

# Uninsurable Risk and the Determination of Real Interest Rates: An Investigation using UK Indexed Bonds

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## Abstract

This paper investigates the empirical performance of a new class of uninsurable risk models in the context of UK indexed bond market. Using closed form expressions for pricing kernels, we test the ability of three consumption-based models to price indexed bonds in the UK, and find that the standard general equilibrium, complete markets model is soundly rejected in favour of two uninsurable-risk models. Of the latter, a model prohibiting all insurance appears to perform better than a model permitting partial insurance. Using the estimated bond price equation, impulse response analysis is undertaken to understand the effects of various macroeconomic fundamental shocks on real interest rates. In contrast to the estimates that typically arise in equity markets, the estimated coefficient of relative risk aversion and the resulting bond risk premia are found to be small in this class of models with uninsurable risk.

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## 1. Introduction

The influence on financial asset prices of macroeconomic factors and uninsurable risk has generated increasing interest among financial economists in recent years. While the majority of research in this area has focused on linking macro factors to stock prices and exchange rates, much less attention has been paid to explaining bond prices and the term structure of real and nominal yields. Recent macro-finance papers focusing on bonds include Piazzesi and Schneider (2007), who consider the effect of aggregate consumption growth and inflation on nominal and real yields under Epstein and Zin (1991) preferences, and Eraker (2008), who also uses Epstein and Zin preferences, and investigates the role of jumps in explaining both the equity premium and the term structure. Rudebusch (2010) gives an excellent survey of three types of macro-finance model employed in the recent literature to explain the relation between macro economic variables and the term structure.

The role of uninsurable risk was introduced to an optimising framework by Constantinides and Duffie (1996), and among recent empirical investigations Kocherlakota and Pistaferri (2007, 2009) look at two uninsurable-risk settings: (i) an incomplete market environment (INC henceforth) in which idiosyncratic consumption risks are completely uninsurable, and (ii) an environment in which these risks are insurable but only partly so, due to incentive constraints on agents' truth revelation about their private shocks. Kocherlakota and Pistaferri label this market structure 'private information pareto optimal' (PIPO), since its consumption allocation is constrained Pareto opti-

mal. They also demonstrate that the PIPO structure performs much better in terms of explaining the equity premium and real exchange rate puzzles than the representative agent complete market and INC models.<sup>1</sup>

This is the first paper we are aware of that attempts to relate uninsurable risk to real interest rates.<sup>2</sup> While real rates are arguably the most fundamental of interest rates rate (since inflation and various risk premia have to be added to them in order to price nominal bonds and equities) the difficulty of measuring their *ex ante* values has made research in this area problematic. A partial solution to this problem is provided by inflation-indexed bonds since the real value of their expected coupons is almost wholly independent of expected inflation. A number of papers have attempted to model real rates using such bonds, including Barr and Campbell (1997) and Piazzesi and Schneider (2007). Of these the latter is closest to the present paper in that it links real rates to consumption growth via the usual Euler equation. Our paper differs from theirs in several respects however. In particular, their

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<sup>1</sup>This stochastic discount factor used by K-P (2007, 2009) differs from the pricing kernels proposed by Constantinides and Duffie (1996), Sarkissian (2003), Basu and Wada (2006), and Semenov (2008) in an important respect. K-P use the cross-sectional distribution of consumption in *levels*, rather than the cross-sectional distribution of consumption in *growth rates*, as the driving process for the pricing kernel. The use of the cross-sectional distribution of consumption in *growth rates* as in Constantinides and Duffie (1996) has a counterfactual implication that the cross-sectional distribution of consumption in *levels* is not stationary, implying that the Gini coefficient of consumption distribution goes to infinity.

<sup>2</sup>Basu et al. (2011) investigate the implications of uninsurable risk to address various financial market puzzles including the low risk free rate puzzle as in Weil (1989). However, they do not explore the implications of uninsurable risk for term structure of real interest rates which is the major focus of this paper.

paper uses a two-variable VAR in inflation and consumption growth while our VAR includes these variables along with an additional variable to reflect uninsurable risk; Piazzesi and Schneider calibrate the two parameters of their asset pricing model to the short and long ends of an estimated nominal yield curve while we estimate the parameters by maximum likelihood based on fitting the model to market prices of bond. Finally and most importantly, they do not explore the implications of uninsurable consumption risk for bond price behaviour, which is the central aim of this paper.

As in Basu *et al.* (2011) the pricing kernels are derived using a lognormal process for the cross-sectional distribution of consumption which has strong empirical validity.<sup>3</sup> The bond price is thus a loglinear function of three state variables whose expected future values are constructed from a vector autoregression. The lognormality of the consumption process yields a simple analytical form for bond prices in the tradition of affine yield curve models.<sup>4</sup> Two of the macroeconomic state variables, namely aggregate consumption

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<sup>3</sup> Battistin *et al.* (2009) establish that the cross-sectional consumption distribution in both the US Consumer expenditure Survey (CEX) and the UK British Family Expenditure Survey (FES) is approximately log normal within demographically homogeneous groups. This is due to the fact that the Gibrat's law applies to consumption. Brzozowski *et al.* (2009) provide further empirical evidence that the cross-sectional distribution of consumption within cohort groups in Canada may be approximated by a log-normal distribution. Blundell and Lewbel (1999) also provide powerful empirical evidence of log-normality of the cross-sectional distribution of consumption in a variety of data sets. Attanasio *et al.* (2004) assume log-normality of the cross-sectional distribution of household consumption when studying the evolution of inequality in consumption in the US both within cohorts and for the all population.

<sup>4</sup>See for example, Campbell *et al* (1997, Ch. 11) for a comprehensive exposition of this class of affine yield models.

growth and the growth rate of the cross sectional log variance of consumption (which represents the uninsurable risk) come directly from the underlying equilibrium model. A third factor, inflation, is included because we are fitting the market prices of indexed bonds, which are likely to depend on expected inflation due to the imperfect nature of their indexation. The estimated VAR is then used to provide the expectations proxies that allow us to estimate the structural parameters (agent's risk aversion and the rate of time preference) by fitting the closed form price equation to market data.

Our results are consistent with the common finding that the standard complete market model is not consistent with the data. The models with uninsurable risk fare much better. The estimates of the coefficient of risk aversion and the resulting bond risk premia are found to be small but these are consistent with the low returns earned on UK indexed bonds during our sample period. The impulse response analysis of the equations reveals that: a rise in inflation lowers real interest rates of nearly all maturities (due to the news it carries about future consumption); a rise in economic growth raises real interest rates, which is consistent with the permanent income hypothesis, and greater uninsurable consumption risk lowers real rates via the precautionary saving motive.

We use the estimates of the real yield curve provided by the Bank of England as a check on the plausibility of the yields implied by our estimated models. Although there are discrepancies between the Bank's rates and ours, particularly at the short end of the curve, the incomplete market models come

closer to the Bank estimates than does the standard RA model. While the relatively simple models that we estimate do not tell the complete story, they do provide support for the proposition that uninsurable risks have a significant impact on real interest rates.

The paper is organized as follows. The following section lays out the basic setup for the three pricing kernels. Section 3 presents the applications of these pricing kernels to UK indexed-bond prices. Section 4 discusses the estimation methods and the data. Section 5 presents the estimation results. Section 6 concludes and suggests areas for future research.

## 2. Theoretical models.

### 2.1. Three Pricing Kernels

Our benchmark case is the traditional complete market model with homogeneous agents. With a power utility function (with risk aversion parameter  $\gamma$ ), the stochastic discount factor is given by:

$$M_{t+1}^{RA} = \frac{\beta c_{t+1}^{-\gamma}}{c_t^{-\gamma}} \quad (1)$$

where  $\beta$  is the subjective discount factor and  $c_t$  is the aggregate consumption at date  $t$ .

In two influential papers (2007, 2009) Kocherlakota and Pistaferri (K-P hereafter) introduce consumer heterogeneity and uninsurable risk for two distinct market environments: (i) incomplete market (*INC*) where private skill shocks are uninsurable, (ii) partial insurance environment where the private

skill shocks are partially insured by an insurance company who stipulate long term contracts with agents subject to a truth revelation constraint for eliciting efforts and private skill shocks. The latter environment is constrained Pareto efficient and K-P call it private information Pareto optimal (*PIPO*) environment.

Using the law of large numbers K-P demonstrate that the pricing kernels for these two market environments can be written as:

$$M_{t+1}^{INC} = \frac{\beta \sum_i c_{it+1}^{-\gamma} prob(i)}{\sum_i c_{it}^{-\gamma} prob(i)} \quad (2)$$

$$M_{t+1}^{PIPO} = \frac{\beta \sum_i c_{it}^{\gamma} prob(i)}{\sum_i c_{it+1}^{\gamma} prob(i)} \quad (3)$$

where  $c_{it}$  is the consumption of individual  $i$  at date  $t$  and  $prob(i)$  is the cross section probability of the occurrence of the  $i$ th household in the population. In the absence of any information frictions and heterogeneity ( $x_t = 0$  for all  $t$ ), both (2) and (3) reduce to (1).

## 2.2. Lognormal Parameterization of Consumption Processes

In a similar spirit as in Sarkissian (2003), we consider a lognormal parameterization of the post-trade consumption process. We represent the *post-trade*



allocation of consumption as follows.<sup>5</sup> The  $i$ th investor's consumption is:

$$c_{i,t} = \delta_{i,t} \cdot c_t \quad (4)$$

where  $\delta_{i,t}$  is the  $i$ th investor's share in aggregate consumption,  $c_t$ . This specification basically means that the log of individual consumption is the sum of the log of aggregate per capita consumption and the uninsurable consumption due to the idiosyncratic uninsurable skill shock. De Santis (2007) also assumes this log-additive specification to estimate the welfare cost of business cycles.

We assume the following lognormal process for  $\delta_{i,t}$ :

$$\delta_{i,t} = \exp\left(u_{i,t}\sqrt{x_t} - \frac{x_t}{2}\right) \quad (5)$$

where  $u_{i,t}$  is standard normal i.i.d shock, and  $x_t$  is the cross sectional variance of log consumption.

The  $s^{th}$  raw moment of the cross sectional distribution of consumption is given by:

$$E_i(c_{i,t}^s) = c_t^s \exp\left(\frac{(s^2 - s)}{2}x_t\right) \quad (6)$$

Note that, by construction, aggregate consumption is the sum of individ-

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<sup>5</sup>Sarkissian (2003) writes the post trade allocation in terms of consumption growth rate while we write here in terms of level of consumption. The motivation for doing this is to apply this post-trade allocation to the Kocherlakota-Pistaferri (2007, 2009) discounting methodology. The Kocherlakota-Pistaferri incomplete market discount factor is based on the growth rates of the cross sectional moments of consumption in level while Sarkissian (2003) and also Semenov (2008) use the Constantinides-Duffie (1996) discount factor which is based on the cross sectional average of the intertemporal marginal rates of substitution.

ual consumption, which can be checked by setting  $s = 1$ . We now address the issue of whether or not it satisfies the optimality conditions. We follow the same reverse engineering approach as in Basu et al (2011): If there is a unique pricing kernel that supports this allocation of consumption, then it must also support the individual optimality conditions.

### 2.3. Lognormal Pricing Kernels

Substituting (6) into (2), and evaluating at  $s = -\gamma$ , gives the following pricing kernel for the *INC* environment :

$$M_{t+1}^{INC} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \exp \left( \frac{\gamma(\gamma + 1)}{2} (x_{t+1} - x_t) \right) \quad (7)$$

Similarly, substitution of (6) into (3), and evaluating at  $s = \gamma$ , gives:

$$M_{t+1}^{PIPO} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \exp \left( \frac{-\gamma(\gamma - 1)}{2} (x_{t+1} - x_t) \right) \quad (8)$$

#### 2.3.1. Long-run risk models.

Pricing kernels (7) and (8) involve a latent variable  $x_t$  which gives rise to heterogeneity in consumption streams. The latent variable  $x_t$  captures idiosyncratic risk in consumption and it washes out in the aggregate. Thus by construction this latent variable  $x_t$  will not show up in the per capita consumption variable.<sup>6</sup> Our pricing kernels are thus fundamentally different from the long run risk model of Bansal and Yaron (2004) and Piazzesi and Schneider (2007). The latter deal with long run aggregate risk which does

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<sup>6</sup>To verify this set  $s = 1$  in (6) and we obtain  $E_i(c_{i,t}) = c_t$ .

not wash out in the aggregate while our focus is exclusively on idiosyncratic risk which does.<sup>7</sup>

### 2.3.2. Taste-shock models.

Our model looks similar to the multiplicative taste shock model such as Campbell (1986). There is, however, a fundamental difference between them. Our consumption process (??) is driven by idiosyncratic income shocks with a microeconomic foundation laid out by K-P (2007, 2009). Our two pricing kernels (7) and (8) involve the interaction between the risk aversion parameter ( $\gamma$ ) and the latent variable  $x_t$  which captures the fundamental features of the two uninsurable risk models of K-P (2007, 2009). Campbell's taste shock model yields a pricing kernel without any such interaction. In the appendix, we show that the multiplicative taste shock model of Campbell yields a pricing kernel that is different from ours.

As in Campbell, let the taste shock impact the power utility function of the  $i$ th consumer as follows :

$$U(c_{it}) = \lambda_{it} \cdot \frac{c_t^{1-\gamma} - 1}{1 - \gamma}$$

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<sup>7</sup>Bansal and Yaron (2004) employ a non-expected utility (Epstein-Zin (...)) type preference while we focus on simple expected utility model. To make a valid comparison between their pricing kernel and ours, we need to reduce their model to an expected utility paradigm. It is easy to check that in Bansal and Yaaron (2004), if one sets their parameter  $\theta = 1$ , their pricing kernel (their equation 1) reduces to a standard RA pricing kernel  $\beta \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma}$  which stands in sharp contrast with our INC and PIPO pricing kernels (7) and (8). This clearly demonstrates the stark difference between the pricing kernel of long run risk model and the KP pricing kernels that we employ in our model.

where  $\lambda_{it}$  represents the taste shock, and  $c_t$  is average consumption.

For a power utility function the intertemporal marginal rate of substitution (IMRS) of the  $i$ th consumer is:

$$M_{t+1}^i = \beta \frac{\lambda_{it+1}}{\lambda_{it}} \cdot \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma}$$

Given any generic asset return (say  $R_{t+1}$ ), the return equation facing the  $i$ th consumer can be written in a generic form as:

$$E_t M_{t+1}^i R_{t+1} = 1$$

Using the law of iterated expectation we can rewrite the above as:

$$E_t E_i M_{t+1}^i R_{t+1} = 1$$

where subscript  $i$  stands for the cross section aggregation. This is the same as:

$$E_t \beta \left\{ E_i \frac{\lambda_{it+1}}{\lambda_{it}} \right\} \cdot \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1} = 1$$

The immediate implication is that the pricing kernel for the taste shock model (call it  $M_{t+1}^{taste}$ ) is:

$$M_{t+1}^{taste} = \beta \left\{ E_i \frac{\lambda_{it+1}}{\lambda_{it}} \right\} \cdot \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \quad (9)$$

The exact pricing kernel depends on how one posits the process for the

taste shock. The additional term  $E_i \frac{\lambda_{it+1}}{\lambda_{it}}$  in (A.1) is the cross sectional first moment of the growth rate of taste shock which captures taste heterogeneity. Note the important difference between the taste shock pricing kernel (A.1), and the incomplete market pricing kernels (7) and (8). The additional terms in the latter two incomplete market pricing kernels are functions of  $\gamma$  and the latent variable  $x_t$ . The interaction between the risk aversion parameter  $\gamma$  and the cross sectional variance term  $x_t$  captures the essence of the uninsurable risk as in Kocherlakota and Pistefferri (...). On the other hand, Campbell's taste shock pricing kernel shows no such interaction between risk aversion and the latent variable. The additional term in (A.1) is free from  $\gamma$ .

The taste shock models are, however, empirically useful because they depart from the standard representative model pricing kernels and have the potential to solving many asset pricing puzzles. The exact form of the pricing kernel, however, depends on the cross sectional distribution of taste shocks.<sup>8</sup>

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<sup>8</sup>In some special cases, a taste shock model may reduce to a representative agent pricing kernel. For example, if the taste shock follows a lognormal random walk such as:

$$\lambda_{it+1} = \lambda_{it} \exp(u_{it} \sqrt{x_t} - \frac{x_t}{2})$$

with a standard normal deviate  $u_{it}$ , then  $E_i \frac{\lambda_{it+1}}{\lambda_{it}} = 1$ , it is easy to verify that the  $M_{t+1}^{taste}$  reduces to a RA pricing kernel. On the other hand, our INC and PIPO pricing kernels can reduce to RA pricing kernels only when the cross sectional variances in consumption  $x_t$  is time invariant.

### 3. Application to UK Indexed Bonds

#### 3.1. Pricing pure-real zero-coupon bonds

We start by considering the real price,  $P_{nt}^R$ , of a zero-coupon bond with maturity  $n$ , and develop this into the nominal price of the imperfectly indexed coupon bonds that are traded in the UK.  $P_{nt}^R$  can be written as follows for each of the market environments,  $h = RA, INC, PIPO$ :

$$P_{nt}^R = E_t [P_{n-1,t+1}^R M_{t+1}^h] \quad (10)$$

Assuming log normality we get the following expression for the log real price of a perfectly indexed zero-coupon bond of maturity  $n$ ,

$$p_{nt}^R = E_t[m_{t+1}^h + p_{n-1,t+1}^R] + \frac{1}{2}Var_t[m_{t+1}^h + p_{n-1,t+1}^R] \quad (11)$$

where

$$m_{t+1}^{RA} = \ln(\beta) - \gamma g_{t+1} \quad (12)$$

$$m_{t+1}^{INC} = \ln(\beta) - \gamma g_{t+1} + \left(\frac{\gamma(\gamma+1)}{2}\right) v_{t+1} \quad (13)$$

$$m_{t+1}^{PIPO} = \ln(\beta) - \gamma g_{t+1} - \left(\frac{\gamma(\gamma-1)}{2}\right) v_{t+1} \quad (14)$$

and  $g_{t+1} \equiv c_{t+1} - c_t$ ,  $v_{t+1} \equiv x_{t+1} - x_t$ .<sup>9</sup>

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<sup>9</sup>In K-P's (2007, 2009) setup, there are both aggregate and individual shocks and the former are completely hedged by a set of aggregate-shock contingent claims. In our bond economy, if these contingent claims do not exist in addition to bonds, PIPO market environment is not constrained pareto optimal and the use of the PIPO discount factor is not justified. In order to avoid this problem, we assume that there are both these contingent claims and bonds traded but for brevity we focus on bonds and do not present

### 3.2. Pricing imperfectly indexed coupon bonds

UK indexed bonds are indexed to the change in goods prices<sup>10</sup> over a base period starting 8 months before their issue date, and ending 8 months before their redemption date.<sup>11</sup> We approximate this eight-month lag by 3 calendar quarters because we are using quarterly data. Thus the total inflation compensation for an  $n$ -period zero-coupon indexed bond is  $Q_{t+n-3}/Q^*$ , where  $Q^*$  is the goods price for the bond's base period, which leaves the bond's real price exposed to inflation over final 3 periods of its life. Thus equation (10) becomes

$$P_{nt}^R = E_t \left[ \left( \prod_{s=1}^n M_{t+s}^h \right) \frac{Q_{t+n-3}}{Q^*} \frac{1}{Q_{t+n}} \right] \quad (15)$$

from which we get the nominal price of the bond as,

$$P_{nt}^{Nom} = \frac{Q_t}{Q^*} E_t \left[ \left( \prod_{s=1}^n M_{t+s}^h \right) \frac{Q_{t+n-3}}{Q_{t+n}} \right] \quad (16)$$

After log-linearizing (16) and denoting the lower cases as the log of upper cases, gives the nominal price as

$$p_{nt}^{Nom} = (q_t - q^*) + E_t[z_{n,t+1}] + \frac{1}{2} Var_t[z_{n,t+1}] \quad (17)$$

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a fully specified model which is available from the author upon request.

<sup>10</sup>Measured by the Retail Prices Index (RPI).

<sup>11</sup>The indexation method for UK bonds changed in 2005 (after the end of our sample). For bonds issued since that date the indexation lag is 3 months.

where

$$z_{n,t+1} = \sum_{s=1}^n m_{t+s} - \sum_{s=0}^2 \pi_{t+n-s} \quad (18)$$

and  $\pi_{t+s} = q_{t+s} - q_{t+s-1}$ .

The nominal price, in natural units, of a bond that pays a quarterly coupon<sup>12</sup>  $C$  can then be expressed as a linear combination of zero coupon log prices as follows:

$$P_{nt}^{Nom,c} = \sum_{s=1}^n \exp(p_{st}^{Nom})C + \exp(p_{nt}^{Nom}) \quad (19)$$

This price is exposed to changes in current inflation to the extent that it influences expectations of future inflation and the consumption components of the stochastic discount factor.

