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Default and Credit Constraints in General Equilibrium

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Abstract

A recent strand of the literature convincingly demonstrates that both the sovereign default and the household default are properly explained by models in which agents face uninsurable income shocks, while being allowed to strategically default on their debt. We develop a similar model in a general equilibrium setting, and we prove that the market equilibrium features over-borrowing as well as too much default: if default may be individually optimal, it can deter the ex ante welfare. We show that regulating the credit market through borrowing constraints contributes to increase the aggregate welfare. More precisely, we characterize the path of optimal credit constraint in general equilibrium and for infinite horizon economy.

1 Introduction

The positive and normative implications of strategic default have recently attracted a lot of attention. Athreya (2002), Livshits (2007) and Chaterjee et al (2007) for instance study the default decision of households. Aguiar and Gopinath (2006), Arrelano (2008) and many others study the default decision of countries, using the seminal contribution of Eaton and Gersovitz (1981). These papers use a common incomplete contract framework, in which debt contracts cannot be written conditionally on the future realization of a given risky variable, such as income for example. This framework is appealing because empirical studies have shown that the incompleteness of insurance contracts seems to be a relevant assumption for many risks (see Zeldes (1989) for households or Eaton and Gersovitz (1981), Bai and Zhang (2011) at the country level). Moreover, the default decision and the related credit risk are well explained by economic variables (see as the quantitative work cited below). However, little is known about market efficiency in general equilibrium when strategic default is allowed. In partial equilibrium, Zame (1993) has proven that the option to default helps to complete contracts and can thus increase welfare. Nevertheless, there are only few results about the optimality of the allocation when strategic default is allowed in general equilibrium. In particular, we raise the question whether market economies generate the efficient level of default.[1]

In this paper, we investigate the efficiency of market allocation in general equilibrium when agents face uninsurable income risks and where strategic default is allowed. We show that this economy generates both excess borrowing and excess default in equilibrium. The reason for this result is that the price of public debt does not convey the right default incentives. Private debt is used by some agents as a self-insurance device against bad idiosyncratic shocks. This social value of private debt is not correctly internalized by private agents who default. Indeed, if they were not defaulting, the amount of safe assets would increase in the economy what would decrease the equilibrium incentives to default, because more risk sharing can be achieved in the economy. In words, default destroy liquidity, defined as the assets used by agents to self-insure, and the social cost of default is not correctly priced. The introduction of a simple financial regulation, such as credit constraints, can increase welfare because they prevent excess default and increase liquidity.

More precisely, we study a growing economy where risk adverse agents face idiosyncratic income shocks. There is no aggregate shocks, but certain financial frictions limit the amount of risk sharing. More precisely, we follow the literature in assuming the structure introduced by Eaton and Gersovitz (1981). Agents have access to one period non-contingent contracts. The history of past shocks is known when contracts are signed and contracts are exclusive. These assumptions avoid contract externalities, which would create inefficiencies in risk sharing arrangements (see Attar et al. (2012) for a discussion of those inefficiencies). At each period, agents can choose to default. In this latter case, we simply assume that they are excluded from the economy for ever. Our results do not depend on the shape of the default cost, as soon as agents are prevented from borrowing for a given period of time after their default. In addition, there are financial intermediaries which collect savings and which are able to fully diversify their risk across agents. As a result, the cost of the debt includes a premium which is the default probability. This framework is common to many papers because it minimizes the distortion generated by the introduction of equilibrium default, with respect to the standard general economy model.

We first study a three-period version of this economy to identify the social cost of default. In this economy, we show that introducing a simple credit constraint can reduce the equilibrium amount of default and increase welfare. We then move to the infinite horizon economy to study the optimal shape of credit constraints. It is known that this economy is difficult to characterize with standard dynamic programming tools because first order conditions are not sufficient to characterize the

[1] Livshits (2008) and Chaterjee et Al (2007) analyze how changes in the default costs affect average welfare. Nevertheless, they do not characterize the efficiency to market allocation for a given cost of default.

equilibrium. Indeed, the optimal individual amount of savings depends on the expected (discrete) default decision in all states of the world and at all future dates[2]. We deal carefully with non-convexities by studying agents with quasi-linear preferences, which allows us to characterize the different types of equilibria. In this economy, we derive the properties of optimal credit constraints using the ex-ante welfare function of Aiyagari (1994) as the social welfare function. We show that optimal credit constraints are not too-tight in the sense of Alvarez and Jermann (2000). They are the minimum credit constraints which prevent agents from defaulting in some states of the world. Thanks to this property, we are able to present numerical examples, in which credit constraints improve welfare compared to the *laissez-faire* economy. Optimal credit constraints typically bind for agents facing a bad idiosyncratic shocks for a given number of periods until they allow agents to default if they face additional bad idiosyncratic shocks. In general, the optimal outcome is thus a mix of credit constraint and default. In words, the credit constraints that would prevent agents from defaulting in all states of the world yield too little risk sharing. The absence of credit constraints reduces aggregate liquidity and risk sharing because of excess default. Both credit constraints and default are useful. Finally, we find that the duration of the binding credit constraints for agents experiencing bad idiosyncratic shock is a quantitative question.

This paper is first related to general equilibrium models with uninsurable idiosyncratic risks. The literature is often called the Bewley-Hugget-Aiyagari literature or the heterogeneous agents literature. In this literature, many papers impose an exogenous credit constraints with no default. Recent papers endogenize the credit constraints, by setting the credit constraints which prevent agents to default in all states of the world. (See Zhang (1997), Krueger and Perri (2006), Ábrahám and Cárceles-Poveda (2010) or Bai and Zhang (2010) who study carefully the recursive formulation of the problem). We show that we can increase welfare compared to the allocations studied in these economies by allowing default. Indeed, in our setup the optimal credit constraints do not prevent default in all states of the world.

Second, this paper is related on the literature on equilibrium default. At this stage models used in international finance often study partial equilibrium for which the riskless interest rate is exogenous (Eaton and Gersovitz 1981, Aguiar and Gopina 2006, Arellano 2008, Karaivanov and Wright 2010, Chatterjee and Eyigungor 2011, among many others). In partial equilibrium, we show that credit constraints can never increase welfare. Thus, our results are due the general equilibrium effect of default on the volume of liquidity and the real interest rate. The literature on household default (Athreya 2002, Livhists et al. 2007, Chatterjee 2007) studies general equilibrium models but the efficiency of the market allocation is not characterized due to the non-convexities, which

[2] To circumvent this problem, it is sometimes assumed that agents can choose assets in given finite set (Chatterjee et al. 2007, Chatterjee and Eyingungor 2001 for instance).

prevent the investigation of the general case.[3] Finally, this paper is also related to work trying to identify conditions for excess borrowing in equilibrium, such as Lorenzoni (2008).

The structure of the paper is the following. The second Section presents the environment. The third Section studies a finite horizon case. The fourth Section is the infinite horizon economy.

2 The Environment

Time is discrete $t = 1, \ldots, T$ where T may be equal to $+\infty$. The economy is populated by a growing continuum of agents of length Ω_t at date t. The growth rate of the population is constant and equal to g. Agents are indexed by $i \in [0, \Omega_t]$ and they are distributed according to the uniform distribution G_t over the segment $[0, \Omega_t]$. The number of agents in period t is thus simply $\Omega_t = \int_0^{\Omega_t} dG_t(i)$. There are two goods in the economy, a perishable consumption good and labor which is supplied by agents.

Each agent has preferences over streams of consumption $(c_t)_{t\geq 0}$ and labor supply $(l_t)_{t\geq 0}$ which is additive and time separable. As Scheinkmann and Weiss (1986), the period utility function is assumed to be quasi-linear: $u(c) - l$. We explain below the implications of linearity in the disutility of labor.

Agents can be in two states called unemployment and employment. When employed, agents can freely adjust their labor supply to produce consumption goods with a simple linear technology $y = l$. When unemployed, agents must work a fixed amount of time δ to produce an amount of goods δ. We make the following assumption

Assumption 1: $u'(\delta) > 1$.

We show below Assumption 1 insures that unemployed agents would always want to work more if they could.

The employed status is denoted as z_t^i. If it is employed then $z_t^i = 1$ and $z_t^i = 0$ otherwise. We denote as $z^{i,t}$ the history of employment status for agent i up to date t : $z^{i,t} = \{z_1^i, ..., z_t^i\}$. The change in employment status is stochastic and follows a first order Markov chain with transition matrix T :

$$T = \begin{bmatrix} \alpha & 1-\rho \\ 1-\alpha & \rho \end{bmatrix}$$

At each date t, we assume that the new mass of agents $g\Omega_{t-1}$ enter the economy as employed agents. We introduce a growing population to have simple stationary distribution in all the cases we consider.

[3] There is also a literature in dynamic contract theory which studies default as an optimal outcome in partial equilibirum (Hopenhayn, Werning 2008). Instead, we restrict the space of contract to allow for a general equilibrium analysis.

2.1 The Market Economy

We now introduce the informal frictions, which prevent agents to attain full risk sharing. We follow the literature introduced by Eaton and Gersovitz (1981) and used in Aguiar and Gopinath (2006) Arrelano (2008), Chatterjee et al (2007) or Livshits (2007) and many others. The financial frictions are the outcome of five assumptions.

1. In each period, there is a positive mass of financial intermediaries, who live for one period.
2. Only financial intermediaries have the technology to lend to agents. Financial intermediaries repay their debt.
3. The employment status of agents is not observable, when agents have to repay their debt.
4. The employment status and the previous volume of debt is observable when agents borrow.
5. Contracts are exclusive.

The first two assumptions exclude bilateral trade. Agents who save, will lend to financial intermediaries, who will lend to agents diversifying their risks. As there is no aggregate risks in the economy, financial intermediaries can offer a risk free rate to lenders and will charge a credit risk to borrowers. As financial intermediaries live for one period, only one period contracts are considered.[4] The third restriction concerns the information structure. It is assumed that when agents have to repay their debt, their employment status is not observable. As a consequence, contracts can not be written contingent on the next period employment status.

These three restrictions, one period contracts and non-contingent contract are common to the whole literature on heterogeneous agents, the so-called Bewely-Aiyagari-Huggett literature. They generate ex-post heterogeneity across agents because full-risk sharing can not be attained. In this literature, default is not allowed and the first three assumptions are enough to characterize the equilibrium contract. When default is allowed, two additional assumptions must be introduced.

