

# Dynamic Agency and Real Options\*

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## Abstract

We present a model integrating dynamic moral hazard and real options. A risk averse manager can exert costly hidden effort to increase productivity growth of a firm. In addition, the risk neutral owners of the firm can irreversibly increase the firm's capital stock. In contrast to the literature, moral hazard may accelerate or delay investment relative to the first best depending on the severity of the moral hazard problem. When the agency problem is more severe, the firm will invest at a *lower* threshold than in the first best case because investment acts as substitute for effort. This mechanism provides an explanation for over-investment that does not rely on “empire building” preferences. Effort decreases after investment, however pay performance sensitivity increases after investment when the agency problem is less severe and the growth option is large.

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# 1 Introduction

How firms make real investment decisions is a central topic in the study of corporate finance. The real options framework argues that firms may delay investment because of a valuable option-to-wait. In the standard real options model, firm cash flows are generated without any agency conflicts. In reality, firms must often motivate managers to exert effort to grow cash flows and when managerial effort is costly and unobservable, a moral hazard problem arises. In this paper, we investigate how this moral hazard problem affects investment timing decisions. On the one hand, moral hazard will decrease the value of undertaking an irreversible investment option and will thus delay investment. On the other hand, moral hazard will decrease the value of *not* investing. In other words, moral hazard decreases the value of the option-to-wait and accelerates investment. Since the optimal time to invest equates the benefit of investing with the direct cost of investing plus the opportunity cost of the option-to-wait, it is not immediately clear if moral hazard will accelerate or delay investment. We show that when the moral hazard problem is less severe, investment is delayed. In contrast, when the moral hazard problem is more severe, investment is accelerated. Thus we are able to generate both over- and under-investment in the context of the same moral hazard model.

To arrive at this result, we construct a continuous time dynamic moral hazard model in which an investor contracts a manager to run a firm. The firm produces cash flow according to a linear production technology. The manager of the firm can exert effort to increase the expected growth rate of productivity. This effort is costly to the manager and hidden to the investors causing a moral hazard problem in that the manager can potentially gain utility by exerting less effort than would be optimal from the perspective of the investor. The optimal contract will then expose the manager to firm performance to provide her with incentives to exert effort. Incentivizing the manager is costly, as it reduces risk sharing between the manager and the investor. In addition to the moral hazard problem, the investor has an option to irreversibly increase the firm's capital.

In our model, effort and capital investment are substitutes as means of increasing the

firm's future cash flows. However, implementing higher effort entails an incentive cost while investment is entirely observable and contractable. Thus, the degree of substitutability of effort and investment depends on the incentive cost of effort. As a result, moral hazard can both accelerate and delay investment timing.

The existing models of moral hazard and investment largely consider contracts that implement effort at the first-best level. Consequently, agency conflicts distort only capital investment, and the typical result that has been replicated in a number of settings is that moral hazard decreases or delays investment (e.g. Grenadier and Wang (2005); DeMarzo and Fishman (2007a); Biais et al. (2010); DeMarzo et al. (2012)). In contrast, we solve for optimal effort under moral hazard that in general differs from the first-best effort. Indeed, the effort implemented under moral hazard is typically lower than under the first-best problem. However, capital investment may be increased or decreased depending on the severity of agency conflicts, the size of the growth option, and other parameters. In particular, if the manager's cost of exerting effort is high (which is one proxy for agency conflicts in our setup) and providing the agent with sufficient incentives is costly for the investors, then capital investment may substitute for effort and investment is accelerated under agency. However, if the cost of effort is low, then both effort and investment are decreased under agency.

While much of the previous theoretical literature predicts that moral hazard should curtail investment, a large empirical literature has documented that firms often over-invest (e.g. Morck et al. (2012)). A classic theoretical explanation for over investment is that managers enjoy private benefits from engaging in new projects (e.g. Jensen (1986); Stulz (1990)). The intuition is that if managers prefer to run larger enterprises, i.e. they wish to build "empires," and shareholder control is not perfect, firms will over invest. This intuition is directly at odds with basic ingredients of much of the literature on moral hazard: investment imposes a cost on managers because running larger firms is more costly. As a result, it seems hard to reconcile the empirical finding that firms sometimes over invest with the moral

hazard story. Our dynamic moral hazard setup can lead to over-investment without evoking empire-building preferences or delegated investment decisions. The investor will choose to invest at a lower threshold under moral hazard than under the first best case even though effort is more costly to implement in the larger firm. Again, the key is that effort decreases after investment making investment an attractive alternative to effort. In this sense, the firm invests to lower effort and incentive costs.

In addition to implications for the investment behavior of firms, our model also admits results for pay performance sensitivity. When the moral hazard problem is less severe, the optimal contract will call for the manager to exert full effort before and after investment. As a result, pay performance sensitivity will actually increase after investment. If, however, the moral hazard problem is more severe, the optimal contract will call for the manager to significantly decrease effort after investment and pay performance sensitivity decreases after investment.

An interesting feature of the results of our model is that the qualitative nature of the distortion to the investment timing and the effect of investment on pay performance sensitivity are linked. On the one hand, when investment leads to an increase in pay performance sensitivity, it must also be the case that investment is delayed relative to the first best case. On the other hand, when investment leads to a decrease in pay performance sensitivity, as is often empirically the case (e.g. Murphy (1999)), investment must be accelerated relative to the first best case.

This paper also makes a technical contribution to the dynamic contracting literature. We present a tractable framework for integrating real options and dynamic moral hazard. Specifically, and in contrast to models with risk neutral agents, our model will feature no wealth effects because the manager has constant absolute risk aversion (CARA) preferences and can privately save. This allows us to additively separate the effect of productivity and the manager's promised utility on firm value. This in turn means that productivity alone is a sufficient statistic for investment and the investment problem reduces to the solution

of an ODE. If in contrast we considered a risk neutral agent, the profitability of investment would depend on both productivity and the managers promised utility and the investment problem would not reduce to one state variable.

This paper contributes to growing literature on the intersection of dynamic agency conflicts and investment under uncertainty. On the dynamic contracting side, the classic contributions of Holmstrom and Milgrom (1987) and Spear and Srivastava (1987) introduced the notions that providing agents with incentives may take place over many periods. More recently, a number of papers have built on the continuous time approach of Sannikov (2008) to characterize optimal dynamic contracts in a variety of settings. For example, DeMarzo and Sannikov (2006) consider the design of corporate securities when the manager may divert cash. Piskorski and Tchisty (2010) and Piskorski and Tchisty (2011) consider the optimal design of mortgages when lenders face stochastic interest rates or house prices are stochastic. He (2009) considers optimal executive compensation when firm size follows a geometric Brownian motion. Most closely related to our dynamic agency problem is the capital-structure model of He (2011) which allows for a risk-averse agent.

On the investment side, DeMarzo and Fishman (2007a), Biais et al. (2010) and DeMarzo et al. (2012) consider dynamic moral hazard with investment. One important distinction between our paper and both Biais et al. (2010) and DeMarzo et al. (2012) is that their setups yield first-best effort even under moral hazard, and as such the substitutability of effort and investment is not present in their models. A closely related paper to ours is Grenadier and Wang (2005). They consider a real option exercise problem in the presence of a static moral hazard problem and find that when there is an additional adverse selection problem over managerial ability, real option exercise is delayed. We consider a dynamic moral hazard problem and find that real option exercise may be either delayed or accelerated. Finally, Philippon and Sannikov (2007) consider real options in a dynamic moral hazard setting similar to ours. However, they consider i.i.d. productivity shocks and a risk neutral agent. They find that moral hazard can only delay investment.

## 2 The Model

In this section we present our model of dynamic moral hazard and real options. It will resemble that of He (2011) in that we will consider an agent (the firm's manager) with constant absolute risk averse (CARA) preferences who can affect firm productivity growth by exerting costly hidden effort. In addition, we endow the firm with an irreversible investment opportunity in the spirit of the classic real options model of McDonald and Siegel (1986).

### 2.1 Technology and Preferences

Time is continuous, infinite, and indexed by  $t$ . The risk free rate is  $r$ . A risk-neutral investor employs a risk-averse manager to operate a firm. Firm cash flows are  $K_t X_t dt$  where  $K_t$  is the level of capital at time  $t$  and  $X_t$  is a productivity shock with dynamics given by

$$dX_t = a_t \mu X_t dt + \sigma X_t dZ_t$$

where  $a_t \in [0, 1]$  is the manager's effort and  $Z_t$  is a standard Brownian motion. Managerial effort here corresponds to any action that increases the growth rate – not the current level – of productivity. The firm starts with capital  $K_0 = k$  and has a one time expansion option to increase capital to  $\hat{k}$  at cost  $p$ . In the notation that follows a hat will indicate a post investment quantity.

