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Beyond Investment-Cash Flow Sensitivities: Using Indirect Inference to Estimate Costs of External Funds*

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Abstract

This paper estimates costs of external finance, applying indirect inference to a dynamic structural model where the corporation endogenously chooses investment, distributions, leverage and default. The corporation faces double taxation, costly state verification in debt markets, and linear-quadratic costs of external equity. Consistent with direct evidence on underwriter fee schedules, behavior is best explained by rising marginal costs of external equity, starting at 3.9%. Contrary to the notion that corporations are debt conservative, debt issuance is consistent with small (12.2%) and statistically insignificant bankruptcy costs. Investment-cash flow sensitivities are not a sufficient statistic for financing costs. In fact, the estimated model implies the cash flow coefficient increases in bankruptcy costs, but decreases in external equity costs.

The most studied question in empirical corporate finance over the past fifteen years is investment-cash flow sensitivities. In their influential paper, Fazzari, Hubbard, and Petersen (FHP) (1988) argue the significance of cash flow in determining corporate investment demonstrates capital markets are imperfect. If this were the extent of their claim, it is doubtful that investment-cash flow sensitivities would have attracted so much attention. After all, few would argue that the necessary conditions identified by Modigliani and Miller (1958) for financial irrelevance are, in fact, satisfied. However, there is considerable debate regarding the *magnitude*

*Preliminary and incomplete. Please do not cite without authors' permission. Hennessy is from the University of California at Berkeley. Whited is from the University of Wisconsin, Madison.

of financial frictions. This brings up the second, more intriguing, part of FHP's claim, that investment-cash flow sensitivities are monotonically increasing in the size of financing frictions. If this claim were indeed true, then corporate finance theorists and empiricists could look to a single number, the coefficient on cash flow in investment regressions, in order to gauge the size of financial frictions.

The position that investment-cash flow sensitivities are informative about the magnitude of financial frictions has been questioned. Kaplan and Zingales (1997) argue, "there is no strong theoretical reason for investment-cash flow sensitivities to increase monotonically with the degree of financing constraints." Market power is commonly cited as an alternative explanation for the predictive power of cash flow. Hayashi (1982) shows that market power drives a wedge between marginal and average q. However, Abel and Eberly (1994) show that average q is still a sufficient statistic for investment when operating profits and capital adjustment costs share the same degree of homogeneity. Stepping away from this special case, Gomes (2001) and Abel and Eberly (2004) show that financial frictions are not necessary to generate significant cash flow effects when profits are concave in capital and the investment cost schedule is linear.[1] Similarly, Cooper and Ejarque (2001) conclude that market power is sufficient to replicate existing regression results. Measurement error is frequently mentioned as potentially explaining the significance of cash flow in ordinary least squares regressions. For example, Erickson and Whited (2001) find that cash flow is insignificant under measurement error-consistent GMM estimation.

The sheer duration of the debate suggests that corporate finance economists will not reach consensus regarding the magnitude of financial frictions based on investment-cash flow sensitivities. In retrospect, it seems there was never any hope that this line of research could deliver conclusive evidence regarding the magnitude of financial frictions. To see this, consider the central financial frictions identified by FHP (1988): corporate and personal taxation; bankruptcy and agency costs associated with debt; and costs of external equity, which may be fixed, proportional, or nonlinear according to various theories. Even if one could identify constrained firms and perfectly measure marginal q, it is clearly impossible to infer the magnitudes of the diverse costs of external funds based on a single regression coefficient.

This *underidentification* problem suggests the need for alternative approaches to the inference problem

[1] See also Alti (2003).

under consideration. To this end, the present paper offers a model-based procedure for estimating the magnitude of the costs of external debt and equity. First, we formulate a general model of dynamic corporate investment and financial policy under uncertainty, imposing minimal functional form assumptions and capturing the financial frictions identified by FHP (1988). FHP present a dynamic model of financing and investment in the presence of taxes and proportional costs of external equity. We extend their model by allowing for uncertainty, debt finance, default, corporate saving, and linear-quadratic costs of external equity.

With the model in-hand, we employ the indirect inference technique in Gourieroux, Monfort, and Renault (1993) and Gourieroux and Monfort (1996). This estimation procedure determines which vector of financial friction parameters best explains observed financial behavior, i.e. minimizes the distance between moments generated by the simulated model and a broad set of real-world data moments. By using an array of moments, indirect inference overcomes the underidentification problem cited above. We find that corporate financial behavior is best explained by rising costs of external equity, starting at 3.9% for the first dollar raised, in conjunction with small (12.2%) bankruptcy costs.

One may question why indirect inference is necessary, given that other research provides more direct estimates of financing costs. For example, Weiss (1990) estimates that legal and other professional fees amount to 2.8% of the book value of assets in default. On the equity side, Altinkilic and Hansen (2000) examine the shape of underwriter fee schedules. A potential shortcoming of such estimates is that they cannot measure indirect costs perceived by corporations. For example, Weiss does not measure the indirect costs of bankruptcy, such as the loss of sales predicted by Titman (1984). Similarly, underwriter fees may not fully reflect the lemons premia predicted by Myers and Majluf (1984). We attempt to infer both direct and indirect costs of external funds based on observed financing behavior.

In fact, our estimates complement existing direct evidence on financing costs. Comparison of our parameter estimates with the direct estimates facilitates a rough test of the null hypothesis of maximizing behavior. For example, our low point estimate of bankruptcy costs is evidence in favor of the null that corporations are not "debt conservative." Similarly, our evidence indicates that corporations behave "as if" facing a low, convex cost of external equity. This is roughly consistent with the underwriting fee schedules estimated by Altinkilic and Hansen (2000), again supporting the null of maximizing behavior.

The model also allows us to identify which moments matter, i.e. which moments are informative about the magnitude of the various financing frictions. As intuition would suggest, the debt to asset ratio and the propensity to hold cash are informative about bankruptcy costs. The frequency, mean, variance, and skewness of equity issuance are informative costs of external equity. The cash flow coefficient is not a summary statistic for financing costs, nor is it monotonic in the various frictions. The cash flow coefficient increases in bankruptcy costs, but decreases in external equity costs.

We now discuss closely related papers. Cooley and Quadrini (2001) analyze a firm that can issue defaultable debt and faces proportional costs of external equity. Their model of the debt market greatly influenced that presented in our paper. Our model is a bit more general, allowing for corporate and personal taxation and linear-quadratic costs of external equity. Our empirical objective is to use indirect inference to estimate which parameters best explain observed financing patterns. In contrast, Cooley and Quadrini show that existing stylized facts regarding firm growth and exit can be explained by their model when one imposes a reasonable parameterization.

Cooper and Ejarque (2003) is most similar to the present paper in terms of methodology. They use indirect inference to estimate costs of external equity. There is no taxation, no debt, and costs of external equity are linear. Cooper and Ejarque do sketch the broad outlines of a model with corporate saving and riskless debt at the conclusion of their paper. However, no estimation is performed on this model. They state, "The model is very difficult to estimate due to the additional state variable and the need for a fine state space. Further developments along this line seems warranted in order to more fully integrate capital market and investment decisions at the firm level." The present paper overcomes the dimensionality problem. Net worth is the only endogenous state variable. Cooper and Ejarque attempt to fit the following moments: standard deviation of profits; average q; serial correlation of investment; and regression coefficients from an OLS regression of investment on average q and cash flow. Our empirical focus is very different, as we attempt to match moments relating to financial policy.

Hennessy and Whited (2004) present a dynamic model with corporate and personal taxation, proportional costs of external equity, and credit rationing. An exogenous credit constraint ensures debt is riskless.[2] In

[2] We thank David Mauer for encouraging us to relax this assumption given that a nontrivial subset of corporations do not face strict credit rationing.

the event of a cash deficit, the firm is required to sell capital at a fire-sale price. The primary objective of Hennessy and Whited (2004) is to show that a rational trade-off model can be reconciled with existing corporate finance "anomalies." The present paper contains a more general model of financial frictions, allowing the data to speak for itself in terms of unobservable parameters.

Leary and Roberts (2004) assume that the firm's objective is to keep the leverage ratio within an exogenous band. A dynamic duration model is used to make inferences about the nature of restructuring costs. They conclude that a combination of fixed plus weakly convex costs of adjustment best explains observed hazard rates. Their results are informative about the nature of financial frictions, but leave open the question of magnitudes. Rauh (2004) uses mandatory pension contributions as a potentially exogenous innovation to internal funds.[3] He finds a significant negative response of capital expenditures to required contributions. This evidence may serve as a reasonable basis for rejecting the null hypothesis of perfect capital markets, but does not address the nature and magnitude of capital market imperfections.

The remainder of the paper is organized as follows. Section 1 sets up the model. Section 2 derives the optimal financial and investment policies. Section 3 describes the numerical solution to the model and presents a baseline simulation. Section 4 describes the indirect inference procedure and presents the estimation results. Section 5 concludes.

1. Economic Environment

A. Operating Profits

Time is discrete and the horizon infinite. There are two control variables, the capital stock (k) and the market value of one-period debt (b). Capital decays exponentially at rate δ. Negative values of b are properly interpreted as corporate saving. Variables with primes denote future values and minus signs denote lagged values. Subscripts denote partial derivatives.

An objective of the theoretical model is to specify the firm's problem in terms of primitives. We consider a firm with market power employing a constant returns to scale production technology in two inputs: capital

[3] Earlier papers by Blanchard, Lopez-de-Silanes, and Shleifer (1994) and Lamont (1997) also examine windfalls.

and labor (l). Cooper and Ejarque (2001) find that, in the context of indirect inference estimation, the failure to account for market power causes one to incorrectly impute concavity in the profit function to convexity in the adjustment cost function. By analogy, failure to account for market power would cause us to confound concavity of the profit function with convex costs of external funds. The firm faces demand, productivity, and wage shocks. The timing assumption is that new capital becomes productive with a one-period lag. This means that k' is chosen before next period's shocks are observed. In contrast, the variable labor input is chosen optimally after next period's shocks are observed. Assumption 1 summarizes.

