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Stochastic Idiosyncratic Operating Risk and Real Options: Implications for Stock Returns

Harjoat S. Bhamra*
University of British Columbia

Kyung Hwan Shim†, ‡
University of New South Wales

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Abstract

We combine real options and stochastic idiosyncratic operating risk in a simple equity valuation model of firms to capture the cross-sectional variation of stock returns associated with idiosyncratic return volatility. Our model is able to simultaneously explain two main disparate empirical anomalies: the positive contemporaneous relation between risk-adjusted returns and idiosyncratic return volatility, and the poor risk-adjusted performance of stocks with high idiosyncratic risks, among some others. The model further predicts that (i) risk-adjusted returns increase (decrease) following large rises (drops) in idiosyncratic return volatility – the switch effect – and that (ii) the anomalies and the switch effect are stronger for firms that are more abundant in real options and undergo larger changes in idiosyncratic return volatility. Simulations and empirical analysis strongly support these predictions.

Keywords: Idiosyncratic return volatility, cross section of stock returns, asset pricing, real options, growth options, stochastic volatility, regime switching, mixed jump-diffusion processes.

*University of British Columbia, Sauder School of Business, 2053 Main Mall, Vancouver BC, Canada, V6T 1Z2, Email: harjoat.bhamra@sauder.ubc.ca; Tel: 1 (604) 822 3314.

†School of Banking and Finance, Australian School of Business, University of New South Wales, Sydney NSW, Australia, 2052, Email: k.shim@unsw.edu.au; Tel: 61 (0)2 9385 5852.

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1 Introduction

Modern portfolio theory and the capital asset pricing model (CAPM) suggest that investors diversify idiosyncratic risks and only systematic risk is priced in equilibrium. The empirical evidence on idiosyncratic return volatility ($IVol$) and stock returns is not readily explained by this simple intuition. One strand of the literature (e.g., Duffee (1995)) establishes that monthly realized $IVol$ is contemporaneously positively related with stock returns.[1] A different strand (e.g., Ang, Hodrick, Xing, and Zhang (2006)), on the other hand, establishes that portfolios of stocks with high end-of-month realized $IVol$ significantly under-perform their low $IVol$ counterparts in the future on a risk-adjusted basis, casting doubts on the notion of a positive risk premium for idiosyncratic risks.[2,3] Yet, a third strand of the literature establishes that the negative intertemporal relation between $IVol$ and stock returns is due to short-term reversal of returns among a subset of small stocks (e.g., Huang, Liu, Rhee, and Zhang (2009) and Fu (2009)). Given the lack of consensus, it is not surprising that progress in delivering a unified explanation for the above empirical findings has been difficult.

In this paper, we explain these seemingly disparate empirical regularities in a simple equity valuation model of firms with real options and stochastic idiosyncratic operating risk. We demonstrate that, contrary to conventional wisdom, the presence of stochastic idiosyncratic operating risk creates a close relationship between stocks' risk-adjusted returns and $IVol$ if firm valuations incorporate convexities in the firms' output price, a feature that we attribute to the firms' real options. A firm's productive assets – assets-in-place – have linear valuations in the profit flow and the firm's idiosyncratic operating volatility is not a state variable in their valuations. Therefore, we argue that the relation between risk-adjusted stock returns and $IVol$ is entirely attributed to the firms' reliance on real options and their exposure to stochastic idiosyncratic operating risk. We demonstrate that the proposed mechanism reconciles the conflicting anomalies related to $IVol$ in the cross-section of stock returns.

The intuition for our results is as follows. Assuming a Markov regime -switching process for firm-specific operating risk allows the value of a firm's real options to relate positively

[1]Spiegel and Wang (2006), Fu (2009) and Huang, Liu, Rhee, and Zhang (2009) also report a positive correspondence between idiosyncratic return volatility and stock returns at the firm or portfolio level.

[2]Ang, Hodrick, Xing, and Zhang (2009) also report similar findings using international return data.

[3]Earlier empirical papers investigating idiosyncratic volatility and returns in the cross section are Lintner (1965), Tinic and West (1986) and Lehmann (1990).

with the volatility regime.[4,5] The arrival of a switch in idiosyncratic operating risk induces a discrete change in the valuation and a concurrent jump in the return of the firm's real options which relates positively with the sign of the switch. Therefore, a real option's expected return is composed of two state dependent terms: a continuous drift term that prevails between switches, and a sporadic jump term that occurs upon the arrival of a switch in the firm's idiosyncratic operating risk. The condition that these two components must amount to expected return, combined with the inverse correspondence between return jumps and the volatility regime before the switch, helps establish a positive relation between the continuous drift term and idiosyncratic risks. In normal times absent of jumps, a growth option's realized returns is on average larger than expected when volatility is high, and lower than expected when volatility is low. In the language of asset pricing, realized returns larger than expected translates to positive risk-adjusted returns. Therefore, real options tend to exhibit risk-adjusted returns that correlate positively with $IVol$, consistent with the first strand of the literature.

The same mechanism is used to justify the observed poor future performance of high $IVol$ stocks. Real options exhibit mean-reversion in risk-adjusted returns in tandem with movements in firm-specific operating risks. In asset pricing tests, employing standard portfolio approaches ensures that sorting and grouping options on month-end realized $IVol$ is akin to grouping options on the firms' most recent idiosyncratic volatility regime. A portfolio of real options with high recent $IVol$ experiences abnormally low future returns reflecting the downward jump and the lower continuous returns that eventuates upon the arrival of a switch in the firms' idiosyncratic operating risk. A similar mechanism applies to the superior risk-adjusted performance of low $IVol$ stocks. Therefore, real options tend to exhibit risk-adjusted returns that correlate negatively with past realized $IVol$, consistent with the second strand of the literature.

We validate our intuition with numerical simulations. Using a panel of simulated data of real option returns, we are able to recreate results qualitatively similar to Duffee (1995) and Ang, Hodrick, Xing, and Zhang (2006), with more pronounced effects in model specifications in which we use a larger spread in idiosyncratic operating risk between regimes. When we specify a single volatility regime – the standard specification in most real option models – we find that the model generates no link between $IVol$ and real option returns.

[4] A 2-regime Markov switching process is assumed for tractability, but it is not with loss of generality. Qualitatively, our results should persist in a more general structure insofar as idiosyncratic volatility exhibits mean reversion.

[5] Guo, Miao, and Morellec (2005) and Hackbarth, Miao, and Morellec (2006) also develop a 2-regime Markov switching process in state dynamics to investigate investment and capital structure decisions, respectively.

Simulations validate that our explanation is the driving mechanism behind the results, not other potentially opaque features of the model.

Our model also helps understand the findings that the negative intertemporal relation between $IVol$ and future stock returns is largely explained by the return reversal of stocks with high $IVol$ among a subset of small stocks (e.g., Huang, Liu, Rhee, and Zhang (2009)) and Fu (2009)). More specifically, small stocks with high $IVol$ exhibit stronger positive contemporaneous correlation with returns, subsequently leading to stronger short-term reversals and lower abnormal returns. Pontiff (2006) offers an explanation based on high transaction costs and limits to arbitrage to point to the persistence of low returns among small and high $IVol$ stocks. An alternative possible explanation can be based on Daniel, Hirshleifer, and Subrahmanyam (1998), who offer an explanation of cognitive bias and persistent mispricings in financial markets. Our model is able to generate stock return reversals through the dynamics of the volatility structure embedded in the operations of small firms that posses growth opportunities. Our explanation is based on a rational theory of firms that face uncertain operating environments and observable firm characteristics, rather than on market imperfections or investors' cognitive biases.

The novel contribution of this paper is to highlight that real options, in conjunction with stochastic firm-specific operating risk, can explain the conflicting empirical relation between risk-adjusted returns and $IVol$. The bulk of our empirical analysis is focused on reporting this link and verifying the predictions of our theory. To investigate our conjecture on the positive contemporaneous relation between $IVol$ and returns, we revisit Grullon, Lyandres, and Zhdanov (2010) by recreating many of their empirical proxies for firms' reliance on growth options, and additionally, creating some of our own proxies. Our regression specifications are similar to those of Grullon, Lyandres, and Zhdanov as well, with additional specifications in which we include the difference between the stocks' 70th and 30th percentile in-sample breakpoint values of $IVol$ as an additional explanatory variable to proxy for the spread in idiosyncratic operating risk across volatility regimes. Using a battery of real option intensity proxies, we find that the positive contemporaneous relation between returns and changes in $IVol$ is stronger among firms that are more likely to incorporate real options and experience more extreme changes in $IVol$, results that lend strong support to our model.

In order to investigate our conjecture on the poor performance of high $IVol$ stocks, we revisit Ang, Hodrick, Xing, and Zhang (2006) by sorting and grouping stocks into portfolios based on the level of their month-end realized $IVol$, and independently, on the firms' real option proxy, and on the difference between the stocks' 70th and 30th in-sample percentile breakpoint values of $IVol$. As in asset pricing tests, we assess the portfolio performances by investigating value-weighted risk-adjusted returns. Again, using a battery of real option intensity proxies, we find that the poor future performance of high $IVol$ stocks is more pronounced among firms that are more likely to incorporate real options and experience more extreme changes in $IVol$, results that are in strong agreement with our model.

The model we propose captures additional empirical testable features. To the extent that stock returns incorporate firms' real options and stochastic firm-specific operating risks, stocks that experience an up (down) $IVol$-switch episode should be associated with higher (lower) post-switch risk-adjusted returns to reflect their returns in the new regime. In order to investigate this novel conjecture, we compute the difference in 5-month risk-adjusted average returns around the month in which a stock's $IVol$ undergoes a sudden change larger than the difference between its 70th and 30th in-sample percentile breakpoint values. Using a battery of real option intensity proxies, we find that in the up-switch sample the difference between post and pre-switch returns is positive, while in the down-switch sample the difference in returns is negative. We also find that this switch effect is amplified for more real option intensive firms and firms that experience more extreme changes in $IVol$. Here again, the results are in strong support of our theory.

Corporate investment decisions are commonly modeled in the context of growth options,[6] and growth options are shown to affect firm risks. Berk, Green, and Naik (1999) were among the first to establish a link between corporate investment-based characteristics and firm betas to explain anomalous regularities in the cross section of stocks.[7,8] We contribute to this literature by integrating a new dimension – stochastic operating risk – with real options to explain the empirical regularities related to $IVol$. A common theme in the extant literature investigates how much real options contribute to the firm's market beta relative to the firm's assets-in-place, a feature present in our model as well. Our contribution hinges on how firm-specific risks in operations affect the firms' market risk through the firms' real options. In our model, the idiosyncratic volatility of a firm' output price serves as an additional state variable that affects the market beta of the firm's real options, but not the market beta of the firm's assets-in-place, a channel previously not considered as a link between firm observable characteristics and expected stock returns.

We also make a contribution to the broader literature on the cross section of stock

[6] This approach was first pioneered by MacDonald and Siegel (1985), MacDonald and Siegel (1986) and Brennan and Schwartz (1985), and later adopted by many others. Dixit and Pindyck (1994) summarizes the body of literature.

[7] Fama and French (1992) provide evidence on the ability of size and book-to-market to explain returns. Fama and French (1996) provides a cross-sectional landscape view of how average returns vary across stocks.

[8] Further work in this area have also focused on real options to build a bridge between firms' characteristics and market betas (e.g., Carlson, Fisher, and Giammarino (2004), Zhang (2005), Sagi and Seashole (2007) and Garlappi and Yan (2011), among many others).

returns. By explicitly considering time varying firm-specific operating risk allows us to propose a novel channel between the operating environment that firms face and the firms' stock returns, providing fertile grounds for additional research. The novel features of our model yield additional testable predictions on the correspondence between $IVol$ and risk-adjusted stock returns, some of which we test empirically in this paper.[9]

To our knowledge little inroads have been made to link idiosyncratic operating risk to asset pricing, a void we hope to fill with this paper. The economics literature has recognized that firm level idiosyncratic technology shocks aggregate to create macroeconomic effects (e.g. Caballero and Pindyck (1996)) and Bloom (2009)). Also, it has been shown that the presence of idiosyncratic risks in a competitive industry of firms with growth options translates to the firms' potential to retain monopolistic rents (e.g. Ch. 8 of Dixit and Pindyck (1994)) because a firm that experiences a positive idiosyncratic technology shock experiences a unique advantage that is not shared with the competitors.[10] Incorporating time varying idiosyncratic operating risks ensures that firms have time varying potential to retain monopolistic rents, motivating the importance of idiosyncratic operating risk as a determinant of firm value and stock returns.[11]

Lastly, this paper has predictable implications for the distribution of stock returns. The literature has reported that asset returns must exhibit both stochastic volatility and discontinuous jumps to fit their empirical distributions (e.g. Das and Sundaram (1999)). A model that incorporates both would be able to generate adequate kurtosis and skewness in both conditional and unconditional return distributions. Short of announcements effects, little work has been done on the sources that drive stochastic volatility and jumps in returns. Our model parsimoniously generates skewness and fat tails in return distributions and suggests that the empirical distributions stem from the operating environment that firms face. This novel channel we propose to understand return distributions offers fertile grounds for further research.

The rest of the paper is organized as follows: Section II presents the model environment. Section III derives closed-form solutions for the valuations and returns. Section IV discusses model simulation results. Section V reports the empirical methodology along with the results. Section VI concludes. The Appendix contains all the proofs and other technical details omitted in the main body of the paper.

[9] Schwert (2003) highlights the merits of structural models in deriving new testable hypotheses.

[10] Idiosyncratic shocks contrasts from aggregate shocks since the latter are shared among competing firms.

[11] This paper does not explicitly consider a competitive industry equilibrium with multiple firms as in Caballero and Pindyck (1996) or Dixit and Pindyck (1994). In an earlier draft, we considered an industry equilibrium model of firms with entry and exit and idiosyncratic operating risk similar to the one consider in this paper. The qualitative implications for $IVol$ and stock returns are similar.

2 Model

We construct a growth option model similar in spirit to the models in Garlappi and Yan (2008) and Carlson, Fisher, and Giammarino (2004).[12] This section describes the firms' economic environment.

2.1 The Environment

We consider two types of firms. Mature firms are producing units in the economy, produce at capacity, and can not change their operating scale. In contrast, young firms produce at a lower operating scale, but have the option to make an irreversible investment to increase production and become mature. Firms are all equity financed. The price of the output for each firm follows a geometric Brownian motion

$$dP = \mu P dt + P\sigma_{P,i} dB_1 + \sigma_A P dB_2 \tag{2.1}$$

where μ is the growth rate of the product price, σ_A is its market volatility, $\sigma_{P,i}$ is its idiosyncratic volatility, and dB_1 and dB_2 are the increments of two independent Brownian motions.

We allow firms to have random and time varying potential to realize monopolistic rents by allowing idiosyncratic operating risks to be time varying.[13] The idiosyncratic volatility parameter $\sigma_{P,i}$ follows a 2-state Markov switching process [14]

$$\triangle\sigma_{P,i} = \begin{cases} \sigma_{P,H} - \sigma_{P,L} & , \text{ with prob. } \lambda_H dt, \quad \text{if } i = L \\ 0 & , \text{ with prob. } 1 - \lambda_H dt, \quad \text{if } i = L \\ \sigma_{P,L} - \sigma_{P,H} & , \text{ with prob. } \lambda_L dt, \quad \text{if } i = H \\ 0 & , \text{ with prob. } 1 - \lambda_L dt, \quad \text{if } i = H \end{cases} \tag{2.2}$$

where $\sigma_{P,H} - \sigma_{P,L} \geq 0$, and λ_L and λ_H are known transition parameters between high and low volatility regimes H and L. The switches between the two regimes $\triangle\sigma_{P,i}$ are independent Poisson processes and independent across firms. Both P and the volatility

[12] With no loss of generality, we rely specifically on growth options to incorporate convexity of firm valuations in the firms' output price. Other forms of real options would accommodate similar features.

[13] Dixit and Pindyck (1994) and Caballero and Pindyck (1996) show that the presence of idiosyncratic operating risks in a perfectly competitive industry translates to firms' retaining monopolistic rents – a firm that experiences a positive idiosyncratic technology shock experiences an advantage that cannot be stolen by its competitors, while a positive aggregate shock is shared among competitors. Allowing time varying idiosyncratic operating risks ensures that firms have time varying potential to realize monopolistic rents.

[14] Assuming a 2-state Markov switching process is not without generality. A model with a more general volatility structure is possible, but at a cost of analytical tractability.

regime i are observable for any given firm.[15] We subscript quantities with $i \in \{H, L\}$ to denote their dependence on the idiosyncratic volatility regime at any given time.

All sources of operating uncertainty are driven by the uncertainty in the prices of the firms' output. Investors in the stock market can hedge market uncertainty in the firms' operations. Let M_t denote the price of a riskless bond with dynamics

$$dM = rMdt \tag{2.3}$$

and let S be a risky asset with dynamics

$$dS = \mu_S S dt + \sigma_S S dB_2 \tag{2.4}$$

We treat the traded asset S as having a beta of one, so $\lambda = \frac{\mu_S - r}{\sigma_S}$ is the market price of risk. The proportion of S held in a replicating portfolio determines the beta of the portfolio. This greatly simplifies firm valuation and the determination of market risk.

2.2 The Value of a Mature Firm

A mature firm's value is composed of the profit stream from production of the output. The cost of producing a unit of output is c per unit of time. ξ_M denotes the scale of production, therefore the profit per unit of time is $\pi_M(P) = \xi_M(P - c)$. The equity value of a mature firm can therefore be expressed as follows

$$V_M(P) = \xi_M \left(\frac{P}{r - \mu^*} - \frac{c}{r} \right) \tag{2.5}$$

where $\mu^* = \mu - \sigma_A \lambda < r$. The firm value is the present value of a growing risky perpetuity, less the present value of a riskless perpetuity.

2.3 The Value of a Young Firm

Young firms produce at a lower capacity than mature firms, i.e. $\xi_Y < \xi_M$, but possess a perpetual option to increase production scale by $\xi = \xi_M - \xi_Y$ upon making a one time irreversible investment of I. For simplicity, we assume that financing is done by equity. The flexibility that growth firms can increase production implies that their firm value are made

[15] Conditioned on being in the high volatility state, the probability that $\triangle\sigma_{P,i}$ will switch to the low volatility regime in the next short interval dt is $\lambda_L dt$. $\lambda_H dt$ is defined similarly. Based on standard properties of Poisson processes, the expected duration that the process dP will stay in the high volatility regime H and the low volatility regime L are λ_L^{-1} and λ_H^{-1}, respectively. The proportion of time spent in the high and low volatility regimes are $\frac{\lambda_H}{\lambda_H + \lambda_L}$ and $\left(1 - \frac{\lambda_H}{\lambda_H + \lambda_L}\right)$ respectively.

up of assets currently in production, or assets-in-place, and the growth option. The value of the assets-in-place have the same functional form as equation (2.5) with ξ_M replaced by ξ_Y. The growth option has the following Bellman equation prior to exercise

$$Y_i(P) = e^{-rdt} E^{\mathbb{Q}} \left[Y_i(P + dP, \sigma_{P,i} + \triangle\sigma) \right] \tag{2.6}$$

and a value realization upon exercise net of cost of $Y_i(P) = \xi \left(\frac{P}{r-\mu^*} - \frac{c}{r} \right) - I$, where $E^{\mathbb{Q}}[.]$ denotes the expectation operator under the $\mathbb{Q}$ measure. At any given time, the value of the young firm is dependent on its output price P and its idiosyncratic volatility regime i. Optimal exercise requires to choose when to invest, which occurs at time τ_i. τ_i is distinct across volatility regimes because the value of the growth opportunity in general differs across volatility regimes. Define P_i^* the price level at which a young firm exercises its growth option. The choice of P_i^* describes the strategy for a young firm, and the strategy chosen that satisfies optimality conditions maximizes the firm's value.

2.4 Expected Returns

A version of the conditional CAPM holds in our model given the structure of the environment. In the cross section, firms differ in their maturity, the price of their output, and the idiosyncratic volatility regime in effect. Therefore, the firms' betas and expected returns also differ based on the realization of their state variables. The instantaneous expected return of a young firm's stock is given by

$$E \left[\frac{\pi_Y(P)dt + dV_{Y,i}(P)}{V_{Y,i}(P)dt} \right] = r + \beta_{Y,i}(P)\sigma_S\lambda \tag{2.7}$$

Equation (2.7) helps to examine the effects of the firm's state variables on the interplay between stock returns and idiosyncratic return volatility.

3 Model Solution

This section describes the model solution. We describe the valuation and the expected return of a growth option for a young firm, followed by the model's empirical predictions.

3.1 Valuation

The following proposition states the option's value and the optimal exercise policies.

Proposition 1 *If the price process is given by (2.1) and (2.2), then the value of a growth option in the region of low values of P, $P \in (0, P_1)$, is*

$$F_H(P) = \frac{B_{L,1} P^{\beta_{2,1}} q_L(\beta_{2,1})}{\lambda_H} + \frac{B_{L,2} P^{\beta_{2,2}} q_L(\beta_{2,2})}{\lambda_H} \tag{3.1}$$

if P is in the high volatility regime, and

$$F_L(P) = B_{L,1} P^{\beta_{2,1}} + B_{L,2} P^{\beta_{2,2}} \tag{3.2}$$

if P is in the low volatility regime.

In the region of intermediate values of P, $P \in (P_1, P_2)$, the option value is

$$G_H(P) = \frac{\lambda_L}{\lambda_L + r}\left(\xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I\right) + C_{H,1} P^{\beta_{1,1}} + C_{H,2} P^{\beta_{1,2}} \tag{3.3}$$

if P is in the high volatility regime, and

$$G_L(P) = \xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) \tag{3.4}$$

if P is in the low volatility regime. Moreover, the optimal exercise boundaries P_1 and P_2 are the solution to the following system of equations

$$\begin{aligned} C_{H,1} P_1^{\beta_{1,1}} + C_{H,2} P_1^{\beta_{1,2}} - \frac{\lambda_L}{\lambda_L + r}\left(\xi\left(\frac{P_1}{r-\mu^*} - \frac{c}{r}\right) - I\right) \\ = \frac{B_{L,1} P_1^{\beta_{2,1}} q_L(\beta_{2,1})}{\lambda_H} + \frac{B_{L,2} P_1^{\beta_{2,2}} q_L(\beta_{2,2})}{\lambda_H} \end{aligned} \tag{3.5}$$

$$\begin{aligned} \beta_{1,1} C_{H,1} P_1^{\beta_{1,1}} + \beta_{1,2} C_{H,2} P_1^{\beta_{1,2}} + \frac{\xi P_1 \lambda_L}{(r-\mu^*)(\lambda_L + r)} \\ = \frac{\beta_{2,1} B_{L,1} P_1^{\beta_{2,1}} q_L(\beta_{2,1})}{\lambda_H} + \frac{\beta_{2,2} B_{L,2} P_1^{\beta_{2,2}} q_L(\beta_{2,2})}{\lambda_H} \end{aligned} \tag{3.6}$$

where the expressions for $B_{L,1}$,$B_{L,2}$,$C_{H,1}$, $C_{L,1}$, $q_L(\beta)$, $\beta_{1,1}$, $\beta_{1,2}$, $\beta_{2,1}$ and $\beta_{2,2}$ are given in the Appendix.