The manager has constant absolute risk aversion (CARA) preferences over consumption. She values a stream of consumption  $\{c_t\}$  and effort  $\{a_t\}$  as

$$E \left[ \int_0^{\infty} e^{-rt} u(c_t, a_t) dt \mid \{a\} \right]$$

where  $u(c, a) = -e^{-\gamma(c - XKg(a))} / \gamma$  is the manager's instantaneous utility for consumption and effort and  $XKg(a)$  is the manager's cost of effort in units of consumption. This specification captures the notion that the manager's effort costs are increasing in both capital employed by

the firm as well as in productively. In other words it is more costly for the manager to increase productivity growth when the firm is larger or more productive. We assume the manager's normalized cost of effort  $g(a)$  is continuously differentiable, increasing, and convex,  $g(a) \in \mathcal{C}^1([0, 1])$ ,  $g'(a) \geq 0$ ,  $g''(a) > 0$ , and  $g'(0) = 0$ . When we consider specific parameterizations of the model we will assume a simple quadratic functional for  $g(a)$ . In addition to facing a cost of effort, the manager may save at the risk free rate  $r$ . We assume that the manager begins with zero savings. The manager's savings and effort are unobservable to the investor.

## 2.2 Contracts

A contract consists of a compensation rule, a recommended effort level, and an investment policy denoted  $\Pi = (\{c_t, a_t\}_{t \geq 0}, \tau)$ . The compensation rule  $\{c_t\}$  and recommended effort  $\{a_t\}$  are stochastic processes adapted to the filtration of public information  $\mathcal{F}_t$ . For simplicity, we will drop the subscript  $t$  whenever we are referring to the entire process of either consumption or effort from now on. The investment policy  $\tau$  is  $\mathcal{F}_t$ -stopping time that dictates when the firm exercises the option to increase capital. We assume that the investors can directly control investment and will pay the cost of investment. Note that the time  $t$  cash flow to the investor under a contract  $\Pi$  is given by

$$dD_t = X_t K_t dt - c_t dt - \mathbb{I}(t = \tau)p.$$

where  $D_t$  denotes cumulative cash flow to the investor.

Since the agent can privately save, the compensation  $c_t$  specified by the contract need not be equal to the manager's time  $t$  consumption. Denote the manager's accumulated savings as  $S_t$  and the agent's actual time  $t$  consumption and effort by  $\tilde{c}_t$  and  $\tilde{a}_t$  respectively.

Given a contract  $\Pi$ , the manager chooses a consumption and effort plan to maximize her

utility from the contract:

$$W(\Pi) = \max_{\{\tilde{c}, \tilde{a}\}} E \left[ \int_0^\infty -\frac{1}{\gamma} e^{-\gamma(\tilde{c}_t - X_t K_t g(\tilde{a}_t)) - rt} dt \right] \quad (1)$$

$$\text{such that } dS_t = rS_t dt + (c_t - \tilde{c}_t) dt, S_0 = 0$$

$$dX_t = \tilde{a}_t \mu X_t dt + \sigma X_t dZ_t$$

$$K_t = k + (\hat{k} - k) \mathbb{I}(t \geq \tau).$$

The dynamics of savings  $S_t$  reflect that the difference between compensation  $c_t$  and consumption  $\tilde{c}_t$  goes to increase (or decrease) savings while the savings balance grows at the risk free rate  $r$ . In addition to the dynamics for  $S_t$  given above, we impose the following transversality condition on the consumption process  $\tilde{c}_t$

$$\lim_{t \rightarrow \infty} E \left[ e^{-rt} \int_0^t (c_s - \tilde{c}_s) ds \right] = 0. \quad (2)$$

The dynamics of productivity  $X_t$  reflect that the expected growth rate of productivity depends on the actual effort  $\tilde{a}_t$  of the manager. Finally, the time  $t$  capital stock of firm depends on the investment policy set forth in the contract.

Given an initial outside option of the manager  $w_0$ , the investor then solves the problem

$$B(X_0, w_0) = \max_{\{c, a, \tau\}} E \left[ \int_0^\infty e^{-rt} dD_t \right] \quad (3)$$

$$\text{such that } dX_t = \tilde{a}_t \mu X_t dt + \sigma X_t dZ_t$$

$$K_t = k + (\hat{k} - k) \mathbb{I}(t \geq \tau)$$

$$w_0 \leq E^{\{(\tilde{c}, \tilde{a}), \tau\}} \left[ \int_0^\infty -\frac{1}{\gamma} e^{-\gamma(\tilde{c}_t - X_t K_t g(\tilde{a}_t)) - rt} dt \right]$$

where  $\{\tilde{c}, \tilde{a}\}$  solves problem (1).

We call a contract  $\Pi$  *incentive compatible* and *zero savings* if the solutions  $\{\tilde{c}_t\}$  and  $\{\tilde{a}_t\}$  to Problem (1) are equal to the payment rule and recommended effort plan given in the



contract. As is standard in the literature, we focus on contracts in which the solution to problem (1) is to follow the recommended action level and maintain zero savings by virtue of the following revelation-principle result.

**Lemma 1.** *For an arbitrary contract  $\tilde{\Pi}$ , there is an incentive compatible contract and zero savings contract  $\Pi$  that delivers at least as much value to the investor.*

### 3 Solution

The solution will follow the now standard martingale representation approach developed by Sannikov (2008). The first step is to give a necessary and sufficient condition for a contract to implement zero savings. We then represent the dynamics of the manager's continuation utility (the expected present value of her entire path of consumption) as the sum of a deterministic drift component and some exposure to the unexpected part of productivity growth shocks via the martingale representation theorem. With these dynamics in hand, we go on to characterize the incentive compatibility condition as a restriction on the dynamics of continuation utility. Given the dynamics of continuation utility and the dynamics of productivity implied by incentive compatibility, we can represent the investor's optimal contracting problem as a dynamic program resulting in a system of ODEs for investor value together with boundary conditions that pin down the investment policy. In the appendix, we provide verification that the solution to this system of ODEs indeed achieves the optimum investor value.

#### 3.1 The no savings condition

In this subsection, we characterize a necessary and sufficient condition for the manager to choose consumption equal to payment and thus maintain zero savings or debt. In words, the condition states that the manager's marginal utility for consumption is equal to her marginal utility for savings. To determine the agent's marginal utility for an additional unit of savings,

we first consider the impact of an increase in savings on the manager's optimal consumption and effort plan going forward. Suppose  $\{\tilde{c}, \tilde{a}\}$  solves problem (1) for a given contract that implements zero savings. Now suppose we simply endowed the manager with savings  $\mathcal{S} > 0$  at some time  $t > 0$ , how would her consumption and effort plan respond? Due to the absence of wealth effects implied by the manager's CARA preferences, the optimal consumption plan for  $s \geq t$  would be just  $\tilde{c}_s + r\mathcal{S}$  while the effort plan would remain unchanged. Thus, an increase in savings from zero to  $\mathcal{S}$  increases the managers utility flow by a factor of  $e^{-\gamma r\mathcal{S}}$  forever.<sup>1</sup> To make this intuition formal, it is useful to define the managers continuation utility for a given contract when following the recommended effort policy and accumulating the savings  $\mathcal{S}$  up to time  $t$ ,

$$W_t(\Pi, \{X_s, K_s\}_{s \leq t}; \mathcal{S}) = \max_{\{\tilde{c}_t, \tilde{a}_t\}} E \left[ \int_t^\infty -\frac{1}{\gamma} e^{-\gamma(\tilde{c}_s - X_s K_s g(\tilde{a}_t)) - r(s-t)} ds \mid \{X_s, K_s\} \right] \quad (4)$$

$$\text{such that } dS_s = rS_s ds + (\tilde{c}_s - c_s) ds \quad S_t = \mathcal{S}$$

$$dX_s = \tilde{a}_s \mu X_s ds + \sigma X_s dZ_s$$

$$K_s = k + (\hat{k} - k) \mathbb{I}(s \geq \tau).$$

The definition of continuation utility and the intuition given above lead to the following lemma.

