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Liquidity Trap and Excessive Leverage

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Abstract

We investigate the role of debt market policies in mitigating liquidity traps driven by deleveraging. When constrained agents engage in deleveraging, the interest rate needs to fall to induce unconstrained agents to pick up the decline in aggregate demand. However, if the fall in the interest rate is limited by the zero lower bound, aggregate demand is insufficient and the economy enters a liquidity trap. In such an environment, agents' *ex-ante* leverage and insurance decisions are associated with aggregate demand externalities. The competitive equilibrium allocation is constrained inefficient. Welfare can be improved by macroprudential policies such as debt limits and mandatory insurance requirements.

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1 Introduction

Leverage has been proposed as a key contributing factor to the recent recession and the slow recovery in the US. Several authors have documented the dramatic increase of leverage in the household sector before 2006 as well as the subsequent deleveraging episode. Using county-level data, Mian and Sufi (2010) have argued that household deleveraging is responsible for much of the job losses between 2007 and 2009. This view has recently been formalized in a number of theoretical models, e.g., Hall (2011), Eggertsson and Krugman (2012), and Guerrieri and Lorenzoni (2012). These models have emphasized that the interest rate needs to fall when constrained agents engage in deleveraging to induce unconstrained agents to make up for the lost aggregate demand. However, the nominal interest rate cannot fall below

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zero given that hoarding money provides an alternative to holding bonds -a phenomenon also known as the *liquidity trap.* When inflation is sticky, the lower bound on the nominal rate also prevents the real interest rate from declining sufficiently to clear the goods market, plunging the economy into a demand-driven recession.

An important question concerns the optimal policy response to these episodes. The US treasury and the Federal Reserve have responded to the recent recession by utilizing fiscal stimulus and unconventional monetary policies. These policies are (at least in part) supported by a growing theoretical literature. These contributions have understandably taken an ex-post perspective–characterizing the optimal policy once the economy is in the trap. Perhaps more surprisingly, they have also largely ignored the debt market even though the problems are thought to have originated in the debt market.[1] As a result, a number of policy questions remain unanswered. Do agents take on efficient levels of debt in the run-up to deleveraging episodes? Do they take on efficient levels of insurance?

To address these questions, we present a stylized model of the liquidity trap driven by deleveraging. The distinguishing feature of our model is that agents endogenously accumulate leverage, even though they anticipate the upcoming deleveraging episode. When some agents have a sufficiently strong reason to borrow, there is a demand-driven recession when the anticipated deleveraging episode materializes.[2]

Our main result is that it is desirable to use *preventive policies* to slow down the accumulation of leverage in such instances. In the run-up to such episodes, borrowers who behave individually rationally undertake *excessive leverage* from a social point of view. A simple debt market policy that restricts leverage (coupled with appropriate ex-ante transfers) could make all agents better off. This result obtains whenever the deleveraging episode is severe enough to trigger a liquidity trap –assuming that the liquidity trap cannot be fully alleviated by ex-post policies.

The mechanism behind the constrained inefficiency is an *aggregate demand externality* that applies in environments in which output is influenced by aggregate demand. When this happens, the decisions of economic agents that affect aggregate demand also affect aggregate output, and therefore other agents' income. Agents do not take into account these general equilibrium effects, which may lead to inefficiencies. In our economy, the liquidity trap ensures that output is influenced by demand and that it is below its (first-best) efficient

[1] Several papers capture the liquidity trap in a representative household framework which leaves no room for debt market policies (see Eggertsson and Woodford (2003), Werning (2012), and Correira et al. (2012)). An exception is Eggertsson and Krugman (2011), which features debt but does not focus on debt market policies.

[2] We do not claim that most agents in the US expected the deleveraging episode–the evidence suggests otherwise. But some agents and, more importantly, regulators might have taken such a possibility into account, especially in 2006 and 2007.

level. Moreover, greater ex-ante leverage leads to a greater reduction in aggregate demand and a deeper recession. Borrowers who choose their leverage (and lenders who finance them) do not take these negative demand externalities into account, leading to excessive leverage.

Our second main result establishes that borrowers are also *underinsured* with respect to deleveraging episodes. Intuitively, borrowers do not take into account the positive aggregate demand externalities their insurance purchases would bring about. A mandatory insurance requirement (coupled with ex-ante transfers) could make all households better off. Among other things, this result provides a rationale for indexing mortgage contracts to house prices.

Next we investigate the scope for preventive monetary policy. We show that contractionary monetary policy during periods when borrowers accumulate their debt holdings will not generally help to reduce leverage. In particular, this type of policy is inferior to debt market policies in the sense that it cannot implement the constrained efficient allocations that can be obtained using simple debt limits. However, an increase in the inflation target can reduce the incidence of liquidity traps.

Finally, we endogenize the debt limit faced by borrowers by assuming that debt is collateralized by financial assets, creating the potential for fire-sale effects. This introduces a new feedback loop into the economy. First, a decline in asset prices reduces the borrowing capacity of agents and force them to delever. Second, in a liquidity trap, deleveraging leads to a demand-induced decline in output that pushes down asset prices even further. This suggests that episodes of deleveraging that involve collateral assets that decline in price are particularly severe. Furthermore, the model also provides a channel through which asset price declines hurt all agents in an economy through aggregate demand effects, even if they do not hold financial assets.

The remainder of this paper is structured as follows. The next subsection discusses the related literature. Section 2 introduces the key aspects of our environment. Section 3 characterizes an equilibrium that features an anticipated demand-driven recession. Section 4 presents our main result about excessive leverage. Section 5 generalizes the model to incorporate uncertainty and presents our second main result about underinsurance. Section 6 discusses the role of preventive monetary policies in our environment. Section 7 endogenizes the debt limit and clarifies the relationship between aggregate demand and fire sale externalities, and Section 8 concludes. The appendix contains generalizations of main results and omitted proofs.

1.1 Related literature

Our paper is related to a long economic literature studying the zero lower bound on nominal interest rates and liquidity traps, starting with Hicks (1937) and Krugman (1998) in simple

IS/LM-style frameworks to investigations in a New Keynesian framework by e.g. Eggertsson and Woodford (2003, 2004). A growing recent literature has investigated the optimal fiscal and monetary policy response to liquidity traps (see e.g. Eggertsson, 2009; Correia et al., 2011; Werning, 2012). Our contribution to this literature is that we focus on debt market policies, mainly from an ex-ante perspective.

Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2012) describe how financial market shocks that induce borrowers to delever lead to a decline in interest rates, which in turn can trigger a liquidity trap. Our framework is most closely related to Eggertsson and Krugman because we also model deleveraging between a set of impatient borrowers and patient lenders, which enables us to obtain many of our results in closed form. Whereas this literature focuses on the positive implications of episodes of deleveraging and monetary and fiscal responses, our contribution is to show that deleveraging-induced liquidity traps lead to aggregate demand externalities. Among other things, our paper calls for novel policy actions in debt markets that are significantly different from the more traditional monetary and fiscal policy responses to liquidity traps.

The aggregate demand externality that we identify in our paper is similar to the externalities described by Farhi and Werning (2012ab, 2013) and Schmitt-Grohe and Uribe (2012abc). The broad idea is that, when output is influenced by aggregate demand, decentralized allocations are inefficient because agents do not internalize the impact of their actions on aggregate demand. In Farhi and Werning (2012ab), output responds to aggregate demand because prices are sticky and countries are in a currency union (and thus, under the same monetary policy). They emphasize the inefficiencies in cross-country insurance arrangements. Schmitt-Grohe and Uribe identify a similar externality that is driven by the downward rigidity of nominal wages. In our model, output is demand-determined because of a liquidity trap, and we emphasize the inefficiencies in household leverage in a closed economy setting. Farhi and Werning (2013) develop a general theory of aggregate demand externalities in the presence of nominal rigidities and constraints on monetary policy, with applications including liquidity traps and currency unions. Our framework falls into this broad class of aggregate demand externalities, but we focus in depth on the externalities created by deleveraging in a liquidity trap.

Our results on excessive borrowing and risk-taking also resemble the recent literature on pecuniary externalities, including Caballero and Krishnamurthy (2003), Lorenzoni (2008), Bianchi and Mendoza (2010), Jeanne and Korinek (2010ab) and Korinek (2011). In those papers, agents do not internalize the impact of individual decisions on aggregate prices, and a planner can improve on outcomes by moving asset prices in a way that relaxes financial constraints. The aggregate demand externality of this paper works through a completely different channel – individual agents do not internalize that their private deleveraging reduces aggregate demand, and the interest rate cannot decline sufficiently to induce borrowers to make up for the lost demand and clear markets, creating an inefficient labor wedge. A planner internalizes that reducing leverage ex-ante supports aggregate demand during episodes of deleveraging and reduces the labor wedge.

2 Environment and equilibrium

The economy is set in infinite discrete time $t \in \{0, 1, ...\}$, with a single consumption good. There are two types of households, borrowers and lenders, denoted by $h \in \{b, l\}$. There is an equal measure of each type of households, normalized to 1/2. Households are symmetric except that borrowers have a weakly lower discount factor than lenders, $\beta^b \leq \beta^l$. Let d_{t+1}^h denote the outstanding debt (or savings if negative) of household h for date $t+1$. Let r_{t+1} denote the real interest rate between dates t and $t+1$.

Our first key ingredient is a tightening of borrowing constraints, which is fully anticipated in our baseline framework. For simplicity, households can choose d_1^h without any constraints at date 0. From date 1 onwards, households are subject to an exogenous borrowing constraint: $d_{t+1}^h \leq \phi$, where $\phi > 0$ denotes an exogenous debt limit as in Aiyagari (1994), or more recently, Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2012). This can be thought of as corresponding to a financial shock, e.g. a drop in collateral values or loan-to-value ratios, that tightens borrowing constraints. (In Section 5 we will introduce uncertainty about the financial shock; in Section 7 we will endogenize the tightening of the constraint because of falling asset prices.)