First, the fourth restriction states that the current volume of debt and the current employment status is observable when the contract is written. This information is used by financial intermediaries to derive the equilibrium probability of default. Alternatively, one could assume that the whole history of idiosyncratic shocks is observable. Second, restriction 5 insures that contracts can be signed conditional on the total volume of debt made by each agents. An alternative assumption would be that the total volume of debt is contractible This assumption precludes contract

[4] Krueger and uhlig (2006) studies intertemporal contract in a related environment, under the assumption of intermediaries can commit to future payment whereas agenst can not.

externalities, which would introduce additional inefficiencies.[5]

Competition among financial intermediaries insure that they make no profit in equilibrium. The outcome of these assumptions is that loans of different size q' provided to households with different histories $z^{i,t}$ are different assets with different prices. Denote as $p_t\left(q', z^{i,t}\right)$ the period t price of a promise to pay back one unit of goods in period $t+1$ made by an agents of history $z^{i,t}$ who promise to pay back a total amount q'. Define as $P_t\left(q', z^{i,t}\right)$ the probability that an agents with history $z^{i,t}$ making a loan of size q' in period t default in period $t+1$. Finally, define as p_t the price of a promise to repay a unit of goods in period $t+1$, with zero credit risk, i.e. the safe price. The zero profit condition insures that the price $p_t\left(q', z^{i,t}\right)$ is

$$p_t^i\left(q', z^{i,t}\right) = p_t\left(1 - P_t^i\left(q', z^{i,t}\right)\right)$$

This market arrangement allows for equilibrium default, which is priced accordingly.

2.2 The State

The State is benevolent. It has no information about private agents. It can thus not implement transfers conditional on each agent productive status, otherwise full-insurance could be achieved. State intervention can only be achieved by introducing some constraints on financial intermediaries.

The State does not observe the characteristics of the agents but only the borrowed amounts. The only regulation that the State can introduce is a constraint on financial intermediary to impose a credit limit to households.

3 A Finite Horizon Example

In this section we study finite horizon example where the interaction of default and financial regulation can be studied. We study a three-period economy, $T = 3$. In this case, we need to make an additional assumption to allow for trade. We assume that there is a technology, which allows lenders to force employed agents to repay their debt. Unemployed agents repay their debt if they prefer to pay their debt instead of living in autarky.

Assumption 1: *Employed agents can be forced to repay their debt. Formally,* $P_t^i\left(a', \{z^{i,t-1}, e\}\right) = 0$

This assumption is first necessary to allow for positive trade in any period. Indeed, if all agents were given the possibility to default in an economy with T periods, $T < +\infty$, they would always default the last period, because there is no gain to repay the debt in period T. As a consequence,

[5] An alternative assumption would be to assume that contracts are non-exclusive, but that the total volume of debt is contractible. See Attar et al. (2011) for a general discussion of the role covenants with asymmetric information and no exclusivity.

there is no credit in period $T-1$ and by backward induction, no credit in any period. Second, the restriction of the option to default to unemployed agents is motivated by earlier result within the Eaton and Gersovitz framework such as Arellano (2008). It is known that when market are incomplete agents who may want to default are agents facing the bad idiosyncratic shock, what is not the case when markets are complete (see Alvarez and Jermann (2000)). As a consequence, this assumption allows studying the default incentive of agents with bad income shocks in a finite horizon economy, which is the relevant case. Finally, in the next section we dispose of this assumption and show that the mechanisms identify in this current Section are valid.

As the goal of this Section is to provide the most transparent example, we study an economy with the following parameter restrictions.

Assumption 2: $\mathrm{u}(\cdot) = \ln(\cdot)$, $\beta = 1$, $N_1^e = N_1^u = 1$.

There is a unit mass of both unemployed and employed agents at date 1, the discount factor is equal to 1, and we focus on the log case. To simplify notations we simply denote the agents for which $z^{i,t} = \{z_1^i, ..., z_t^i\}$ as $z_1^i, ..., z_t^i$ agents. For instance, e agents are agents employed in period 1, and euu agents are agents who were employed in period 1, and unemployed in periods 2 and 3. From the assumption about the probabilities of transition, we obtain the number of each type of agents (with obvious notations).

$$N_2^e = N_1^e \alpha + N_1^u (1-\rho) = \alpha + 1 - \rho \tag{1}$$

$$N_2^{eu} = 1 - \alpha \tag{2}$$

$$N_2^{uu} = \rho \tag{3}$$

$$N_3^e = \alpha N_2^e + (1-\rho)\left(N_2^{eu} + N_2^{uu}\right) \tag{4}$$

3.1 Autarky

The welfare in autarky of any agents is easy to determine. When an agent is unemployed, its instantaneous utility is simply $U \equiv u(\delta) - \delta$. When an agent is employed, he or she solves $\max_{c,l} u(c) - l$, subject to$c = l$. The solution is simply $u'(c) = 1$. Hence, $c = l = u'^{-1}(1)$. The instant utility is $E \equiv u\left(u'^{-1}(1)\right) - u'^{-1}(1)$. Denote as W_t^s the intertemporal welfare in period t of an agent in state $s = e, u$. In the last period one finds $W_3^u = U$ and $W_3^e = E$. In periods 2 and 1, one has

$$W_2^e = E + \beta\left(\alpha W_3^e + (1-\alpha) W_3^u\right) \text{ and } W_2^u = U + \beta((1-\rho) W_3^e + \rho W_3^u)$$
$$W_1^e = E + \beta\left(\alpha W_2^e + (1-\alpha) W_2^u\right) \text{ and } W_1^u = U + \beta\left((1-\rho) W_2^e + \rho W_2^u\right)$$

3.2 The First Best

In the complete market economy, the agents are able to buy insurance and equalize marginal utility in all state of the world. As the discount factor is 1, the price of a one period bond is 1 both in period 1 and 2.

$$p_1^{FB} = p_2^{FB} = 1$$

3.3 The Default Equilibrium

In this environment, three types of equilibria are possible. We study them in Appendix. For the sake of brevity, we focus on the relevant equilibrium where uu agents default in period 2. We follow a guess and verify strategy to construct this equilibrium. We start from period 3 and move backward.

In period 3, an employed agent with a net asset holding q always repays its debt, by assumption. Its utility, denoted as $V_3^e(q)$, is $V_3^e(q) \equiv \max_{c,l} u(c) - l$ subject to $c = l + q$. The solution is simply $V_3^e(q) = E + q,$. The utility $V_3^u(q)$ of an unemployed agents in period t with a net asset holding q

$$V_3^u(q) = \max\left(u(\delta + q), u(\delta)\right) \tag{5}$$

As a direct consequence, unemployed agents in period 3 always default on their debt (i.e., when $q < 0$).

At date 2, an employed agent who has an asset q enjoys an intertemporal utility $V_2^e(q)$ defined by

$$V_2^e(q) = \max_{c,l,q'} u(c) - l + \beta\alpha V_3^e\left(q'\right) + \beta(1-\alpha) V_3^u\left(q'\right)$$
$$\text{s.t.} \quad c + p_2 q' = l + q$$

These agents are the high income agents and save in equilibrium. They buy claims at the price p_2, where p_2 is the price in period 2 of a safe promise to repay one unit of goods in period 3. The choice of asset denoted as q_2^e solves the following Euler equation

$$p_2 = \beta\alpha + \beta(1-\alpha) u'\left(\delta + q_2^e\right)$$

If e type-agents save in period 2, eu ones have to borrow in period 2. As a consequence, eu agents default in period 3 if they stay unemployed, with a probability ρ. The price of their debt is thus $(1-\rho) p_2$. eu agents in period 2 with an asset q have the intertemporal welfare $V_2^{eu}(q)$

$$V_2^{eu}(q) = \max_{q_2'} u(c) - \delta + \beta(1-\rho) V_3^e(q\prime) + \beta\rho W_3^u$$
$$(1-\rho) p_2 q\prime + c = \delta + q$$

The asset choice of agents eu, denoted as q_2^{eu}, is given by the simple Euler equation

$$p_2 u'\left(\delta + q - (1-\rho)\, p_2 q_2^{eu}\right) = 1$$

Remember that uu agents are assumed to default, which has to be checked in equilibrium. Finally and following the same reasoning, one finds that the asset choices in period 1 are given by

$$V_1^e = \max_{q'} u(c) - l - p_1 q' + \beta\alpha V_2^{ee}\left(q'\right) + \beta(1-\alpha)\, V_2^{eu}\left(q'\right),$$

with the constraint $p_1 q' + c = l$. As a consequence, the asset demand q_1^e of employed agents in period 1 satisfies

$$p_1 = \beta\alpha + \beta(1-\alpha)\, u'\left(\delta + q_1^e - (1-\rho)\, p_2 q_2^{eu}\right)$$

Finally, the important choice is the asset demand of agents u in period 1. These agents face a price schedule $p_1(q', u)$, which may not be continuous. By assumption, these agents default in period 2, if they remain unemployed. This happens with a probability ρ. We thus assume that, when facing the equilibrium price schedule $p_1(q', u)$, u agent choose an amount $q^{'} < 0$ such that they default if they stay unemployed. Their default probability is thus ρ and the equilibrium price they face on their debt is $p_1(q', u) = (1-\rho)\, p_1$. Their program is

$$V_1^u = \max_{q'} u(c) + \beta(1-\rho)\, V_2^{ue}\left(q'\right) + \beta\rho W_2^{uu}$$
$$\text{s.t.} \quad c = \delta + (1-\rho)\, p_1(-q)$$

To show that this equilibrium exists, one has to derive the equilibrium price schedule $p_1(q', u)$, and to show that agents effectively choose an amount such that they can default in period 2.

Competition ensures that intermediaries make no profit in period 1 and in period 2. The no profit conditions in period 1 and 2 can be written as

$$N_1^e p_1 q_1^e + (1-\rho)\, N_1^u p_1 q_1^u = 0$$
$$N_2^{ee} p_2 q_2^e + N_2^{ue} p_2 q_2^e + (1-\rho)\, N_2^{eu} p_2 q_2^{eu} = 0$$

In words, the financial intermediaries sell claims to employed agents in period 1 at a price p_1 to buy the claims of unemployed agents at a price $(1-\rho)\, p_1$. As $1-\rho$ unemployed agents repay their debt next period (those who become employed), the intermediaries get just enough resources to pay back their debt. The same reasoning apply for period 2.

We now characterize this equilibrium and then provide existence conditions.