**Lemma 2** (He (2011)). *Let  $W_t(\Pi, \{X_s, K_s\}_{s \leq t}; \mathcal{S})$  be the solution to problem (4), then*

$$W_t(\Pi, \{X_s, K_s\}_{s \leq t}; \mathcal{S}) = e^{-\gamma r\mathcal{S}} W_t(\Pi, \{X_s, K_s\}_{s \leq t}; 0). \quad (5)$$

Equation (5) allows us to determine the manager's marginal utility for savings under a contract that implements zero savings

$$\frac{\partial}{\partial \mathcal{S}} W_t(\Pi, \{X_s, K_s\}_{s \leq t}; \mathcal{S}) \Big|_{\mathcal{S}=0} = -\gamma r W_t(\Pi, \{X_s, K_s\}_{s \leq t}; 0). \quad (6)$$

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<sup>1</sup>Since utility is always negative the factor  $e^{-\gamma r\mathcal{S}} < 1$  represents an increase in utility.

Since we are focused on zero savings contracts, we will now drop the arguments and refer simply to continuation utility  $W_t$ . For the manager to maintain zero savings, her marginal utility of consumption must be equal to her marginal utility of savings

$$u_c(c_t, a_t) = -\gamma r W_t \quad (7)$$

which, together with the CARA form of the utility function, implies the convenient no savings condition

$$u(c_t, a_t) = r W_t. \quad (8)$$

Thus, for a contract to implement zero savings, the manager's flow of utility from the contract must be equal to the risk free rate  $r$  times her continuation utility. This is intuitive, in order for the manager to have no incentive to save, the contract must deliver the the risk free yield of her continuation utility in units of utility flow. For the remainder of the paper we will only consider contracts that satisfy the no savings condition given by Equation (8).

### 3.2 A Martingale Representation and Incentive Compatibility

Now that we have characterized a necessary and sufficient condition for a contract to implement zero savings, we turn our attention to the incentive compatibility condition. For an arbitrary incentive compatible and zero savings contract, consider the following process

$$F_t = E_t \left[ \int_0^\infty e^{-rs} u(c_s, a_s) ds \right]. \quad (9)$$

This process is clearly a martingale with respect to the filtration of public information  $\mathcal{F}_t$ , thus the martingale representation theorem implies that there exists a progressively measurable process  $\beta_t$  such that

$$dF_t = \beta_t (-\gamma r W_t) e^{-rt} \left( \frac{dX_t - a_t \mu X_t dt}{X_t} \right). \quad (10)$$

Now note that that  $F_t$  is related to the manager's continuation utility  $W_t$  (under the recommended consumption and effort plan) by

$$dW_t = (rW_t - u(c_t, a_t))dt + e^{rt}dF_t. \quad (11)$$

Combining the no savings condition (8) with Equations (10) and (11) gives the following dynamics for the manager's continuation utility

$$dW_t = \beta_t(-\gamma r W_t) \left( \frac{dX_t - a_t \mu X_t dt}{X_t} \right). \quad (12)$$

The process  $\beta_t$  is the sensitivity of the manager's continuation utility to unexpected shocks to the the growth rate of productivity. Since a deviation from the recommended effort policy will result in an unexpected (from the investor's perspective) shock to productivity growth,  $\beta_t$  measures the incentives of the manager to deviate from the contracts recommended effort policy.

For a given contract, Problem (1) implies that the manager chooses her current effort to maximize the sum of her instantaneous utility  $u(c_t, a_t)dt$  and the expected change in her continuation utility  $W_t$ . The manager's expected change in continuation utility from deviating from the recommended effort policy  $a_t$  to the policy  $\tilde{a}_t$  is

$$E[dW_t|\tilde{a}] = \beta_t(-\gamma r W_t)(\tilde{a} - a_t)\mu dt.$$

Thus, incentive compatibility requires that

$$a_t = \arg \max_{\tilde{a}} \{u(c_t, \tilde{a}) + \beta_t(-\gamma r W_t)(\tilde{a} - a_t)\mu\}. \quad (13)$$

Taking a first order condition for Problem (13) yields

$$-u_c(c_t, a_t)X_t K_t g'(a_t) + \beta_t(-\gamma r W_t)\mu = 0.$$

It is straightforward to show that this is a necessary and sufficient condition for the manager's optimal effort plan. Now recall that the no savings condition is  $u_c(c_t, a_t) = (-\gamma r W_t)$ , in other words the  $(-\gamma r W_t)$  term translates the sensitivity the manager's continuation utility to unexpected growth shocks to marginal utility units, which in turn implies that

$$\beta_t = \frac{1}{\mu} X_t K_t g'(a_t). \quad (14)$$

Intuitively, the sensitivity  $\beta_t$  that is required for incentive compatibility is exactly the agents marginal cost of effort in units of marginal utility of consumption. We can state the following lemma characterizing incentive-compatible no-savings contracts.

**Lemma 3.** *A contract is incentive compatible and no savings if and only if the solution  $W_t$  to Problem (4) has dynamics given by*

$$dW_t = -\frac{1}{\mu} \gamma r W_t \sigma X_t K_t g'(a_t) dZ_t. \quad (15)$$

### 3.3 First Best

As a benchmark, we first solve the model assuming effort is observable so that there are no agency conflicts. In this case, the investor simply pays the manager her cost of effort. Additionally, if the agent's promised utility is  $W$ , its certainty equivalent,  $-\ln(-\gamma r W)/\gamma r$ , can be paid out immediately. Thus, the investor's first best value function  $B_{FB}$  depends linearly on the certainty equivalent of  $W$ , or  $B_{FB}(X, W) = b_{FB}(X) + \ln(-\gamma r W)/\gamma r$ , for some function  $b_{FB}(X)$ . We will often refer to this function as the investor's gross value function to indicate that is equal to the investor's value gross of the certainty equivalent owed to the manager. To solve for the investor's first best value function, we can simply maximize the value of cash flows from the firm less the (direct) cost of effort. The investor's

post-investment gross value function  $\hat{b}_{FB}$  then solves the following HJB equation:

$$r\hat{b}_{FB} = \max_{a \in [0,1]} \left\{ X\hat{k}(1 - g(a)) + a\mu X\hat{b}'_{FB} + \frac{1}{2}\sigma^2 X^2\hat{b}''_{FB} \right\}. \quad (16)$$

Recall that a hat refers to a post investment quantity. The first two terms in the brackets are instantaneous cash flows and the cost of effort and the other two terms reflect the impact of the dynamics of  $X$  on the value function. The optimal effort level is then given by

$$\hat{a}_{FB} = \arg \max_{a \in [0,1]} \left\{ -X\hat{k}g(a) + a\mu X\hat{b}'_{FB} \right\}. \quad (17)$$

As all flows are proportional to  $X$ , the solution is also expected to be linear in  $X$  and as a result the optimal effort level given by Equation (20) will be constant in  $X$ . We can then solve Equation (16) to find the investor's first best gross value function:

$$\hat{b}_{FB}(X) = \frac{1 - g(\hat{a}^{FB})}{r - \hat{a}^{FB}\mu} X\hat{k}. \quad (18)$$

Before investment, the firm cash flows and the cost of effort are proportional to the lower level of capital,  $k$ . The HJB equation for the pre-investment gross firm value,  $b_{FB}(X)$ , is thus:

$$rb_{FB} = \max_{a \in [0,1]} \left\{ Xk(1 - g(a)) + a\mu Xb'_{FB} + \frac{1}{2}\sigma^2 X^2b''_{FB} \right\}. \quad (19)$$

The optimal effort level prior to investment is then given by

$$a_{FB} = \arg \max_{a \in [0,1]} \left\{ -Xkg(a) + a\mu Xb'_{FB} \right\}. \quad (20)$$

Note that  $b'_{FB}$  is not constant due to the curvature implied by the option to invest. Consequently, optimal effort prior to investment  $a^{FB}$  will not necessarily be constant in  $X$ . To solve the first best gross firm value prior to investment, we must identify a set of boundary conditions in addition to the HJB equation. At a sufficiently high level of  $X$ , denoted by

$\bar{X}_{FB}$ , the firm will pay the investment cost  $p$  to increase the capital to  $\hat{k}$ . The firm value at  $\bar{X}_{FB}$  must satisfy the usual value-matching and smooth-pasting conditions:

$$b_{FB}(\bar{X}_{FB}) = \hat{b}_{FB}(\bar{X}_{FB}) - p, \quad (21)$$

$$b'_{FB}(\bar{X}_{FB}) = \hat{b}'_{FB}(\bar{X}_{FB}). \quad (22)$$

Additionally, the firm value is equal to zero as  $X$  reaches its absorbing state zero. Thus

$$b_{FB}(0) = 0. \quad (23)$$

### 3.4 Optimal Contracting and Investment

We now present a heuristic derivation of optimal contracts in the full moral hazard case. First we characterize the payment rule to the manager. Recall that the no savings condition in Equation (8) provides a link between instantaneous utility and continuation utility. This allows us to express the manager's compensation as a function of the current state of the firm  $(X_t, K_t)$ , the recommended effort level  $a_t$  and her continuation utility  $W_t$  as follows:

$$c_t = X_t K_t g(a_t) - \frac{1}{\gamma} \ln(-\gamma r W_t). \quad (24)$$

The first term in Equation (24) is the manager's cost of effort in consumption units while the second is the risk free rate times the certainty equivalent of her continuation utility. In other words, the contract pays the manager her cost of effort plus the yield on her continuation utility.