#### 4. Estimation Method and Data

Our focus is on maximum likelihood estimation of the log-linearized bond pricing models described above. We use a ‘panel’ of observed prices consisting of a time-series of a selection of about six bonds in each period. The structural parameters can be estimated from a single cross-section, or from a time-series of prices for a single bond. Subject to parameter stability, the simultaneous use of both cross-sectional and time-series data should increase the efficiency of the estimates and provide a sharper test of the model than

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<sup>12</sup>UK indexed coupons are paid 6-monthly. We fit this into our quarterly model by assuming half of the 6-monthly coupon to be paid each quarter. This introduces a small error due to the overvaluation of each coupon that accompanies our assumption that half of it is paid earlier than it is in reality.



we get from either cross-section or time-series estimation alone.

#### 4.1. A vector autoregressive model for the state variables

The nominal coupon bond price  $P_{nt}^{Nom,c}$  in (19) through (17) depends on expectations of the three state variables; consumption growth ( $g$ ), the change in the cross-sectional variance of consumption ( $v$ ), and inflation ( $\pi$ ), which we generate from a separately estimated vector autoregression as explained below.

Let  $w_t$  be a vector of state variables

$$w_t = \begin{pmatrix} g_t \\ v_t \\ \pi_t \end{pmatrix} \quad (20)$$

where all variables are in logs.

We assume the state vector to be autoregressive

$$w_{t+1} = A + Bw_t + \epsilon_{t+1} \quad (21)$$

where

$$\epsilon_{t+1} \sim N(0, \Omega_\epsilon) \quad \forall t$$

We define a set of coefficient vectors  $\phi_R$  to be consistent with equations (12) to (14) as follows:

$\phi_R^{RA}$	$\phi_R^{INC}$	$\phi_R^{PIPO}$
$-\gamma$	$-\gamma$	$-\gamma$
$0$	$\frac{\gamma(\gamma+1)}{2}$	$\frac{-\gamma(\gamma-1)}{2}$
$0$	$0$	$0$

and define a second set  $\phi_L$ , to capture the effects of inflation, as

$$\phi_L = \phi_R + \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

The log of the pricing kernels can then be written as,

$$m_{n,t+1}^h = \ln(\beta) + \phi_R' w_{t+1} \quad (22)$$

and, from (18),

$$E_t[z_{n,t+1}] = \sum_{i=1}^{n-3} \phi_R' E_t[w_{t+i}] + \sum_{i=n-2}^n \phi_L' E_t[w_{t+i}] + \ln(\beta) \quad (23)$$

$$Var_t[z_{n,t+1}] = \sum_{i=1}^{n-3} \phi_R' \Omega_{t+i} \phi_R + \sum_{i=n-2}^n \phi_L' \Omega_{t+i} \phi_L \quad (24)$$

where

$$E_t[w_{t+i}] = \tilde{B}_i A + B w_t \quad (25)$$

$$\Omega_{t+i} = \sum_{j=0}^{i-1} B^j \Omega_\epsilon B^{j'} \quad \forall t \quad (26)$$

and

$$\tilde{B}_i = \sum_{j=0}^{i-1} B^j$$

After substituting (23) and (24) into (17), the real price of the indexed zero-coupon bond can then be expressed in familiar affine form as:

$$p_{nt}^R = G_n + H_n w_t \quad (27)$$

where

$$G_n = \ln(\beta) + \left( \sum_{i=1}^{n-3} \phi'_R \tilde{B}_i A + \sum_{i=n-2}^n \phi'_L \tilde{B}_i A \right) + \frac{1}{2} \left( \sum_{i=1}^{n-3} \sum_{j=0}^{i-1} \left( \phi'_R B^j \Omega_\epsilon B^{j'} \phi_R \right) + \sum_{i=n-2}^n \sum_{j=0}^{i-1} \left( \phi'_L B^j \Omega_{t+i} B^{j'} \phi_L \right) \right) \quad (28)$$

$$H_n = \sum_{i=1}^{n-3} \phi'_R B + \sum_{i=n-2}^n \phi'_L B \quad (29)$$

The log nominal price follows as  $p_{nt}^{Nom} = p_{nt}^R + q_t$  which we substitute into (19) to obtain our estimation equation.<sup>13</sup>

$$P_{nt}^{Nom,c} = \sum_{s=1}^n \exp(p_{st}^R + q_t) C + \exp(p_{nt}^R + q_t) \quad (30)$$

We first estimate the vector autoregression for the state variables in order to obtain estimates of  $A$ ,  $B$  and  $\Omega_\epsilon$ , and then use maximum likelihood to estimate the parameters ( $\beta$  and  $\gamma$ ) of the asset pricing models by fitting equation (30) to market prices. The pricing errors are assumed to be normally

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<sup>13</sup>The details of the derivation are presented in Appendix Appendix B.

and independently distributed, and homoskedastic across both maturities and time.

Using all of the available data in this way greatly increases the number of degrees of freedom, but does so at the cost of imposing parameter constancy over the sample. Some degree of persistence in the parameter values seems reasonable, so our approach offers a potential efficiency gain over the familiar approach of estimating the yield curve parameters for each period independently. To allow for the possibility that the parameters change with changes in the policy regime we also estimate the model over a number of sub samples, as discussed below.

#### *4.2. Data*

We use bond price data from the UK Debt Management Office. Since all indexed bonds with a maturity of 8 months or less, are pure nominal bonds we select only bonds with a residual maturity of 2 years or more. The number of indexed bonds in the market in any quarter is very small, ranging from 7 to 9. We select 6 bonds in each period, aiming for as even a spread as possible across the maturities from 1 to 25 years. When choosing between bonds with similar maturities, we select the one with the largest issue size.

Aggregate real consumption data are from the Office for National Statistics, and the cross sectional variances of the log of real consumption are from the Family Expenditure Survey (FES).<sup>14</sup> Data are quarterly for the period

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<sup>14</sup>The FES was replaced by the Expenditure and Food Survey, which also covered the National Food Survey, in April 2001.

1983Q1 to 2004Q4 and are seasonally unadjusted.<sup>15</sup>

#### *4.3. Sub-samples and Monetary Policy Regimes*

We estimate the model over the full sample 1983Q2 to 2004Q4, and over the following sub-samples:

Sub sample	Monetary policy regime
1983Q2 to 1992Q3	Monetary-growth and exchange-rate targets.
1992Q4 to 2004Q4	Inflation target: all stages.
1992Q4 to 1997Q2	Inflation target: early stages.
1997Q3 to 2004Q4	Inflation target with Bank of England independence.

From 1983 to 1992 the UK sought to anchor inflation first with control of monetary aggregates, then by using an informal combination of monetary and exchange rate targets, and finally with a 2-year membership of the European Exchange Rate Mechanism (ERM). In the post-ERM period inflation was targeted directly by the Treasury, and then, from 1997, by the newly independent Bank of England. sub-sample estimation provides an informal check of the model's robustness. While it is unlikely that the preference parameters  $\beta$  and  $\gamma$  would change as a result of monetary regime changes, we might expect some instability in their estimates if the model is misspecified.

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<sup>15</sup>The details of the computation of the cross sectional variances are presented in Appendix Appendix A.

#### *4.4. Bank of England estimates of real yields.*

We compare the implied real yields from our models with those estimated by the Bank of England. UK indexed bonds do not provide unambiguous estimates of real rates however, due to the way in which they are indexed: their yields can be calculated only once we have data for expected inflation. The Bank of England calculates zero-coupon real yields conditional on their preferred method of dealing with inflation expectations, and we use these yields as a market benchmark for ours. Both sets of yields are based on fitting a model to market prices of indexed bonds. The principal difference between them is that we attempt to model the yield curve using a consumption-based asset pricing model; the Bank data are generated from a curve-fitting exercise that is not based on economic foundations. The Bank's objective is of course different from ours: we seek to estimate and explain real yields while the Bank seeks only to estimate them. Further, the parameters of the Bank model are re-estimated every day, in contrast to ours, which are held constant throughout each sample.

The Bank data are not complete: there are several missing observations at most maturities in our sample period 1983Q3 to 2004Q4, but these gaps do not appear at the same dates for each maturity. Further, the earliest Bank estimates are for 1985Q1, while our implied yields start in 1983Q3. The implications of this are discussed where Bank data are used below.

## 5. Results

### 5.1. Maximum Likelihood Estimates of $\beta$ and $\gamma$

Estimates of the coefficients  $\beta$  and  $\gamma$  are presented in Table B.1, along with likelihood values, and t-statistics in parentheses. The traditional representative agent model does not perform well. While the estimates of  $\beta$  are reasonable (implying a discount rate of about 1.5%) and highly significant, the estimates of  $\gamma$  are generally imprecise, with small t values, and are sometimes negative.

The estimates for INC and PIPO are substantially better, with statistically significant estimates of  $\beta$  and  $\gamma$  throughout. The estimates for  $\gamma$  are rather small, however, at about 0.2.<sup>16</sup>

In terms of the likelihood values, the RA model does not perform as well as the other two, but the results for the latter are too close to each other to allow us to choose between them. To the extent that a choice can be reached, it seems that the INC specification performs slightly better in the first half of the sample, while the PIPO specification is slightly better in the second. In terms of explaining the prices of indexed bonds however, it is clear that the incomplete-markets models have something to offer over the representative agent model.

We measure the goodness of fit to bond prices using Nagelkerke's (1991)

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<sup>16</sup>Estimation using GMM, following Hansen and Singleton (1982), and using the VAR factors as instruments, produced similar results for  $\gamma$ , with full-sample estimates of  $-0.48, 0.18, 0.23$  for RA, INC and PIPO respectively. Basu et al. (2011) also get small estimates of  $\gamma$  for the INC model.

generalized  $R^2$  ,

$$R^2 = 1 - \left( \frac{L(0, 0)}{L(\hat{\beta}, \hat{\gamma})} \right)^{\frac{2}{n}}$$

where  $n$  is the number of bonds in the sample, and  $L$  is the value of the likelihood function.

The results (which are not separately reported) are very similar for each model at around 0.05 for the full sample, and ranging from 0.1 to 0.25 for the sub-samples. The PIPO model has the highest of the three  $R^2$ s in each case. The measures suggest that there is a lot of variation in the prices of indexed bonds that is not accounted for by the factors underlying our models. Nevertheless, the models can explain 5% to 25% of the variation in prices, without letting the parameters of the model change from one period to the next, as is standard practice in market applications of no-arbitrage models.

## *5.2. Testing the plausibility of the model-implied yields.*

### *5.2.1. Moments of fitted and actual yields.*

In this section we ask whether the yields implied by the consumption-based models are consistent with those estimated by the Bank of England. It should be noted that the Bank's estimates are not definitive rates, they are estimates based on a specific methodology, which may or may not be more accurate than ours. Our models' implied yields for period are constructed using equation (30) after adapting the coefficients  $G_n$  and  $H_n$  to remove the effects of the lagging indexation. Yields are then calculated from the implied prices for a range of maturity values  $n$ .



The results show that the consumption-based models overestimate both the level and volatility of real yields at short maturities but that these errors diminish at longer maturities. The discrepancy in the level of short yields is likely to be due to the fact that the asset pricing models have only two parameters, and the majority of our price data is for bonds of longer maturities. Thus the estimated parameters are generated primarily by the fit to long-bond prices. The Bank data, by contrast, are based on a model that has many more parameters and which can fit both long and short ends of the curve. The discrepancy in the volatilities will reflect both this difference in the number of parameters and the fact that the asset pricing models' estimates are held constant throughout the sample, with the result that movements in the factors can create relatively large pricing errors for the poorly fitted short-bond prices. The Bank model on the other hand can alter its estimated parameters to match the prices of short-dated bonds.<sup>17</sup>

### *5.2.2. Steady-state yield curves.*

Figures B.1 to B.5 show our estimated steady yield curves plotted against sample average curves from the Bank of England. We calculate steady-state yields based on sub-sample-specific averages for the factors. The Bank curves are based on periods for which a complete curve is available. Consequently,

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<sup>17</sup>For each maturity we select only those dates for which Bank of England data are available; these dates are then used for selecting our implied yields also. Thus, for example, the 2.5-year full sample starts in 1985Q1 while that for 20-year yields starts in 1986Q3. There are further missing periods within these sample i.e. they do not occur only at the start of the samples.

the Bank curves miss out several observations that are included in the steady-state curves, which will contribute to the differences between the curves.

Piazzesi and Schneider (2007) report that the UK 2-year to 10-year real-yield curve sloped down from 6.12% to 4.12% on average in the period from January 1983 to November 1995, and from 2.59% (at 2.5 years) to 2.41% from December 1995 to March 2006. Figure B.1 suggests that the UK real-yield curve was upward sloping on average from 1986 to 2004. The difference is probably due to the different sample periods.

The average Bank curves differ from the average yields quoted in Table B.2 due to the treatment of missing values in the Bank data. Our Bank average curves in the figures above are based only on those periods for which a complete yield curve is available. Hence, for example, the full-sample average real yields at 2.5, 10 and 20 years from the Bank data, using only those periods for which a complete curve is available, are 2.70%, 2.91% and 2.94%; the averages based on all of the observations available for each maturity individually are 2.99%, 3.22% and 3.02%.

The main feature of the figures is that all of our estimated curves slope down, while the Bank's data include upward, downward and relatively flat curves.

### 5.3. Risk premia.

The expected 1-period return on an  $n$ -period pure real bond is

$$E_t[r_{n,t+1}] = -E_t[m_{t+1}] - \frac{1}{2}Var_t[m_{t+1} + p_{n-1,t+1}] \quad (31)$$

from which we get the expected excess return ( $\tilde{r}$ ) of an  $n$ -period bond over that of an  $l$ -period bond as

$$E_t[\tilde{r}_{n,l,t+1}] = -(Cov_t[m_{t+1}, p_{n-1,t+1}] - Cov_t[m_{t+1}, p_{l-1,t+1}]) \quad (32)$$

$$= -Cov_t[\phi'_R w_{t+1}, (H_{n-1} - H_{l-1})w_{t+1}] \quad (33)$$

$$= -(H_{n-1} - H_{l-1})\Omega_\epsilon \phi_R \quad (34)$$

In the case of a one-period, and therefore riskless, bond we have  $H_{1-1} = H_0 = 0$  from which we get the *ex ante* risk premium on an  $n$  period bond as

$$E_t[\tilde{r}_{n,1,t+1}] \equiv \rho_n = -H_{n-1}\Omega_\epsilon \phi_R$$

Our assumption that the errors in the VAR are homoskedastic (i.e. that  $\Omega_\epsilon$  is constant) results in *ex ante* risk premia that are time invariant. Table B.3 shows that the point estimates of these *ex-ante* risk premia are almost zero.

Our estimate of  $\gamma$  is surprisingly small when compared with the estimates of earlier papers. The earlier estimates of  $\gamma$  depend very much on the choice of financial assets and asset holders. Using US aggregate data Hansen-Singleton (1983) obtained an estimate on the low side (0.7 to 0.8). The seminal paper of Mehra and Prescott (1985) find  $\gamma$  to be implausibly high to resolve the equity premium puzzle. Using micro UK household data, Attanasio et al. (2002) find that the bond holder's risk aversion is generally lower than

that of stock holders. Vissing-Joregensen (2002) get a higher estimate of the elasticity of intertemporal substitution for US bond holders which, in a power utility function means lower risk aversion for bond holders. Basu et al. (2011) address four financial market puzzles and find a low estimate of  $\gamma$  when an investor participates in stock, bond and foreign exchange markets when faced with uninsurable risk, as in the present setting.

In our present study we exclusively focus on the UK bond investors. Our low estimate of  $\gamma$  is consistent with the study of Attanasio et al. (2002) who find that UK bond holders have lower risk aversion. In addition, we use a model with uninsurable risk similar to Basu et al. (2011) who also find that the estimate of  $\gamma$  is rather low for this class of incomplete market models, particularly the INC market structure.

The small risk aversion parameter, and the resulting small implied ex ante and ex post risk premia, raise the question of why investors appear to demand so little compensation for real term-structure risks. The poor returns on indexed bonds are noted in Dimson *et al* (2002) who find that indexed bonds returned 1.25% p.a. less than Treasury bills for the period 1981 to 2000. They offer 2 possible explanations:

‘...their poor performance stems from unexpected increases in the real rate of interest particularly in the early 1980s.’

and

‘Since most inflation-indexed bonds have low coupons, and since there is no capital gains tax on UK government bonds, they are attractive to high-rate taxpayers relative to most conventional bonds. The low returns on inflation-indexed bonds may therefore also partly reflect the influence of tax clienteles’ (Dimson *et al* (2002), page 86.)

Our results suggest a third possibility i.e. that the low returns may have arisen from a low aversion to risk on the part of indexed-bond investors. This explanation may complement Dimson *et al*’s tax-clienteles argument. It also suggests that, while unexpected increases in real rates may be part of the explanation, they are not a necessary part since low returns could arise from low risk aversion even if interest expectations were, on average, correct.

#### 5.4. Real-rate responses to factor shocks.

##### 5.4.1. Impulse effects.

We examine the impulse responses of real interest rates to shocks to the factors in the form of 1-period ahead expectations of consumption growth, the change in cross-sectional consumption variance, and inflation.

The system of equations can be represented as,

$$p_{n,t}^R = G_n + H_n w_t \quad n = 1, 2, \dots \quad (35)$$

$$w_t = A + Bw_{t-1} + \epsilon_t \quad (36)$$

from which we get the prices as functions of the history of the factor shocks  $\epsilon$  as

$$p_{n,t}^{R,z} = G_n + H_n(I - B)^{-1}A + H_n(I - BL)^{-1}\epsilon_t \quad (37)$$

Real interest rates at all maturities  $n$  follow directly from this equation.

The  $\epsilon_t$  terms are mutually correlated so we recast these as linear functions of three orthogonal random terms  $\xi_t$  and measure the response of real rates to shocks to the latter. Thus we assume that,

$$\epsilon_{1t} = c_{11}\xi_{1t} + c_{12}\xi_{2t} + c_{13}\xi_{3t} \quad (38)$$

$$\epsilon_{2t} = c_{21}\xi_{1t} + c_{22}\xi_{2t} + c_{23}\xi_{3t} \quad (39)$$

$$\epsilon_{3t} = c_{31}\xi_{1t} + c_{32}\xi_{2t} + c_{33}\xi_{3t} \quad (40)$$

This leads to the familiar problem that we cannot identify all 9  $c_{ij}$  coefficients from the 6 independent coefficient estimates in  $\hat{\Omega}_\epsilon$ . We deal with this in the usual way with a Cholesky decomposition of the covariance matrix  $\Omega_\epsilon$  i.e. we impose zero-restrictions on  $c_{12}$ ,  $c_{13}$  and  $c_{23}$ . This is equivalent to assuming that the shock  $\xi_{1t}$  influences all three variables,  $\xi_{2t}$  influences only the latter two, and  $\xi_{3t}$  influences only the third. Since the ordering of the  $\epsilon$ s is not unique (we could put the 3 variables in the VAR in any order), and because we have no prior information as to the real-world ordering under these identifying restrictions (if in fact any is correct), we present results for four of the six possible orderings; for the remaining two the reordered  $\Omega$  is

not positive definite and, therefore, there is no Cholesky decomposition.

Thus we define

$$\epsilon_t = C\xi_t \quad (41)$$

$$E[\xi_t\xi_t'] = I \quad (42)$$

where  $C$  is lower-triangular

Hence,

$$\Omega_\epsilon = CC' \quad (43)$$

Substituting (41) into the bond price equations we get:

$$p_{n,t}^R = G_n + H_n(I - B)^{-1}A + H_n(I - BL)^{-1}C\xi_t \quad n = 1, 2, \dots \quad (44)$$

In order to give the shocks to the  $\xi$  a clearer economic meaning we scale them such that they generate a 1 percentage point increase in each of the factors in turn. For example, in Table ??, the first row shows the effects of a shock to  $\xi_{1t}$  such that expected consumption growth increases by 1%. In line with the ordering of the VAR, this same  $\xi_{1t}$  shock also generates a contemporaneous 30.71% increase in the change of the cross-sectional variance of consumption, and a 0.12% decline in inflation. The qualitative effects on yields of all of the factor shocks turn out to be robust to changes in the order

of the factors.

Table B.4 presents the impulse responses of yields at maturities of 2, 10 and 20 years to factor shocks, based on the full-sample estimates. Figures B.6 to B.29 show responses for the full-sample INC and PIPO models corresponding to the impulse responses in the table.

The effect of a shock to inflation that is not accompanied by shocks to the other two factors can be seen from lines 3 and 6 of Table B.4.<sup>18</sup> For both the INC and PIPO models, there are small falls in real rates at all maturities. Results for 3-month rates (not reported) show that the short real rate does not respond to changes in expected inflation since agents are assumed to optimise their utility over real magnitudes, and there are no changes in the consumption factors in the utility function. At all longer maturities however, the effect of a current inflation shock on expectations of future consumption growth and variance do have an impact on real rates by altering the utility value of future real returns. The negative response of *ex-ante* real rates is consistent with results found in Barr and Campbell (1997) and others, and provide a possible explanation for their results. This negative impact of expected inflation on real rates arises for all of the VAR orderings, although with inflation placed at position 1 or 2 in the VAR the associated contemporaneous shocks to the other factors generate negative responses in the 3-month rate also.