Lemma 1 *The period* 1 *price schedule for unemployed agents is characterized by a unique* $q_1^{\lim} < 0$ *such that*

$$p_1\left(q', u\right) = \begin{cases} p_1 \;\; \textit{if } q' \geq q_1^{\lim} \\ p_1(1-\rho) \;\; \textit{if } q' < q_1^{\lim} \end{cases}$$

The period 2 price schedule is

$$p_2\left(q', u\right) = \begin{cases} p_2 \text{ if } q' \geq 0 \\ p_2\left(1-\rho\right) \text{ if } q' < 0 \end{cases}$$

In the conjectures equilibrium, the price schedule is a simple step function. For instance, u agents sell their promise at a high price is they borrow an amount higher than the limit $q_1^{\lim}$. They sell their promise at a low price is they borrow more than $q_1^{\lim}$ because they default if they stay unemployed. The credit risk is priced by the discount $1-\rho$. With such a price schedule, the equilibrium exists if u agents choose an amount $q' < q_1^{\lim}$, what has to be checked in equilibrium.

Proposition 2 *We have*

1. $p_1 < 1 < p_2$*,*

2. *If a default equilibrium exists, it is unique.*

The previous proposition summarizes the distortion by ranking the two prices of the one-period safe asset in period 1 and 2, namely p_1 and p_2. The price in period 1 higher than price in the first best, 1, and the price in period 2 is lower than the first best price, 1. In words, there are too much liquidity in period 1 (or the liquidity is too cheap) and not enough liquidity in period 2. As uu agents default in period 2, they do not issue debt which can be used by other agents as liquidity. Their default thus reduce the amount of liquidity in period 2. In period 1, u agents borrow a lot, anticipating that they repay only if they become productive.

Item 2) of the proposition shows the uniqueness of a default equilibrium. Its does not exclude the possibility of other types of equilibria. The conditions for global uniqueness are provided in Appendix. Finally, we also provide the conditions for existence of a default equilibrium. Two conditions have to be fulfilled. First, uu have to choose to default. Second, u agents have to choose an amount of debt q_1^u such that they default if they become uu. We derive all the necessary conditions for the existence and the global uniqueness of the default equilibrium, such that no other type of equilibrium exist. The following proposition insures that there no equilibrium in which uu agents do not default.

Proposition 3 *In equilibrium, agents unemployed for* 2 *periods always default in period* 2*.*

The previous proposition states that there is no general equilibrium, in which uu agents do not default in period 2. We prove it by contradiction. We construct a deviating equilibrium, in which uu agents do not default. We derive general equilibrium implications and exhibit a contradiction. The reason for such an equilibrium not to exist is the assumption $\beta = 1$. For this high discount factor, the price of claims is high in period 1. It induces agents to borrow a lot what makes the default option attractive for bad idiosyncratic shocks.

The default equilibrium is the only relevant equilibrium to study the role of policies which affect the incentive to default. Indeed, strategic default only concerns agents uu. Default in period 3 is indeed not strategic. At a general level, one can easily characterize the inefficiency of the market economy. In an equilibrium where uu agents default, the prices do not reflect the social value of the period 2 debt they could issue if they stay in the economy. Employed agents can not coordinate to increase the price of the period 2 debt to provide the incentives for uu to stay in the economy.

3.4 The role of credit constraints

In this equilibrium, the default of uu agents in period 2 reduces the scope for risk-sharing between period 2 and 3 as some agents leave the economy. More precisely, the volume of asset traded between period 2 and period 3 is reduced because of default. The introduction of a regulation preventing agents uu from defaulting may increase risk-sharing between period 2 and 3 at the cost of reducing trade and hence risk-sharing between period 1 and 2.

To prevent default in period 2, the State can impose a limit on the balance sheet of intermediaries, which will appear as a credit constraints for households. Limiting the indebtness of agents in period 1 might prevent agents from defaulting in period 2, hence increasing the risk sharing. We thus study the welfare effect of the introduction of credit limit in period . Assume that the State impose a size D to contracts made by financial intermediaries in period 1.

To perform welfare analysis, we use the ex-ante welfare criterion, which is the equilibrium intertemporal welfare $W^{ex_ante}$of an agent which has the probability $N^e/\left(N^e+N^u\right)$ to be employed in period 1 and the probability $N^u/\left(N^e+N^u\right)$ to be unemployed in period 1. Hence, $V^{ex\_ante}=\left(V_1^e+V_1^u\right)/2$.

We now construct equilibria for which the credit constraint D binds in period 1 and for which uu agents do not default in period 2. As before, the provide below the conditions for which such an equilibrium exists. In period 3, e and u agents have the same program as in the previous section. In period 2, e and eu agents have the same program has before. Without default, uu agents now participate to the economy and have the program

$$V_2^{uu}\left(q_1^u\right)=\max_{q_2^{uu}} u\left(c\right)-\delta+\beta\left(1-\rho\right)V_3^{eue}\left(q_2^{uu}\right)+\beta\rho W_3^u$$

$$\left(1-\rho\right)p_2q_2^{uu}+c=\delta+q_1^u$$

In period 1, the program of agents e is the same as before, while the program of agents u can be expressed as:

$$V_1^u = \max_{q_1^u} u(c) + \beta(1-\rho) V_2^{ue}(q_1^u) + \beta\rho V_2^{uu}(q_1^u)$$
$$p_1 q_1^u + c = \delta$$
$$q_1^u \geq -D$$

Non-profit conditions in period 1 and 2 are now

$$N_1^e p_1 q_1^e + N_1^u p_1 q_1^u = 0$$
$$N_2^{ee} p_2 q_2^e + N_2^{ue} p_2 q_2^e + (1-\rho)\left(N_2^{eu} p_2 q_2^{eu} + N_2^{uu} p_2 q_2^{uu}\right) = 0$$

Compared to the case, with default, period 1 debt of u agents is now safe and sold at the price p_1 In period 2 uu agents participate to the economy and sell their debt at a price $(1-\rho)\, p_2$.

To next proposition derives some results concerning the effect of credit constraints on prices

Proposition 4 *If uu agent do not default and if $1-\alpha-\rho>0$, then an increases in the borrowing limit D, implies that p_2 increases, p_1 decreases, and q_2^e decreases.*

$$\frac{\partial p_1}{\partial D} < 0, \quad \frac{\partial p_2}{\partial D} > 0$$

The previous proposition characterizes the effect of a relaxation of credit constraints when they prevent uu agents from defaulting. The proposition provides results for the case where $\alpha+\rho<1$, which implies that the productive and non-productive states are not very persistent. This case is used in the numerical example below and implies that agents demand for insurance will be high, what is the relevant case to identify the mechanisms. In this case, a relaxation of the credit constraint (a higher D) reduces the first period price of the safe asset and increases the second period price of the safe asset, $p_{2.}$ Indeed, a relaxation of the credit constraint increases the volume of debt in period 1, what decreases its price, because e agents have to accept to hold a higher volume of assets. The evolution of prices in period 2 is driven by the behavior of eu agents in period 2. Those agents are wealthier in period 2, when the credit constraints is relaxed because they were able to buy more assets in period 1. As a consequence, they borrow less to smooth consumption between period 2 and 3. As a consequence, the supply of debt is reduced and the price of debt in period 2 is higher.

The overall effect of a change in the credit constraints on total welfare is summarized by the following expression.

$$\frac{\partial V^{ex\_ante}}{\partial D} = \underbrace{p_1 u'(\delta + p_1 D) - (1-\rho) - \frac{\rho}{p_2}}_{\text{Relaxation of the credit constraint}} + \underbrace{D\frac{\partial p_1}{\partial D}\left(u'(\delta + p_1 D) - 1\right)}_{\text{Redistribution in period 1}} + \underbrace{(1+\alpha-\rho)\, q_2^e \frac{\partial p_2}{\partial D}\left(\frac{1}{p_2} - 1\right)}_{\text{Redistribution in period 1}} \tag{6}$$

Three terms appear. The first one is the relaxation of the credit constraints for u agents in period 1. This term is positive and would be obtained in partial equilibrium. Note that the relaxation of the credit constraints has direct effects on the welfare of e agents in period 1 because their portfolio is optimized at the equilibrium price (technically, this is the envelop theorem). The two other terms reflect general equilibrium effects due to changes in equilibrium prices. First, the change in the price of debt in period 1 yields a welfare change between agents e and u agents in period 1. A lower price of debt in period 1 ($\frac{\partial p_1}{\partial D}$) makes it less costly for e agents to self-insure and decreases the value of the debt of u agents. The total welfare effect is proportional to the volume of asset D, and to the difference in the marginal utility of the two types of agents ,$u'(\delta + p_1 D) - 1$. Second, the change in price in period 2 generates a welfare change among agents in period 2, which can be interpreted among the same lines. The difference in marginal utilities across employed and unemployed agents in period 2 boils down to the term $\frac{1}{p_2} - 1$.

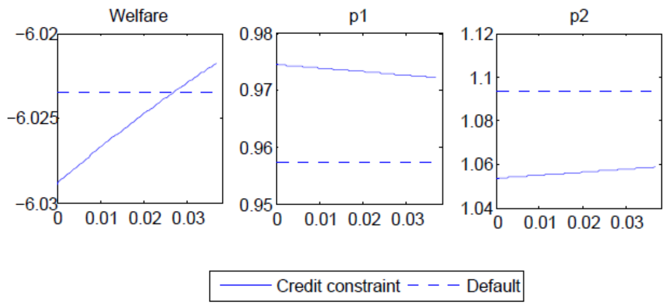
When the derivative $\frac{\partial V^{ex\_ante}}{\partial D}$ is positive, the welfare improving credit constraint is the highest debt limit $D^{\lim}$ such that uu do not default. When $D \frac{\partial V^{ex\_ante}}{\partial D} is$ higher, the welfare is reduced because credit constraint limits risk-sharing in period 1, without contributing to keep agents in the economy in period 2. When the derivative $\frac{\partial V^{ex\_ante}}{\partial D}$ is negative, the optimal credit constraint is 0. In this case, the introduction of credit constraints does not contribute to increase welfare.

To assess the usefulness of credit constraints, one has to compare the highest welfare V^{ex_ante} which can be attained, when the credit constraint is set at its optimal value $D^* \in [0, D^{\lim}]$ to the ex ante welfare V^{ex_ante} reached in the decentralized economy where uu agents default in period 2.

