexist. A very interesting aspect of the results is the following. In constructing equilibrium with $\lambda \in (0, 1)$ we need to guarantee $\lambda < 1$, but this is actually never binding: as λ increases we hit the incentive constraint for money to be accepted before we reach $\lambda = 1$. For example, in the region where $\lambda \in (0, 1)$ exists uniquely, as c increases we get more thieves, but before the entire population resorts to crime people stop producing for cash..

4.2 Banking

We now reintroduce banks.¹⁶ Bellman's equations are

$$\begin{aligned} rV_m &= (1 - M)(1 - \lambda)x(u + V_0 - V_1) + (1 - M)\lambda\gamma(V_0 - V_1) + V_1 - V_m \\ rV_d &= (1 - M)(1 - \lambda)x(u + V_0 - V_1) + V_1 - V_d \\ rV_p &= Mx(V_1 - V_p - c) + (1 - Mx)(V_0 - V_p) \\ rV_t &= M_0\gamma(V_1 - V_t - z) + (1 - M_0\gamma)(V_0 - V_t), \end{aligned}$$

¹⁶The analysis in this subsection is somewhat complicated algebraically, if not conceptually, compared to the previous models. Readers may skip to the next section with little loss in continuity, but we do want to highlight the interesting economic result that, with endogenous theft, checks cannot drive cash out of circulation.

where $V_1 = \max\{V_m, V_d - a\}$ and $V_0 = \max\{V_p, V_t\}$. Here equilibrium also requires

$$\lambda = 1 \Rightarrow V_t \geq V_p; \lambda = 0 \Rightarrow V_t \leq V_p; \text{ and } \lambda \in (0, 1) \Rightarrow V_t = V_p,$$

and, as above,

$$\theta = 1 \Rightarrow V_d - a \geq V_m; \theta = 0 \Rightarrow V_d - a \leq V_m; \text{ and } \theta \in (0, 1) \Rightarrow V_d - a = V_m.$$

In this section, since things are more complicated, we analyze possible equilibria one at a time. In principle there are nine qualitatively different types of equilibria, since each endogenous variable λ and θ can be 0, 1, or $\Phi \in (0, 1)$, but we can quickly rule out all but three possibilities.

Lemma 4 *The only possible equilibria are: $\theta = 0$ and $\lambda = 0$; $\theta = 0$ and $\lambda \in (0, 1)$; and $\theta \in (0, 1)$ and $\lambda \in (0, 1)$.*

Proof: Clearly $\lambda = 1$ cannot be an equilibrium, as then no one accepts money. If $\lambda = 0$ then there are no thieves, so money is safe and $\theta = 0$. Finally, if $\theta = 1$ then $V_t = 0$ and so we cannot have $\lambda > 0$. ■

Proposition 4 *$\theta = 0$ and $\lambda = 0$ is an equilibrium iff either: $x < x^*$ and $c \in [0, c_0]$; or $x > x^*$ and $c \in [0, c_1]$.*

Proof: Given $\lambda = 0$, it is clear that $\theta = 0$ is a best response. Hence the only conditions we need are $V_1 - V_0 - c \geq 0$ and $V_p \geq V_t$. With $\theta = 0$ these are equivalent to the conditions from the model with no banks. ■

To proceed with equilibria where $\lambda > 0$, define

$$c_{11} = \frac{-B_0 \pm \sqrt{B_0^2 - 4A_0C_0}}{2A_0}$$

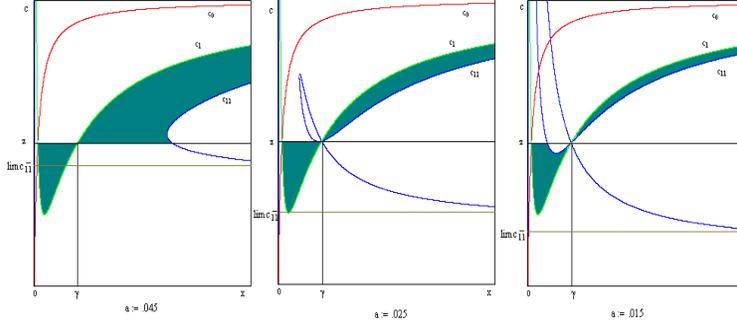


Figure 9: The function in Lemma 5

where

$$\begin{aligned}
 A_0 &= \gamma x^2[r + (1 - M)x + M\gamma] \\
 B_0 &= -[(1 - M)\gamma xu + (x - \gamma)\hat{a}](x - \gamma)x - [2(r + x) - M(x - \gamma)]\gamma^2 x z \\
 C_0 &= (x - \gamma)^2 x u \hat{a} + [(1 - M)\gamma xu + (x - \gamma)\hat{a}](x - \gamma)\gamma z + (r + x)\gamma^3 z^2.
 \end{aligned}$$

As we will see, c_{11} is the solution to a quadratic equation describing the incentive condition for θ . As such, depending on the sign of $B_0^2 - 4A_0C_0$, generically c_{11} either has two real values, call them c_{11}^- and c_{11}^+ , or none. See Figure 9, which shows c_0 and c_1 as well as c_{11} . The figure is drawn assuming $\hat{a} < u\gamma(1 - M)$, which implies $\lim_{x \rightarrow \infty} c_{11}^- = \frac{\hat{a}}{\gamma(1 - M)} < u$; the other case is similar. In the left panel, drawn for large \hat{a} , c_{11} exists only for x close to 1, and in particular does not exist in the neighborhood of $x = \gamma$ (for still bigger \hat{a} , c_{11} would not even appear in the figure). As \hat{a} shrinks, c_{11} exists for more values of x , and at some point it exists for x in the neighborhood of γ , as in the middle panel; notice c_{11}^- and c_{11}^+ happen to coalesce at $x = \gamma$. As \hat{a} shrinks further, c_{11} exists for all $x > 0$, as in the right panel. One can show the following.