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<sup>18</sup>The effects on yields are independent of the ordering of the other two variables because the shock to inflation does not impact on them.



Increases in expected consumption growth lead to increases in real rates for both models (with the exception of the 2-year rates for the INC model) irrespective of the ordering of the factors: higher consumption growth lowers agent's incentive to save and financial markets respond by offering a higher real yield as the demand for real bonds declines.

Positive shocks to the cross-sectional variance of consumption cause real rates to fall in all cases, which is consistent with the Euler equations (2) and (3) given that the estimated  $\gamma < 1$  in both the INC and PIPO models. A higher cross-sectional variance in consumption means that consumers face greater uninsurable risk. In an both INC and PIPO environments with zero or partial insurance, consumers increase their saving for precautionary reasons, and this increase in the supply of loanable funds drives down real interest rates. In a PIPO environment, with partial insurance, an opposing effect on saving will be at work. Greater consumption inequality (i.e. higher variance of consumption) lowers the agency cost because it is cheaper to provide incentives to poor people. This lower agency cost may create a wealth effect that lowers the incentive to save. Thus we expect the effect on the real interest rate to be weaker in the PIPO environment.

#### *5.4.2. Response paths.*

Figures B.6 to B.29 show the response paths of real rates following the impulse effects presented in Table B.4. The figures add two pieces of information to the results in the Table. First, the yield response paths are non-monotonic, which reflects the presence of complex roots in the VAR,

and second, the responses all die out within 2 years.

To summarize the main results: A positive shock to inflation lowers real rates as a consequence of its effects on consumption growth and consumption variance; a positive shock to consumption growth raises real interest rates as markets compete for reduced savings, and a positive shock to uninsurable consumption risk lowers real rates as the precautionary motive drives agents to save more.

Piazzesi and Schneider (2007) present a similar impulse response analysis for US real rates. They obtain a very clean result about real interest rates: growth and inflation surprises move short-maturity real rates in opposite directions but have only small effects on long real rates. In particular, a positive inflation surprise decreases short-maturity real rates (with a half-life of about 5 years), while a growth surprise increases them (with a half life of about 6 months).<sup>19</sup> Both of our incomplete markets models produce impact effects of the same sign as Piazzesi and Schneider (2007) (see lines 3, 6 and 12 of Table B.4), and have only small effects on long rates. Our response paths are non-monotonic however, in contrast to those of P-S, and die out completely within about 2 years.

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<sup>19</sup>The half-lives are our estimates based on the charts presented in Piazzesi and Schneider (2007) p405.

## 6. Summary and conclusions

This paper tests three consumption-based asset pricing models applied to indexed bonds in the UK. We employ a three factor model of log normal bond pricing. Our novelty lies in deriving closed form expressions for the pricing kernels of the new class of uninsurable risk models and integrating this with a lognormal affine form bond pricing function. This innovation allows us to derive the price function of indexed coupon bonds in an estimable form with a convenient marriage between VAR based representation of the state variables and the bond price equation. Our central equation is a log linear bond price equation in which expected values of the state variables are constructed from a parsimonious VAR involving three macroeconomic variables, namely the growth rate of aggregate consumption, cross section variance of consumption and the rate of inflation.

Not surprisingly we find that the standard complete markets model with homogenous agents do not fare well with inflation-indexed bonds. Our results appear to lend support to the new class of uninsurable risk models. These uninsurable-risk models can account for about 20% of the variation in indexed bond prices in sub-samples of our data. The impulse response analysis with the estimated bond price equation reveals that a rise in inflation lowers the real interest rates of almost all maturities while rise in aggregate consumption growth rate raises real interest rates. An increase in uninsurable risk, on the other hand, lowers the real interest rates. These impulse response results agree with our basic economic intuition thus lending further support

to these models.

Comparisons of the implied real yield curves from our incomplete markets models with those estimated by the Bank of England suggest that our models generate real rates that are about 1.6% greater at the short (2.5-year) end of the curve and about 30 basis points greater at the long (20-year) end. Our models also generate greater volatility in short rates but the volatility of long rates is broadly the same as the Bank's estimates. The differences between the two sets of estimates are likely to lie in: the fact that the Bank re-estimates the parameters of its curve every day, while we impose parameter constancy with each sample period; differences in the way in which inflation expectation are generated (we use a backward-facing VAR while the Bank uses forward-facing break-even rates), and the Bank estimates are not constrained to be functions of consumption and inflation. The last of these points is crucial in the sense that while our estimated models suggest that consumption growth and incomplete markets explain part of the behaviour of real rates they clearly do not tell the full story if we regard the Bank's estimates as more representative of actual real rates than ours.

Our results give rise to new questions and challenges. Why, for example, are the estimated coefficient of relative risk aversion and the resulting bond risk premia so small in the UK indexed bond market? One possible explanation is that UK bond market is extremely segmented and populated with near risk neutral institutional investors. An alternative explanation may be that our utility function could be misspecified. A more general func-

tion, combining the uninsurable risk features with the separation between risk aversion and intertemporal substitution in consumption as in Piazzesi and Schneider (2007) may lead to larger estimates of the degree of risk. To the best of our knowledge however, there is as yet no theory that integrates these incomplete market models with nonexpected utility maximization, and is likely to be an interesting avenue for further research.

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## Appendix A. Construction of the Cross Sectional Distribution of Consumption

We construct the cross sectional variance of real consumption using the records of daily expenditure from the Family Expenditure Survey (FES) conducted by the Office for National Statistics (ONS). The data we use are based on the expenditure of approximately 6,500 households for a period of 2 weeks in every quarter.

Our procedure mimics Kocherlakota and Pistaferri (2009, 2007). First, the household-wide consumption of nondurables and services is calculated by adding the nominal consumption of nondurables and services for each individual in the household. We follow the definition of nondurable and services of Attanasio and Weber (1995). Second, since the household consumption data are two week durations only, we multiply them by 6.5 to arrive at quarterly frequency. Third, we divide this quarterly consumption expenditure of each household by the number of people in the household in that quarter to arrive at the quarterly nominal, consumption of nondurables and services per member of each household unit. Fourth, by dividing the quarterly data by the quarterly CPI for all items (not seasonally adjusted) (the CPI is from the OECD main economic indicators) with the basis of 2005:Q1, we arrive at the quarterly real per capita consumption for all the relevant households.

### *Appendix A.1. Measurement errors*

KP (2009) alert us to measurement errors from the use of cross section expenditure data. In our context, if these measurement errors appear mul-

tiplicatively they do not impact the pricing kernels. To see this define the measured consumption as:

$$\hat{c}_{i,t} = c_{i,t} \exp(\xi_{i,t})$$

where the measurement error  $\xi_{i,t}$  is stationary, i.i.d. across households, and uncorrected with  $z_t$ , we get:

$$\hat{x}_t - \hat{x}_{t-1} = x_t - x_{t-1}$$

Since we work with the first difference of the variance of log consumption, the measurement error is not an issue.

*Appendix A.2. Pricing kernel from Campbell's (1984) taste shock model*

As in Campbell let the taste shock impact the power utility function of the  $i$ th consumer as follows :

$$U(c_{it}) = \lambda_{it} \cdot \frac{c_t^{1-\gamma} - 1}{1-\gamma}$$

where  $\lambda_{it}$  represents the taste shock, and  $c_t$  is average consumption.

For a power utility function the intertemporal marginal rate of substitution (IMRS) of the  $i$ th consumer is:

$$M_{t+1}^i = \beta \frac{\lambda_{it+1}}{\lambda_{it}} \cdot \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma}$$

Given any generic asset return (say  $R_{t+1}$ ), the return equation facing the

ith consumer can be written in a generic form as:

$$E_t M_{t+1}^i R_{t+1} = 1$$

Using the law of iterated expectation we can rewrite the above as:

$$E_t E_i M_{t+1}^i R_{t+1} = 1$$

where subscript  $i$  stands for the cross section aggregation. This is the same as:

$$E_t \beta \left\{ E_i \frac{\lambda_{it+1}}{\lambda_{it}} \right\} \cdot \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1} = 1$$

The immediate implication is that the pricing kernel for the taste shock model (call it  $M_{t+1}^{taste}$ ) is:

$$M_{t+1}^{taste} = \beta \left\{ E_i \frac{\lambda_{it+1}}{\lambda_{it}} \right\} \cdot \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \quad (\text{A.1})$$

The exact pricing kernel depends on how one posits the process for the taste shock. The additional term  $E_i \frac{\lambda_{it+1}}{\lambda_{it}}$  in (A.1) is the cross sectional first moment of the growth rate of taste shock which captures taste heterogeneity. Note the important difference between the taste shock pricing kernel (A.1), and the incomplete market pricing kernels (7) and (8). The additional terms in the latter two incomplete market pricing kernels are functions of  $\gamma$  and the latent variable  $x_t$ . The interaction between the risk aversion parameter  $\gamma$  and the cross sectional variance term  $x_t$  captures the essence of the uninsurable

risk as in Kocherlakota and Pistefferri (2007, 2009). On the other hand, Campbell's taste shock pricing kernel shows no such interaction between risk aversion and the latent variable. The additional term in (A.1) is free from  $\gamma$ . This shows the clear difference between these two classes of pricing kernels.

The taste shock models are, however, empirically useful because they depart from the standard representative model pricing kernels and have the potential to solving many asset pricing puzzles. The exact form of the pricing kernel, however, depends on the cross sectional distribution of taste shocks.<sup>20</sup>

## Appendix B. Derivation of the estimated price equations.

### Appendix B.1. From zero-coupon to coupon bonds.

We price a coupon bond as the sum of the prices of its coupons and redemption payment. I.e.

$$P_{nt}^{N,C} = \sum_{s=1}^n P_{st}^{Nom} C + p_{nt}^{Nom} \quad (\text{B.1})$$

hence, since

$$P_{st}^{Nom} = \exp(p_{st}^{Nom}) \quad (\text{B.2})$$

---

<sup>20</sup>In some special cases, a taste shock model may reduce to a representative agent pricing kernel. For example, if the taste shock follows a lognormal random walk such as:

$$\lambda_{it+1} = \lambda_{it} \exp(u_{it} \sqrt{x_t} - \frac{x_t}{2})$$

with a standard normal deviate  $u_{it}$ , then  $E_i \frac{\lambda_{it+1}}{\lambda_{it}} = 1$ , it is easy to verify that the  $M_{t+1}^{taste}$  reduces to a RA pricing kernel. On the other hand, our INC and PIPO pricing kernels can reduce to RA pricing kernels only when the cross sectional variances in consumption  $x_t$  is time invariant.

etc, we have the log price of a coupon bond as

$$P_{nt}^{N,c} = \sum_{s=1}^n \exp(p_{st}^{Nom})C + \exp(P_{nt}^{Nom}) \quad (\text{B.3})$$

where  $p_{st}^{Nom}$  and  $P_{nt}^{Nom}$  have the forms in the previous section.

*Appendix B.2. Incorporating the macroeconomic factors.*

First, we introduce a vector notation for the factors i.e.

$$w_t = \begin{pmatrix} g_t \\ v_t \\ \pi_t \end{pmatrix} \quad (\text{B.4})$$

where  $g_t = c_t - c_{t-1}$ ,  $v_t = x_t - x_{t-1}$  and  $\pi_t = q_t - q_{t-1}$  and  $q_t$  is the RPI.

Now introduce 2 selection vectors to allow us to pick out different combinations of the factors i.e.

$$\phi_R = \begin{pmatrix} -\gamma \\ \pm \left( \frac{\gamma(\gamma \pm 1)}{2} \right) \\ 0 \end{pmatrix} \quad (\text{B.5})$$

$$\phi_L = \begin{pmatrix} -\gamma \\ \pm \left( \frac{\gamma(\gamma \pm 1)}{2} \right) \\ -1 \end{pmatrix} \quad (\text{B.6})$$

*Appendix B.3. The stochastic discount factors.*

For the stochastic discount factors (13) and (14),

$$M_{t+i} = \beta G_{t+i}^{-\gamma} \exp \left[ \left( \frac{\gamma(\gamma \pm 1)}{2} \right) (x_{t+i} - x_{t+i-1}) \right] \quad (\text{B.7})$$

$$\Rightarrow m_{t+i} = \ln \beta - \gamma g_{t+i} \pm \left[ \left( \frac{\gamma(\gamma \pm 1)}{2} \right) (x_{t+i} - x_{t+i-1}) \right] \quad (\text{B.8})$$

$$= \ln \beta - \gamma g_{t+i} \pm \left[ \left( \frac{\gamma(\gamma \pm 1)}{2} \right) v_{t+i} \right] \quad (\text{B.9})$$

$$= \ln \beta + \phi'_R w_{t+i} \quad (\text{B.10})$$

Now substitute this expression for  $m$  into the price equation (??) to get,

$$\begin{aligned} p_{nt}^n - (q_t - q^*) &= E_t (\phi'_R w_{t+1} + \dots + \phi'_R w_{t+n-3} + \phi'_L w_{t+n-2} + \phi'_L w_{t+n-1} + \phi'_L w_{t+n}) + \\ &\quad \frac{1}{2} \text{Var}_t(\dots) + n \ln \beta \end{aligned} \quad (\text{B.11})$$

The terms  $\phi'_R w_{t+1} + \dots + \phi'_R w_{t+n-3}$  come directly from the equation for  $m$  above. The others,  $\phi'_L w_{t+n-2} + \phi'_L w_{t+n-1} + \phi'_L w_{t+n}$ , are a combination of the  $m$  terms and inflation, for the last 3 months of the bond's life i.e. the period after the indexation ends, and the bond's real value is exposed to inflation.

So a convenient alternative way to write  $z$  is,

$$z_{n,t+1} = (\phi'_R w_{t+1} + \dots + \phi'_R w_{t+n-3} + \phi'_L w_{t+n-2} + \phi'_L w_{t+n-1} + \phi'_L w_{t+n}) \quad (\text{B.12})$$

*Appendix B.4. Time series projections for the factors.*

For the case of a VAR(1) we have,

$$\begin{aligned} w_{t+1} &= A + Bw_t + \epsilon_{t+1} \\ &= A_n + B_n w_t + \eta_{t+n} \end{aligned} \quad (\text{B.13})$$

where

$$A_n = (I + B + \dots + B^{n-1})A \quad (\text{B.14})$$

$$B_n = B^n \quad (\text{B.15})$$

$$\eta_{t+n} = \epsilon_{t+n} + B\epsilon_{t+n-1} + \dots + B^{n-1}\epsilon_{t+1} \quad (\text{B.16})$$

It follows that, introducing  $\Omega_{t+n} \equiv \text{Var}_t(\eta_{t+n})$ , which we assume to be constant w.r.t  $t$ ,

$$E_t(w_{t+n}) = A_n + B_n w_t \quad (\text{B.17})$$

$$\begin{aligned} \text{Var}_t(w_{t+n}) &= \text{Var}_t(\eta_{t+n}) \\ &= \Omega + B_1 \Omega B_1' + \dots + B_{n-1} \Omega B_{n-1}' \\ &= \Omega_{t+n} \end{aligned} \quad (\text{B.18})$$

More compactly,

$$\Omega_{t+i} = \sum_{j=0}^{i-1} B_j \Omega_\epsilon B_j' \quad \forall t, \text{ and } i = 1 \dots n \quad (\text{B.19})$$



*Appendix B.5. Derivations of  $E_t(z)$  and  $Var_t(z)$ .*

Given that

$$z_{n,t+1} = + (\phi'_R w_{t+1} + \dots + \phi'_R w_{t+n-3} + \phi'_L w_{t+n-2} + \phi'_L w_{t+n-1} + \phi'_L w_{t+n}) \quad (\text{B.20})$$

we get,

$$\begin{aligned}
E_t(z_{n,t+1}) &= E_t[\phi'_R(w_{t+1} + \dots + w_{t+n-3}) + \phi'_L(w_{t+n-2} + w_{t+n-1} + w_{t+n})] \\
&= \phi'_R((A_1 + B_1w_t) + \dots + (A_{n-3} + B_{n-3}w_t)) + \\
&\quad \phi'_L((A_{n-2} + B_{n-2}w_t) + (A_{n-1} + B_{n-1}w_t) + (A_n + B_nw_t)) \\
&= \phi'_R(A_1 + \dots + A_{n-3}) + \phi'_L(A_{n-2} + A_{n-1} + A_n) + \\
&\quad \phi'_R(B_1w_t + \dots + B_{n-3}w_t) + \phi'_L(B_{n-2}w_t + B_{n-1}w_t + B_nw_t) \\
&= \phi'_R(A_1 + \dots + A_{n-3}) + \phi'_L(A_{n-2} + A_{n-1} + A_n) + \\
&\quad \phi'_R(B_1 + \dots + B_{n-3})w_t + \phi'_L(B_{n-2} + B_{n-1} + B_n)w_t \\
&= [\phi'_R(A_1 + \dots + A_{n-3}) + \phi'_L(A_{n-2} + A_{n-1} + A_n)] + \\
&\quad [\phi'_R(B_1 + \dots + B_{n-3}) + \phi'_L(B_{n-2} + B_{n-1} + B_n)]w_t \\
&= \left( \sum_{i=1}^{n-3} \phi'_R A_i + \sum_{i=n-2}^n \phi'_L A_i \right) + \\
&\quad \left( \sum_{i=1}^{n-3} \phi'_R B_i + \sum_{i=n-2}^n \phi'_L B_i \right) w_t \tag{B.21}
\end{aligned}$$

$$\begin{aligned}
Var_t(z_{n,t+1}) &= \phi'_R (\Omega_{t+1} + \dots + \Omega_{t+n-3}) \phi_R + \phi'_L (\Omega_{t+n-2} + \dots + \Omega_{t+n}) \phi_L \\
&= \phi'_R \left( \sum_{j=0}^0 B_j \Omega_\epsilon B'_j + \dots + \sum_{j=0}^{n-3-1} B_j \Omega_\epsilon B'_j \right) \phi_R + \\
&\quad \phi'_L \left( \sum_{j=0}^{n-2-1} B_j \Omega_\epsilon B'_j + \dots + \sum_{j=0}^{n-1} B_j \Omega_\epsilon B'_j \right) \phi_L \quad (B.22)
\end{aligned}$$

*Appendix B.6. The final equation for a zero-coupon indexed bond.*

Recall,

$$p_{nt}^{Nom} - (q_t - q^*) = n \ln \beta + E_t(z_{n,t+1}) + \frac{1}{2} Var_t(z_{n,t+1}) \quad (B.23)$$

we can substitute for the conditional expectations and variances of  $w$  that appear in  $z$ . The expectations introduce a series of terms in the constant  $A$ , which when added to the constant conditional variance, gives us the constant term in the price equation.

The time-varying element i.e. the terms in the factors  $w_{t+i}$  are all functions of  $w_t$ . Hence, the real price,  $p_{n,t}^R \equiv p_{nt}^{Nom} - (q_t - q^*)$ , is

$$p_{n,t}^R = G_n + H_n w_t \quad (B.24)$$

where

$$G_n = \ln(\beta) + \left( \sum_{i=1}^{n-3} \phi'_R A_i + \sum_{i=n-2}^n \phi'_L A_i \right) + \frac{1}{2} \left( \sum_{i=1}^{n-3} \sum_{j=0}^{i-1} (\phi'_R B_j \Omega_\epsilon B'_j \phi_R) + \sum_{i=n-2}^n \sum_{j=0}^{i-1} (\phi'_L B_j \Omega_\epsilon B'_j \phi_L) \right) \quad (\text{B.25})$$

$$H_n = \sum_{i=1}^{n-3} \phi'_R B_i + \sum_{i=n-2}^n \phi'_L B_i \quad (\text{B.26})$$

Table B.1: Estimation results.

	RA			INC			PIPO		
	$\beta$	$\gamma$	LF	$\beta$	$\gamma$	LF	$\beta$	$\gamma$	LF
1983-2004	0.9950 (11.55)	-0.0144 (-0.3115)	5.099	0.9958 (23.59)	0.1328 (11.16)	5.792	0.9962 (17.41)	0.1727 (8.012)	5.767
1983-1992	0.9948 (11.59)	-0.0071 (-0.1666)	4.983	0.9955 (18.63)	0.1319 (7.613)	5.468	0.9957 (13.69)	0.1607 (5.423)	5.434
1992-2004	0.9959 (3.05)	0.0501 (0.3478)	5.237	0.9962 (14.61)	0.1336 (8.043)	6.154	0.9967 (10.04)	0.1897 (5.356)	6.171
1992-1997	0.9964 (2.82)	0.3363 (2.635)	6.517	0.9941 (20.60)	0.1371 (12.59)	8.160	0.9947 (15.20)	0.2085 (7.320)	8.308
1997-2004	0.9975 (1.91)	0.0609 (0.4200)	6.583	0.9977 (9.24)	0.1353 (4.580)	7.692	0.9983 (3.66)	0.2004 (3.094)	7.737

LF is the value of the log-likelihood function. Figures in parentheses are t-statistics for  $H_0(\beta) = 1$  and  $H_0(\gamma) = 0$ .