To summarize the trade-off identified in this Section, introducing credit constraints creates more liquidity in period 2 but may limit risk sharing in period 1. The optimal credit constraint balances the gain of more risk-sharing in period 1 and adverse redistributive effects. We will show that these trade-off also exist in the infinite horizon economy. Before, we present a simple numerical example where the introduction of credit constraints is welfare improving.

3.5 A numerical example

We set $\delta = 0.8, \alpha = 1/2$, $\rho = 0.1$. In this economy, one can check that the only market equilibrium is a default equilibrium where uu agents. As $1 - \alpha - \rho > 0$, the conditions of Proposition 4 are fulfilled. The introduction of credit constraint in period 1 increases welfare, and the optimal credit constraints is the highest credit constraint which prevent agents from defaulting. These results can be easily represented graphically. Figure 1 plots the welfare the period 1 and period 2 prices, p_1 and p_2 as a function of the credit limit D for $D \in [0, D^{\lim}]$. In this example, the optimal credit limit is the highest one, which prevents uu from defaulting.



Welfare, period 1 and period 2 prices as function of the credit limit D. The dash-line plots the same values for the economy without credit limit.

The loosening of the credit constraint increases period 1 average intertemporal welfare to a level which is above the one obtained in the market economy without credit constraint. The effect of loosening the credit limit is to decrease period 1 price, because more assets are available in period 1 and to increase the price of period 2 debt, as discussed in Proposition 4. Recall that in the first best, the period 1 and 2 prices are equal to 1. A a consequence, the introduction of credit constraints bring both prices closer to their first best value and restore the ability of agents to smooth consumption intertemporally. The general equilibrium effect of such price movements is to increase total welfare, as explained in the discussion of equation (6).

The goal of this simple example is to provide the intuition why introducing credit constraints can increase welfare in general equilibrium. They avoid that some agent leave the economy, because their debt has a social value due to market incompleteness. This example is based on some parameter restrictions and on a specific assumption for trade to occur in a finite horizon case. We now study the infinite horizon economy and to derive the optimal path of credit constraints.

4 Infinite horizon

4.1 First best

In the first best, all marginal utilities from consumption are equal to 1. Each agent consume $c^{FB} = u'^{-1}(1)$. From transition probabilities and the growth rate of the population, we find that the number of employed agents is

$$n = g\left(1 + g - \alpha - (1-\rho)(1-\alpha)\frac{1}{1+g-\rho}\right)^{-1}$$

Resource constraints imply that the labor supply of employed agents is $l = \left(c^{FB} - (1-n)\,\delta\right)/n$.

4.2 Autarky

In autarky, agents do not have access to any type of saving technology. Agents only choose their labor supply, if possible. Employed agents solve the static program $\max_{c,l} u(c) - l$ subject to $c = l$. Unemployed agents are constrained at $c = l = \delta < u'^{-1}(1)$. We deduce, with obvious notations

$$c_{aut,u} = \delta \text{ and } l_{aut,u} = 0$$
$$c_{aut,e} = l_{aut,e} = u'^{-1}(1)$$

We denote V^e_{aut} and V^u_{aut} the intertemporal welfare of an employed and unemployed agents in autarky, respectively. From the previous expressions of labor supply and consumption, we deduce the following expressions for intertemporal welfare:

$$\begin{bmatrix} V^e_{aut} \\ V^u_{aut} \end{bmatrix} = \left(I_2 - \beta \begin{bmatrix} \alpha & 1-\alpha \\ 1-\rho & \rho \end{bmatrix} \right)^{-1} \begin{bmatrix} u\left(u'^{-1}(1)\right) - u'^{-1}(1) \\ u(\delta) - \delta \end{bmatrix} \tag{7}$$

where I_2 is the 2×2 identity matrix. Since $\beta < 1$, the inverse in (7) exists.

4.3 Market equilibrium

This Section characterizes the allocation when the horizon is infinite and when agents have the possibility to default. As the program is not convex and first order conditions are not sufficient to characterize the equilibrium, we first study the structure of the equilibrium.

First, the program of an agent i in period t can be written in compact form. If an agent has not defaulted in the past, denote as I_t the default decision of agent i in period t, $I^i_t = 0$ if the agent default in period t and $I^i_t = 1$ if the agent chooses not to default. As a consequence, $\prod_{j=0}^{k} I^i_{t+j} = 0$ if the agents have defaulted between period t and period $t+k$, and $\prod_{j=0}^{k} I^i_{t+j} = 1$ if the agent has never defaulted. With this notation, the program of agent i in period t, if he or she has not defaulted before is

$$W_t\left(a^i_{t-1}, z_t\right) = \max_{\{a^i_{t+k}, c^i_{t+k}, l^i_{t+k}, e^i_{t+k}, I^i_{t+k}\}_{k\geq 0}} \sum_{k=0}^{\infty} \beta^k \left(u\left(c^i_{t+k}\right) - l^i_{t+k}\right) \tag{8}$$

subject to the following constraints, for all $k \geq 0$:

$$\tilde{p}^i_{t+k}\left(a^i_{t+k},\, z_{t+k}\right) a^i_{t+k} + c^i_{t+k} = l^i_{t+k} + I^i_{t+k}\, a^i_{t+k-1}, \tag{9}$$

$$l^i_{t+k} = \left(1 - z^i_{t+k}\right)\delta + z^i_{t+k} e^i_{t+k}, \tag{10}$$

$$a^i_{t+k} = 0 \text{ if } \prod_{j=0}^{k} I^i_{t+j} = 0, \tag{11}$$

$$c^i_{t+k},\, e^i_{t+k} \geq 0,\ I^i_{t+k} \in \{0,1\},\, a^i_{t+k} \geq \bar{a} \tag{12}$$

In each period, the agent chooses its asset net holding a^i_t, which can be negative if he or she borrows, its consumption level c^i_t, its labor supply l^i_t, and its default decision I^i_t. When the agent chooses to default in period t, $I^i_t = 0$, she loses her financial income if $a^i_{t-1} > 0$ or do not repay her debt if $a^i_{t-1} < 0$. The price of the claims of agent i borrowing an amount a^i_t in state z^i_t is $\tilde{p}^i_t\left(a^i_t, z^i_t\right)$. Equality (9) is the budget constraint. The right hand side is the sum of labor and financial income. Equality (10) states that when the agent is employed $z^i_t = 1$, he or she can freely adjust the labor supply, when she is unemployed, $z^i_t = \delta$, the labor supply is $l^i_t = \delta$. Finally, the constraint (11) states that when the agent has defaulted in the past, she is excluded from the financial markets for ever. The last constraint is the transversality condition. The constraints (12) are non negativity constraints, for consumption and labor supply, the definition of the discrete default choice, and a constraint which states that debt must be higher than a finite value to avoid Ponzi games. The debt limit is assumed to be very high and will not bind in any equilibrium under consideration. It is thus a convenient substitute to transversality conditions in this setup.

Due to competition, the expected return on the claim of the agent i who has not defaulted in the past is the same as the one on the safe asset. If p^s_t is the price of the safe asset in period t, one must have

$$\tilde{p}^i_t\left(a^i_t, z_t\right) = p^s_i\left(1 - \pi^i_{t+1}\right)$$

where π^i_t is the probability in period t that the agents default in period $t+1$. This one can be written formally as

$$\pi^i_t = \mu_t\left(z^i_{t+1} | I^i_{t+k+1}\left(a^i_t, z^i_{t+1}\right) = 0\right)$$

where μ_t is the measure in period t over individual states in period $t+1$. The equilibrium of financial market is simply $\int_{i \in \Omega_t} p\left(a_i, z^i\right) a_i G\left(di\right) = 0$. In this Section, we focus on stationary equilibria, where the price of the safe asset is constant and denoted as p. As a consequence the price schedule will be constant over time $\tilde{p}^i_t\left(a, z\right) = \tilde{p}\left(a, z\right)$.

The problem (8)–(12) is not convex because the price schedule $\tilde{p}^i_t\left(a^i_t, z_t\right)$ may not be continuous in a^i_t. In words, according to the level of debt, agents may face different prices and thus agents may compare different level of debt optimized for the various prices they may face. First order conditions are not sufficient to characterize optimality. Although the problem is not convex, it can be written in a recursive form (See Stockey and Lucas, Theorem 9.4). We define the following program:

$$V\left(a,D,z\right)=\max_{a',c,l,I}u\left(c\right)-l+\beta V\left(a',D',z'\right) \tag{13}$$

subject to

$$\tilde{p}\left(a',z\right)a'+c=l+Ia, \tag{14}$$

$$l=\left(1-z\right)\delta+ze, \tag{15}$$

$$D'=D\times I \tag{16}$$

$$a'=0 \text{ if } D'=0, \tag{17}$$

$$c,\ e\geq 0,\ \ I\in\{0,1\} \tag{18}$$

The prime represents the next period value of the variables. The state variable D is an indicator equal to 1 if the agents have not defaulted in the past and equals to 0 otherwise. The solution of the problem defines the policy rules which characterize the default decision for each type of agents $z=0,1$ for each level of asset a.

Consider employed agents for which $z=1$. As in a standard program, the labor choice is not an intertemporal choice. Hence, the first order condition for the labor choice yields

$$c=u'^{-1}\left(1\right)$$

Consumption is thus independent of past wealth, which only affects the labor supply. To see the implication of this result consider the choice of an employed agent who chooses not to default in the current period (what will be the case in equilibrium), its intertemporal welfare can be written as

$$V\left(a,1,1\right)=a+\max_{a'}u\left(u'^{-1}\left(1\right)\right)-\left(u'^{-1}\left(1\right)+\tilde{p}\left(a',1\right)a'\right)\beta V\left(a',D',z'\right) \tag{19}$$

In words, the value function is linear in current wealth a and the asset choice a' does not depend on the current amount of wealth. All employed agents who do not default thus choose the same amount of wealth. As shown below, this property will considerable simply the structure of the equilibrium, which allows us to derive new results. Finally, as the agents always have the option to default we have $V\left(a,D,1\right)\geq V_{aut}^{e}$ and $V\left(a,D,0\right)\geq V_{aut}^{u}$.

From the stationary assumptions on can easily compute the share of each agents in all employment status. First, the share of employed agents participating to financial market in period t is

$$n=g\left(1+g-\alpha-(1-\rho)(1-\alpha)\frac{1-\left(\frac{\rho}{1+g}\right)^{M}}{1+g-\rho}\right)$$

The share of unemployed agents for at any period $k \geq 1$ is equal to

$$n_k^u = \frac{(1-\alpha)\rho^{k-1}}{(1+g)^k} n$$

By convention, we denote $n_0^u \equiv n$ the number of employed agents, i.e. unemployed for 0 period.