Lemma 5 (a) For $x > \gamma$, if c_{11} exists then $c_{11}^+ < c_1$; for $x < \gamma$, if c_{11} exists

then $c_1 < c_{11}^-$; if c_{11} exists in the neighborhood of $x = \gamma$ then $c_{11} = c_1$ at $(x, c) = (\gamma, z)$. (b) For $x > \gamma$, $c_{11}^+ \rightarrow c_1$ as $\hat{a} \rightarrow 0$; for $x < \gamma$, $c_{11}^- \rightarrow c_1$ as $\hat{a} \rightarrow 0$.

Proposition 5 $\theta = 0$ and $\lambda \in (0, 1)$ is an equilibrium iff either: $x > \gamma$, $c \in [z, c_1)$, and $c \notin (c_{11}^-, c_{11}^+)$; or $x < \gamma$, $c \in (c_1, z]$, and $c \notin (c_{11}^-, c_{11}^+)$.

Proof: In this case the conditions for $\lambda \in (0, 1)$ are exactly the same as in the model without banks, but we now have to additionally check $V_m - V_d + a \geq 0$ to guarantee $\theta = 0$. Algebra implies this condition holds iff $A_0c^2 + B_0c + C_0 \geq 0$, which is equivalent to $c \notin (c_{11}^-, c_{11}^+)$. ■

The region where equilibrium with $\theta = 0$ and $\lambda \in (0, 1)$ exists is shown by the shaded area in Figure 9. The economics is simple: in addition to the conditions for $\lambda \in (0, 1)$ from the model with no banks, we also have to be sure now that people are happy carrying cash instead of checks, which reduces to $c \notin (c_{11}^-, c_{11}^+)$. For large \hat{a} this is not much of a constraint. As \hat{a} gets smaller we eliminate more of the region where $\lambda \in (0, 1)$ is an equilibrium. When $\hat{a} \rightarrow 0$ the relevant branch of c_{11} converges to c_1 , and this equilibrium vanishes.

Now consider equilibria with $\theta \in (0, 1)$ and $\lambda \in (0, 1)$. To begin, we have the following result.

Lemma 6 *If there exists an equilibrium with $\lambda \in (0, 1)$ and $\theta \in (0, 1)$, then*

$$\lambda = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A(1-M)} \text{ and } \theta = 1 - \frac{x[\hat{a} - (1-M)\lambda\gamma c]}{\gamma[\hat{a} - (1-M)\lambda\gamma z]}$$

where $A = \gamma xu$, $B = -[(1-M)\gamma xu + M\gamma xc + (x - \gamma)\hat{a}]$, and $C = (r + x)\hat{a}$.

Proof: In this equilibrium we have $V_1 = V_m = V_d - a$ and $V_0 = V_p = V_t$. Solving Bellman's equations and inserting the value functions into these conditions gives us two equations in λ and θ . One is a quadratic that can be

solved for $\lambda = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A(1-M)}$. The other gives us θ as a function of the solution for λ . ■

In order to reduce the number of possibilities, we concentrate on the smaller root for λ in Lemma 6.¹⁷ To see when such an equilibrium exists, define

$$\begin{aligned}
c_3 &= \frac{-k + \sqrt{4(r+x)\gamma xu \hat{a}}}{M\gamma x} \\
c_4 &= \frac{-(1-M)^2\gamma xu + [2(r+x) - (x-\gamma)(1-M)]\hat{a}}{(1-M)M\gamma x} \\
c_5 &= \frac{[r + Mx + (1-M)\gamma]\hat{a}}{(1-M)M\gamma x} \\
c_6 &= \frac{k}{[2(r+x) - Mx]\gamma} \\
c_7 &= \frac{k_{\pm} \pm \sqrt{k^2 - 4[r + (1-M)x]\gamma xu \hat{a}}}{2[r + (1-M)x]\gamma} \\
c_8 &= \frac{-k + 2(r+x)\gamma z}{Mx\gamma} \\
c_9 &= \frac{xu \hat{a} - kz + (r+x)\gamma z^2}{M\gamma xz} \\
c_{10} &= \frac{(x-\gamma)k + 2(r+x)\gamma^2 z}{[2(r+x) - M(x-\gamma)]x\gamma}
\end{aligned}$$

where we let $k = (1-M)\gamma xu + (x-\gamma)\hat{a}$ to reduce notation. We now prove some properties of the c_j 's and relate them to c_0 , c_1 and c_{11} , continuing to assume $\hat{a} < \gamma(1-M)u$.

Lemma 7 (a) $x = 0 \Rightarrow c_3 = c_4 = c_5 = c_8 = c_9 = c_{10} = \infty$ and $c_6 = -\frac{\hat{a}}{2r} < 0$.

