

Government Spending Multipliers under Zero Lower Bound: Evidence from Japan*

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Abstract

Using a rich data set on government spending forecasts, we estimate the effects of unexpected government spending both when the nominal interest rate is near zero lower bound (ZLB) and outside of ZLB period in Japan. The output multiplier is 1.5 on impact in the ZLB period while it is 0.7 outside of the ZLB period. We argue that this result is not driven by the amount of slack in the economy. We estimate a positive but mild response of inflation in both periods. We consider a standard New Keynesian model and examine two popular alternatives that can generate the ZLB period: fundamental and confidence shocks. A calibrated model with a fundamental-driven ZLB period can match our empirical findings. A model with a confidence-driven ZLB period can match our finding if government spending stays elevated for a long time even after the ZLB period. However, the government spending is not that persistent in the data.

JEL classification: E32, E62, E5.

Keywords: fiscal stimulus, multiplier, government spending, zero lower bound.

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1 Introduction

Is the output multiplier on government spending large during the periods when nominal interest rate is at the zero bound? The recent global financial crisis, which forced the central banks in many developed countries to reduce their short-term nominal interest rates to the zero lower bound, brought this question to the center of policy debates.

Recent theoretical literature provides a number of answers. For example, [Woodford \(2010\)](#), [Eggertsson \(2011\)](#) and [Christiano et al. \(2011\)](#) show that the multiplier can be large in a standard New Keynesian model in which the ZLB period is caused by a fundamental shock. In this environment, temporary government spending is inflationary which stimulates private consumption and investment by decreasing the real interest rate. As a result, the output multiplier can be larger than one. At the same time, [Mertens and Ravn \(2014\)](#) argue that the output multiplier during the ZLB period is quite small in a New Keynesian model in which ZLB period is caused by non-fundamental confidence shocks. In this situation, government spending shocks are deflationary which increases real interest rate and reduces private consumption and investment. This results in the output multiplier necessarily smaller than one and even smaller than that outside of the ZLB period.

Empirical estimation of the multiplier during the ZLB periods is challenging. One reason is that in most countries, the ZLB periods are short and often coincide with large recessions, making it difficult to distinguish evidence of the ZLB period from that of the recession. For example, [Auerbach and Gorodnichenko \(2012a\)](#) find that the multiplier is significantly larger in recession than in expansion using post-WWII data in the United States. [Ramey and Zubairy \(2014\)](#) extend U.S. data back to 1889, which includes ZLB periods, and find that the high value of the multiplier is sensitive to including the World War II period in the sample.

This paper contributes to the literature both empirically and theoretically. In the empirical part, we estimate the effects of government spending shocks on the economy when the nominal interest rate is at the zero lower bound (the ZLB period) and outside of the ZLB period (the normal period) using Japanese data between 1980Q1 and 2014Q1. We use the fact that Japan has more information on the ZLB periods than other countries. The nominal interest rate in Japan has been near the zero bound since 1995Q4. During this period, Japan goes through four business cycles, so we can potentially distinguish evidence coming from the ZLB period from evidence coming from recessions. We exploit a rich dataset that includes not only standard macroeconomic variables but also forecasts of government spending and other variables. We address the concern that government spending

can be anticipated by constructing *unexpected* government spending changes. In addition, we use data on inflation forecasts to study the behavior of ex-ante real interest rates after government spending shocks.

Using [Jorda \(2005\)](#) local projection method, we find that the output multiplier is 1.5 on impact in the ZLB period while it is 0.7 in the normal period.¹ At longer horizons, the output multiplier increases to over two in the ZLB period while it becomes negative in the normal period. We estimate that the government spending shocks increase both private consumption and investment during the ZLB period. Unemployment rate decreases in the ZLB period, while it does not respond significantly during the normal period. Inflation responses are mild in both periods. Expected inflation increases but also mild in both periods. Nominal interest rate in the normal period increases significantly while it remains constant in the ZLB period. This result implies that the real interest rate does not increase as much in the ZLB period as in the normal period.