The next task is to calculate the value of the firm to the investor before and after the investment. This amounts to expressing the investor's optimization problem given in (3) as a system of HJB equations. First, we consider the investor's problem post investment. An application of Ito's formula plus the dynamics of  $X_t$  and  $W_t$  yields the following HJB

equation for the value function  $\hat{B}$  post investment:

$$r\hat{B} = \max_{a \in [0,1]} \left\{ X\hat{k}(1 - g(a)) + \frac{1}{\gamma} \ln(-\gamma rW) + a\mu X\hat{B}_X + \frac{1}{2}\sigma^2 X^2 \hat{B}_{XX} + \sigma^2 X^2 \hat{k}g'(a)(-\gamma rW)\hat{B}_{XW} + \frac{1}{2} \left( \frac{1}{\mu} \sigma X \hat{k}g'(a) \right)^2 (-\gamma rW)^2 \hat{B}_{WW} \right\} \quad (25)$$

We guess that  $\hat{B}(X, W) = \hat{b}(X) + \frac{1}{\gamma r} \ln(-\gamma rW)$ . Again, we will refer to  $\hat{b}(X)$  as the investor gross firm value as it measures the investor's valuation of the firm gross of the certainty equivalent promised to the manager. Then  $B_{XW} = 0$  and  $B_{WW} = \frac{-1}{\gamma rW^2}$ . This leaves the following HJB equation for  $\hat{b}(X)$

$$r\hat{b} = \max_{a \in [0,1]} \left\{ X\hat{k}(1 - g(a)) - \frac{1}{2}\gamma r \left( \frac{1}{\mu} \sigma X \hat{k}g'(a) \right)^2 + a\mu X\hat{b}' + \frac{1}{2}\sigma^2 X^2 \hat{b}'' \right\} \quad (26)$$

It is instructive to note the difference between Equations (16) and (26). In the first best case, the investor need only compensate the manager for her cost of effort, while in the moral hazard case the investor must also bear some incentive costs of effort given by the third term on the right hand side of Equation (26). The incentive cost of effort is proportional to the square of the level of cash flows  $X\hat{k}$  and thus the value function  $\hat{b}(X)$  is no longer linear in  $X$  as in the first best case. The optimal effort plan post investment solve

$$\hat{a}^*(X) = \arg \max_{a \in [0,1]} \left\{ -X\hat{k}g(a) - \frac{1}{2}\gamma r \left( \frac{1}{\mu} \sigma X \hat{k}g'(a) \right)^2 + a\mu X\hat{b}' \right\}. \quad (27)$$

Equations (26) and (27) together provide an ODE for the value function  $\hat{b}$ . It is left to specify the boundary conditions that determine a solution to the ODE. The first boundary condition is that the firm, gross of the consumption claim to the agent, must be valueless when productivity is zero as this is an absorbing state:

$$\hat{b}(0) = 0. \quad (28)$$



The second boundary condition obtains by noting that the cost of positive effort goes to infinity as  $X$  goes to infinity, and as a result the optimal effort goes to zero. Thus, the value function must approach a linear function consistent with zero effort as  $X$  goes to infinity:

$$\lim_{x \rightarrow \infty} \left| \hat{b}'(X) - \frac{\hat{k}}{r} \right|. \quad (29)$$

We now turn to the pre-investment firm value  $B$ . A similar argument to the above leads to the HJB equation for the pre-investment firm value:

$$rBdt = \max_{a, I \in \{0,1\}} \left\{ \left[ Xk(1 - g(a) + \frac{1}{\gamma} \ln(-\gamma r W_t) + a\mu X B_X + \frac{1}{2} \sigma^2 X^2 B_{XX} + \sigma^2 X^2 k g'(a) (-\gamma r W) B_{XW} + \frac{1}{2} \left( \frac{1}{\mu} \sigma X k g'(a) \right)^2 (-\gamma r W)^2 B_{WW} \right] dt + I(\hat{B} - B - p) \right\}. \quad (30)$$

The difference between Equations (25) and (30), as indicated by the last term in Equation (30), is that the investor must now optimally choose whether or not to invest. Clearly,  $I = 0$  whenever  $B > \hat{B} - p$ . Again, we guess that  $B(X, W) = b(X) + \frac{1}{\gamma r} \ln(-\gamma r W)$ . This allows a simplification of Equation (30). In the region where  $I = 0$ , it becomes:

$$rb = \max_{a \in [0,1]} \left\{ Xk(1 - g(a)) - \frac{1}{2} \gamma r \left( \frac{1}{\mu} \sigma X k g'(a) \right)^2 + a\mu X b' + \frac{1}{2} \sigma^2 X^2 b'' \right\}. \quad (31)$$

Note that the dependence of the investor's value on the manager's continuation utility before and after the investment cancels. The optimal pre-investment effort solves

$$a^*(X) = \arg \max_{a \in [0,1]} \left\{ -Xk g(a) - \frac{1}{2} \gamma r \left( \frac{1}{\mu} \sigma X k g'(a) \right)^2 + a\mu X b' \right\}. \quad (32)$$

Equations (31) and (32) yield an ODE for the pre-investment value function  $b$ . It is similar to the post-investment ODE in Equations (26) and (27) but for the level of employed capital. Its solution is determined by investment-specific boundary conditions. As in the first-best case, the optimal investment policy will be a threshold  $\bar{X}$  in productivity at which the investor

will increase the capital of the firm. Again the usual smooth pasting and value matching conditions apply:

$$b(\bar{X}) = \hat{b}(\bar{X}) - p \quad (33)$$

$$b'(\bar{X}) = \hat{b}'(\bar{X}). \quad (34)$$

Additionally, as  $X$  reaches zero, the gross firm value is zero:

$$b(0) = 0. \quad (35)$$

We collect our results on the optimal contract in the following proposition.

**Proposition 1.** *The optimal contract is given by the payment rule (24), together with the effort plan before and after investment given in (32) and (27) respectively, and investment time  $\tau = \min\{t : X_t \geq \bar{X}\}$  such that the investor's gross firm value before and after investment,  $b$  and  $\hat{b}$ , solve (31)-(35) and (26)-(29).*

At this point, our choice to endow the manager with CARA preferences and the ability to privately save should be clear. These features of the model allow us to additively separate the dependence the investors value on productivity  $X_t$  and the manager's continuation utility  $W_t$ . As a result, the investment problem reduces to the ODE in (31)-(35). If we had considered a risk neutral manager, the the resulting investment problem be substantially more complex with two state variables and the optimal investment threshold as a curve in  $(X_t, W_t)$  space.

## 4 Implications

We now proceed to discuss the implications of the optimal contract characterized in Proposition 1. We will numerically illustrate these implications using particular parameterizations

of our model and the following function form for the normalized cost of effort

$$g(a) = \frac{1}{2}\theta a^2. \quad (36)$$

Following He (2011), we use a risk free rate of  $r = 5\%$  and a standard deviation of productivity growth of  $\sigma = .25$ . We choose a slightly lower upper bound on the growth rate of productivity of  $\mu = 4\%$  reflecting the that in our model the the growth rate of productivity is bounded below by 0 due to the non-negativity of effort and the multiplicative specification for the effect of effort on productivity while some calibrations (e.g. Goldstein et al. (2001)) find negative average growth rates. The parameter of risk aversion  $\gamma$  is set equal to 1. Investment increases capital from  $k = 0.5$  to  $\hat{k} = 1$  at cost 1 per unit of new capital. The cost of effort parameter is  $\theta = 1$ . We choose parameters for the cost of effort and investment so that the two are close substitutes.