Assumption 1. The firm faces a stochastic constant elasticity demand schedule

$$q^d(p, \widehat{z}) \equiv \left[\frac{\widehat{z}}{p}\right]^{\eta}.$$

The production function has constant returns to scale

$$q^s(k, l, \widetilde{z}) = \widetilde{z} k^{\phi} l^{1-\phi}.$$

Labor inputs are variable, and the stochastic wage rate is ω. Capital requires a one-period time-to-build.

Under Assumption 1, the profit function admits a concave representation[4]

$$\begin{aligned} \text{Operating Profit} &= z\pi(k) \qquad (1) \\ \pi(k) &\equiv k^{\alpha} \\ \alpha &\equiv \frac{\phi(\eta-1)}{1+\phi(\eta-1)} \\ z &\equiv \widehat{z}\left[\frac{1+\phi(\eta-1)}{\eta}\right]\left[\frac{\widehat{z}(1-\phi)(1-\eta^{-1})\widetilde{z}^{\frac{1}{1-\phi}}}{\omega}\right]^{\frac{(1-\phi)(\eta-1)}{1+\phi(\eta-1)}}. \end{aligned}$$

Assumption 2 imposes some structure on the shock z.

Assumption 2: The shock z takes values in the compact set $Z \equiv [\underline{z}, \overline{z}]$, $0 \leq \underline{z} < \overline{z} < \infty$, with its Borel subsets $\mathcal{Z}$. The Markovian transition function $Q : Z \times \mathcal{Z} \to [0, 1]$ has no atoms, satisfies the Feller property, and is monotone (increasing).

[4]Simply evaluate operating profits at the optimal labor input.

B. Tax System

Fazzari, Hubbard and Petersen (1988) cite the tax system as being a potentially important factor affecting the financing hierarchy and cost of funds schedule. Our goal is to parsimoniously model the salient features of the U.S. corporate income tax.

Investors are risk neutral, and the risk-free asset earns a pre-tax rate of return equal to r. The tax rate on interest income at the individual level is τ_i, implying investors use $r(1-\tau_i)$ as their discount rate. Corporate taxable income is equal to operating profits less economic depreciation less interest expense plus interest income. Consistent with the U.S. tax code, interest expense is computed as the product of the promised yield ($\widetilde{r}$) and the amount borrowed. As shown by Graham (1996a, 1996b), loss limitations create nonlinearities. Following Leland and Toft (1996), loss limitations are treated as a kink in the tax schedule. The tax rate when income is positive (τ_c^+) exceeds the tax rate when income is negative (τ_c^-). Letting χ be an indicator for positive taxable income, the corporate tax bill is

$$T^c(k', b', z, z') \equiv [\tau_c^+ \chi + \tau_c^- (1-\chi)] * [z'\pi(k') - \delta k' - \widetilde{r}(k', b', z)b'] . \tag{2}$$

An equilibrium bond pricing identity, derived below, is used to pin down $\widetilde{r}$. For now, it should be noted that the promised yield only hinges upon variables observable to the lender at the time of loan inception, and excludes the realized shock (z'). If the corporation saves, it earns r pre-tax, thus

$$b' < 0 \Rightarrow \widetilde{r}(k', b', z) = r \quad \forall (k', z). \tag{3}$$

The taxation of distributions is complicated by the fact that corporations pay out cash through dividends and share repurchases. Corporations should use share repurchases to disgorge cash if the marginal shareholder is a taxable individual due to the lower statutory rate historically accorded to capital gains, tax deferral advantages, and the tax free step-up in basis at death. Green and Hollifield (2003) present a model of optimal share repurchases. The first shareholders to sell into a tender offer are those with the lowest amount of locked-in capital gains. Under the optimal strategy, the effective tax rate on capital gains is only 60% of the statutory rate.

Complete substitution of repurchases for dividends is limited by the fact that the IRS prohibits replacing dividends with systematic repurchases. Given the historical reluctance of the IRS to challenge repurchase programs, the optimal plan would seem to entail a modest percentage of dividends. Another factor that may mitigate the substitution of repurchases for dividends is concern over SEC prosecution for stock price manipulation. SEC Rule 10b-18 provides safe harbor for firms adhering to certain restrictions on the timing and amount of shares repurchased. Cook et al. (2003) document that most corporations conform to the SEC restrictions.

To capture these effects, we model the corporation as perceiving an increasing marginal tax rate on distributions. Intuitively, under an optimal distribution program, small distributions are implemented via share repurchases. Shareholders with high basis are the first to tender, implying that the capital gains tax triggered by the repurchase is low. As the firm increases the amount distributed, there are two effects. First, the basis of the marginal tendering shareholder is reduced. Second, the firm may be inclined to increase the percentage paid out as dividends due to the IRS and SEC regulations cited above. Both effects raise the marginal tax rate on distributions.

The marginal distribution tax rate is parameterized as follows

$$\tau_d(x) \equiv \overline{\tau}_d * [1 - e^{-\phi x}]. \tag{4}$$

In contrast, Hennessy and Whited (2004) assume distributions are taxed at a constant rate.[5] The total distribution tax liability at the shareholder level is

$$T^d(X) \equiv \int_0^X \tau_d(x) dx. \tag{5}$$

There is zero tax triggered on the first dollar distributed, while the limiting marginal tax rate reaches $\overline{\tau}_d$. Intuitively, such convexity creates an incentive for the corporation to smooth distributions. This insight is exploited in the indirect inference estimation of ϕ.

Assumptions regarding the tax system are summarized below.

Assumption 3: Corporate taxes are computed according to (2), where $0 < \tau_c^- < \tau_c^+ < 1$. At the individual level, interest income is taxed at rate $\tau_i \in (0, \tau_c^+)$. The marginal tax rate on distributions to shareholders is determined by (4), where $\overline{\tau}_d \in (0, 1)$.

[5]We thank Richard Roll for suggesting that we relax this assumption in light of tax rate heterogeneity.

C. Costs of External Equity and Debt

The main costs of external equity discussed by FHP (1988) are: tax costs; adverse selection premia; and flotation costs. The tax cost associated with external equity is implicit in our parameterization of the tax system, which allows for double-taxation. Myers and Majluf (1984) show that informational asymmetries can raise or lower the cost of external equity. The precise implications of this theory for the perceived cost of external equity are quite sensitive to the nature of the equilibrium one constructs and the type of firm being considered. For example, when the parameters of the problem are such that a pooling equilibrium can be supported, both types of firms issue equity, with low (high) quality firms receiving financing on better (worse) than fair terms. The more general conclusion the profession seems to have taken away from the model of Myers and Majluf is that equity issuance may send a negative signal to the market regarding insiders' assessment of firm quality. For example, the model presented in FHP (1988) treats the "lemons premium" as proportional. Atlinkilic and Hansen (2000) provide detailed evidence regarding underwriter fees, finding that average costs are U-shaped due to fixed costs and widening spreads for larger offerings.

The cost of external equity function is linear-quadratic, capturing the effect of flotation costs and lemons premia.

Assumption 4: The cost of external equity is equal to Λ, where

$$\Lambda(x) \equiv \lambda_0 + \lambda_1 x + \lambda_2 x^2$$

$$\lambda_i \geq 0 \quad i = 0, 1, 2.$$

Indirect inference is used to estimate the three unknown parameters of the cost of external equity function.

The borrowing technology consists of a standard one-period debt contract, analogous to that derived in the costly state verification models of Townsend (1978) and Gale and Hellwig (1985). The intermediary faces perfect competition. In order for him to verify net worth, he must incur a cost. If the promised debt payment is delivered, the intermediary does not verify and the original shareholders retain control. In the event of default, the intermediary verifies net worth. The informed intermediary then enters into renegotiations with the firm. The intermediary has full ex post bargaining power and extracts all bilateral surplus by demanding a payment that leaves the firm indifferent between continuing or not. Allowing for ex post renegotiation is not critical, and one could simply assume that the intermediary liquidates rather than engaging in renegotiations. However, liquidating in this manner is value destroying. Additionally, this specification of the renegotiation process implies that the firm has an infinite life, which is convenient from a computational perspective. The verification cost function is parameterized as follows.

Assumption 5: Verification costs are equal to $\xi(1-\delta)k'$.

Indirect inference is used to estimate the magnitude of "bankruptcy costs" (ξ). It should be noted that this is not the only cost of debt incorporated in the model. Since there is some probability of default, equity recognizes that the lender may capture a portion of the return to capital accumulation. This is the debt overhang effect first analyzed by Myers (1977).

2. Model

A. Equity's Problem

The variable w denotes *realized net worth*

$$w(k',b',z,z') \equiv (1-\delta)k' + z'\pi(k') - T^c(k',b',z,z') - (1+\widetilde{r}(k',b',z))b'. \tag{6}$$

There is a single endogenous state variable $\widetilde{w}$ which denotes *revised net worth*. Revised net worth is equal to realized net worth if the firm does not default. In default, realized net worth is revised due to negotiations between the intermediary and firm. The precise nature of the adjustment is discussed in the next subsection, which treats debt market equilibrium.

To clarify the discussion below, it is useful to derive the firm's external funding requirement for a given desired capital stock (k'). Consider first a firm that did not default in the prior period. The direct cost of the investment is

$$k' - (1-\delta)k. \tag{7}$$

Liquid internal funds are equal to

$$z\pi(k) - T^c(k,b,z^-,z) - (1+\widetilde{r}(k,b,z^-))b. \tag{8}$$

The external funding requirement is equal to investment cost less liquid internal funds, which, in turn, is equal to the desired capital stock less revised net worth:

$$k' - (1-\delta)k - [z\pi(k) - T^c(k, b, z^-, z) - (1+\widetilde{r}(k, b, z^-))b] = k' - \widetilde{w}(k, b, z^-, z). \tag{9}$$

The external equity requirement is equal to

$$k' - \widetilde{w}(k, b, z^-, z) - b'. \tag{10}$$

Of course, when this amount is negative, the distribution to shareholders is positive. Next consider a firm that defaulted on the prior period's debt obligation. Once again, the external funding requirement is equal to the desired capital stock less revised net worth, while the external equity requirement is given by equation (10).