Our second key ingredient is a lower bound on the real interest rate:

$$r_{t+1} \geq 0 \text{ for each } t \geq 1. \tag{1}$$

We obtain this ingredient from two assumptions related to nominal variables. Consider the cashless limit economy described in Woodford (2003). Let P_t denote the nominal price of the consumption good at date t and i_{t+1} denote the nominal interest rate.

Assumption (A1). There is a zero lower bound on the nominal interest rate:

$$i_{t+1} \geq 0 \text{ for each } t \geq 0. \tag{2}$$

This assumption captures a no-arbitrage condition between money and government bonds.

Assumption (A2). The nominal interest rate, i_{t+1}, is set according to a standard Taylor rule with a zero inflation target adjusted for the ZLB constraint.

In our setting, this assumption implies that the inflation expectations starting date 1 are equal to the inflation target (for a more formal treatment, see Eq. $(A.1)$ and the associated discussion in Appendix A):

$$P_{t+1}/P_t = 1 \text{ for each } t \geq 1. \tag{3}$$

Combining Eqs. (2) and (3) with the Fisher equation, $1 + r_{t+1} = (1 + i_{t+1})\, E_t \left[\frac{P_t}{P_{t+1}}\right]$, leads to the lower bound in (1).

Remark The only role of the Taylor rule in our setting is to generate *the stickiness of inflation expectations* in Eq. (3). This prediction could be obtained in at least two other ways. First, stickiness of nominal prices or wages, as in New Keynesian models, would generate a very similar prediction. In such models, Eq. (3) would represent the limit case in which the fraction of agents who can adjust their prices goes to zero. Second, a boundedly rational model in which individuals' inflation expectations are based on limited or past information would also generate a similar prediction. In recent work, Malmendier and Nagel (2013) document that individuals' inflation expectations are in fact influenced by their personal experience, and thus, are not fully determined by monetary policy. In the extreme, suppose individuals' inflation expectations are fully determined by history in which case Eq. (3) would hold (after appropriately adjusting the inflation target). We have chosen to emphasize the Taylor rule both because it is widely used by central banks and because it is the *ex-post efficient* policy in this setting when there is some cost to inflation.

The demand side of the model is described by households' consumption-savings decision. For the baseline model, we assume households' state utility function over consumption $\tilde{c}_t^h$ and labor n_t^h takes the particular form, $u\left(\tilde{c}_t^h - v\left(n_t^h\right)\right)$. We define $c_t = \tilde{c}_t^h - v\left(n_t^h\right)$ as *net consumption*. Households' optimization problem can then be written as:

$$\begin{aligned} \max_{\left\{c_t^h, d_{t+1}^h, n_t^h\right\}_t} & \quad \sum_{t=0}^{\infty} \left(\beta^h\right)^t u\left(c_t^h\right) \\ \text{s.t. } c_t^h &= e_t^h - d_t^h + \frac{d_{t+1}^h}{1 + r_{t+1}} \text{ for all } t, \\ \text{where } e_t^h &= w_t n_t^h + \Pi_t - v\left(n_t^h\right) \text{ and } d_{t+1}^h \leq \phi_{t+1} \text{ for each } t \geq 1. \end{aligned} \tag{4}$$

Here w_t denotes wages, Π_t denotes profits from firms that are described below, and e_t^h denotes households' *net income*, that is, their income net of labor costs. The preferences, $u\left(\tilde{c}_t^h - v\left(n_t^h\right)\right)$, provide tractability but are not necessary for our main results about ex-ante inefficiency (see Appendix A.4). As noted in Greenwood, Hercowitz and Huffman (GHH, 1988), the specification implies that there is no wealth effect on labor supply. As a result, the efficient output level is constant.

The supply side is described by a linear technology that can convert one unit of labor to one unit of the consumption good. The efficient level of net income is then given by:

$$e^* \equiv \max_{n_t} n_t - v\left(n_t\right).$$

However, the equilibrium does not necessarily feature efficient production due to the constraint in (1). When this constraint binds, the interest rate is too high relative to its market clearing level. Since the interest rate is the price of current consumption good (in terms of the future consumption good), an elevated price in the market for current goods leads to a demand shortage and a rationing of supply.

To capture the possibility of rationing, we modify the supply side of the Walrasian equilibrium to accommodate the constraint in (1). In particular, we consider a competitive goods sector that solves the following optimization problem:

$$\Pi_t = \max_{n_t} n_t - w_t n_t \quad \text{s.t.} \quad \begin{cases} 0 \leq n_t, & \text{if } r_{t+1} > 0 \\ 0 \leq n_t \leq \frac{\tilde{c}_t^b + \tilde{c}_t^l}{2}, & \text{if } r_{t+1} = 0 \end{cases}. \tag{5}$$

When the real interest rate is positive, $r_{t+1} > 0$, the sector optimizes as usual. When the interest rate is at its lower bound, $r_{t+1} = 0$, the sector is subject to an additional constraint that supply cannot exceed the aggregate demand for goods, $\frac{\tilde{c}_t^b + \tilde{c}_t^l}{2}$. When this constraint binds, the sector is making positive profits, and firms are in principle willing to increase their output. However, their output is rationed (by a mechanism we leave unspecified) due to a shortage of aggregate demand. The equilibrium output is then determined by aggregate demand at the the zero interest rate, $r_{t+1} = 0$. We also assume households have equal ownership of firms so that each household receives profits, $\Pi_t = n_t - w_t n_t$.[3]

Definition 1 (Equilibrium)**.** *The equilibrium is a path of allocations,* $\left\{c_t^h, d_{t+1}^h, n_t^h, e_t^h\right\}_t$*, real prices and profits,* $\left\{w_t, r_{t+1}, \Pi_t\right\}_t$*, nominal prices* $\left\{P_t, i_{t+1}\right\}$ *such that: households solve problem* (4)*, nominal prices are consistent with (A1)-(A2), the final good sector solves problem* (5) *and markets clear.*

We also make the standard assumptions about preferences: that is $u\left(\cdot\right)$ and $v\left(\cdot\right)$ are both strictly increasing, $u\left(\cdot\right)$ is strictly concave and $v\left(\cdot\right)$ is strictly convex, and they satisfy the Inada-type conditions $\lim_{c \to 0} u'\left(c\right) = \infty, v'\left(0\right) = 0$ and $\lim_{n \to \infty} v'\left(n\right) = \infty$. In addition, we assume $\frac{u'\left(2e^*\right)}{u'\left(e^* + \phi\left(1-\beta^l\right)\right)} < \beta^l$, which allows for the constraint on the real rate to bind. Finally, we simplify the notation as follows. First note that households' labor supply in equilibrium is the same, $n_t^h = \left(v'\right)^{-1}\left(w_t\right)$, which implies that their net income, e_t^h, is also the same.

[3] Note that we modify the Walrasian equilibrium in the goods market but not in the labor market (which is assumed to be competitive as usual). This is because the direct effect of the constraint on the real interest rate is to create a demand shortage in the goods market. This constraint is consistent (at least in principle) with a Walrasian equilibrium in the labor market.

Hence, we let $e_t = n_t - v(n_t)$ denote this common value of net income. Second, the market clearing for debt implies $d_t^l = -d_t^b$. Hence, we drop the superscript on debt and denote the debt level of borrowers in a given period by $d_t^b = d_t$, and that of lenders by $d_t^l = -d_t$.

3 An anticipated demand-driven recession

This section characterizes the decentralized equilibrium and describes a recession that is anticipated by households. The next section analyzes the efficiency properties of this equilibrium. We consider equilibria in which borrowers' debt constraint binds at all future dates, that is, $d_{t+1} = \phi$ for each $t \geq 1$. In our setting, a sufficient condition for this is $d_1 \geq \phi$. We will make assumptions so that we are always in this case.

Steady state First consider dates $t \geq 2$. At these dates, the economy is in a steady-state. Since borrowers are constrained, the real interest rate is determined by the discount factor of lenders and is constant at $r_{t+1} = 1/\beta^l - 1 > 0$. At a positive interest rate, aggregate demand is not a constraining factor and firms are optimizing as usual so that equilibrium wages are given by $w_t = 1$ [cf. problem (5)]. The optimization problem of households (4) then implies that their net income is at its efficient level and consumption is given by:

$$c_t^b = e^* - \phi\left(1 - \beta^l\right) \text{ and } c_t^l = e^* + \phi\left(1 - \beta^l\right) \text{ for } t \geq 2 \tag{6}$$

Deleveraging Next consider date $t = 1$. Borrowers' consumption is given by $c_1^b = e_1 - \left(d_1 - \frac{\phi}{1+r_2}\right)$. In particular, the larger the outstanding debt level d_1 is relative to the debt limit, the more borrowers are forced to reduce their consumption. The resulting slack in aggregate demand needs to be absorbed by an increase in lenders' consumption: $c_1^l = e_1 + d_1 - \frac{\phi}{1+r_2}$. In view of the Euler equation of lenders $\frac{u'\left(c_1^l\right)}{\beta^l u'\left(e^*+\phi\left(1-\beta^l\right)\right)} = 1 + r_2$, the increase in lenders' consumption is mediated through a decrease in the real interest rate, r_2. The key observation is that the lower bound on the real interest rate effectively sets an upper bound on lenders' consumption, $c_1^l \leq \bar{c}_1^l$, given by the solution to

$$u'\left(\bar{c}_1^l\right) = \beta^l u'\left(e^* + \phi\left(1 - \beta^l\right)\right). \tag{7}$$

The equilibrium in period 2 then depends on the relative size of two terms:

$$d_1 - \phi \lesseqgtr \bar{c}_1^l - e^*.$$

The left hand side is the amount of deleveraging borrowers are forced into in a financial shock state (when the real rate is at its constrained level). The right hand side is the

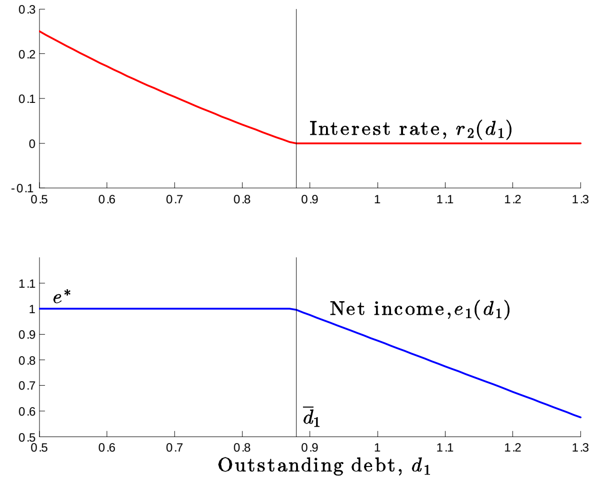

Figure 1: Interest rate and net income at date 1 as a function of outstanding debt d_1.

maximum amount of demand the unconstrained agents can absorb when the real rate is at its lower bound. If the left side is smaller than the right side, then the equilibrium features $r_2 \geq 0$ and $e_1 = e^*$. In this case, the effects of deleveraging on aggregate demand are offset by a reduction in the real interest rate and aggregate supply is at its efficient level e^*. The left side of Figure 1 (the range corresponding to $d_1 \leq \overline{d}_1$) illustrates this outcome.