We first exhibit the structure of the allocation in the equilibrium with default.

Proposition 5 *At each date t*

1) all employed agents who have not defaulted save the same amount denoted as a_0

2) agents who are unemployed for exactly k periods (and thus employed at period $t-k$), who have not defaulted, save the amount a_k,

The previous proposition states that all employed agents save the same amount denoted as a_0. There is thus no end-of-period wealth heterogeneity among employed agents, who have no defaulted in the past. This result is an implication of the linearity in the labor disutility, which is known to reduce heterogeneity (Scheinkman and Weiss 1986, Lagos and Wright 2005 and Challe and Ragot 2012 for a recent analysis). A direct implication is that the amount of savings of agents will depend only on the number of consecutive periods of unemployment. All agents unemployable for k periods save an amount a_k. As a consequence, the structure of wealth heterogeneity generated by the model boils down to a series $\{a_k\}_{k=0,\ldots,\infty}$ a saving rate for the various types of agents (having defaulted or not). An additional implication is that the price of claims depends only on the number of periods of unemployment in equilibrium, the equilibrium price is thus $\{p_k^u\}_{k=0,\ldots,\infty}$. By convention p_0^u, is the price faced by employed agents. This equilibrium outcome is much simpler than the outcome in general case, which is a density function defined over $\mathbb{R}$. The equilibrium on financial markets can be simply written as

$$\sum_{k=0}^{\infty} n_k^u p_k^u a_k = 0$$

The next proposition summarizes the price schedule faced by each type of agents, employed and unemployed. This price schedule depends on the equilibrium default probability, which is determined by their current employment status and the amount borrowed.

Proposition 6 *if $1 > p > \beta$, there exists $A^e < A^u < 0$ such that the price schedule is*

$$p\left(a', u\right) = \begin{cases} p \text{ if } a' \geq A^u \\ p\left(1-\rho\right) \text{ if } A^e \leq a' < A^u \\ 0 \text{ if } a' < A^e \end{cases} \quad \text{and } p\left(a', e\right) = \begin{cases} p \text{ if } a' \geq A^u \\ p\alpha \text{ if } A^e \leq a' < A^u \\ 0 \text{ if } a' < A^e \end{cases}$$

The proposition is proven under the condition that the equilibrium price is between 1 and β, what is the relevant case[6]. The price schedule is a step function which has three levels. Consider

[6] One can construct equilibria where this condition is not fufilled. These equilibria do not seem quantitatively relevant

an agent in period t, if she borrows less than a first quantity A^u then she will not default next period, whatever her employment status. In this case, the price of the claim is the price of a safe claim. p. If she borrows strictly more than A^u but less than a lower quantity A^e, she will default if she is unemployed next period, but she will not default if she is employed next period. As a consequence, the price of the claims is the price of the safe claim corrected by the probability to be employed next period, which is the probability to repay ($1-\rho$ for unemployed agents, and α for employed agents). Finally, for amount borrowed strictly higher than A^e, the agents default both if she is employed or unemployed. In this case, the price of the claim is 0. The technical part of the proof is the result that $A^e < A^u < 0$.

An implication of the price schedule is that employed agents never default. Indeed, if an agent borrow a positive amount which induces default if she is employed next period, then she will default in all states and the price of the claim is 0, what is a contradiction.

We next characterize the type of equilibrium in this economy. We show that there is only two types of equilibria. First, unemployed agents do not default. In this case, their borrowing amount reaches the negative amount A^u in a finite number of periods and stays at its value. Second, agents default after a finite number of unemployment periods. The first type of equilibrium is an equilibrium with incomplete market and endogenous credit constraints. Agents can choose not too default even if they have the option to do so. This allows them to have a higher intertemporal welfare if they are employed. The next proposition summarizes the structure of this two types of equilibrium.

Proposition 7 *We face one of these situations:*

1. *either there exists $K < +\infty$, such that $a_k = A^u$, for $k \geq K$,*
2. *or there exists $M < +\infty$, such that $A^e \leq a_M < A^u$ and agents unemployed for $M+1$ periods default.*

We next turn to the properties of equilibria with default. Our goal is to study the properties of the credit constraints which maximize welfare.

4.4 Credit constraints

The State can not implement a direct transfer to agents according to their employment status. It can nevertheless impose a financial regulation on financial intermediaries to limit credit. We now characterize the optimal credit constraint solving a Ramsey program for the State. We then show how to implement these credit constraints by a simple financial regulation.

First, assume that the State can impose a credit constraint $B_k < 0$ for agents unemployed during k periods. The program of private agents now depends on the number of consecutive periods of unemployment, which is denoted N_t^i in period t. For employed agents, we adopt the convention $N_t^i = 0$. The choices of agents in period t depends on her initial wealth, her employment status, her default decision in the past, and the number of consecutive periods of unemployment. This program can be written recursively as

$$V\left(a_{t-1}^i, N_t^i, D_{t-1}^i\right) = \max_{a_t^i, c_t^i, l_t^i, e_t^i, I_t^i} u\left(c_t^i\right) - l_t^i + E\beta V\left(a_t^i, z_{t+1}^i, D_t^i, N_{t+1}^i\right) \tag{20}$$

subject to

$$\tilde{p}_t^i\left(a_t^i, N_{t-1}^i, z_t\right) a_t^i + c_t^i = l_t^i + I_t^i a_{t-1}^i, \tag{21}$$

$$l_t^i = \left(1 - z_t^i\right)\delta, + z_t^i e_t^i, \tag{22}$$

$$a_t^i = 0 \text{ if } D_t^i = 0, \tag{23}$$

$$a_t^i \geq B_{N_t^i} \tag{24}$$

$$D_t^i = D_{t-1}^i \times I_t \tag{25}$$

$$N_t^i = \left(1 - z_t^i\right)\left(N_{t-1}^i + 1\right) \tag{26}$$

$$I_t^i \in \{0, 1\} \tag{27}$$

$$\lim_{t\to\infty} \beta^t u'\left(c_t^i\right) a_t^i = 0 \tag{28}$$

Note that the number of periods of unemployment has to be introduced has a state variable because it affects the credit constraint faced by agents. As before, competition insures that the price schedule is given by $\tilde{p}_t^i\left(a_t^i, N_t^i, z_t\right) = p_i\left(1 - \pi_t^i\left(a_t^i, N_t^i, z_t\right)\right)$ where
$\pi_t^i\left(a_t^i, N_t^i, z_t\right) = \mu_t\left(z_{t+1}^i | I_{t+k+1}^i\left(a_t^i, \left(1 - z_{t+1}^i\right)\left(N_t^i + 1\right), z_{t+1}^i\right) = 0\right)$.

We assume that the State maximizes the ex-ante welfare in the sense of Aiyagari (1994). This welfare criterion is the expected welfare where the probability of an agent to be in a given state is equal to the share of the population in this state. This welfare criteria is similar to a utilitarian welfare objective. To simply the notations denote as $V_k^u(a) \equiv V(a, k, 1)$ the welfare of an unemployed agents for k periods holding an amount of wealth a and not having defaulted in the past. With obvious notations, we note $V^e(a) \equiv V(a, 0, 1)$. The ex-ante welfare satisfies, in steady state

$$\begin{aligned}(1+g)V = \sum_{k=1}^{\infty}\left[(1-\rho)n_k^u V^e(a_k) + \rho n_k^u V_{k+1}^u(a_k)\right] \\ + \alpha n^e V^e(a_0) + (1-\alpha)n^e V_1^u(a_0) + gV^e(0)\end{aligned} \tag{29}$$

The program of the State is to maximize V with respect to $\{B_k\}_{k=1..\infty}$, when agents solve the problem (20) subject to constraints (21)-(28) and subject to the financial market equilibrium.

The next proposition summarizes the main properties of the optimal credit constraints.

Proposition 8 *1. If the optimal credit constraints after N periods of unemployment and that agents default after $M \leq +\infty$ periods of unemployment then*

$$V_{k+1}^{u}\left(B_k\right) = V_{aut}^{u} \text{ for } k = N....M-1. \tag{30}$$

2. *If $M < +\infty$, B_k is decreasing for $k = N....M-1$.*

3. *If $M = +\infty$, $B_k = B$ for $k \geq N$.*

The first item of Proposition (8) states that if the optimal credit constraints bind for an agent unemployed for k periods, then these credit constraints are such that the agent is indifferent between defaulting or not if she stays unemployed the following period. In other words, the credit constraints are not too tight in the sense of Alvarez and Jermann (2000). The credit constraints is the maximum amount of debt which prevents default the following period. This results allows characterizing the shape of the optimal credit constraints. Result 2 states that, if unemployed agents default in equilibrium, then the binding credit constraints are decreasing as agents stay unemployed. The credit constraints have to become looser and looser for unemployed agents to reach their autarky welfare while repaying a growing amount of debt. Item 3 states that if the optimal credit constraints are such that the agents do not default in equilibrium, then the credit constraints are constant for all unemployed agents. This type of equilibrium is the one studied by Zhang (1997) and recently by Bai and Zhang (2011), among others. It corresponds to an incomplete market environments with endogenous credit constraints.

To understand the trade-off at stake, which yield binding optimal credit constraints the next Proposition analyzes the effect of credit constraints in partial equilibrium, for a given value of $p > \beta$.

Proposition 9 *Partial equilibrium. If $p > \beta$ is given, then:*

1. *Credit constraint can never increase welfare*

2. *In an equilibrium where credit constraint binds after $N > 0$ periods and where agents default after $M \leq +\infty$ periods of unemployment, and where (30) holds, B_k increases $N \leq k \leq M$ when M increases.*

The first result is that credit constraint do not increase welfare in partial equilibrium. Credit constraints can be useful because of their effect on asset prices. The second result allows identifying the extensive and intensive effect of credit constraints: If credit constraints are defined to prevent more and more agents from defaulting, then each agents can borrow less (B_k becomes less negative).

In other words, when credit constraints are used to increase liquidity by using the extensive margin (preventing more and more agents to default), then there is a negative effect on the intensive margin. The interaction between these two effects defines a optimum value of M for which the volume of debt is the total volume of debt allows for the highest amount of risk-sharing.

The previous proposition characterizes an optimal path for credit constraints. We conjecture that this path can be implemented by imposing a maximum amount of default on intermediaries equal to $\rho n_M^u L_t$ in period t. In this case, financial intermediaries have to set credit limit to prevent agents from defaulting. Competition insures that the credit limit set by financial intermediaries are not too tight. We still have to prove this conjecture.