Table B.2: Moments of estimate yields.  
2.5 year

		BoE	RA	INC	PIPO
1985-2004	Mean	2.99	4.62	4.56	4.56
	Var	0.796	2.13	1.62	1.54
1985-1992	Mean	3.40	5.49	5.48	5.47
	Var	0.651	2.54	2.10	2.01
1992-2004	Mean	2.69	3.97	3.88	3.89
	Var	0.686	1.09	0.553	0.529
1992-1997	Mean	3.11	4.88	4.74	4.74
	Var	0.251	1.36	0.658	0.591
1997-2004	Mean	2.45	3.33	3.27	3.27
	Var	0.771	0.852	0.492	0.487
10 year					
		BoE	RA	INC	PIPO
1985-2004	Mean	3.22	3.77	3.71	3.71
	Var	0.667	1.71	1.32	1.26
1985-1992	Mean	3.93	4.57	4.55	4.55
	Var	0.0980	2.04	1.75	1.70
1992-2004	Mean	2.76	3.23	3.14	3.13
	Var	0.495	0.983	0.527	0.505
1992-1997	Mean	3.49	4.14	4.03	4.02
	Var	0.101	1.30	0.668	0.608
1997-2004	Mean	2.28	2.56	2.50	2.50
	Var	0.174	0.686	0.432	0.429
20 year					
		BoE	RA	INC	PIPO
1985-2004	Mean	3.02	3.36	3.32	3.34
	Var	0.770	1.12	0.765	0.751
1985-1992	Mean	4.02	4.16	4.23	4.28
	Var	0.0726	1.13	1.04	0.982
1992-2004	Mean	2.76	3.09	3.00	2.99
	Var	0.628	0.957	0.513	0.492
1992-1997	Mean	3.65	3.99	3.89	3.88
	Var	0.0574	1.27	0.650	0.593
1997-2004	Mean	2.18	2.42	2.37	2.36
	Var	0.146	0.667	0.421	0.418

Table B.3: Implied *ex-ante* risk premia.

	RA	INC	PIPO
1983-2004	-1.035e-007	-4.115e-006	-9.450e-006
1983-1992	-2.535e-008	-4.062e-006	-8.099e-006
1992-2004	-1.245e-006	-4.160e-006	-1.156e-005
1992-1997	-5.603e-005	-4.364e-006	-1.416e-005
1997-2004	-1.839e-006	-4.257e-006	-1.300e-005

Table B.4: Impulse responses to factor shocks.

Cholesky order.	Factor shocks.			Yields: INC			Yields: PIPO		
	Growth	Variance.	Inflation	2 year	5 year	10 year	2 year	5 year	10 year
1.000 2.000 3.000	1.000	26.54	-0.1183	0.0131	0.0053	0.0027	0.0222	0.0091	0.0045
1.000 2.000 3.000	0.0000	1.000	7.971e-005	-0.0005	-0.0002	-9.300e-005	-0.0004	-0.0002	-9.004e-005
1.000 2.000 3.000	0.0000	0.0000	1.000	-0.0016	-0.0007	-0.0004	-0.0041	-0.0018	-0.00087
2.000 1.000 3.000	0.0037	1.000	-0.0002	-0.0011	-0.0005	-0.0002	-0.0011	-0.0004	-0.0002
2.000 1.000 3.000	1.000	0.0000	-0.1204	0.0254	0.0103	0.0051	0.0342	0.0139	0.0069
2.000 1.000 3.000	0.0000	0.0000	1.000	-0.0016	-0.0007	-0.0004	-0.0041	-0.0018	-0.0009
1.000 3.000 2.000	1.000	7.812	-0.1183	0.0131	0.0053	0.0027	0.0223	0.0091	0.0045
1.000 3.000 2.000	0.0000	0.3635	1.000	-0.0022	-0.0002	-0.0005	-0.0046	-0.0020	-0.0010
1.000 3.000 2.000	0.0000	1.000	0.0000	-0.0016	-0.0007	-0.0003	-0.0015	-0.0006	-0.0003
3.000 2.000 1.000	-0.1000	-1.424	1.000	-0.0004	-0.0005	-0.0007	-0.0068	-0.0028	-0.0014
3.000 2.000 1.000	0.0014	1.000	0.0000	-0.0004	0.0103	-8.586e-005	-0.0004	-0.0002	-8.043e-005
3.000 2.000 1.000	1.000	0.0000	0.0000	0.0252	-0.0007	0.0051	0.0337	0.0136	0.0068

*Results from impChart.gss June 2011*



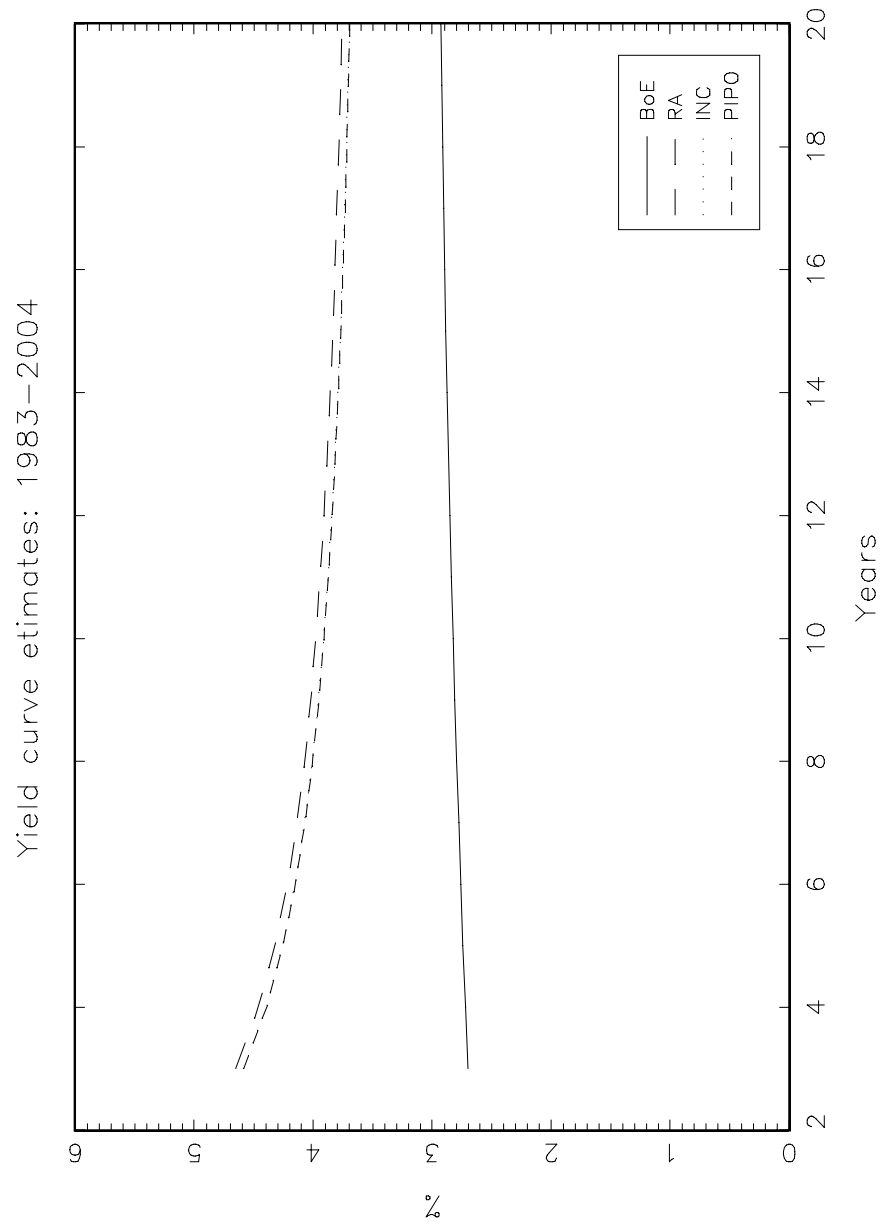


Figure B.1: Steady-state yield curves.

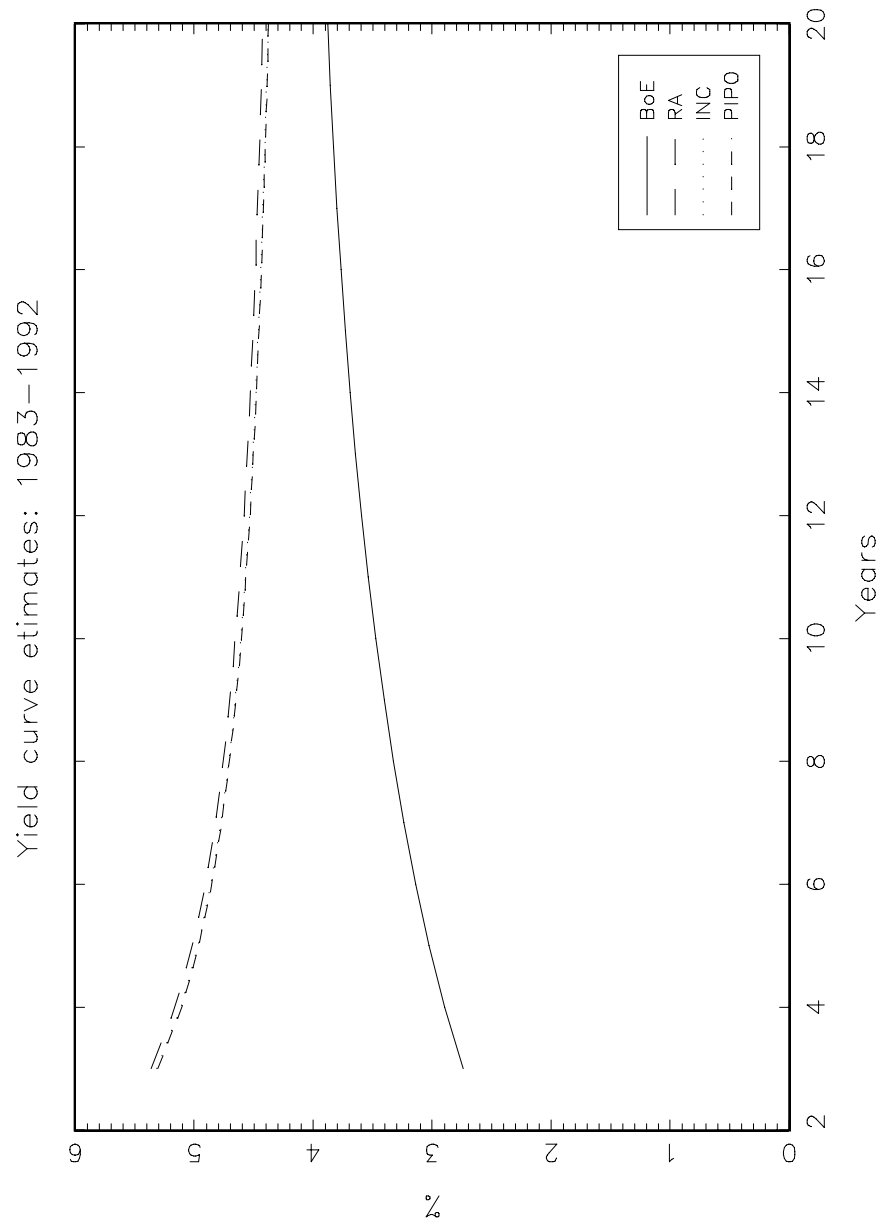


Figure B.2: Steady-state yield curves.

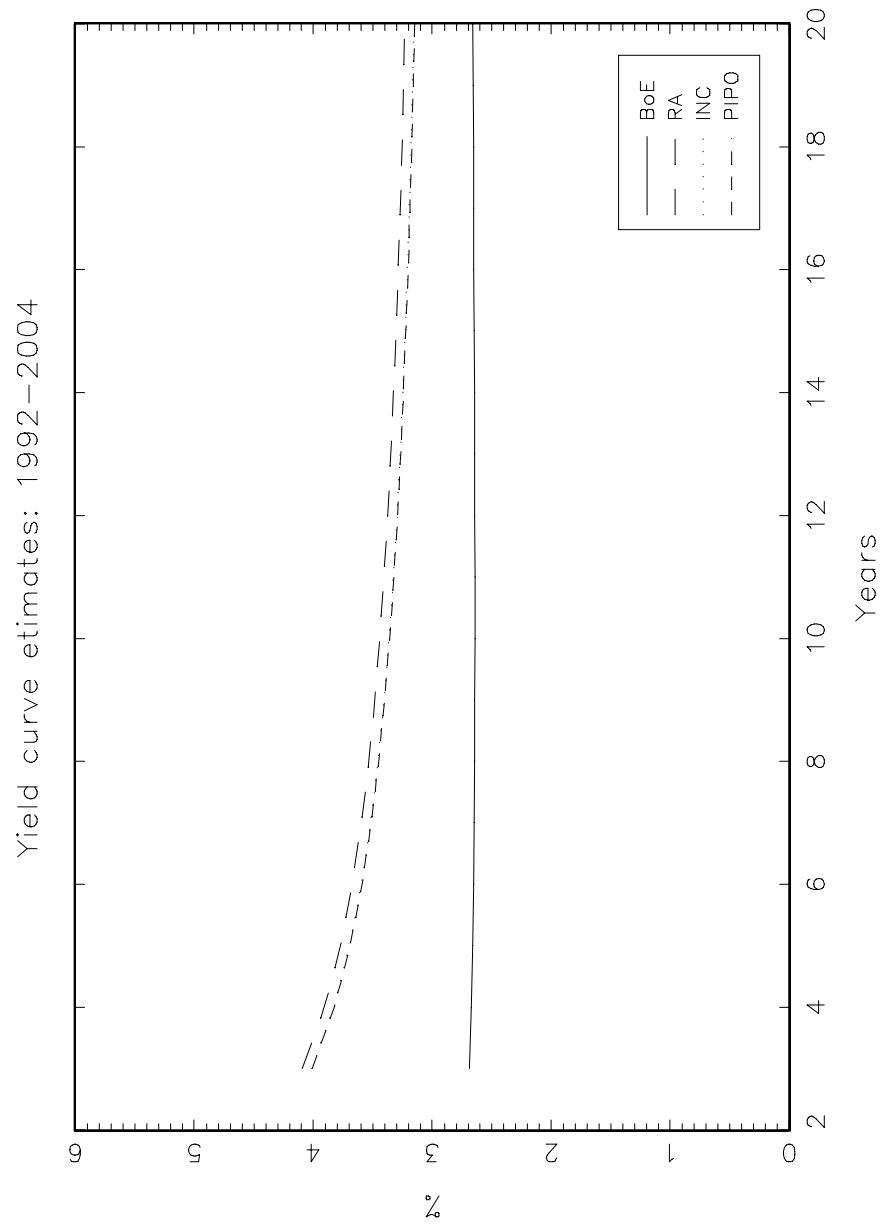


Figure B.3: Steady-state yield curves.

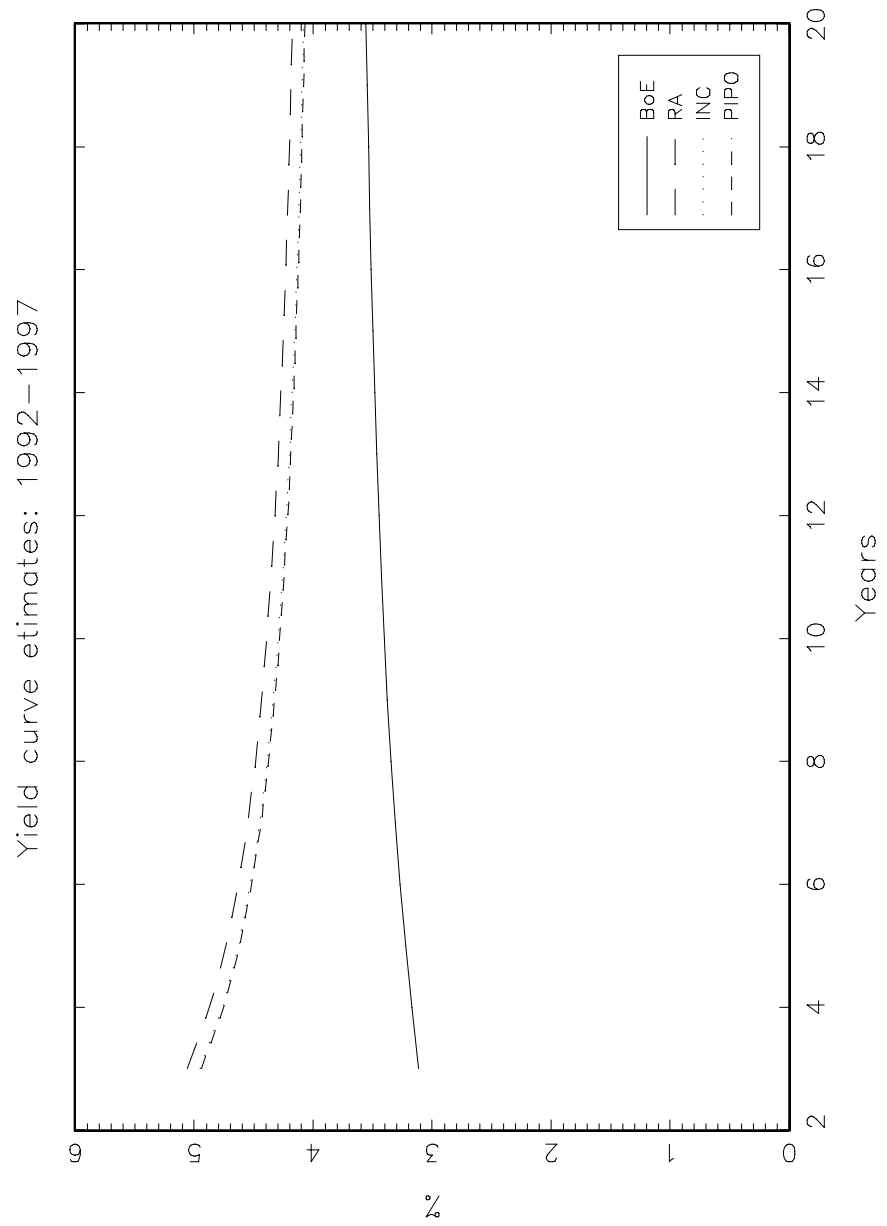


Figure B.4: Steady-state yield curves.

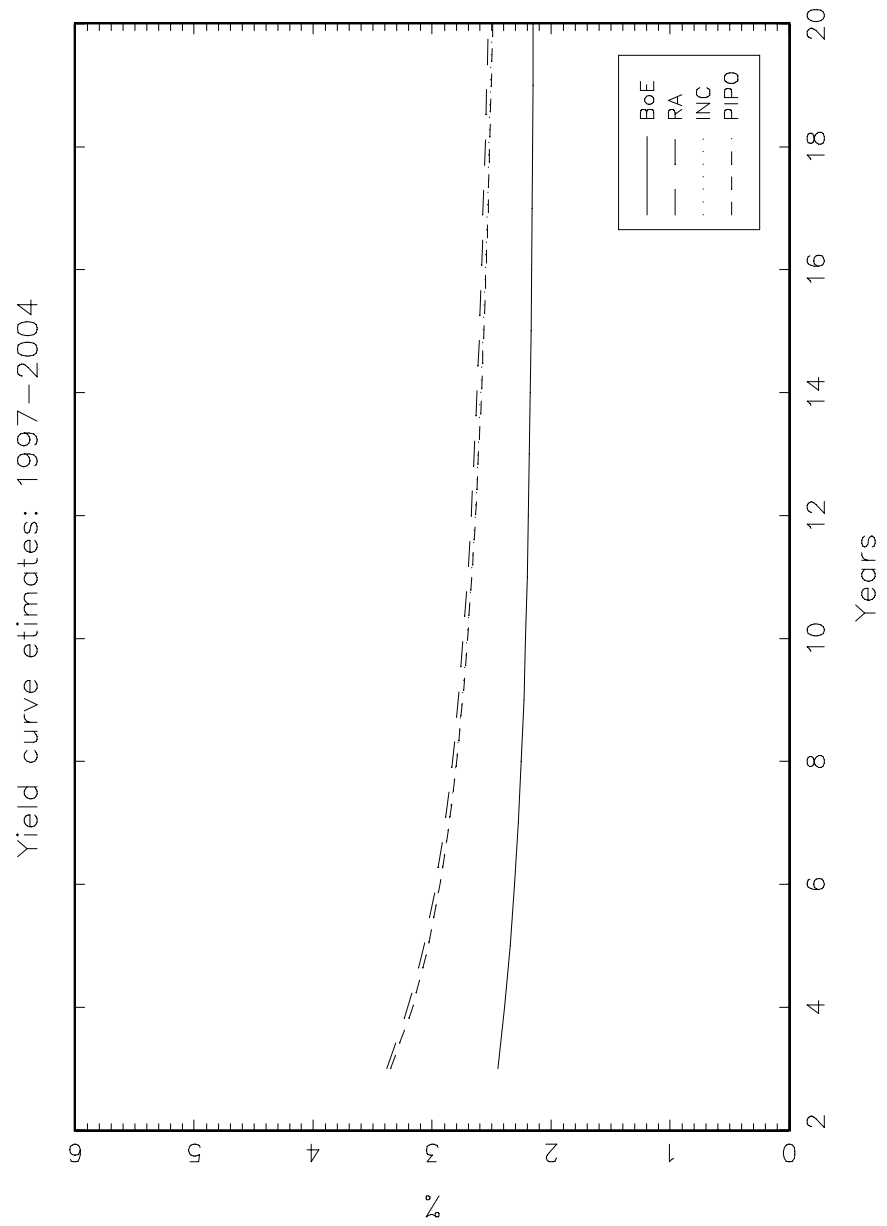


Figure B.5: Steady-state yield curves.