Otherwise, equivalently when the outstanding debt level is strictly above a threshold

$$d_1 > \overline{d}_1 = \phi + \overline{c}_1^l - e^*, \tag{8}$$

then the constraint on the real rate binds, $r_2 = 0$. The interest rate cannot fall sufficiently to induce lenders to consume the efficient level of output. In this case, households' net consumption is given by $c_1^b = e_1 - d_1 + \phi$ and $c_1^l = \overline{c}_1^l$. Firms' demand for labor is determined by aggregate demand for consumption, $n_t = \frac{\tilde{c}_t^b + \tilde{c}_t^l}{2}$. Hence, households' net income, $e_1 = n_1 - v(n_1)$, is also determined by aggregate demand for net consumption:

$$e_1 = \frac{c_1^b + c_1^l}{2} = \frac{e_1 - (d_1 - \phi) + \overline{c}_1^l}{2}.$$

After rearranging this expression, the equilibrium level of net income is given by:

$$e_1 = \overline{c}_1^l + \phi - d_1 < e^*. \tag{9}$$

In words, there is a demand shortage and rationing in the goods market, which in turn lowers wages and employment in the labor market, creating a demand driven recession.

The right side of Figure 1 (the range corresponding to $d_1 \geq \bar{d}_1$) illustrates this outcome. Note also that a greater level of outstanding debt level leads to a greater recession, which will be important in the welfare analysis that follows. Intuitively, greater deleveraging by constrained agents induces a greater shock to aggregate demand, which in turn translates into a greater recession.

Date 0 Allocations We next turn to households' financial decisions at date 0. We conjecture an equilibrium in which the net income is at its efficient level, $e_0 = e^*$. Since households are unconstrained, both of their Euler equations hold:

$$1 + r_1 = \frac{u'\left(c_0^l\right)}{\beta^l u'\left(c_1^l\right)} = \frac{u'\left(c_0^b\right)}{\beta^b u'\left(c_1^b\right)}. \tag{10}$$

The equilibrium debt level, d_1, and the interest rate, r_1, are determined by these equations. We next identify two conditions under which households choose a sufficiently high debt level that triggers a recession, $d_1 > \bar{d}_1$.

Condition 1. *There is a deleveraging-induced recession in period* 1 *if the borrower is sufficiently impatient or sufficiently indebted in period* 0*. Specifically, for any debt level* d_0 *there is a threshold level of impatience* $\bar{\beta}^b(d_0)$ *such that the economy experiences a recession in period* 1 *if* $\beta^b < \bar{\beta}^b(d_0)$*. Conversely, for any level of impatience* β^b *there is a threshold debt level* $\bar{d}_0\left(\beta^b\right)$ *such that the economy experiences a recession in period* 1 *if* $d_0 > \bar{d}_0\left(\beta^b\right)$*.*

We derive the relevant threshold levels in Appendix A. Under these conditions, the appendix establishes that the economy experiences a demand driven recession and liquidity trap at date 1.[4]

4 Excessive leverage

This section analyzes the efficiency properties of equilibrium and presents our main result. We first illustrate the aggregate demand externalities in our setting. We then illustrate that the equilibrium can be Pareto improved *even ex post*, that is, starting date 1. Although this result is special, it clearly illustrates the potential strength of aggregate demand externalities. We then present our main result about ex-ante inefficiencies.

[4] When the initial debt level d_0 is too high, the deleveraging of borrowers may also push the economy into the zero lower bound in period 0. The relevant threshold can be derived analogously. We abstract away from these issues by allowing r_0 to fall below 0.

4.1 Aggregate demand externalities

We consider a constrained planner that can affect the amount of debt d_1 that individuals carry into period 1 but cannot interfere thereafter. We focus on constrained efficient allocations with $d_1 \geq \phi$, so that conditional on d_1, the economy behaves as we analyzed in the previous section for date 1 onwards.

Let $V^h(d_1; D_1)$ denote the utility of a household of type h conditional on entering period 1 with an individual level of debt d_1 and an aggregate level of debt D_1. The aggregate debt level D_1 enters household utility because it determines the interest rate or net income at date 1. More specifically, we have:

$$\begin{aligned} V^b(d_1, D_1) &= u\left(e_1(D_1) - d_1 + \frac{\phi}{1 + r_2(D_1)}\right) + \sum_{t=2}^{\infty} \left(\beta^b\right)^t u\left(c_t^b\right) \\ V^l(d_1, D_1) &= u\left(e_1(D_1) + d_1 - \frac{\phi}{1 + r_2(D_1)}\right) + \sum_{t=2}^{\infty} \left(\beta^l\right)^t u\left(c_t^l\right) \end{aligned} \tag{11}$$

where $r_2(D_1)$ and $e_1(D_1)$ are characterized in the previous section and the continuation utilities from date 2 onwards do not depend on d_1 or D_1 [cf. Eq. (6)].

In equilibrium, we will find that $D_1 = d_1$ since individual agents of type h are symmetric. But taking D_1 explicitly into account is useful to illustrate the externalities. In particular, the private marginal value of debt for an individual household is given by $\frac{\partial V^h}{\partial d_1} = u'\left(c_1^h\right)$, whereas the social marginal is $\frac{\partial V^h}{\partial d_1} + \frac{\partial V^h}{\partial D_1}$. Hence, the externalities from leverage in this setting are captured by $\frac{\partial V^h}{\partial D_1}$, which we characterize next.

Lemma 1. *(i) If $D_1 \in [\phi, \bar{d}_1)$, then $\frac{\partial V^h}{\partial D_1} = \begin{cases} -\eta u'\left(c_1^h\right) < 0, \text{ if } h = l \\ \eta u'\left(c_1^h\right) > 0, \text{ if } h = b \end{cases}$, where $\eta = \frac{-r_2'(D_1)\phi}{(1+r_2)^2} \in (0,1)$.*

(ii) If $D_1 > \bar{d}_1$, then

$$\frac{\partial V^h}{\partial D_1} = -u'\left(c_1^h\right) < 0, \text{ for each } h \in \{b, l\}. \tag{12}$$

The first part of the lemma illustrates the usual pecuniary externalities. It concerns the case in which the debt level is not large so that output is not influenced by demand, that is $e_1(D_1) = e^*$. An increase in leverage then reduces the interest rate (to counter the reduction in demand), which in turn generates a redistribution from lenders to borrowers. Hence, deleveraging imposes positive pecuniary externalities on borrowers but negative pecuniary externalities on lenders. In fact, since markets between date 0 and 1 are complete, these two effects "net out" from an ex-ante point of view. In particular, the date 0 equilibrium is constrained efficient in this region (see Proposition 2 below).

The second part of the lemma illustrates the novel force in our model, *aggregate demand externalities*, and contrasts them with pecuniary externalities. In this case, the debt level is sufficiently large so that the economy is in a liquidity trap, which has two implications. First, the interest rate is fixed, $r_2(D_1) = 0$, so that the pecuniary externalities do not apply. Second, output is below its efficient level, $e_1(D_1) < e^*$, and it is decreasing in leverage, D_1, in view of a reduction in aggregate demand [cf. Eq. (9)]. Consequently, an increase in leverage imposes negative externalities on all agents, $\frac{\partial V^h}{\partial D_1} < h$ for each h, which we refer to as aggregate demand externalities.

A noteworthy feature about this externality is that it affects all agents, even though the zero lower bound only limits the consumption of lenders. This is because the reduced demand from lenders at the zero lower bound pushes down incomes and therefore hurts borrowers through the same channel as it hurts lenders. This feature suggests that, unlike pecuniary externalities, aggregate demand externalities can lead to constrained inefficiencies in our setting, which we verify next.

4.2 Ex-post inefficiency and debt writedowns

The equilibrium in our baseline setting can be Pareto improved by an ex-post debt writedown. To see this, suppose lenders forgive some of borrowers' outstanding debt so that leverage is reduced from d_1 to the threshold, $\overline{d}_1$, given by Eq. (8). By our earlier analysis, the recession is avoided and the net income increases to its efficient level, e^*. Borrowers' net consumption and welfare naturally increases after this intervention. Less obviously, lenders' net consumption remains the same at the upper bound, $\bar{c}_1^l$. We thus obtain the following result.

Proposition 1 (Ex-post Inefficiency)**.** *The equilibrium under condition 1 is ex-post constrained Pareto inefficient (given state preferences $u\left(c_t^h - v\left(n_t^h\right)\right)$). In particular, reducing all borrowers' outstanding debt at date* 1 *to* $\overline{d}_1$ *in Eq.* (8) *strictly increases borrowers' welfare without affecting lenders' welfare.*

A debt writedown increases borrowers' welfare both directly, $-\frac{\partial V^b}{\partial d_1} = u'\left(c_1^b\right) > 0$, and indirectly through aggregate demand externalities, $-\frac{\partial V^b}{\partial D_1} = u'\left(c_1^b\right) > 0$. In contrast, the direct effect on lenders' welfare is negative, $-\frac{\partial V^l}{\partial d_1} = -u'\left(c_1^l\right) < 0$, while the indirect effect through aggregate demand externalities is positive. Lemma 1 shows that the externalities are sufficiently strong to fully counter the direct effect, $-\frac{\partial V^l}{\partial D_1} = u'\left(c_1^l\right) > 0$. Hence, the net effect on lenders' welfare is zero, which provides an alternative proof of Proposition 1. Intuitively, the reduction in aggregate debt mitigates the recession and increases agents' income just enough to leave lenders indifferent despite a reduction in their assets.