The construction of equilibrium proceeds in two steps. In this subsection equity's problem is formulated, while the next subsection analyzes the debt market. Consider first, the feasible policy correspondence

$$\Gamma : Z \to K \times B.$$

Without loss of generality, attention can be confined to compact K. The maximum allowable capital stock $\overline{k}$ is determined by

$$\overline{z}\pi'(\overline{k}) - \delta \equiv 0 \tag{11}$$

$$\Rightarrow \overline{k} = \left[\frac{\overline{z}\alpha}{\delta}\right]^{\frac{1}{1-\alpha}}.$$

Since $k > \overline{k}$ is not economically profitable, let

$$K \equiv [0, \overline{k}]. \tag{12}$$

Under the maintained assumption that $\tau_c^+ > \tau_i$, the optimal value of b is bounded below at some finite level, denoted $\underline{b} \in (-\infty, 0)$. To see this, note that for firms with positive taxable income, the after-tax return on corporate saving is below that available to the shareholder investing on his own account. As the firm's cash balance increases, the precautionary motive for retention becomes negligible and funds should be distributed. The upper bound on debt, i.e. the debt capacity of the firm, is denoted as $\overline{b}(k, z)$. Below, we show that debt capacity is finite.

The feasible policy correspondence can be expressed as

$$\Gamma(z) \equiv \{(k', b') : k' \in K \quad and \quad b' \in [\underline{b}, \overline{b}(k', z)]\}.$$

Let $C(\Theta)$ denote the space of all bounded and continuous functions on an arbitrary set Θ. The Bellman operator (T) corresponding to an abstract formulation of the equity's problem is

$$(Tf)(\widetilde{w}, z) \equiv \max_{(k',b')\in\Gamma(z)} \Phi_d[\widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k')] - \Phi_i[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] + \left[\frac{1}{1 + r(1-\tau_i)}\right] \int_Z f[\widetilde{w}(k', b', z, z'), z']Q(z, dz'). \quad (13)$$

subject to:

$i.$ Γ compact, convex, continuous and nondecreasing in z

$ii.$ $\widetilde{r} \in C(K \times B \times Z)$

$iii.$ $\widetilde{w}(k', b', z, z') \equiv \max\{\underline{w}(z'), w(k', b', z, z')\}$

$iv.$ $\underline{w} \in C(Z),\ \underline{w}(z') < 0 \quad \forall z' \in Z,$ and nonincreasing.

The second constraint states that equity faces a continuous schedule determining the promised yield demanded by the intermediary. The third and fourth constraints state that revised net worth is bounded below by some schedule $\underline{w}$. The next subsection analyzes endogenous default and debt renegotiation. It will be shown that $\underline{w}$ necessarily satisfies condition (iv). The model is then closed by constructing a debt market equilibrium, pinning down a continuous $\widetilde{r}$ function.

The following Lemma will prove useful

LEMMA 1: *The operator* $T : C(\widetilde{W} \times Z) \to C(\widetilde{W} \times Z)$ is a contraction mapping with modulus $[1+r(1-\tau_i)]^{-1}$.
Proof. See Appendix.

Proposition 1 indicates that the value function exists, while Proposition 2 tells us that the value function can be determined by iterating on the Bellman equation, starting from an arbitrary conjecture regarding the solution.

PROPOSITION 1: *There is a unique continuous function* $V : \widetilde{W} \times Z \to \Re_+$ *satisfying*

$$V(\widetilde{w}, z) = \max_{k', b' \in \Gamma(z)} \Phi_d[\widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k')] - \Phi_i[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] + \left[\frac{1}{1 + r(1 - \tau_i)}\right] \int_Z V(\widetilde{w}(k', b', z, z'), z') Q(z, dz'). \tag{14}$$

Proof. Follows from Lemma 1 and the Contraction Mapping Theorem.

PROPOSITION 2: *For arbitrary* $v^0 \in C(\widetilde{W} \times Z)$, the sequence

$$v^{n+1} \equiv T(v^n)$$

converges to V, with

$$d_\infty(v^n, V) \leq \left[\frac{1}{1 + r(1 - \tau_i)}\right]^n * d_\infty(v^0, V).$$

Proof. Follows from Lemma 1 and the Contraction Mapping Theorem.

Propositions 3 and 4 establish some useful and intuitive properties of the value function.

PROPOSITION 3: *For each* $z \in Z$, *the equity value function* $V(\cdot, z) : \widetilde{W} \to \Re_+$ *is strictly increasing.*

Proof. See Appendix.

PROPOSITION 4: *For each* $\widetilde{w} \in \widetilde{W}$, *the equity value function* $V(\widetilde{w}, \cdot) : Z \to \Re_+$ *is nondecreasing.*

Proof. See Appendix.

B. Debt Market Equilibrium

In the event of default and renegotiation, original shareholders are pushed down to their reservation value of zero. Equity does not default if realized net worth is positive, since a positive continuation value can then be achieved even if the promised debt payment is made. There is some z'-contingent critical value of realized net worth, denoted $\underline{w}(z') < 0$, such that equity is just indifferent between defaulting and making the contractual payment. The endogenous default schedule $\underline{w}(\cdot)$ is defined implicitly by the following equation

$$V(\underline{w}(z'), z') = 0 \qquad \forall \quad z' \in Z. \tag{15}$$

Proposition 5 establishes some useful and intuitive properties of the default schedule.

PROPOSITION 5: *The default schedule* $\underline{w}$: $Z \rightarrow \Re$ *is a negative valued, continuous, and nonincreasing function.*

Proof. If revised net worth is positive, so too is equity value, thus establishing negativity. Since V is strictly monotonic and continuous in its first argument, the inverse $\underline{w} = V^{-1}(0)$ is well defined and continuous. Weak monotonicity of $\underline{w}$ follows from Propositions 3 and 4.

Figure 1 depicts the default decision, plotting realized net worth and the default schedule as functions of the realized shock, z'. Since $w(k', b', z, \cdot)$ is strictly increasing, and $\underline{w}(\cdot)$ is nonincreasing, the two functions have at most one point of intersection, which is denoted $z_d(k', b', z)$. For shock values on the interval $[z_d(k', b', z), \overline{z}]$ the firm does not default. To see this, note that

$$z' > z_d(k', b', z) \Rightarrow w(k', b, z, z') > \underline{w}(z') \Rightarrow V(w(k', b', z, z'), z') > 0. \tag{16}$$

Alternatively, if $z' < z_d(k', b', z)$, equity prefers debt renegotiations, since *revised* net worth exceeds *realized* net worth.

If debt is sufficiently low, equity does not default. To see this, note that the z'_d is implicitly defined as follows

$$w(k', b', z, z'_d) = \underline{w}(z'_d). \tag{17}$$

This condition may not be satisfied by any $z' \in Z$ if b' is sufficiently low. Returning to Figure 1, higher values of k' and z shift the $w(k', b', z, \cdot)$ schedule up, thus lowering the default threshold z'_d. Intuitively, high values of the capital stock imply that the realized shock must be very low in order to induce default. Similarly, high values of z are associated with lower bond yields $(\widetilde{r})$, which implies that worse shocks are required to induce default. On the other hand, high values of b' shift the $w(k', b', z, \cdot)$ schedule down, thus increasing the default threshold. Proposition 6 summarizes.

PROPOSITION 6: *The critical shock inducing default,* $z_d : K \times B \times Z \rightarrow Z$, *is a continuous function, decreasing in its first and third arguments, and increasing in its second.*

Proof. See equation (17). Continuity follows from w and $\underline{w}$ being continuous. Monotonicity in the various arguments follows from monotonicity of w.

In the event of renegotiation, the intermediary recovers a payment sufficient to drive net worth down to $\underline{w}(z')$. The intermediary's recovery in default, net of verification costs, is equal to

$$R(k', b', z, z') = (1-\xi)(1-\delta)k' + z'\pi(k') - T^c(k', b', z, z') - \underline{w}(z'). \tag{18}$$

The required bond yield is determined by a zero profit condition for the intermediary

$$b' = \left[\frac{1}{1+r(1-\tau_i)}\right]\left[[1+(1-\tau_i)\widetilde{r}(k', b', z)]b'\int_{z_d(k',b',z)}^{\overline{z}} Q(z, dz') + \int_{\underline{z}}^{z_d(k',b',z)} R(k', b', z, z')Q(z, dz')\right]. \tag{19}$$

Holding fixed the pair (k', z), for modestly risky debt $\widetilde{r}$ must be increasing in b'. However, there are limits to how much the firm can raise through debt, as it eventually reaches a debt capacity where further increases in $\widetilde{r}$ actually reduce b'. Attention is confined to pairs $(\widetilde{r}, b')$ where debt value is increasing in the promised yield, since other pairs are dominated on efficiency grounds. Along this region, equation (19) can be inverted. The required bond yield is

$$\widetilde{r}(k', b', z) = \left[\frac{1}{1-\tau_i}\right]\left[\frac{1+r(1-\tau_i) - \int_{\underline{z}}^{z_d(k',b',z)}[R(k', b', z, z')/b']Q(z, dz')}{\int_{z_d(k',b',z)}^{\overline{z}} Q(z, dz')}\right]. \tag{20}$$

This analysis closes the model, since the bond market equilibrium is consistent with the maximization problem posited for the firm (13). Constraints *iii* and *iv* are implicit in the bond pricing equation. Equation (20) implies that the function $\widetilde{r}$ is continuous, thus satisfying *ii*. The fact that Γ is nondecreasing follows from maintained assumption that Q is monotone (increasing). This property of the transition function ensures that debt capacity is increasing in z. Other properties of Γ follow by construction.