We further demonstrate that including forecast data when identifying government spending shocks can change the estimated multiplier in a non-trivial way and it is important to control for the expectational effects. We find that some of the government spending shocks identified without forecast data are expected, especially in the normal period. In fact, the output multiplier obtained without controlling for forecast data is smaller than our baseline estimate in the normal period. Finally, we argue that the difference between the multiplier in the ZLB period and that in the normal period is not driven by the effects of government spending in recessions. One reason is that only nearly half of the ZLB period is recession.

In the theoretical part, we examine two popular theoretical alternatives that can generate the ZLB period. The first alternative is a standard New Keynesian model in which the ZLB period is driven by a fundamental subjective discount rate shock. The second alternative is the same model but with the ZLB period driven by a non-fundamental confidence shock. We show that in the first case a calibrated model can match our empirical results. Then, we show that even when the ZLB period is driven by non-fundamental confidence shocks, the multiplier can be larger than one, in contrast to the previous literature. In addition, we demonstrate that if the government spending shock stays elevated even after the ZLB period for a long time, the model can match our empirical results. However, because the fiscal shocks are not very persistent in the Japanese data, we conclude that the model with a fundamental-driven ZLB fits our empirical findings better.

¹The local projection method estimates impulse response functions by directly projecting a variable of interest on lags of variables usually entering a VAR. This method avoids restrictions present in the VAR analysis. See [Jorda \(2005\)](#) and [Stock and Watson \(2007\)](#) for more details.

Specifically, we first show that a New Keynesian model calibrated using Japanese data generates an output multiplier larger than one in the ZLB period caused by a fundamental shock, a smaller multiplier in the normal period, and mild responses of inflation in both periods which matches our empirical estimates. In the normal period, monetary policy responds to an increase in government spending by raising interest rate. The result is a mild response of inflation, private consumption declines and the output multiplier is less than one. In the ZLB period caused by fundamental shocks, monetary policy does not react to government spending shocks. Inflation expectation increases and the real interest rate decreases, stimulating private consumption. Therefore, the output multiplier is larger than one without a large increase in inflation. We conclude that the New Keynesian model where the ZLB period occurs due to fundamental shocks can explain the difference in the multipliers depending on monetary policy regimes that we document in the data.

There are two key assumptions in the New Keynesian model that allow us to match our empirical evidence. First, we assume a heterogenous labor market in our model. This assumption increases the degree of complementarities between price setters optimal choices, resulting in a sufficiently flat Phillips curve. Second, we assume that government spending are elevated only during the ZLB period. This ensures that government spending has the largest impact on the economy.

We then consider a New Keynesian model where the zero lower bound is caused by a self-fulfilling state of low confidence. In this case, the economy ends up in a “deflationary trap” with zero nominal interest rate and deflation because agents unexpectedly change their beliefs.² In a recent paper, [Aruoba et al. \(2013\)](#) estimate a New Keynesian model for Japan and conclude that Japan is more likely to be in the deflationary trap rather than the liquidity trap driven by fundamental shocks. [Mertens and Ravn \(2014\)](#) argue that the output multiplier in a deflationary trap is substantially smaller than the high values possible in liquidity trap driven by fundamental shocks. Moreover, their deflationary trap multiplier is smaller than one.

However, we show that the output multiplier can be larger than one in a deflationary trap. The key assumption is that government spending is expected to stay elevated even after exiting the ZLB. In a deflationary trap, temporary positive “aggregate demand” shocks are contractionary.³ This property of the equilibrium underlies [Mertens and Ravn \(2014\)](#) result that the output multiplier is small in a deflationary trap when the government spending shock has the same duration as

²[Benhabib et al. \(2001a,b\)](#) show that there is a second steady state in a standard New Keynesian model when the policy rate actively responds to inflation but is also constrained by the zero lower bound.

³This unusual prediction of a deflationary trap equilibrium parallels a non-standard prediction of the equilibrium in which liquidity trap is caused by fundamental shocks: negative aggregate supply shocks are expansionary. See [Eggertsson \(2008\)](#) for details.

the ZLB period. Once we allow government spending shocks to last longer than the deflationary trap, the output multiplier becomes bigger. Intuitively, if government spending stays elevated after exiting the ZLB, agents expect that consumption will decrease and inflation may also decrease. These expectations reduce their incentives to consume in a deflationary trap. If this effect is strong enough, a persistent government spending shock can act as a negative aggregate demand shock, which is expansionary in a deflationary trap. If the negative aggregate demand shock is large, the output multiplier is positive and bigger than one.