Proof: *See Appendix.* ∎

The proposition states that a young firm has separate investment policies in each volatility regime. As a consequence, there are three distinct regions in the range of possible values of P to consider to value the option (Guo, Miao, and Morellec (2005)). In the region where $P \in (0, P_1)$, the option's intrinsic value is less valuable than the value to keep it alive. Therefore, the option is not exercised in either high or low volatility regime and the value is given by (3.1) and (3.2). In the region where $P \in (P_1, P_2)$, the option's intrinsic value is less valuable than the value to keep it alive only in the high volatility regime.[16] Consequently, the option is kept alive in the high volatility regime only and the value is given by (3.4) and (3.3). Lastly, in the region where $P > P_2$, the option's intrinsic value exceeds the value to keep it alive in both volatility regimes and the value reflects the incremental production of the new assets-in-place which is given by $\xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right)$.

More importantly, the proposition reveals that the distinct valuation feature of the growth option is in stark contrast to the value of a firm's assets-in-place or the value of a mature firm. Equation (2.5) shows that the value of a mature firm is independent of the volatility regime of the firm's output price process. Because the value of the assets-in-place of a young firm has the same functional form as the value of a mature firm, the dependence of a young firm's value on the volatility regime is attributed entirely to the firm's growth options. This property will prove to have important implications for the interplay between idiosyncratic return volatility and stock returns and the extent to which this relation exists among stocks of firms with different real option intensities.

Insert Figure 1 here

Figure 1 reports the the option values $F_H(P)$ and $F_L(P)$ and the exercise thresholds where $P \in (0, P_1)$. Figure 1 (a) shows that except when $P = 0$, the value of the growth option is greater in the high volatility regime and has a higher exercise threshold. Figure 1 (b) assumes a smaller difference in volatility parameters between regimes, and Figure 1 (c) assumes a single volatility regime, which is the usual specification in standard real option models. Comparing the plots across panels reveals that the difference in option valuations and exercise boundaries are larger the greater the spread in volatility between volatility regimes.

3.2 Equity Beta and Expected Returns

This section shows that the distinct valuations of a growth option between volatility regimes has important implications for a growth option's risk and return dynamics.

[16] This property hinges on standard option pricing results that the value of an option is increasing in the volatility of the underlying asset.

3.2.1 Return Dynamics

The next proposition characterizes the return dynamics of a growth option in the region $P \in (0, P_1)$.

Proposition 2 *If $F_H(P)$ and $F_L(P)$ are given by equations (3.1) and (3.2), and P and $\Delta\sigma_{P,i}$ follow the dynamics given in (2.1) and (2.2), then $F_H(P)$ and $F_L(P)$ have the following dynamics*

$$\frac{dF_H(P)}{F_H(P)} = a_H(P)dt + b_H(P)dB_i + \nu_H(P)dz_H \tag{3.7}$$

$$\frac{dF_L(P)}{F_L(P)} = a_L(P)dt + b_L(P)dB_i + \nu_L(P)dz_L \tag{3.8}$$

where $dB_i = \frac{\sigma_{P,i}dB_1 + \sigma_A dB_2}{\sigma_i}$ and $\sigma_i = \sqrt{\sigma_{P,i}^2 + \sigma_A^2}$, and dz_H and dz_L are independent infinitesimal increments of Poisson processes with mean arrival rates λ_L and λ_H respectively, and $a_H(P)$, $b_H(P)$, $\nu_H(P)$, $a_L(P)$,$b_L(P)$ and $\nu_L(P)$ are expressions given in the Appendix.

Proof: *See Appendix.* ∎

The first two terms of (3.7) and (3.8) correspond to the drift and diffusion terms common in standard continuous diffusion models. The third term corresponds to the proportionate change in the value of the firm if a switch in volatility arrives. The distinct value functions across volatility regimes implies that a switch in volatility should concur with a discontinuous jump in the value of the firm. Therefore, the Poisson processes dz_H and dz_L are perfectly functionally dependent on the Poisson process for the volatility parameter $\Delta\sigma_{P,i}$. This dependance occurs because of the option's direct dependence on the volatility regime of the output price process. This property of real options contrasts starkly with the return dynamics of a mature firm or the returns of the assets-in-place of a young firm. Because mature firms and the assets-in-place of young firms have valuations that are not directly dependent on the volatility regime of the price process, they do not exhibit discontinuous jumps from switches in volatility.

It is worthwhile distinguishing the return dynamics (3.7) and (3.8) from other standard mixed jump-diffusion processes common in option pricing with jumps (e.g. Merton (1976)). Standard jump models in option pricing assume a mixed jump-diffusion process with jumps drawn from a known distribution and a known jump amplitude for the underlying asset. In contrast, our model does not assume that the underlying asset – the value gain from the increase in production scale – exhibits jumps in value. Instead, the underlying asset exhibits switches in idiosyncratic operating risks resulting in deterministic jumps in the value of the option. Furthermore, the time series pattern of the value process also differ from standard jump-diffusion models. Equations (3.7) and (3.8) establish that between jumps, the growth option returns of a young firm will be yield $a_H(P)$ and $a_L(P)$ per unit of time in the high and low volatility regimes respectively. However, on average, every $1/\lambda_H$ and $1/\lambda_L$ units of time, the value will take a sudden jump. Conditioned on one of the volatility regimes, the return process obeys the laws of motion pertaining to the volatility regime in effect. If a sudden switch arrives, the option experiences a concurrent discontinuous jump in return, after which the return process obeys the laws of motion pertaining to the new volatility regime. The dynamics stay the same until the next switch arrives. This feature of the model helps establish a close relationship between idiosyncratic return volatility and stock returns that conforms with the extant empirical findings.

3.2.2 Implications for the Relation between Idiosyncratic Return Volatility and Risk-Adjusted Return

Following the results from Propositions 1 and 2, this section describes how real options and switches in the volatility of the output price process combine to capture the relation between stock returns and idiosyncratic return volatility.

Corollary 3 *If the return dynamics of $F_i(P)$ are given by equations (3.7) and (3.8) in Proposition 2, then $\nu_H(P) \leq 0 \leq \nu_L(P)$. Moreover, the difference $\nu_L(P) - \nu_H(P) \geq 0$ is increasing in the difference in volatility parameters across volatility regimes $\sigma_{P,H} - \sigma_{P,L}$.*

The interpretation of the corollary is intuitive. If the value of a growth option is greater in the high volatility regime than in the low volatility regime, then conditional on the high volatility regime the arrival of a down switch results in a lower valuation and a concurrent negative jump in return. On the other hand, conditional on the low volatility regime, the arrival of an up switch coincides with a positive jump in return.

The difference in the sign of the jump terms between volatility regimes serves to further clarify the determinants of stock returns.

Proposition 4 *If $F_H(P)$ and $F_L(P)$ are given by equations (3.1) and (3.2), and P and $\Delta\sigma_{P,i}$ follow the dynamics given in (2.1) and (2.2), then a growth option has a beta given by*

$$\beta_H(P) = \frac{\sigma_A \left(\beta_{2,1} B_{L,1} P^{\beta_{2,1}} q_L(\beta_{2,1}) + \beta_{2,2} B_{L,2} P^{\beta_{2,2}} q_L(\beta_{2,2})\right)}{\sigma_S \left(B_{L,1} P^{\beta_{2,1}} q_L(\beta_{2,1}) + B_{L,2} P^{\beta_{2,2}} q_L(\beta_{2,2})\right)} \tag{3.9}$$

$$\beta_L(P) = \frac{\sigma_A \left(\beta_{2,1} B_{L,1} P^{\beta_{2,1}} + \beta_{2,2} B_{L,2} P^{\beta_{2,2}}\right)}{\sigma_S \left(B_{L,1} P^{\beta_{2,1}} + B_{L,2} P^{\beta_{2,2}}\right)} \tag{3.10}$$

and has an instantaneous expected return (conditional CAPM) given by

$$E\left[\frac{dF_H(P)}{F_H(P)dt}\right] = a_H(P) + \lambda_L \nu_H(P) \tag{3.11}$$

$$E\left[\frac{dF_L(P)}{F_L(P)dt}\right] = a_L(P) + \lambda_H \nu_L(P) \tag{3.12}$$

where the expressions for $B_{L,1}$,$B_{L,2}$,$q_L(\beta)$, $\beta_{2,1}$, $\beta_{2,2}$, $a_H(P), a_L(P), \nu_H(P)$ *and* $\nu_L(P)$ *are expressions given in the Appendix.*

Furthermore, in the specific case where a young firm is exposed to idiosyncratic risks only, i.e. $\sigma_A = 0$, $a_H(P) \geq r \geq a_L(P)$.

Lastly, $a_H(P) - a_L(P) \to 0$ *as* $\sigma_H - \sigma_L \to 0$.

Proof: *See Appendix*

Insert Figure 2 here

Proposition 4 conveys the central idea of our model. A growth option's beta and expected return differ between volatility regimes, a characteristic inherited from its value function.[17] Figure 2 (a) shows that the difference in betas between the high and low volatility regimes, i.e. $\beta_H(P) - \beta_L(P)$, is non-positive and larger the greater the value of P and the greater the spread in volatility betweens volatility regimes. On the other hand, the difference in beta is identically zero if the model is reduced to a single volatility regime. The explanation is intuitive. A growth option inherits risks from the underlying asset. Consequently, the option's market risk as a proportion of total risk should be greater in the lower volatility regime, which translates to a larger beta.[18] Looked another way, the value of a growth option is greater in the high idiosyncratic volatility regime, which implies a lower exposure to aggregate risk and lower a beta.[19] The difference in betas should become more pronounced the larger the spread in volatility between regimes and as the option moves closer in-the-money.[20]

[17] As discussed earlier, the beta and the expected return of a mature firm or the assets-in-place of a young firm is not dependant on the volatility regime.

[18] Strictly speaking, a portfolio that replicates the market risk of a real option entails holding a larger proportion of the portfolio's value in the market asset S in the low volatility regime due to the option's larger market elasticity to value ratio, i.e. $\frac{F_i'(P)}{F_i(P)}$, hence the larger market beta.

[19] Our results are consistent with Johnson (2004), who shows that raising uncertainty about the underlying asset value while holding the asset risk premium constant lowers the expected returns of a levered firm.

[20] The value of an option becomes more sensitive to the value of the underlying as the option moves closer in-the-money since it becomes more likely that the option will be exercised. As a consequence, an

Expressions (3.11) and (3.12) show that the expected return for a growth option in both regimes is composed of two state dependent terms: a continuous drift term and a probability weighted jump term that in expectation occurs sporadically. The existence of the sporadic jump term induced by the switches in volatility have important implications for returns. Given the results that the direction of the jump is inversely related with the initial volatility regime in effect, the condition that the continuous and the jump terms must add up to expected return helps establish a positive relation between $a_i(P)$ and $\sigma_{P,i}$. The proposition further clarifies this idea by assuming zero market risk. If the output price process does not have any market risk, i.e. $\sigma_A = 0$, then the sum of the continuous and sporadic jump terms must add up to the risk-free rate, in which case the relation $a_H(P) \geq r \geq a_L(P)$ holds, establishing a positive correspondence between $a_i(P)$ and $\sigma_{P,i}$.

The proposition, in conjunction with corollary 3, allows us to express the dynamics of an option's realized return in excess of expectation. More specifically, at any given time and conditional on a regime $i \in \{H, L\}$, an option's realized return in excess of expectation is the difference between (3.7) and (3.11), and (3.8) and(3.12), i.e.

$$\begin{aligned}\frac{dF_i(P)}{F_i(P)} - E\left[\frac{dF_i(P)}{F_i(P)}\right] &= a_i(P)dt + b_i(P)dB_i + \nu_i(P)dz_i - [a_i(P) + \lambda_{i'}\nu_i(P)]dt \\ &= b_i(P)dB_i - \lambda_{i'}\nu_i(P)dt + \nu_i(P)dz_i \qquad (3.13)\end{aligned}$$

where the subscript i' denotes the regime L if the current regime is H and vice versa. Since dz_i and dB_i in (3.13) have expected values $\lambda_i' dt$ and 0, respectively, the law of iterated expectations ensures that the expected returns in excess of expectation is zero. However, (3.13) has interesting dynamics. Between switches, an option's realized return in excess of expectation is equal to the sum of the first two terms of (3.13), i.e. $b_i(P)dB_i - \lambda_{i'}\nu_i(P)dt$, since $dz_i = 0$. $b_i(P)dB_i$ has an expected value of zero and $-\lambda_{i'}\nu_i(P)$ prevails. That is, in 'normal times' when jumps do not occur, on average, a growth option's realized returns is larger than expected during times of high idiosyncratic volatility, and lower than expected during times of low idiosyncratic volatility. In the language of asset pricing, realized returns larger than expected translates to positive risk-adjusted returns. Therefore, options tend to exhibit positive risk-adjusted returns during times of high idiosyncratic volatility and negative risk-adjusted returns during times of low idiosyncratic volatility. When a switch in volatility regime arrives, i.e. $dz_i = 1$, the third term of expression (3.13) dominates and the option's risk-adjusted return takes a large jump in the same direction as the switch. Conditioned on an initial regime, the arrival of a switch results in a jump in risk-adjusted

option will inherit more of the risks embedded in the underlying asset as the value of the asset increases amplifying the difference in betas between regimes.

return that relates inversely with the initial regime, establishing a negative correspondence between risk-adjusted returns and past realized volatility.

The remaining panels of Figure 2 report in P space the difference in diffusion, continuous drift, and jump terms for an option's return process between volatility regimes. Figure 2 (b) shows the difference in the diffusion terms $b_H(P) - b_L(P)$. Not surprisingly, the difference in the option's return volatility is positively related with the difference in volatility of the output price process. Figure 2 (c) shows that the difference in the continuous drift terms $a_H(P) - a_L(P)$ is positive and also larger the greater the spread in volatility between regimes. Taken together, the results point to the conclusion that a growth option's continuous drift term is positively related with the diffusion term. Figure 2 (d) reveals that the difference in the jump terms $\nu_H(P) - \nu_L(P)$ is negative and larger the greater the difference in volatility parameter between regimes. Lastly, the difference in all quantities are identically zero if the volatility values are the same in both regimes, coinciding with the outcome of a single regime model.

4 Simulations

In this section, we rely on numerical simulations in order to investigate if our model can produce results consistent with the main empirical findings in Duffee (1995) and Ang, Hodrick, Xing, and Zhang (2006). Our goal is to create a laboratory as an environment in which to analyze the effects of real options and idiosyncratic volatility in the cross section of stock returns.

We simulate a large panel of growth options using the solution equations from our model. All quantities are simulated on a daily frequency in order to keep the analysis as close as possible to the empirical papers in the literature. We begin by simulating a single time series path of S_t using the process in equation (2.4), and 500 separate time series paths of P_t values and volatility regimes using processes (2.1) and (2.2) in order to generate a panel.[21] Each simulated sample path of P_t corresponds to the price series for the commodity output of a firm in the cross section. We use a time horizon of 50 years for each price series and assume that there are 20 trading days in each month. That corresponds to a total of 12,000 (50 years $\times$ 12 months $\times$ 20 days) observations for each price series. We compute the returns on growth options using the return dynamics (3.7) and (3.8) shown in Proposition 2. Once a full panel of sample returns is simulated, we use the simulated data to carry out the main empirical analysis conducted by Duffee and Ang et al. and store the results. For this part of the analysis, we consider only the returns on growth options prior

[21]See Hanson (2007) for a good description of simulations of mixed jump-diffusion processes.

to reaching their activation thresholds, since activation corresponds to the termination of an option.[22] We then repeat the entire process 100 times to arrive at a sample of 100 sets of simulated results and estimates. Then t-tests are carried out the usual way in order to investigate the estimates' statistical significance. The entire simulation process is repeated using three different sets of values for $\sigma_{P,H}$ and $\sigma_{P,L}$ in order to investigate the effects of the spread in idiosyncratic volatility on the cross section of returns. Our choice of parameter values for the model are summarized in Table 1.

Insert Table 1 here

In order to replicate the empirical papers, we need to compute the risk- adjusted returns and idiosyncratic return volatility of the simulated options at the monthly frequency. To this end, for each growth option j and month t, we define the option's monthly idiosyncratic return volatility as the standard deviation of the daily excess returns relative to the CAPM. More specifically, we define the daily risk-adjusted return as

$$\varepsilon_{j,\tau} = r_{j,\tau} - r/240 - \beta_{j,\tau}\left[r_{S,\tau} - r/240\right] \tag{4.1}$$

where $r_{j,\tau}$ is the simulated daily return on growth option j, $r_{S,\tau}$ is the simulated daily returns on S, and $\beta_{j,\tau}$ is the option's simulated daily beta. Then, we define the growth option's idiosyncratic volatility as $IVol_{j,t} = \sqrt{var(\varepsilon_{j,\tau})}$ where $\tau \in (t-1, t]$, and the change in idiosyncratic volatility as $\Delta IVol_{j,t} = IVol_{j,t} - IVol_{j,t-1}$. We define the growth option's monthly excess returns as the simple sum of the daily excess returns during the month, .i.e $\tau \in (t-1, t]$,

$$r^e_{j,t} = \sum_{\tau=1}^{20} \varepsilon_{j,\tau} \tag{4.2}$$

For options that are terminated within a month, $r^e_{j,t}$ is computed from available daily observations prior to termination within the month. In order to mitigate the potential mechanical effects of return skewness on the correlation between monthly realized returns and volatility in regressions[23], we separately compute monthly $IVol_{j,t}$, $\Delta IVol_{j,t}$ and excess

[22]The lack of continuation for terminated options means that our simulations over-sample price paths that never reach the activation threshold, resulting in a downward bias in option returns. While the return levels are uniformly downward biased, the simulated relation between option returns and volatilities are not.

[23]Skewness in realized returns can arise from jumps in returns induced by switches in the volatility regime (Chen, Hong, and Stein (2001)).

returns using logarithms of daily excess returns. More specifically, we define

$$Log(\varepsilon_{j,\tau}) = Log(1 + r_{j,\tau}) - Log(1 + r/240) - \beta_{j,\tau}\left[Log(1 + r_{S,\tau}) - Log(1 + r/240)\right] \quad (4.3)$$

then compute $IVol_{j,t}$, $\Delta IVol_{j,t}$, and $r^e_{j,t}$ as described before. The simulated log excess returns are used in cross sectional regressions, while simple excess returns are used to form portfolio risk-adjusted returns.

Insert Figure 3 here

Figure 3 shows simulated sample paths of P, $F(P)$, $IVol$, realized month-end simple returns and realized month-end risk-adjusted simple returns for a single firm j using the base case set of model parameters. Panels (a) and (b) of the figure show that the growth option value $F(P)$ follows a similar patten with the underlying state variable P, as expected. Panels (c) to (f) show that the realized month-end returns, risk-adjusted returns and $IVol$ appear to be regime dependent, as expected from the derived model equations.

4.1 The Positive Return-Volatility Relation

In this section, we investigate if our model is qualitatively able to reproduce results similar to the main empirical finding in Duffee (1995).

Duffee (1995) runs regressions to establish that there exists a positive contemporaneous relationship between stock returns and idiosyncratic return volatility in the cross section. We investigate this return-volatility relation by fitting Fama and MacBeth (1973) monthly cross-sectional regressions of option returns on contemporaneous changes in idiosyncratic return volatility using our simulated data. The regression is repeated for different simulated samples in order to investigate the role that the spread in volatility parameters between regimes have on the volatility-return relation. The cross sectional regression model for month t is

$$r^e_t = \gamma_{0,t}\iota + \gamma_{1,t}\Delta IVol_t + \eta_t \quad (4.4)$$

where r^e_t is a vector of $r^e_{j,t}$ and $\Delta IVol_t$ is a vector of $\Delta IVol_{j,t}$ of all the firms $j \in J$, and ι is a vector of ones.[24] A positive contemporaneous relation between returns and $\Delta IVol$ consistent with results in Duffee (1995) translates to estimates that $\gamma_{1,t} > 0$. Furthermore, if real options and switches in idiosyncratic volatility regime as shown in our model can be driving forces in the positive contemporaneous relationship between returns and idiosyn-

[24]Grullon, Lyandres, and Zhdanov (2010) also regress returns on $\Delta IVol_t$ to establish a positive return-volatility relation in the cross section.

cratic return volatility in the cross section, then we should expect positive $\gamma_{1,t}$ estimates for the simulated samples where $\sigma_{P,H} > \sigma_{P,L}$, and an estimate of 0 for the sample under a single volatility regime.

Insert Table 2 here

The results of fitting regression (4.4) are reported in the first columns of each panel in Table 2. The table shows that the return-volatility relation is positive and highly statistically significant in the simulated samples where $\sigma_{P,H} > \sigma_{P,L}$. These results confirm the predictions from our model discussed earlier that firms' real options and switches in idiosyncratic volatility of the profit shock variables can generate results consistent with Duffee. Furthermore, the table shows that the positive relation is stronger and more significant for the simulated sample with larger spreads between $\sigma_{P,H}$ and $\sigma_{P,L}$, but negligible and insignificant for the sample with no spread. These results confirm that the positive return-volatility relation in the simulations is driven by the stochastic idiosyncratic volatility feature of our model, and not by other potentially opaque features.

These results confirm that our real option/stochastic volatility model may play a significant role in explaining the positive contemporaneous correlation between risk-adjusted stock returns and idiosyncratic return volatility as shown to exist in the cross section.

4.2 The Negative Return-Volatility Relation

In this section, we investigate if our model is qualitatively able to reproduce results similar to the main empirical findings in Ang, Hodrick, Xing, and Zhang (2006).

Ang, Hodrick, Xing, and Zhang use portfolio-based asset pricing tests to establish that high $IVol$ stocks significantly underperform their low $IVol$ counterparts on a risk adjusted basis. Similarly, we investigate the relative performance of high $IVol$ assets using our simulated data. At the end of each month, we sort growth options on the level of realized volatility $IVol$ over the past month into decile portfolios. Then, we compute value-weighted one month holding period portfolio returns using the options' monthly excess returns as defined in equation (4.3). The portfolios are rebalanced at the end of each month. This procedure is repeated for different simulated samples in order to investigate the role that the spread in volatility parameters between regimes have on the volatility-return relation.

If the real options and stochastic volatility features of our model can be driving forces in the relative poor performance of high $IVol$ stocks, then we should expect lower risk-adjusted returns of high $IVol$ portfolios than low $IVol$ portfolios for the simulated samples where $\sigma_{P,H} > \sigma_{P,L}$, but no difference in risk-adjusted portfolio returns for the simulated sample corresponding to the model under a single volatility regime.

Insert Table 3 here

Table 3 reports the mean portfolio risk-adjusted returns along with their t-statistics. $IVol$ ranks are listed across columns with the difference in risk-adjusted returns between the highest and lowest $IVol$ decile portfolios reported in the last column. Figure 4 provides a visual illustration of the monthly risk-adjusted returns of the ten decile portfolios reported in table 3. The table shows that the difference between the top and bottom decile $IVol$ portfolios has a highly statistically significant and negative risk-adjusted returns for simulated samples where $\sigma_{P,H} > \sigma_{P,L}$. These results confirm the predictions discussed earlier that our model can generate results consistent with Ang, Hodrick, Xing, and Zhang. Furthermore, the table shows that the poor performance of the long-short portfolio is stronger and more significant for the simulated samples with larger spreads between $\sigma_{P,H}$ and $\sigma_{P,L}$, but negligible and insignificant for the sample corresponding to a single volatility regime. This confirms that the main features of our model are the driving force behind the results, and not other potentially opaque features of the model.