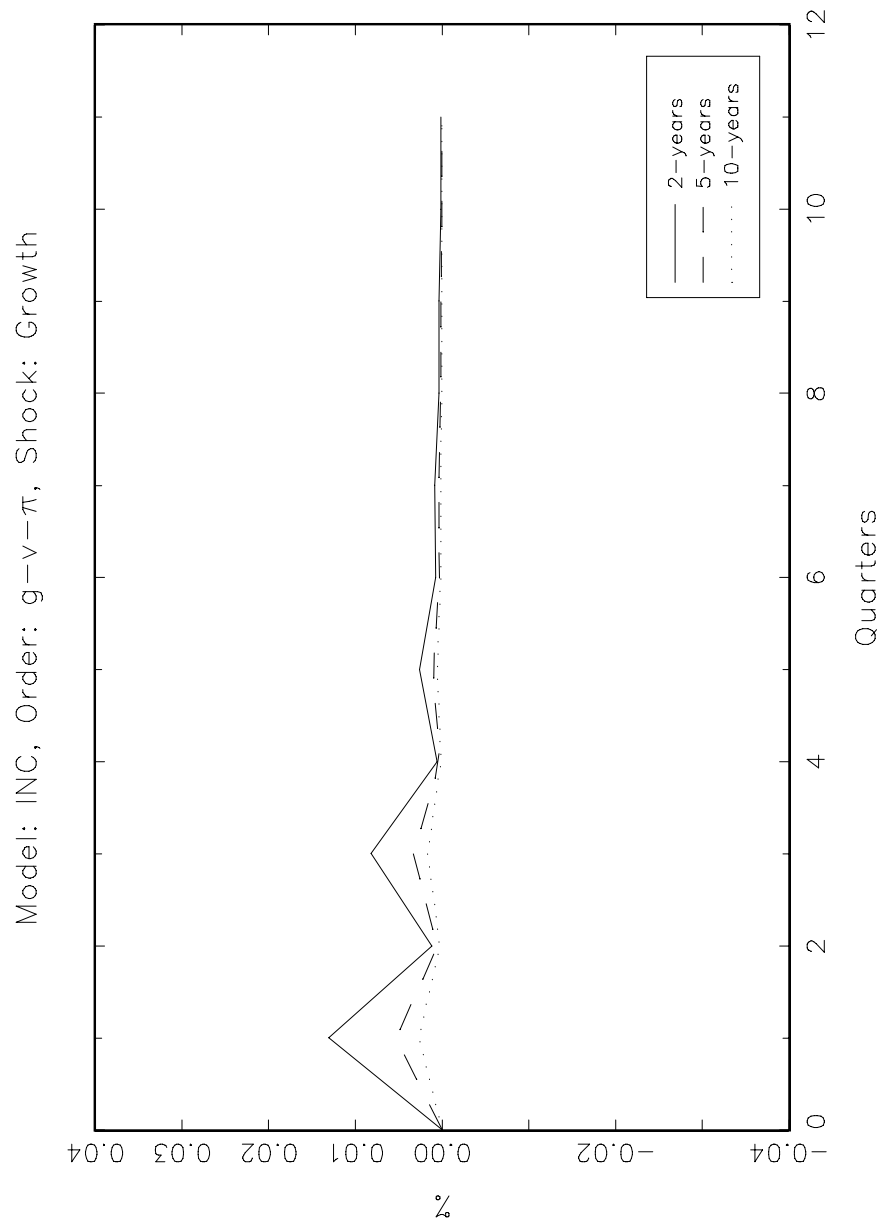


Figure B.6: Real rate responses to factors shocks.

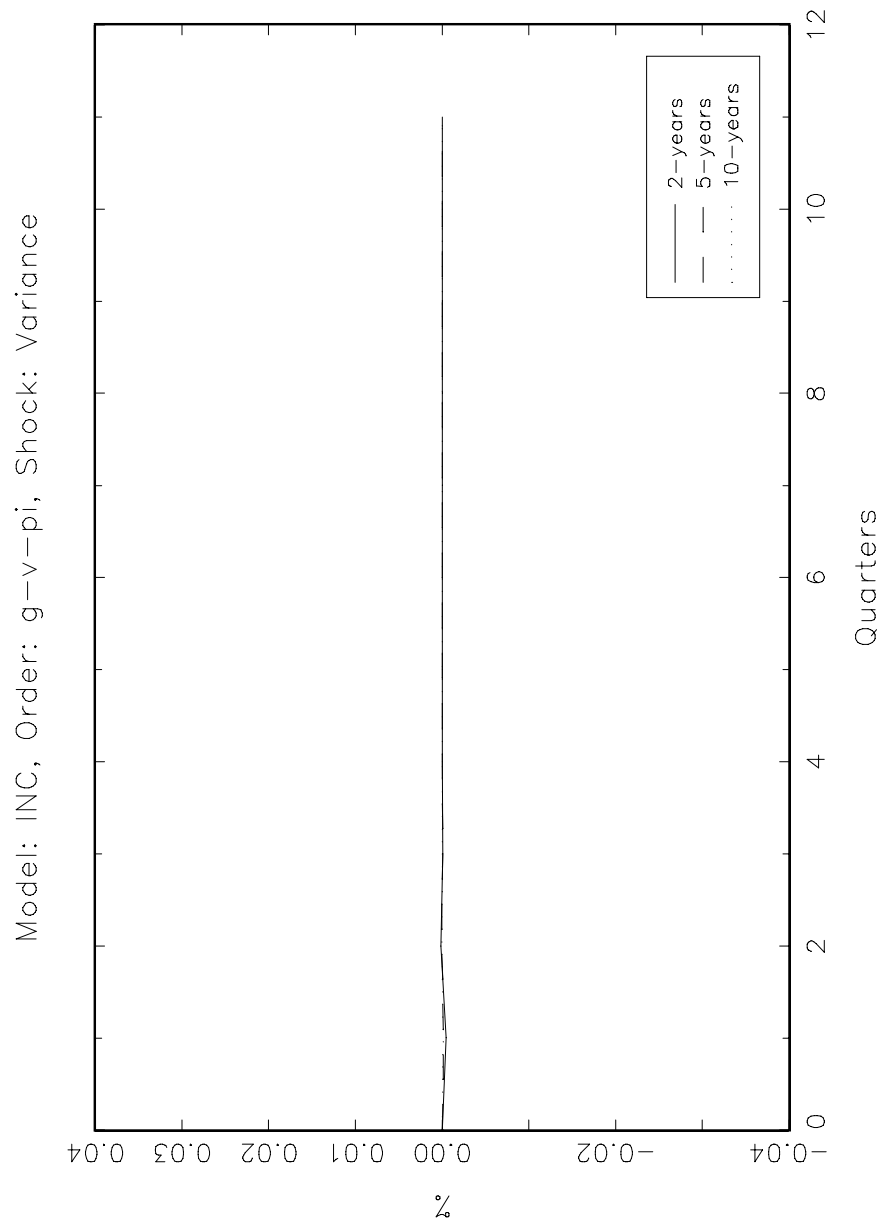


Figure B.7: Real rate responses to factors shocks.

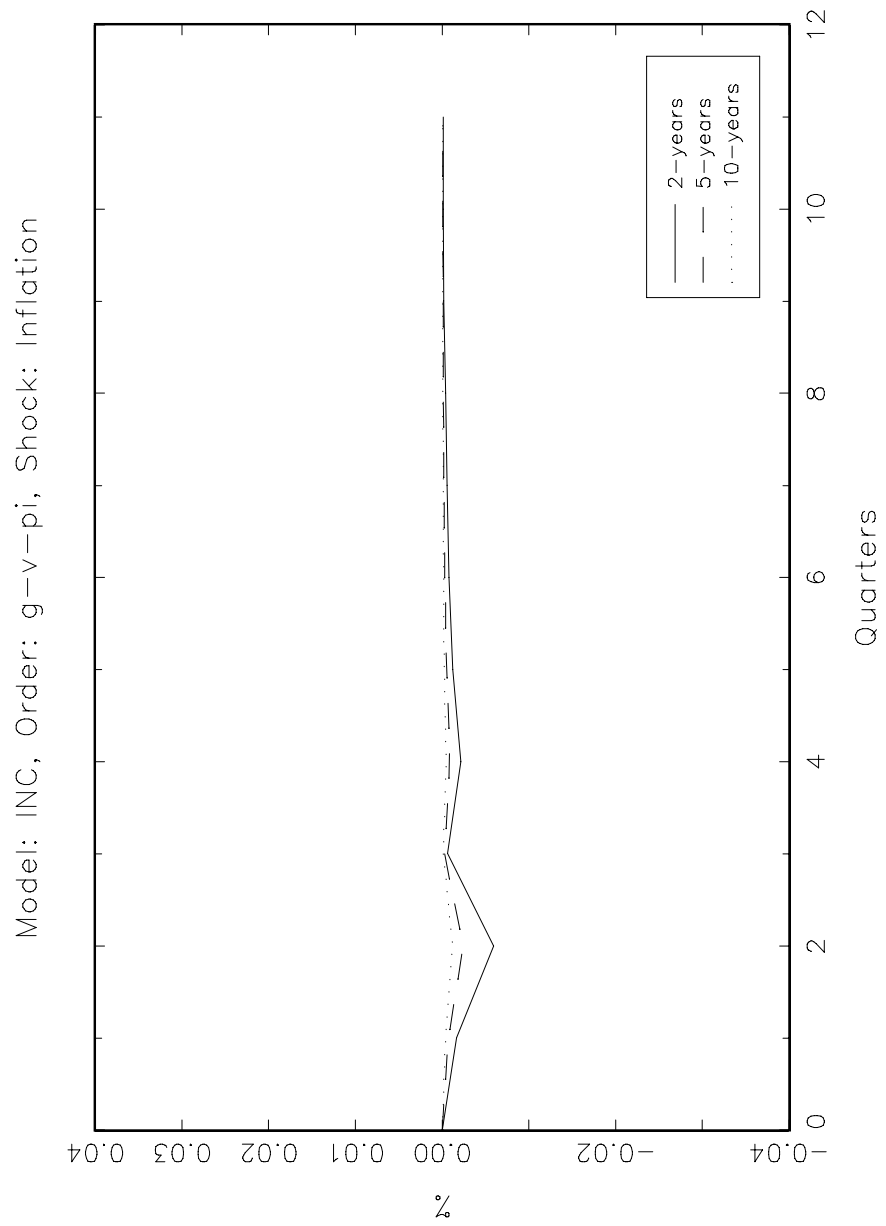


Figure B.8: Real rate responses to factors shocks.



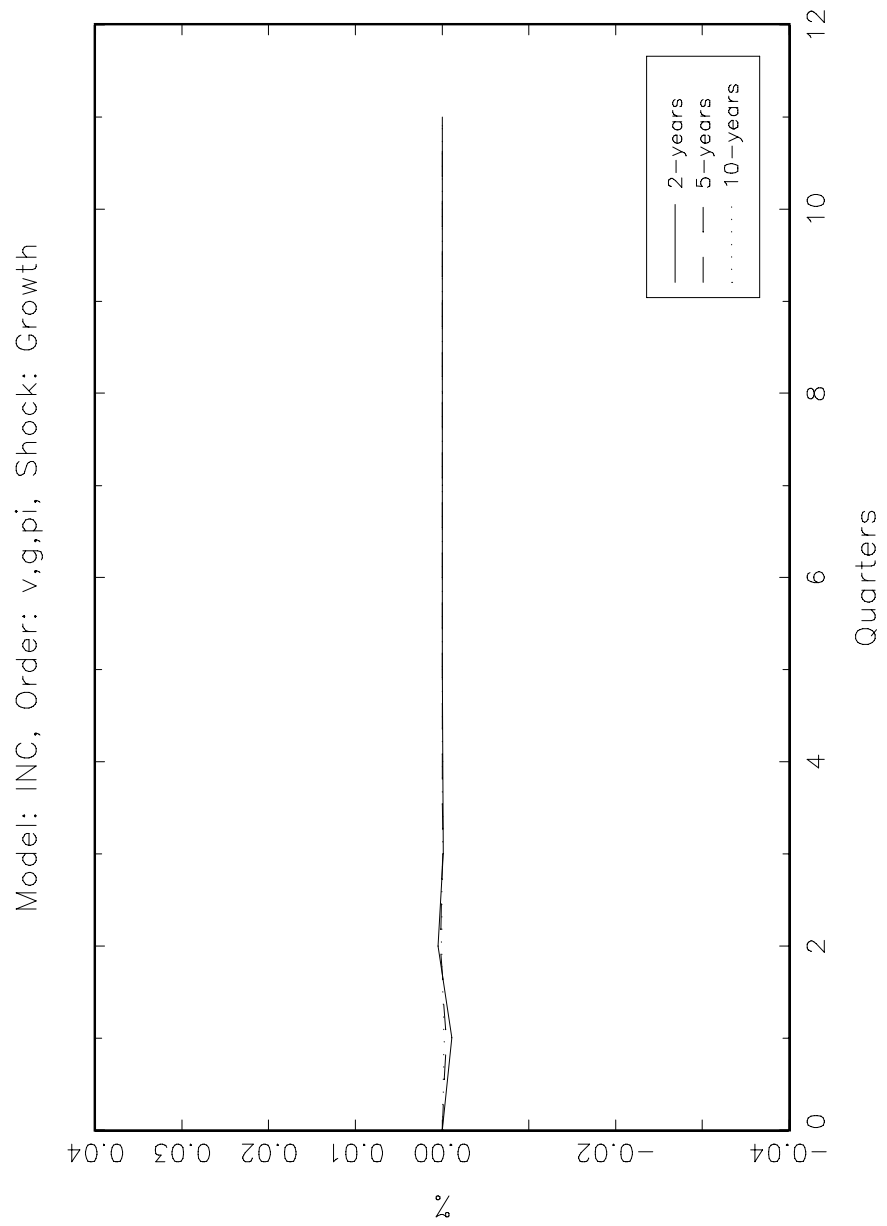


Figure B.9: Real rate responses to factors shocks.

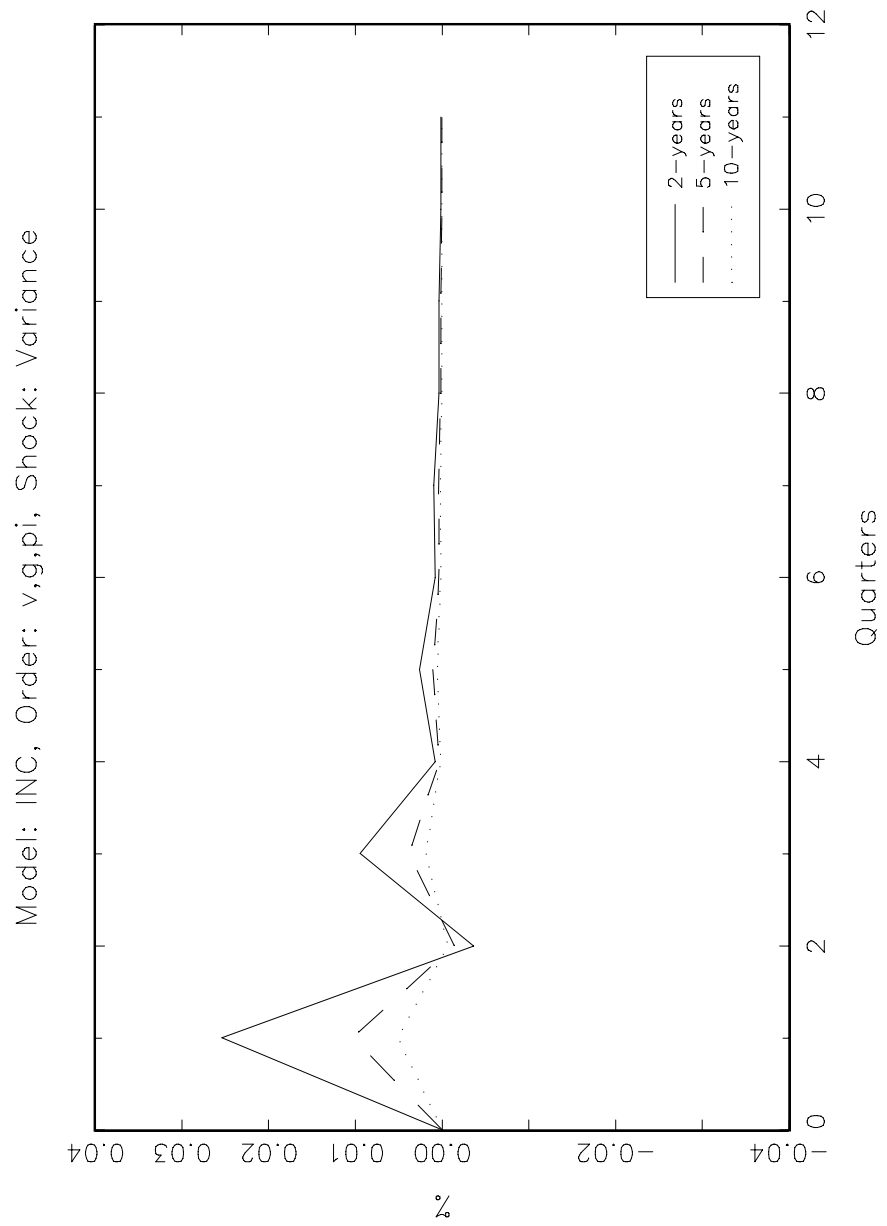


Figure B.10: Real rate responses to factors shocks.

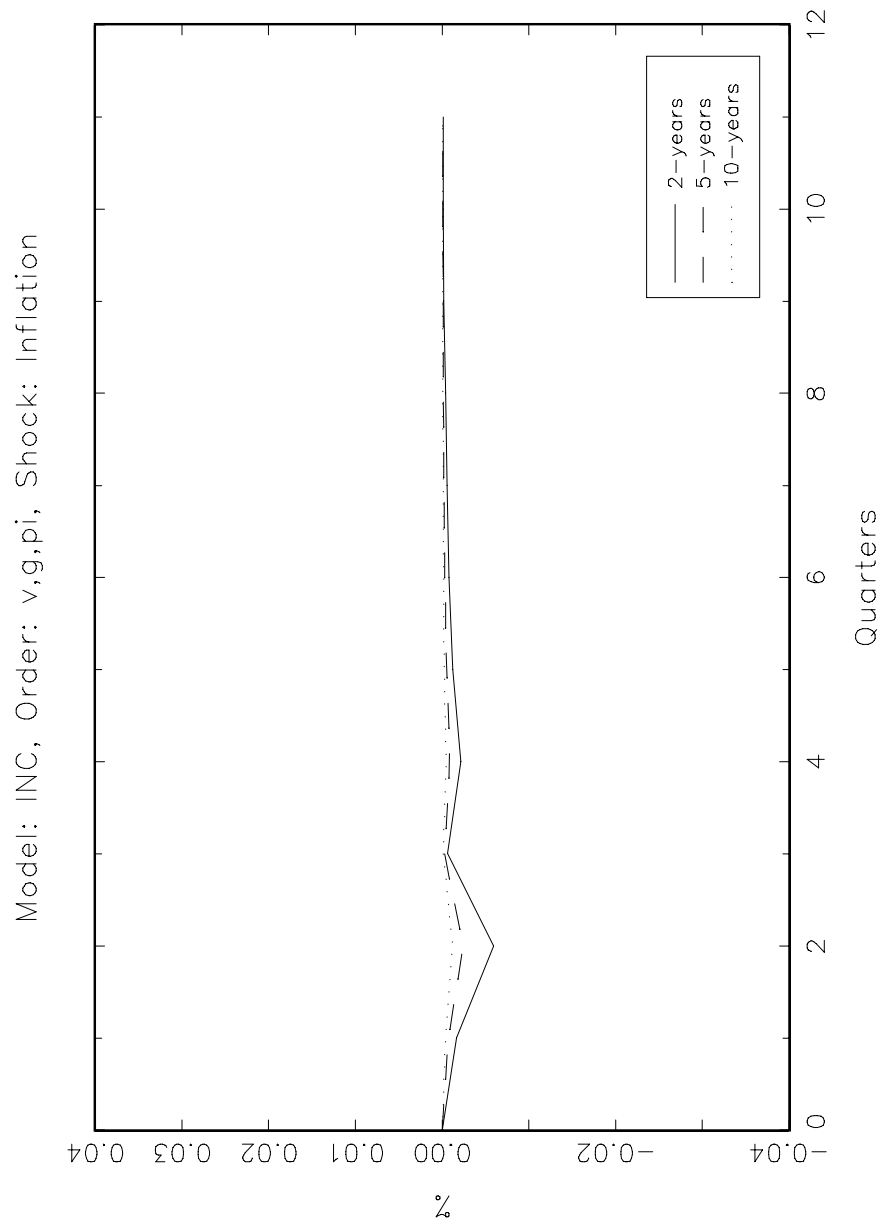


Figure B.11: Real rate responses to factors shocks.