Aggregate demand externalities are sufficiently strong in part because of the GHH form of lenders' state-preferences, $u\left(c_1^l - v\left(n_1\right)\right)$. This form ensures that, when the real rate is at its lower bound, lenders' consumption net of their disutility of labor is a purely forward looking variable that is independent of the current state of the economy [cf. Eq. (7)]. Consequently, a debt write-down increases lenders' income net of the additional disutility of labor by precisely the same amount as the write-down, as long as the zero lower bound is binding. That said, debt write-downs in a liquidity trap lead to strong positive externalities more generally, even if they do not always generate a Pareto improvement.[5]

4.3 Ex-ante inefficiency and excessive leverage

Ex-post debt writedowns might be difficult to implement in practice for a variety of reasons, e.g., legal restrictions, concerns with moral hazard, or concerns with the financial health of intermediaries (assuming that some lenders are intermediaries). An alternative way to reduce deleveraging is to prevent the accumulation of debt in the first place. To capture this possibility, suppose households' date 0 leverage choices are subject to an additional constraint, $d_1^h \leq D_1$, where D_1 is an endogenous debt limit (which will also be the equilibrium debt limit, hence the abuse of notation). To trace the constrained efficient frontier, we also allow for a transfer of wealth, T_0, at date 0 from lenders to borrowers so that the outstanding debt becomes $d_0 - T_0$.

Our main result characterizes the constrained efficient allocations in this setting, which also illustrates the Pareto inefficiency of the equilibrium described in Section 3. Consider a planner that chooses the period 0 allocations of households, $\left(c_0^h, n_0^h\right)_h$, as well as the debt level next period, D_1, and leaves the remaining allocations starting date 1 to the market. We will see that the allocations chosen by this planner can be implemented with the simple policies described above. Let $W^h\left(D_1\right) = V^h\left(D_1; D_1\right)$ capture type h agents' continuation payoffs at date 1 in equilibrium [cf. Eqs. (11)]. The constrained planning problem can then be written as:

$$\max_{\left(\left(c_0^h, n_0^h\right)_h, D_1\right)} \sum_h \gamma^h \left(u\left(c_0^h\right) + \beta^h W^h\left(D_1\right)\right) \text{ s.t. } \sum_h c_0^h = \sum_h n_0^h - v\left(n_0^h\right),$$

where $\gamma^h \neq 0$ captures the relative welfare weight assigned to type h agents.

[5] Appendix A.4 illustrates this by characterizing the equilibrium corresponding to standard separable preferences, $u(c) - v(n)$. In this case, lenders' consumption (as opposed to net consumption) remains constant, which necessitates sufficiently strong general equilibrium effects to counter a debt writedown. Despite strong externalities, the debt writedown does not lead to a Pareto improvement in this case because it also increases lenders' disutility of labor, $v\left(n_1^l\right)$.

The optimality conditions for consumption and leverage can be combined to give:

$$\frac{u'\left(c_0^l\right)}{\beta^l\left(u'\left(c_1^l\right)+\partial V^l/\partial D_1\right)}=\frac{u'\left(c_0^b\right)}{\beta^b\left(u'\left(c_1^b\right)-\partial V^b/\partial D_1\right)} \text{ for each } D_1 \neq \bar{d}_1. \tag{13}$$

That is, the planner equates relative marginal utilities at dates 0 and 1 while also taking externalities into account. For $D_1 > \bar{d}_1$, given the result $\partial V^h/\partial D_1 < 0$ for each h, the planner perceives date 1 social marginal utility to be lower for lenders' consumption and higher for borrowers' consumption relative to a standard Euler equation (10). For $D_1 = \bar{d}_1$, the optimality condition holds as inequality because the function V^h has a kink induced by the lower bound $r_2 \geq 0$. We next present our main result.

Proposition 2 (Excessive Leverage)**.** *An allocation $\left(\left(c_0^h, n_0^h\right)_h, D_1\right)$, with $D_1 \geq \phi$, is constrained efficient if and only if labor supply and output at date 0 is efficient, i.e., $v'\left(n_0^h\right)=1$ for each h; and the consumption and debt allocations satisfy one of the following:*

(i) $D_1 < \bar{d}_1$ and the Euler equation (10) *holds.*

(ii) $D_1 = \bar{d}_1$, and the following distorted Euler equation holds:

$$\frac{u'\left(c_0^l\right)}{\beta^l u'\left(c_1^l\right)} \leq \frac{u'\left(c_0^b\right)}{\beta^b u'\left(c_1^b\right)}. \tag{14}$$

Moreover, every constrained efficient allocation of this type can be implemented as a competitive equilibrium with the debt limit, $d_1^h \leq \bar{d}_1$ for each h, combined with an appropriate ex-ante transfer, T_0.

The first part illustrates that equilibrium allocations in which $d_1 < \bar{d}_1$ are constrained efficient. This part verifies that pecuniary externalities alone do not generate inefficiencies in this setting.

The second part, which is our main result, concerns equilibria in which $d_1 \geq \bar{d}_1$ and aggregate demand externalities are active (on the margin). Constrained efficient allocations in this region are characterized by the debt level, $D_1 = \bar{d}_1$, and the distorted Euler equation in (14). In particular, the competitive equilibrium under Condition 1, which features $d_1 > \bar{d}_1$, is constrained inefficient.

To build intuition for the inefficiency, it is useful to characterize explicitly the Pareto improving policy in this case. By Proposition 1, the debt limit $D_1 = \bar{d}_1$ (weakly) increases all households' welfare starting date 1. However, it also changes households' date 0 consumption: Lenders' net consumption is higher (since they save less) and borrowers' net consumption is lower (since they borrow less). An appropriate initial transfer to borrowers ensures that households' date 0 net consumption is also unchanged. Thus, the resulting allocation is a Pareto improvement over the equilibrium. Intuitively, borrowers that choose their leverage (or equivalently, lenders that finance them) do not take into account the adverse general equilibrium effects on demand and output at date 1. A debt limit internalizes these externalities and leads to an ex-ante Pareto improvement.

Unlike Proposition 1, our ex-ante inefficiency results apply quite generally. Appendix A.4 illustrates this by characterizing the equilibrium corresponding to standard separable preferences, $u(c) - v(n)$. In this case, aggregate demand externalities are given by

$$\frac{\partial V^h}{\partial D_1} = u'\left(c_1^h\right)\frac{\partial n_1^h}{\partial D_1}\tau_1 \text{ for } D_1 > \overline{d}_1, \tag{15}$$

where $\tau_1 = 1 - \frac{v'\left(n_1^h\right)}{u'\left(c_1^h\right)} > 0$ is the labor wedge at date 1. The labor wedge is positive since the economy experiences an inefficient demand driven recession (given $D_1 > \overline{d}_1$). Eq. (15) then illustrates that the strength of the externalities for type h households depends on their employment response to leverage. The appendix also establishes that $\frac{\partial n_1^l}{\partial D_1} + \frac{\partial n_1^h}{\partial D_1} < 0$, that is, aggregate employment always declines with leverage due to the shortage of aggregate demand. Combining these observations with the planner's optimality condition (13), it follows that constrained efficient allocations with $D_1 \geq \overline{d}_1$ satisfy the distorted Euler equation (14) also in this case. Proposition blah in the appendix formalizes this result and establishes the constrained inefficiency of competitive equilibrium whenever $d_1 > \overline{d}_1$.[6]

5 Uncertainty and underinsurance

We next analyze the efficiency of households' insurance arrangements before deleveraging episodes. This requires extending our earlier analysis to incorporate uncertainty. We assume that the economy is in one of two states $s \in \{H, L\}$ from date 1 onwards. The states differ in their debt limits. State L captures a deleveraging state with a debt limit as before, $\phi_{t+1,L} \equiv \phi$ for each $t \geq 1$. State H in contrast captures an unconstrained state similar to date 0 of the earlier analysis, that is, $\phi_{t+1,H} = \infty$ for each $t \geq 1$. We let π_s^h denote the belief of type h households for state s. We assume $\pi_L^h > 0 \ \forall h$ so that the deleveraging episode is anticipated by all households.

We simplify the analysis by assuming that starting date 1, both types of households have the same discount factor $\beta^b = \beta^l = \beta$.[7] At date 0, however, borrowers are weakly more impatient than lenders, $\beta_0^b \leq \beta = \beta_0^l$. In addition, we also assume borrowers are

[6] The only part of Proposition 2 that does not generalize is that the recession is completely avoided in the region $D_1 \geq \overline{d}_1$. With separable preferences, there are constrained efficient allocations that partially mitigate the recession.

[7] This ensures that the equilibrium is non-degenerate in the high state H. Alternatively, we could impose a finite debt limit in $\phi_{t+1,H} < \infty$.

(weakly) more optimistic than lenders about the likelihood of the unconstrained state, $\pi_H^b \geq \pi_H^l$. Neither of these assumptions is necessary, but since impatience/myopia and excessive optimism were viewed as important contributing factors to many deleveraging crises, they enable us to obtain additional interesting results.

At date 0, households are allowed to trade in a complete market of one-period ahead Arrow securities. Let $q_{1,s}$ denote the price of an Arrow security that pays 1 dollar in state $s \in \{H, L\}$ of date 1. Let $d_{1,s}^h$ denote the security issuance of household h contingent on state $s \in \{H, L\}$. Household h raises $\sum_s q_{1,s}^h d_{1,s}^b$ dollars at date 0. Observe that the real interest rate at date 0 satisfies $1 + r_1 = 1/\sum_s q_{1,s}$. Given this notation, the optimization problem of households and the definition of equilibrium generalize to uncertainty in a straightforward way.