## 4.1 Effort

In this section, we discuss the implications of the optimal contract for managerial effort. For the quadratic effort cost, we can characterize the optimal effort policy by a simple first order condition:

$$\hat{a}^*(X) = \min \left\{ \frac{\mu^3 \hat{b}'(X)}{\theta \hat{k}(\mu^2 + \gamma r \sigma^2 X \hat{k})}, 1 \right\}, \quad (37)$$

$$a^*(X) = \min \left\{ \frac{\mu^3 b'(X)}{\theta k(\mu^2 + \gamma r \sigma^2 X k)}, 1 \right\}. \quad (38)$$

Implemented effort is time-varying with productivity, depends on the primitive parameters of the model and on the presence of growth opportunities. Figure 1 illustrates some of the key properties of the optimal effort. Efforts in young (pre-investment), mature (post-investment) and small no-growth (permanently small) firms are plotted at two levels of the cost of effort  $\theta$ . Effort implemented in the mature and no-growth firms decreases and goes to zero as  $X$

approaches infinity. This is because the cost of providing incentives grows more in  $X$  than does the benefit of effort. A related effect makes effort decrease in response to exogenous changes in capital (that is, abstracting from growth options; to see this compare efforts of the no-growth and mature firms).

Effort implemented in the young firm is above that of the mature firm due to two reasons. First, the young firm employs low level of capital which makes the total cost of effort lower for a given level of productivity  $X$ . This property manifests itself also in the fact that effort (weakly) decreases at the moment of investment. If effort is interior immediately before and after investment, then using equations (37) and (38) yields

$$\frac{\hat{a}^*(\bar{X})}{a^*(\bar{X})} = \frac{k(\mu^2 + \gamma r \sigma^2 \theta \bar{X} k)}{\hat{k}(\mu^2 + \gamma r \sigma^2 \theta \bar{X} \hat{k})} < 1. \quad (39)$$

where the smooth pasting condition (34) at the investment threshold removes the dependence of efforts on  $b'(X)$  and  $\hat{b}'(X)$ . The effort is constant at investment only in the case when post-investment effort is full.

The second reason for high effort in young firms is due to the presence of growth options. As is standard in real-options models, growth options increase the sensitivity of the firm value to productivity shocks as the firm approaches the investment threshold. As the optimal effort increases in  $b'(X)$  (see Equation (38)), this suggest that effort may increase in  $X$  in the young firm. Intuitively, the prospect of capital investment makes contracting high effort additionally attractive from the investor's point of view.

An important feature of our model is that the level of effort implemented under agency conflicts is in general different than the first-best level. Due to the cost of providing sufficient incentives to extend effort, the manager is contracted to exert a weakly lower effort than under the first best (the efforts under agency and in the first-best problem can only be equal if the effort implemented under agency is full). However, the extent to which the moral hazard problem decreases effort for the young pre-investment and mature post-investment

firms is not uniform and depends on parameters, notably on the cost of effort.

Figure 2 illustrates the relation between effort under agency and in the first-best problem for two different levels of the cost of effort  $\theta$ . In the case of low  $\theta$ , the pre-investment firm value is not much affected by agency because effort can still be at the first-best level (see the left panel of Figure 2). If costs of effort  $\theta$  is relatively high, then agency conflicts decrease the implemented effort and so the pre-investment firm value. The effect of agency on the mature firm is less differential in  $\theta$  because the effort is already relatively low. Note that the timing of capital investment is determined by the relation of the pre-investment and post-investment firm values and so a deferential effect of agency on implemented efforts in pre-investment and post-investment firms can explain investment behavior.

## 4.2 Investment Behavior

In this subsection we discuss the optimal investment behavior of the firm. Previous models of investment with moral hazard have largely agreed upon a central result: agency conflicts delay or curb investment. For example, DeMarzo and Fishman (2007a) and DeMarzo et al. (2012) find that dynamic moral hazard problems decrease the rate of investment, while Grenadier and Wang (2005) find that moral hazard combined with adverse selection delays the exercise of real options. In contrast to the extant literature, we find that agency conflicts can also accelerate investment relative to first best. This result is driven by the fact that although moral hazard decreases the value of the firm after investment, it also decreases the value of not investing. Moreover, the investment decision is driven by the difference between firm value with and without increased capital. Under conditions discussed below, increasing moral hazard increases this difference, and as a result, decreases the investment threshold.

The key intuition is that effort and investment are (imperfect) substitutes.<sup>2</sup> One period of high effort leads to one period of high expected growth of cash flows. In a similar way, an investment in additional capital yields growth in cash flows. A key difference between

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<sup>2</sup>The substitutability of effort and investment was first emphasized in Holmstrom and Weiss (1985).

these methods of increasing cash flow growth is that effort is unobservable while investment is contractable. Thus, the relative cost of these two technologies depends on the severity of the moral hazard problem. Intuitively, when the moral hazard problem is severe, investment is a relatively cheap way of growing cash flows. Figures 3-5 show the investment threshold for the moral hazard and first best cases over a range of parameter values. When normalized the cost of effort  $\theta$ , the manager's risk aversion  $\gamma$ , or the volatility of growth  $\sigma$  are low, the moral hazard problem is less severe. In this case higher effort is not too costly to implement and the investment threshold is higher for the moral hazard case than for the first best. In contrast, when any of these parameters are high, implementing high effort is costly relative to investment and the investment threshold for the moral hazard case is below that of the first best case.

In order to make this intuition precise, we study the comparative static properties of the firm value before and after investment. Specifically, we consider the following comparative state

$$\frac{\partial}{\partial \gamma} [\hat{b}(X) - b(X)]$$

for  $X$  close to  $\bar{X}$ . If this comparative static is negative, then an increase in  $\gamma$  decreases the difference between the firm before and after investment, in other words investment is less attractive and will be delayed. However, when this comparative static is positive, an increase in  $\gamma$  increases the profitability of investment and investment is accelerated. To compute the derivative above we apply the method of comparative statics developed by DeMarzo and Sannikov (2006). For convenience, we define the differential operator  $\mathcal{L}$  as follows

$$\mathcal{L}(x, k, f) = xk(1 - g(a^*(x, k, f))) - \frac{1}{2}\gamma r(\sigma xkg'(a^*(x, k, f)))^2 + a^*(x, k, f)\mu f'(x) + \frac{1}{2}\sigma^2 x^2 f''$$

where

$$a^*(x, k, f) = \arg \max_{a \in [0,1]} \left\{ -xkg(a) - \frac{1}{2}\gamma r(\sigma xkg'(a))^2 + a\mu f'(x) \right\}.$$

The HJB equations (26) and (31) then imply that

$$r \frac{\partial \hat{b}}{\partial \gamma} = \frac{\partial}{\partial \gamma} \mathcal{L}(X, \hat{k}, \hat{b}), \quad (40)$$

$$r \frac{\partial b}{\partial \gamma} = \frac{\partial}{\partial \gamma} \mathcal{L}(X, k, b). \quad (41)$$

Thus, an application of the Feynman-Kac formula gives

$$\frac{\partial}{\partial \gamma} [\hat{b}(X) - b(X)] = E \left[ \int_0^\tau e^{-rt} \frac{\partial}{\partial \gamma} (\mathcal{L}(X, \hat{k}, \hat{b}) - \mathcal{L}(X, k, b)) dt \middle| X_0 = X \right]. \quad (42)$$

Signing this comparative static for an arbitrary  $X < \bar{X}$  is not possible. However, we are only interested in difference between the before and after investment for  $X$  arbitrarily close to  $\bar{X}$  as this indicates the affect of a change in agency costs on the investment threshold. An application of the Bounded Convergence Theorem gives

$$\lim_{X \rightarrow \bar{X}} \frac{\partial}{\partial \gamma} [\hat{b}(X) - b(X)] = \lim_{X \rightarrow \bar{X}} E \left[ \int_0^\tau e^{-rt} \left[ \frac{\partial \hat{\mathcal{L}}(X, \hat{b}; \gamma)}{\partial \gamma} - \frac{\partial \mathcal{L}(X, b; \gamma)}{\partial \gamma} \right] dt \right] \quad (43)$$

$$= E \left[ \lim_{X \rightarrow \bar{X}} \int_0^\tau e^{-rt} \left[ \frac{\partial \hat{\mathcal{L}}(X, \hat{b}; \gamma)}{\partial \gamma} - \frac{\partial \mathcal{L}(X, b; \gamma)}{\partial \gamma} \right] dt \right] \quad (44)$$

$$= \left( \frac{\partial \hat{\mathcal{L}}(\bar{X}, \hat{b}; \gamma)}{\partial \gamma} - \frac{\partial \mathcal{L}(\bar{X}, b; \gamma)}{\partial \gamma} \right) dt. \quad (45)$$

Thus when

$$\frac{\partial \hat{\mathcal{L}}(\bar{X}, \hat{b}; \gamma)}{\partial \gamma} - \frac{\partial \mathcal{L}(\bar{X}, b; \gamma)}{\partial \gamma} \geq 0 \quad (46)$$

a small increase in the manager's risk aversion  $\gamma$  leads to an increase in the difference between  $\hat{b}(\bar{X})$  and  $b(\bar{X})$ . By the value matching condition, this means that the investment threshold must decrease. An application of the envelope theorem allows us to evaluate Equation (46) to find

$$\text{sign} \left( \frac{\partial \bar{X}}{\partial \gamma} \right) = \text{sign} \left( kg'(a^*(\bar{X})) - \hat{k}g'(\hat{a}^*(\bar{X})) \right). \quad (47)$$