The model in which the duration of government spending shocks is sufficiently longer than that of the ZLB period can match our empirical estimates of the output multiplier and inflation response in the ZLB period in Japan. In this calibration, government spending shocks are almost permanent with half-life of 28 years. This is in contrast to the ZLB caused by fundamental shocks, where spending should be less persistent to generate a multiplier larger than in the normal period. Our empirical estimate of the persistence of government spending shocks in Japanese data suggests that the model in which the ZLB occurs due to fundamental shocks fits our empirical findings better.

Related Literature. Our paper contributes to a large body of work in macroeconomics that estimates the effects of government spending shocks on the economy. The relevant literature is large. See, for example, [Ramey \(2011a\)](#) for a survey. The papers in this literature often find the output multiplier to be smaller than one. We also estimate the output multiplier to be smaller than one outside of the ZLB period in Japan.

A recent literature estimates the output multiplier in different states of the economy. For example, [Auerbach and Gorodnichenko \(2012a,b, 2014\)](#) explore the difference in the output multiplier during recessions and expansions using U.S. data, OECD data and Japanese data. Our paper instead focuses on comparing the multiplier in the zero lower bound period and in the normal period. We argue the difference is not due to the nonlinear effects of government spending during expansion and recession. We also exploit more data available for Japan. For example, we include quarterly forecast data of government spending in order to control for expectations for our whole sample between 1980Q1 and 2014Q1. Furthermore, we adjust the published government spending data to exclude transfers. We also examine the type of models consistent with our empirical evidence.

Few papers estimate the output multiplier in the zero lower bound periods. [Ramey \(2011b\)](#) estimates for the United States data from 1939 through 1951 that the multiplier is not higher during that sample. [Crafts and Mills \(2012\)](#) estimate that the multiplier is below one in the U.K. during

the 1922 -1938 period when interest rate is near zero. A recent paper by [Ramey and Zubairy \(2014\)](#) examining the United States with the ZLB period during 1932Q2-1951Q1 and 2008Q4-2013Q4 finds that the multiplier is higher during the ZLB periods than during the normal periods if they exclude World War II from the sample. They also point out that the main government spending shocks during the ZLB periods occur after the start of WWII and at the start of the Korean War in 1950, which can confound the effects of government spending shocks in states with rationing with those in states with the ZLB. Unlike them, our paper uses Japanese data with almost 20 years of ZLB and provide a new evidence that the multiplier is larger than one in the ZLB period. We also argue that this multiplier in the ZLB period is not driven by the effects of government spending in recession. We further examine the transmission mechanism of government spending in different theoretical models.

A recent literature estimates local multiplier using data from different regions with common monetary policy. The local multiplier measures the changes in relative output of one region to others in response to an increase in relative government spending. For example, [Nakamura and Steinsson \(2014\)](#) estimate the local multiplier for the U.S. states and [Bruckner and Tuladhar \(2014\)](#) for Japanese prefecture data. However, [Nakamura and Steinsson \(2014\)](#) note that this local multiplier is not the same as the aggregate multiplier in the ZLB. The reason is that the long-term real interest rate falls in the ZLB setting while it does not in the regions with common monetary policy. Our paper, in contrast, focuses on the aggregate multiplier in the ZLB period.

We are also related to the literature testing the ZLB predictions of New Keynesian models. [Wieland \(2013\)](#) examines if negative aggregate supply shocks, proxied by oil price shocks and the Great East Japan earthquake, are expansionary during the ZLB periods. He finds that oil price spikes decrease output but also decrease the real interest rate in the ZLB period. He concludes that these results are not consistent with a calibrated standard New Keynesian model with a fundamental-shock-driven ZLB period. We focus on the effects of government spending shocks in the ZLB period and in the normal period. We provide support for a New Keynesian model with our parameterization in which the ZLB period is caused by fundamental shocks. We also complement the work of [Dupor and Li \(2015\)](#) by focusing on the responses of both output and inflation to the government spending shocks. While [Dupor and Li \(2015\)](#) argue that inflation does not move sufficiently enough for the New Keynesian mechanism to generate larger multiplier under ZLB, we show that the multiplier can be large and consistent with the empirical evidence even without much response from inflation in a model with a sufficiently flat Phillips Curve. Our model and

analyses build on [Woodford \(2010\)](#), [Eggertsson \(2011\)](#), [Christiano et al. \(2011\)](#) and [Mertens and Ravn \(2014\)](#).