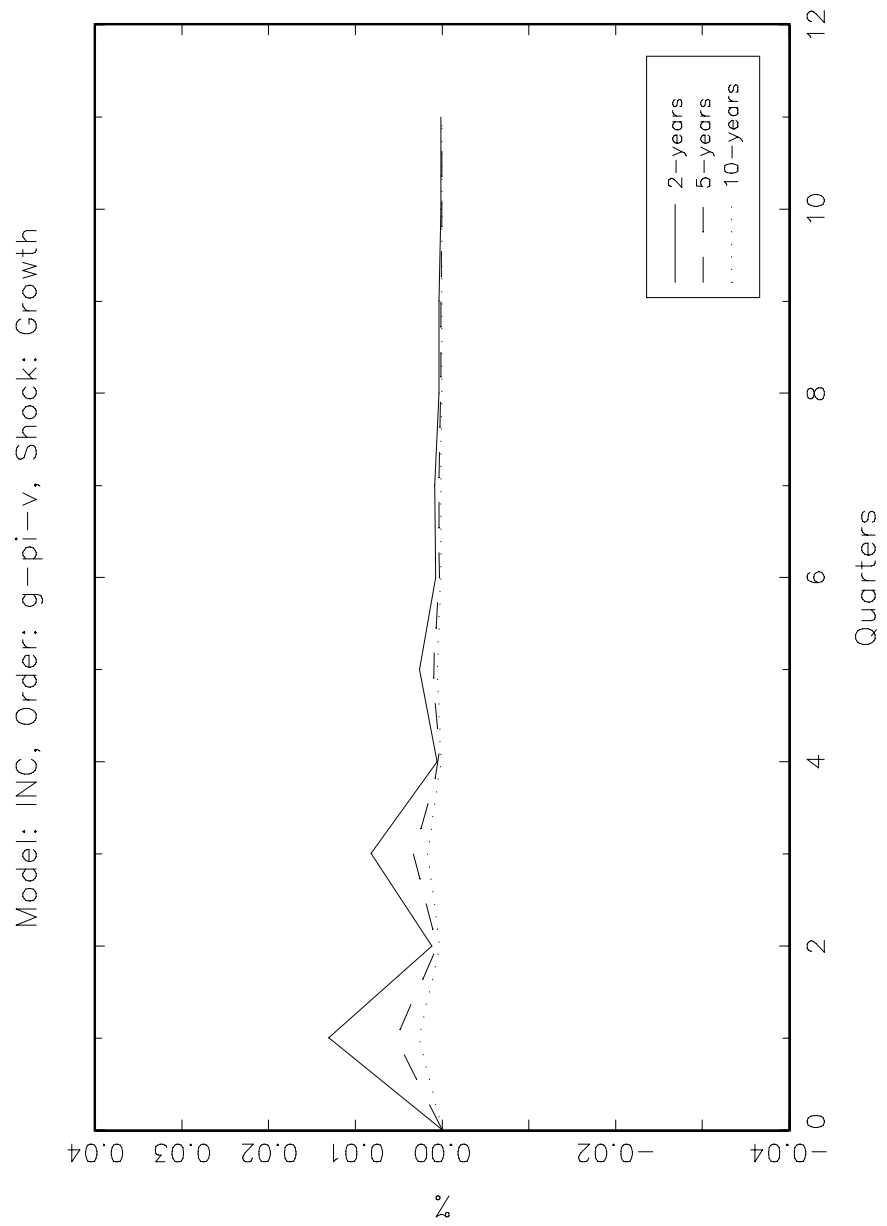


Figure B.12: Real rate responses to factors shocks.

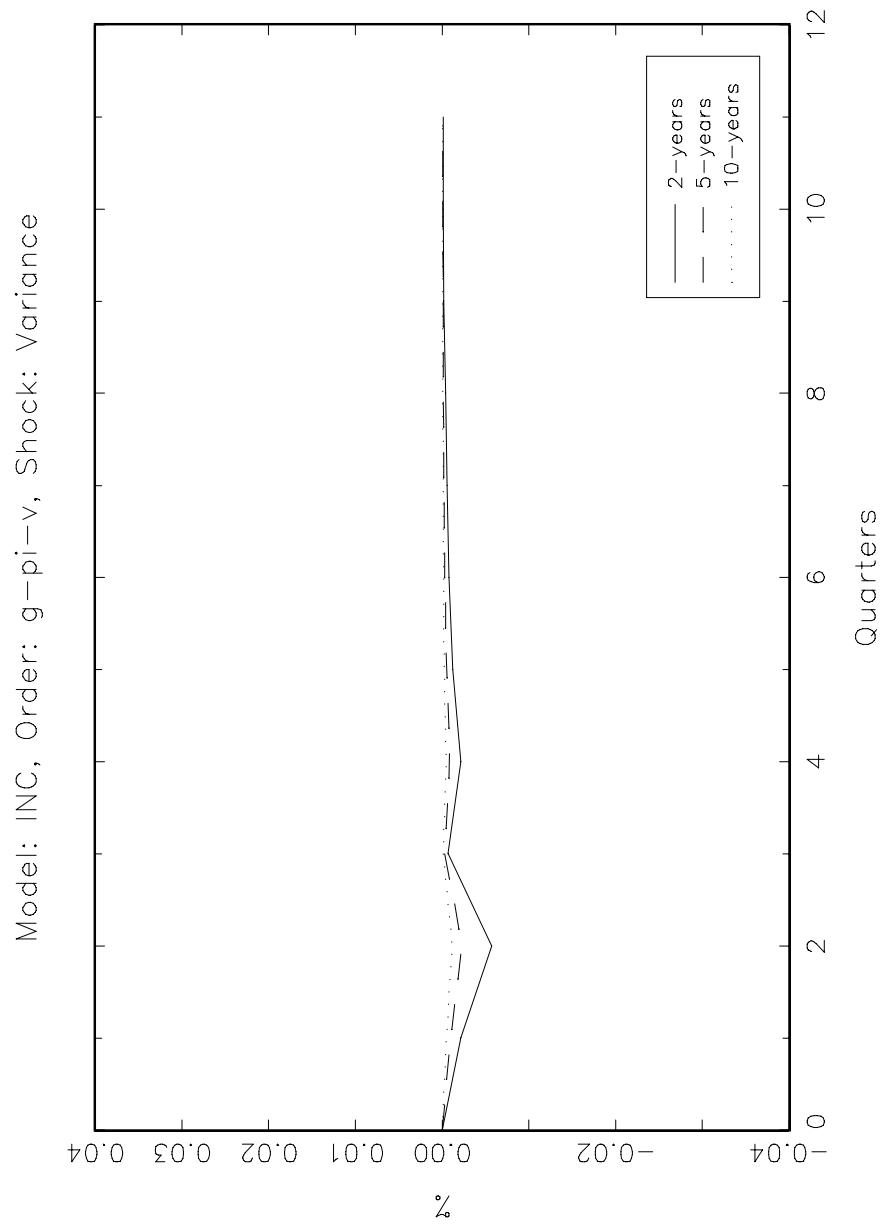


Figure B.13: Real rate responses to factors shocks.

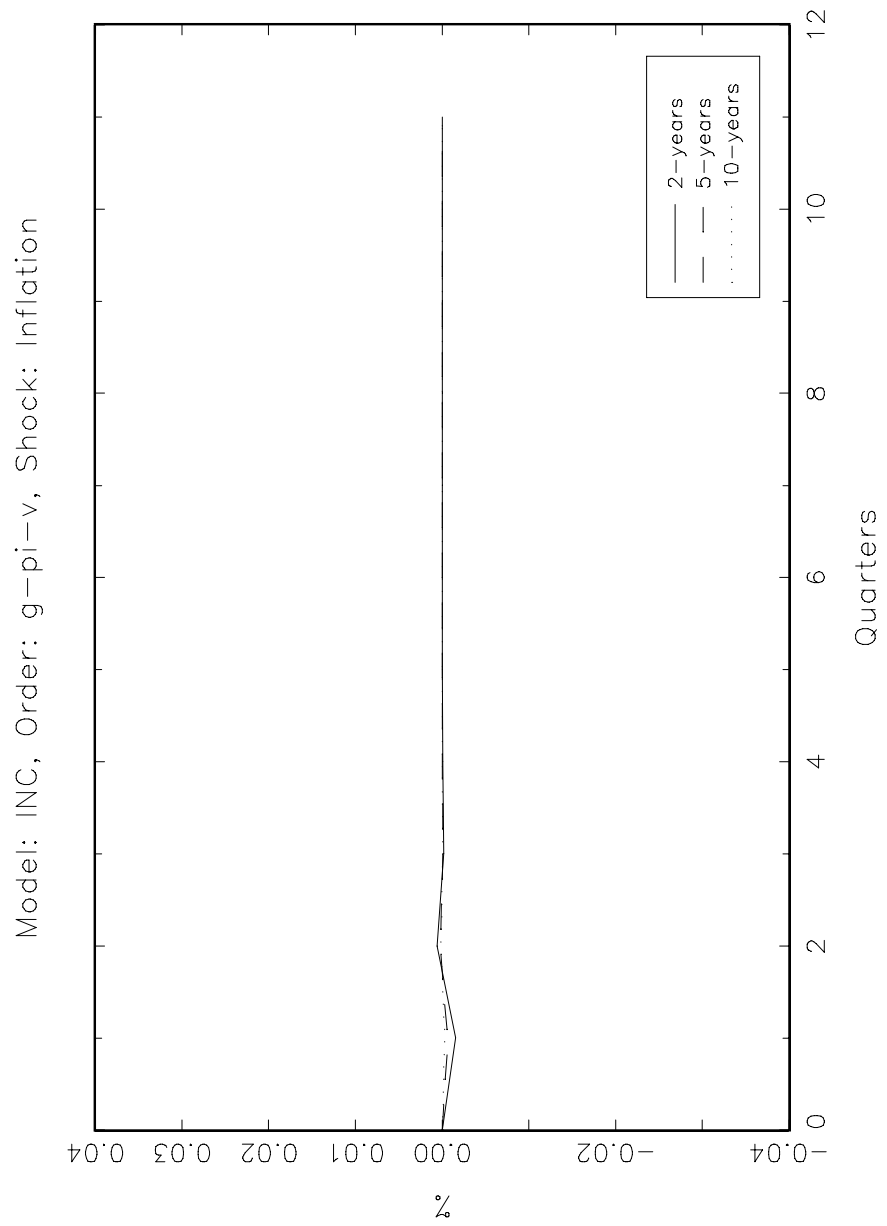


Figure B.14: Real rate responses to factors shocks.

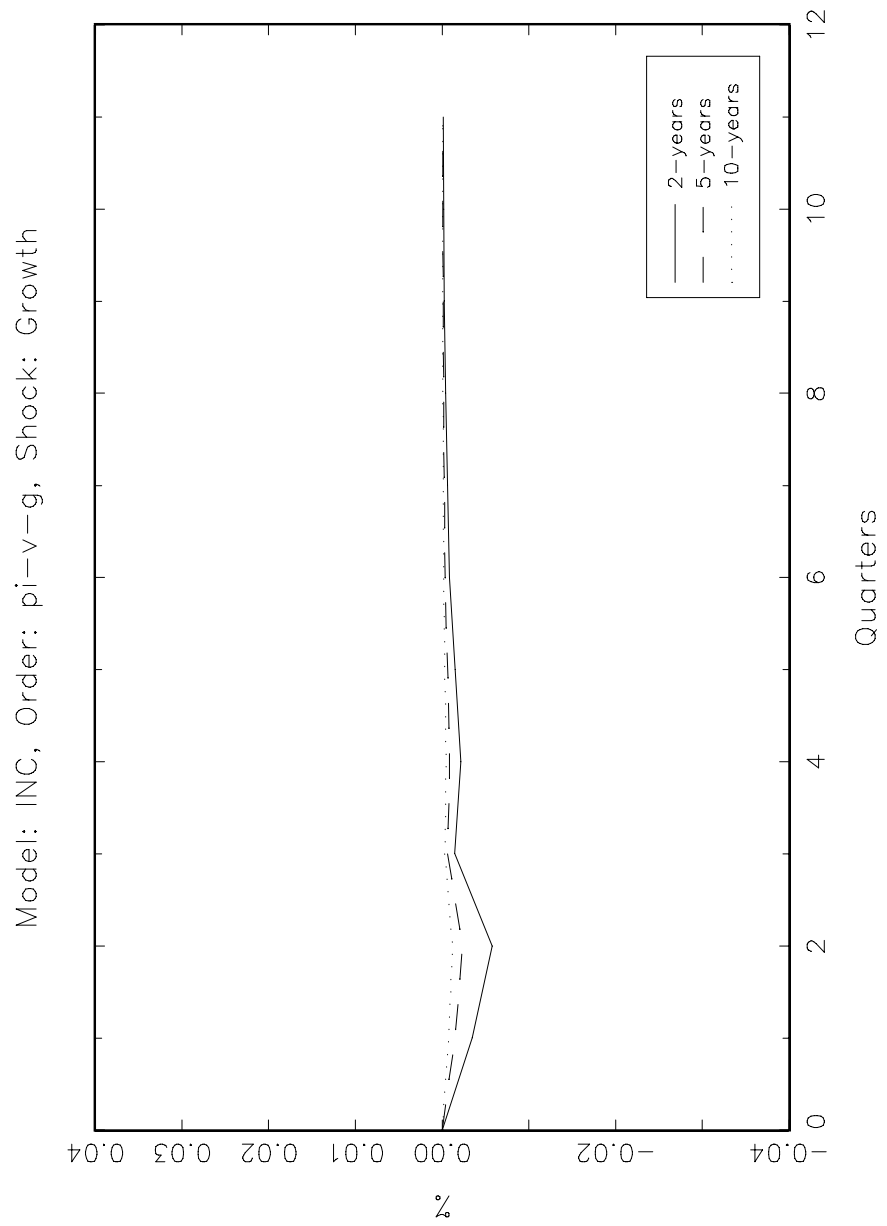


Figure B.15: Real rate responses to factors shocks.

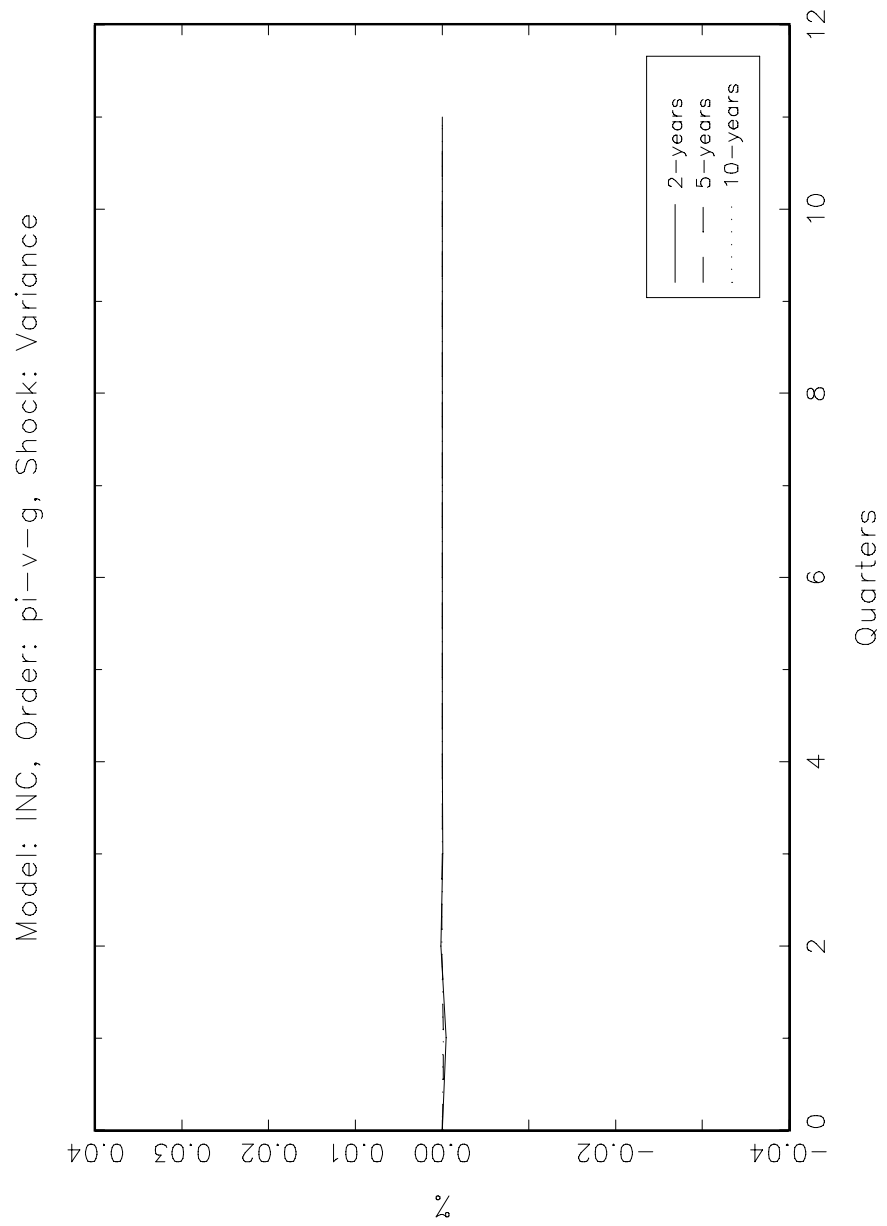


Figure B.16: Real rate responses to factors shocks.



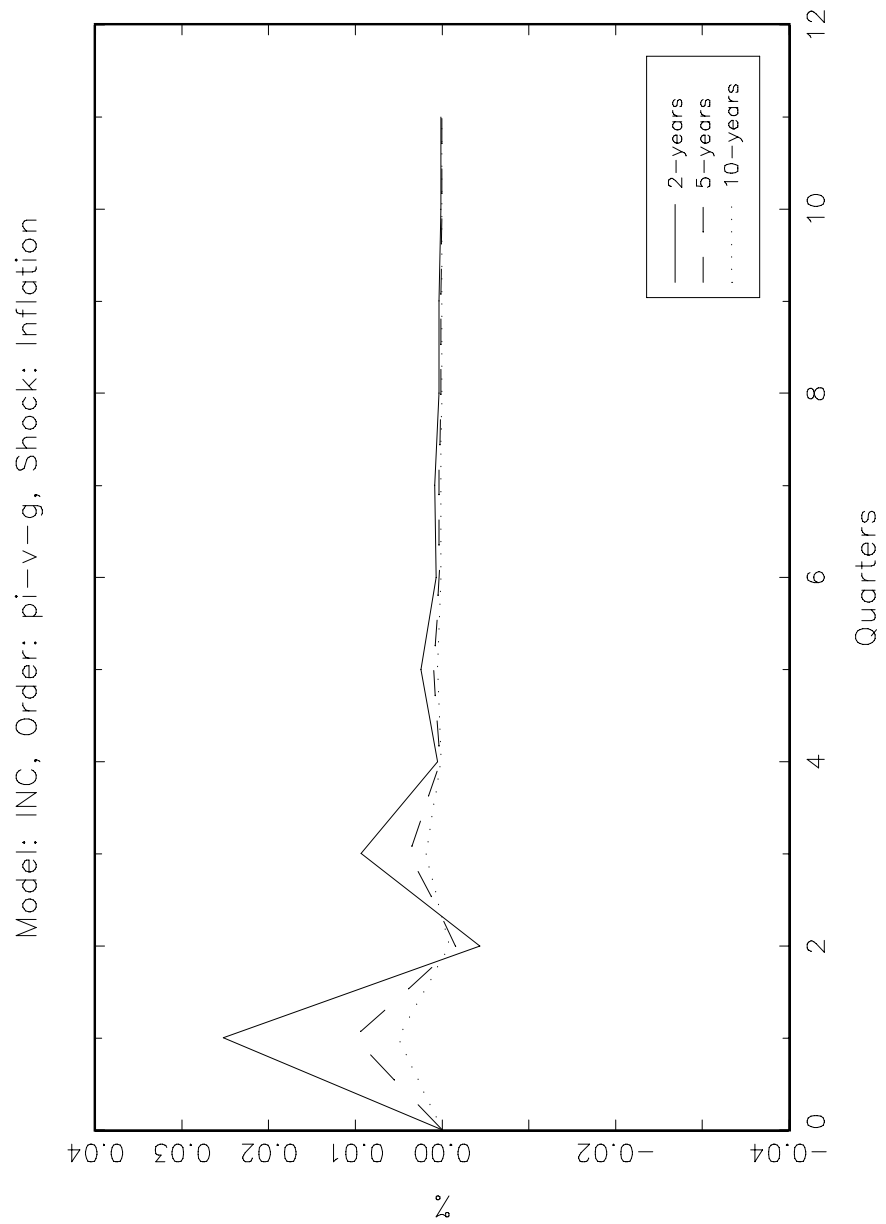


Figure B.17: Real rate responses to factors shocks.