We formally state this result in the following proposition

**Proposition 2.** *An increase in  $\gamma$  decreases the investment threshold  $\bar{X}$  if and only if marginal cost of effort at the optimum drops by a sufficiently large amount at investment, i.e. if and only if*

$$\frac{g'(\hat{a}^*(\bar{X}))}{g'(a^*(\bar{X}))} \leq \frac{k}{\hat{k}}. \quad (48)$$

Proposition 2 highlights one of our main findings: increased moral hazed problems do not necessarily lead to delayed or decreased investment. In fact, in our model, an increase in managerial risk aversion can lead to a decrease in the investment threshold. Much of the literature on agency conflicts and investment following Jensen (1986) has focused on problems of free cash flow, in which managers may invest funds in pet projects that are not beneficial to shareholders. This view posits that a central conflict between managers and shareholders is that manager’s want to invest even when it’s not in the benefit of shareholders to do so, i.e. manager’s wish to “empire-build.” At the same time another strand of the literature (e.g. DeMarzo and Fishman (2007a) and DeMarzo et al. (2012)), has focused on the assumption that motivating managers to apply effort is more costly for larger firms. This view implies that managers have either have no preferences over investment or prefer less investment. Consequently, moral hazard in effort models typically predict that investment is curtailed or delayed. As such, it seems hard to reconcile this type of moral hazard with empirical evidence that firms sometimes over-invest. Proposition 2 shows that over-investment may be perfectly natural in a standard moral hazard setting without empire building preferences if we allow for flexible effort.

Proposition 2 shows that it possible for moral hazard to accelerate real option exercise and provides some guidance for when such acceleration may take place. Specifically, an increase in the managers risk aversion  $\gamma$ , which in turn makes incentive provision more costly, accelerates investment when the marginal cost of effort at the optimum is greater before investment than after investment. Thus, the effect of  $\gamma$  on investment timing depends on how the optimal effort changes when the firm invests. To investigate this effect further it useful to consider quadratic effort cost given by Equation (36). For this special case, we



can characterize the optimal effort policy by a simple first order condition:

$$\hat{a}^*(X) = \min \left\{ \frac{\mu^3 \hat{b}'(x)}{\theta \hat{k}(\mu^2 + \gamma r \sigma^2 X \hat{k})}, 1 \right\}, \quad (49)$$

$$a^*(X) = \min \left\{ \frac{\mu^3 b'(x)}{\theta k(\mu^2 + \gamma r \sigma^2 X k)}, 1 \right\}. \quad (50)$$

When optimal effort is interior both before and after investment, i.e. when  $a^*(\bar{X}), \hat{a}^*(\bar{X}) < 1$ ,

Equation (46) simplifies to

$$\frac{\mu^2 + \gamma r \sigma^2 \theta \bar{X} k}{\mu^2 + \gamma r \sigma^2 \theta \bar{X} \hat{k}} \leq 1, \quad (51)$$

which is always satisfied. When  $a^*(\bar{X}, k, b) = 1$ , i.e. when the optimal contract calls for the manager to exert full effort before investment, Equation (46) simplifies to

$$\hat{a}^*(\bar{X}) = \frac{\mu^3 \hat{b}'(\bar{X})}{\hat{k} \theta (\mu^2 + \gamma r \sigma^2 \theta \bar{X} \hat{k})} \leq \frac{k}{\hat{k}}. \quad (52)$$

This condition states that the if optimal managerial effort drops immediately after investment by a sufficiently large (small) amount, then increasing (decreasing) agency costs decreases (increases) the investment threshold.

This reasoning begs the following question. Under what circumstances would optimal effort decrease after investment by a sufficiently large amount such that increasing the agency problem accelerates investment? Consider the optimal effort choice under first best ( $\gamma = 0$ ). Intuitively, when the cost of effort  $\theta$  is high, optimal effort will be interior both before and after investment. Using a similar technique to compute comparative statics to the one employed above, it is possible to show that effort weakly decreases with the severity of the agency problem

$$\frac{\partial a^*}{\partial \gamma}, \frac{\partial \hat{a}^*}{\partial \gamma} \leq 0, \quad (53)$$

which implies that if optimal effort is interior in the first best, it will be interior in the presence of agency conflicts as well. Thus, when the cost of effort is high, an increase in the severity

of the agency problem decreases the investment threshold and accelerates investment. When the cost of effort  $\theta$  is small, first best effort prior to investment will be high, i.e. full effort will be employed. In this case an increase in the severity of the agency problem from  $\gamma = 0$  will accelerate investment if and only if

$$\mu \hat{b}'_{FB}(\bar{X}) \leq \theta k. \quad (54)$$

Moreover, we have seen that in the first best case  $\hat{b}(X)$  takes a simple linear. We can then state the following proposition:

**Proposition 3.** *Suppose the cost of effort is quadratic and given by Equation (36). If the cost of effort  $\theta$  is small, specifically when*

$$\theta < \frac{2\mu \hat{k}^2}{k(2r\hat{k} - \mu k)}, \quad (55)$$

*an increase in  $\gamma$  will delay investment when  $\gamma$  is small and accelerate investment when  $\gamma$  is large. Moreover, the investment threshold  $\bar{X}$  will be above the first best threshold  $\bar{X}_{FB}$  when  $\gamma$  is small and below when  $\gamma$  is large.*

*When the cost of effort  $\theta$  is large, an increase in  $\gamma$  will always accelerate investment. Moreover, the investment threshold  $\bar{X}$  will always be below the first best threshold  $\bar{X}_{FB}$*

Proposition (36) shows that for the special case of quadratic effort costs, our model generates both accelerated and delayed investment. Returning to Figure 4, we can see that when  $\gamma$  is small, the investment threshold increases in  $\gamma$ . Since at  $\gamma = 0$  the investment threshold under moral hazard and first best are equivalent, this means that for small  $\gamma$  the investment threshold is higher under moral hazard than under first best. Investment is then delayed relative to first best. For large enough  $\gamma$ , the sign of the comparative static of the investment threshold with respect to  $\gamma$  is negative, and the investment threshold under moral hazard is lower than under first best. Investment is then accelerated relative to first

best.

### 4.3 Pay Performance Sensitivity

The certainty equivalent of agent's promised continuation utility,  $V_t = -1/(\gamma r) \ln(-\gamma r W_t)$ , can be interpreted as her financial wealth. From Ito's lemma,  $dV_t = \frac{1}{2}\gamma r \sigma^2 \beta_t^2 dt + \sigma \beta_t dZ_t$ , so it is clear that  $\beta_t$  represents the manager's dollar sensitivity to the unexpected productivity shocks. The sensitivity of the agent's dollar value,  $V_t$ , to the changes of the value of the firm,  $b(X_t)$ , is a measure of the agent's pay-performance sensitivity (PPS) in our model. Looking at the nondeterministic components of the two values (that is, the volatility terms), pre-investment PPS is given by

$$PPS_t = \frac{\sigma \beta_t dZ_t}{\sigma X_t b'(X_t) dZ_t} = \frac{g'(a^*(X_t))k}{\mu b'(X_t)} = \begin{cases} \frac{\mu^2}{\mu^2 + \gamma r \sigma^2 \theta X_t k} & \text{if } a^*(X_t) < 1 \\ \frac{\theta k}{\mu b'(X_t)} & \text{if } a^*(X_t) = 1, \end{cases} \quad (56)$$

where the last equation in the case of  $a^*(X_t) < 1$  uses Equation (32). (Post-investment  $\widehat{PPS}_t$  is characterized by similar equations substituting appropriately  $\hat{k}$ ,  $\hat{a}^*$ , and  $\hat{b}$ .) Equation (56) shows that pay-performance sensitivity is the ratio of the agent's dollar incentives and the slope of the firm value in the productivity level. PPS depends crucially on the level of implemented effort. If the level of effort is interior, then PPS is independent of the shape of the value function, because the optimal effort chosen by the investor makes the manager's dollar incentives exactly linear in the slope of the firm value function. At full effort, PPS retains the inverse relation on the slope of the firm value function.

Equation (56) implies that if the effort is interior, then PPS decreases in the level of productivity,  $X_t$ , and also in the level of capital,  $K_t$ . This is due to the fact that the cost of providing incentives increases in the firm size more than the benefit of effort; the intuition is similar to that in He (2011).