Insert Figure 4 here

We provide further support for our model by investigating the performance of high $IVol$ options by fitting Fama and MacBeth (1973) monthly cross-sectional regressions of option returns on lag idiosyncratic return volatility using our simulated data. The regression model for the for month t is

$$r_t^e = \gamma_{0,t}\iota + \gamma_{1,t} IVol_{t-1} + \eta_t \tag{4.5}$$

Our model's predictions point to negative $\gamma_{1,t}$ estimates for the simulated samples where $\sigma_{P,H} > \sigma_{P,L}$, and a zero $\gamma_{1,t}$ estimate for the simulated sample under a single volatility regime.

The results of fitting regression (4.5) are reported in the second columns of each panel in Table 2. The table shows that there is a negative and highly statistically significant return-lag volatility relation in the simulated samples where $\sigma_{P,H} > \sigma_{P,L}$. Furthermore, the table shows that this negative relation is stronger and more significant for the samples simulated under larger spreads between $\sigma_{P,H}$ and $\sigma_{P,L}$, but negligible and insignificant for the sample corresponding to a single volatility regime. These results reaffirm the earlier results from portfolio analysis.

Taken together, the reported results confirm that our real option/stochastic volatility model may play a significant role in explaining both the positive contemporaneous correlation between risk-adjusted returns and $IVol$, and the poor risk-adjusted performance of high $IVol$ stocks as shown to exist in the cross section of stocks, conflicting empirical puzzles that seem at odds with standard asset pricing arguments.

5 Empirical Analysis

In this section, we empirically test the predictions of our model and show empirical support in the data.

5.1 Data, Variable Descriptions and Summary Statistics

We obtain daily and monthly stock returns from CRSP daily and monthly return files, respectively. Daily and monthly factor returns and risk-free rates are collected from Ken French's website.[25] Our sample period is from January, 1971[26] to December, 2010 for all market-based variables. All our accounting variables are from annual COMPUSTAT files. Following the literature, we eliminate utility (SIC codes between 4900 and 4999) and financial (SIC codes between 6000 and 6999) companies. We also eliminate observations with share price of zero, negative book equity values, and returns of firms with less than one year of accounting data on annual COMPUSTAT files. In order to remove the effects of delistings on stock returns, we eliminate return observations within one year of delisting for stocks whose first delisting code digit is different from 1. Lastly, we consider only ordinary shares traded on the NYSE, AMEX and Nasdaq with primary link to companies on COMPUSTAT with domestic data source. After computing monthly idiosyncratic return volatility as described below, our sample size is over 1 million monthly observations with non-missing return and idiosyncratic return volatility values.

5.1.1 Idiosyncratic Volatility

Testing our hypothesis requires a measure of idiosyncratic volatility of the firms' fundamental shock variables. We motivate our choice for this variable following Grullon, Lyandres, and Zhdanov (2010). Theoretically, the value of a firm's real options is dependent on the firm's fundamental shock variable, however the latter is not easily observable for many

[25] http:// mba.tuck.dartmouth.edu/ pages/faculty/ken.french/data library.html

[26] Casual observation reveals that the annual number of firm observations on COMPUSTAT is relatively low prior to the 70's after applying the reported filters in addition with non-missing Sales and Net Income data.

firms. If firm values incorporate real option values, then the idiosyncratic return volatility of a firm's stocks should be related to the volatility of the firm's underlying shock variable.[27] Following this argument, a number of papers, such as Leahy and Whited (1996) and Bulan (2005), and Grullon, Lyandres, and Zhdanov (2010) have used measures of volatility of stock returns as proxies for underlying volatility. We adopt the same approach in this paper.

Following Ang, Hodrick, Xing, and Zhang (2006) and Ang, Hodrick, Xing, and Zhang (2009), among others, we estimate the idiosyncratic return volatility ($IVol$) for the stock of firm j during the month t as the standard deviation of the firm's daily return during month t, i.e. $\tau \in (t-1, t]$, relative to the Fama and French 3 factor model

$$r_{j,\tau} = \alpha_i + \beta_{j,MKT} MKT_\tau + \beta_{j,SMB} SMB_\tau + \beta_{j,HML} HML_\tau + \varepsilon_{j,\tau} \tag{5.1}$$

More specifically, $IVol$ at month t is defined as $IVol_{j,t} = \sqrt{var(log(1 + \varepsilon_{j,\tau}))}$ where $\varepsilon_{j,\tau}$, $\tau \in (t-1, t]$, is the estimated residual from regression (5.1). As in Grullon, Lyandres, and Zhdanov (2010), we use the logarithm of the residuals in order to mitigate the potential mechanical effect of return skewness (see Duffee (1995) and Kapadia (2007)) on the relation between return and contemporaneous return volatility.[28] The change in $IVol$, $\Delta IVol_{j,t}$, is the difference between the estimated volatility in month t and that in month $t-1$

$$\Delta IVol_{j,t} = IVol_{j,t} - IVol_{j,t-1} \tag{5.2}$$

5.1.2 Firm Characteristics

We require several observable firm characteristics known in the literature to determine stock returns as controls when conducting cross sectional return regressions. These characteristics are the firms' log market equity, log book-to-market, past stock returns, stock CAPM beta and trading volume. Following Fama and French (1993), the market value of equity is defined as the share price at the end of June times the number of shares outstanding. Book equity is stockholders' equity minus preferred stock plus balance sheet deferred taxes and investment tax credit if available, minus post-retirement benefit asset if available. If missing, stockholders' equity is defined as common equity plus preferred stock par value. If these variables are missing, we use book assets less liabilities. Preferred stock, in order of availability, is preferred stock liquidating value, or preferred stock redemption value, or

[27]For a firm that is all-equity financed, there is a perfect relationship between the firms' shock variable volatility and the firm's stock return volatility.

[28]Chen, Hong, and Stein (2001) also find that using simple returns induces a pronounced correlation between skewness and contemporaneous volatility.

preferred stock par value. The denominator of the book-to-market ratio is the December closing stock price times the number of shares outstanding. We match returns from January to June of year t with COMPUSTAT-based variables of year $t-2$, while the returns from July until December are matched with COMPUSTAT variables of year $t-1$. This matching scheme is conservative and ensures that the accounting information-based observables are contained in the information set prior to the realization of the market-based variables. We employ the same matching scheme in all our matches involving accounting related variables and CRSP-based variables. We define past returns as the buy-and-hold gross compound returns minus 1 during the six-month period starting from month $t-7$ and ending in month $t-2$. Following Karpoff (1987), trading volume is trading volume normalized by the number of shares outstanding during month t. Lastly, stock CAPM beta is the estimated coefficient from rolling regressions of monthly stock excess returns on the market factor's excess returns. We use a 60-month window every month requiring at least 24 monthly return observations in a given window, and use the procedure suggested in Dimson (1979) with a lag of one month in order to remove biases from thin trading in the estimations.

5.1.3 Real Option Proxies

Following the argument in the literature (e.g., Grullon et al.) that the option to invest is the most common type of real options (e.g., Brennan and Schwartz (1985), MacDonald and Siegel (1986), Majd and Pindyck (1987), and Pindyck (1988) among many others), we examine if the relation between the firms' stock returns and $IVol$ is driven by the effects of stochastic operating risk on growth option values by comparing the strength of this relation across firms with different intensities of investment opportunities. To this end, we require several firm characteristics to proxy for the firms' reliance on growth options. We follow Grullon, Lyandres, and Zhdanov (2010) and consider firm size as an inverse measure of growth opportunities. Larger firms tend to be more mature and have larger proportions of their values from assets-in-place, while smaller firms tend to derive value from growth opportunities (e.g., Brown and Kapadia (2007) and Carlson, Fisher, and Giammarino (2004)). We define two measures of firm size: the book value of total assets and the market value of equity. Our third (inverse) proxy for growth options is firm age. Older, more established firms tend to derive larger proportions of profits and firm value from assets-in-place (Lemmon and Zender (2010), Carlson, Fisher, and Giammarino (2004)). Age is defined as the log of the difference between the month of observation and the month in which the stock first appeared in the CRSP monthly return files.

In addition to size and age, we consider a second set of growth option proxies. We construct categorical variables by combining other observable characteristics with the belief that they may capture real option intensities in ways not captured by size and age alone. The results using these categorical variables serve as additional robustness checks and supplement the results from age and size. To this end, we define profit growth as the sum of the growth rates from years $t+2$ to $t+5$ of the firms' operating profits. Sales growth and investment growth are defined similarly except with total sales and investments in property plant and equipment, respectively. Then in each year, we categorize firms as high future profit, high future sales, and high future investment growth if they have future profit growth, sales growth and investment growth that exceed the top tercile breakpoint values among NYSE firms for each of these three variables. We motivate the use of these variables as bases for real option proxies because we believe that future profit, sale and investment growth correlate with future investment opportunities and growth options. As in Grullon, Lyandres, and Zhdanov (2010), we acknowledge that future performance measures as proxies for growth suffer from look-ahead biases. However, these proxies can still be useful for our study because we do not focus on developing predictive models in this paper. In order to alleviate concerns of spurious correlations between contemporaneous surprises in growth and monthly returns, month t returns are matched with growth variables starting two years following the returns data, i.e. growth data starting from $t+2$ and ending in $t+5$. We follow the same approach each year to define small, growth and young based on the firms' positioning among NYSE firms on total asset value, market-to-book and age, with the exception that the matching with monthly return data is done the same way as firm characteristics are matched with monthly returns. Once these characteristic dummies are computed, we define a new set of categorical variables by combining these dummies (e.g. small and growth, small and young, young and high profit growth) to expand our set of categorical variables.

It is natural to think that firms in certain industries possess more real options than others, and firm valuations from real options may be captured by their industry membership. Following Grullon, Lyandres, and Zhdanov (2010), for our third set of proxies of real option intensity, we consider three main classifications of industry membership based on Fama and French (1997) 49 industries. We define firms with membership in Fama and French (FF) industries 27 (precious metals), 28 (mining), and 30 (oil and natural gas) as natural resource firms. We classify firms in FF industries 22 (electrical equipment), 32 (telecommunications), 35 (computers), 36 (computer software), 37 (electronic equipment), and 38 (measuring and control equipment) as high-tech firms. Membership in FF industries 12 (medical equipment) and 13 (pharmaceutical products) are defined as biotechnology or pharmaceutical firms. Lastly, we define firms with membership in any one of the three aforementioned classifications as all-growth industry firms.

Lastly, we consider a real option intensity measure that reflects the sensitivity of the firms' equity to the volatility of the firms' underlying shock variable. Following the structural model of Merton (1974) and Merton (1992), the equity of a firm resembles a call option on the firm's assets and the strike price corresponds to the firm's total face value of debt due upon maturity. Basing on the knowledge that an option's vega captures the option's sensitivity to the volatility of the underlying state variable, the stocks of large vega firms implied by their capital structure should have greater sensitive to the idiosyncratic risks of the firms' assets. As a consequence, the relation between $IVol$ and stock returns should be stronger for high vega firms. To test this hypothesis, for each firm j and year n, we define the firm's vega based on the firm's capital structure and the Black and Scholes's formula

$$vega_{j,n} = V_{j,n} N'(d_{j,n})\sqrt{5} \tag{5.3}$$

where $d_{j,n} = \frac{ln\left(\frac{V_{j,n}}{D_{j,n}}\right)+\left(r_{f,n}-\frac{\sigma_{j,n}^2}{2}\times 5\right)}{\sigma_{j,n}\sqrt{5}}$, $N'(x) = \frac{exp(-x^2/2)}{\sqrt{2\pi}}$, $r_{f,n}$ is the annualized risk free rate, $\sigma_{j,n}$ denotes firm j's annualized six-month rolling window idiosyncratic volatility based on the Fama French 3 factor model, $V_{j,n}$ denotes the sum of the firm's market equity value and book value of debt, and $D_{j,n}$ is the firm's book value of debt. When calculating vega, we match all our accounting-based variables with market-based variables the same way as described in section 5.1.2 of the paper. For simplicity, we assume in (5.3) that firms have a debt maturity of 5 years. Because vega is relatively invariant over most of the range of debt and firm values[29], it is useful to classify real option intensity based on vega values in relation to other firms in the cross section. To this end, we follow the same approach as before for all our categorical variables. More specifically, in each year we classify firms to be high vega firms if they have vega values in the top tercile where the tercile breakpoint values are found among NYSE firms in the sample. Then, the high vega dummies are matched with monthly return data the same way as firm characteristics are matched with monthly returns. Firm vega is useful in our study because it is the only criterion under consideration that is not necessarily related to firms' future growth opportunities.

5.1.4 Summary Statitics

Table 4 reports summary statistics for the main variables in our study. Mean (median) excess return in our sample is 0.9976% (-0.41%) per month or about 11.9712% (-4.92%) annual. Mean (median) daily idiosyncratic stock return volatility $IVol$ is 2.9476% (2.2782%) or about 44.0171% (34.0208%) annual. Our $IVol$ estimates are similar to those reported in

[29] An option's vega is greatest when the option is at the money, and relatively low and invariant over the remainder of possible values of the underlying state variable (Hull (2011)).

Ang, Hodrick, Xing, and Zhang (2006) and Grullon, Lyandres, and Zhdanov (2010). Mean (median) month-to-month change in $IVol$ is -.0023% (-0.011%). The standard deviation is 2.1096% and similar to the figure reported in Grullon, Lyandres, and Zhdanov (2010).

Insert Table 4 here

5.2 Switches in Idiosyncratic Volatility and Stock Returns

The results of our model offers novel testable predictions on the relation between idiosyncratic volatility and stock returns. If firm values are partially composed of real options, and subject to finite and discrete volatility regime switches, then stocks that experience up switches in idiosyncratic volatility should experience greater risk-adjusted returns after the switch than before the switch. Conversely, stocks that realize a down switch in volatility should experience lower post-switch risk-adjusted returns. The difference between post and pre-switch returns should be more pronounced among more real option intensive firms and firms that experience more extreme changes in idiosyncratic volatility (the switch effect hereafter). We empirically test these predictions and provide supporting evidence in this section.

For each stock in our sample, we compute the stock's 30th and 70th percentile in-sample values of $IVol$. We consider these values to be the thresholds that determine the volatility regimes for the stock. We define an up switch event in $IVol$ in a given month if the stock's previous month's $IVol$ was below its 30th percentile breakpoint value and if the current month's $IVol$ exceeds its 70th percentile breakpoint value, capturing the notion of a change in volatility regime. A down switch event is defined similarly.

Once all the up and down switch events are identified for each stock, we compute the 5-month average of the risk-adjusted returns ending in the month prior to the month of the switch event, and the 5-month average of the risk-adjusted returns beginning from the month after the switch event. Then we investigate if the difference between the post and pre-switch event average of the risk-adjusted returns are related to firms' real option characteristics. More specifically, risk-adjusted returns are based on the Fama and French (1993) 3 factor model

$$r^*_{j,t} = r_{j,t} - r_{f,t} - \sum_{k=1}^{3} \widehat{\beta_{j,k}} F_{k,t} \tag{5.4}$$

where $r_{j,t}$ is the return on stock j in month t, $r_{f,t}$ is the risk-free rate, and $F_{k,t}$ denotes one of the 3 Fama and French factors (market excess return, size, and book-to-market factors) and $k \in (1, 3)$. We estimate the factor loadings $\widehat{\beta_{j,k}}$ for individual stocks using monthly rolling regressions with a 60-month window every month requiring at least 24 monthly return observations in a given window. We use the procedure suggested in Dimson (1979) with a lag of one month in order to remove biases from thin trading in the estimations. The difference in 5-month averages of the risk-adjusted returns between post and pre-switch events when a switch episode occurs in month t is calculated as

$$r_{j,t}^{Diff} = \sum_{\tau=t+1}^{t+6} r_{j,\tau}^{*} - \sum_{\tau=t-6}^{t-1} r_{j,\tau}^{*} \tag{5.5}$$

Once the difference in returns are computed, for each real option criteria, we run separate Fama MacBeth cross-sectional regressions of differences in average risk-adjusted returns on the measure of real option for the sample of up switch events and the sample of down switch events. The cross sectional regression model for the sample of switch events for month t is

$$r_t^{Diff} = \gamma_0 \iota + \gamma_1 RO_{t-1} + \eta_t \tag{5.6}$$

where r_t^{Diff} is the vector of differences in the average of the risk-adjusted returns relative to the event month t, ι is a vector of ones, and RO_{t-1} is the vector of measures of real option intensity. Our model's predictions translate to tests that $\gamma_0 > 0$ and $\gamma_1 > 0$ (or $\gamma_1 < 0$ for inverse RO proxies) for the up switch event sample, and $\gamma_0 < 0$ and $\gamma_1 < 0$ (or $\gamma_1 > 0$ for inverse RO proxies) for the down switch event sample.

Insert Table 5 here

The results of estimating (5.6) are presented in Table 5.[30] The table shows that up switches in $IVol$ are positively (i.e. $\gamma_0 > 0$) related to differences in risk-adjusted returns and down switches are negatively (i.e. $\gamma_0 < 0$) related to differences in risk-adjusted returns, the relation being highly statistically significant in all of the specifications using different criteria for real option intensity. This result is consistent with our model's predictions on the switch effect. In order to analyze the effects of real options on this relation, the table shows that the loadings on the inverse real option proxies measured by total asset size, market equity values and age are positive for the down switch sample, and negative for the up switch sample, indicating that the switch effect is stronger for more growth option intensive firms.

The reported results using categorical variables as proxies for real option intensity are

[30]Results using unadjusted returns are available from the author upon request, but they are not materially different from the results using risk-adjusted returns.

also in favor of the predictions of our real options/volatility model, with greater significance for the up switch sample than for the down switch sample. The only exception occurs when the high vega dummy is used as a real option criteria, whose coefficient estimate is positive and significant for the down switch sample. However, if real option intensity is measured as small in size and high vega, the coefficient estimates are significant and consistent with the model's predictions for both the up and down switch event samples. The coefficient estimate when dummies for young and high vega firms are used also work in favor of the model's predictions for the up switch sample. For the down switch sample, the estimate is positive but not statistically significant. Based on these results, we argue that vega alone is not a strong measure of firms' real options unless it is combined with other real option characteristics to identify real option reliant firms. Another reason for these results may be that the stocks of younger and smaller firms are more sensitive to the firms' shock variables than older and larger firms even if their vegas are large, in line with the view that the stocks of small and young firms experience larger reaction to operating risk when coupled with heavy borrowing than larger and older firms.[31]

The results reported using industry dummies are weakly in favor of the real options/ stochastic volatility model. The only industry classification working in favor of the model's predictions is the all-growth option industries, while any one of the natural resources, high tech or bio tech firms turns out to be statistically not significant. A possible reason for the relatively weaker results using industry dummies is that industry classification alone is a weak proxy for real options since firms within industries may vary widely in their real option intensities.

The real option/stochastic volatility model also predicts that the switch effect should be more pronounced for more real option intensive firms and firms that experience larger changes in $IVol$. We perform tests of this hypothesis in our second set of regressions. To this end, denote $\overline{\Delta IVol_j}$ the difference between the 70th and 30th percentile in-sample breakpoint values of $IVol$ for stock j. $\overline{\Delta IVol_j}$ captures the magnitude of changes in $IVol$ for the firm j. For each real option criteria and each of the down and up switch event samples, we run separate Fama MacBeth cross-sectional regressions. The regression model for month t is

$$r_t^{Diff} = \gamma_0 \iota + \gamma_1 \overline{\Delta IVol} + \gamma_2 \overline{\Delta IVol} \times RO_{t-1} + \eta_t \tag{5.7}$$

where r_t^{Diff}, ι and RO_{t-1} are as defined previously, and $\overline{\Delta IVol}$ is a vector of $\overline{\Delta IVol_j}$. Our model's predictions translate to tests that $\gamma_1 > 0$ and $\gamma_2 > 0$ (or $\gamma_2 < 0$ for inverse RO proxies) for the up switch event sample, and $\gamma_1 < 0$ and $\gamma_2 < 0$ (or $\gamma_2 > 0$ for inverse RO

[31] Anecdotal evidences seem to point to this possibility.

proxies) for the down switch event sample.

Insert Table 6 here

The results of estimating (5.7) are presented in Table 6. The table shows that the coefficient estimates for $\overline{\Delta IVol}$ (i.e. γ_1) is positive and highly significant for the up switch sample and negative and highly significant for the down switch sample in virtually all of the regression specifications. The only exception occurs in the down switch sample if age is used as a real option criteria whose coefficient estimate is negative but not statistically significant. These results provide conclusive evidence in support of our model's predictions on the switch effect.

The table also reports the coefficient estimates for the interaction terms between $\overline{\Delta IVol}$ and GO (i.e. γ_2) from estimating (5.7). The loadings when firm size is measured by total asset value is positive for the down switch sample and negative for the up switch sample, with significance at the 10% and 5% levels respectively. The loadings when firm size is measured by market capitalization value is negative for the down switch sample but not significant, and negative for the up switch sample and highly statistically significant. We conclude that size as an inverse measure of real options provide results that agree with our model's predictions that the switch effect should be stronger for more real option intensive firms and firms that experience larger changes in $IVol$. The interaction with age, on the other hand, provides inconclusive evidence since the estimated loadings are not significant.

Some of the results reported using categorical variables as proxies for real option intensity are also in favor of our real options/volatility model, with greater significance for the up switch sample than for the down switch sample. When the dummies for future profit growth, future sale growths, future investment growth, or small firms with greater future growth are used as proxies for real option intensity, the estimated coefficients are positive for the up switch sample with varying levels of significance. For the down switch sample, none of the estimates are significant even though the estimates are the predicted sign for a number of the specifications. Lastly, using industry dummies as proxies of real option intensity reveals that even though the estimates for most of the specifications are in support of the model's predictions, none of them are statistically significant. We conclude from these results that there is stronger support for the predictions of our model in the up switch sample than in the down switch sample.

Overall, the results in Tables 5 and 6 demonstrate that the empirical evidences on the switch effect are in strong agreement and support the main predictions of our real options/stochastic volatility model.

5.3 Contemporaneous Relation Between Idiosyncratic Volatility and Returns

In addition to the novel predictions on the switch effect, the model also offers a new explanation for the existing empirical findings on the relation between idiosyncratic return volatility and stock returns. In relation to the first extant empirical finding, our model provides an explanation for the positive contemporaneous relation between idiosyncratic return volatility and risk-adjusted returns first documented by Duffee (1995). We argue that the observed positive relation is consistent with our model's positive correspondence between the firms' idiosyncratic volatility regime and realized risk-adjusted returns as discussed in the previous section of the paper. Furthermore, empirically the jumps in returns eventuated from the arrival of extreme changes in idiosyncratic volatility within a month should further strengthen the return-volatility relation observed at the monthly frequency. Therefore, if firms own real options and are subject to changes in idiosyncratic volatility, then the positive return-volatility relation should be more pronounced among more real option intensive firms and firms that experience larger changes in idiosyncratic volatility. We test this hypothesis and provide supporting evidence in this section.