The equilibrium in state L of period 1 conditional on debt level $d_{1,s}$ is the same as described as before. In particular, the interest rate is zero and there is a demand driven recession as long as the the outstanding debt level is sufficiently large, $d_{1,L} > \bar{d}_1$. The equilibrium in state H jumps immediately to a steady-state with interest rate $1 + r_{t+1} = 1/\beta > 0$ and consumption $c_{t,H}^h = e^* - (1 - \beta)\, d_{1,H}^h \; \forall t \geq 1$.

The main difference concerns households' date 0 choices. In this case, households' allocations satisfy not only the analogue of the Euler equation (10) but also a *full-insurance equation* across the two states:

$$\frac{q_{1,H}}{q_{1,L}} = \frac{\pi_H^l}{\pi_L^l} \frac{u'\left(c_{1,H}^l\right)}{u'\left(c_{1,L}^l\right)} = \frac{\pi_H^b}{\pi_L^b} \frac{u'\left(c_{1,H}^b\right)}{u'\left(c_{1,L}^b\right)}. \tag{16}$$

We next describe under which conditions households choose a sufficiently high debt level for state L to trigger a recession, $d_{1,L} > \bar{d}_1$:

Condition 2. There is a deleveraging-induced recession in state L of period 1 if the borrower is either (i) sufficiently impatient or (ii) sufficiently indebted or (iii) sufficiently optimistic in period 0. Specifically, for any two of the parameters $\left(\beta_0^b, d_0, \pi_L^b\right)$, we can determine a threshold for the third parameter such that $d_{1,L} > \bar{d}_1$ if the threshold is crossed, i.e. if $\beta_0^b < \bar{\beta}_0^b \left(d_0, \pi_L^b\right)$ or $d_0 > \bar{d}_0 \left(\beta_0^b, \pi_L^b\right)$ or $\pi_L^b < \bar{\pi}_L^b \left(\beta_0^b, d_0\right)$.

The thresholds are characterized in more detail in Appendix A.3. The first two cases of the condition are analogous to Condition 1 in Section 3: if borrowers have a strong reason to take on leverage, they also place some of their debt in state L, even though this triggers a recession. The last case identifies a new factor that could exacerbate this outcome. If borrowers assign a sufficiently low probability to state L, relative to lenders, then they naturally have more debt outstanding in state L as opposed to state H. In each scenario,

$d_{1,L} > \bar{d}_1$ and there is a recession in state L of date 1.

To analyze constrained efficiency of this equilibrium, consider a planner that chooses households' allocations at date 0 and the outstanding leverage at date 1, but leaves the remaining allocations to the market. As before, we will see that the allocations chosen by this planner can be implemented with simple debt market policies. Let $W_s^h(D_{1,s}) = V_s^h(D_{1,s}, D_{1,s})$ denote agents' continuation utility in equilibrium starting state s of date 1 [cf. Eq. (11)]. The planning problem can be written as:

$$\max_{\left(\left(c_0^h, n_0^h\right)_h, (D_{1,s})_s\right)} \sum_h \gamma^h \left(u\left(c_0^h\right) + \beta^h \sum_s^h W_s^h(D_{1,s}) \right) \text{ s.t. } \sum_h c_0^h = \sum_h n_0^h - v\left(n_0^h\right).$$

The planner's optimality condition for insurance can be written as:

$$\frac{\pi_H^l}{\pi_L^l} \frac{u'\left(c_{1,H}^l\right)}{u'\left(c_{1,L}^l\right) + \partial V_L^l / \partial D_1} = \frac{\pi_H^b}{\pi_L^b} \frac{u'\left(c_{1,H}^b\right)}{u'\left(c_{1,L}^b\right) - \partial V_L^b / \partial D_1} \text{ for } D_1 \neq \bar{d}_1.$$

Combining this expression with Lemma 1, we obtain the main result of this section.

Proposition 3 (Underinsurance)**.** *An allocation $\left(\left(c_0^h, n_0^h\right)_h, (D_{1,s})_s\right)$, with $D_{1,L} \geq \phi$, is constrained efficient if and only if labor supply and output at date 0 is efficient, i.e., $v'\left(n_0^h\right) = 1$ for each h; households' substitution between date 0 and state h of date 1 is efficient, $\frac{\beta^h \pi_H^h u'\left(c_1^h\right)}{u'\left(c_0^h\right)} = 1$ for each h; and the remaining consumption and leverage allocations satisfy one of the following:*

(i) $D_{1,L} < \bar{d}_1$ and the full insurance equation (16) *holds.*

(ii) $D_{1,L} = \bar{d}_1$ and the distorted insurance equation holds:

$$\frac{\pi_H^l}{\pi_L^l} \frac{u'\left(c_{1,H}^l\right)}{u'\left(c_{1,L}^l\right)} \leq \frac{\pi_H^b}{\pi_L^b} \frac{u'\left(c_{1,H}^b\right)}{u'\left(c_{1,L}^b\right)}. \quad (17)$$

Moreover, every constrained efficient allocation of this type can be implemented as a competitive equilibrium with the mandatory insurance requirement, $d_{1,L}^h \leq \bar{d}_1$ for each h, combined with an appropriate ex-ante transfer, T_0.

The second part illustrates our main result with uncertainty: Constrained efficient allocations satisfy the distorted insurance condition in (17) (in addition to the distorted Euler equation). Moreover, these allocations can be implemented with an endogenous limit on an agent's outstanding debt in state L, $d_1^b \leq D_{1,L}$. Since this policy is equivalent to an insurance requirement that restricts agents' losses in the deleveraging state, we refer to it as a mandatory insurance requirement. In particular, the competitive equilibrium under

Condition 1, which features $d_{1,L} > \overline{d}_1$, is constrained inefficient and can be Pareto improved with a simple insurance requirement.[8]

This result identifies a distinct type of inefficiency in our setting. Borrowers in a competitive equilibrium not only take on excessive leverage, but they also buy too little insurance with respect to severe deleveraging episodes. Intuitively, they do not take into account the positive aggregate demand externalities their insurance purchases would bring about. One application of this result is to mortgage insurance. There has long been proposals to index mortgages to house prices (e.g. Shiller, 1993). However, households do not seem to be particularly interested in such instruments. Proposition 3 provides a rationale for making this type of insurance mandatory–especially with respect to severe and national house price declines of the type the US recently experienced.

An alternative reason for the underinsurance of borrowing contracts is provided by borrowers' optimism. Our analysis under condition (iii) illustrates that borrowers' optimism and aggregate demand externalities are *complementary* sources of underinsurance. In particular, optimism generates a first source of underinsurance relative to a common belief benchmark. This type of underinsurance, which is efficient according to borrowers' own beliefs, contributes to leverage and makes the aggregate demand externalities more likely to emerge. These externalities in turn generate a second source of underinsurance, which is socially inefficient even if borrowers' welfare is calculated according to their own optimistic beliefs.

6 Preventive monetary policies

The analysis so far has focused on preventive policies in financial markets, e.g., debt limits or mandatory insurance requirements. A natural question is whether preventive monetary policies could also be desirable to mitigate the inefficiencies in this environment. In this section, we analyze respectively the effect of the inflation target and the interest rate policy in our environment.

Blanchard, Dell'Ariccia and Mauro (BDM, 2010), among others, emphasized that a higher inflation target could be useful to avoid or mitigate the liquidity trap. We capture

[8] The inefficiency of competitive equilibrium also generalizes to an economy in which financial markets are incomplete so that households only have access to noncontingent debt. This amounts to imposing the constraint $d_1 \equiv d_{1,L} = d_{1,H}$ for households' problem in competitive equilibrium as well as for the constrained planning problem. The main difference in this case is that, since the interest rate r_2 in state H is variable, the planner that sets D_1 considers not only the aggregate demand externalities in state L but also the pecuniary externalities in state H. In fact, since agents' marginal utilities across states H and L are not equated, these pecuniary externalities by themselves could generate inefficiencies. In general, pecuniary externalities of this type could lead to too little or too large leverage. However, in our setting with two continuation states and GHH preferences, aggregate demand externalities are sufficiently powerful that the equilibrium features too much leverage.

this by replacing Eq. (3) with $P_{t+1}/P_t = 1+\zeta$ for each $t \geq 1$, where $\zeta > 0$ corresponds to the higher inflation target. In this case, the constraint on the real rate is relaxed, that is, we have $r \geq -\frac{\zeta}{1+\zeta}$ instead of $r \geq 0$. Consequently, a greater level of leverage is necessary to plunge the economy into a demand driven recession, consistent with BDM (2010). Our analysis adds further that this policy might also improve social welfare because the AD externalities emerge only when the real rate is constrained. These welfare benefits should of course be weighed against the various costs of a higher steady-state inflation.

It has also been emphasized that the interest rate policy could be used as a preventive measure against financial crises (see Woodford (2012) for a detailed discussion). To analyze this policy, consider the baseline setting without uncertainty in which the equilibrium features excessive leverage. Since low interest rates are generally thought to stimulate leverage, a contractionary policy that raises interest rates is a natural candidate for a preventive measure. Even though this model does not feature nominal rigidities, we can capture the effects of this policy by introducing a linear tax at date 0. In particular, suppose the final good profits in problem (5) is replaced by $n_0(1-\tau_0) - w_0 n_0$ for $\tau_0 > 0$ (and suppose the tax rebates $T_0 = n_0\tau_0$ are distributed lump sum to households). This policy generates a recession $e_0 < e^*$ similar to contractionary monetary policy. The date 0 equilibrium without debt limits satisfies the following analogue of the Euler equation in (10):

$$1 + r_1 = \frac{u'\left(e_0 + d_0 - \frac{d_1}{1+r_1}\right)}{\beta^l u'\left(\bar{c}_1^l\right)} = \frac{u'\left(e_0 - d_0 + \frac{d_1}{1+r_1}\right)}{\beta^b u'\left(\bar{c}_1^l - 2\left(d_1 - \phi\right)\right)}, \tag{18}$$

which determines the leverage, d_1, and the interest rate, r_1. It can be seen that $r_1(e_0)$ is a decreasing function of e_0, that is, a contractionary policy is indeed associated with a higher interest rate.