C. Optimal Policies

To simplify the exposition, this subsection assumes V is concave and once differentiable.[6] In order to characterize optimal financial policy, hold k' fixed and consider the choice of b'. For simplicity, suppose $k' > \widetilde{w}$, implying that the firm requires external funding. Let b'_0 denote the amount of debt required to finance the investment program

$$b'_0 \equiv k' - \widetilde{w}. \tag{21}$$

[6]This assumption is not utilized in the numerical analysis. Concavity and differentiability can be established if $\lambda_0 = 0$ and restrictions are imposed on Q. In particular, see Proposition 3 in Cooley and Quadrini (2001).

Now consider the effect of a small positive perturbation in b' on the right-side of the Bellman equation

$$\begin{aligned}\frac{\partial V}{\partial b'} &= \Phi_i[1+\Lambda_1(k'-\widetilde{w}-b')] + \Phi_d[1-\tau_d(\widetilde{w}+b'-k')] \\ &\quad -\int_{z'_d}^{\overline{z}} \frac{\left[1+(1-\tau_c)\left(\widetilde{r}(k',b',z)+b'\frac{\partial \widetilde{r}}{\partial b'}\right)\right] * V_1(w',z')}{1+r(1-\tau_i)} Q(z,dz'),\end{aligned} \tag{22}$$

The first line in (22) represents the marginal benefit to shareholders today from increasing debt. If $b' < b'_0$, then $\Phi_i = 1$ and debt replaces external equity. If $b' > b'_0$, then $\Phi_d = 1$ and marginal debt finances higher distributions. Figure 2 graphs the marginal benefit schedule under linear-quadratic costs of external equity. The MB schedule is strictly declining. The first units of debt substitute for high levels of external equity, which are the most costly. At the opposite extreme, high levels of debt are used to finance distributions to shareholders, who face an increasing τ_d schedule. The MB schedule exhibits a jump at b'_0 whenever $\lambda_1 > 0$.

The second line in (22) represents the marginal cost (MC) of debt service. The upward slope of the MC schedule is caused by four factors. First, higher amounts of debt increase the probability that interest expense will only be deductible at the lower rate τ_c^-. As in the detailed micro-simulations performed by Graham (2000), the expected marginal corporate tax rate in our model is flat at τ_c^+ up to a kink point, where it then becomes downward sloping. Second, Proposition 6 shows that increasing b' increases the probability of default, a standard effect. To compensate for higher default risk, the lender demands a higher promised yield ($\widetilde{r}$). Third, increases in $\widetilde{r}$ raise the cost of servicing intra-marginal units of debt. Finally, since V is concave, the shadow value of funds devoted to debt service increases with b'.

Figure 2 depicts the optimal financial policy for various positions of the MC schedule. Consider first the lowest MC schedule. The optimal b' is at the intersection of MC_L and the MB schedule. This firm issues a high amount of debt in order to finance distributions at the margin. For the firm facing the MC_M schedule, the optimal policy entails financing the entire investment with debt, with no equity issued and no distribution to shareholders. The optimal policy for the firm facing the high marginal cost schedule depends on the magnitude of the fixed costs of external equity. If $\lambda_0 < S$, the optimal choice of b' is at the intersection of the MC_H and MB schedules. The firm employs a relatively low amount of debt, using external equity to make up the rest of the financing gap. At the optimal policy, the marginal costs of debt and external equity are equated. If $\lambda_0 > S$, the firm will not issue equity, finding that the fixed costs swamp the marginal benefits.[7]

The analysis suggests the following insights which inform the choice of moments utilized in the indirect inference estimation. Under high fixed costs (λ_0), equity market access is lumpy, as firms avoid small flotations. If λ_1 is high, firms cluster around zero distributions. High values of λ_2 limit the variance and skewness of equity issuance. A high degree of curvature in the τ_d schedule limits the variance of distributions. Finally, high bankruptcy costs (ξ) shift up the MC schedule, reducing optimal leverage. In the next section, we perform numerical comparative statics on the model (under an exogenous parameterization) in order to clarify the effect of various frictions on various model-generated moments.

Consider next the effect of a perturbing the capital stock (k'). At an interior solution, the optimal investment policy satisfies

$$\frac{\partial V}{\partial k'} = -[\Phi_i(1+\Lambda_1(k'-\widetilde{w}-b'))+\Phi_d(1-\tau_d(\widetilde{w}+b'-k'))]\left[1-\frac{\partial b'}{\partial k'}\right]+\int_{z'_d}^{\overline{z}} \frac{V_1(w',z')\left(\frac{\partial w'}{\partial k'}+\frac{\partial w'}{\partial b'}\frac{\partial b'}{\partial k'}\right)}{1+r(1-\tau_i)} Q(z,dz') = 0. \tag{23}$$

Referring to equation (23), consider those firms for whom debt is the marginal source of funds. In Figure 2, debt is the marginal source of funds for those facing the MC_M schedule and those facing the MC_H schedule with high fixed costs of equity issuance ($\lambda_0 > S$). For such firms, $\partial b'/\partial k' = 1$, so the optimality condition simplifies to

$$\int_{z'_d}^{\overline{z}} V_1(\widetilde{w}',z')\left[z'\pi_1(k') - [\widetilde{r}(k',b',z)+\delta] - b'\left(\frac{\partial \widetilde{r}}{\partial k'}+\frac{\partial \widetilde{r}}{\partial b'}\right)\right] Q(z,dz') = 0. \tag{24}$$

Stiglitz (1973) analyzes optimal financial policy and investment in a setting with no uncertainty and no default. He proves that for debt-financed investment: 1) the firm's first-order condition is unaffected by the corporate income tax; and 2) the marginal revenue product of capital is equated to $r+\delta$. Condition (24) shows that the first of Stiglitz' results carries over to a dynamic environment with uncertainty and default. Stiglitz' second result must be modified, given that the required bond yield is not constant. The optimal investment policy accounts for intra-marginal effects associated with changes in k' and b'.

Next consider firms with strictly positive equity issuance or strictly positive distributions. Rewriting (23)

[7] If $\lambda_0 = S$, the optimal policy is not unique. In such cases, it is assumed that equity is not issued. Formally, this is a measurable selection from the optimal policy correspondence.

yields

$$\left[\Phi_i(1+\Lambda_1(k'-\widetilde{w}-b'))+\Phi_d(1-\tau_d(\widetilde{w}+b'-k'))+\int_{z'_d}^{\overline{z}}\frac{V_1(w',z')\frac{\partial w'}{\partial b'}}{1+r(1-\tau_i)}Q(z,dz')\right]\frac{\partial b'}{\partial k'} \tag{25}$$

$$-[\Phi_i(1+\Lambda_1(k'-\widetilde{w}-b'))+\Phi_d(1-\tau_d(\widetilde{w}+b'-k'))]+\int_{z'_d}^{\overline{z}}\frac{V_1(w',z')\frac{\partial w'}{\partial k'}}{1+r(1-\tau_i)}Q(z,dz')=0.$$

For such firms, the first bracketed term in (25) is zero. Therefore, the optimal investment plan satisfies

$$\Phi_i(1+\Lambda_1(k'-\widetilde{w}-b'))+\Phi_d(1-\tau_d(\widetilde{w}+b'-k'))=\int_{z'_d}^{\overline{z}}\frac{V_1(w',z')\left[1+(1-\tau_c)(z'\pi_1(k')-\delta-b'\frac{\partial\widetilde{r}}{\partial k'})\right]}{1+r(1-\tau_i)}Q(z,dz'). \tag{26}$$

Intuitively, firms issuing equity are just indifferent between financing incremental investment with debt or equity. Hence, the marginal cost of funds is $1+\Lambda_1$, which is equated with the expected discounted marginal benefit from installed capital. Similarly, firms making positive distributions are indifferent between financing incremental investment with debt or a reduction in distributions. Hence, the term $1-\tau_d$ represents the marginal cost of investing. Clearly, the opportunity cost of investing is lower for firms making distributions than those issuing equity, thus encouraging capital accumulation.

3. Benchmark Simulation

This section presents a simulation of the model based on reasonable parameter values that are gleaned from previous studies. The intent is to provide the reader with an intuitive understanding of the mapping between the model and moments, before proceeding to our estimates of the underlying parameters. The analysis also serves as a robustness check of the comparative static properties of the model under alternative parameterizations.

A. Design

The shock z follows an $AR(1)$ process in logs:

$$\ln(z')=\rho\ln(z)+\varepsilon', \tag{27}$$

where $\varepsilon' \sim N\left(0, \sigma_e^2\right)$. We transform (27) into a discrete-state Markov chain using the method in Tauchen (1986), letting z have 5 points of support in $\left[-3\sigma_e \middle/ \sqrt{1-\rho^2}\right., \left.3\sigma_e \middle/ \sqrt{1-\rho^2}\right]$. We set $\alpha = 0.623$, which is mid-way between the point estimates of Cooper and Ejarque (2003) and Hennessy and Whited (2004). Also following Hennessy and Whited (2004), we set $\sigma_e = 0.118$ and $\rho = 0.740$.

The state space for (k, p, z) is discrete. The capital stock, k, lies in the set

$$\left[\overline{k}, \overline{k}\left(1-\delta\right)^{1/2}, \overline{k}\left(1-\delta\right), \ldots, \overline{k}\left(1-\delta\right)^{10}\right],$$

where $\overline{k}$ is defined by (12). The state space for b has the same number of points as the state space for k. We set the maximal value equal to k^{α}/r and the minimal value equal to the opposite of the maximal value. The maximal value represents a crude guess of the value of the firm. These state spaces for k and b appear to be sufficient for our purposes in that the optimal policy never occurs at an endpoint of either state space.

Next we define the tax environment. For $\bar{\tau}_d$, we use the estimate in Graham (2000) of 12%. We set the parameter ϕ in (4) equal to 0.02. We also set the tax rate on interest income, τ_i, equal to the Graham (2000) estimate of 29.6%. We set the maximal corporate tax rate $\tau_c^+ = 40\%$, which is close to the average combined state and federal tax rates. We set $\tau_c^- = 20\%$.