(b) $c'_3 < 0$, $c'_4 < 0$, $c'_5 < 0$, $c'_6 > 0$, $c'_8 < 0$, $c'_9 < 0$. (c) $c_7 \geq 0$ exists iff $x \geq x_7 \in (0, 1)$; if c_7 exists then $c_7^{-'} < 0$, $c_7^{+'} > 0$ and $(c_7^+, c_7^-) \rightarrow \left(\frac{\hat{a}}{\gamma(1-M)}, u\right)$ as $x \rightarrow \infty$; $c_7 \in (c_3, c_0)$ and $(c_7^+, c_7^-) \rightarrow (c_0, 0)$ as $\hat{a} \rightarrow 0$. (d) $c_{10} = c_1$ at $(x, c) = (\gamma, z)$; c_6 , c_8 and c_{10} all cross at $c = z$, and c_7 , c_9 and c_{11} all cross at $c = z$, although the values of x at which these crossing occur need not be in

¹⁷In examples we found it was not impossible to have an equilibrium with λ given by the larger root, but only for a very small set of parameters.

$(0, 1)$. (e) $c_3 = c_6 = c_7$ at a point where c_7 is tangent to c_3 ; $c_3 = c_{10} = c_{11}$ at a point where c_{11} is tangent to c_3 .

The c_j are shown in (x, c) space in Figure 10 for various values of a progressively decreasing to 0. The shaded area is the region where equilibrium with $\theta \in (0, 1)$ and $\lambda \in (0, 1)$ exist, as proved in the next Proposition.

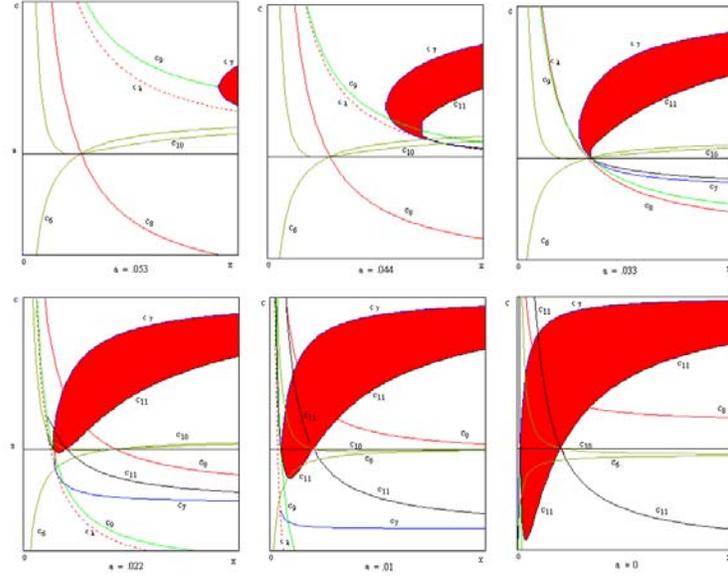


Figure 10: Proposition 6 equilibria

Proposition 6 $\theta \in (0, 1)$ and $\lambda \in (0, 1)$ is an equilibrium iff all of the following conditions are satisfied: (i) $c > c_\lambda = \max\{c_3, \min\{c_4, c_5\}\}$; (ii) $c > \max\{c_8, c_9\}$ and either $c < c_6$ or $c \in (c_7^-, c_7^+)$; and either (iii-a) $x > \gamma$, $c > \max\{z, c_{10}\}$, and $c \notin (c_{11}^-, c_{11}^+)$, or (iii-b) $x < \gamma$ and either $c > c_{10}$ or $c \in (c_{11}^-, c_{11}^+)$.

Proof: The previous lemma gives us λ as a solution to a quadratic equation and θ as a function of λ , assuming that an equilibrium with $\theta \in (0, 1)$ and $\lambda \in (0, 1)$ exists. We now check the following conditions: when does a solution

to this quadratic in λ exist; when is that solution in $(0, 1)$; when is the implied θ in $(0, 1)$; and when is $V_1 - V_0 \geq c$.

First, a real solution for λ exists iff $B^2 - 4AC \geq 0$, which holds iff either of the following hold:

$$\begin{aligned} c &\geq \frac{-[(1-M)\gamma xu + (x-\gamma)(1+r)a] + \sqrt{4(r+x)(1+r)\gamma xua}}{M\gamma x} = c_3 \\ c &\leq \frac{-[(1-M)\gamma xu + (x-\gamma)(1+r)a] - \sqrt{4(r+x)(1+r)\gamma xua}}{M\gamma x} = c'_3 \end{aligned}$$

Second, given that it exists, $\lambda > 0$ iff $B < 0$ iff

$$c > \frac{-(1-M)\gamma xu - (x-\gamma)(1+r)a}{M\gamma x} = c''_3.$$

It is easy to check that $c_3 > c''_3 > c'_3$, and so $\lambda > 0$ exists iff $c \geq c_3$.

It will be convenient to let $W = V_1 - V_0$. Subtracting the first two Bellman equations, we have $V_m - V_d = \frac{(1-M)\lambda\gamma}{1+r}W$, and hence by the equilibrium condition for $\theta \in (0, 1)$, $V_m - V_d = -a$, we have $\omega = \frac{\hat{a}}{\lambda(1-M)\gamma} = \frac{-B + \sqrt{B^2 - 4AC}}{2(r+x)\gamma}$ after inserting λ . We can now see that $\lambda < 1$ is equivalent to $\omega > \frac{\hat{a}}{(1-M)\gamma}$. Analysis shows this holds iff $c > \min\{c_4, c_5\}$. Hence, there exists a $\lambda \in (0, 1)$ satisfying the equilibrium conditions iff

$$c > c_\lambda = \max\{c_3, \min\{c_4, c_5\}\}.$$

We now proceed to check $\theta \in (0, 1)$ and $\omega \geq c$. Rearranging Bellman's equations gives us

$$1 - \theta = \frac{M_0}{M} = \frac{x(\omega - c)}{\gamma(\omega - z)}.$$

Hence, we conclude the incentive condition $\omega \geq c$ and $\theta < 1$ both hold iff $\omega > \max(c, z)$. Algebra shows that $\omega > c$ holds iff $c < c_6$ or $c_7^- < c < c_7^+$, and that $\omega > z$ holds iff $c > \min\{c_8, c_9\}$.