In particular, following Grullon, Lyandres, and Zhdanov (2010) we estimate monthly cross-sectional Fama and MacBeth (1973) regressions of individual stock returns on changes in idiosyncratic volatility and growth options using the various alternative criteria to classify real option intensity. The regression model for the cross section of stocks for time period t is

$$r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \Delta IVol_t + \gamma_2 \Delta IVol_t \times RO_{t-1} + \gamma_3 X_{t-1} + \eta_t \qquad (5.8)$$

where r_t is the vector returns, $r_{f,t}$ is the riskless rate, ι is a vector of ones, $\Delta IVol_t$ is a vector of changes in $IVol_t$, RO_{t-1} is a vector of one of the firms' characteristics used to capture real option intensity, and X_{t-1} is a matrix with columns corresponding to vectors of controls for firm characteristics related to size, book-to-market, past returns, trading volume and stock beta. Our main hypothesis is that the positive relation between stock returns and idiosyncratic volatility should be stronger for firms whose value incorporates more real options. Therefore, our hypothesis translates to tests that $\gamma_1 > 0$ and $\gamma_2 > 0$ ($\gamma_2 < 0$ for inverse RO proxies).

Insert Table 7 here

Table 7 reports the coefficient estimates from estimating (5.8) using our proxies for real options. The table shows that there is a highly statistically significant positive volatility-return relation in all specifications. Not surprisingly, and consistent with a majority of the empirical papers, the coefficients on the market factor loading and on the log book-to-market are both significantly positive, while the coefficients on log size are significantly negative in all specifications. The coefficients on contemporaneous volume are positive and highly statistically significant, consistent with Karpoff (1987) and Grullon, Lyandres, and Zhdanov (2010). The coefficients on past six month cumulative return are insignificant and negative in all specifications, and consistent with some specifications reported in Cooper, Huseyin, and Schill (2008) and Grullon, Lyandres, and Zhdanov (2010).[32]

In the first two columns of the top panel, in which firm size – measured by firms' equity market value and separately by total asset value – is used as an inverse proxy for real options, the table shows that the estimates of γ_2 are negative and highly statistically significant. In the third column, in which age is used as inverse proxy for real options, the table shows that the estimate of γ_2 is negative but not statistically significant.

The table also reports the results when the categorical variables are used as proxies for real option intensity. The estimate of γ_2 for the high vega dummy is positive and highly statistically significant. This result is interesting because vega is the only proxy for real option intensity under consideration that is not necessarily related to the firms' life cycle or growth prospects. In specifications where real options are proxied by future growth opportunities, measured by the high future investment or the high future sales growth dummies, the table reveals that the relation between return and $IVol$ is stronger for more real option intensive firms. The only exception occurs when the dummy for high future profit growth is used, where the estimate is not statistically significant. However, the table shows that when the high future profit growth dummy is combined with the small size dummy, the estimate of γ_2 is positive and highly statistically significant. The table reports a similar pattern of stronger positive relation between returns and volatility when the high future investment growth, high future sales growth, or high vega dummies are combined with the small size dummy to form proxies of real option intensity. An explanation for these stronger effects is that firms that combine more than one real option characteristic incorporate more real option in their valuations perhaps because they possess more real options, leading to more favorable results in line with the predictions of our model.

The remainder of the table reports regression results using industry categorical variables as proxies for real option intensity. The table shows that the estimates of the γ_2 coefficient are positive when dummies for natural resources, high technology and bio technology

[32]Grullon, Lyandres, and Zhdanov show that the coefficient on past returns is sensitive to the set of other independent factors included in Fama Macbeth regressions.

firms are used as proxies for real options, however only the dummy for natural resources gives statistically significant results. The estimate when stocks that belong to any one of the growth option industries is used is positive and highly statistically significant. One explanation for why the γ_2 estimates lack significance when some industry dummies are used is that industry membership alone may be imperfect proxies for real options since firms within industries vary widely in their real option characteristics. In sum, the results reported in Table 7 lend strong support to our model's predictions.

The real options/stochastic volatility model also shows that the positive volatility-return relation should be stronger for more real option intensive firms and firms that experience larger jumps in volatility between volatility regimes. We test this hypothesis in our second set of Fama and MacBeth (1973) regressions. The regression model for the cross section of stocks for time period t is

$$r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \overline{\Delta IVol} + \gamma_2 \Delta IVol_t + \gamma_3 \overline{\Delta IVol} \times \Delta IVol \times RO + \gamma_3 X_{t-1} + \eta_t \qquad (5.9)$$

where r_t, $r_{f,t}$, ι, $\Delta IVol_t$, RO_{t-1}, X_{t-1} and $\overline{\Delta IVol}$ are as defined previously. Our main hypothesis is that the positive relation between stock returns and idiosyncratic volatility should be stronger for firms whose values incorporate more real options and firms that experience larger volatility switches. Therefore, our hypothesis translates to tests that $\gamma_3 > 0$ (or $\gamma_3 < 0$ for inverse RO proxies).

Insert Table 8 here

The results of estimating (5.9) are presented in Table 8. The table shows that the estimate of γ_1 is negative and highly statistically significant for all the regression specifications, implying that the the premium for the jump size is negative in the cross section of stock returns. In other words, stocks that experience larger jumps in $IVol$ between volatility regimes on average earn lower stock returns. This finding is interesting and relates to the $IVol$ puzzle of Ang, Hodrick, Xing, and Zhang (2006), an empirical puzzle that is investigated in the context of our real option/stochastic volatility model in the next section. The table also shows that the estimate of γ_2 is positive and highly statistically significant for all specifications, in agreement with the results in relation to the estimations of (5.8).

Table 8 shows that the estimates of γ_3 are positive (negative for inverse real option proxies) and highly statistically significant for virtually all specifications using different real option proxies. The only exceptions are the coefficient estimates when age, the dummies for young, small and young, and young and high vega are used as real option proxies, which are not statistically significant. The remainder of the table reports regression results using industry categorical variables as proxies for real option intensity. The estimates of the γ_3 coefficient are positive for natural resources, high technology and bio technology firms, however only natural resources is statistically significant. The estimate when stocks that belong to any one of the growth option industries is used is positive and highly statistically significant. Collectively, the results in Table 8 lend strong support to our model's predictions that the positive stock return-volatility relation is related to the firms' real options and uncertainty in the firms' operating risk.

Taken together, we conclude that the results in Table 7 and 8 are consistent with the real option/stochastic volatility model of our paper.

5.4 The Poor Future Performance of High Idiosyncratic Volatility Stocks

Ang, Hodrick, Xing, and Zhang (2006) report that portfolios of high $IVol$ stocks significantly under-perform their low $IVol$ counterparts on a risk-adjusted basis (the $IVol$-puzzle hereafter). Our model also provides an explanation for this finding. Our model predicts that real options exhibit mean-reversion in risk-adjusted returns in tandem with movements in firm-specific operating risks. If firm values incorporate real options and firms are subject to finite and discrete switches in idiosyncratic operating risk, then conducting asset pricing tests employing standard portfolio approaches ensures that sorting and grouping stocks on month-end realized idiosyncratic return volatility ($IVol$) is akin to grouping stocks on the firms' most recent idiosyncratic operating volatility regime. A portfolio of high recent $IVol$ stocks experiences abnormally low future returns reflecting the options' down jump and the lower continuous returns that eventuates upon the arrival of an extreme change in the firms' idiosyncratic operating risk. A similar mechanism explains the superior risk-adjusted performance of low $IVol$ stocks. Consequently, stocks exhibit risk-adjusted returns that correlate negatively with past $IVol$. Furthermore, our model predicts that these effects should be more pronounced for more real option intensive firms and firms that experience more extreme changes in $IVol$. We test these hypotheses and provide empirical support in this section.

More specifically, at each month-end we sort stocks into terciles ranked on the idiosyncratic return volatility $IVol$. Then, independently, at the end of each June, we sort stocks into three terciles on the basis of the real option characteristics age, size (total assets) and size (market equity), and into high/low vega, high/low future profit growth, high/low future sales growth, young/old, high/low future investment rate based on the 33th and 66th percentile breakpoint values of size, age and future growth variables of NYSE firms in our sample. Then we form value-weighted portfolio returns for each two-way classification and assess their performance over the following month. This approach corresponds to the 1/0/1 (formation period/waiting period/holding period) strategy of Ang, Hodrick, Xing, and Zhang (2006) which most of their analysis is concentrated on.

The performance of the portfolios are assessed on a risk-adjusted basis relative to the Fama and French 3 factor model. More specifically, we estimate the intercepts from running time-series regressions of portfolio excess returns on the three Fama French factors (i.e. market risk premium, size, and book-to-market)

$$r_t - r_{f,t} = \gamma_0 + \gamma_1 MKTRF_t + \gamma_2 SMB_t + \gamma_3 HML_t + \epsilon_t \tag{5.10}$$

where r_t is the portfolio return, $r_{f,t}$ is the monthly riskless rate, $MKTRF$, SMB, and HML are the Fama and French (1993) three factors that proxy for the market risk premium, size and book-to-market factors respectively. In order to investigate the extent to which real option intensity contributes to the $IVol$-puzzle, we also estimate the alphas for the long-short portfolios along each one of the real option intensity rank classification. The long-short portfolio returns along a fixed rank classification are the returns of the portfolio that is long the highest-ranked $IVol$ stock portfolio and short the lowest-ranked $IVol$ stock portfolio. All portfolios are rebalanced monthly.

Tables 9 to 11 report average portfolio alphas along with Newey West robust t-statistics after doing two-way independent sorts on $IVol$ and the measure for real option intensity. $IVol$ ranks are listed across columns with the long-short portfolio reported in the last column, while real option ranks are listed down the rows. In order to facilitate interpretation of the economic significance, the reported portfolio alphas are annualized after multiplying the intercept estimates by 12. All other reported statistics are unadjusted.

Insert Table 9 here

The first three panels of Table 9 show that the $IVol$-puzzle is more pronounced and statistically more significant for the smaller two tercile groups when firm size is measured by total asset value. The $IVol$-puzzle for the largest tercile stocks is not significant. A similar pattern is present for size portfolios, if size is measured by market equity value, and age portfolios. The $IVol$-puzzle is not significant at the 5% level for the largest and oldest tercile stocks, but are large and significant for younger and smaller stocks, results that are in agreement with the predictions of the real option/stochastic volatility model. Table 9 also reports that the relative poor performance of high $IVol$ stocks are more pronounced and statistically more significant for the high vega firms than for the low vega firms. This finding is enlightening because vega is the only proxy for real option intensity under consideration that is not necessarily related to firms' future growth opportunities.

Insert Table 10 here

Perhaps what is more striking are the results in Table 10. The table shows that the $IVol$-puzzle is even stronger and more significant for small and high vega firms than for high vega firms alone. These results support the argument that firm values incorporate real options other than future growth opportunities and that the $IVol$-puzzle relates to those real options as well. The results also support the notion that the $IVol$-puzzle is stronger among stocks of firms that combine more than one real option characteristic, perhaps because these firms have more real options. The other panels in the table point to that conclusion as well. While the $IVol$-puzzle is not conclusively stronger for high future profit, high future sale or high future investment growth firms as reported in Table 9, the $IVol$-puzzle is stronger and more significant for these firms if they are also small in size. Similar results also hold for young firms and firms that are both young and have high vega. In sum, the $IVol$-puzzle seems to be more pronounced for more real option intensive firms, and for firms that are characterized by more than one real option criteria.

Insert Table 11 here

Table 11 reports the strength of the $IVol$ puzzle across real option intensive industries. The first two panels of the table show that the $IVol$-puzzle is more pronounced for natural resources and high technology stocks. The difference in alphas between the high $IVol$ and low $IVol$ portfolios is large and significant at the 5% level for natural resources and high tech stocks, but lower and not significant at the 5% for stocks that do not belong to these industries. These results are in agreement with the predictions of our model. However, the third panel of the table shows that the $IVol$-puzzle is not significant among bio tech stocks, but significant for other stocks. The last panel shows that the $IVol$-puzzle is statistically less significant for stocks with membership in any one of the three growth option industries. From these results, we conclude that the evidence on real option industry membership and the $IVol$-puzzle is weakly in favor of the predictions of our model. As mentioned earlier, industry dummies may be weak proxies for real option intensity because firms within industries can vary widely in the amount of real options they have.

In order to investigate how the $IVol$-puzzle relates to the size of changes in operating risk, in addition to the two-way independent sorts discussed earlier, we also independently sort and rank stocks on $\overline{IVol}$ into terciles. Then we form long-short portfolios for each two-way classification of the real option intensity criteria and $\overline{\Delta IVol}$ and assess their performance over the following month relative to the Fama and French 3 factor model. As before, a long-short portfolio is a portfolio that is long the highest-ranked $IVol$ stock portfolio and short the lowest-ranked $IVol$ stock portfolio. Portfolios are rebalanced monthly.

Tables 12 to 14 report average portfolio alphas, along with Newey West robust t-statistics, of the long-short portfolios after doing 3-way independent sorts. Reported alphas are annualized (all other statistics are unadjusted). The $\overline{\Delta IVol}$ ranks are listed down the rows and the ranks for real option proxies are listed across columns. In order to facilitate the investigation of the $IVol$- puzzle within each tercile rank of $\overline{IVol}$ after controlling for real option intensity, the tables also report the average alphas of the real option intensity-ranked long-short portfolios for each $\overline{\Delta IVol}$ rank classification.

Insert Table 12 here

The first three panels of Table 12 show that the $IVol$-puzzle is monotonically stronger and more significant for the higher $\overline{\Delta IVol}$ tercile than for the lower terciles if real option intensity is proxied by size and age. These results support our predictions that the $IVol$-puzzle should be more pronounced for firms that experience more extreme changes in $IVol$. The table also shows that the $IVol$-puzzle is stronger and highly statistically significant among the youngest firms and firms that have the largest $\overline{\Delta IVol}$, which also support the predictions our model. However, the $IVol$-puzzle seems to be more pronounced for larger firms among the top $\overline{\Delta IVol}$ tercile stocks. These findings are not in direct support of our model. As inverse proxies for real options, our model predicts that the $IVol$-puzzle should be stronger for smaller firms. However, consistent with the predictions of our model, the $IVol$-puzzle remains both statistically and economically significant for smaller firms.

If high future profit, high future sale or high future investment growth dummies are used to categorize real option intensity, the main conclusions are similar. While the $IVol$-puzzle is stronger for the high $\overline{\Delta IVol}$ stocks independently of the real option characteristics, the $IVol$-puzzle seems to be weaker for high future growth firms. The latter is not in direct support of our model. One possible explanation for the weaker results for high growth firms is that the $IVol$-puzzle may be confounded by the strong positive stock returns of the firms with high future growth prospects. To the extent that high future growth correlates with high expected future earnings growth, and investors incorporate earnings growths expectations into stock returns during the portfolios' evaluation month, this is likely to be a confounding factor that may weaken the $IVol$- puzzle.

Insert Table 13 here

While the $IVol$-puzzle does not seem stronger for high future growth stocks within the high $\overline{\Delta IVol}$ stocks, Table 13 shows that the $IVol$-puzzle is stronger for high growth firms that are also small. A similar pattern is present for the stocks of firms that are small and young, and firms that are small and have a high vega. These results support the argument made previously that the $IVol$-puzzle is more pronounced for firms that combine more than one real option characteristics.

Insert Table 14 here

Table 14 reports the strength of the $IVol$-puzzle for the different real option intensive industries and $\overline{IVol}$ ranks. The table shows that the $IVol$-puzzle is monotonically stronger and statistically more significant for larger $\overline{IVol}$ than for smaller $\overline{IVol}$ stocks independently of industry membership, results that are consistent with our predictions. The table also shows that the strength of the $IVol$-puzzle and its statistical significance correlates with membership in natural resources, bio tech and all-growth industries, and firms that have the larger $\overline{\Delta IVol}$, results that are also in direct support of the predictions of our model. The only industry membership working against the model's predictions is the high tech group, which exhibit a weaker $IVol$-puzzle than the low tech stocks within the high $\overline{IVol}$ group. Although the $IVol$-puzzle is not stronger among the high tech stocks, the puzzle still remains both statistically and economically significant.

Overall, the results in tables 9 to 14 demonstrate that the relative poor performance of high $IVol$ stocks is more pronounced for stocks of firms with characteristics that are likely to be related to real options and experience more extreme changes in $IVol$, results in line with our real option/stochastic idiosyncratic volatility model. Taken together with our regression results, we find strong evidence that our model may explain the two conflicting empirical $IVol$-puzzles that previously seemed to be at odds with conventional asset pricing theories.

6 Conclusions

Recent empirical evidences on the correspondence between stock returns and idiosyncratic return volatility at the firm level have been mixed at best. While some point to a positive relation, others report that the relation is negative. In this paper, we propose a new economic explanation for the conflicting findings in a simple equity valuation model of firms involving real options and stochastic operating risk. More generally, we motivate why idiosyncratic risk may appear to be priced in the cross section of stock returns.

In this paper, we introduce a 2-regime Markov switching process for the idiosyncratic volatility of the firms' output price in order to incorporate uncertainty in operating risk. The value of a real option is convex in the output price and its valuation does not distinguish between market and idiosyncratic operating risks, a feature that contrasts starkly from the valuation of the firm's assets-in-place. We show that the value of a firm's real options relate positively with the volatility regime, giving rise to regime dependency of the firm's stock return and market beta. The time-series dynamics of the realized and expected returns induced by the volatility structure results in an interplay between risk-adjusted returns and idiosyncratic return volatility consistent with observations in the cross section. We verify our intuition with numerical simulations, followed with our own predictions and empirical tests. We find that the results are strongly supportive of our model.

To maintain tractability, our model is devoid of a more general structure for the firms' idiosyncratic operating risk. In the 2-regime structure, the operating volatility of a firm leaps between the two regimes to generate the results aligned with the empirical observations. Qualitatively, our results should persist in a more general structure insofar as idiosyncratic volatility exhibits mean reversion. Work establishing this conjecture seems to be an interesting extension to our paper.

A number of papers have reported that asset returns must exhibit heteroscedasticity as well as discontinuous movements to fit their empirical distributions. Previously, the literature has relied on liquidity and other microstructure related aspects to explain the distributional properties of stock returns. Our model, on the other hand, suggests that the observed stock return distributions stems from the operating environment that firms face. Our model has the capability to parsimoniously generate skewness and fat tails in return distributions, providing fertile grounds for additional research. Further research in this direction is highly merited.

Lastly, we believe that our model imparts an important linkage between corporate investment environments and other stock return regularities. Our model implies that jumps in stock returns should coincide with large changes in idiosyncratic volatility in predictable ways. Due to space constraint, we refrained from investigating the relation between stock returns and return skewness potentially generated by jumps that have been reported to exist in the literature. The framework developed in this paper is capable of shedding new insights in this growing literature as well. We hope that the ideas developed here will spark further interest in research addressing the observations involving three-way interactions between stock returns, volatility, and skewness.

7 Appendix

This section describes the valuation approaches used to value the assets-in-place and growth options of the firms, followed by the proofs of the propositions and corollaries stated in Section II of the paper.

7.1 Valuation Approaches

It simplifies valuation if we reexpress the dynamics of the price (2.1) more concisely by letting

$$dP = \mu P dt + \sigma_i P dB_i \tag{7.1}$$

where $dB_i = \frac{\sigma_{P,i} dB_1 + \sigma_A dB_2}{\sigma_i}$ and $\sigma_i = \sqrt{\sigma_{P,i}^2 + \sigma_A^2}$. Then it can be shown that $\text{Cov}\left(dB_i, dB_2\right) = \frac{\sigma_A}{\sigma_i} dt$, $\text{Cov}\left(\frac{dP}{P}, \frac{dS}{S}\right) = \sigma_S \sigma_A dt$, and $\rho_i = \frac{\sigma_A}{\sigma_i}$.

Let $Y(P, \sigma_{P,i}) = Y_i(P)$ be the value function of an asset that is twice-differentiable in P where P follows the process in equation (7.1) and $\sigma_{P,i}$ follows the process (2.2). At this stage, $Y_i(P)$ can be the value of a growth option, the assets-in-place of a young firm, or the value of a mature firm. The generalized Ito's Lemma (Malliaris (1988)) implies that the value of $Y_i(P)$ follows the process

$$\frac{dY_i(P)}{Y_i(P)} = \frac{\mu P Y_i'(P) + \frac{1}{2} P^2 \sigma_i^2 Y_i''(P)}{Y_i(P)} dt + \frac{\sigma_i P Y_i'(P)}{Y_i(P)} dB_i + \frac{(Y_{i'}(P) - Y_i(P))}{Y_i(P)} dz_i \tag{7.2}$$

The first two terms on the right hand side of the equation are the standard form for Ito's Lemma. The third term is the jump of the value of $Y_i(P)$ when σ_i switches from regime i to regime i'. Equation (7.2) can be written as

$$\frac{dY_i(P)}{Y_i(P)} = [\mu_{Y_i}(P) - \lambda_i \gamma_i(P)] dt + \sigma_{Y_i}(P) dB_i + \gamma_i(P) dz_i \tag{7.3}$$

where

$$\mu_{Y_i}(P) = \left[\frac{\mu P Y_i'(P) + \frac{1}{2} P^2 \sigma_i^2 Y_i''(P)}{Y_i(P)}\right] + \lambda_i \gamma_i(P) \tag{7.4}$$

$$\sigma_{Y_i}(P) = \frac{\sigma_i P Y_i'(P)}{Y_i(P)} \tag{7.5}$$

$$\gamma_i(P) = \frac{Y_{i'}(P) - Y_i(P)}{Y_i(P)} \tag{7.6}$$

We describe three valuation approaches that result in consistent values for $Y_i(P)$.