Next we parameterize the financial frictions. Here, following Gomes (2001), we set $\lambda_1 = 0.028$. We set $\lambda_0 = 1.2$, which gives us a ratio of fixed costs to equity issuance close to the 0.35% figure in Altinkilic and Hansen (2000). We set $\lambda_2 = 0.005$, to represent the increasing marginal costs of equity issuance found in Altinkilic and Hansen (2000). We set $\xi = 10\%$, to account for indirect and direct costs of bankruptcy. The real risk-free interest rate is $r = 2.5\%$.

The model is solved via iteration on the Bellman equation, which produces the value function $V(\tilde{w}, z)$ and the policy function $(k', b') \equiv h\left(\widetilde{w}, z\right)$. The numerical solution proceeds in two steps. First, we guess $\tilde{r}\left(k', b', z\right) = r$, and solve for the value function given this guess. Second, we use the solution for the value function to identify default states and then recalculate $\tilde{r}\left(k', b', z\right)$ according to (20). We then iterate on this two-step procedure until the value function converges.

The model simulation proceeds by taking a random draw of the z shock and then computing $V(\tilde{w}, z)$ and $h(\tilde{w}, z)$. In the model simulation, the space for z is expanded to include 20 points, with interpolation used to find corresponding values of V, k, and b. The model is simulated for 1000 time periods, with the first fifty observations dropped in order to allow the firm to work its way out of a possibly sub-optimal starting point.

Knowledge of h and V also allows us to compute interesting quantities such as cash flow, Tobin's q, debt, and distributions. Specifically, we define our variables to mimic the sorts of variables used in the literature.

Ratio of investment to the "book value" of assets	$(k' - (1-\delta)k)/\,k$
Ratio of cash flow to the book value of assets	$(zk^{\alpha} - T^{c}(k', b', z, z') - b)/\,k$
Tobin's q	$(V(\tilde{w}, z) + b')/\,k' + u$
Ratio of debt to the "market value" of assets	$b'/\,(V(\widetilde{w}, z) + b')$
Ratio of equity issuance to the book value of assets	$(k' - \widetilde{w} - b')/\,k$

As discussed by Erickson and Whited (2000), computation of average q using real-world data sets involves numerous judgment calls and imputations. Of course, this produces measurement error. In contrast, there is no measurement error when average q is computed from a structural model. Since it is impossible to remove measurement error from the real-world data, we put the model on equal footing by adding a pseudo-normal error term, denoted u, to model-generated q. We set $\sigma_u = 2.4$. The implied R^2 from the regression of $(V + b)/\,k + u$ on $(V + b)/\,k$ is approximately 0.4—a figure in line with the estimates in Erickson and Whited (2000). All variables are normalized by the book value of assets, except for debt, which is normalized by the market value of the firm.

B. Results

Table I presents the results from simulating the model. The first column provides moments from the simulated data. The rest of the table provides elasticities of these moments with respect to the underlying structural parameters, providing the reader with a sense of how various moments vary when financial frictions change.

Each elasticity is calculated by simulating the model twice: once with a value of the parameter of interest fifty percent below its baseline value, and once with a value fifty percent above its baseline value. The change in the moment is calculated as the difference between results from the two simulations. This difference is divided by the change in the underlying structural parameter between the two simulations. The result is then multiplied by the ratio of the baseline structural parameter to the baseline moment.

First we discuss the more interesting moments. The simulated firm issues equity 4.65% of the time.

Conditional upon issuing equity, the average flotation is equal to 6.34% of total assets. This average has a substantial variance and is highly skewed. Unlike equity issuance, distributions have a low variance. The average debt to asset ratio is 11.56%. The firm has negative leverage, i.e. holds cash, approximately 24% of the time. The coefficient on q is quite close to most of the estimates found in the literature. The coefficient on cash flow, however, is substantially smaller than most empirical estimates.

Next we turn to the elasticities. Not surprisingly, the frequency and size of equity issuance are quite sensitive to the three parameters that determine the cost of external equity. Leverage and cash holding are sensitive to verification/bankruptcy costs, ξ, and to the parameters governing the driving process for z: ρ and σ_e. Intuitively, the more variable are the shocks, the less desirable is debt given costs of default, and the higher is the precautionary savings motive.

It is interesting to note that the cash flow coefficient is decreasing in all parameters of the external equity cost function and increasing in bankruptcy costs. This result underscores the idea that one number, cash flow sensitivity, cannot capture the magnitude of all external financial frictions.

4. Indirect Inference Estimation

A. Data

Our data are from the full coverage 2002 Standard and Poor's COMPUSTAT industrial files. We select a sample by first deleting firm-year observations with missing data. Next, we delete observations in which total assets, the gross capital stock, or sales are either zero or negative. To avoid rounding errors, we delete firms whose total assets are less than two million dollars and gross capital stocks are less than one million dollars. Further, we delete observations that fail to obey standard accounting identities. Finally, we omit all firms whose primary SIC classification is between 4900 and 4999 or between 6000 and 6999, since our model is inappropriate for regulated or financial firms. We end up with an unbalanced panel of firms from 1993 to 2001 with between 592 and 1128 observations per year. We truncate our sample period below at 1993, because our tax parameters are relevant only for this period.

B. Methodology

Because our model has no closed-form solution, we opt for an estimation technique based on simulation. Specifically, we estimate the structural parameters of the model via the indirect inference method proposed in Gourieroux, Monfort, and Renault (1993) and Gourieroux and Monfort (1996). This procedure chooses the parameters to minimize the distance between model-generated moments and the corresponding moments from actual data. Because the moments of the model-generated data depend on the structural parameters utilized, minimizing this distance will, under certain conditions discussed below, provide consistent estimates of the structural parameters. Another appealing feature of this approach is that it allows us to establish a link between our model and existing evidence on investment-cash flow sensitivities.

We now give a brief outline of this procedure. The goal is to estimate a vector of structural parameters, b, by matching a set of *simulated moments*, denoted as m, with the corresponding set of actual *data moments*, denoted as M. The candidates for the moments to be matched include simple summary statistics, OLS regression coefficients, and coefficient estimates from non-linear reduced-form models.

Without loss of generality, the data moments to be matched can be represented as the solution to the maximization of a criterion function

$$\hat{M}_N = \arg\max_M J_N\left(Y_N, M\right),$$

where Y_N is a data matrix of length N. For example, the sample mean of a variable, x, can be thought of as the solution to minimizing the sum of squared errors of the regression of x on a constant. We estimate $\hat{M}_N$ and then construct S simulated data sets based on a given parameter vector. For each of these data sets, we estimate m by maximizing an analogous criterion function

$$\hat{m}^s_{N'}\left(b\right) = \arg\max_m J_{N'}\left(Y^s_{N'}, m\right),$$

where $Y^s_{N'}$ is a simulated data matrix of length N'. Note that we express the simulated moments, $\hat{m}^s_{N'}\left(b\right)$, as explicit functions of the structural parameters, b. The indirect estimator of b is then defined as the solution to the minimization of

$$\hat{b} = \arg\min_{b} \left[\hat{M}_N - \frac{1}{S} \sum_{s=1}^{S} \hat{m}_{N'}^{s}(b) \right]' \hat{W}_N \left[\hat{M}_N - \frac{1}{S} \sum_{s=1}^{S} \hat{m}_{N'}^{s}(b) \right]$$

$$\equiv \arg\min_{b} \hat{G}_N' \hat{W}_N \hat{G}_N,$$

where $\hat{W}_N$ is a positive definite matrix that converges in probability to a deterministic positive definite matrix W. In our application, a consistent estimator of W is given by $\left[N \operatorname{var}\left(\hat{M}_N\right)\right]^{-1}$. Since our moment vector consists of both means and regression coefficients, we use the influence-function approach in Erickson and Whited (2000) to calculate this covariance matrix. Specifically, we stack the influence functions for each of our moments and then form the covariance matrix by taking the sample average of the inner product of this stack.

The indirect estimator is asymptotically normal for fixed S. Define $J \equiv \operatorname{plim}_{N\to\infty}(J_N)$. Then

$$\sqrt{N}\left(\hat{b} - b\right) \xrightarrow{d} \mathcal{N}\left(0, \operatorname{avar}(\hat{b})\right),$$

where

$$\operatorname{avar}(\hat{b}) \equiv \left(1 + \frac{1}{S}\right) \left[\frac{\partial J}{\partial b \partial m'} \left(\frac{\partial J}{\partial m} \frac{\partial J'}{\partial m} \right)^{-1} \frac{\partial J}{\partial m \partial b'} \right]^{-1}. \tag{28}$$

Further, the technique provides a test of the overidentifying restrictions of the model, with

$$\frac{NS}{1+S} \hat{G}_N' \hat{W}_N \hat{G}_N$$

converging in distribution to a χ^2 with degrees of freedom equal to the dimension of M minus the dimension of b.

The success of this procedure relies on picking moments m that can identify the structural parameters b. In other words, the model must be identified. Global identification of a simulated moments estimator obtains when the expected value of the difference between the simulated moments and the data moments equal zero if and only if the structural parameters equal their true values. A sufficient condition for identification is a one-to-one mapping between the structural parameters and a subset of the data moments of the same dimension. Although our model does not yield such a closed form mapping, we select our moments based on the underlying theory. In particular, we exploit moments that the underlying theoretical model indicates *should* be informative about the various frictions.

We use a minimization algorithm, simulated annealing, that avoids local minima. Finally, we perform an informal check of the numerical condition for local identification. Let $\hat{m}^s_{bN'}$ be a subvector of m with the same dimension as b. Local identification implies that the Jacobian determinant, $\det\left(\partial \hat{m}^s_{N'}(b)/\partial b\right)$, is non-zero. This condition can be interpreted loosely as saying that the moments, m, are informative about the structural parameters, b; that is, the sensitivity of m to b is high. If this were not the case, not only would $\det\left(\partial \hat{m}^s_{N'}(b)/\partial b\right)$ be near zero, but the sample counterpart to the term $\partial J/\partial b \partial m'$ in (54) would be as well—a condition that would cause the parameter standard errors to blow up.