For the last part, $\theta > 0$ holds iff $(x - \gamma)\omega < xc - \gamma z$. When $x > \gamma$, $\theta > 0$ is therefore equivalent to $\omega < \frac{xc - \gamma z}{x - \gamma}$, which holds iff $c > c_{10}$ and $c \notin (c_{11}^-, c_{11}^+)$. Moreover, notice that when $x > \gamma$, $c < z$ implies $\omega < \frac{xc - \gamma z}{x - \gamma} < c$, and therefore, we need $c > z$ as an extra constraint. When $x < \gamma$, $\theta > 0$ is equivalent to $\omega > \frac{xc - \gamma z}{x - \gamma}$, which is equivalent to $c > c_{10}$ or $c_{11}^- < c < c_{11}^+$. This completes the proof. ■

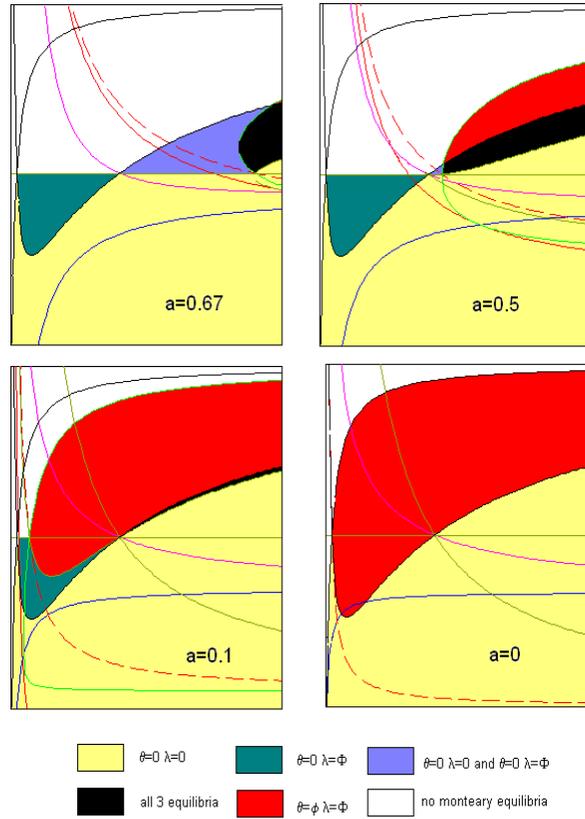


Figure 11: Equilibrium set for different a

Figure 11 puts together everything we have learned in this section and shows the equilibrium set for decreasing values of a . For big a the equilibrium set is like the model with no banks. As a decreases, equilibria with $\theta > 0$ emerge, and

we expand the set of parameters for which there exists a monetary equilibrium. Thus, for relatively high values of c there cannot be a monetary equilibrium without banks, because too many people would be thieves, but once banks are introduced, if a is not too big agents will deposit their money into checking accounts and monetary equilibria can exist. It is important to emphasize that the fall in a actually has two effects in this regard: the direct effect is that it makes it cheaper for agents to keep their money safe; the indirect effect is that as more agents put their money in the bank the number of thieves changes.

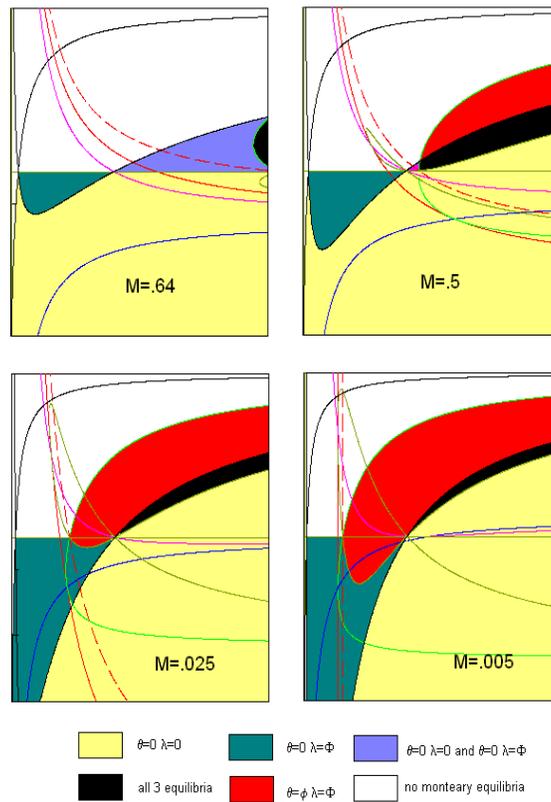


Figure 12: Equilibrium set for different M

Figure 12 shows what happens as M decreases. As in Section 3.2, lower

M raises the demand for banking and makes it more likely to have equilibria with $\theta > 0$; however, in this model this result cannot be due to the number of thieves $(1 - M)\lambda$ mechanically increasing with a fall in M , since λ is endogenous. Perhaps the most interesting thing about the model with endogenous λ is that as long as $a > 0$, no matter how small, we can never have $\theta = 1$, since $\theta = 1$ implies $\lambda = 0$ but $\lambda = 0$ implies $\theta = 0$. Therefore money will always circulate. The general point is that as we reduce the cost of alternatives to money as means of payment, there can be general equilibrium effects that make the demand for these substitutes fall, or that make cash seem better, and the net effect may be that cash is never driven entirely out of circulation.