The rest of the paper proceeds as follows. Section 2 presents our empirical evidence including the identification strategy, the data we use, the baseline results about the effects of government spending changes on the aggregate economy and several robustness checks. We then relate our empirical findings to the theoretical literature. Section 3 analyzes a calibrated New Keynesian model in which the ZLB period is caused by either fundamental shocks or non-fundamental shocks. Section 4 concludes.

2 Measurement of Multipliers

2.1 Specification and Identification

Our baseline empirical strategy relies on *both* the institutional information about government spending *and* the real-time information regarding expectations of fiscal variables. The institutional information approach assumes that government spending does not respond to output within a quarter. [Blanchard and Perotti \(2002\)](#) and many subsequent studies have used this assumption to identify government spending shocks in a structural vector autoregression (SVAR) in which government spending is ordered first. However, the subsequent research such as [Ramey \(2011b\)](#) has criticized this approach that the identified government spending shocks in these SVARs may not represent unanticipated changes in government spending. The reason is that the variables in the SVAR, which includes government spending, tax revenues and output, do not control for expected changes in government spending.⁴ To identify a more precise measure of *unexpected* government spending shocks, we include a measure of the expected government spending to purge fiscal variables of the predicted government spending shocks.

We then implement the identification of government spending shocks using the local projection method by [Jorda \(2005\)](#).⁵ This method estimates the impulse response functions directly by projecting a variable of interest on lags of variables capturing information available in a given time period. Our two-step estimation is as follows. First, we identify the unexpected innovations in

⁴[Auerbach and Gorodnichenko \(2012a\)](#) show that there are some forecastability in the shocks identified using information in the VAR using United States data.

⁵This implementation has been used in [Auerbach and Gorodnichenko \(2012a\)](#), [Auerbach and Gorodnichenko \(2012b\)](#), [Ramey and Zubairy \(2014\)](#) and others.

government spending by estimating the following specification:

$$\Delta \ln G_t = \alpha + \gamma F_{t-1} \Delta \ln G_t + \psi(L)y_{t-1} + \epsilon_t, \quad (1)$$

where $\Delta \ln G_t$ is the log difference of government spending, $F_{t-1} \Delta \ln G_t$ is the one period ahead forecast of $\Delta \ln G_t$, and y_t is a vector of controls. The residuals, $\hat{\epsilon}_t$, are the *unexpected* government spending orthogonal to the expected component of government spending and information in the control variables. In the second step, we estimate the following specification at each horizon h :

$$x_{t+h} = \alpha_h^x + \beta_h^x shock_t + \psi_h^x(L)y_{t-1} + \epsilon_{t+h}^x \text{ for } h = 0, 1, 2, \dots \quad (2)$$

where x_t is a variable of interest and $shock_t$ is the government spending shocks, proxied by the estimated $\hat{\epsilon}_t$. Then, β_h^x is the response of x at horizon h to an unexpected government spending shock.⁶ In all of the following results, the standard controls, y_t , include the growth rates of government spending ($\Delta \ln G_t$), tax revenue ($\Delta \ln T_t$) and output ($\Delta \ln Y_t$) unless otherwise noted. We include four lags for the standard controls.

To estimate the effects of government spending on output in both normal and ZLB periods, we estimate equation (2) for two variables of interest: output and government spending, i.e.:

$$\begin{aligned} \frac{Y_{t+h} - Y_{t-1}}{Y_{t-1}} &\approx \ln Y_{t+h} - \ln Y_{t-1}, \\ \frac{G_{t+h} - G_{t-1}}{Y_{t-1}} &\approx (\ln G_{t+h} - \ln G_{t-1}) \frac{G_{t-1}}{Y_{t-1}}. \end{aligned}$$

We note that both output and government spending change are converted to the same unit before estimation to calculate the output multiplier.⁷

We define the output multiplier at each horizon h as the integral of the output response divided by the integral of the government spending response. The output multiplier measures the cumu-