7.1.1 Method I: Hedge Portfolio Approach

The first valuation approach follows Merton (1976) and involves the construction of a hedge portfolio that eliminates market risk. To this end, denote the proportion of a portfolio invested in the market asset S, asset $Y_i(P)$ and the riskless asset M as w_1, w_2 and $w_3 = 1 - w_1 - w_2$, respectively. The instantaneous rate of return on the portfolio is given by

$$\frac{dW}{W} = w_1 \frac{dS}{S} + w_2 \frac{dY_i}{Y_i} + (1 - w_1 - w_2) r dt \tag{7.7}$$

$$= [w_1(\mu_S - r) + w_2(\mu_{Y_i}(P) - r) + r - w_2 \lambda_i \gamma_i(P)] \, dt \tag{7.8}$$

$$+ w_1 \sigma_S dB_2 + w_2 \sigma_{Y_i}(P) dB_i + w_2 \gamma_i(P) dz_i$$

where we have substituted (7.3), (2.3) and (2.4) into (7.7) to arrive at (7.8). It is not possible to make this portfolio riskless[33]. Instead, we choose the portfolio weights w_1^* and w_2^* to eliminate market risk only. This leads to the following process for the hedge portfolio

$$\frac{dW}{W} = w_1^* \frac{dS}{S} + w_2^* \frac{dY_i}{Y_i} + (1 - w_1^* - w_2^*) r dt \tag{7.9}$$

$$= [w_1^*(\mu_S - r) + w_2^*(\mu_{Y_i}(P) - r) + r - w_2^* \lambda_i \gamma_i(P)] \, dt \tag{7.10}$$

$$+ w_1^* \sigma_S dB_2 + w_2^* \sigma_{Y_i}(P) dB_i + w_2^* \gamma_i(P) dz_i \tag{7.11}$$

Since the risk of volatility switch is purely firm specific, the jump risk in Y_i is perfectly diversifiable and does not command a risk premium (Merton (1976)). This implies that the market price of jump risk induced by switches in volatility is zero. In this case, the expected rate of return for the portfolio that has hedged market risk must equal the risk free rate, r. This implies that

$$w_1^*(\mu_S - r) + w_2^*(\mu_{Y_i}(P) - r) + r = r \tag{7.12}$$

and

$$w_1^* \sigma_S + w_2^* \sigma_{Y_i}(P) \frac{\sigma_A}{\sigma_i} = 0 \tag{7.13}$$

where we have used the knowledge that $dB_i = \frac{\sigma_{P,i} dB_1 + \sigma_A dB_2}{\sigma_i}$ in equation (7.11). Equation (7.12) together with equation (7.13) implies that

$$\sigma_S \mu_{Y_i}(P) = -\frac{r \sigma_A \sigma_{Y_i}(P)}{\sigma_i} + \frac{\sigma_A \mu_S \sigma_{Y_i}(P)}{\sigma_i} + r \sigma_S \tag{7.14}$$

[33] As in Merton (1976), the jump risk in the hedge portfolio is unhedgeable

Substituting equations (7.4) and (7.5) into (7.14), and simplifying gives the fundamental valuation equation

$$\frac{1}{2}P^2\sigma_i^2Y_i''(P) + (\mu - \sigma_A\lambda)\,PY_i'(P) + \lambda_{i'}\,(Y_{i'}(P) - Y_i(P)) = rY_i(P) \tag{7.15}$$

where we have substituted in the market Sharpe ratio $\lambda = \frac{\mu_S - r}{\sigma_S}$. The differential equation (7.15) serves as the backbone for the derivation of all valuations in Section II of the paper.

7.1.2 Method II: Constantinides (1978) 2 Step Approach

An alternative and more direct approach to deriving the valuation equation (7.15) for any asset $Y_i(P)$ is based on Constantinides (1978). The first step in the approach calls for the replacement of the drift of $\frac{dS}{S}$, μ, by $\mu^* = \mu - \lambda \text{Corr}\left(\frac{dP}{P}, \frac{dS}{S}\right)\sigma_i = \mu - \lambda\rho_i\sigma_i = \mu - \lambda\sigma_A$. The second step evaluates the stream of cash flows of $Y_i(P)$ as if the market price of risk were zero, i.e., discount expected cash flows at the riskfree rate. To this end, the Bellman equation for asset Y_i is given by[34]

$$Y_i(P) = \frac{1}{1 + r\triangle t}E\left[Y(P + \triangle P, \sigma_i + \triangle\sigma)\right] \tag{7.16}$$

The expectation on the right hand side evaluates to

$$E\left[Y_i(P + \triangle P)\right] = \left\{\lambda_{i'}\triangle tE\left[Y_{i'}(P + \triangle P)\right] + (1 - \lambda_{i'}\triangle t)E\left[Y_i(P + \triangle P)\right]\right\} \tag{7.17}$$

The first term is the asset's probability weighted expected value if there is a switch in volatility regime and the second term corresponds to the asset's probability weighted expected value under the current volatility regime. One can arrive at equation (7.15) after substituting equation (7.17) into (7.16), multiplying both sides by $1 + r\triangle t$, letting $\triangle t$ go to zero, applying Ito's Lemma, and substituting μ by μ^*.

7.1.3 Method III: The Risk Neutral Approach

The third approach is related to the valuation approach by Constantinides (1978) and consistent with both valuation approaches described above. The traded assets M and S allow us to define a new measure under which the process $dB_i^* = \rho_i\lambda dt + dB_i$ is a brownian motion under the $\mathbb{Q}$ measure. For this risk neutral measure, the price dynamics satisfy $dP = \mu^* P dt + \sigma_i P dB_i^*$, where $\mu^* = \mu - \sigma_i\rho_i\lambda = \mu - \sigma_A\lambda$. Then the valuation of any asset merely requires that the stream of the asset's cash flows be discounted under the

[34] Y_i does not pay cash flows in the interim

$\mathbb{Q}$ measure. This approach is used to determine the value of mature firms (2.5) and the asset-in-place of young firms in Section II of the paper.

7.2 Value of a Mature Firm

To value a mature firm, it requires only that we discount the operating cash flows under the risk neutral measure $\mathbb{Q}$, which implies

$$V_M(P) = \mathbb{E}^{\mathbb{Q}}\left[\int_0^\infty e^{-rs}\xi_M(P_{t+s} - c)ds\right] \tag{7.18}$$

where $\mathbb{E}^{\mathbb{Q}}$ is the expectation under the $\mathbb{Q}$ measure. Evaluating the integral results in the value function (2.5). A similar value function also constitutes the reward when a young firm invests and increases production scale, with the exception that ξ_M is substituted by ξ.

7.3 Value of a Growth Firm

Let $G_i(P)$ denote the value of a growth firm in the region where $P \in (P_1, P_2)$ and when the volatility regime i is in effect. In this region of P values, the growth option is in-the-money only if the low volatility regime is in effect. The option value function is given by

$$G_L(P) = \xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) \tag{7.19}$$

Using valuation equation (7.15), the value of a growth option in the high volatility regime obeys the following equation

$$\frac{1}{2}P^2\sigma_i^2G''_H(P) + (\mu - \sigma_A\lambda)PG'_H(P) + \lambda_L\left[\xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I - G_H(P)\right] = rG_H(P) \tag{7.20}$$

Equation (7.20) is the standard valuation equation commonly seen in the growth option literature with the exception of the last term. The last term corresponds to the probability weighed change in the value of the option due to a change in regime from high to low and immediate activation. The payoff from activation, net of investment cost I and opportunity cost $G_H(P)$, is $\left[\xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I - G_H(P)\right]$.

Now we address the region of low values of P, i.e. $P \in (0, P_1)$. In this region the option is out-of-the money in both volatility regimes and kept alive. Let $F_i(P)$ denote the value of a growth option in the region where $P \in (0, P_1)$ and the i regime is in effect. Following the same steps as above that led to the differential equation (7.20), we arrive at the following pair of differential equations

$$\frac{1}{2}P^2\sigma_H^2 F_H''(P) + (\mu - \sigma_A\lambda)\, PF_H'(P) + \lambda_L\,(F_L(P) - F_H(P)) = rF_H(P) \tag{7.21}$$

$$\frac{1}{2}P^2\sigma_L^2 F_L''(P) + (\mu - \sigma_A\lambda)\, PF_L'(P) + \lambda_H\,(F_H(P) - F_L(P)) = rF_L(P) \tag{7.22}$$

As before, the differential equations are similar to those in standard diffusion models with the exception that they include an additional component that captures the possibility of a change in the volatility regime of the decision variable P. This term equals $\lambda_L\,(F_L(P) - F_H(P))$ if the high volatility state is in effect, and $\lambda_H\,(F_H(P) - F_L(P))$ otherwise.

With this last pair of valuation equations, we have all the tools required for all the valuations in the paper.

7.4 Proof of Proposition 1

The solution method follows Guo (2001) and Guo and Zhang (2004). Consider first the system of differential equations composed of equations (7.21) and (7.22). It is easy to show that the system has the following characteristic function

$$q_H(\beta) \times q_L(\beta) - \lambda_L\lambda_H = 0 \tag{7.23}$$

where $q_H(\beta)$ and $q_L(\beta)$ are given by the following quadratic equations

$$q_H(\beta) = -\beta\mu^* - \frac{1}{2}(\beta - 1)\beta\sigma_H^2 + \lambda_L + r \tag{7.24}$$

$$q_L(\beta) = -\beta\mu^* - \frac{1}{2}(\beta - 1)\beta\sigma_L^2 + \lambda_H + r \tag{7.25}$$

The characteristic function has four distinct roots $\beta_{2,1} > \beta_{2,2} > 0 > \beta_{2,3} > \beta_{2,4}$ (Guo (2001)) such that the general form of the solutions to (7.21) and (7.22) are given by

$$F_H(P) = \sum_{i=1}^{4} B_{H,i}(P)P^{\beta_{2,i}} \quad \text{and} \quad F_L(P) = \sum_{i=1}^{4} B_{L,i}(P)P^{\beta_{2,i}}$$

The valuation problem is greatly simplified if we reduce the number of terms in the general solutions. Given the signs of $\beta_{2,1}$, $\beta_{2,2}$, $\beta_{2,3}$, and $\beta_{2,4}$, and the property that the option value must approach zero if P approaches zero, the constants multiplying the negative powers of

P must be zero. Therefore, the solutions take the simplified form given by

$$F_H(P) = B_{H,1}P^{\beta_{2,1}} + B_{H,2}P^{\beta_{2,2}} \tag{7.26}$$

$$F_L(P) = B_{L,1}P^{\beta_{2,1}} + B_{L,1}P^{\beta_{2,2}} \tag{7.27}$$

Substituting equations (7.26) and (7.27) into the differential equations (7.21) and (7.22) results in the following equations

$$0 = P^{\beta_{2,1}}\left(\lambda_L B_{L,1} - q_H(\beta_{2,1})B_{H,1}\right) + P^{\beta_{2,2}}\left(\lambda_L B_{L,2} - q_H(\beta_{2,2})B_{H,2}\right)$$
$$0 = P^{\beta_{2,1}}\left(\lambda_H B_{H,1} - q_L(\beta_{2,1})B_{L,1}\right) + P^{\beta_{2,2}}\left(\lambda_H B_{H,2} - q_L(\beta_{2,2})B_{L,2}\right)$$

The next step involves solving for the unknown constants. After solving for $B_{H,1}$ and $B_{H,2}$ and substituting into $F_H(P)$ gives equation (3.1) in the proposition.

Now turning out attention to equation (7.20), the solution has the following form

$$G_H(P) = C_{H,1}P^{\beta_{1,1}} + C_{H,2}P^{\beta_{1,2}} + \phi(P) \tag{7.28}$$

where $C_{H,1}$ and $C_{H,2}$ are constants of integration, $\phi(P)$ is a particular solution and $\beta_{1,1}$ and $\beta_{1,2}$ are the two real roots of the following quadratic equation

$$q_H(\beta) = -\beta\mu^* - \frac{1}{2}(\beta - 1)\beta\sigma_H^2 + \lambda_L + r$$

In particular, if $q_H(0) = r + \lambda_L \neq 0$ one can choose

$$\phi(P) = \frac{\lambda_L}{\lambda_L + r}\left(\xi\left(\frac{P}{r - \mu^*} - \frac{c}{r}\right) - I\right) \tag{7.29}$$

and the complete solution is given by (3.3) in the proposition.

It remains to determine the constants of integration $B_{L,1}$, $B_{L,2}$, $C_{H,1}$, $C_{H,2}$, and the exercise policies P_1 and P_2. To complete the solution, we make use of the following boundary conditions

$$V(P_1) - I = F_L(P_1) \tag{7.30}$$

$$V'(P)|_{P=P_1} = F_L'(P)|_{P=P_1} \tag{7.31}$$

$$V(P_2) - I = G_H(P_2) \tag{7.32}$$

$$V'(P)|_{P=P_2} = G_H'(P)|_{P=P_2} \tag{7.33}$$

$$G_H(P_1) = F_H(P_1) \tag{7.34}$$

$$G_H'(P)|_{P=P_1} = F_H'(P)|_{P=P_1} \tag{7.35}$$

The value matching conditions (7.30) and (7.32) impose an equality between the option's intrinsic value and the option's value at the optimal exercise values of P in the two volatility regimes. These conditions merely mean that upon activation the owner foregoes the value of the option in exchange for the net benefits of exercising the option, $V(P) - I$. The smooth pasting conditions (7.31) and (7.33) ensure the optimality of the exercise policies P_1 and P_2 (Dixit and Pindyck (1994)). Lastly, the conditions (7.34) and (7.35) ensure that the value of the option is continuous and smooth around P_1.

We turn to each of the conditions (7.30)–(7.35) above. At $P = P_1$ the value matching condition is given by $V(P_1) - I = F_L(P_1)$ and the smooth pasting condition is given by $V'(P)|_{P=P_1} = F_L'(P)|_{P=P_1}$. More explicitly, the two conditions (7.30) and (7.31) can be written as

$$\xi\left(\frac{P_1}{r-\mu^*} - \frac{c}{r}\right) - I = B_{L,1}P_1^{\beta_{2,1}} + B_{L,2}P_1^{\beta_{2,2}} \tag{7.36}$$

$$\frac{\xi P_1}{r-\mu^*} = \beta_{2,1}B_{L,1}P_1^{\beta_{2,1}} + \beta_{2,2}B_{L,2}P_1^{\beta_{2,2}} \tag{7.37}$$

At $P = P_2$ the value matching condition is given by $V(P_2) - I = G_H(P_2)$ and the smooth pasting condition is given by $V'(P)|_{P=P_2} = G_H'(P)|_{P=P_2}$. More explicitly, the two conditions (7.32) and (7.33) can be written as

$$\xi\left(\frac{P_2}{r-\mu^*} - \frac{c}{r}\right) - I = C_{H,1}P_2^{\beta_{1,1}} + C_{H,2}P_2^{\beta_{1,2}} + \frac{\lambda_L}{\lambda_L + r}\left(\xi\left(\frac{P_2}{r-\mu^*} - \frac{c}{r}\right) - I\right) \tag{7.38}$$

$$\frac{\xi P_2}{r-\mu^*} = \beta_{1,1}C_{H,1}P_2^{\beta_{1,1}} + \beta_{1,2}C_{H,2}P_2^{\beta_{1,2}} + \frac{\lambda_L}{\lambda_L + r}\left(\frac{\xi P_2}{r-\mu^*}\right) \tag{7.39}$$

We can use the first four conditions (7.36)–(7.39) to solve for $B_{L,1}$, $B_{L,2}$, $C_{H,1}$ and $C_{H,2}$. The expressions in closed form are

$$B_{L,1} = \frac{\beta_{2,2}}{\beta_{2,2} - \beta_{2,1}} P_1^{-\beta_{2,1}} \left(\xi \left(\frac{P_1}{r - \mu^*} \left(1 - \frac{1}{\beta_{2,2}} \right) - \frac{c}{r} \right) - I \right) \tag{7.40}$$

$$B_{L,2} = \frac{\beta_{2,1}}{\beta_{2,2} - \beta_{2,2}} P_1^{-\beta_{2,2}} \left(\xi \left(\frac{P_1}{r - \mu^*} \left(1 - \frac{1}{\beta_{2,1}} \right) - \frac{c}{r} \right) - I \right) \tag{7.41}$$

$$C_{H,1} = \frac{\beta_{1,2}}{(\beta_{1,2} - \beta_{1,1})} \frac{r}{(\lambda_L + r)} P_2^{-\beta_{1,1}} \left(\xi \left(\frac{P_2}{r - \mu^*} \left(1 - \frac{1}{\beta_{1,2}} \right) - \frac{c}{r} \right) - I \right) \tag{7.42}$$

$$C_{H,2} = \frac{\beta_{1,1}}{(\beta_{1,1} - \beta_{1,2})} \frac{r}{(\lambda_L + r)} P_2^{-\beta_{1,2}} \left(\xi \left(\frac{P_2}{r - \mu^*} \left(1 - \frac{1}{\beta_{1,1}} \right) - \frac{c}{r} \right) - I \right) \tag{7.43}$$

Continuity and smoothness of the value functions at $P = P_1$ requires that $F_H(P_1) = G_H(P_1)$ and $F'_H(P)|_{P=P_1} = G'_H(P)|_{P=P_1}$. These conditions are the equation (3.5) and (3.6) in the proposition.

Conditions (3.5) and (3.6) and the constants of integration $B_{L,1}$, $B_{L,2}$, $C_{H,1}$ and $C_{H,2}$ are expressed in terms of the exercise boundaries P_1 and P_2. Therefore, the equations compose a system of two equations and two unknowns variables, P_1 and P_2, which are solved numerically for each set of parameters of the model. This completes the proof of Proposition 1 of the paper.

To implement the solution of the model, it is required to only determine the values of P_1 and P_2 numerically for any reasonable set of parameter values. ∎

7.5 Proof of Proposition 2

Direct application of Ito's Lemma on $F_H(P)$ and $F_L(P)$ results in equations (3.7) and (3.8) where

$$a_H(P) = \left(\frac{1}{2} \sigma_H^2 P^2 F''_H(P) + \mu P F'_H(P) \right) \Big/ F_H(P) \tag{7.44}$$

$$b_H(P) = \sigma_H P F'_H(P) / F_H(P) \tag{7.45}$$

$$a_L(P) = \left(\frac{1}{2} \sigma_L^2 P^2 F''_L(P) + \alpha P F'_L(P) \right) \Big/ F_L(P) \tag{7.46}$$

$$b_L(P) = \sigma_L P F_L(P) / F_L(P) \tag{7.47}$$

$$\nu_H(P) = (F_L(P) - F_H(P)) / F_H(P) \tag{7.48}$$

$$\nu_L(P) = (F_H(P) - F_L(P)) / F_L(P) \tag{7.49}$$

Substituting in the value functions (3.1) and (3.2) and their derivatives into expressions (7.44) to (7.49) results in

$a_H(P) =$

$$\frac{\beta_{2,1} B_{L,1} P^{\beta_{2,1}} q_L(\beta_{2,1}) \left(\frac{1}{2}\left(\beta_{2,1}-1\right)\sigma_H^2+\mu\right) + \beta_{2,2} B_{L,2} P^{\beta_{2,2}} q_L(\beta_{2,2}) \left(\frac{1}{2}\left(\beta_{2,2}-1\right)\sigma_H^2+\mu\right)}{B_{L,1} P^{\beta_{2,1}} q_L(\beta_{2,1}) + B_{L,2} P^{\beta_{2,2}} q_L(\beta_{2,2})} \tag{7.50}$$

$$b_H(P) = \frac{\sigma_H \left(\beta_{2,1} B_{L,1} P^{\beta_{2,1}} q_L(\beta_{2,1}) + \beta_{2,2} B_{L,2} P^{\beta_{2,2}} q_L(\beta_{2,2})\right)}{B_{L,1} P^{\beta_{2,1}} q_L(\beta_{2,1}) + B_{L,2} P^{\beta_{2,2}} q_L(\beta_{2,2})} \tag{7.51}$$

$$\nu_H(P) = \frac{B_{L,1} P^{\beta_{2,1}} \left(\lambda_H - q_L(\beta_{2,1})\right) + B_{L,2} P^{\beta_{2,2}} \left(\lambda_H - q_L(\beta_{2,2})\right)}{B_{L,1} P^{\beta_{2,1}} q_L(\beta_{2,1}) + B_{L,2} P^{\beta_{2,2}} q_L(\beta_{2,2})} \tag{7.52}$$

$$a_L(P) = \frac{\beta_{2,1} B_{L,1} P^{\beta_{2,1}} \left(\frac{1}{2}\left(\beta_{2,1}-1\right)\sigma_L^2+\mu\right) + \beta_{2,2} B_{L,2} P^{\beta_{2,2}} \left(\frac{1}{2}\left(\beta_{2,2}-1\right)\sigma_L^2+\mu\right)}{B_{L,1} P^{\beta_{2,1}} + B_{L,2} P^{\beta_{2,2}}} \tag{7.53}$$

$$b_L(P) = \frac{\sigma_L \left(\beta_{2,1} B_{L,1} P^{\beta_{2,1}} + \beta_{2,2} B_{L,2} P^{\beta_{2,2}}\right)}{B_{L,1} P^{\beta_{2,1}} + B_{L,2} P^{\beta_{2,2}}} \tag{7.54}$$

$$\nu_L(P) = \frac{B_{L,1} P^{\beta_{2,1}} \left(q_L(\beta_{2,1}) - \lambda_H\right) + B_{L,2} P^{\beta_{2,2}} \left(q_L(\beta_{2,2}) - \lambda_H\right)}{\lambda_H \left(B_{L,1} P^{\beta_{2,1}} + B_{L,2} P^{\beta_{2,2}}\right)} \tag{7.55}$$

This completes the proof. ∎

7.6 Proof of Proposition 4

The (conditional) CAPM beta for a growth option can be computed in two different but equivalent ways. In the first approach, we find the beta of a growth option by forming a replicating portfolio with state dependent and time varying weights in the traded assets S and M that exactly reproduces the systematic risk of the option. The proportion of portfolio value held in S determines the beta of the option. To this end, take equation (7.2) and substitute in $dB_i = \frac{\sigma_{P,i} dB_1 + \sigma_A dB_2}{\sigma_i}$. By inspection we can see that the diffusion term of the common risk factor can be eliminated by holding $\frac{F_i'(P)\sigma_A P}{\sigma_S S}$ units of the stock in the hedge portfolio. Multiplying the number of stocks by S and dividing by the portfolio value $F_i(P)$, we get the weight of the hedge portfolio invested in the tradeable asset. Since the tradeable asset has a beta of one, the beta of the growth option is given by $\beta_{\text{CAPM}} = \frac{F_i(P)'\sigma_A P}{\sigma_S S}\frac{S}{F_i(P)} = \frac{F_i'(P)\sigma_A P}{\sigma_S F_i(P)}$. Substituting in $F_i(P)$ from equations (3.1) and (3.2) and their derivative gives (3.9) and (3.10).

Alternatively, one can find the CAPM beta by computing the option's return elasticity with respect to the returns of the tradeable asset. The elasticity is $\frac{\text{Cov}[\text{dF}_i/F_i, \text{dS}/S]}{\text{Var}[\text{dS}/S]} = \frac{\sigma_{F,i}\sigma_A}{\sigma_i \sigma_S}$. Substituting in (7.5), $F_i(P)$ from equations (3.1) and (3.2), and their derivative gives (3.9) and (3.10). This completes the proof for expressions (3.9) and (3.10) of the proposition.[35]

[35] There is yet a third approach as shown in Sagi and Seashole (2007). Sagi and Seashole show that

The proof of expressions (3.11) and (3.12) follow directly from Proposition 2 and taking expectations. Expressions (3.11) and (3.12) are consistent with the expected returns given by a conditional version of the CAPM. To show this, substitute in (7.50), (7.53), (7.52) and (7.55) into (3.11) and (3.12) and simplify. The resulting expressions equate to the expressions from substituting (3.9), (3.10) into $r + \beta_i(P)\sigma_S\lambda$ and simplifying.