An intriguing observation is that the PPS may increase after investment. This is not

immediately expected because we know from Section 4.1 that contracted effort decreases at investment which could call for lesser exposure of the agent to the firm value shocks. An increase in PPS at investment happens when the (observable and contactable) growth option creates a large sensitivity of the pre-investment firm value to productivity shocks for which the agent does not have to be compensated. In such risky firms is then optimal to set relatively low PPS. Investment then triggers a decrease of implemented effort but also a decrease in sensitivity of the firm value to productivity shock which may a result an increase in PPS. The next proposition specifies when this is the case.

**Proposition 4.** *The agent's pay-performance sensitivity increases at investment if*

$$\frac{g'(\hat{a}^*(\bar{X}))}{g'(a^*(\bar{X}))} > \frac{k}{\hat{k}}$$

*and decreases otherwise.*

In words, PPS increases at investment when the drop of the implemented effort at investment is sufficiently small relative to the inverse of the size of the growth option. This is the case for firms with low costs of effort (low agency conflicts) and large growth opportunities.

It is interesting to note that the condition in Proposition 4 is closely related to the condition given in Proposition 2 for the negative sign of the effect of risk aversion on investment threshold. This means that the agent's PPS response to investment can be linked to the distortion to investment timing due to agency conflicts. Specifically, our model makes the prediction that PPS decreases at investment if moral hazard conflicts accelerate investment and increases at investment if moral hazard conflicts delay investment.

## 5 Conclusion

We have presented a model of real options and dynamic moral hazard. We find that the affect of agency conflicts on investment timing depend on the severity of the conflict. When

the moral hazard problem is less severe, the optimal contract will implement high effort, but delay investment. When the moral hazard problem is more severe, the optimal contract will implement lower effort but will call for accelerated investment. The finding that moral hazard may accelerate investment is new and provides an alternative to empire-building based explanations of over investment.

The affect of investment on pay performance sensitivity also depends on the severity of the moral hazard problem. When the moral hazard problem is less severe, pay performance sensitivity increases after investment. When the moral hazard problem is more severe, pay performance sensitivity decreases with investment. These result provide a link between pay performance sensitivity and the nature the distortion on investment timing imposed by moral hazard.

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## A Proofs

*Proof of Lemma 1.* Consider an arbitrary contract  $\Pi = (\{c_t, a_t\}, \tau)$  and suppose the solution to managers optimization problem (1) for this contract is given by  $\{\tilde{c}_t, \tilde{a}_t\}$  and the manager's associated value for this contract is  $\tilde{W}_0$ .

Now consider the alternative contract  $\tilde{\Pi} = (\{\tilde{c}_t, \tilde{a}_t\}, \tau)$ . Note that under this contract the manager again gets utility  $\tilde{W}_0$  from the consumption effort pair  $\{\tilde{c}_t, \tilde{a}_t\}$ . We claim that the solution to manager's optimization problem (1) is again  $\{\tilde{c}_t, \tilde{a}_t\}$ . Indeed suppose it is not and that there is a alternative feasible consumption effort pair  $\{\check{c}_t, \check{a}_t\}$  such that this policy yields utility  $\check{W}_0 > \tilde{W}_0$  to the manager. The consumption effort pair  $\{\check{c}_t, \check{a}_t\}$  is also feasible under the original contract  $\Pi$  since

$$\begin{aligned} \lim_{t \rightarrow \infty} E \left[ e^{-rt} \int_0^t (c_s - \check{c}_s) ds \right] &= \lim_{t \rightarrow \infty} \left( E \left[ e^{-rt} \int_0^t (c_t - \tilde{c}_t) dt \right] + E \left[ e^{-rt} \int_0^t (\tilde{c}_s - \check{c}_s) ds \right] \right) \\ &= \lim_{t \rightarrow \infty} E \left[ e^{-rt} \int_0^t (c_s - \tilde{c}_s) ds \right] + \lim_{t \rightarrow \infty} E \left[ e^{-rt} \int_0^s (\tilde{c}_s - \check{c}_s) ds \right] \\ &= 0. \end{aligned}$$

Thus, the manager could achieve utility  $\check{W}_t > \tilde{W}_t$  under the original contract  $\Pi$ , a contradiction.

Finally note that the investor is achieves the same value under the new contract  $\tilde{\Pi}$  as under the original contract  $\Pi$ , since effort and investment are unchanged, and the transversality condition implies that the two consumption streams have the same present value. □

*Proof of Lemma 2.* Suppose  $S_t = \mathcal{S}$  and recall the definition of  $W_t(\Pi, \{X_s, K_s\}_{s \leq t}; \mathcal{S})$  and let  $\{\tilde{c}, \tilde{a}\}$  solve problem (4). We claim that  $\{\tilde{c} - rS, \tilde{a}\}$  solves problem (4) for  $S_t = 0$ . This plan gives the manager utility of  $W_t(\Pi, \{X_s, K_s\}_{s \leq t}; 0) = e^{\gamma r \mathcal{S}} W_t(\Pi, \{X_s, K_s\}_{s \leq t}; \mathcal{S})$ . Suppose there is some alternative  $\{\check{c}, \check{a}\}$  that yields a higher utility to the agent  $\check{W}_t(\Pi, \{X_s, K_s\}_{s \leq t}; 0) > W_t(\Pi, \{X_s, K_s\}_{s \leq t}; 0)$ . Now consider the plan  $\{\check{c} + rS, \check{a}\}$  and note that this plan is feasible

under  $S_t = \mathcal{S}$  but under this plan the manager can achieve utility

$$\begin{aligned}\check{W}_t(\Pi, \{X_s, K_s\}_{s \leq t}; 0) &= e^{-\gamma r \mathcal{S}} \check{W}_t(\Pi, \{X_s, K_s\}_{s \leq t}; 0) \\ &\geq e^{-\gamma r \mathcal{S}} W_t(\Pi, \{X_s, K_s\}_{s \leq t}; 0) \\ &= W_t(\Pi, \{X_s, K_s\}_{s \leq t}; 0)\end{aligned}$$

which contradicts the optimality of the  $\{\tilde{c}, \tilde{a}\}$ .  $\square$

*Proof of Proposition 1.* We show that the candidate policies are indeed optimal for the investor. Note that the compensation policy  $c_t$  is pinned down by the no savings condition. Define the stopped gain process by

$$G_t = \int_0^t e^{-rs} (X_s K_s - c_s) ds + e^{-rt} B(X_t, W_t) + \mathbb{I}(t \geq \tau) (e^{-rt} (\hat{B}(X_t, W_t) - B(X_t, W_t)) - e^{-r\tau} p).$$

When the  $W_t$  evolves according to (15), Ito's formula gives the following dynamics

$$\begin{aligned}e^{rt} dG_t &= \left( X_t K_t - \frac{1}{2} \theta X_t K_t a_t^2 + \frac{1}{\gamma} \ln(-\gamma r W_t) + a_t \mu X_t \mathcal{B}_X + \frac{1}{2} \sigma^2 X_t^2 \mathcal{B}_{XX} + \right. \\ &\quad \left. \theta \sigma^2 X_t^2 K_t^2 a_t (-\gamma r W_t) \mathcal{B}_{XW} + \frac{1}{2} (\sigma \theta X_t K_t a_t)^2 (-\gamma r W_t)^2 \mathcal{B}_{WW} - r \mathcal{B} \right) dt + \\ &\quad (\mathcal{B}_X - \gamma r W_t \theta a_t X_t K_t \mathcal{B}_W) \sigma X_t dZ_t + \mathbb{I}(t = \tau) (\hat{B}(X_t, W_t) - \mathcal{B}(X_t, W_t) - p) \quad (57)\end{aligned}$$

where  $\mathcal{B}(X_t, W_t) = B(X_t, W_t) + \mathbb{I}(t \geq \tau) (\hat{B}(X_t, W_t) - B(X_t, W_t))$ . Note that the drift term is clearly negative for any alternative policy while it is zero for the candidate policy. Now examine the last term. Under the candidate policy this term is zero due to the value matching condition. Under any alternative investment time  $\tau$ , this term is negative due to the smooth pasting condition and the concavity of  $\hat{B}(X_t, W_t) - B(X_t, W_t)$ . As a result the process  $G_t$  is a martingale under the proposed contract and a super martingale otherwise. The rest of the argument proceeds along the standard lines.  $\square$