⁶Another way to implement [Jorda \(2005\)](#) in one step is to identify the unexpected government spending shocks and their effects on the variable of interest x by modifying equation (2) as follows:

$$x_{t+h} = \alpha_h + \beta_h^x \Delta \ln G_t + \gamma_h^x F_{t-1} \Delta \ln G_t + \psi_h^x(L)y_{t-1} + \epsilon_{t+h}^x \text{ for } h = 0, 1, 2, \dots$$

In this case, β_h^x can still be interpreted as the response of x to a shock in government spending orthogonal to the one-period ahead forecast and controls. One advantage of the baseline approach over this one-step approach is the computational efficiency if we add several controls for expectations, so we choose to follow [Auerbach and Gorodnichenko \(2012a\)](#). We check in the Robustness section that this one-stage implementation does not affect our results.

⁷We can also convert government spending change by potential output. We discuss the results using this alternative normalization in the Robustness section.

lative output gain relative to government spending during a given period. We follow [Mountford and Uhlig \(2009\)](#) and [Ramey and Zubairy \(2014\)](#) and choose this definition of multiplier as it has several advantages over other definitions used in the literature. We calculate the output multiplier M_h as follows:

$$M_h = \frac{\sum_h \beta_h^Y}{\sum_h \beta_h^G},$$

where β_h^Y and β_h^G are the impulse responses of output and government spending at horizon h , respectively. We obtain the standard errors for M_h by estimating all of the regressions jointly as one panel regression and using the [Driscoll and Kraay \(1998\)](#) standard errors to account for autocorrelated errors.⁸

2.2 Data

We use Japanese quarterly data between 1980Q1 and 2014Q1 in the baseline estimation. There are two reasons to use Japanese data to examine the effects of government spending on the economy in the ZLB period. First, Japan has more information about the ZLB period than other countries. As plotted in [Figure 1](#), nominal interest rate in Japan has stayed around zero ever since the fourth quarter of 1995, so there are about 20 years of data for study. Second, within the ZLB period, Japan has four business cycles, so we can potentially tell if the estimated multiplier is driven by the nonlinear effects of government spending in different states of the business cycle. This feature makes Japan an attractive case to study as other countries including the United States often have the zero lower bound period coinciding with recessions, making it difficult to distinguish the effects of government spending in the zero lower bound period from those in recession.

The data for government spending (or government purchases) is the sum of adjusted government consumption and public investment. Adjusted government consumption is calculated as total government consumption excluding transfer of goods.⁹ Tax data come from the national account starting from 1980Q1 which is the sum of direct and indirect taxes less subsidies.¹⁰ Finally, output

⁸We thank Valerie Ramey and Sarah Zubairy for their advice on the implementation.

⁹After 1980, the total government consumption includes both transfers (payment to households on medical services is an example) and consumption (payment for textbooks is an example). Therefore, we construct the “adjusted government consumption” by excluding transfers from total government consumption from 1980. Prior to 1980, Japan adopted System of National Account 1968, which has a different definition of government consumption. Our adjusted government consumption series is similar to the data of government spending prior to 1980. Japan also has data for “actual final” consumption of government spending after 1980. The definition of this series is the most narrow and it accounts for less than 8% of output. We show in the Robustness section that the estimates using actual final government spending or the unadjusted measure of government consumption are similar to the baseline results.

¹⁰This series is almost identical to the series constructed by adding taxes on production and imports and taxes on income and wealth etc. less subsidies from [Doi et al. \(2011\)](#).

data are taken from the National Accounts. All variables are deflated by GDP deflator.