It is worth noting that indirect inference offers an important advantage over OLS and IV as a basis for parameter estimation. In particular, it does not suffer from simultaneity problems, since it does not require the zero-correlation restrictions that are necessary to identify OLS and IV regressions. Rather, as in a standard GMM estimation, it merely requires at least as many moments as underlying structural parameters.

To generate simulated data comparable to COMPUSTAT, we create $S = 6$ artificial panels, containing 10,000 *i.i.d.* firms.[8] We simulate each firm for 50 time periods and then keep the last nine, where we pick the number "nine" to correspond to the time span of our COMPUSTAT sample. Dropping the first part of the series allows us to observe the firm after it has worked its way out of a possibly suboptimal starting point.

One final issue is unobserved heterogeneity in our data from COMPUSTAT. Recall that our simulations produce *i.i.d.* firms. Therefore, in order to render our simulated data comparable to our actual data we can either add heterogeneity to the simulations, or take the heterogeneity out of the actual data. We opt for the latter approach, using fixed firm and year effects in the estimation of all of our data moments.

In order to estimate the eight unknown parameters $(\lambda_0, \lambda_1, \lambda_2, \xi, \phi, \sigma_e, \rho, \sigma_u)$ we must match at least eight model-generated moments with corresponding data moments. The parameters governing production, α and δ, are not estimated given that our focus is on financing. In addition, numerous other studies have already estimated these parameters. As discussed above, tax rate parameters are based upon estimates from Graham (2000).

We use twelve data moments in order to have an overidentified model. We start with the average,

[8] Michaelides and Ng (2000) find that good finite sample performance of an indirect inference estimator requires a simulated sample that is approximately ten times as large as the actual data sample.

variance, and skewness of the ratio of equity issuance to assets. We also use the frequency of equity issuance; the fraction of firm years in which the firm neither issues equity nor distributes; the fraction of firm years in which the firm saves rather than borrows; the variance of the ratio of distributions to assets; and the average ratio of net debt to total assets, where net debt is defined as total long-term debt less cash.

The next two moments are the two slope coefficients from a regression of the ratio of investment to the capital stock on cash flow and Tobin's q. These three variables are calculated as in Erickson and Whited (2000). The final two moments capture important features of the driving process for z. We estimate a first-order panel autoregression of operating income on lagged operating income using the technique in Holtz-Eakin, Newey, and Rosen (1988). Operating income is defined as COMPUSTAT item #13 divided by item #6. The two moments that we match from this exercise are the autoregressive parameter and the shock variance.[9]

Assets are COMPUSTAT item #6, equity issuance is item #108 minus item #115, and net debt is item #9 plus item #34 minus item #1, and distributions are the sum of COMPUSTAT items 19 and 21 plus any negative equity issuance.

C. Parameter Estimates and Comparative Statics

The results from this estimation exercise are in Tables II-IV. Table II compares the actual moments with those from the simulated model. We match most of the moments well. Indeed, one cannot reject the null hypothesis that the simulated moments equal the actual data moments. The χ^2 test of this null hypothesis reported in Table III (the test of the model overidentifying restrictions) does not produce a rejection at even the 10% level.

We have slight difficulty matching two of our twelve moments. This first is the frequency of equity issuance, with the simulated firm issuing equity fifty percent more often than the average real firm. Because high fixed costs of equity issuance should lower the frequency of issues, we suspect this result is due to the low point estimate for fixed costs of equity issuance (λ_0) reported in Table III.

[9] As required by the Holtz-Eakin, Newey, and Rosen (1988) technique, we account for fixed effects via differencing our autoregression. For our other regressions, we simply remove firm-level means from the data. We opt for this method simply because it is the method most used in the empirical literature we are trying to understand.

The second moment that we have difficulty in matching is the investment-cash flow sensitivity: our model-generated sensitivity of investment to cash flow is just over half that of the corresponding figure seen in the data. We conjecture that this result is in part due to relatively low estimated measurement error variance for q reported in Table III. This estimate implies that 55% of the variation in "true q" can explained by "observed q." This figure is somewhat higher than the estimates in Erickson and Whited (2000); that is, our estimate of the measurement error variance is lower. Since cash flow and true q are positively correlated, the lower the measurement error variance, the lower the coefficient on cash flow.

Table III contains the point estimates of the structural parameters. As noted above, our estimate of λ_0 is quite small and insignificantly different from zero. This result stands in contrast to that in Altinkiilc and Hansen (2000), who find significant fixed costs. Our estimates of λ_1 and λ_2, are, however, significantly different from zero and in line with their study, which finds that average variable costs of equity issuance are 4.4%. To calculate a comparable figure we take the ratio of total variable costs to equity issuance, finding an average value of 5.8%. The remaining wedge between these estimates could be easily be accounted for by adverse selection premia over and above those capitalized into underwriter fee schedules. Finally, our positive estimate of λ_2 mirrors the increasing marginal costs found in Altinkilic and Hansen.

The point estimate of bankruptcy costs is ξ=12.2%. Taken at face value, this point estimate casts doubt on the conventional wisdom that firms are debt conservative. Firms do not behave "as if" facing implausibly large bankruptcy costs. In fact, one cannot reject the null that bankruptcy costs are zero. However, this parameter estimate is noisy. Perhaps this is not surprising given that the structural model attempts to match the heterogeneous behavior of real-world firms who undoubtedly face different costs of bankruptcy and different risks to underlying cash flows.

Table IV is analogous to Table I, with the sole difference being that the moments and elasticities in Table IV are based upon a simulation of the model parameterized using the estimates from Table III. Once again, the debt to asset ratio and the propensity to hold cash are informative about bankruptcy costs. The frequency, mean, variance, and skewness of equity issuance are informative costs of external equity. This is consistent with the theoretical model presented in Section II.

The cash flow coefficient is not a catch-all for financing costs, nor is it monotonic in the various frictions.

It increases in bankruptcy costs, but decreases in external equity costs. Explaining the behavior of the cash flow coefficient is difficult. However, we would argue that this is to be expected. Real world firms optimize over time and over various quantities, e.g. investment, distributions, and leverage. Changes in the cost of external funds will bring about subtle changes at each margin of choice, rendering it humanly impossible to infer changes in auxiliary moments, such as regression coefficients.

5. Conclusions

This paper proposed an alternative to investment-cash flow sensitivities as a basis for estimating the magnitude of financial frictions faced by corporations. Starting with primitives, we first presented a dynamic structural model endogenizing all relevant choice variables of the firm: investment, distributions, leverage and default. This model is then taken to the data using indirect inference. We estimate which constellation of financial frictions best explains observed financing behavior, i.e. minimizes the distance between model-generated moments and real-world data moments. Consistent with direct evidence on underwriter fee schedules, behavior is best explained by rising marginal costs of external equity, starting at 3.9%. Contrary to the notion that corporations are debt conservative, debt issuance is consistent with small (12.2%) and statistically insignificant bankruptcy costs. Finally, the cash flow coefficient is not a summary statistic for financing costs, nor is it monotonic in the various frictions. It increases in bankruptcy costs, but decreases in external equity costs.

The model can be generalized along a number of dimensions, which would move us closer to a fully realistic depiction of the firm's problem. First, our model features single-period debt. This may serve as a reasonable approximation for the majority of firms that rely primarily upon short-maturity bank debt. Still, larger and more mature firms do issue longer maturity debt. One would like to permit the firm to choose the *type* of debt, in addition to the amount. Second, the investment cost schedule is linear in our model, ruling out the sorts of irreversibilities that are central to the real options literature. Bridging the gap between the two literatures is necessary. Finally, on the empirical front, it might be asking too much of any model to match data moments across industries. It would be interesting to see how well the model performs on an industry-by-industry basis.

Appendix

Proof of Lemma 1

In the interest of brevity and keeping our notation consistent with that in Stokey and Lucas (1989), let

$$\begin{aligned} F(\widetilde{w},k',b',z) &\equiv \Phi_d[\widetilde{w}+b'-k'-T^d(\widetilde{w}+b'-k')]-\Phi_i[k'-\widetilde{w}-b'+\Lambda(k'-\widetilde{w}-b')] \\ \beta &\equiv \frac{1}{1+r(1-\tau_i)}. \end{aligned}$$

We will show that, without loss of generality, the set of possible endogenous state variables can be treated as a compact set. For Lemma 1, $\widetilde{W} \times Z$ is treated as compact. Weierstrass' Theorem ensures that each $f \in C(\widetilde{W} \times Z)$ is bounded.

Partitioning the constraint correspondence as follows

$$\begin{aligned} \Gamma^+(z) &\equiv \{(k',b') \in \Gamma(z) : \widetilde{w}+b'-k' \geq 0\} \\ \Gamma^-(z) &\equiv \{(k',b') \in \Gamma(z) : \widetilde{w}+b'-k' \leq 0\}, \end{aligned}$$

we may express the Bellman operator (T) for this problem as follows, for arbitrary $f \in C(\widetilde{W} \times Z)$:

$$(Tf)(\widetilde{w},z) \equiv \max \left\{ \begin{array}{ll} \max\limits_{(k',b')\in\Gamma^-(z)} & -[k'-\widetilde{w}-b'+\Lambda(k'-\widetilde{w}-b')]+\beta\int_Z f[\widetilde{w}(k',b',z,z'),z']Q(z,dz'), \\ \max\limits_{(k',b')\in\Gamma^+(z)} & \widetilde{w}+b'-k'-T^d(\widetilde{w}+b'-k')+\beta\int_Z f[\widetilde{w}(k',b',z,z'),z']Q(z,dz') \end{array} \right\}$$

where the constraints are as specified in (13).