5 Prices

We now relax the assumption of indivisible goods in the decentralized market, and endogenize prices by letting agents with money make take-it-or-leave-it offers for some amount of output q .¹⁸ Since we mainly want to illustrate the method and show the main results carry through, we keep λ exogenous. Now, without banks, Bellman's equations become

$$\begin{aligned} rV_1 &= (1 - M)(1 - \lambda)x[u(q) + V_0 - V_1] + (1 - M)\lambda\gamma(V_0 - V_1) \\ rV_0 &= M(1 - \lambda)x[V_1 - V_0 - c(q)] + M\lambda\gamma(V_1 - V_0 - z), \end{aligned}$$

where $u(q)$ is the utility from consuming and $c(q)$ the disutility from producing q units. We assume $u(0) = 0$, $u(\bar{q}) = \bar{q}$ for some $\bar{q} > 0$, $u' > 0$, $u'' < 0$, and normalize $c(q) = q$.

Given $c(q) = q$, this implies $q = V_1 - V_0$, and so

$$rV_0 = M\lambda\gamma(V_1 - V_0 - z).$$

¹⁸Money is still indivisible here. This approach follows Shi (1995) and Trejos and Wright (1995), although they actually use symmetric Nash bargaining while we use take-it-or-leave-it offers; both are special cases of the general analysis in Rupert et al. (2001).

Rearranging Bellman's equations, we have $q = C_M(q)$ where

$$C_M(q) = \frac{(1-M)(1-\lambda)xu(q) + M\lambda\gamma z}{r + (1-M)(1-\lambda)x + \lambda\gamma}$$

is the same as the threshold C_M defined in the model with indivisible goods, except that $u(q)$ replaces u . We also need to check the participation condition $V_0 \geq 0$. This holds iff $q \leq C_A(q)$, where

$$C_A(q) = \frac{(1-M)[\lambda\gamma + (1-\lambda)x]u(q)}{r + (1-M)[\lambda\gamma + (1-\lambda)x]} - \frac{\lambda\gamma z}{(1-\lambda)x}$$

is the same as the threshold C_A , except $u(q)$ replaces u . One can show $q \leq C_A(q)$ reduces to $q \geq z$.

A monetary equilibrium exists iff the solution to $q = C_M(q)$ satisfies $q \leq C_A(q)$, or equivalently $q \geq z$. A particularly simple case is the one with $z = 0$, since then the participation condition holds automatically and there always exists a unique monetary equilibrium $q \in (0, \bar{q})$. The equilibrium price level is $p = 1/q$, and one can check that, as long as z is not too big, $\partial q/\partial \lambda < 0$ and $\partial q/\partial \gamma < 0$. So more crime means money is less valuable and prices are higher.

With banks, we have

$$rV_m = (1-M)(1-\lambda)x[u(q) + V_0 - V_1] + (1-M)\lambda\gamma(V_0 - V_1) + V_1 - V_m$$

$$rV_d = (1-M)(1-\lambda)x[u(q) + V_0 - V_1] + V_1 - V_d$$

$$rV_0 = M(1-\theta)\lambda\gamma(V_1 - V_0 - z),$$

where $V_1 = \max\{V_m, V_d - a\}$. Equilibrium again requires $q = V_1 - V_0$ and $V_0 \geq 0$, and now also

$$\theta = 1 \Rightarrow V_d \geq V_m; \theta = 0 \Rightarrow V_d \leq V_m; \text{ and } \theta \in (0, 1) \Rightarrow V_d = V_m.$$

Consider first $\theta = 0$. We need the same conditions as in the case with no banks, $q = C_M(q)$ and $q \geq z$, but now we additionally need $V_m \geq V_d - a$.

This latter condition holds iff $q \leq C_1(q)$, where C_1 is the same as in the model with indivisible goods, except $u(q)$ replaces u . One can show $q \leq C_1(q)$ reduces to $q \leq \hat{a}/(1-M)\lambda\gamma = C_4$. In what follows we write q_0 for the value of q in equilibrium with $\theta = 0$. Then the previous condition has a natural interpretation as saying that for $\theta = 0$ we need the cost of banking \hat{a} to exceed the benefit, which is avoiding the expected loss $(1-M)\lambda\gamma q_0$.

Now consider $\theta = 1$. Then $q = V_1 - V_0$ implies $q = C_2(q)$, where

$$C_2(q) = \frac{(1-M)(1-\lambda)xu(q) - \hat{a}}{r + (1-M)(1-\lambda)x}$$

is the same as above, except $u(q)$ replaces u . For small values of \hat{a} there are two solutions to $q = C_2(q)$ and for large \hat{a} there are none. Hence, we require \hat{a} below some threshold, say \hat{a}_2 , in order for there to exist a $q = C_2(q)$ consistent with this equilibrium. Since $\theta = 1$ the participation condition $V_0 \geq 0$ holds automatically, but we still need to check $V_m \leq V_d - a$. This holds iff $q \geq C_3(q)$, where C_3 is the same as in the model with indivisible goods, except $u(q)$ replaces u . The condition $q \geq C_3(q)$ can be reduced to $q \geq C_4 = \hat{a}/(1-M)\lambda\gamma$. Writing q_1 for the equilibrium value of q when $\theta = 1$, this says that we need the cost of banking \hat{a} to be less than the benefit, which is again avoiding the expected loss $(1-M)\lambda\gamma q_1$. Again this requires \hat{a} to be below some threshold, say \hat{a}_1 . Hence, whenever $\hat{a} \leq \min\{\hat{a}_1, \hat{a}_2\}$, $q = C_2(q)$ exists and satisfies all the equilibrium conditions.