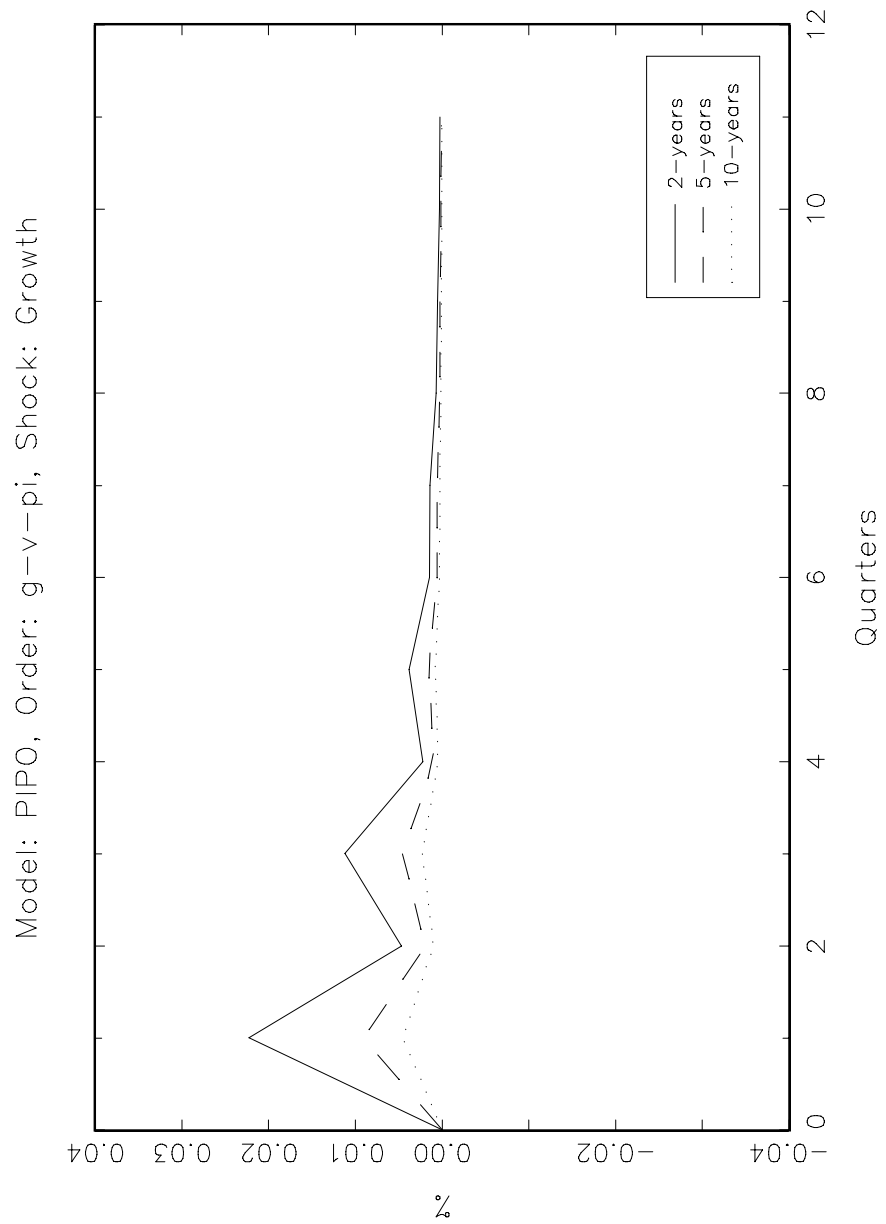


Figure B.18: Real rate responses to factors shocks.

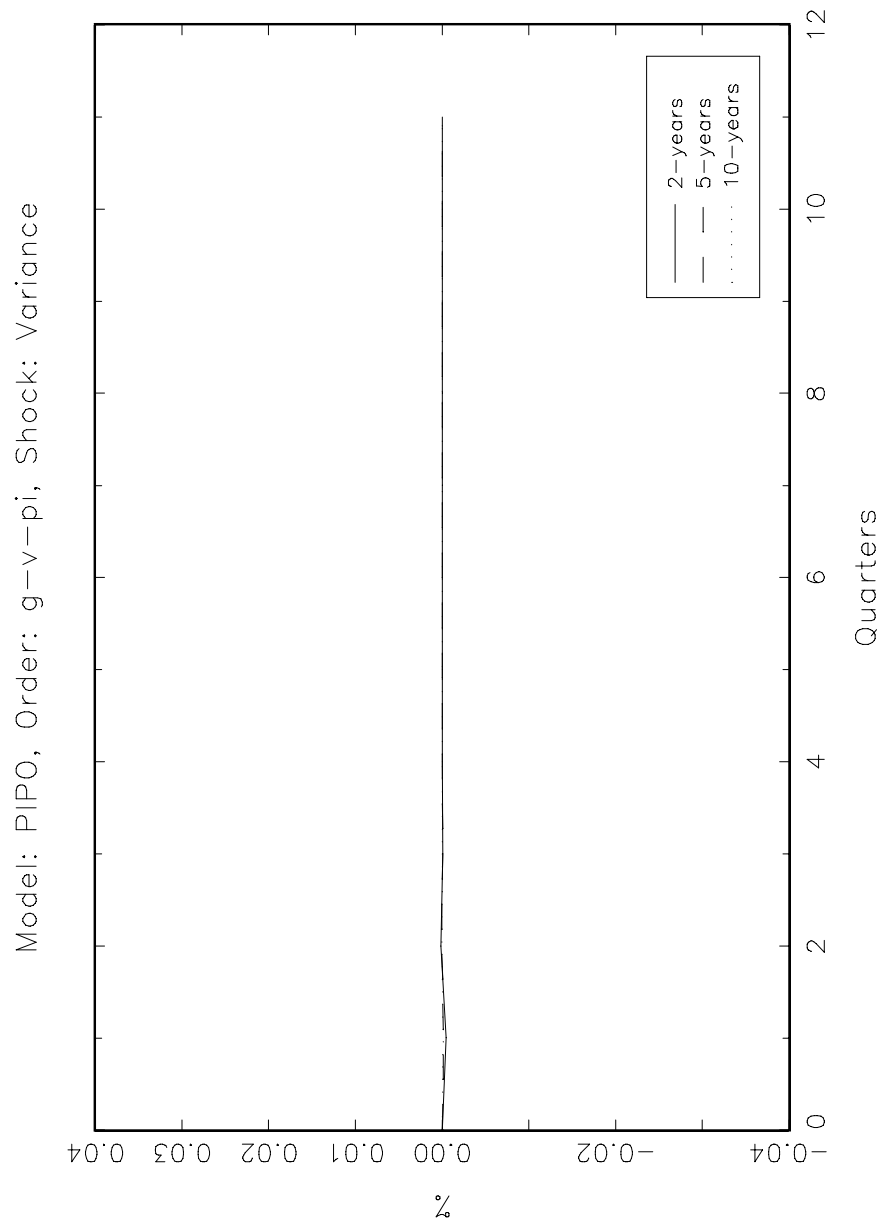


Figure B.19: Real rate responses to factors shocks.

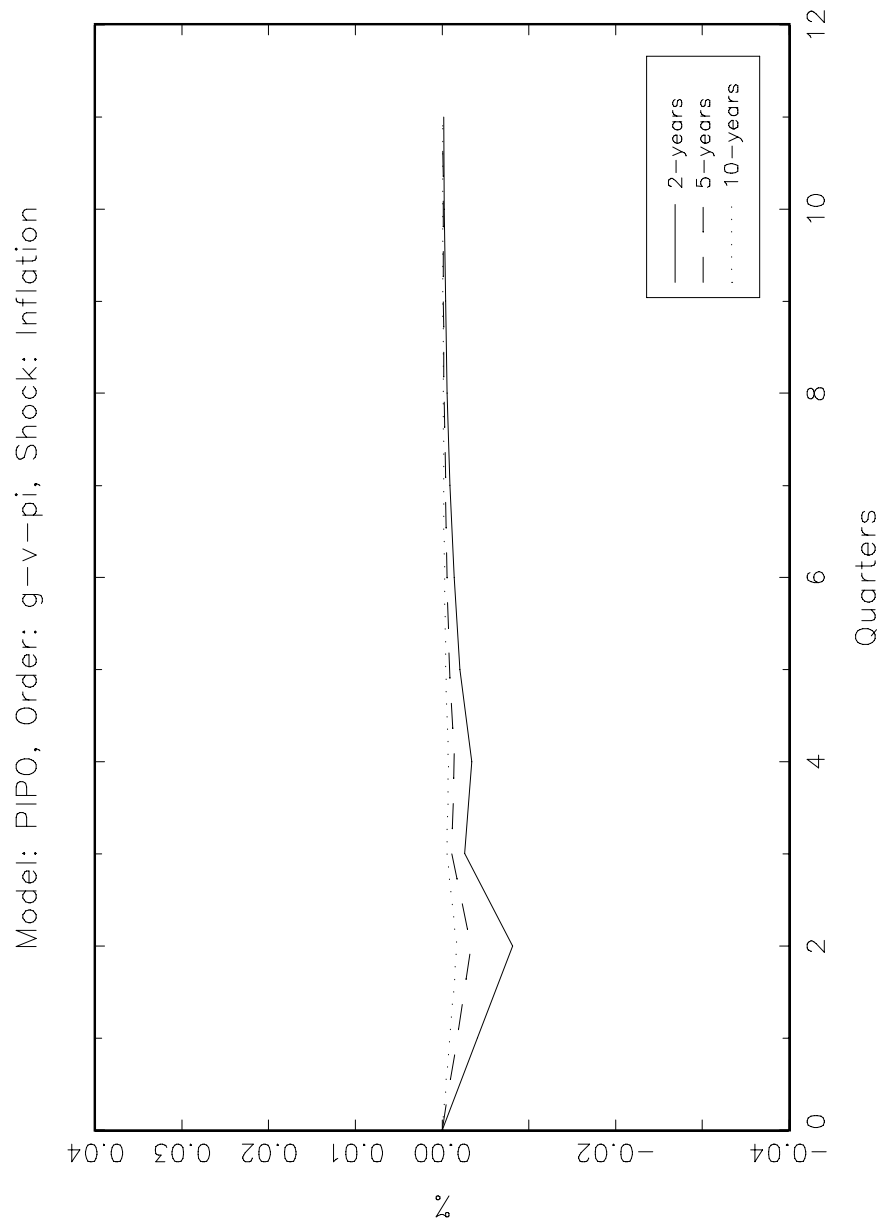


Figure B.20: Real rate responses to factors shocks.

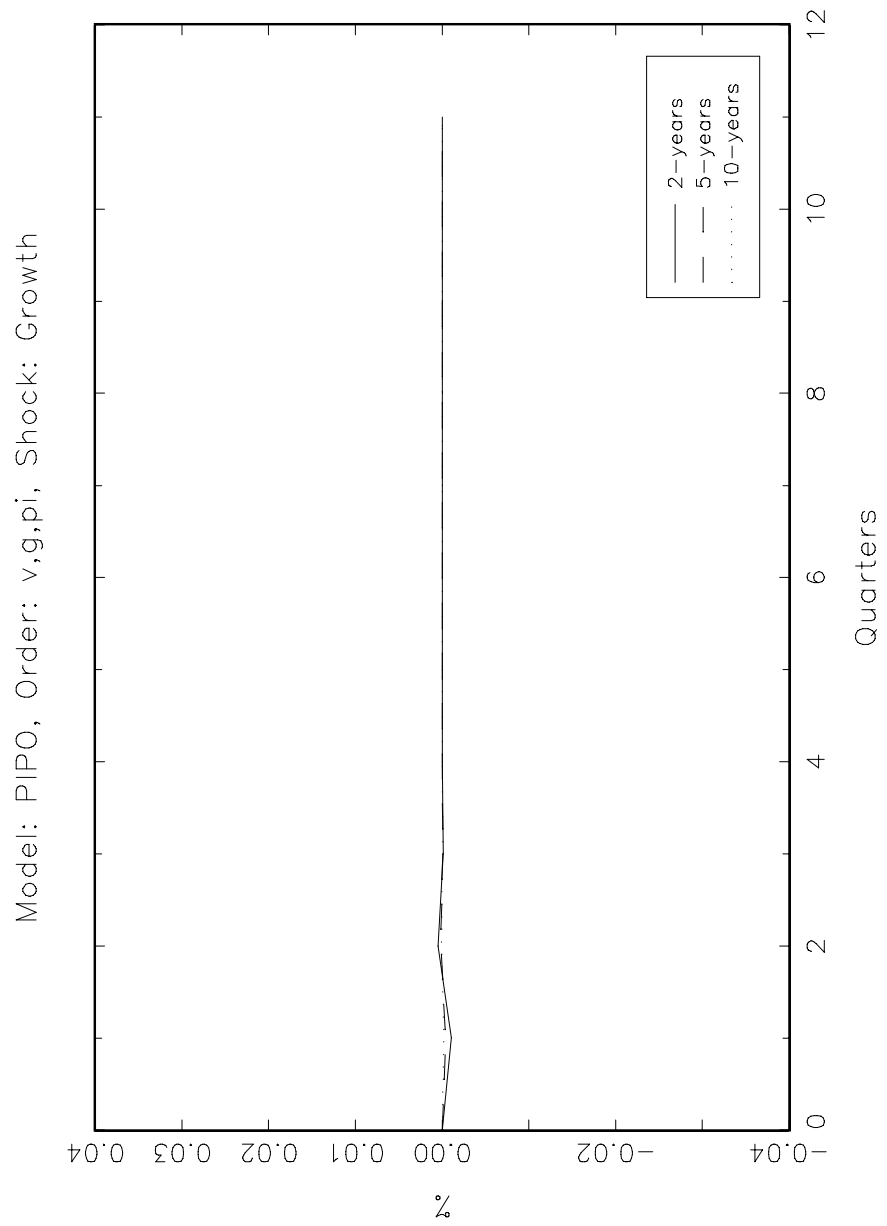


Figure B.21: Real rates responses to factors shocks.

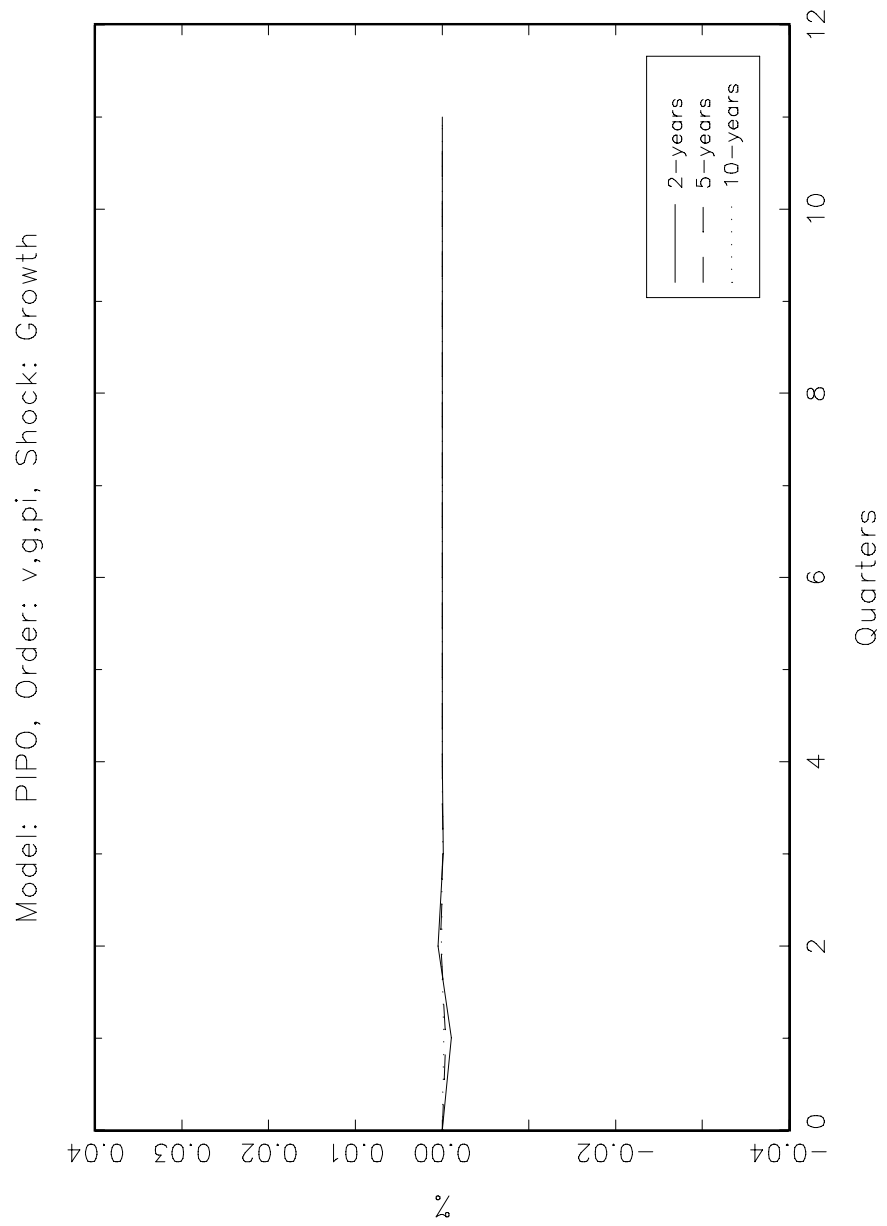


Figure B.22: Real rates responses to factors shocks.

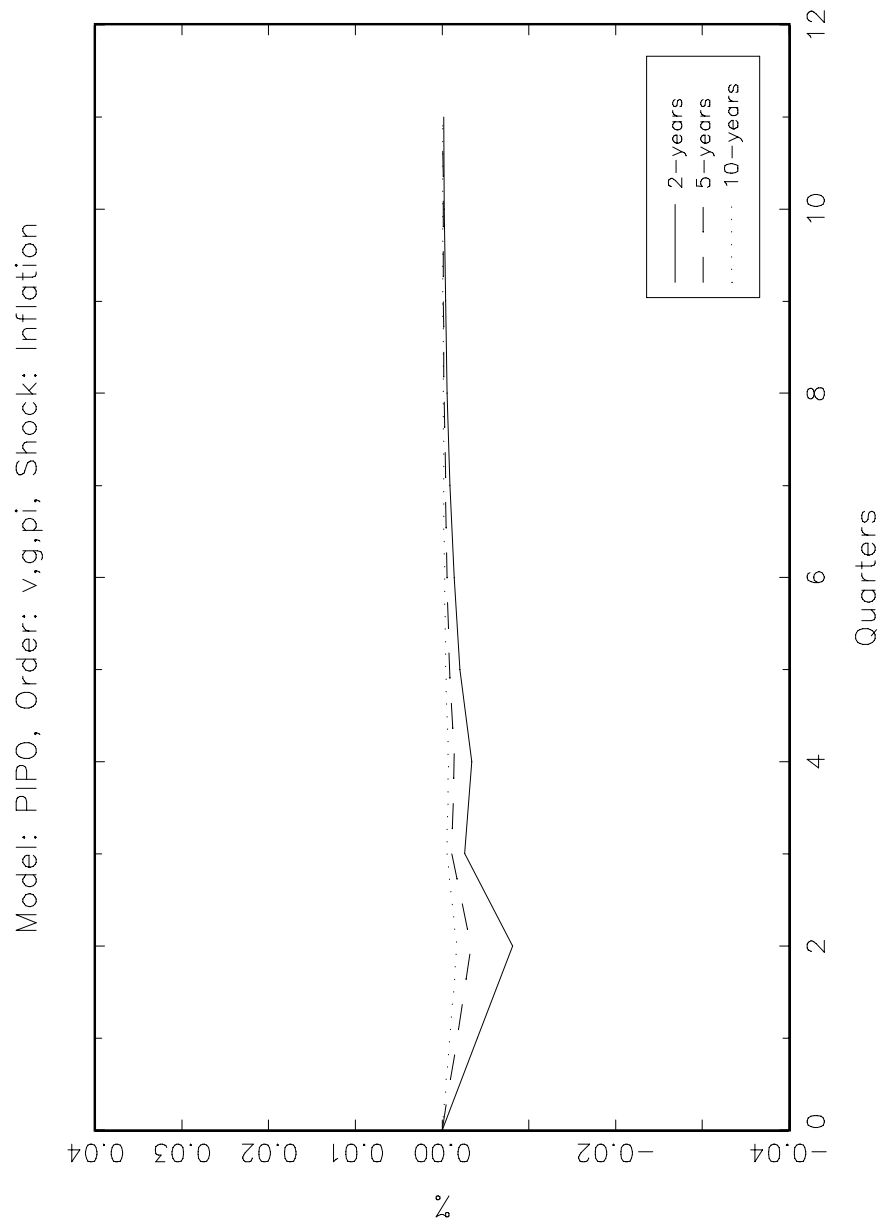


Figure B.23: Real rates responses to factors shocks.