For the proof of the second claim of the corollary, we make use of the differential equations (7.21) and (7.22) which $F_H(P)$ and $F_L(P)$ must satisfy. Substituting in 0 everywhere in place of σ_A, taking equations (7.21) and (7.22) and dividing them by $F_H(P)$ and $F_L(P)$ respectively gives

$$r = a_H(P) + \lambda_L \nu_H(P) \tag{7.56}$$

$$r = a_L(P) + \lambda_H \nu_L(P) \tag{7.57}$$

Setting the right hand side of expression (7.56) equal to the right hand side of expression (7.57) gives

$$a_H(P) + \lambda_L \nu_H(P) = a_L(P) + \lambda_H \nu_L(P) \tag{7.58}$$

Since $\nu_H(P) \leq 0 \leq \nu_L(P)$, it follows that $a_H(P) \geq r \geq a_L(P)$. This proves the second claim of the proposition.

If $\sigma_H - \sigma_L \to 0$ then the solution approaches the solution to a single volatility regime model. Therefore, both $\nu_L(P)$ and $\nu_H(P) \to 0$, and $a_H(P) - a_L(P) \to 0$. This proves the third claim of the proposition.

This completes the proof. ∎

the expected excess return is given by $(\mu - \mu^*) \frac{\partial \log F_i(P)}{\partial \log P} = (\mu - \mu^*) \frac{F_i'(P)}{F_i(P)}$ where $(\mu - \mu^*)$ is the difference between the unadjusted and risk adjusted mean returns of P. In our set up, $(\mu - \mu^*) = \rho_i \sigma_i \lambda = \sigma_A \lambda$. Substituting in $F_i(P)$ from equations (3.1) and (3.2), adding r, evaluating the derivatives and simplifying results in the instantaneous expected excess returns stated in the proposition.

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Table 1: Model Inputs. This table reports the parameter values used to solve and simulate the real option/stochastic idiosyncratic volatility model developed in Section 3. Base case parameter values are distinguished with an asterisk $*$ if more than one value is reported for a variable.

Model Parameters		
Price Dynamics Parameters	Variable Description	Values
$\sigma_{P,H}$	Output price idiosyncratic volatility in the high regime	0.3,0.4,0.5*
$\sigma_{P,L}$	Output price idiosyncratic volatility in the low regime	0.1*,.02,0.3
λ_H	Transition parameter from low to high regime	0.1
λ_L	Transition parameter from high to low regime	0.1
μ	Drift of output price process	0.04
σ_A	Market volatility of output price process	0.20
Market Parameters	Variable Description	Values
r	Riskless rate	0.05
μ_S	Drift of tradeable asset (Market)	0.1
σ_S	Diffusion of tradeable asset (Market)	0.25
Firm's Profit Function Parameters	Variable Description	Values
c	Variable cost per unit of output	10
ξ	Difference in production scale between mature and young Firms	1.1
I	Investment cost of young firm to become mature	$\frac{1.5\times(\xi-1)\times c}{r-\mu^*}$
Simulation Parameters	Variable Description	Values
N	Number of samples	100
n	Number of firms in each sample	500
T	Number of years	50
nt	Number of days in each year	240

Table 2: Simulation Results. The table reports coefficient estimates along with their t-statistics for the regression model $r_t^e = \gamma_{0,t}\iota + \gamma_{1,t}\Delta IVol_t + \eta_t$ in the first column of panels (a), (b) and (c), and estimates for the regression model $r_t^e = \gamma_{0,t}\iota + \gamma_{1,t}IVol_{t-1} + \eta_t$ in the second column of panels (a), (b) and (c) using data simulated from the real option/stochastic idiosyncratic volatility model developed in Section 3 of the paper. Panels (a), (b) and (c) report separate model estimates corresponding to the simulated samples where $\sigma_{P,H} = 0.5$,$\sigma_{P,L} = 0.1$, $\sigma_{P,H} = 0.4$,$\sigma_{P,L} = 0.2$ and $\sigma_{P,H} = \sigma_{P,L} = 0.3$ respectively. T-statistics are reported in square brackets.

	(a) $\sigma_{P,H} = 0.5$,$\sigma_{P,L} = 0.1$		(b) $\sigma_{P,H} = 0.4$,$\sigma_{P,L} = 0.2$		(c) $\sigma_{P,H} = 0.3$,$\sigma_{P,L} = 0.3$	
Intercept	-0.0128^{***}	0.0015^{***}	-0.0113^{***}	0.0022^{***}	-0.0107^{***}	-0.0108^{***}
	[-111.6866]	[47.0927]	[-108.6885]	[28.9533]	[-113.4347]	[-50.7982]
$\Delta IVol_t$	0.0830^{***}		0.0451^{***}		-0.0003	
	[76.6009]		[45.1841]		[-0.2890]	
$IVol_{t-1}$		-0.1171^{***}		-0.1068^{***}		0.0002
		[-147.4614]		[-109.9917]		[0.1707]

Table 3: Simulation Results. The table reports the mean portfolio risk-adjusted returns along with their t-statistics using the return data simulated based on the real option/stochastic idiosyncratic volatility model developed in Section 3 of the paper. Growth options are ranked and sorted into decile portfolios based on the level of realized idiosyncratic return volatility $IVol$ over the past month. Then, value-weighted one-month holding period mean portfolio returns are computed using the monthly excess returns as defined in equation (4.3) of the paper. The portfolios are rebalanced at the end of each month. $IVol$ ranks are listed across columns with the difference in risk-adjusted returns between the highest and lowest $IVol$ decile portfolios reported in the last column. The table reports separate portfolio risk-adjusted returns corresponding to the simulated samples where $\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$, $\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$ and $\sigma_{P,H} = \sigma_{P,L} = 0.3$. T-statistics are reported in square brackets.

	IVol Decile Portfolios										
	1	2	3	4	5	6	7	8	9	10	10-1
$\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$	−0.0005***	−0.0005***	−0.0007***	−0.0007***	−0.0028***	−0.0118***	−0.0126***	−0.0120***	−0.0123***	−0.0112***	−0.0107***
	[-6.1821]	[-7.8346]	[-9.3421]	[-10.2025]	[-9.9710]	[-27.0494]	[-25.9232]	[-25.5117]	[-28.5445]	[-27.2305]	[-26.3279]
$\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$	−0.0024***	−0.0025***	−0.0029***	−0.0031***	−0.0041***	−0.0085***	−0.0090***	−0.0087***	−0.0089***	−0.0081***	−0.0057***
	[-15.3298]	[-16.1259]	[-22.3011]	[-20.4976]	[-16.5190]	[-20.3931]	[-22.5918]	[-23.5657]	[-26.2542]	[-24.5645]	[-18.8308]
$\sigma_{P,H} = 0.3, \sigma_{P,L} = 0.3$	−0.0057***	−0.0058***	−0.0053***	−0.0057***	−0.0053***	−0.0054***	−0.0052***	−0.0058***	−0.0053***	−0.0053***	0.0004
	[-20.4492]	[-19.7872]	[-18.3362]	[-21.3927]	[-20.0214]	[-21.1910]	[-24.1009]	[-22.2099]	[-23.3239]	[-20.6195]	[1.5955]

Table 4: This table reports summary statistics for excess stock returns, return volatilities $IVol$, month-to-month changes in $IVol$, or $\Delta IVol$, and proxies for firms' real option intensity. Stock return data are from CRSP. The sample period is from January, 1971 to December, 2010 for all market based variables. All our accounting variables are from annual COMPUSTAT files. Utilities (SIC codes between 4900 and 4999) and financials (SIC codes between 6000 and 6999) are excluded. A stock's excess returns is the difference between its monthly stock return and the risk-free rate. Volatility and its change refer to monthly volatility of log daily risk-adjusted returns where risk-adjustment is based on the Fama and French 3-factor model. Market equity and total assets are in millions of dollars. Age is in months since first appearance in monthly CRSP files. Future investment, profit and sales growth are the sum of the growth rates from year $t+2$ to $t+5$ of firms' investments in property plant and equipment, of firm's operating profits, and of firms' sales, respectively. *vega* is computed for each firm according to (5.3) in the paper.

market variables	Mean	StdDev	P5	Median	P95	N
excess return	0.009976	0.180828	-0.22309	-0.0041	0.272627	1041266
$IVol$	0.029476	0.024979	0.0079	0.022782	0.072884	1038601
$\Delta IVol$	-2.3E-05	0.021096	-0.02552	-0.00011	0.026111	1035935

Real Option variables	Mean	StdDev	P5	Median	P95	N
log(market equity)	4.694734	2.106019	1.5389081	4.521163	8.389149	1040478
log(total assets)	4.804593	2.009753	1.789757	4.62188	8.352702	1041266
log(age)	3.953142	1.540425	0	4.290459	5.746203	1041266
investment growth	0.996235	18.22423	-0.64226	0.225036	2.237907	871778
profit growth	-0.55037	80.99137	-6.71653	0.353252	4.689659	871779
sales growth	1.579677	79.57993	-0.46927	0.29381	1.83045	868519
vega	2.84E-69	1.49E-67	9.63E-110	9.89E-81	1.88E-70	1041104

Table 5: This table reports coefficient estimates along with their t-statistics of Fama and MacBeth (1973) cross sectional regressions of firm level differences in the 5-month average of the risk-adjusted returns between post and pre-$IVol$ switch monthly events on the real option proxies. The regression equation is $r_t^{Diff} = \gamma_0 \iota + \gamma_1 RO_{t-1} + \eta_t$. The construction of the real option proxies are described in the paper. The estimates for separate real option proxies used in the regression are reported across columns. Separate regression estimates are reported for up and down switch samples. The reported estimates are time series averages of the monthly coefficient estimates. Newey and West (1987) robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R squared.

		RO Proxy									
switch	Coeff.	size (total asset)	size (mkt equity)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
down	Intercept	-0.0201***	-0.0175***	-0.0111**	-0.0062***	-0.0068***	-0.0079***	-0.0079***	-0.0080***	-0.0076***	-0.0074***
		[-6.3508]	[-5.4812]	[-2.0936]	[-6.8905]	[-6.6695]	[-7.8954]	[-7.7187]	[-8.7388]	[-8.6914]	[-8.1784]
	RO	0.0024***	0.0018***	0.0005	0.0063***	-0.0016	0.0021	-0.0007	0.0025	-0.0116**	-0.0071**
		[4.4752]	[3.1156]	[0.3552]	[-3.1565]	[-1.0178]	[1.2240]	[-0.3579]	[1.5724]	[-2.5072]	[-2.1897]
	RSQR	0.0326***	0.0329***	0.0265***	0.0268***	0.0266***	0.0252***	0.0288***	0.0244***	0.0482***	0.0376***
up	intercept	0.0158***	0.0182***	0.0150***	0.0052***	0.0040***	0.0036***	0.0044***	0.0030**	0.0048***	0.0047***
		[5.3102]	[6.9155]	[3.2173]	[5.2992]	[3.0968]	[3.0411]	[3.9123]	[2.4548]	[4.9601]	[4.7785]
	RO	-0.0022***	-0.0027***	-0.0024**	0.0023	0.0039***	0.0059***	0.0053***	0.0076***	0.0082**	0.0095***
		[-4.4951]	[-6.2128]	[-2.0830]	[1.2730]	[2.7997]	[3.8766]	[3.1845]	[5.2559]	[2.2245]	[3.4609]
	RSQR	0.0283***	0.0279***	0.0285***	0.0252***	0.0251***	0.0269***	0.0255***	0.0261***	0.0408***	0.0349***

		RO Proxy					*RO* Industries			
switch	Coeff.	small high profit	small high sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Industry
down	Intercept	-0.0062***	-0.0068***	-0.0069***	-0.0069***	-0.0077***	-0.0083***	-0.0085***	-0.0079***	-0.0082***
		[-7.1963]	[-7.6766]	[-7.6615]	[-8.1033]	[-8.3749]	[-9.2471]	[-8.5055]	[-8.8771]	[-8.5319]
	RO	-0.0088***	-0.0045	-0.0108***	-0.0036	0.001	-0.0015	0.0013	-0.001	0.0002
		[-3.1267]	[-1.4208]	[-3.5391]	[-0.8044]	[0.2391]	[-0.3588]	[0.5947]	[-0.3122]	[0.1036]
	RSQR	0.0394***	0.0388***	0.0384***	0.0439***	0.0357***	0.0333***	0.0273***	0.0288***	0.0296***
up	Intercept	0.0045***	0.0043***	0.0051***	0.0036***	0.0054***	0.0056***	0.0057***	0.0056***	0.0041***
		[3.9771]	[4.0485]	[5.1847]	[3.2879]	[5.2951]	[5.1394]	[5.5452]	[5.5774]	[3.8737]
	RO	0.0088***	0.0107***	0.0077***	0.0164***	0.0079***	0.002	0.0025	0.0028	0.0043**
		[3.0261]	[3.4846]	[2.6551]	[4.7291]	[2.7409]	[0.5846]	[1.3052]	[0.8773]	[2.1847]
	RSQR	0.0365***	0.0368***	0.0324***	0.0397***	0.0325***	0.0299***	0.0237***	0.0265***	0.0290***

Table 6: This table reports coefficient estimates along with their t-statistics of Fama and MacBeth (1973) cross sectional regressions of firm level differences in the 5-month average of the risk-adjusted returns between post and pre-$IVol$ switch monthly events on the difference between the high and low $IVol$ values, and the real option proxies. The regression equation is $r_t^{Diff} = \gamma_0 \iota + \gamma_1 \overline{\Delta IVol} + \gamma_2 \overline{\Delta IVol} \times RO_{t-1} + \eta_t$. The construction of the real option proxies are described in the paper. The results using different real option proxies in the regression are reported across columns. We estimate separate monthly regressions for the up and down switch samples. The reported estimates are the time series averages of the monthly coefficient estimates. Newey and West (1987) robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R squared.

		RO Proxy									
switch	Coeff.	size (total asset)	size (mkt equity)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
down	Intercept	-0.0018	0.0011	0.0015	0.0002	-0.0003	0	0.0023	0.0004	0.0014	0.0011
		[-0.6873]	[0.4085]	[0.7341]	[0.1085]	[-0.1137]	[0.0180]	[1.0268]	[0.1644]	[0.6783]	[0.5057]
	$\overline{\Delta IVol}$	-1.0054***	-0.8526***	-0.2454	–0.6433***	-0.5962**	-0.7080***	-1.0003***	-0.7249***	-0.8262***	-0.8361***
		[-3.8117]	[-3.4519]	[-0.4000]	[-2.8142]	[-2.5609]	[-3.1400]	[-4.6543]	[-3.2998]	[-3.9831]	[-3.8818]
	$\overline{\Delta IVol} \times RO$	0.1072*	-0.0046	-0.1626	-0.1921	0.0088	0.1432	0.2439	0.1334	-0.2349	0.0228
		[1.7705]	[-0.0742]	[-1.0051]	[-1.2213]	[0.0611]	[0.9226]	[1.4789]	[0.8459]	[-1.0029]	[0.1089]
	RSQR	0.0882***	0.0912***	0.0877***	0.0871***	0.0967***	0.0964***	0.0898***	0.0958***	0.0959***	0.0962***
		[19.6250]	[16.1655]	[17.3481]	[17.1345]	[18.9544]	[19.4351]	[17.9502]	[17.5652]	[17.1672]	[19.9459]
up	Intercept	-0.0003	0.0037	-0.0035	-0.0043*	-0.0066***	-0.0056**	-0.003	-0.0059**	-0.0038*	-0.0024
		[-0.1080]	[1.4158]	[-1.6219]	[-1.8809]	[-2.8511]	[-2.4964]	[-1.2945]	[-2.4858]	[-1.7371]	[-1.1340]
	$\overline{\Delta IVol}$	0.9565***	0.9889***	1.3251**	0.8610***	0.9418***	0.8203***	0.6773***	0.7773***	0.7808***	0.6350***
		[3.9990]	[4.4507]	[2.1528]	[4.0885]	[4.4276]	[4.2999]	[3.1748]	[3.9858]	[3.9437]	[3.2559]
	$\overline{\Delta IVol} \times RO$	-0.1229**	-0.2472***	-0.2528	-0.0527	0.2297*	0.4044***	0.1471	0.6317***	0.0379	0.2846
		[-2.2675]	[-3.9057]	[-1.0236]	[-0.3575]	[1.7174]	[2.7888]	[1.2512]	[4.7608]	[0.1882]	[1.5988]
	RSQR	0.0721***	0.0775***	0.0740***	0.0736***	0.0821***	0.0810***	0.0689***	0.0810***	0.0809***	0.0784***

		RO Proxy					*RO* Industry			
switch	Coeff.	small high profit	small high sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Industries
down	Intercept	-0.0007	0.0001	0	-0.0002	0.0015	0.0009	0.0003	0.0006	0.0005
		[-0.2803]	[0.0229]	[0.0216]	[-0.0799]	[0.6820]	[0.4145]	[0.1555]	[0.2746]	[0.2131]
	$\overline{\Delta IVol}$	-0.5243**	-0.6459***	-0.7449***	-0.6130***	-0.8933***	-0.7969***	-0.7817***	-0.7619***	0.8022***
		[-2.2498]	[-2.8497]	[-3.5372]	[-2.8459]	[-4.1053]	[-3.6453]	[-3.6182]	[-3.5135]	[-3.6652]
	$\overline{\Delta IVol} \times RO$	-0.209	-0.0493	-0.1334	-0.0914	0.3063	-0.0335	0.0778	0.0886	0.1131
		[-1.0722]	[-0.2414]	[-0.7354]	[-0.3688]	[1.3479]	[-0.0943]	[0.4351]	[0.3732]	[0.7114]
	RSQR	0.1005***	0.1007***	0.0930***	0.1045***	0.0941***	0.0937***	0.0913***	0.0888***	0.0934***
		[20.2486]	[20.3574]	[18.3941]	[18.4894]	[16.2996]	[17.5190]	[19.7699]	[19.0352]	[17.9798]
up	Intercept	-0.0059**	-0.0054**	-0.0032	-0.0048**	-0.0031	-0.0037*	-0.0042*	-0.0037	-0.0032
		[-2.3840]	[-2.1412]	[-1.4104]	[-2.0238]	[-1.4008]	[-1.6704]	[-1.6530]	[-1.4657]	[-1.4908]
	$\overline{\Delta IVol}$	0.9227***	0.8880***	0.7471***	0.7969***	0.7031***	0.7761***	0.8545***	0.7441***	0.6572***
		[4.2713]	[4.2814]	[3.7131]	[4.1374]	[3.4753]	[4.0162]	[3.7309]	[3.6893]	[3.5573]
	$\overline{\Delta IVol} \times RO$	0.1712	0.3161*	0.0841	0.6159***	0.0601	0.339	0.0486	0.0432	0.1858
		[0.9805]	[1.7296]	[0.5751]	[3.2721]	[0.3100]	[1.0305]	[0.3155]	[0.1791]	[1.1833]
	RSQR	0.0847***	0.0841***	0.0701***	0.0842***	0.0777***	0.0753***	0.0723***	0.0711***	0.0773***

Table 7: This table reports coefficient estimates along with their t-statistics of Fama and MacBeth (1973) cross sectional regressions of firm level excess returns on the estimated loading on the market factor (β^{CAPM}), beginning of year log book-to-market ($Log(BM)$), log market equity ($Log(ME)$), six-month lagged return from months -7 to -2 relative to the month of observation ($Lag(r)$), monthly trading volume normalized by the number of shares outstanding ($trade$), month-to-month change in firm level idiosyncratic volatility ($\Delta Ivol$), real option proxy, and the interaction between the real option proxy and $\Delta Ivol_t$. The construction of the real option proxies are described in the paper. The regression model is $r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \Delta IVol_t + \gamma_2 \Delta IVol_t \times RO_{t-1} + \gamma_3 X_{t-1} + \eta_t$ in the paper. The real option proxies used in the regression are reported across columns. The reported estimates are the time series averages of the monthly coefficient estimates. Newey and West (1987) robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R squared.

	RO Proxy									
Coeff.	size (mkt equity)	size (total assets)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
Intercept	0.0402***	0.0404***	0.0405***	0.0404***	0.0417***	0.0414***	0.0404***	0.0417***	0.0403***	0.0403***
	[6.7509]	[6.8148]	[6.8501]	[6.8507]	[7.0746]	[7.0291]	[6.8406]	[7.0813]	[6.8330]	[6.8257]
$Log(BM)$	0.0057***	0.0059***	0.0057***	0.0057***	0.0052***	0.0052***	0.0057***	0.0052***	0.0057***	0.0058***
	[7.7458]	[8.0586]	[7.7807]	[7.6285]	[6.8186]	[6.8100]	[7.7920]	[6.7996]	[7.7387]	[7.7815]
$Log(ME)$	-0.0034***	-0.0035***	-0.0035***	-0.0035***	-0.0037***	-0.0037***	-0.0035***	-0.0037***	- -0.0035***	-0.0035***
	[-5.5311]	[-5.6697]	[-5.7620]	[-5.7513]	[-6.1040]	[-6.0733]	[-5.7558]	[-6.1205]	[-5.7169]	[-5.7682]
β^{CAPM}	0.0015**	0.0015**	0.0015**	0.0016**	0.0014*	0.0015*	0.0015**	0.0014*	0.0016**	0.0015**
	[2.2340]	[2.1974]	[2.0700]	[2.2294]	[1.8310]	[1.9049]	[2.1691]	[1.8073]	[2.2730]	[2.1722]
$trade$	0.0157***	0.0156***	0.0155***	0.0155***	0.0154***	0.0153***	0.0155***	0.0154***	0.0155***	0.0155***
	[14.4667]	[14.3834]	[14.2095]	[14.3831]	[13.4373]	[13.4022]	[14.2209]	[13.4620]	[14.3394]	[14.2345]
$Lag(r)$	-0.0031	-0.0031	-0.0031	-0.0031	-0.0031	-0.003	-0.0031	-0.0032	-0.0032	-0.003
	[-1.1424]	[-1.1612]	[-1.1747]	[-1.1768]	[-1.0903]	[-1.0371]	[-1.1534]	[-1.1049]	[-1.1820]	[-1.1219]
$\Delta IVol$	2.3623***	2.2474***	1.1341***	1.0903***	1.2728***	1.1797***	1.1865***	1.1680***	1.1066***	1.2181***
	[17.5356]	[15.8777]	[6.9660]	[9.8669]	[12.1527]	[11.3973]	[10.1845]	[10.9610]	[10.5451]	[12.3035]
$RO \times \Delta Ivol$	-0.3125***	-0.2647***	-0.0094	0.3748***	0.0217	0.2497***	0.0321	0.3273***	0.5297***	0.3139***
	[-10.790]	[-9.8134]	[-0.1847]	[5.0322]	[0.3632]	[3.7902]	[0.3178]	[5.0967]	[5.2866]	[3.4861]
RSQR	0.1150***	0.1128***	0.1099***	0.1107***	0.1104***	0.1111***	0.1099***	0.1110***	0.1127***	0.1107***

	RO Proxy					*RO* Industry			
Coeff.	small high profit	small highs sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Option Industries
Intercept	0.0416***	0.0413***	0.0405***	0.0416***	0.0403***	0.0403***	0.0403***	0.0403***	0.0404***
	[7.0506]	[7.0154]	[6.8674]	[7.0715]	[6.8187]	[6.8169]	[6.8440]	[6.8730]	[6.8530]
$Log(BM)$	0.0053***	0.0052***	0.0058***	0.0052***	0.0058***	0.0057***	0.0057***	0.0058***	0.0057***
	[6.8579]	[6.8384]	[7.8430]	[6.7970]	[7.7727]	[7.7483]	[7.7597]	[7.8780]	[7.7787]
$Log(ME)$	-0.0037***	-0.0036***	-0.0035***	-0.0037***	-0.0035***	-0.0035***	-0.0035***	-0.0035***	-0.0035***
	[-6.0680]	[-6.0359]	[-5.7614]	[-6.0951]	[-5.7286]	[-5.7625]	[-5.7303]	[-5.7558]	[-5.7524]
β^{CAPM}	0.0015*	0.0015**	0.0016**	0.0014*	0.0015**	0.0016**	0.0015**	0.0015**	0.0015**
	[1.8461]	[2.0167]	[2.2027]	[1.8043]	[2.1244]	[2.2400]	[2.1177]	[2.0997]	[2.1287]
$trade$	0.0154***	0.0153***	0.0155***	0.0154***	0.0155***	0.0154***	0.0155***	0.0155***	0.0154***
	[13.4108]	[13.4492]	[14.2725]	[13.4390]	[14.1888]	[14.1937]	[14.2384]	[14.3645]	[14.2776]
$Lag(r)$	-0.003	-0.0029	-0.003	-0.003	-0.003	-0.0032	-0.0032	-0.0031	-0.0031
	[-1.0544]	[-1.0095]	[-1.1569]	[-1.0560]	[-1.1339]	[-1.1729]	[-1.1851]	[-1.1741]	[-1.1711]
$\Delta IVol$	1.1829***	1.1282***	1.1573***	1.1504***	1.2322***	1.2437***	1.2796***	1.2852***	1.1634***
	[11.4545]	[10.8841]	[11.6522]	[10.8892]	[12.0615]	[12.0660]	[12.8921]	[12.8305]	[11.8727]
$RO \times \Delta Ivol$	0.4097***	0.6017***	0.3011***	0.5906***	0.1640**	0.3629***	0.0536	0.0483	0.2804***
	[4.6175]	[7.4742]	[3.5690]	[7.7597]	[2.0284]	[3.5133]	[0.6618]	[0.4384]	[4.2373]
RSQR	0.1116***	0.1122***	0.1111***	0.1122***	0.1108***	0.1106***	0.1102***	0.1101***	0.1102***

Table 8: This table reports coefficient estimates along with their t-statistics of Fama and MacBeth (1973) cross sectional regressions of firm level excess returns on the estimated loading on the market factor (β^{CAPM}), beginning of year log book-to-market ($Log(BM)$), log market equity ($Log(ME)$), six-month lagged return for months -7 to -2 relative to the month of observation ($Lag(r)$), monthly trading volume normalized by the number of shares outstanding ($trade$), month-to-month change in firm level idiosyncratic volatility ($\Delta Ivol$), the difference between the high and low $IVol$ regimes, the real option proxy, and the interaction between the real option proxy and the difference in $IVol$ regime $\overline{\Delta Ivol}$. The construction of the real option proxies are described in the paper. The regression model is $r_t - r_{f,t} = \gamma_0\iota + \gamma_1\overline{\Delta IVol} + \gamma_2\Delta IVol_t + \gamma_3\overline{\Delta IVol} \times \Delta IVol \times RO + \gamma_3 X_{t-1} + \eta_t$ in the paper. The real option proxies used in the regression are reported across columns. The reported estimates are the time series average of the monthly coefficient estimates. Newey-West robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R squared.