Perhaps surprisingly, $d_1(e_0)$ is not necessarily an increasing function, that is, a contractionary policy does not necessarily reduce leverage. Intuitively, there are two counteracting forces. Raising the interest rate tends to induce borrowers to take on smaller leverage conditional on their income. However, raising the interest rate also reduces borrowers' income (by contracting the output), which induces them to take on greater leverage to smooth their consumption. In fact, if $u(\cdot)$ has weakly decreasing risk aversion (for instance if it lies in the commonly used CRRA family), then the second force dominates and a contractionary policy in this model leads to *greater leverage*, exacerbating aggregate demand externalities.

We could also construct variants of this model in which a contractionary policy decreases the outstanding leverage, d_1 (for instance, by making borrowers' preferences less concave). However, interest rate policy is unlikely to be the ideal instrument even in these variants. To see this, recall that the Pareto dominating allocation in Proposition 2 satisfies the distorted

Euler equation. In contrast, a contractionary policy continues to satisfy the regular Euler equation in (18). One way to interpret this difference is that the interest rate policy creates a single wedge for intertemporal substitution, whereas the constrained efficient allocation requires separate wedges for borrowers and lenders. Put differently, the interest rate policy does not create the right wedges, and thus, it could at best be a crude solution to the problem of excessive leverage. In contrast, macroprudential policies, e.g., debt limits or insurance requirements, naturally reduce leverage and internalize aggregate demand externalities.

7 Aggregate demand and fire-sale externalities

In this section we endogenize the debt limit faced by borrowers by assuming that debt is collateralized by a financial asset, creating the potential for fire-sale effects. This introduces a new feedback loop into the economy: first, a decline in asset prices reduces the borrowing capacity of agents and forces them to delever; secondly, in a liquidity trap, endogenous deleveraging leads to a demand-induced decline in output that pushes down asset prices further. As a result, deleveraging involving collateral assets that decline in price may be particularly severe.

We modify our earlier setup by assuming that borrowers hold one unit $a_t = 1$ of a tree that pays a dividend y_t every period but only if owned by borrowers (so the tree cannot be sold to lenders). The tree trades at a market price of p_t. Borrowers are subject to a moral hazard problem in that they have the option to abscond with their loans after the market for loans has closed. In order to alleviate the moral hazard problem, they pledge their trees as collateral to lenders. When a borrower absconds with her loan, lenders can detect this and can seize up to a fraction $\phi_{t+1} < 1$ of the collateral and sell it to other borrowers. Borrowers will refrain from absconding if their debt satisfies

$$d_{t+1}/\left(1+r_{t+1}\right) \leq \phi_{t+1} a_{t+1} p_t.$$

Deleveraging may now be driven by two separate forces: declines in the "leverage" parameter ϕ_{t+1} and declines in the price of the collateral asset p_t. We will see shortly that declines in the leverage parameter are generally amplified by asset price declines.

In the following, we make two simplifying assumptions. First, starting in period $t = 2$, we assume that the output from the tree is a constant y and there are no further shocks. Second, we let the discount factors of the two agents $\beta^b = \beta^l = \beta$. Together, these two assumptions imply that the economy will be in a steady state starting in period 2 in which debt is constant at $d_t = d_2$ and the asset price and consumption satisfy $p_t = \frac{\beta}{1-\beta} y$, $c_t^b = y + e^* - (1-\beta)\, d_2$, $c_t^l = e^* + (1-\beta)\, d_2$ for $t \geq 2$ respectively.

We next consider the equilibrium at date 1 at which the asset's dividend is given by some $y_1 \leq y$. As before, if the debt level is sufficiently large, that is, $d_1 > \overline{d}_1$ for some threshold $\overline{d}_1$, then the economy is at a liquidity trap. In particular, borrowers are constrained, $d_2 = \phi_2 p_1$, the interest rate is at zero, $r_2 = 0$, and output is below its efficient level, $e_1 < e^*$. Moreover, the equilibrium is determined lenders' Euler equation at the zero interest rate:

$$u'(e_1 + d_1 - \phi_2 p_1) = \beta u'(e^* + (1-\beta)\phi_2 p_1). \tag{19}$$

The difference is that the asset price also enters this equation since higher prices increase the endogenous debt limit, which influences aggregate demand and output. The asset price is in turn characterized by:

$$p_1 = \frac{\beta y}{1-\beta} \frac{u'\left(c_2^b\right)}{(1-\phi_2) u'\left(c_1^b\right) + \phi_2 \beta u'\left(c_2^b\right)}, \text{ where } \left\{ \begin{array}{c} c_2^b = e^* + y - (1-\beta)\phi_2 p_1 \\ c_1^b = e_1 + y_1 - d_1 + \phi_2 p_1 \end{array} \right. . \tag{20}$$

Note that the asset price is increasing in current consumption, c_1^b, as well as in the exogenous collateral limit, ϕ_2 (since $u'\left(c_1^b\right) > \beta u'\left(c_2^b\right)$ in view of the constraint). The latter effect can be understood from a collateral value channel: A higher ϕ_2 implies the asset is more useful to relax the borrowing constraint which raises its price.

The equilibrium is characterized by two equations, (19) and (20), in two unknowns (e_1, p_1). The first equation describes an increasing relation, $e_1^{demand}(p_1)$. Intuitively, a higher price raises the endogenous debt level, which in turn raises aggregate demand and output. Under regularity conditions, the second equation also describes an increasing relation $e_1^{pricing}(p_1)$. Intuitively, a higher net income, e_1, increases current consumption, which in turn raises the asset demand and price. Any intersection of these two curves, that also satisfies $\frac{\partial e_1^{pricing}}{\partial p_1} > \frac{\partial e_1^{demand}}{\partial p_1}$, is a stable equilibrium.

To analyze welfare, consider the externalities from leverage, $\frac{\partial V^h}{\partial D_1}$, which can now be written as:

$$\begin{aligned}
\frac{\partial V^l}{\partial D_1} &= u'\left(c_1^l\right) \frac{\partial e_1}{\partial D_1}, \\
\frac{\partial V^b}{\partial D_1} &= u'\left(c_1^b\right) \frac{\partial e_1}{\partial D_1} + \phi_2 \frac{\partial p_1}{\partial D_1}\left[u'\left(c_1^b\right) - \beta u'\left(c_2^b\right)\right],
\end{aligned}$$

where $\frac{\partial e_1}{\partial D_1}$ and $\frac{\partial p_1}{\partial D_1}$ are jointly obtained from expressions (19) and (20) and are both negative under the assumptions made earlier. Note that the expression for both types of households features aggregate demand externalities. The expression for borrowers features in addition fire sale externalities. Intuitively, a higher debt level lowers borrowers' consumption, which in turn lowers the asset price. The low price in turn tightens borrowing constraints and further reduces borrower' welfare. Recall also that a low price further reduces aggregate demand and output, which in turn generates even lower prices, and so on.

It follows that endogenizing the financial constraint as a function of asset prices reinforces the problems of excessive leverage and underinsurance through two channels. First, it introduces fire-sale externalities that operate on borrowers' welfare in the same direction as aggregate demand externalities. Second, it also exacerbates aggregate demand externalities by tightening borrowing constraints further. The latter effect also illustrates an interesting mechanism through which asset price declines hurt all agents in the economy via aggregate demand effects, even if they do not hold financial assets. In this model, lenders do not hold the asset, but they are nonetheless hurt by the price decline because it leads to more deleveraging and magnifies the recession.

8 Conclusion

When borrowers are forced to delever, the interest rate might fail to decline sufficiently to clear the goods market, plunging the economy into a liquidity trap. This paper analyzed the role of preventive policies in the run-up to such episodes. In this context, we have illustrated that the competitive equilibrium allocations feature excessive leverage and underinsurance. A planner can improve welfare by implementing leverage restrictions or mandatory insurance requirements. If feasible, ex-post debt writedowns are also associated with positive demand externalities.

A growing literature on financial crises has emphasized various other factors that encourage excessive leverage and underinsurance, including optimism, moral hazard and pecuniary externalities. We conjecture that these distortions are complementary to the aggregate demand externalities that we investigate in this paper. For example, optimistic beliefs imply that households take on excessive leverage and do not want to insure, which makes it more likely that the economy enters the high-leverage conditions under which the lower bound on the interest rate binds and aggregate demand externalities matter. An interesting future direction is to analyze further the interaction between these amplification channels and the aggregate demand externalities.

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A Appendix: Extensions and omitted proofs

A.1 Equilbrium without uncertainty

This section completes the characterization of the equilibrium described in Section 3.