Note that the conditions for the two equilibria considered so far are not mutually exclusive: $\theta = 0$ requires $C_4 \geq q_0$ and $\theta = 1$ requires $C_4 \leq q_1$, but the equilibrium values q_0 and q_1 are not the same. Hence these equilibria may overlap, or there could be a region of parameter space where neither exists. In either case it is interesting to consider $\theta \in (0, 1)$. This requires $V_m = V_d - a$, which reduces to $q = C_4$. As in the model with q fixed, we can now solve for M_0

and check $M_0 \in (0, M)$. Recall that with q fixed there were two possibilities for $M_0 \in (0, M)$: either $z < C_4$, $c \in [C_3, C_1]$ and $c \leq C_4$; or $z > C_4$, $c \in [C_1, C_3]$ and $x \geq \tilde{x}$. It is easy to check that now the latter possibility violates the condition $V_0 \geq 0$, leaving the former possibility. Hence, $\theta \in (0, 1)$ is an equilibrium when $q = C_4 > z$ and

$$C_3(C_4) \leq C_4 \leq C_1(C_4).$$

More can be said about this model. For instance, one can describe the regions of parameter space where the various equilibria exist, and how these regions change with a or M , as in the previous sections. We leave this as an exercise. The point was to show the results are robust to having divisible goods.

6 Fractional Reserves

Here we relax the 100% required reserve ratio and allow banks to make loans.¹⁹ There is a demand for loans because $1 - M$ agents start each period without purchasing power. Now any of them can go to the bank and ask for a unit of money, either as cash or an entry in his bank account. Rather than assuming the loan is repaid in the future, here the agent pays ρ units of general goods up front when the money is extended, and ρ will in equilibrium equate loan demand and supply. Thus our banks are providing *liquidity* rather than credit.²⁰ Let V_n be the value function of an agent with no money deciding whether to take out a loan: $V_n = \max\{V_0, V_1 - \rho\}$. Let χ be the fraction of such agents who do take out a loan. Clearly, we must have $\chi < 1$, since we cannot have monetary equilibria where everyone is a buyer and no one is a seller. Hence, $V_n = V_0 \geq V_1 - \rho$.

The (exogenous) required reserve ratio is $\alpha \in (M, 1)$. There is still a cost a

¹⁹For simplicity, in this section we only consider the case where λ is exogenous and the specialized night goods are indivisible.

²⁰Given the environment, it would be equivalent to have dynamic loan contracts, but this adds nothing.

for managing each dollar on deposit, but there is no cost for managing loans, without loss in generality. Banks charge ϕ for deposit services, although since they can lend out the money it is not necessarily the case that $\phi = a$. Now zero profit implies $r\rho L + \phi D = aD$, where L is the measure of agents with loans and D is the measure with deposits.²¹ It is obvious that banks will lend out as much as possible since there is no uncertainty regarding withdrawals in this model. Hence the required reserve ratio is binding: $L = (1 - \alpha)D$. As long as $D > 0$, zero profit requires

$$(1 - \alpha)r\rho + \phi = a.$$

This implies $\phi < a$, and ϕ could even be negative (interest on checking accounts).

We next present some accounting identities. As above, M_0 denotes the measure of agents with cash and M_1 the measure with cash or demand deposits. Loans plus the original stock of money sum to $L + M = M_1$, as do deposits plus cash held by individuals, $D + M_0 = M_1$. Combining these equations with $L = (1 - \alpha)D$ leads to the identity

$$\alpha M_1 + (1 - \alpha)M_0 = M,$$

which we will use below. If θ is again the proportion of agents with money who deposit it, given α and M banks can “initially” make $\theta(1 - \alpha)M$ loans, but then a fraction θ of these get deposited, and so on. We therefore have the textbook money multiplier:

$$M_1 = M + \theta(1 - \alpha)M + \theta^2(1 - \alpha)^2M + \dots = \frac{M}{1 - \theta(1 - \alpha)}.$$

Also, $M_0 = (1 - \theta)M_1 = (1 - \theta)M / [1 - \theta(1 - \alpha)]$.

²¹Since the fee for deposit services ϕ is received each period while the revenue from a loan ρ is received only once, we need to multiply the latter by r to get the units right.

Bellman's equations are

$$\begin{aligned} rV_m &= (1 - \lambda)(1 - M_1)x(u + V_0 - V_1) + \lambda(1 - M_1)\gamma(V_0 - V_1) + V_1 - V_m \\ rV_d &= (1 - \lambda)(1 - M_1)x(u + V_0 - V_1) + V_1 - V_d, \\ rV_0 &= (1 - \lambda)M_1x(V_1 - V_0 - c) + \lambda M_0\gamma(V_1 - V_0 - z), \end{aligned}$$

where in equilibrium $V_n = \max\{V_0, V_1 - \rho\} = V_0$, $V_1 = \max\{V_m, V_d - \phi\}$, $V_1 - V_0 \geq c$ and $V_0 \geq 0$. These equations are the same as in Section 3, except M_1 replaces M (that model was a special case with $\alpha = 1$). Given this, in principle there are nine qualitatively different types of equilibria, since χ and θ can each be 0, 1, or $\Phi \in (0, 1)$, but we can quickly reduce things to three possibilities.