	RO Proxy									
Coeff.	size (mkt equity)	size (total assets)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
Intercept	0.0620***	0.0627***	0.0635***	0.0636***	0.0628***	0.0622***	0.0633***	0.0626***	0.0635***	0.0633***
	[12.6667]	[12.9225]	[13.0910]	[13.1015]	[13.0393]	[12.8871]	[12.9775]	[12.9719]	[13.0683]	[12.9677]
$Log(BM)$	0.0043***	0.0044***	0.0044***	0.0043***	0.0039***	0.0040***	0.0043***	0.0039***	0.0043***	0.0044***
	[5.9892]	[6.0253]	[6.0236]	[5.8810]	[5.2759]	[5.3703]	[6.0379]	[5.3164]	[5.9482]	[6.1117]
$Log(ME)$	-0.0058***	-0.0059***	-0.0059***	-0.0059***	-0.0059***	-0.0058***	-0.0059***	-0.0059***	-0.0059***	-0.0059***
	[-11.668]	[-11.886]	[-11.888]	[-11.951]	[-12.321]	[-12.203]	[-11.845]	[-12.268]	[-11.906]	[-11.880]
β^{CAPM}	0.0016**	0.0017**	0.0017**	0.0017***	0.0016**	0.0017**	0.0017**	0.0017**	0.0017***	0.0017***
	[2.3965]	[2.4694]	[2.5253]	[2.5956]	[2.1575]	[2.2686]	[2.4378]	[2.1917]	[2.5962]	[2.5885]
$Lag(r)$	-0.0042	-0.0041	-0.0041	-0.0041	-0.0039	-0.0038	-0.0041	-0.004	-0.0041	-0.0041
	[-1.6123]	[-1.5606]	[-1.5916]	[-1.5743]	[-1.4138]	[-1.3623]	[-1.6018]	[-1.4368]	[-1.5860]	[-1.5573]
$trade$	0.0158***	0.0158***	0.0159***	0.0159***	0.0157***	0.0157***	0.0158***	0.0157***	0.0159***	0.0158***
	[14.8382]	[14.7683]	[14.8815]	[14.9759]	[14.1370]	[14.0990]	[14.8001]	[14.1068]	[14.9459]	[14.8546]
$\overline{\Delta IVol}$	-0.9410***	-0.9714***	-1.0111***	-1.0136***	-0.9507***	-0.9369***	-0.9915***	-0.9412***	-1.0108***	-1.0003***
	[-9.4354]	[-9.6958]	[-10.129]	[-10.100]	[-8.9019]	[-8.7162]	[-9.9101]	[-8.8688]	[-10.092]	[-9.9407]
$\Delta IVol$	1.8416***	1.6986***	1.0350***	1.1713***	1.2416***	1.1677***	1.2704***	1.1767***	1.1665***	1.2368***
	[16.2397]	[12.7028]	[8.1893]	[10.8873]	[12.1765]	[11.3403]	[12.5016]	[11.2056]	[11.3075]	[12.3132]
$\overline{\Delta IVol} \times \Delta IVol \times RO$	-10.149***	-7.0635***	-2.5813	10.4454***	5.7354*	15.9548***	-1.0437	18.5515***	13.4981***	11.4536***
	[-6.4869]	[-5.3356]	[-0.5099]	[2.6592]	[1.6515]	[4.5518]	[-0.2828]	[5.1675]	[3.1356]	[2.6497]
RSQR	0.1176***	0.1164***	0.1166***	0.1180***	0.1170***	0.1175***	0.1169***	0.1173***	0.1191***	0.1173***

	RO Proxy					*RO* Industry			
Coeff.	small high profit	small highs sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Option Industries
Intercept	0.0629***	0.0623***	0.0636***	0.0628***	0.0634***	0.0627***	0.0627***	0.0627***	0.0631***
	[13.0392]	[12.9103]	[13.0859]	[12.9952]	[13.0304]	[12.9122]	[12.8832]	[13.0103]	[13.0997]
$Log(BM)$	0.0040***	0.0040***	0.0044***	0.0039***	0.0044***	0.0044***	0.0044***	0.0044***	0.0044***
	[5.2952]	[5.3880]	[6.0411]	[5.3075]	[5.9657]	[6.0308]	[6.0725]	[6.1201]	[6.0726]
$Log(ME)$	-0.0059***	-0.0058***	-0.0059***	-0.0059***	-0.0059***	-0.0059***	-0.0059***	-0.0058***	-0.0059***
	[-12.312]	[-12.215]	[-11.882]	[-12.270]	[-11.959]	[-11.903]	[-11.811]	[-11.871]	[-11.952]
β^{CAPM}	0.0017**	0.0017**	0.0017**	0.0017**	0.0018**	0.0018***	0.0018**	0.0016**	0.0017**
	[2.1808]	[2.3133]	[2.4464]	[2.1928]	[2.5693]	[2.6381]	[2.5412]	[2.3988]	[2.4645]
$Lag(r)$	-0.0038	-0.0037	-0.0041	-0.0039	-0.0041	-0.0042	-0.0042	-0.0041	-0.0041
	[-1.3929]	[-1.3437]	[-1.5979]	[-1.4052]	[-1.5618]	[-1.6174]	[-1.5995]	[-1.6019]	[-1.5898]
$trade$	0.0157***	0.0157***	0.0158***	0.0157***	0.0158***	0.0158***	0.0157***	0.0158***	0.0158***
	[14.1190]	[14.1104]	[14.8476]	[14.1089]	[14.8011]	[14.8076]	[14.8051]	[14.9660]	[14.8848]
$\overline{\Delta IVol}$	-0.9580***	-0.9432***	-1.0027***	-0.9531***	-1.0019***	-0.9704***	-0.9810***	-0.9709***	-0.9873***
	[-8.9675]	[-8.7742]	[-9.9681]	[-8.9661]	[-9.9511]	[-9.6355]	[-9.7776]	[-9.5971]	[-9.6676]
$\Delta IVol$	1.2115***	1.1590***	1.2214***	1.1849***	1.2578***	1.2651***	1.2737***	1.2776***	1.1733***
	[11.9304]	[11.2169]	[12.0872]	[11.3227]	[12.4545]	[12.3600]	[12.7952]	[12.8883]	[11.9514]
$\overline{\Delta IVol} \times \Delta IVol \times RO$	14.6836***	24.4176***	5.9097	22.5754***	3.2036	19.3544***	5.77	8.6091	13.0368***
	[3.3967]	[6.0754]	[1.3540]	[5.6070]	[0.7889]	[3.3040]	[1.4192]	[1.2756]	[4.2008]
RSQR	0.1178***	0.1180***	0.1176***	0.1179***	0.1173***	0.1158***	0.1168***	0.1160***	0.1167***

Table 9: The table reports Fama and French (1993) alphas along with robust Newey and West (1987) t-statistics in square brackets of the portfolios of stocks sorted on idiosyncratic return volatility $IVol$ and real option proxies. $IVol$ Ranks are reported across the columns and the ranks of the real option proxies are reported down the rows. The column labeled '3-1' refers to FF-3 alphas of the portfolios that are long and short the top and bottom $IVol$ portfolios respectively within each rank of the real option proxy. Idiosyncratic return volatility is computed relative to FF-3. We use daily data over the previous month and rebalance monthly. We sort stocks into three portfolios sorted by idiosyncratic volatility relative to the FF-3 model. Then, independently, at the end of each June, we sort stocks into three tercile on the basis of the real option characteristics age, size (total assets) and size (mkt equity), and into high/low vega, high/low future profit growth, high/low future sales growth, young/old, high/low future investment rate based on 33th/66th NYSE breakpoints. The row labeled 'Mean' corresponds to the FF-3 alpha for the portfolio equally weighted in the real option proxy sorted portfolios within each $IVol$ rank group. All portfolios are value weighted.

		Ivol Rank			
RO Proxy	*RO* Proxy Rank	1	2	3	3-1
size (total assets)	1	4.0975**	4.0300**	-6.5365***	-10.6340***
		[2.4911]	[1.9908]	[-2.8647]	[-4.3472]
	2	4.1199***	2.5739*	-5.4489***	-9.5688***
		[4.0848]	[1.8638]	[-2.7191]	[-4.1640]
	3	1.1176**	1.0809	-3.3977	-4.5153
		[1.9880]	[0.7594]	[-1.0931]	[-1.4095]
	Mean	3.1117***	2.5616**	-5.1277**	-8.2394***
		[3.9365]	[2.1641]	[-2.5398]	[-3.7403]
size (mkt equity)	1	3.4891**	7.1679***	-1.4118	-4.9009**
		[2.1164]	[4.2677]	[-0.6068]	[-2.0704]
	2	2.0518**	2.4464***	-5.7961***	-7.8479***
		[2.2206]	[2.6878]	[-3.0515]	[-3.3100]
	3	1.1703**	1.1891	-4.3537	-5.5241*
		[2.0816]	[0.8592]	[-1.5122]	[-1.8321]
	Mean	2.2371***	3.6012***	-3.8539**	-6.0910***
		[2.7423]	[3.7955]	[-1.9974]	[-2.7314]
age	1	0.7141	1.4974	-5.4860**	-6.2001**
		[0.5620]	[0.8703]	[-2.0730]	[-2.2095]
	2	0.0977	2.2573	-6.8932***	-7.0886***
		[0.0690]	[1.1694]	[-2.9674]	[-3.0333]
	3	0.2338	1.0879	-1.3004	-0.999
		[0.1729]	[0.7286]	[-0.4450]	[-0.3091]
	Mean	0.4196	1.6689	-4.5701**	-4.9897**
		[0.5937]	[1.3031]	[-2.1691]	[-2.2072]
high vega	0	1.4045**	2.0348*	-2.361	-3.7655*
		[2.2822]	[1.7913]	[-1.1747]	[-1.6816]
	1	-1.1113	-2.3174	-10.4032***	-9.2920***
		[-0.7644]	[-1.2174]	[-3.6044]	[-2.9722]
	Mean	0.1466	-0.1413	-6.3821***	-6.5287***
		[0.1741]	[-0.1068]	[-2.9884]	[-2.7877]
high profit	0	0.9203	-0.3679	-3.9303	-4.8505*
		[1.3045]	[-0.3151]	[-1.6076]	[-1.8132]
	1	2.1091*	2.2445	-2.0649	-4.1739
		[1.8580]	[1.1791]	[-0.7984]	[-1.5930]
	Mean	1.5147**	0.9383	-2.9976	-4.5122*
		[2.3362]	[0.7270]	[-1.2911]	[-1.8501]
high sale	0	0.5376	-2.7208**	-8.0188***	-8.5564***
		[0.7579]	[-2.2648]	[-3.3133]	[-3.3929]
	1	4.8059***	6.1847***	2.9237	-1.8821
		[3.9921]	[3.2661]	[1.0957]	[-0.6379]
	Mean	2.6718***	1.732	-2.5475	-5.2193**
		[3.9133]	[1.3646]	[-1.0939]	[-2.0502]
high inv	0	-1.6825**	-5.2831***	-11.3864***	-9.7039***
		[-2.3719]	[-3.9863]	[-4.4539]	[-3.5168]
	1	8.6144***	10.8817***	8.8514***	0.237
		[9.5657]	[6.3204]	[3.5488]	[0.0895]
	Mean	3.4659***	2.7993**	-1.2675	-4.7334**
		[5.8032]	[2.2492]	[-0.5834]	[-2.0071]

Table 10: The table reports Fama and French (1993) alphas along with robust Newey and West (1987)t-statistics in square brackets of the portfolios of stocks sorted on idiosyncratic return volatility $IVol$ and real option proxies. $IVol$ Ranks are reported across the columns and the ranks of the real option proxies are reported down the rows. The column labeled '3-1' refers to FF-3 alphas of the portfolios that are long and short the top and bottom $IVol$ portfolios respectively within each rank of the real option proxy. Idiosyncratic return volatility is computed relative to FF-3. We use daily data over the previous month and rebalance monthly. We sort stocks into three portfolios sorted by idiosyncratic volatility relative to the FF-3 model. Then, independently, at the end of each June, we sort stocks into three tercile on the basis of the real option characteristics age, size (total assets) and size (mkt equity), and into high/low vega, high/low future profit growth, high/low future sales growth, young/old, high/low future investment rate based on 33th/66th NYSE breakpoints. The row labeled 'Mean' corresponds to the FF-3 alpha for the portfolio equally weighted in the real option proxy sorted portfolios within each $IVol$ rank group. All portfolios are value weighted.

		Ivol Rank			
RO Industry	*RO* Industry dummy	1	2	3	3-1
young	0	1.3132	1.1363	-3.6190*	-4.9322**
		[1.2116]	[0.6678]	[-1.7697]	[-2.1477]
	1	0.4351	-0.1468	-7.3830***	-7.8181***
		[0.3662]	[-0.0634]	[-2.7331]	[-2.9091]
	Mean	0.8742	0.4948	-5.5010***	-6.3752***
		[1.0866]	[0.3128]	[-2.6674]	[-2.8934]
small high vega	0	1.3118**	1.443	-4.1226**	-5.4345**
		[2.2125]	[1.2015]	[-2.0369]	[-2.4447]
	1	7.2256***	-1.3516	-9.4742***	-16.6999***
		[3.1348]	[-0.5017]	[-2.7450]	[-5.1945]
	Mean	4.2687***	0.0457	-6.7984***	-11.0672***
		[3.5833]	[0.0289]	[-3.0578]	[-5.0682]
small growth	0	1.3399**	1.1866	-3.5023	-4.8422**
		[2.3010]	[0.9753]	[-1.5847]	[-2.0239]
	1	2.622	-1.3573	-17.2875***	-19.9095***
		[1.2730]	[-0.4825]	[-5.9209]	[-7.4748]
	Mean	1.9809*	-0.0854	-10.3949***	-12.3758***
		[1.8978]	[-0.0533]	[-4.8105]	[-5.8970]
small high profit	0	1.1321*	0.9205	-4.8299**	-5.9620***
		[1.9005]	[0.7740]	[-2.2867]	[-2.6013]
	1	6.6400***	5.1844	-5.9994	-12.6394***
		[3.2080]	[1.5296]	[-1.4277]	[-3.1488]
	Mean	3.8861***	3.0525	-5.4146*	-9.3007***
		[3.5113]	[1.6193]	[-1.9624]	[-3.4349]
small high sale	0	1.1484*	0.8917	-4.9013**	-6.0497**
		[1.8996]	[0.7600]	[-2.2452]	[-2.5556]
	1	6.9615***	7.2097***	-4.2678	-11.2293***
		[3.5611]	[2.6765]	[-1.1800]	[-3.1720]
	Mean	4.0549***	4.0507***	-4.5846*	-8.6395***
		[3.9697]	[2.6368]	[-1.9041]	[-3.5687]
small young	0	1.3810**	1.2084	-3.6206*	-5.0016**
		[2.3321]	[1.0050]	[-1.7557]	[-2.2157]
	1	3.2628	-0.4925	-12.5558***	-15.8186***
		[1.4685]	[-0.2145]	[-3.7289]	[-5.5878]
	Mean	2.3219**	0.358	-8.0882***	-10.4101***
		[2.0742]	[0.2500]	[-3.4986]	[-5.0069]
small high inv	0	1.0777*	0.8581	-5.3113**	-6.3890***
		[1.7810]	[0.7173]	[-2.4564]	[-2.7210]
	1	13.4407***	15.6392***	-0.1405	-13.5812***
		[6.5670]	[4.2039]	[-0.0450]	[-4.3761]
	Mean	7.2592***	8.2486***	-2.7259	-9.9851***
		[6.8613]	[3.9692]	[-1.2189]	[-4.5802]
young high vega	0	1.2383**	1.8331	-3.2394	-4.4778*
		[2.1660]	[1.4618]	[-1.5166]	[-1.9169]
	1	-2.2457	-1.7817	-11.4735***	-9.2278***
		[-1.0604]	[-0.6608]	[-3.4854]	[-2.6007]
	Mean	-0.5037	0.0257	-7.3565***	-6.8528***
		[-0.4390]	[0.0147]	[-3.2038]	[-2.8329]

Table 11: The table reports Fama and French (1993) alphas along with robust Newey and West (1987) t-statistics in square brackets of the portfolios of stocks sorted on idiosyncratic return volatility $IVol$ and real option proxies. $IVol$ Ranks are reported across the columns and the ranks of the real option proxies are reported down the rows. The column labeled '3-1' refers to FF-3 alphas of the portfolios that are long and short the top and bottom $IVol$ portfolios respectively within each rank of the real option proxy. Idiosyncratic return volatility is computed relative to FF-3. We use daily data over the previous month and rebalance monthly. We sort stocks into three portfolios sorted by idiosyncratic volatility relative to the FF-3 model. Then, independently, at the end of each June, we sort stocks into whether they belonged to the Natural Resources, High Technology, and Bio Technology industries, and whether they belonged to any one of the three industries (All G.O. Industries) on the basis of Fama and French (1997) industry classifications. The row labeled 'Mean' corresponds to the FF-3 alpha for the portfolio equally weighted in the industry sorted portfolios within each $IVol$ rank group. All portfolios are value weighted.

		Ivol Rank			
RO Industry	*RO* Industry dummy	1	2	3	3-1
Natural Resources	0	0.7477	0.4518	-3.7675*	-4.5152*
		[0.9722]	[0.3995]	[-1.7082]	[-1.8697]
	1	3.2293	2.3808	-4.5156	-7.7450**
		[1.5656]	[0.6126]	[-0.9362]	[-2.0684]
	Mean	1.9885**	1.4163	-4.1416	-6.1301**
		[2.1253]	[0.6784]	[-1.4653]	[-2.4363]
High Tech	0	1.4748*	0.7158	-2.7805	-4.2553*
		[1.9199]	[0.5762]	[-1.3525]	[-1.8781]
	1	1.4316	1.2535	-6.0880*	-7.5196**
		[0.8085]	[0.4088]	[-1.7998]	[-2.4055]
	Mean	1.4532*	0.9847	-4.4343*	-5.8875***
		[1.7676]	[0.5740]	[-1.9353]	[-2.5934]
Bio Tech	0	0.6634	1.2607	-5.4760**	-6.1394***
		[1.0983]	[0.8937]	[-2.4337]	[-2.6350]
	1	5.4876***	5.0094	0.8018	-4.6858
		[3.1616]	[1.5461]	[0.2237]	[-1.3247]
	Mean	3.0755***	3.1350*	-2.3371	-5.4126**
		[3.3905]	[1.7568]	[-1.0509]	[-2.2298]
All G.O. Industries	0	0.0523	-0.6041	-5.3537***	-5.4061***
		[0.0518]	[-0.5033]	[-3.2074]	[-2.9169]
	1	2.8762***	3.0485	-2.9505	-5.8266*
		[2.9638]	[1.2202]	[-0.9131]	[-1.8956]
	Mean	1.4642***	1.2222	-4.1521**	-5.6163***
		[2.7541]	[0.9161]	[-2.1409]	[-2.7146]

Table 12: The table reports Fama and French (1993) alphas, along with robust Newey and West (1987) t-statistics in square brackets, of the portfolios that are long and short the top and bottom $IVol$ portfolios respectively within each rank of the real option proxy and difference in $IVol$ values between the high and low regimes, $\overline{\Delta IVol}$. The columns labeled 'Mean(1 to 3)' and 'Mean(0 to 1)' correspond to the FF-3 alpha for the portfolio equally weighted in the real option proxy sorted portfolios within each $\overline{\Delta IVol}$ rank group. Idiosyncratic volatility is computed relative to FF-3. All portfolios are value weighted We use daily data over the previous month and rebalance monthly. We sort stocks into three portfolios sorted by idiosyncratic volatility relative to the FF-3 model. At the end of each June, we independently sort stocks into three tercile on the basis of the real option characteristics age, size (total assets) and size (mkt equity), and into high/low vega, high/low future profit growth, high/low future sales growth, young/old, high/low future investment rate based on 33th/66th NYSE breakpoints. Independent sorts of $\overline{\Delta IVol}$ are done once using in-sample values of $IVol$ for each stock and then grouping stocks on the basis of the 33th/66th NYSE $\overline{\Delta IVol}$ breakpoint values.