Taylor rule and the inflation Target The nominal interest rate is set according to a standard Taylor rule adjusted for the zero lower bound, given by:

$$\begin{aligned} \log\left(1+i_{t+1}\right) &= \max\left(0, \log\left(1+r_{t+1}^{n}\right)+\psi \log \frac{P_{t}}{P_{t-1}}\right) \text{ for each } t, \quad \text{(A.1)} \\ \text{where } 1+r_{t+1}^{n} &= \min_{h \in\{b, l\}} \frac{u^{\prime}\left(c_{t}^{h}\right)}{\beta^{h} u^{\prime}\left(c_{t+1}^{h}\right)} \text{ and } \psi>1. \end{aligned}$$

We first verify that this rule implies Eq. (3). Recall that starting date 2 the real interest rate is constant and given by $r_{t+1}=1/\beta^{l}-1>0$. Assumption (A2) then implies that there is zero inflation, that is, $P_{t}=P_{t-1}$ for each $t \geq 2$ as long as the nominal interest rate, $i_{\tilde{t}+1}$, is positive and finite for each $\tilde{t} \geq 2$. To see this, suppose $P_{t}>P_{t-1}$ for some $t \geq 2$. Then, the Taylor rule along with the Fisher equation, $1+r_{t+1}=\left(1+i_{t+1}\right) \frac{P_{t}}{P_{t+1}}$, implies $\frac{P_{t+1}}{P_{t}}=\left(\frac{P_{t}}{P_{t-1}}\right)^{\psi}$. Given the Taylor coefficient $\psi>1$, repeating this argument implies $\lim_{\tilde{t} \rightarrow \infty} \frac{P_{\tilde{t}+1}}{P_{\tilde{t}}}=\infty$, which yields a contradiction. A similar contradiction is obtained if $P_{t}<P_{t-1}$ for some $t \geq 2$.[9]

Conditions for date 1 Recession We next establish that there is a demand driven recession under condition 1. Consider households' debt choices at date 0. We conjecture that a recession is triggered at date 1 but not at date 0, that is, $e_{1}<e^{*}$ and $e_{0}=e^{*}$. A recession at date 1 would indeed be triggered if borrowers and lenders find it optimal to increase the amount of debt carried into period 1 beyond $\bar{d}_{1}$, that is if the marginal rates of substitution of the two agents at a debt level $d_{1}=\bar{d}_{1}$ satisfy

$$\begin{aligned} \left.\frac{u^{\prime}\left(c_{0}^{l}\right)}{\beta^{l} u^{\prime}\left(c_{1}^{l}\right)}\right|_{d_{1}=\bar{d}_{1}} &< \left.\frac{u^{\prime}\left(c_{0}^{b}\right)}{\beta^{b} u^{\prime}\left(c_{1}^{b}\right)}\right|_{d_{1}=\bar{d}_{1}} \\ \text{or} \quad 1+\bar{r}_{1}=\frac{u^{\prime}\left(e^{*}+d_{0}-\bar{d}_{1}/\left(1+\bar{r}_{1}\right)\right)}{\beta^{l} u^{\prime}\left(e^{*}+\bar{d}_{1}-\phi\right)} &< \frac{u^{\prime}\left(e^{*}-d_{0}+\bar{d}_{1}/\left(1+r_{1}^{l}\right)\right)}{\beta^{b} u^{\prime}\left(e^{*}-\bar{d}_{1}+\phi\right)} \quad \text{(A.2)} \end{aligned}$$

[9] We abstract away from the equilibria with self-fulfilling deflationary traps and inflationary panics (see Cochrane, 2011).

where we define r_1^l as the interest rate at which lenders are willing to supply $\bar{d}_1$, which is determined by the implicit equation on the left-hand side of the condition. Observe that the left-hand side of the inequality is decreasing in d_0 and the right-hand side is increasing in d_0 and decreasing in β^b. It follows that the inequality is satisfied if d_0 is sufficiently high and if β^b is sufficiently low. Specifically, for a given level β^b of impatience of the borrower, there is a threshold level $d_0\left(\beta^b\right)$ beyond which the inequality holds. Conversely, for a given debt level d_0, there is a threshold level of impatience $\beta^b\left(d_0\right)$ below which the inequality holds. In short, there will be a deleveraging-induce recession at date 1 if the borrower is sufficiently impatient and sufficiently indebted in period 0.[10]

For any given β^b, there is also a threshold level of period 0 debt $\tilde{d}_0 >> \bar{d}_0\left(\beta^b\right)$ beyond which the borrower delevers sufficiently in period 0 to induce a recession at both dates 0 and 1. We focus our analysis on situations in which period 0 debt satisfies $\tilde{d}_0 > d_0 > \bar{d}_0$ so as to rule out this situation. This completes the characterization of equilibrium.

A.2 Efficiency of equilibrium without uncertainty

This section presents the omitted proofs in Section 4.

Proof of Lemma 1. First consider the case $d_1 > \bar{d}_1$. Eq. (9) implies $\frac{de_1}{dd_1} = -1$. Eq. (11) then implies $\frac{\partial V^h}{\partial D_1} = -u'\left(u_1^h\right) < 0$.

Next consider the case $d_1 < \bar{d}_1$. In this case, differentiating lenders' Euler equation (10), we have:

$$\frac{dr_2}{dd_1} = \frac{u''\left(c_1^l\right)}{\beta^l u'\left(c_2^l\right) - u''\left(c_1^l\right)\phi/\left(1+r\right)^2} < 0.$$

This in turn raises consumption of borrowers and lowers consumption of lenders by:

$$\frac{dc_1^b}{dd_1} = -\frac{\phi}{\left(1+r\right)^2}\cdot\frac{dr_2}{dd_1} \equiv \eta \text{ and } \frac{dc_1^l}{dd_1} = -\eta.$$

It can also be checked that $\eta \in (0,1)$, completing the proof.

Proof of Proposition 1. Presented in the main text.

Proof of Proposition 2. It can be seen that, for each set of Pareto weights $\left(\gamma^b, \gamma^l\right)$, the planner's problem has a unique maximum characterized by the first order conditions:

[10] In these equilibria, the nominal price levels, P_1 and P_0, are indeterminate because the Taylor rule at date 1 is constrained. However, in every Walrasian equilibrium, the triple, (P_1, P_0, i_1), satisfies the Fisher equation, $1 + r_1 = (1 + i_1)\frac{P_0}{P_1}$. The triples, (P_1, P_0, i_1), that are consistent with equilibrium are determined by this equation along with the Taylor rule at date 0, $(1 + i_1) = (1 + r_1)\left(\frac{P_0}{P_{-1}}\right)^{\psi_\pi}$.

the intratemporal condition, $v'\left(n_0^h\right) = 1$ for each h, and the optimality condition (13), and the subgradient or inequality version of the condition for $D_1 = \overline{d}_1$ (given the kink at the objective value). Conversely, any allocation that satisfies these conditions corresponds to a solution to the planner's problem given the relative welfare weight $\frac{\gamma^b}{\gamma^l} = \frac{u'(c_0^b)}{u'(c_0^l)}$. Hence, it suffices to characterize the allocations that satisfy condition (13).

First consider the case $D_1 < \overline{d}_1$. Using Lemma 1 and Eq. (13), condition (13) becomes identical to the Euler equation (10), proving the first part.

Next consider the case $D_1 \geq \overline{d}_1$. Using Lemma 1 and Eq. (13), condition (13) is violated for each $D_1 > \overline{d}_1$. Hence, consider the subgradient version of the condition, which applies at $D_1 = \overline{d}_1$ and which can be written as:

$$\frac{\beta^l \left(u'\left(c_1^l\right) + V_1^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b \left(u'\left(c_1^b\right) - V_1^b\right)}{u'\left(c_0^b\right)} \text{ where } V_1^h \in \partial V^h / \partial D_1,$$

where $\partial V^h / \partial D_1$ now denotes the subgradient of V^h (as a function of D_1). Lemma 1 implies that these subgradients are given by:

$$\frac{\partial V^l}{\partial D_1} = \left[-u'\left(c_1^l\right), -\eta u'\left(c_1^l\right)\right] \text{ and } \frac{-\partial V^b}{\partial D_1} = \left[\eta u'\left(c_1^b\right), u'\left(c_1^b\right)\right],$$

where $\eta < 1$. Combining these observations, it follows that optimal allocations with $D_1 \geq \overline{d}_1$ are characterized by $D_1 = \overline{d}_1$ and the distorted Euler equation $\frac{\beta^l u'(c_1^l)}{u'(c_0^l)} \geq \frac{\beta^b u'(c_1^b)}{u'(c_0^b)}$. It can also be seen that every allocation of this type can be implemented with the debt limit, $D_1 = \overline{d}_1$, combined with an appropriate transfer, T_0. The tranfser depends on D_0, and it is chosen to satisfy households' date 0 budget constraints so that they can each consume, c_0^h, given the interest rate, $1 + r_1$, implied by the constrained efficient allocation, completing the proof of the second part.

As a corollary, it follows that the equilibrium under condition 1 is constrained inefficient. Moreover, as described in the main text, it can be Pareto improved with the debt limit, $D_1 = \overline{d}_1$, along with an appropriate transfer, T_0, that leaves households' date 0 consumption unchanged.

A.3 Equilibrium with uncertainty

Under either condition (i), (ii), or (iii), we claim that there exists an equilibrium in which a recession is triggered only in state L of date 1. The optimality conditions can be written as:

$$\frac{1}{q_{1,L}} = \frac{u'\left(e^* + d_0 - q_{1,H}d_{1,H} - q_{1,L}d_{1,L}\right)}{\pi_L^l \beta^l u'\left(\overline{c}_1^l\right)} = \frac{u'\left(e^* - \left(d_0 - q_{1,H}d_{1,H} - q_{1,L}d_{1,L}\right)\right)}{\pi_L^b \beta^b u'\left(\overline{c}_1^l + 2\left(\phi - d_{1.L}\right)\right)}, \tag{A.3}$$

$$\frac{q_{1,H}}{q_{1,L}} = \frac{\pi_H^l}{\pi_L^l} \frac{u'\left(e^* + \left(1-\beta^l\right) d_{1,H}\right)}{u'\left(\overline{c}_1^l\right)} = \frac{\pi_H^b}{\pi_L^b} \frac{u'\left(e^* - \left(1-\beta^l\right) d_{1,H}\right)}{u'\left(\overline{c}_1^l + 2\left(\phi - d_{1.L}\right)\right)}.$$

These expressions represent 4 equations in 4 unknowns, $d_{1,H}, d_{1,L}, q_{1,L}, q_{1,H}$. Given the regularity conditions, there is a unique solution. For the conjectured allocation to be an equilibrium, we also need the solution to satisfy $d_{1,L} \geq \overline{d}_1$. First consider conditions (i) or (ii), i.e., suppose $\pi_L^b = \pi_L^l$. In this case, a similar analysis as in the certainty case establishes that $d_{1,L} \geq \overline{d}_1$ is satisfied when $\beta^b \leq \overline{\beta}^b$ or when $d_0 \in \left(\overline{d}_0, \tilde{d}_0\right)$ for appropriate thresholds $\overline{\beta}^b, \overline{d}_0, \tilde{d}_0$. Next consider condition (iii), i.e., suppose $\beta^b = \beta^l$ and $d_0 = 0$ but $\pi_L^b \leq \pi_L^l$. It can be checked that $d_{1,L}$ is decreasing in π_L^b, and that $\lim_{\pi^b \to 0} d_{1,L} > \overline{d}_1$ (since $\lim_{\pi^b \to 0} c_{1,L}^b = 0$). Thus, there exists $\overline{\pi}_L^b > 0$. such that $d_{1,L} > \overline{d}_1$ whenever $\pi_L^b < \overline{\pi}_L^b$, proving the claim.