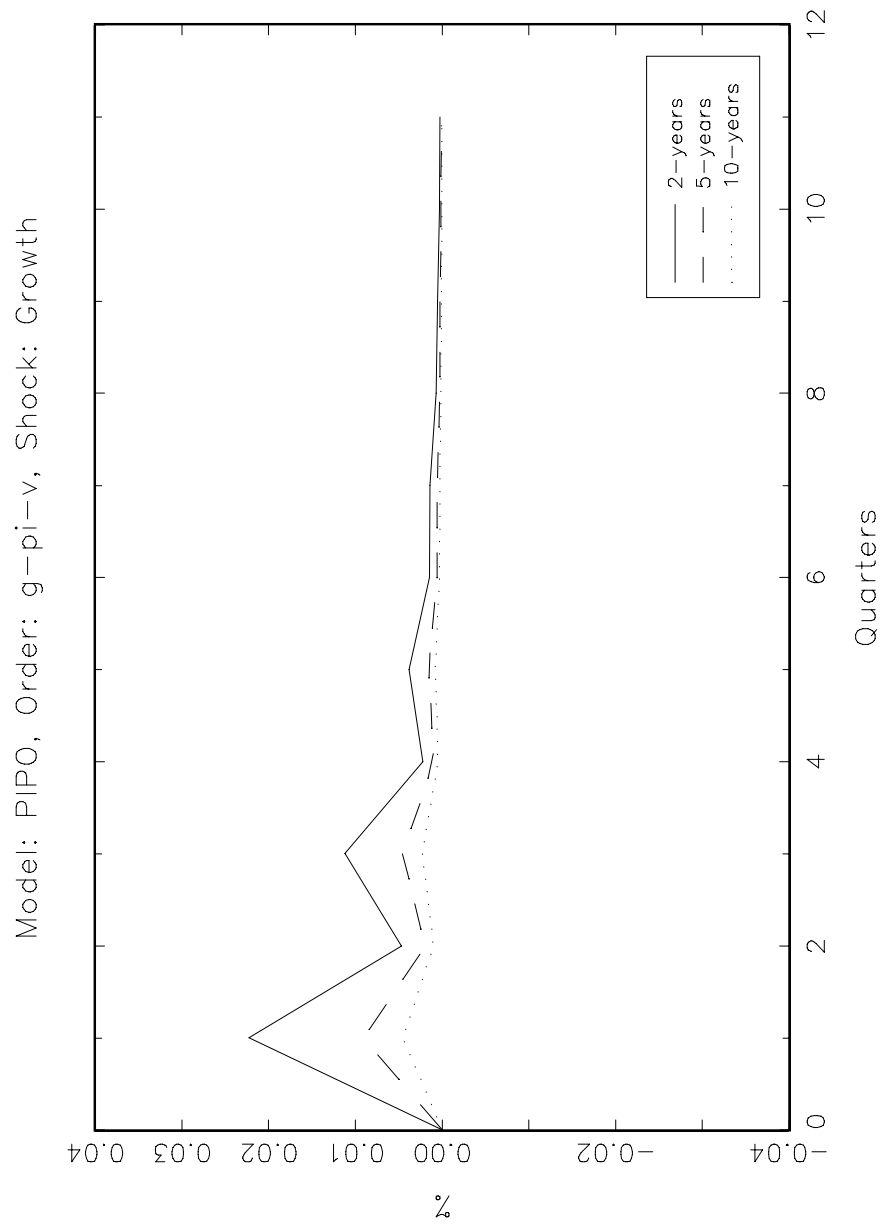


Figure B.24: Real rate responses to factors shocks.



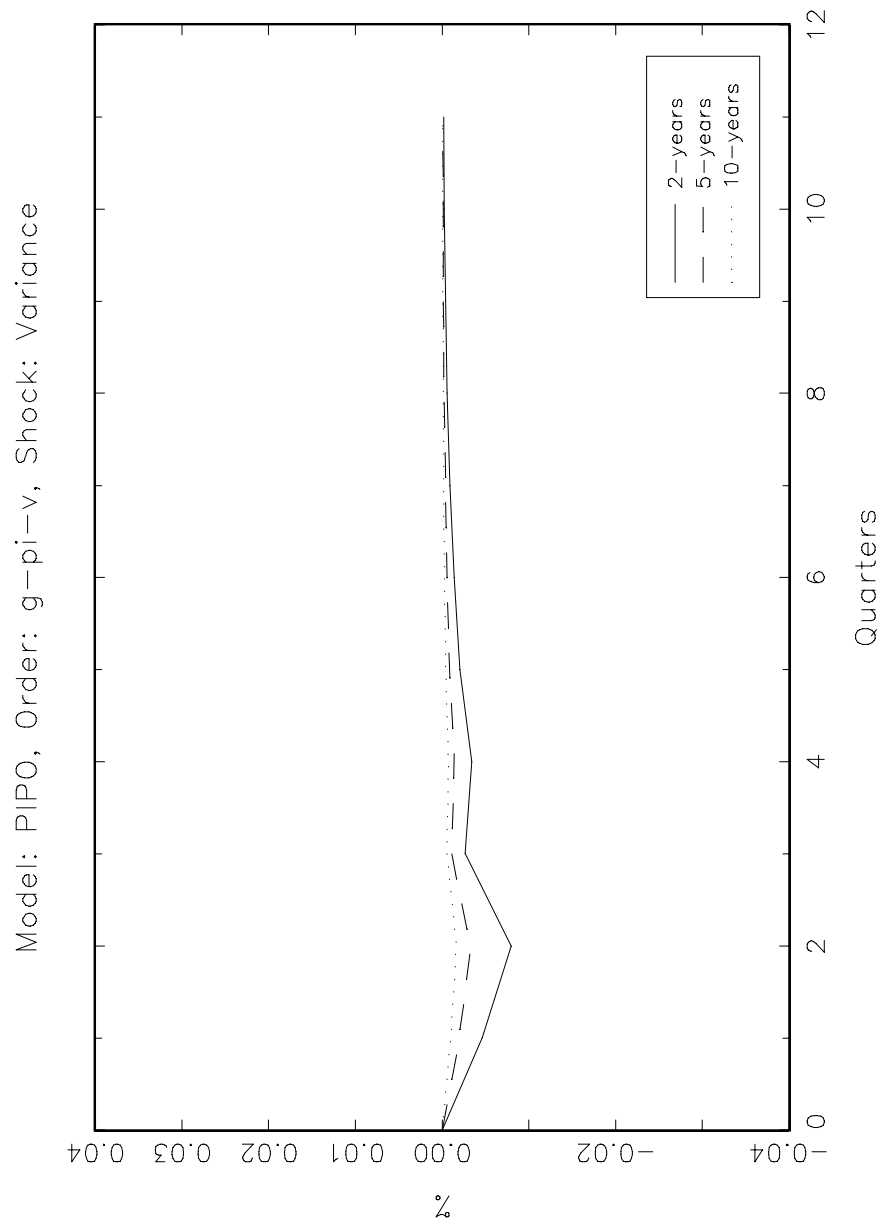


Figure B.25: Real rate responses to factors shocks.

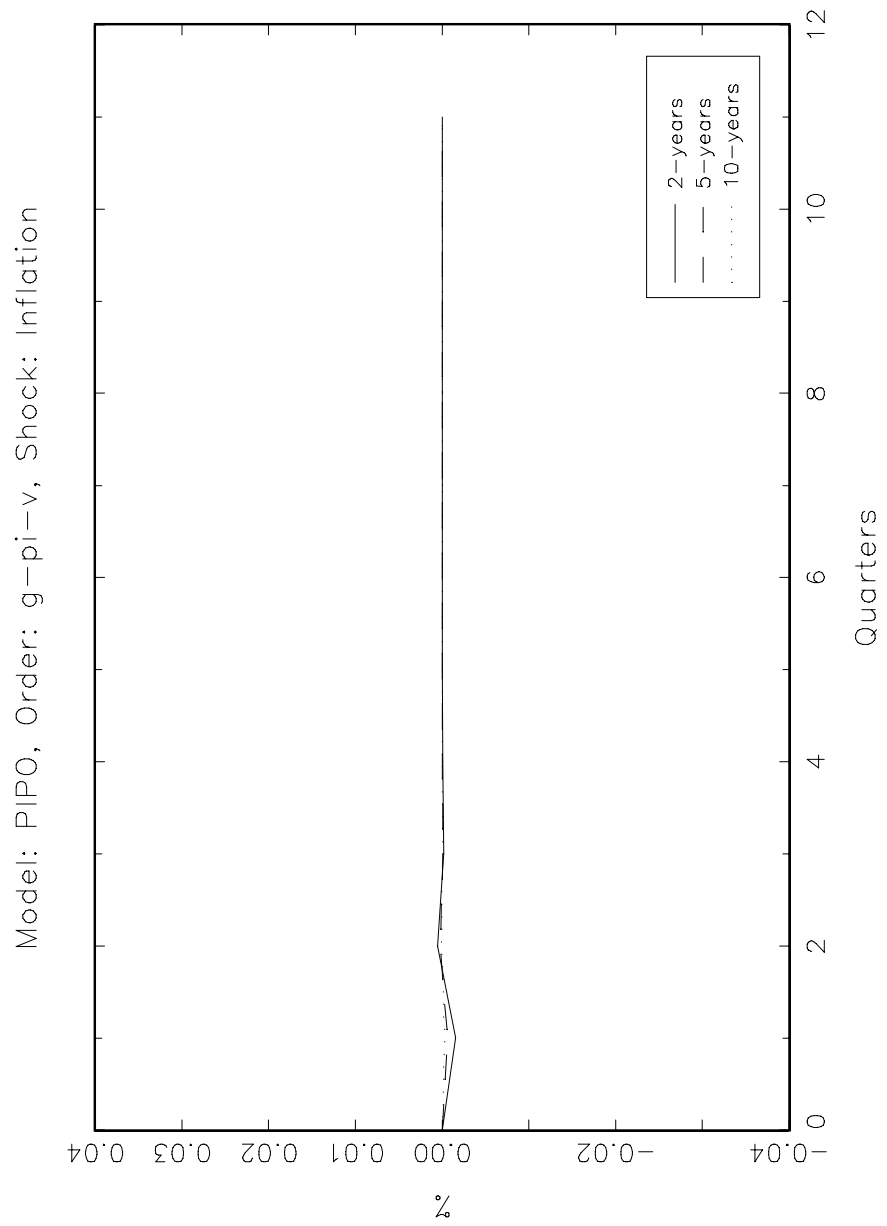


Figure B.26: Real rate responses to factors shocks.

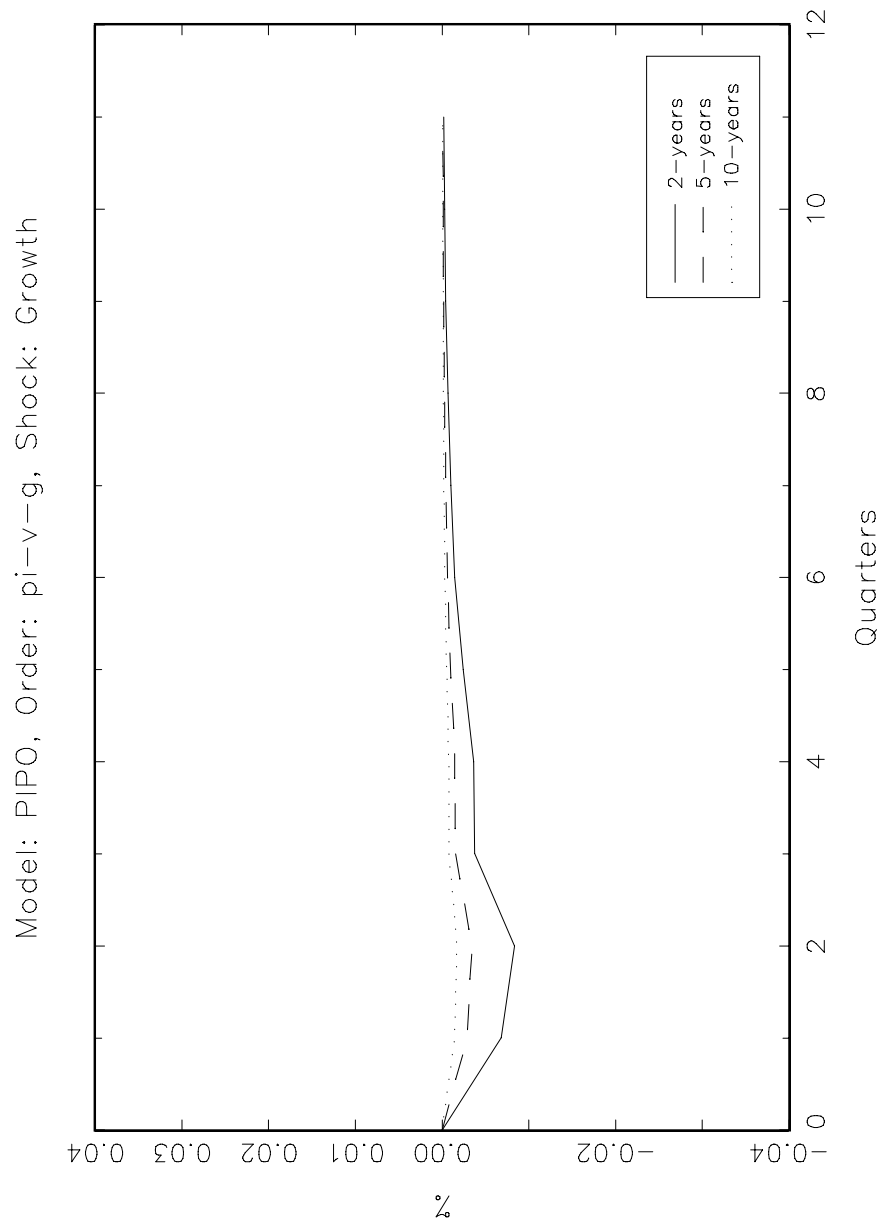


Figure B.27: Real rate responses to factors shocks.

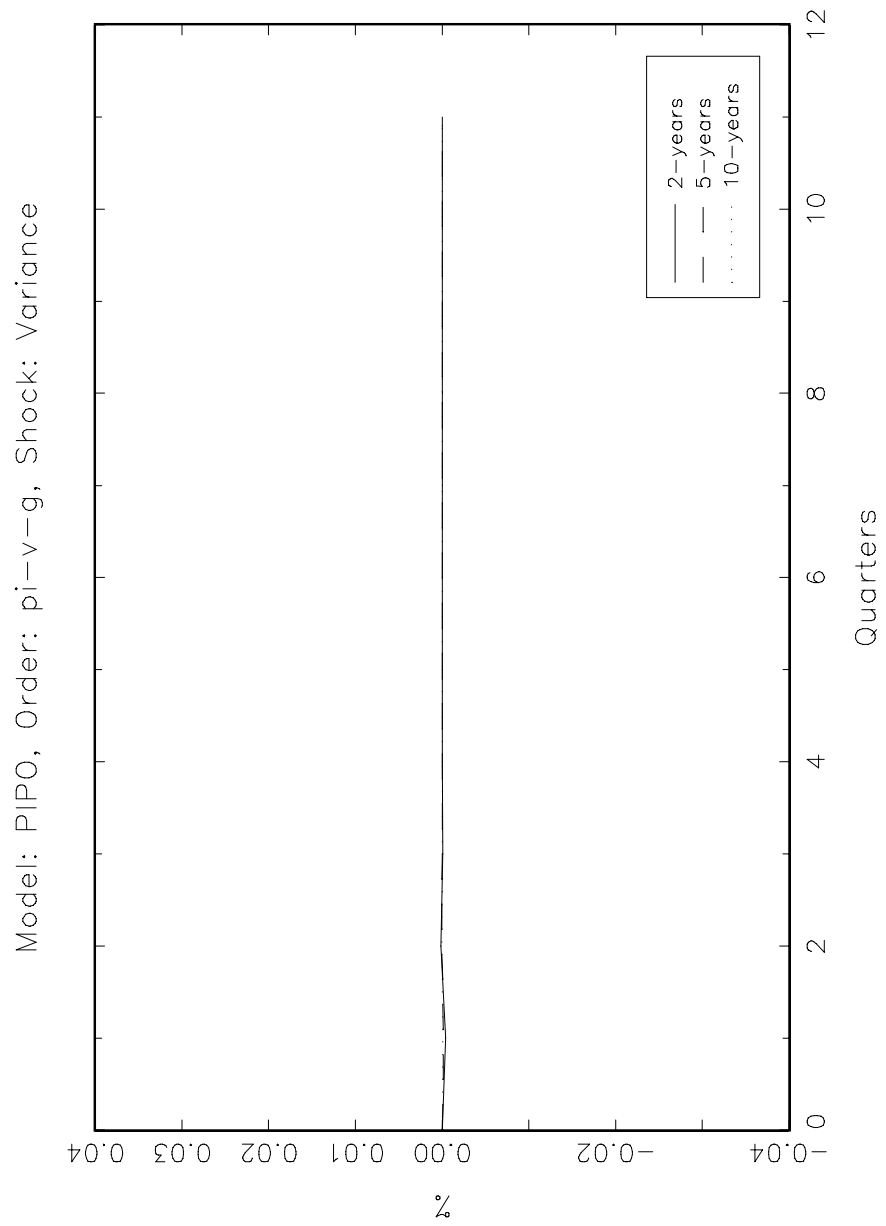


Figure B.28: Real rate responses to factors shocks.

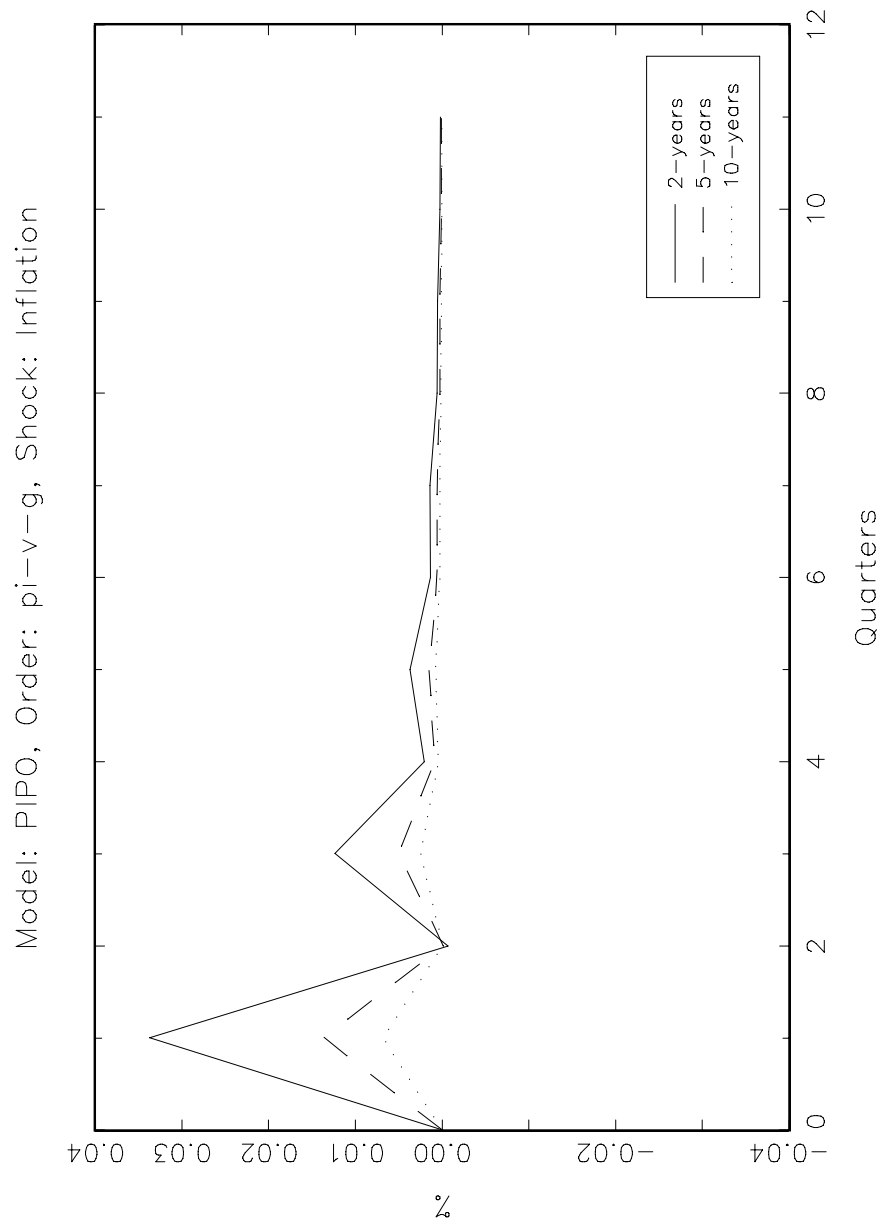


Figure B.29: Real rate responses to factors shocks.