Besides standard macroeconomic variables, we exploit a rich quarterly forecast data set that includes forecast about government spending. Unlike the United States, Japan has short surveys of professional forecasters which contain little to no information about government spending. Therefore, previous studies on Japan such as [Auerbach and Gorodnichenko \(2014\)](#) rely on semiannual forecast from the OECD starting in 1985 and the IMF starting in 2003 to infer about unexpected changes in government spending. An important difference in our paper is that we exploit quarterly forecast data published by the Japan Center for Economic Research (JCER) for many macroeconomic variables including government spending, output and GDP deflator. This dataset starts in 1967Q1 and contains several forecast horizons, ranging from nowcast to eight quarters ahead. Furthermore, the JCER data also contains the initial release and up to seven subsequent revisions of realized data. There are two caveats to this dataset. First, the JCER publishes this dataset three out of four quarters in some years.¹¹ So, in the quarters without updated forecast data, we assume that there is no revision in forecasts, i.e. the one-quarter ahead forecast is replaced by the two-quarter ahead forecast published in the previous quarter. Second, there are missing data in all of 2010, so we use the forecast data published by the Mitsubishi UFJ Financial group between 2010Q1 and 2014Q1.¹² We plot in [Figure 2](#) our one-quarter ahead forecast of the four quarter growth rate of government spending, $F_{t-1}\Delta \ln G_{t-4,t}$, along with the realized government spending, $\Delta \ln G_{t-4,t}$.¹³ Although forecast misses some of the fluctuations such as those in early 2000s, the one-quarter ahead forecast tracks the actual data relatively well. This suggests that the realized government spending may have some predictable components and including these forecast data in the estimation can help us obtain a more pure measure of unexpected government spending shocks. We show in the Robustness section that these forecast data are indeed important to control for the timing of the spending and affect the estimated multipliers.

We define the normal period to be between 1980Q1 and 1995Q3 and the zero lower bound period to be between 1995Q4 and 2014Q1. Although the earliest start date for our data with forecast is 1967Q1, we choose the start of the normal period to be 1980Q1 for three reasons. First, Japan was in the fixed nominal exchange rate regime until 1973. According to [Ilzetzki et al. \(2013\)](#), the fiscal multipliers in a fixed exchange rate regime are higher than those in a flexible exchange rate regime.

¹¹The periods with three forecasts a year are: from 1972 to 1999, from 1995 to 2002, and from 2004 to 2006.

¹²The baseline results do not change if we only use the Mitsubishi UFJ forecast for 2010.

¹³Note that we construct the one-quarter ahead forecast of the four quarter growth rate of government spending using real-time data, i.e. forecasters do not know the final release of government spending in $t - 4$ when making forecast at time $t - 1$.

So there is a break in monetary policy in 1973. Since we focus on periods with homogeneous monetary policy, we exclude the period under the fixed exchange rate regime before 1973. Second, the 1973 oil price crisis affects the real government spending through unexpected changes in the price level, which can bias the estimates of the multipliers.¹⁴ Third, the definition of government spending data changes in 1980.¹⁵ Therefore, we restrict our attention of the normal period to 1980Q1-1995Q3. The zero lower bound period starts when the short-term nominal interest rate goes down to 0.25% in 1995Q4. Since then, the short-term nominal interest rate in Japan has been low, staying under 0.6% as plotted in Figure 1. We then estimate equation (2) for both normal and ZLB periods.

2.3 Empirical Results

This section presents the main result of our empirical analysis using the local projection method to estimate the baseline model with Japanese data during normal and ZLB periods. We analyze the effect of an increase in government spending on output, private consumption and investment, inflation, unemployment rate, nominal interest rate and expected inflation.

2.3.1 Output

We first consider the responses of government spending and output to an increase in government spending by one percent of output in period 0. As plotted in Figure 3, output increases on impact and up to two years in the ZLB period while it increases slightly on impact then decreases significantly in the normal period. The one standard deviation confidence interval bands for these estimates overlap with each other in some horizons. At the same time, the response of government spending is more persistent in the normal period than in the ZLB period.

Since the underlying process of government spending differs between the normal and ZLB periods, we convert the impulse responses to output multipliers. Figure 4 plots the output multipliers and their confidence bands in both normal and ZLB periods. Overall, the output multiplier is substantially larger in the ZLB period than in the normal period. The output multiplier in the normal period is 0.7 on impact. This estimate is in line with previous estimates for the United

¹⁴To the extent that government spending is determined in nominal terms, an unexpected change in the current price level can bias the identification of government spending shocks using nominal government spending deflated by the current price level. In fact, the estimated multiplier for the normal period starting in 1973 is slightly higher than the baseline estimates in longer horizons. However, if we deflate nominal government spending by smoothed inflation or one quarter lagged inflation, the estimate is similar to the baseline.

¹⁵The baseline result does not change if the normal period starts in 1975Q1 using our constructed series of government spending.