4.5 Numerical Example

We propose a calibration for the previous infinite horizon economy. We exhibit the path of credit constraints which maximizes welfare.

We choose the function u to be a CRRA and in particular: $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, in which we calibrate σ to 2.5, i.e. an elasticity of intertemporal substitution of 0.4. We assume that the period is the year, so we choose $\beta = 0.95$. Regarding transition probabilities, we take $\alpha = 0.95$ and $\rho = 0.10$. The home production δ is set up to 0.5, which roughly matches the unemployment benefits in the US. Finally, we choose a rate of growth g of the population equal to 0.5% The table 1 summarizes our calibration.

Parameters	σ	α	ρ	δ	g
Values	2.5	0.95	0.10	0.5	0.5%

Table 1: Calibration of the model

Under this calibration, 94.7% of the population is employed and 0.1% of the population is in autarky, in each period. In this economy we find that agents would default after two periods of unemployment. We maximize the welfare according to the number of period of unemployment M that we impose before default. We find that it is optimal to impose credit constraint after two periods of unemployment and to let agents default after 5 periods of unemployment.

Figure 2 plots the path of asset holdings of unemployed households a_k as a function of the number of periods of unemployment k. The asset holding is a decreasing function of the number of consecutive unemployment periods. After two periods of unemployment, agents face credit constraints. the path of credit constraints is decreasing from period 2 to period 4. Finally, the price of the claim of agents is $p = 0.962$ until they reach four periods of unemployment. The price of claims is then $p\,(1-\rho)$=0.865.

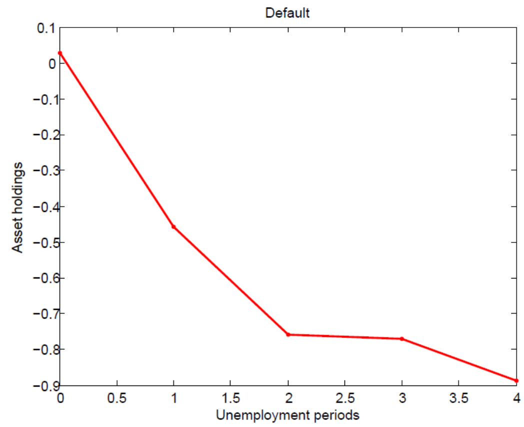


Figure 2: Net asset holdings a_k as a function of the number of period of unemployment.

To exhibit the effects of credit constraints, we perform the following exercise. We consider a number of periods of unemployment M, and we compute the optimal path of credit constraints for agent to default after exactly $M+1$ periods of unemployment. The results are given in figure 3, where we plot ex-ante welfare. We also plot the welfare for the first best allocation as a dashed line to ease comparison. One can observe that the welfare is hump shaped and maximized at $M = 4$.

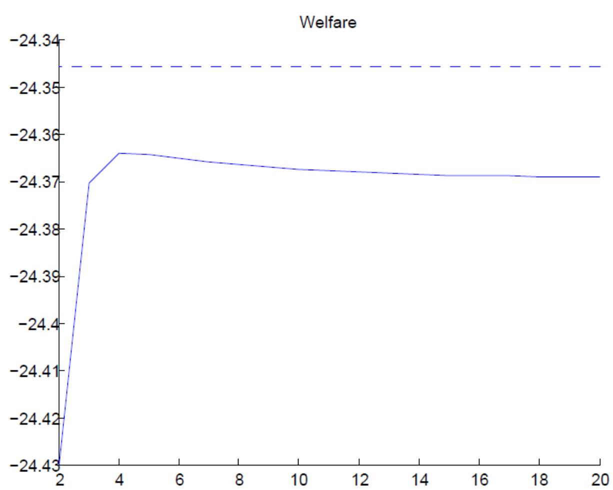


Figure 3: Welfare as function of number of period of unemployment before default.

This numerical example confirms that the optimal financial regulation, which is the optimal path of credit constraint in our model, allows for both credit constraints and equilibrium default.

5 Conclusion

We investigate the normative implications of models of equilibrium default based on the incompleteness of debt contract. This framework, first introduced by Eaton and Gersovitz (1981) has been widely used to study default of countries and households. We show that the equilibrium is not constrained efficient and that market equilibrium can induce too much default in general. The model generates both excess borrowing and excess default. The debt issued by borrowers has a social value because it allows other agents to self-insure, but this value is not perfectly included in prices. Increasing the price of debt may not provide the right default incentives. Thus, a simple financial regulation consists in imposing credit constraints. This regulation can contribute to increase welfare, because it prevents some agents from defaulting. This article has identified the effect and provide a simple calibration. A full quantitative investigation of the optimal financial regulation in this setup may now be useful.

Appendix

A Proof of Lemma 1

The Lemma characterizes the price schedule for unemployed agents, which may default. The price schedule for employed agents can be derived similarly. First, the price schedule $p_2(q', u)$ is a direct implication of the value function (5), as mentioned in the text. Second, the price schedule $p_1(q', u)$ stems the fact that value functions are increasing. To see this, if uu agents do not default for an amount of debt q, then their intertemporal welfare is the consequence of the

$$V_2^{uu}(q) = \max_{q_2'} u(c) - \delta + \beta(1-\rho) V_3^e(q') + \beta\rho V_3^u(q')$$

$$(1-\rho) p_2 q' + c = \delta + q$$

Inspecting this value function, it is easy to show that it is increasing in q. If $q > 0$, then uu agents do not default because the option to default next period always dominate the option to default in period 2. This provides an upper bound for q_1^{lim}. Second, if $q < -\delta$, the uu agents always have to borrow to reach positive consumption. As a consequence, $V_3^e(q\prime) \leq W_3^e$ and $V_3^u(q') = W_3^u$, maximizing for q' we find that $V_2^{uu}(q)$ is not bounded when q tends toward $-\infty$. As a consequence, there exists a unique q_1^{lim}.

B Proof of Proposition 2

The equilibrium is summarized by the following equations

$$p_2 = \alpha + (1-\alpha) u'(\delta + q_2^e)$$

$$p_2 u'(\delta + q_1^e - (1-\rho) p_2 q_2^{eu}) = 1$$

$$p_1 = \alpha + (1-\alpha) u'(\delta + q_1^e - (1-\rho) p_2 q_2^{eu})$$

$$p_1 u'(\delta - (1-\rho) p_1 q_1^u) = 1$$

$$(\alpha + 1 - \rho) q_2^e + (1-\rho)(1-\alpha) q_2^{eu} = 0$$

$$q_1^e + (1-\rho) q_1^u = 0$$

From the first equation, we get $p_2 > \alpha$. The first three equations are the Euler equations, the last two equations are market equilibria, where we have substituted for the number of agents using (1)-(4). After some algebra, we find $p_1 = \alpha + (1-\alpha)/p_2$ and

$$\delta - \frac{1}{p_1}\delta + 1 + (\alpha + 1 - \rho)\frac{p_2}{p_2 - \alpha} - \frac{\alpha + 1 - \rho}{1 - \alpha} p_2 \delta = p_2 \tag{31}$$

Define

$$F(x) = \left(1+\delta+\frac{\alpha+1-\rho}{1-\alpha x} - \frac{\delta}{\alpha+(1-\alpha)x}\right)x - \left(1+\frac{\alpha+1-\rho}{1-\alpha}\delta\right)$$

From (31), we get that if p_2 is an equilibrium price, $x \equiv 1/p_2$ has to satisfy $F(x) = 0$. As the price must be positive and $p_2 > \alpha$, we must have $0 < x < 1/\alpha$. It is easy to show that F is increasing in x and that $F(x) = 0$ has only one solution for the relevant range for x. Moreover, $F(1) > 0$, hence $x < 1$ We deduce $p_2 > 1$ and as $p_1 = \alpha + (1-\alpha)/p_2$ we have $p_1 < 1$. This proves the two items of the Proposition. The condition for this equilibrium to exist is that uu agents are better of if they default than staying in the economy, what can easily be checked.

C Proof of the Proposition 3

We need to characterize the general equilibrium under the assumption that uu agents do not default. In this case, uu participate to the economy and save a net amount q_2^{uiu} in period 2.Following the same steps as in Section 3.3, one finds that economy without default is characterized by the equations

$$p_2 = \alpha + (1-\alpha)u'(\delta + q_2^e)$$

$$\begin{cases} p_2 u'(\delta + q_1^e - p_2 q_2^{eu}) = 1 \text{ if } q_2^{eu} \geq 0 \\ p_2 u'(\delta + q_1^e - (1-\rho)p_2 q_2^{eu}) = 1 \text{ if } q_2^{eu} < 0 \end{cases}$$

$$p_2 u'(\delta + q_1^u - (1-\rho)p_2 q_2^{uu}) = 1$$

$$p_1 = \alpha + (1-\alpha)u'(\delta + q_1^e - (1-\rho)p_2 q_2^{eu})$$

$$p_1 u'(\delta - p_1 q_1^u) = (1-\rho) + \rho u'(\delta + q_1^u - (1-\rho)p_2 q_2^{uu})$$

$$N_2^e q_2^e + N_2^{eu} q_2^{eu} + (1-\rho)N_2^{uu} q_2^{uu} = 0$$

$$N_1^e q_1^e + N_1^u q_1^u = 0$$

The second equation is the Euler equation for eu agents in period 2. They face different prices schedule if they borrow or if they save. We now show that this can not define an equilibrium and that uu agent always want to default in period 2.We focus on the case where $q_2^{eu} < 0$. The case where $q_2^{eu} \geq 0$ is similar.

After some algebra one finds, that the price p_2 has to fulfill

$$\delta(1-p_2) = u'^{-1}\left(\frac{\beta}{p_2}\right) - p_2 u'^{-1}\left(2\frac{p_2}{\beta} - 1\right)$$

There is only one solution which is

$$\begin{aligned} &p_1 = p_2 = 1, && q_2^e = 1-\delta \\ &q_1^u = -(1-\delta), && q_1^e = 1-\delta \\ &q_2^{eu} = 0, && q_2^{uu} = 4(\delta-1) \end{aligned}$$

But this can not be an equilibrium. One can check that $V_2^u(q_1^u) < W_2^u$ and uu agents default in period 2.