A.4 Equilibrium and efficiency with separable preferences

This section characterizes the equilibrium with separable preferences. It also characterizes the constrained efficient allocations and establishes the main result about ex-ante inefficiency of equilibrium also for this setting.

Consider the same model with two differences. First, households' preferences are given by $u(c) - v(n)$, where $u(\cdot)$ and $v(\cdot)$ satisfy the standard assumptions. Second, suppose type h households hold all of the shares of firms in which they work (and none of the shares of other firms) so their equilibrium income is always given by n_1^h. The latter assumption is made only for simplicity and can be relaxed at the expense of additional notation.

As before, consider first dates $t \geq 2$, at which the level of consumer debt is constant at the maximum permissible level $d_t = \phi$ and borrowers pay lenders a constant amount of interest $\left(1 - \frac{1}{1+r_{t+1}}\right)\phi = \left(1-\beta^l\right)\phi$ every period. Households labor supply is given by:

$$u'\left(n^{l*} + \left(1-\beta^l\right)\phi\right) = v'\left(n^{l*}\right), \text{ and } u'\left(n^{b*} + \left(1-\beta^l\right)\phi\right) = v'\left(n^{b*}\right),$$

with $n^{l*} < n^{b*}$ (since $c^{l*} > c^{b*}$).

Now consider date 1. As in the main text, there is a threshold, $\overline{d}_1$, such that $r_2 \leq 0$ only if $d_1 \geq \overline{d}_1$. To characterize this threshold, consider lenders' Euler equation at zero interest rate, $v'\left(\overline{n}_1^l\right) = \beta^l v'\left(n_1^{l*}\right)$,which pins down their labor supply, $\overline{n}_1^l$. Their intratemporal optimality condition, $u'\left(\overline{n}_1^l + \overline{d}_1 - \phi\right) = \beta^l v'\left(n_1^{l*}\right)$, then pins down $\overline{d}_1$.

The equilibrium when $d_1 < \overline{d}_1$ is standard and characterized by the variables, $\left(r_2 > 0, n_1^l, n_1^b\right)$, that satisfy:

$$\begin{aligned} u'\left(n_1^b - d_1 + \frac{\phi}{1+r_2}\right) &= v'\left(n_1^b\right) \\ \text{and } u'\left(n_1^l + d_1 - \frac{\phi}{1+r_2}\right) &= v'\left(n_1^l\right) = \beta^l \left(1+r_2\right) v'\left(n_1^{l*}\right). \end{aligned}$$

For comparison with below, it is useful to note that $\frac{\partial r_2}{\partial d_1} < 0$, $\frac{\delta n_1^l}{\delta d_1} \in (-1, 0)$ and $\frac{\delta n_1^b}{\delta d_1} \in (0, 1)$. In particular, an increase in debt leads to a reduction in the interest rate, a decrease in lenders' labor supply and an increase in borrowers' labor supply (through a wealth effect). Note also that agents' labor supply responses are less than one-for-one (as usual).

The equilibrium when $d_1 > \overline{d}_1$ features the liquidity trap. In this case, $r_2 = 0$ and thus lenders' Euler equation can be written as:

$$u'\left(n_1^l + d_1 - \phi\right) = \beta^l u'\left(n^{l*} + \left(1 - \beta^l\right)\phi\right).$$

This implies

$$n_1^l\left(d_1\right) = \overline{n}_1^l + \overline{d}_1 - d_1,$$

which is the analogue of Eq. (9) in the main text. In particular, lenders' employment and output decreases one-to-one with leverage as in the main text: $\frac{\delta n_1^l}{\delta d_1} = -1$. The equilibrium wage is given by:

$$w_1\left(d_1\right) = \frac{v'\left(n_1^l\right)}{u'\left(n_1^l + d_1 - \phi\right)} = \frac{v'\left(n_1^l\right)}{\beta^l v'\left(n^{l*}\right)} = \frac{v'\left(n_1^l\right)}{v'\left(\overline{n}_1^l\right)} < 1.$$

The last inequality implies the labor wedge is striclty positive $\tau_1 = 1 - w_1\left(d_1\right) > 0$. Note also that the elasticity of the wage with respect to leverage is given by $\frac{\delta w_1/\delta d_1}{w_1/d_1} = \frac{-1}{\eta^l}\frac{d_1}{n_1^l}$, where $\eta^l = \frac{v''(n_1^l)n_1^l}{v'(n_1^l)}$ is the Frish elasticity of lenders' labor supply. This expression illustrates that the wage response is influenced by two factors: The size of debt relative to output and the Frish elasticity.

Borrowers' labor supply (and consumption) is then determined by their intratemporal condition, $v'\left(n_1^b\right) = w_1\left(d_1\right) u'\left(n_1^b - \left(d_1 - \phi\right)\right)$. Implicitly differentiating, and using the expression for the wage response, we obtain:

$$\frac{\delta n_1^b}{\delta d_1}\frac{\left(v''\left(n_1^b\right) - u''\left(c_1^b\right)\right)c_1^b}{w_1 u'\left(c_1^b\right)} = -\frac{1}{\eta^l}\frac{c_1^b}{n_1^l} + \frac{1}{\theta^b},$$

where $\theta^b = 1/\left(\frac{-u''(c_1^b)c_1^b}{u'(c_1^b)}\right)$ is defined as lenders' intertemporal elasticity of consumption.[11] This expression also implies $\frac{\delta n_1^b(d_1)}{\delta d_1} < 1$, which when combined with $\frac{\delta n_1^l}{\delta d_1} = -1$ implies:

$$\frac{\delta n_1^l}{\delta d_1} + \frac{\delta n_1^b}{\delta d_1} < 1. \tag{A.4}$$

In particular, the total employment is decreasing in leverage regardless of the parameters.

Date 0 equilibrium is then characterized by Euler equations (10). Under conditions similar to those in 1, the equilibrium features $d_1 > \overline{d}_1$ and an anticipated recession.

We next analyze the efficiency properties of this equilibrium. First let $V^h(d_1, D_1)$ denote the utility of a household of type h conditional on entering period 1 with an individual level of debt d_1 and an aggregate level of debt D_1. We have:

$$V^b(d_{1,}D_1) = u\left(n_1(D_1) - d_1 + \frac{\phi}{1 + r_2(D_1)}\right) - v(n_1(D_1)) + \sum_{t=2}^{\infty}\left(\beta^b\right)^t u\left(c_t^b\right)$$

for borrowers and a similar expression for lenders. For $D_1 > \overline{d}_1$, taking the first order conditions with respect to D_1 gives the characterization of the externalities, $\frac{\partial V^h}{\partial D_1}$, in Eq. (15).

Next let $W^h(D_1) = V^h(d_{1,}D_1)$ and consider the ex-ante constrained planning problem described in Section 4.3. The first order conditions imply Eq. (13) also in this case. Using the expression for externalities in (15), the planner's optimality condition can be written as:

$$\frac{u'\left(c_0^l\right)}{\beta^l u'\left(c_1^l\right)\left(1 + \frac{\partial n_1^l}{\partial D_1}\tau_1\right)} = \frac{u'\left(c_0^b\right)}{\beta^b u'\left(c_1^b\right)\left(1 - \frac{\partial n_1^b}{\partial D_1}\tau_1\right)} \text{ for } D_1 > \overline{d}_1. \tag{A.5}$$

In view of the inequality in $(A.4)$, this equation also implies the distorted Euler equation in (14). The following result uses these observations to characterize the constrained efficient allocations in this setting.

Proposition 4 (Excessive Leverage with Separable Preferences)**.** *An allocation* $\left(\left(c_0^h, n_0^h\right)_h, D_1\right)$, *with* $D_1 \geq \phi$, *is constrained efficient if and only if labor supply and out-*

[11] Hence, the sign of borrowers' labor supply response to leverage is in general ambiguous. On the one hand, greater leverage lowers borrowers' consumption and increases their marginal utility, which in turn induces them to work more: this is captured by the second term. On the other hand, greater leverage lowers aggregate demand and wages, which in turn induces borrowers to work less: this is captured by the first term. The net effect is likely to be negative if the labor elasticity, η^l, is low relative to the intertemporal elasticity, θ^b (given $\frac{c_1^b}{n_1^l}$ is likely to be close to 1).

put at date 0 is efficient, i.e., $u'\left(c_0^h\right) = v'\left(n_0^h\right)$ *for each* h*; and the consumption and debt allocations satisfy one of the following:*

(i) $D_1 < \bar{d}_1$ *and the Euler equation* (10) *holds.*

(ii) $D_1 \geq \bar{d}_1$*, and the planner's optimality condition in* $(A.5)$*, and thus also the distorted Euler equation in* (14)*, holds. Moreover, every constrained efficient allocation of this type can be implemented as a competitive equilibrium with a debt limit,* $d_1^h \leq D_1$ *for each* h*, combined with an appropriate ex-ante transfer,* T_0*.*

This result is the analogue of our main result, Proposition 2, for separable preferences. The only difference is that debt levels strictly greater than $\bar{d}_1$ can also correspond to constrained efficient allocations, as long as the allocation satisfies the planner's optimality condition in $(A.5)$. In these cases, the planner mitigates but does not completely alleviate the recession. Intuitively, this is because the aggregate demand externalities with separable preferences depend on the size of the labor wedge, τ_1 [cf. Eq. (15)]. At $D_1 = \bar{d}_1$, the labor wedge is small, $\tau_1 = 0$, which implies that the planner might prefer to allow for a mild recession so as to improve ex-ante risk sharing. In contrast, the aggregate demand externalities with the GHH preferences analyzed in the main text are large regardless of the labor wedge [cf. Eq. (12)], which induces the planner to fully avoid the recession.