Lemma 8 *The only possible equilibria are: $\theta = 0$ and $\chi = 0$; $\theta \in (0, 1)$ and $\chi \in (0, 1)$; and $\theta = 1$ and $\chi \in (0, 1)$.*

Proof: Clearly $\chi = 1$ cannot be an equilibrium. For $\chi = 0$ to be an equilibrium we require $\theta = 0$ (for the loan market to clear). For $\chi \in (0, 1)$ we require $\theta > 0$. ■

We study the three possible cases $\theta = 0$, $\theta = 1$ and $\theta \in (0, 1)$, where in each case we know χ from the above Lemma. In the first case, there are no deposits, so $M_0 = M_1 = M$, and $V_1 = V_m$. Recall that in Section 3, where loans were not considered, the condition for such an equilibrium is $c \leq \min\{C_M, C_A, C_1\}$, which corresponds to conditions $V_1 - V_0 \geq c$, $V_0 \geq 0$ and $V_m \geq V_d - \phi$. With the opening of the loan market, the third condition needs to be modified. The maximum amount a borrower is willing to pay is $\bar{\rho} = V_1 - V_0$, and the maximum a depositor is willing to pay is $\bar{\phi} = V_d - V_m$. If the cost a exceeds potential revenue the loan market clears at $D = L = 0$. This happens iff

$$(1 - \alpha)r\bar{\rho} + \bar{\phi} \leq a,$$

which simplifies to

$$c \leq C_1 \equiv \frac{[r + (1 - \lambda)x + \lambda\gamma]\hat{a}}{(1 - \lambda)Mx[\lambda\gamma(1 - M) + (1 - \alpha)r(1 + r)]} - \frac{(1 - M)u}{M} - \frac{\lambda\gamma z}{(1 - \lambda)x}.$$

If $\alpha = 1$, then C_1 reduces to the expression for C_1 in Section 3. Since C_1 is increasing in α , it is more difficult to have equilibrium with $\theta = 0$ when the required reserve ratio is low. Intuitively this is because when banks can make loans, the equilibrium service fee ϕ goes down, making agents more inclined to use banking. We rearrange $c \leq C_1$ as

$$\hat{a} \geq \hat{A}_1 \equiv \frac{\lambda\gamma(1 - M) + (1 - \alpha)r(1 + r)}{r + (1 - \lambda)x + \lambda\gamma} \{(1 - \lambda)x[(1 - M)u + Mc] + \lambda\gamma Mz\}$$

and present results for this version of the model in terms of thresholds for \hat{a} rather than c . Intuitively, when \hat{a} is too high banking is not viable, but the possibility of lending lessens this problem (\hat{A}_1 is decreasing in α) because borrowers share in the cost.

We summarize the results for the case $\theta = 0$ as follows.

Proposition 7 *A unique equilibrium with $\theta = \chi = 0$ exists iff $c \leq \min\{C_M, C_A\}$ and $\hat{a} \geq \hat{A}_1$.*

Now consider the case $\theta = 1$, where every individual with money, including those who just borrowed it, deposits it in the bank.²² This implies $M_0 = 0$, and $M_1 = M/\alpha$, at least given $M < \alpha$ (if $M \geq \alpha$ an equilibrium with $\theta = 1$ cannot exit). Since $\theta = 1$, we have $V_1 = V_d - \phi \geq V_m$. Moreover, the previous lemma implies that $\chi \in (0, 1)$ when $\theta = 1$, so we must have $\rho = V_1 - V_0$. Solving for $V_1 - V_0$ and using the zero profit condition, we get

$$\rho = \frac{(1 - \lambda)x[(1 - M_1)u + M_1c] - \hat{a}}{(1 - \lambda)x - r[(1 + r)(1 - \alpha) - 1]}.$$

²²Recall that, when $\alpha = 1$, $\theta = 1$ could never be an equilibrium in the model with λ endogenous. Here we are looking at the case with λ exogenous, but even if it were endogenous it may now be possible to have $\theta = 1$, because ϕ can be negative once we allow $\alpha < 1$. It is still the case that endogenizing λ would make it more likely for $\theta < 1$, since when θ is big λ will be small.

This is the value of ρ that clears the loan market.

For this equilibrium to exist we need to check two more things, $V_1 - V_0 \geq c$ and $V_d - V_m \geq \phi$ (note that $V_0 \geq 0$ holds automatically when $\theta = 1$). The first of these reduces to

$$\hat{a} \leq \hat{A}_2 \equiv (1 - \lambda)\left(1 - \frac{M}{\alpha}\right)x(u - c) + r[r - \alpha(1 + r)]c,$$

while the second reduces (using the zero profit condition and the equilibrium value of ρ) to

$$\hat{a} \leq \hat{A}_3 \equiv \frac{(1 - \lambda)x[r(1 + r)(1 - \alpha) + \lambda\left(1 - \frac{M}{\alpha}\right)\gamma]\left[\left(1 - \frac{M}{\alpha}\right)u + \frac{M}{\alpha}c\right]}{r + (1 - \lambda)x + \lambda\gamma\left(1 - \frac{M}{\alpha}\right)}.$$

These conditions imply that when \hat{a} is small enough checks completely replace money. Also, a low reserve ratio facilitates the circulation of checks, since \hat{A}_2 and \hat{A}_3 are decreasing in α .