D Proof of the Proposition 4

D.1 Equilibrium and existence conditions

Assume that parameters are such that a default equilibrium exists. Introducing a credit limit D such that uu agents do not default yield

$$p_2 = \alpha + (1-\alpha) u'(\delta + q_2^e)$$
$$p_2 u'(\delta + q_1^e - (1-\rho) p_2 q_2^{eu}) = 1$$
$$p_2 u'(\delta + q_1^u - (1-\rho) p_2 q_2^{uu}) = 1$$
$$p_1 = \alpha + (1-\alpha) u'(\delta + q_1^e - (1-\rho) p_2 q_2^{eu})$$
$$N_2^e q_2^e + (1-\rho) N_2^{eu} q_2^{eu} + (1-\rho) N_2^{uu} q_2^{uu} = 0$$
$$N_1^e q_1^e + N_1^u q_1^u = 0$$
$$q_u^1 = -D$$

The conditions for this equilibrium to exist if first that the credit constraint is actually binding and that uu agents default. we check that the two conditions are fulfilled below. We first characterize the equilibrium. After some algebra, one finds that the price p_2 satisfies

$$(\alpha + 1 - \rho) p_2 \left(\frac{1-\alpha}{p_2 - \alpha} - \delta\right) + (1-\alpha)(\delta + D - p_2) + \rho(\delta - D - p_2) = 0$$

Lemma 10 *1) If $1-\alpha-\rho > 0$, p_2 increases and p_1 decreases, q_2^e decreases when D increases.* $\frac{\partial p_1}{\partial D} < 0, \frac{\partial p_2}{\partial D} > 0$
2) If $\alpha > \rho$ and $1-\alpha-\rho > 0$ then $p_2 > 1$

Define

$$H(p_2, \delta) = \left((\alpha + 1 - \rho)\left(\frac{1-\alpha}{p_2 - \alpha} - \delta\right) - (1 - \alpha + \rho)\right) p_2 + (1-\alpha+\rho)\delta + (1-\alpha-\rho) D$$

p_2 must satisfy $H(p_2, \delta) = 0$ under the constraint $\alpha < p_2 < 1$. If $1-\alpha-\rho > 0$, we must have

$$H(p_2, \delta) = 0$$
$$H_\delta(p_2, \delta) > 0$$
$$H_{p2}(p_2, \delta) = -\frac{(1-\alpha+\rho)\delta + (1-\alpha-\rho) D}{p_2} - (\alpha + 1 - \rho)\frac{1-\alpha}{(p_2-\alpha)^2} p_2 < 0$$

If $\alpha > \rho$

$$H(1, \delta) = 2(\alpha - \rho)(1 - \delta) + (1 - \alpha - \rho) D > 0$$

what concludes the proof.

The condition for the credit constraint to bind is

$$p_1 u'(\delta - p_1 q_1^u) > (1-\rho) + \rho u'(\delta + q_1^u - p_2(1-\rho) q_2^{uu})$$

After some algebra, one finds that this condition is

$$D < \frac{1}{1-\rho+\rho/p_2} - \frac{\delta}{p_1} \tag{32}$$

where p_1 and p_2 are equilibrium prices.

The condition for uu agents not too default is

$$V_2^u = u(\delta - D - (1-\rho) p_2 q_2^{uu}) - \delta + \beta(1-\rho) V_3^e(q_2^{uu}) + \beta\rho W_3^u > W_2^u$$

After some algebra ones finds the condition

$$D < \delta - p_2\left(\ln\frac{\delta}{p_2} + 1\right) \tag{33}$$

The two conditions (32) and (33) are expressed as functions of equilibrium prices.

D.2 Welfare

The welfare of each agents in period 3 is

$$\begin{aligned} V_3^{eee} &= E + q_2^e, & V_3^{eeu} &= u(\delta + q_2^e) - \delta \\ V_3^{eue} &= E + q_2^{eu}, & V_3^{euu} &= U \\ V_3^{uue} &= E + q_2^{uu}, & V_3^{uuu} &= U \\ V_3^{uee} &= E + q_2^e, & V_3^{eeu} &= E + q_2^{uu} \end{aligned}$$

In period 2 we have

$$\begin{aligned} V_2^{ee} &= E + D - p_2 q_2^e + \alpha V_3^{eee} + (1-\alpha) V_3^{eeu} \\ V_2^{eu} &= u(\delta + D - (1-\rho) p_2 q_2^{eu}) - \delta + (1-\rho) V_3^{eue} + \rho V_3^{euu} \\ V_2^{ue} &= E - D - p_2 q_2^e + \alpha V_3^{uee} + (1-\alpha) V_3^{ueu} \\ V_2^{uu} &= u(\delta - D - (1-\rho) p_2 q_2^{uu}) - \delta + (1-\rho) V_3^{uue} + \rho V_3^{uuu} \end{aligned}$$

Finally in period 1,

$$\begin{aligned} V_1^e &= E - p_1 D + \alpha V_2^{ee} + (1-\alpha) V_2^{eu} \\ V_1^u &= u(\delta + p_1 D) - \delta + (1-\rho) V_2^{ue} + \rho V_2^{uu} \end{aligned}$$

and ex-ante welfare is

$$V = V_1^e + V_1^u$$

Using equilibrium values we have

$$\frac{\partial V}{\partial D}=\underbrace{p_1u'\left(\delta+p_1D\right)-\left(1-\rho\right)-\rho u'\left(\delta-D-\left(1-\rho\right)p_2q_2^{uu}\right)}_{\text{Relaxation of the credit constraint}}+D\frac{\partial p_1}{\partial D}\left(u'\left(\delta+p_1D\right)-1\right)$$
$$-\frac{\partial p_2}{\partial D}\left(N_2^{ee}q_2^e+N_2^{eu}\left(1-\rho\right)q_2^{eu}u'\left(\delta+D-\left(1-\rho\right)p_2q_2^{eu}\right)\right)$$
$$-\frac{\partial p_2}{\partial D}\left(N_2^{ue}q_2^e+N_2^{uu}\left(1-\rho\right)q_2^{uu}u'\left(\delta-D-\left(1-\rho\right)p_2q_2^{uu}\right)\right)$$

we find

$$\frac{\partial V}{\partial D}=\underbrace{p_1u'\left(\delta+p_1D\right)-\left(1-\rho\right)-\frac{\rho}{p_2}}_{\text{Relaxation of the credit constraint}}+\underbrace{D\frac{\partial p_1}{\partial D}\left(u'\left(\delta+p_1D\right)-1\right)}_{\text{Redistribution in period 1}}-\underbrace{\left(1-\frac{1}{p_2}\right)\frac{\partial p_2}{\partial D}\left(1+\alpha-\rho\right)q_2^e}_{\text{Redistribution in period 1}}$$

E Proof of Proposition 6

We define the function W^u as follows:

$$W^u\left(a\right)=\max_{a',c}u\left(c\right)-\delta+\beta E_uV\left(a',1,z'\right)$$
$$\text{s.t. } p\left(a',u\right)a'+c=\delta+a,$$

where $E_u$is the expectation operator, conditional on currently being in state u.

The function W^u is the value function of an unemployed agent with a initial wealth a and who does not default in the current period, but who can default the following period. The function W^u may not be defined over $\mathbb{R}$ because agents must have a positive consumption. We circumvent this problem by setting $u\left(c\right)=-\infty$ if $c<0$. This insures that the intertemporal welfare will be equal to $-\infty$ for any consumption plan in which $c<0$ at given point of time. This consumption will not be chosen in equilibrium.

Similarly, we define $W^e$the corresponding value function for employed agents:

$$W^e\left(a\right)=\max_{a',c}u\left(c\right)-l+E_e\beta V\left(a',1,z'\right)$$
$$\text{s.t.}p\left(a',e\right)a'+c=l+a$$
$$l\geq 0$$

where $E_u$is the expectation operator, conditional on currently being employed.

We can first remark that an agent in state $s=e,u$ defaults in the current period if $W^s(a)\leq V_{aut}^s$.

It is obvious to show that the function $V\left(a,z\right)$ is strictly increasing in a. Indeed, any additional wealth can at least be consumed today, which leaves the continuation value unchanged but strictly improves the instantaneous utility what increases welfare. By a similar argument, the two functions W^e and W^u are also strictly increasing in a.

The proof consists in showing that there are two thresholds A^e and A^u such that $W^u(a) \geq V^u_{aut} \Leftrightarrow a \geq A^u$ and $W^e(a) \geq V^e_{aut} \Leftrightarrow a \geq A^e$ and that $A^e \geq A^u$.

First, the two thresholds A^e and A^u have to be negative. If A^u is for example positive, the unemployed agent could today consume A^u in addition to δ and chose the autarky consumption plan (i.e. δ) for the future consumptions. This leaves the agents strictly better off than in autarky, which is a contradiction.

Second, we can rewrite $W^e(a)$ as follows:

$$W^e(a) = a + \max_{c,a'} u(c) - c - p\left(a', e\right) a' + E_e \beta V\left(a', z'\right) \text{ if } p\left(a', e\right) a' + c - a \geq 0$$

It converges to $-\infty$ for a decreasing toward $-\infty$. Since W^e is strictly increasing, we know that there is a unique threshold A^e such that $W^e(A^e) = V^e_{aut}$.

Regarding W^u, we can make a similar argument. We can indeed bound the instantaneous utility $u(\delta + a - pa')$ as follows:

$$u(\delta + a - pa') \leq \begin{cases} u(\delta - pa') + au'(\delta) & \text{if} \quad a' \leq 0 \\ u(\delta) + au'(\delta) & \text{if} \quad a' \geq 0 \end{cases}$$

So, W^u can be bounded by a linear function converging to $-\infty$ for a decreasing toward $-\infty$. There exists thus a unique threshold A^u such that $W^u(A^u) = V^u_{aut}$.

Let us assume that $A^e \geq A^u$. We consider an unemployed agent endowed with A^u at the beginning of the period and choosing the wealth $a'(A^u)$

$$W^u(A^u) = V^u_{aut} = u\left(\delta + A^u - p\left(a'(A^u), u\right) a'(A^u)\right) - \delta + \beta\rho V\left(a'(A^u), 1, 0\right) + \beta(1-\rho) V\left(a'(A^u), 1, 1\right) \tag{34}$$

First, we remark that $a'(A^u) \geq A^u$: otherwise the agent defaults next period in both states (unemployed and employed ones) and the price is $p(a'(A^u), u) = 0$. In turns, the expression of $W^u(A^u)$ simplifies to $W^u(A^u) = u(\delta + A^u) - \delta + \beta\rho V^u_{aut} + \beta(1-\rho) V^e_{aut} < V^u_{aut}$ since $A^u < 0$. It contradicts the definition of A^u. In consequence, $a'(A^u) \geq A^u$ and at least unemployed agents repay their debt.