We first claim that

$$T : C(\widetilde{W},Z) \rightarrow C(\widetilde{W},Z).$$

Fix $f \in C(\widetilde{W},Z)$ and consider first the problem

$$\max_{(k',b')\in\Gamma^+(z)} \quad \widetilde{w}+b'-k'-T^d(\widetilde{w}+b'-k')+\beta\int_Z f[\widetilde{w}(k',b',z,z'),z']Q(z,dz').$$

Continuity of the function $\widetilde{r}$ implies continuity of $\widetilde{w}$. Lemma 9.5$'$ in Stokey and Lucas (SL) (1989), implies that the expectation above is bounded and continuous. From the Theorem of the Maximum, the value function

$$f^+(\widetilde{w},z) \equiv \max_{(k',b')\in\Gamma^+(z)} \quad \widetilde{w}+b'-k'-T^d(\widetilde{w}+b'-k')+\beta\int_Z f[\widetilde{w}(k',b',z,z'),z']Q(z,dz')$$

is continuous, and hence bounded. By the same reasoning, the value function

$$f^{-}(\widetilde{w},z) \equiv \max_{(k',b')\in\Gamma^{-}(z)} \quad -[k'-\widetilde{w}-b'+\Lambda(k'-\widetilde{w}-b')]+\beta\int_Z f[\widetilde{w}(k',b',z,z'),z']Q(z,dz')$$

is continuous, and hence bounded.

We can then write the Bellman operator as

$$(Tf)(\widetilde{w},z) \equiv \max\left\{f^{+}(\widetilde{w},z), f^{-}(\widetilde{w},z)\right\},$$

which is also continuous, and hence bounded. This establishes the first claim.

We next show that T satisfies Blackwell's sufficient conditions for a contraction mapping, stated as Theorem 3.3 in SL. To establish monotonicity, consider arbitrary functions f_1 and f_2 in $C(\widetilde{W}\times Z)$, where $f_1 \leq f_2$ on $\widetilde{W}\times Z$. For $i=1,2$, we can define the same partitioned maximization problems as above, with

$$(Tf_i)(\widetilde{w},z) \equiv \max\left\{f_i^{+}(\widetilde{w},z), f_i^{-}(\widetilde{w},z)\right\}.$$

Let (k'_*, b'_*) be the optimal policies corresponding to the value $f_1^{+}(\widetilde{w},z)$. It follows that

$$\begin{aligned} f_1^{+}(\widetilde{w},z) &= \widetilde{w}+b'_*-k'_*-T^d(\widetilde{w}+b'_*-k'_*)+\beta\int_Z f_1[\widetilde{w}(k'_*,b'_*,z,z'),z']Q(z,dz') \\ &\leq \widetilde{w}+b'_*-k'_*-T^d(\widetilde{w}+b'_*-k'_*)+\beta\int_Z f_2[\widetilde{w}(k'_*,b'_*,z,z'),z']Q(z,dz') \\ &\leq f_2^{+}(\widetilde{w},z). \end{aligned}$$

The first inequality follows from the hypothesis $f_1 \leq f_2$ and the second follows from a standard dominance argument. By the same reasoning

$$\begin{aligned} f_1^{-}(\widetilde{w},z) &\leq f_2^{-}(\widetilde{w},z) \\ &\Rightarrow Tf_1(\widetilde{w},z) \leq Tf_2(\widetilde{w},z). \end{aligned}$$

Now fix scalar $a \geq 0$ and $f \in C(\widetilde{W}\times Z)$. We have

$$\begin{aligned} [T(f+a)](\widetilde{w},z) &\equiv \max\left\{\begin{array}{ll} \max\limits_{(k',b')\in\Gamma^{-}(z)} & -[k'-\widetilde{w}-b'+\Lambda(k'-\widetilde{w}-b')]+\beta\int_Z[f(\widetilde{w}(k',b',z,z'),z')+a]Q(z,dz'), \\ \max\limits_{(k',b')\in\Gamma^{+}(z)} & \widetilde{w}+b'-k'-T^d(\widetilde{w}+b'-k')+\beta\int_Z[f(\widetilde{w}(k',b',z,z'),z')+a]Q(z,dz') \end{array}\right\} \\ &= \beta a+(Tf)(\widetilde{w},z). \end{aligned}$$

This establishes discounting. Hence, T is a contraction mapping.■

Proof of Proposition 3

Let $C'(\widetilde{W} \times Z)$ and $C''(\widetilde{W} \times Z)$ be the space of all functions in $C(\widetilde{W} \times Z)$, that are, respectively, weakly and strictly increasing in their first argument. SL's Corollary 1 to the Contraction Mapping Theorem shows that

$$T[C'(\widetilde{W} \times Z)] \subseteq C''(\widetilde{W} \times Z) \Rightarrow V \in C''(\widetilde{W} \times Z).$$

Fix $f \in C'(\widetilde{W} \times Z)$ and $z \in Z$. Assume that the policy pairs (k_1', b_1') and (k_2', b_2') attain the supremum for the firm starting with revised net worth consider $\widetilde{w}_1$ and $\widetilde{w}_2$, respectively, where $\widetilde{w}_1 > \widetilde{w}_2$. Then

$$\begin{aligned}
(Tf)(\widetilde{w}_1, z) &= F(\widetilde{w}_1, k_1', b_1', z) + \beta \int_Z f[\widetilde{w}(k_1', b_1', z, z'), z']Q(z, dz') \\
&\geq F(\widetilde{w}_1, k_2', b_2', z) + \beta \int_Z f[\widetilde{w}(k_2', b_2', z, z'), z']Q(z, dz') \\
&> F(\widetilde{w}_2, k_2', b_2', z) + \beta \int_Z f[\widetilde{w}(k_2', b_2', z, z'), z']Q(z, dz') \\
&= (Tf)(\widetilde{w}_2, z).
\end{aligned}$$

The first inequality follows from that fact that (k_1', b_1') must weakly dominate (k_2', b_2') for the firm with revised net worth $\widetilde{w}_1$, since both firms have the same choice set $\Gamma(z)$. The second inequality follows from the fact that F is strictly increasing in its first argument. This establishes

$$T[C'(\widetilde{W} \times Z)] \subseteq C''(\widetilde{W} \times Z).■$$

Proof of Proposition 4

Let $C'(\widetilde{W} \times Z)$ be the space of all functions in $C(\widetilde{W} \times Z)$, that are nondecreasing in their second argument. SL's Corollary 1 to the Contraction Mapping Theorem shows that

$$T[C'(\widetilde{W} \times Z)] \subseteq C'(\widetilde{W} \times Z) \Rightarrow V \in C'(\widetilde{W} \times Z).$$

Fix $f \in C'(\widetilde{W} \times Z)$ and $\widetilde{w} \in \widetilde{W}$. Assume that the policy pairs (k_1', b_1') and (k_2', b_2') attain the supremum for the firm starting with the shocks z_1 and z_2, respectively, where $z_1 > z_2$. Then

$$\begin{aligned}(Tf)(\widetilde{w}, z_1) &= F(\widetilde{w}, k_1', b_1', z_1) + \beta \int_Z f[\widetilde{w}(k_1', b_1', z_1, z'), z'] Q(z_1, dz') \\ &\geq F(\widetilde{w}, k_2', b_2', z_1) + \beta \int_Z f[\widetilde{w}(k_2', b_2', z_1, z'), z'] Q(z_1, dz') \\ &\geq F(\widetilde{w}, k_2', b_2', z_2) + \beta \int_Z f[\widetilde{w}(k_2', b_2', z_2, z'), z'] Q(z_2, dz') \\ &= Tf(\widetilde{w}, z_2).\end{aligned}$$

The first inequality follows from that fact that (k_1', b_1') must weakly dominate (k_2', b_2') for the firm facing the shock z_1, since $\Gamma(z_2) \subseteq \Gamma(z_1)$ by hypothesis. The second inequality follows from the fact that F is nondecreasing in z, $\widetilde{w}$ is nondecreasing in its third argument, and Q is monotone.■

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Table I: Sensitivity of Model Moments to Parameters

	Baseline Moments	λ_0	λ_1	λ_2	ϕ	ξ	σ_e	ρ	σ_u
Average Equity Issuance/Assets	0.0634	-0.4010	-1.2576	-0.7251	-0.1288	0.4638	1.3129	-0.6129	0.0000
Variance Equity Issuance/Assets	0.8657	-0.3923	-1.3354	-0.8784	-0.1311	0.0420	0.9579	0.1834	0.0000
Skewness Equity Issuance/Assets	8.2353	-0.3329	-1.3677	-0.9467	-0.1264	0.1317	1.5959	1.3224	0.0000
Frequency of Equity Issuance	0.0465	-0.5667	-0.6302	-0.2677	-0.1081	0.0250	0.3468	-0.2505	0.0000
Frequency of Zero Dividend	0.1542	0.1145	0.0437	-0.0514	-0.0263	0.1564	-0.0575	-1.1716	0.0000
Frequency of Cash Holding	0.2361	-0.1256	0.3860	0.2196	0.7729	0.6360	0.9650	1.1520	0.0000
Variance Distributions/Assets	0.0143	-0.2983	-0.9237	-0.4320	-0.0707	0.0011	2.7675	-0.2092	0.0000
Average Debt-Assets Ratio (Net of Cash)	0.1156	1.0872	1.9597	-0.3136	-0.5485	-1.0192	-1.8896	-2.4520	0.0000
Investment q Sensitivity	0.0165	-0.5130	0.2057	0.0061	0.0145	0.0165	-0.4329	-0.5000	-1.5420
Investment Cash Flow Sensitivity	0.0543	-0.3690	-0.3212	-0.2415	0.0050	0.6708	-1.4352	0.2315	0.8564
Serial Correlation of Income/Assets	0.6433	0.0812	0.1360	0.1068	-0.0224	0.1085	0.1690	1.9624	0.0000
Standard Deviation of the shock to Incomes/Assets	0.1218	0.0686	0.0556	0.0471	-0.0042	0.0005	3.8226	0.1143	0.0000

This table presents elasticities of model moments with respect to the model parameters. The baseline parameters are $\lambda_0 = 1.2$, $\lambda_1 = 0.028$, $\lambda_2 = 0.005$, $\phi = 0.02$, $\xi = 0.1$, $\sigma_\varepsilon = 0.118$, $\rho = 0.740$, and $\sigma_u = 2.4$. Each elasticity is calculated by simulating the model twice: once with a value of the parameter of interest fifty percent below its baseline value, and once with a value fifty percent above its baseline value. Then the change in the moment is calculated as the difference between the results from the two simulations. This difference is then divided by the change in the underlying structural parameter between the two simulations. The result is then multiplied by the ratio of the baseline structural parameter to the baseline moment.