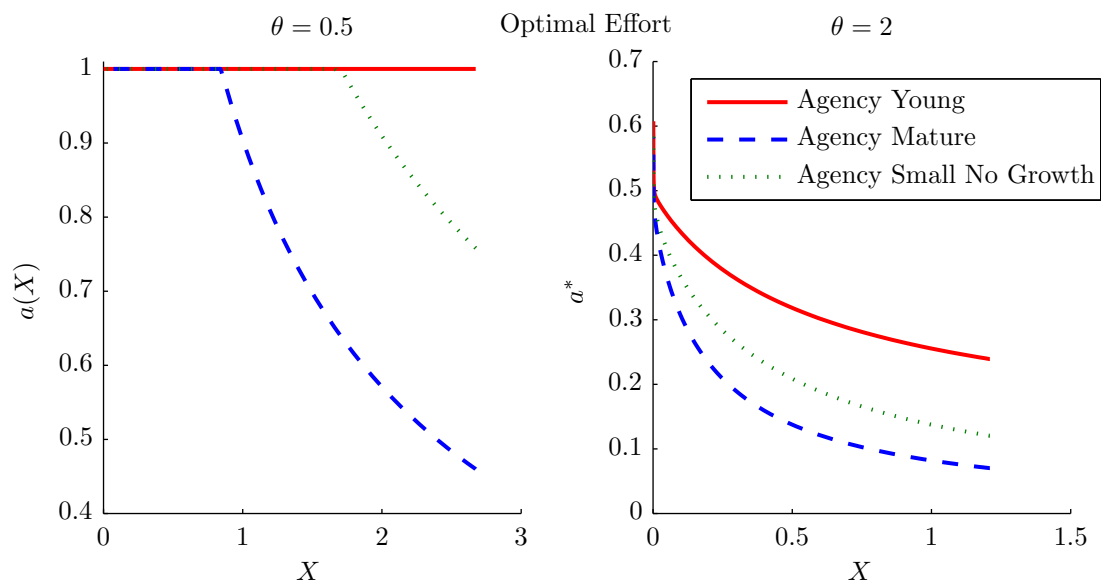


Figure 1: The optimal effort at two levels of the cost of effort,  $\theta = 1$  (left) and  $\theta = 2$  (right).

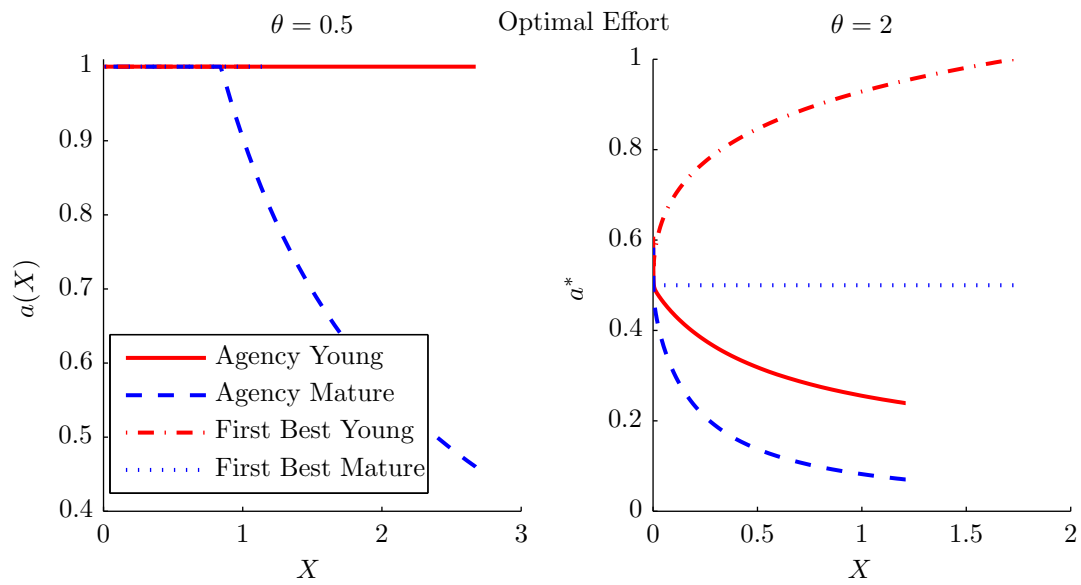


Figure 2: The optimal effort, in the first-best problem and under agency, at two levels of the cost of effort,  $\theta = 1$  (left) and  $\theta = 2$  (right).

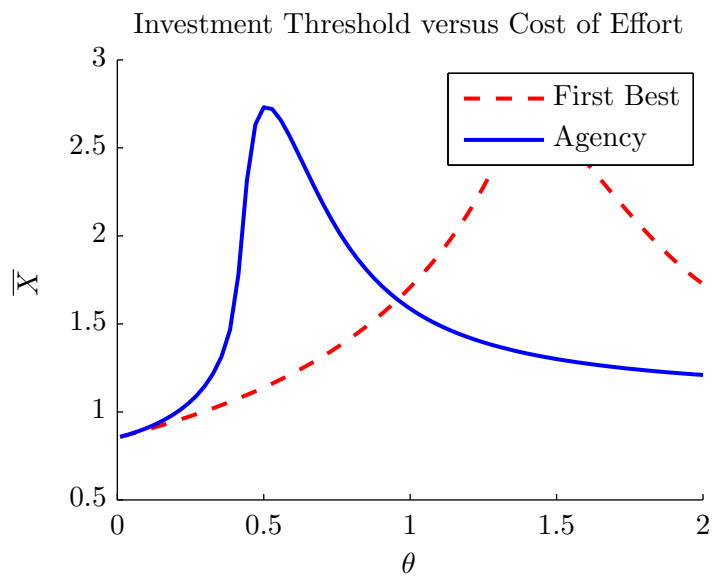


Figure 3: The investment threshold  $\bar{X}$  as a function of the cost of effort. For low costs of effort, the investment threshold in the moral hazard setting is above that of the first best. For high costs of effort, the relationship is reversed.

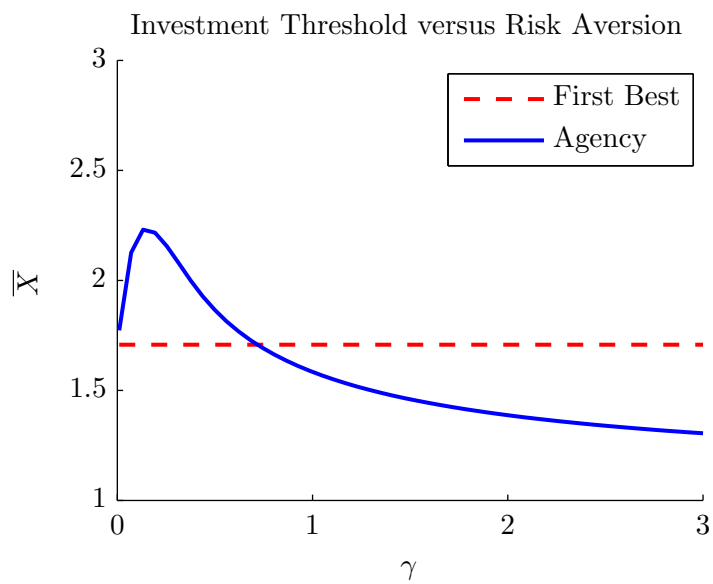


Figure 4: The investment threshold  $\bar{X}$  as a function of the risk aversion.

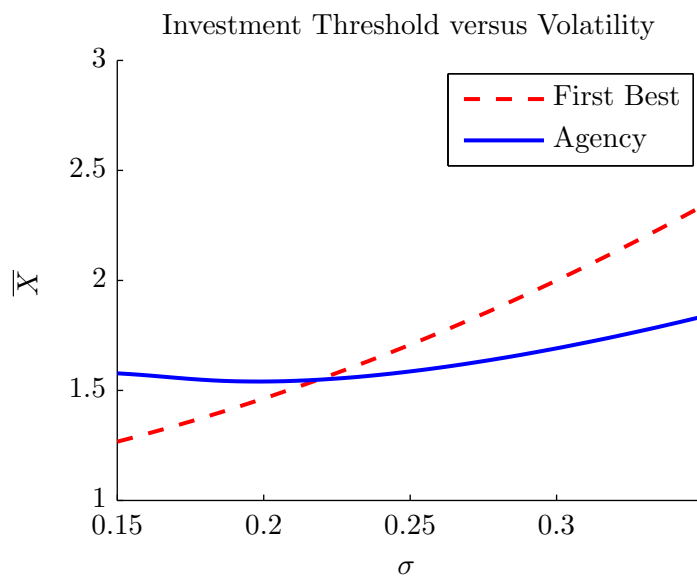


Figure 5: The investment threshold  $\bar{X}$  as a function of the volatility.