		RO Proxy Rank			
RO Proxy	$\overline{\Delta Ivol}$ Rank	1	2	3	Mean(1 to 3)
size (total assets)	1	1.2953	1.9408	-1.7919	0.4814
		[0.7783]	[0.9964]	[-0.8756]	[0.3398]
	2	-1.1459	-5.6068**	-5.9637**	-4.2388**
		[-0.3936]	[-2.1743]	[-2.0455]	[-2.0982]
	3	-9.1933**	-16.1224***	-14.8518***	-13.3892***
		[-2.5293]	[-5.0924]	[-3.6702]	[-4.9292]
size (mkt equity)	1	-0.1546	0.8774	-0.7373	-0.0048
		[-0.1097]	[0.6937]	[-0.3732]	[-0.0040]
	2	0.6286	0.2541	-6.2849**	-1.8008
		[0.2874]	[0.1028]	[-2.2975]	[-1.0100]
	3	-6.3840**	-12.7608***	-19.9126***	-13.0192***
		[-2.0049]	[-4.3610]	[-5.4313]	[-5.0374]
age	1	1.1583	-2.4428	-3.479	-0.2096
		[0.4656]	[-0.8672]	[-1.6132]	[-0.1219]
	2	0.5376	-9.0395***	-3.8205	-3.8516*
		[0.1828]	[-3.0131]	[-1.1623]	[-1.9194]
	3	-13.2842***	-9.7927**	-10.4336**	-11.6222***
		[-3.2330]	[-2.5055]	[-2.3812]	[-3.8341]

		RO Proxy Rank		
RO Proxy	$\overline{\Delta Ivol}$ Rank	0	1	Mean(0 to 1)
high vega	1	0.147	-8.1421	-3.2983
		[0.0618]	[-1.3157]	[-0.9137]
	2	-3.505	-5.6724	-4.5887*
		[-1.4618]	[-1.4065]	[-1.9547]
	3	-6.8298*	-5.0545	-5.9422*
		[-1.9367]	[-1.2537]	[-1.8772]
high profit	1	-0.943	-7.1614	-3.9736
		[-0.3316]	[-1.5875]	[-1.3484]
	2	-1.9006	1.2129	-0.3438
		[-0.6468]	[0.3879]	[-0.1404]
	3	-12.9659***	1.8703	-5.5478
		[-3.1592]	[0.4470]	[-1.6406]
high sale	1	-5.8249**	3.3435	-1.2823
		[-1.9994]	[0.6184]	[-0.3764]
	2	-1.9143	-1.607	-1.7606
		[-0.6081]	[-0.5251]	[-0.7201]
	3	-12.3146***	-2.904	-7.6093**
		[-2.7325]	[-0.7771]	[-2.2865]
high inv	1	-6.8576**	2.6811	-2.2096
		[-2.0994]	[0.6673]	[-0.7627]
	2	-4.5404	0.7809	-1.8797
		[-1.5994]	[0.2560]	[-0.8361]
	3	-9.8621**	-4.5532	-7.2076**
		[-2.3589]	[-1.1472]	[-2.2730]

Table 13: The table reports Fama and French (1993) alphas, along with robust Newey and West (1987) t-statistics in square brackets, of the portfolios that are long and short the top and bottom $IVol$ portfolios respectively within each rank of the real option proxy and difference in $IVol$ values between the high and low regimes, $\overline{\Delta IVol}$. The column labeled 'Mean(0 to 1)' corresponds to the FF-3 alpha for the portfolio equally weighted in the real option proxy sorted portfolios within each $\overline{\Delta IVol}$ rank group. Idiosyncratic volatility is computed relative to FF-3. All portfolios are value weighted. We use daily data over the previous month and rebalance monthly. We sort stocks into three portfolios sorted on idiosyncratic volatility relative to the FF-3 model. At the end of each June, we independently sort stocks on the basis of the real option characteristics age and size (total assets), and into high/low vega, high/low future profit growth, high/low future sales growth, young/old, high/low future investment rate based on 33th/66th NYSE breakpoints. Independent sorts of $\overline{\Delta IVol}$ are done once using in-sample values of $IVol$ for each stock and then grouping stocks on the basis of the 33th/66th NYSE $\overline{\Delta IVol}$ breakpoint values.

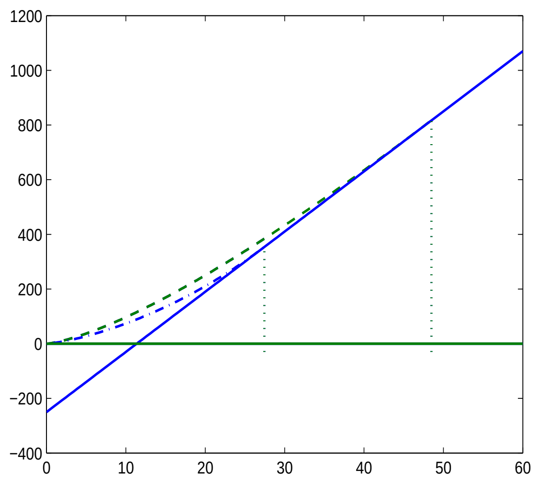
		RO Proxy Rank		
RO Proxy	$\overline{\Delta Ivol}$ Rank	0	1	Mean(0 to 1)
young	1	-2.6092	1.1289	-1.1093
		[-0.9174]	[0.2094]	[-0.3338]
	2	-5.2328**	-1.3772	-3.3045
		[-1.9887]	[-0.4292]	[-1.4599]
	3	-8.4579**	-8.0120**	-8.6963***
		[-2.3154]	[-2.2161]	[-3.2494]
small	1	-2.4541	-27.9701*	-9.3070**
high vega		[-1.0159]	[-1.8613]	[-2.5311]
	2	-2.187	-25.0583***	-13.5731***
		[-0.9645]	[-3.7224]	[-3.7903]
	3	-9.4226***	-12.0750***	-10.7488***
		[-2.7477]	[-3.3645]	[-3.7550]
small	1	-1.2082	4.5421	-0.8958
growth		[-0.4787]	[0.5815]	[-0.2379]
	2	-3.1672	-11.2681***	-7.2177***
		[-1.2881]	[-2.6043]	[-2.8752]
	3	-7.5165**	-16.2439***	-11.8802***
		[-1.9739]	[-5.2359]	[-4.4760]
small	1	-3.5166	-4.1494	-11.5094***
high profit		[-1.3669]	[-0.4946]	[-3.2741]
	2	-2.8234	-7.9637	-5.3935*
		[-1.1582]	[-1.3867]	[-1.7095]
	3	-6.7369*	-6.2536	-6.4952*
		[-1.7856]	[-1.4220]	[-1.9543]
small	1	-4.1531	-5.0725	-9.9128***
high sale		[-1.5812]	[-0.6850]	[-2.8132]
	2	-2.8776	-5.5105	-4.194
		[-1.1796]	[-1.0362]	[-1.3573]
	3	-7.0473*	-7.3758*	-7.2115**
		[-1.8367]	[-1.8315]	[-2.4433]
small	1	-1.8499	-0.1049	-5.7142
young		[-0.7736]	[-0.0137]	[-1.4315]
	2	-2.7867	-8.3822*	-5.5845**
		[-1.1933]	[-1.7600]	[-2.0902]
	3	-8.4557**	-10.3204***	-9.3881***
		[-2.2478]	[-2.8822]	[-3.3053]
small	1	-3.6428	-0.432	-10.0648***
high inv		[-1.3841]	[-0.0552]	[-2.9128]
	2	-2.9884	-8.5901*	-5.7893**
		[-1.2031]	[-1.7891]	[-2.0616]
	3	-7.0524*	-9.6920**	-8.3722***
		[-1.8104]	[-2.5248]	[-2.9166]
young	1	-0.8212	-12.0280*	-5.0852
high vega		[-0.3315]	[-1.7938]	[-1.3339]
	2	-3.0975	6.1775	1.54
		[-1.2775]	[1.1195]	[0.5106]
	3	-8.8177***	0.4351	-4.1913
		[-2.5898]	[0.0944]	[-1.3614]

Table 14: The table reports Fama and French (1993) alphas, along with robust Newey and West (1987) t-statistics in square brackets, of the portfolios that are long and short the top and bottom $IVol$ portfolios respectively within each classification of the real option industries and difference in $IVol$ values between the high and low regimes, $\overline{\Delta IVol}$. Industry classifications are reported across columns and $\overline{\Delta IVol}$ ranks are reported down the rows. The column labeled 'Mean(0 to 1)' corresponds to the FF-3 alpha for the portfolio equally weighted in both industry portfolios within each $\overline{\Delta IVol}$ rank group. Idiosyncratic volatility is computed relative to FF-3. All portfolios are value weighted. We use daily data over the previous month and rebalance monthly. We sort stocks into three portfolios sorted by idiosyncratic volatility relative to the FF-3 model. At the end of each June, we independently sort stocks into whether they belonged to the Natural Resources, High Technology, and Bio Technology industries, and whether they belonged to any one of the three industries (All G.O. Industries) on the basis of Fama and French (1997) industry classifications. Independent sorts of $\overline{\Delta IVol}$ are done once using in-sample values of $IVol$ for each stock and then grouping stocks on the basis of the 33th/66th NYSE $\overline{\Delta IVol}$ breakpoints.

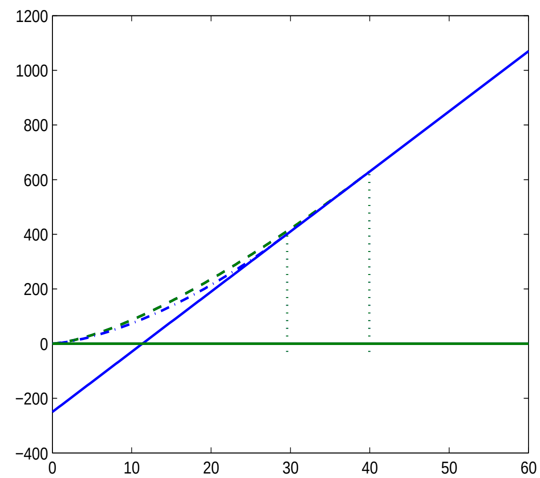
		RO Proxy Rank		
RO Proxy	$\overline{\Delta Ivol}$ Rank	0	1	Mean(0 to 1)
Natural Resources	1	-1.6231	6.8209	3.1442
		[-0.5745]	[1.3028]	[0.9702]
	2	-1.7178	-2.7294	-2.2236
		[-0.6962]	[-0.6421]	[-0.9270]
	3	-7.0048**	-11.2290**	-8.9211***
		[-2.1141]	[-2.5374]	[-3.5191]
High Tech	1	0.3569	0.7912	0.5179
		[0.1429]	[0.1257]	[0.1458]
	2	-0.8473	-8.2843**	-4.5658*
		[-0.3146]	[-2.2199]	[-1.9168]
	3	-10.9527***	-8.6281*	-9.7904***
		[-3.1950]	[-1.6827]	[-3.2519]
Bio Tech	1	-0.8241	7.5278	4.0748
		[-0.2945]	[1.2412]	[1.2224]
	2	-3.0634	-4.2983	-3.6546
		[-1.2751]	[-1.0584]	[-1.4655]
	3	-7.7035**	-12.9701**	-9.6440***
		[-2.1873]	[-2.2989]	[-2.7023]
All G.O. Industries	1	-0.8772	6.3261	2.709
		[-0.4017]	[1.0666]	[0.8223]
	2	-0.7051	-6.5709**	-3.6380*
		[-0.3291]	[-2.1681]	[-1.7147]
	3	-12.3964***	-13.3725***	-12.8844***
		[-4.1275]	[-3.3579]	[-4.8065]

Figure 1: Model Results. The figure shows the option values, and the option exercise policies P_1 and P_2 in P space for low and high volatility regimes. The 45 degree solid line corresponds to the intrinsic value of the real option. Option values in the high and low volatility states are depicted by dashed and dashed dotted curves respectively. The exercise thresholds are depicted by the vertical dotted lines where the lower threshold corresponds to the exercise threshold P_1 if the option is the low volatility regime, and the higher threshold corresponds to the exercise threshold P_2 if the option is the high volatility regime. Panel (a) depicts the model solution corresponding to parameters $\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$, panel (b) depicts the model solution corresponding to parameters $\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$, and panel (c) depicts the model solution corresponding to parameters $\sigma_{P,H} = 0.3, \sigma_{P,L} = 0.3$.

(a) $\sigma_H = 0.5$, $\sigma_L = 0.1$



(b) $\sigma_H = 0.4$, $\sigma_L = 0.2$



(c) $\sigma_H = 0.3$, $\sigma_L = 0.3$

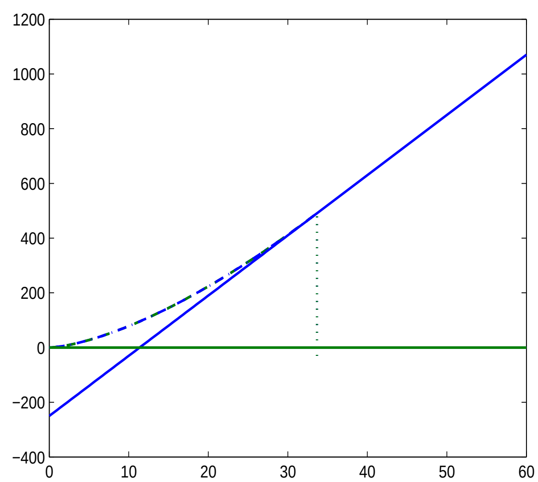


Figure 2: Model Solution. The figure shows the differences in growth option beta, the continuous drift, the jump, and the diffusion terms of the jump-diffusion processes (3.7) and (3.8) between the high and low volatility regimes in P space based on the real option/stochastic volatility model developed in Section 3 of the paper. Panel (a) shows differences in option betas, panel (b) shows differences in the diffusion term, panel (c) shows differences in the continual drift term, and panel (d) shows differences in the sporadic jump term. The figure shows separate results corresponding to different set of model parameters, i.e. $\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$, $\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$ and $\sigma_{P,H} = \sigma_{P,L} = 0.3$.

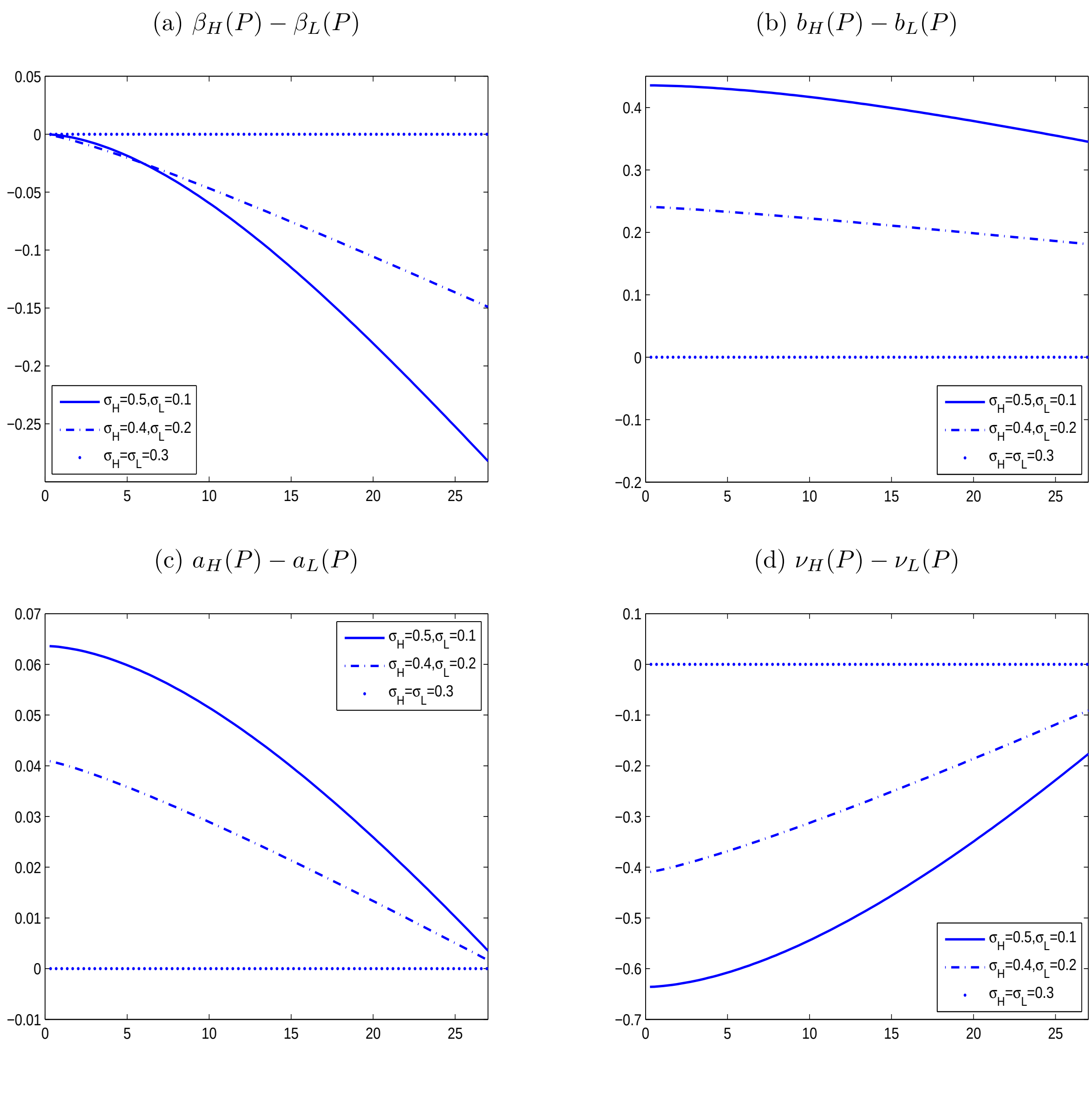


Figure 3: Simulation Results. The figure shows a sample path of simulated variables based on the real option/stochastic volatility model developed in Section 3 of the paper for $\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$. Panel (a) shows a sample path for the output price P at the end of each month, panel (b) shows the corresponding $F_i(P)$ values, panel (c) shows month end idiosyncratic volatility regimes for P where $i = 1$ stands for the high volatility regime and $i = 0$ stands for the low volatility regime, panel (d) shows month end realized idiosyncratic return volatility $IVol$, panel (e) shows month end realized returns for $F_i(P)$, and panel (f) shows month end realized excess returns for $F_i(P)$ computed according to equation (4.3) of the paper.

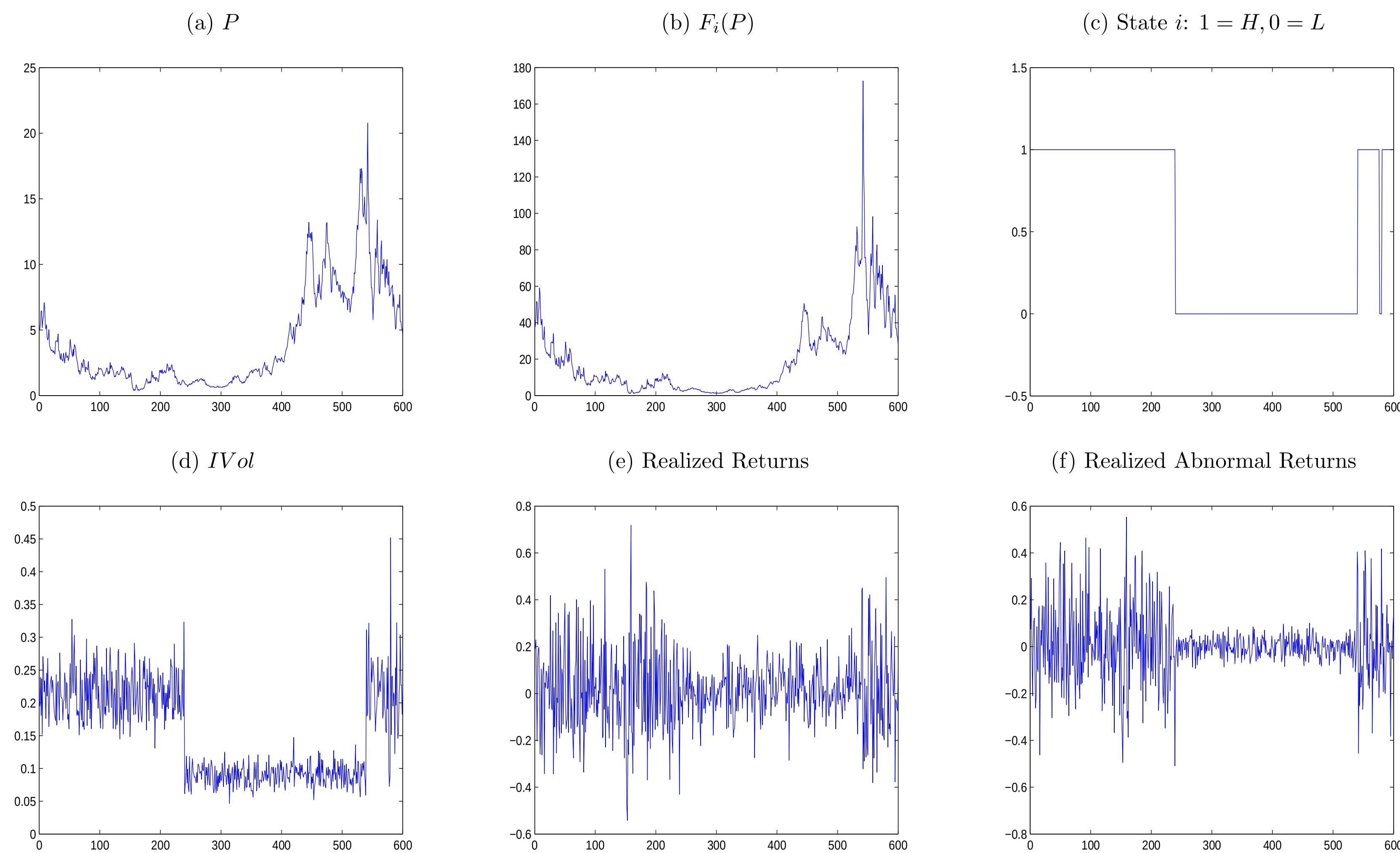


Figure 4: Simulation Results. The figure shows the mean portfolio risk-adjusted returns using growth option return data simulated based on the real option/stochastic volatility model developed in Section 3 of the paper. Growth options are ranked and sorted into decile portfolios based on the level of realized volatility $IVol$ over the past month. Then, value-weighted one month holding period mean portfolio returns are computed using the options monthly excess returns as defined in equation (4.3) of the paper. The portfolios are rebalanced at the end of each month. The figure shows separate portfolio risk-adjusted returns corresponding to the simulated samples where $\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$, $\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$ and $\sigma_{P,H} = \sigma_{P,L} = 0.3$.