Table II: Simulated Moments Estimation: Moment Estimates

	Actual Moments	Simulated Moments
Average Equity Issuance/Assets	0.042	0.056
Variance Equity Issuance/Assets	0.319	0.546
Skewness Equity Issuance/Assets	4.008	3.054
Frequency of Equity Issuance	0.099	0.156
Frequency of Zero Dividends	0.444	0.540
Frequency of Cash Holding	0.394	0.269
Variance Distributions/Dividends	0.001	0.001
Average Debt-Assets Ratio (Net of Cash)	0.075	0.078
Investment q Sensitivity	0.019	0.021
Investment Cash Flow Sensitivity	0.172	0.098
Serial Correlation of Income/Assets	0.583	0.620
Standard Deviation of the shock to Incomes/Assets	0.117	0.102

Calculations are based on a sample of nonfinancial firms from the annual 2002 COMPUSTAT industrial files. The sample period is 1993 to 2001. Estimation is done with the simulated moments estimator in Gourieroux, Monfort, and Renault (1993), which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from these data. The simulated panel of firms is generated from the dynamic partial-equilibrium model in Section II, which characterizes the firm's optimal choice of investment and capital structure in the face of corporate and personal taxes and costs of financial distress. The model is solved by value-function iteration. The simulated panel contains 10,000 firms over 50 time periods, where only the last nine time periods are kept for each firm. This table reports the simulated and estimated moments.

Table III: Simulated Moments Estimation: Structural Parameter Estimates

λ_0	λ_1	λ_2	ξ	ϕ	σ_ε	ρ	σ_u	χ^2
0.369	0.039	0.0007	0.122	0.011	0.097	0.701	6.289	4.338
(0.273)	(0.018)	(0.0002)	(0.373)	(0.038)	(0.084)	(0.329)	(2.075)	(0.362)

Calculations are based on a sample of nonfinancial firms from the annual 2002 COMPUSTAT industrial files. The sample period is 1993 to 2001. Estimation is done with the simulated moments estimator in Gourieroux, Monfort, and Renault (1993), which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from these data. The simulated panel of firms is generated from the dynamic partial-equilibrium model in Section II, which characterizes the firm's optimal choice of investment and capital structure in the face of corporate and personal taxes and costs of financial distress. The model is solved by value-function iteration. The simulated panel contains 10,000 firms over 50 time periods, where only the last nine time periods are kept for each firm. This table reports the estimated structural parameters, with standard errors in parentheses. λ_0, λ_1, and λ_2 are the fixed, linear, and quadratic costs of equity issuance. ϕ governs the shape of the distributions tax schedule, with a lower value for ϕ corresponding to a flatter tax schedule. ξ is the verication parameter, with total verification costs equal to ξ times the capital stock. σ_ε is the standard deviation of the innovation to $\ln(z)$, and ρ is the serial correlation of $\ln(z)$. σ_u is the variance of the measurement error in average Q. χ^2 is a chi-squared statistic for the test of the overidentifying restrictions. In parentheses is its p-value.

Table IV: Sensitivity of Model Moments to Parameters

	Baseline Moments	λ_0	λ_1	λ_2	ϕ	ξ	σ_e	ρ	σ_u
Average Equity Issuance/Assets	0.0557	-1.2647	-2.2407	-2.1076	0.0678	0.6401	1.0012	-0.7273	0.0000
Variance Equity Issuance/Assets	0.5455	-1.9209	-2.9099	-2.1886	0.2676	-0.0424	1.8444	0.1490	0.0000
Skewness Equity Issuance/Assets	3.0537	-2.2983	-1.5460	-3.5017	-0.1624	-0.0590	2.4726	0.8411	0.0000
Frequency of Equity Issuance	0.1557	-1.0526	-2.6140	-1.5848	0.1081	-0.0234	1.7661	-0.2383	0.0000
Frequency of Zero Dividends	0.5395	-0.0039	0.0650	0.0673	-0.0455	0.2007	-0.3203	-1.4709	0.0000
Frequency of Cash Holding	0.2690	-0.2178	0.5998	0.4210	0.4954	0.7452	1.6012	0.4018	0.0000
Variance Distributions/Assets	0.0012	-0.5565	-3.0370	-2.5483	0.1234	-0.0185	2.1674	-0.1946	0.0000
Average Debt-Assets Ratio (Net of Cash)	0.0784	-0.1501	0.0348	0.1770	-0.4190	-1.1087	-2.3411	-0.4708	0.0000
Investment q Sensitivity	0.0205	0.4487	-2.6677	-3.1464	0.0449	0.0213	1.7318	0.1933	-1.7654
Investment Cash Flow Sensitivity	0.0981	-0.8596	-2.9175	-2.1012	0.1527	0.7328	1.4838	0.1788	0.9148
Serial Correlation of Income/Assets	0.6201	0.0670	0.2375	0.1703	-0.0232	0.0021	-0.3369	0.9700	0.0000
Standard Deviation of the shock to Incomes/Assets	0.1015	-0.1175	0.0572	0.1650	-0.0385	-0.0004	2.0271	0.0859	0.0000

This table presents elasticities of model moments with respect to the model parameters. The baseline parameters are given in Table III. Each elasticity is calculated by simulating the model twice: once with a value of the parameter of interest fifty percent below its baseline value, and once with a value fifty percent above its baseline value. Then the change in the moment is calculated as the difference between the results from the two simulations. This difference is then divided by the change in the underlying structural parameter between the two simulations. The result is then multiplied by the ratio of the baseline structural parameter to the baseline moment.

Figure 1: Endogenous Default

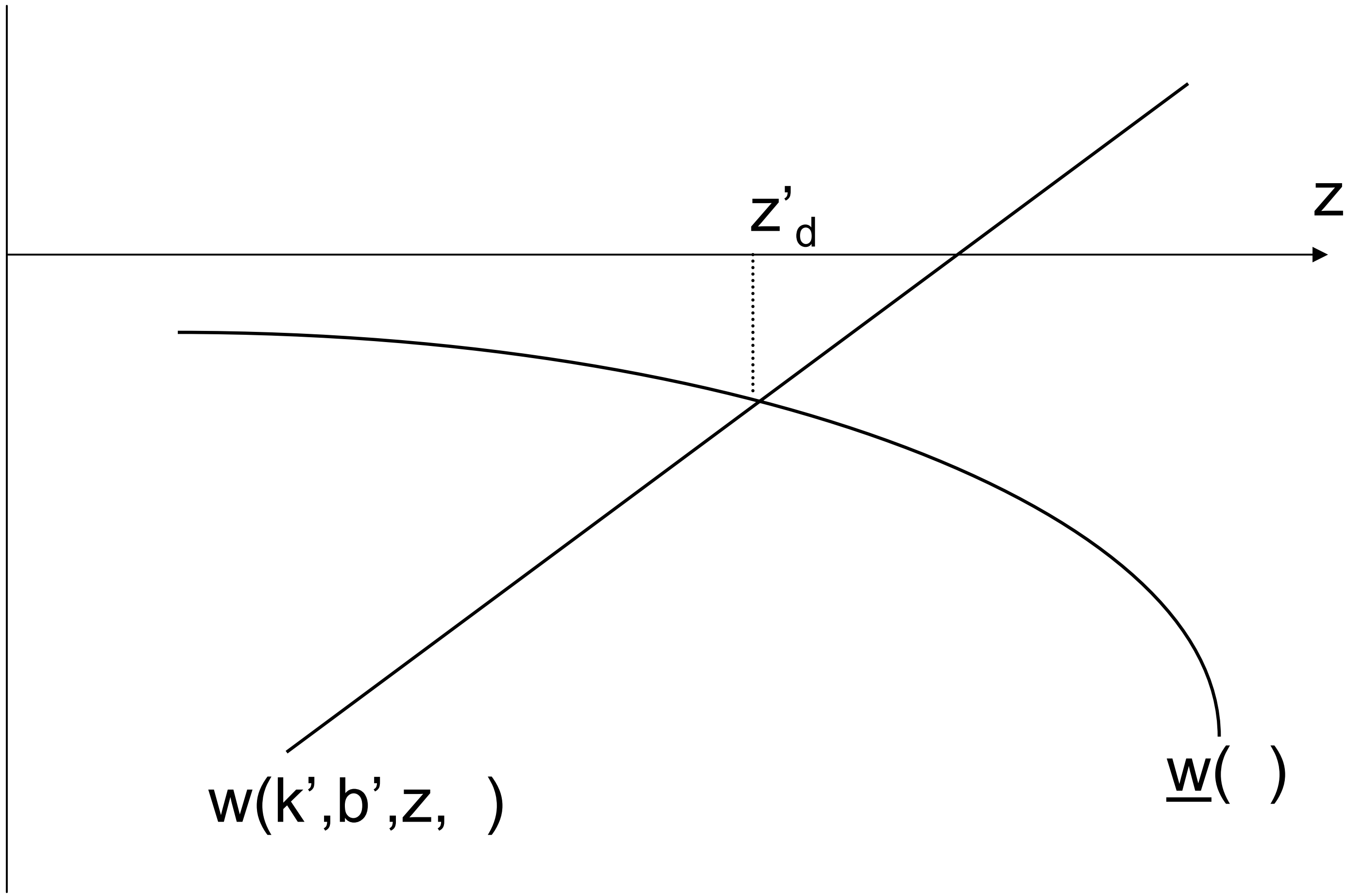

Figure 2: Optimal Financial Policy