Proposition 8 *A unique equilibrium with $\theta = 1$ and $\chi \in (0, 1)$ exists iff $\hat{a} \leq \min\{\hat{A}_2, \hat{A}_3\}$.*

Finally, consider the case $\theta \in (0, 1)$. An individual with money is indifferent between holding cash and depositing it if $\phi = V_d - V_m$. Using the value functions, $\rho = V_1 - V_0$, and the zero profit condition, we get

$$\rho = s(M_1) \equiv \frac{\hat{a}}{r(1 + r)(1 - \alpha) + \lambda\gamma(1 - M_1)}.$$

We interpret this as the (inverse) loan supply function, since it is the result of comparing V_m and V_d . Notice $s'(M_1) > 0$, since higher ρ leads to lower ϕ through the zero profit condition, which induces more deposits.

We can reduce the condition $\rho = V_1 - V_0$ to

$$\rho = d(M_1) \equiv \frac{(1 - \lambda)(1 - M_1)xu + (1 - \lambda)M_1xc + \lambda\gamma zM_0}{r + (1 - \lambda)x + \lambda\gamma(1 - M_1) + \lambda\gamma M_0}.$$

We interpret this as the (inverse) loan demand function. Note that M_0 is a function of M_1 in this relation, using the identity $\alpha M_1 + (1 - \alpha)M_0 = M$ derived above. It can be shown that $d'(M_1) < 0$ iff c is below some threshold c^* .²³ In the analysis below we assume this is satisfied. Figure 13 shows the supply and demand curves in (M_1, ρ) space.

In order to have $\theta \in (0, 1)$ we must have $M_1 \in (M, M/\alpha)$, which as can be seen in Figure 13 means

$$s(M) < d(M) \text{ and } s(M/\alpha) > d(M/\alpha).$$

These conditions reduce to $\hat{a} < \hat{A}_1$ and $\hat{a} > \hat{A}_3$, where \hat{A}_1 and \hat{A}_3 were defined above. We also need the incentive condition $c \leq V_1 - V_0$, and the participation condition $V_0 \geq 0$; for simplicity we assume here that $c > z$, so that the former implies the latter automatically. In terms of demand and supply, we need $d^{-1}(c) \geq s^{-1}(c)$, since then $\rho \geq c$ at the intersection of supply and demand, and since $\rho = V_1 - V_0$ the incentive condition holds. It is easy to check this is satisfied iff

$$\hat{a} \geq \hat{A}_4 \equiv r(1+r)(1-\alpha)c + \lambda\gamma c \frac{r(1-\alpha)c + \lambda\gamma(M-\alpha)(z-c)}{(1-\alpha)(1-\lambda)x(u-c) - \lambda\gamma c + \lambda\gamma\alpha z}.$$

Proposition 9 *Assume $c > \max\{z, c^*\}$. A unique equilibrium with $\theta \in (0, 1)$ and $\chi \in (0, 1)$ exists iff $\hat{A}_3 < \hat{a} < \hat{A}_1$ and $\hat{a} \geq \hat{A}_4$.*

We can use Figure 13 to describe how the equilibrium changes with parameters like a or α . First, $d(M_1)$ is independent of the banking cost a , while $s(M_1)$ shifts up as a increases. Hence M_1 goes down and ρ goes up with an increase in a ; intuitively, the loan volume decreases as some of the increase in cost is passed on

²³For the record, c^* is given by

$$\frac{(1-\lambda)xu\{(1-\alpha)[r+(1-\lambda)x] + \lambda\gamma(M-\alpha)\} + \lambda\gamma z\{\alpha[r+(1-\lambda)x] + \lambda\gamma(\alpha-M)\}}{(1-\lambda)x(1-\alpha)[r+(1-\lambda)x + \lambda\gamma] + \lambda\gamma M}.$$

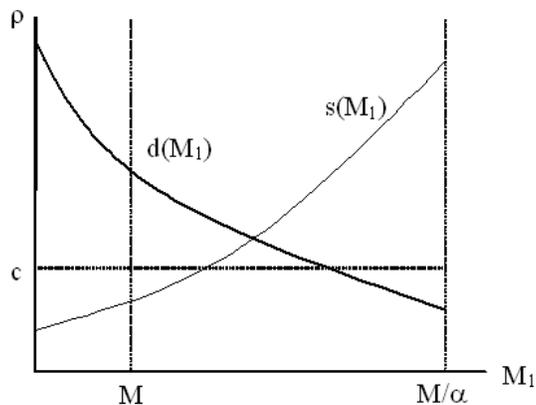


Figure 13: Loan supply and demand

to borrowers. With an increase in the reserve ratio α , one can show $s(M_1)$ shifts up, while $d(M_1)$ shifts up iff $(1-\lambda)[(1-M_1)u+M_1c] > [r+(1-\lambda)x+\lambda\gamma(1-M_1)]z$, which is equivalent to $V_1 - V_0 > z$. For example, if $z < c$ as assumed above, this condition must be satisfied, and so both supply and demand shift up with an increase in α . This means that ρ increases but the effect on M_1 is actually ambiguous. Other results can be derived; we leave these as exercises.

7 Conclusion

We have analyzed some models of money and banking based on explicit frictions in the exchange process. Although simple, we think these models capture something interesting and historically accurate about banking. Various extensions may contribute to our understanding of financial institutions more generally. An obvious generalization is to consider a version with divisible money, which would allow us to address issues concerning, e.g., the effects of inflation on banking. Other possible extensions include using versions of the model to study many of the phenomena addressed elsewhere in the existing literature (bank

runs, delegated monitoring, etc.). The goal here was to provide a first pass at some fairly simple models, where money and banking arise endogenously, and in accord with the economic history.

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