Second, the right hand side of (34) can be seen as a function ψ of $a'(A^u)$. For $a'(A^u) = A^u$, we have:

$$\begin{aligned}
\psi(A^u) &= u\left(\delta + A^u - p\left(A^u, u\right) A^u\right) - \delta + \beta\rho V\left(A^u, 1, 0\right) + \beta(1-\rho) V\left(A^u, 1, 1\right) \\
&= u\left(\delta + A^u(1 - p\left(A^u, u\right))\right) - \delta + \beta\rho V^u_{aut} + \beta(1-\rho) V^e_{aut} \\
&< u(\delta) - \delta + \beta\rho V^u_{aut} + \beta(1-\rho) V^e_{aut} = V^u_{aut}
\end{aligned}$$

Moreover, the function ψ is decreasing in the neighborhood of A^u. Indeed, an increase of ε in A^u implies the utility variation $-\left(p\left(A^u,u\right)-\beta\rho\right)u'\left(\delta+A^u-p\left(A^u,u\right)A^u\right)\varepsilon$, which is negative. The non-profit condition of financial intermediaries in A^u (when unemployed repay but employed do not) implies indeed that $p\left(A^u,u\right)=\rho p$, with $p>\beta$ by assumption.

Since we look for $a'\left(A^u\right)$, such that $\psi(a'\left(A^u\right))=W^u\left(A^u\right)=V^u_{aut}$, we necessarily have $a'\left(A^u\right)<A^u<A^e$, which is a contradiction with our first statement $a'\left(A^u\right)\geq A^u$.

F Proof of the Proposition 7

In this proof we characterize the types of equilibria and we present the fixed point problem for each of them. We characterize the equilibria for a given price, and then gives the equilibrium condition which defines the riskless price p.

We define $N=\min\{k\in\mathbb{N}\cup\{+\infty\},\ a_{k-1}>A^u\}$. We remember that a_k is the asset choice after k periods of unemployment. the asset choice of employed agents is a_0 by convention. An unemployed agent for $k\leq N$ periods face a price of claim equal to p by Proposition 6, because she chooses not default the following period. The first order conditions for asset prices is

$$p=\beta\alpha+\beta\left(1-\alpha\right)u'\left(\delta+a_0-pa_1\right)\tag{35}$$

$$pu'\left(\delta+a_{k-1}-pa_k\right)=\beta\left(1-\rho\right)+\beta\rho u'\left(\delta+a_k-pa_{k+1}\right)\ \ k=1,\ldots,N-2\tag{36}$$

$$pu'\left(\delta+a_{N-2}-pa_{N-1}\right)=\beta\left(1-\rho\right)+\beta\rho u'\left(\delta+a_{N-1}-p(a_N,u)a_N\right)\tag{37}$$

We define $Y_k=u'\left(\delta+a_{k-1}-p(a_k,u)a_k\right)$ for $k=1,\ldots,N$. Equations (35)–(37) imply

$$Y_1=\frac{p-\beta\alpha}{\beta\left(1-\alpha\right)}>1$$
$$Y_k=\frac{pY_{k-1}-\beta\left(1-\alpha\right)}{\beta\rho},\ \ k=2,\ldots,N$$

Since $p>\beta$, the sequence $\left(Y_k\right)_{k=1,\ldots,N}$ is increasing with k. In words, the marginal utility of unemployed households increases with the number of periods of unemployment. All Y_k depend only on the price p and on deep model parameter. We can remark that a_k and a_{k-1} are related through Y_k as follows: $a_{k-1}=u'^{-1}\left(Y_k\right)-\delta+pa_k$.We deduce that $\left(a_k\right)_{k\geq0}$ is a sequence decreasing towards $-\infty$. Therefore, we know that $N<\infty$.

Four cases have now to be considered. Indeed, the presence of the threshold creates some non convexity which may generate corner solutions. In all cases, the structure of the fixed point is the following. first one can express the series a_k, $k=0...N+3$ as function of $W^e\left(0\right)$. Then, one can express $W^e\left(0\right)$ as a function of a_N.

The different cases are the following.

1) $a_{N+i}=A^u$ for $i=1..\infty$

2) $a_{N+1} = A^e$

3) $a_{N+1} = A^u$ and $A^e < a_{N+2} < A^u$

4) $a_{N+1} = A^u$ and $a_{N+2} = A^e$

In case 1) unemployed agents do not default but reach the value A^u after a finite number of periods of unemployment and then unemployed agents stay at A^u forever. In 2) agents default after $N+2$ periods of unemployment. In case 3) and 4) agents default after $N+3$ periods of unemployment. We study case 1) and 2) carefully. Other cases follow the same line.

1) In case 1, after N agent face credit constraints a_{N+i},$i = 1..\infty$ is defined by

$$\begin{aligned} W^u(a_{N+i}) = V^u_{aut} &= u(\delta + A^u - pA^u) - \delta + \beta(1-\rho)V(A^u, 1) + \beta\rho V(A^u, 0) \\ &= u(\delta + (1-p)A^u) - \delta + \beta(1-\rho)(W^e(0) + A^u) + \beta\rho V^u_{aut} + \beta\rho V^u_{aut} \end{aligned}$$

As a consequence,

$$(1-\beta\rho)V^u_{aut} - \beta(1-\rho)W^e(0) = u(\delta + (1-p)A^u) - \delta + \beta(1-\rho)A^u$$

As agent are credit constraints, the derivative of the right hand side is positive and the previous equation defines uniquely A^u as a function of $W^e(0)$.

Knowing that $a_{N+1} = A^u$, one can find recursively a_k, $k = 0..N$. Finally, intertemporal welfare $W^e(0)$ can be found easily. For instance,

$$\begin{aligned} W^e(0) &= u\left(u'^{-1}(1)\right) - u'^{-1}(1) + \beta\alpha W^e(a_0) + \beta(1-\alpha)W^u(a_0) \\ W^u(a_{k-1}) &= u(\delta + a_{k-1} - pa_k) - \delta + \beta(1-\rho)(W^e(0) + a_k) + \beta\rho W^u(a_k), k = 1..N \end{aligned}$$

Using the previous equations, one finds

$$\begin{aligned} &\left(1 - \beta\left(\alpha + \beta(1-\alpha)(1-\rho)\frac{1-(\beta\rho)^N}{1-\beta\rho}\right)\right)W^e(0) \\ &= u(u'^{-1}(1)) - u'^{-1}(1) + \beta^{N+1}(1-\alpha)\rho^N V^u_{aut} \\ &+ \beta(1-\alpha)\sum_{k=0}^{N-1}(\beta\rho)^k\left(u\left(u^{-1}(Y_k)\right) - \delta\right) - (p - \beta\alpha)\left(p^N a_N^N + \sum_{j=0}^{N-1}p^j\left(u\left(u^{-1}(Y_j)\right) - \delta\right)\right) \\ &+ \beta^2(1-\alpha)(1-\rho)\left(a_N^N\frac{p^N - (\beta\rho)^N}{p - \beta\rho} + \sum_{j=1}^{N-1}\frac{p^j - (\beta\rho)^j}{p - \beta\rho}\left(u\left(u^{-1}(Y_k)\right) - \delta\right)\right) \end{aligned}$$

The previous equality defines $W^e(0)$ as a function of A^u. One can jointly solve for $W^e(0)$ and A^u.

In case 2, we have $W^e(a_{N+1}) = V^e_{aut}$. As a consequence, $a_{N+1} = V^e_{aut} - W^e(0)$. One can express recursively a_k as a function of Y_k and $W^e(0)$. Then, it is easy to express $W^e(0)$ as a function of a_{N+1} and $Y_k, k = 1..N$. Finally, one finds the maximum value for $W^e(0)$ which is the solution for this constraints.

Cases 3 and 4 can be characterized along the same lines. Finally, the solution to the households problem is the equilibrium which maximizes $W^e(0)$.

The Proposition summarizes the result. Item 1) of the Proposition is case 1) of this Appendix. item 2) of the Proposition corresponds to case 2) to 4).

G Proof of the Proposition 8

Consider B_i, $N \leq i \leq M-1$ if the constraint $V^u(B_i) \geq V^u_{aut}$ is not binding, one can define a marginal decrease B_i and a marginal increase in B_{i+1} such that $V^u(B_{i+1})$ does not decrease, the volume of asset is unchanged (and so is the price). One can show that this marginal change is increasing intertemporal welfare because it increases the welfare of agents choosing B_i who are credit constrained. It also increases the welfare of agents choosing B_{i+2} because they are wealthier. This is a contradiction. As a consequence, if credit constraints are binding one must have $V^u(B_i) = V^u_{aut}$.

The item 2) of the Proposition is shown studying the path of $B_i, N \leq i \leq M$. For which $V^u(B_i) = V^u_{aut}$. We have

$$V^u(B_i) = V^u_{aut} = u(\delta + B_i - pB_{i+1}) - \delta + \beta(1-\rho)V^e(B_{i+1}) + \beta\rho V^u_{aut}, \text{ for } i < M-1 \quad (38)$$

$$V^u(B_{M-1}) = V^u_{aut} = u(\delta + B_{M-1} - p(1-\rho)B_M) - \delta + \beta(1-\rho)V^e(B_M) + \beta\rho V^u_{aut} \quad (39)$$

The terminal choice of B_M is given by either the condition $pu'(\delta + B_{M-1} - p(1-\rho)B_M) = \beta$ if agents are not constrained by $V^e(B_M) \geq V^e_{aut}$ or by the constrained $V^e(B_M) = V^e_{aut}$. Studying both cases, one finds the path of credit constraints B_k is decreasing, $k = N..M-1$.

Finally, if $M = +\infty$, the value of B is uniquely defined by

$$V^u_{aut} = u(\delta + (1-p)B) - \delta + \beta(1-\rho)V^e(B) + \beta\rho V^u_{aut}$$

H Proof of the Proposition 9

The proof follows the same lines as the previous proof. First, when there is no effect on prices, one can show that the welfare is increasing when there are no credit constraints. To do so, assume that credit constraints are binding in period N. One can show that removing the credit constraint B_N is always welfare increasing. The second part of the Proposition consists in the analysis of the series B_k defined by (38) and (39) for a given price p, when M increases.

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