States and other countries. The output multiplier in the ZLB period is larger: it is 1.5 on impact. This multiplier is larger than that documented in the baseline estimate of [Ramey and Zubairy \(2014\)](#) but it is similar to their estimate when they exclude the WWII period. The difference between the multipliers in the normal period and in the ZLB period are more pronounced at longer horizons. While the output multiplier in the normal period turns significantly negative after the first two quarters, the output multiplier in the ZLB period increases to about 2 after one year. As reported in [Table 1](#), the output multiplier in the normal period is -1.08 and significantly smaller than zero in two-year horizon. In contrast, the output multiplier in the ZLB period increases to 2.38 in one-year horizon and stays well above 2 in two-year horizon. The confidence band of the multipliers do not overlap each other much.

To formally test if the multipliers in two periods are statistically significantly different from each other, we estimate the following specification:

$$x_{t+h} = I_{t-1}^{ZLB} \times (\alpha_{A,h} + \beta_{A,h} shock_t + \gamma_{A,h} F_{t-1} \Delta \ln G_t + \psi(L)y_{t-1}) \\ + (1 - I_{t-1}^{ZLB}) \times (\alpha_{B,h} + \beta_{B,h} shock_t + \gamma_{B,h} F_{t-1} \Delta \ln G_t + \psi(L)y_{t-1}) + \epsilon_{t+h}^x \text{ for } h = 1, 2, \dots,$$

where I_t is one if the economy is in ZLB in period t .¹⁶ We then calculate the difference between the multiplier in the ZLB period, M_h^{ZLB} , and that in the normal period, M_h^{normal} . [Table 1](#) reports the differences of the multipliers, their standard errors and the corresponding p-value over different horizons. We also plot in [Figure 4](#) the difference between M_h^{ZLB} and M_h^{normal} for all horizons between zero and ten quarters and the confidence bands. Although the 90% confidence interval includes zero, the difference is more significant at shorter horizons. The difference is significant at 11% significance level one quarter after the shock and at 13% significance level one year after the shock. This result suggests that there is some evidence that the output multiplier in the ZLB period is larger than that in the normal period.

2.3.2 Private Consumption and Investment

We next examine the effects of government spending on private consumption and investment. We modify equation [\(2\)](#) and estimate the effects of government spending on consumption using the

¹⁶ [Ramey and Zubairy \(2014\)](#) also use this specification to estimate their state-dependent multipliers. If we, instead, use the indicator for current period, I_t , instead, the results do not change.

following two equations:

$$\frac{C_{t+h} - C_{t-1}}{C_{t-1}} = \alpha_h^C + \beta_h^C shock_t + \psi^C(L)y_{t-1} + \epsilon_{t+h}^C$$

$$\frac{G_{t+h} - G_{t-1}}{C_{t-1}} = \alpha_h^{G,C} + \beta_h^{G,C} shock_t + \psi^{G,C}(L)y_{t-1} + \epsilon_{t+h}^{G,C} \text{ for } h = 0, 1, 2, ..$$

where y_t includes both the standard controls and consumption. The impulse response of consumption to an increase in government spending by one percent of consumption is β_h^C and the consumption multiplier is defined as $M_h^C = \frac{\sum_h \beta_h^C}{\sum_h \beta_h^{G,C}}$. The responses of private investment and its multiplier are estimated and defined in the same manner.

The impulse responses of private consumption and investment to an increase in government spending of one percent of consumption and investment, respectively, are plotted in the upper panel of Figure 5. In the normal period, both consumption and investment decline after an increase in government spending, i.e. government spending crowds out private spending. Consumption initially increases on impact but the increase is small. In contrast, in the ZLB period, government spending crowds in private consumption and investment: the peak responses of consumption and investment are about 1.5% at one year horizon. Figure 5 also plots the cumulative multiplier of consumption and investment to government spending at all horizons. The multiplier for consumption is significantly positive in the ZLB period while it is negative in the normal period. The investment multiplier in the ZLB period is also positive and higher than that in the normal period. We formally test and report in Table 1 the differences between the consumption and investment multipliers in the normal period and in the ZLB period. We find that the consumption multiplier is significantly larger in the ZLB period than in the normal period, at 4% significance level after four quarters. The difference in the investment multipliers is less significant as the p-value is about 0.17.

2.3.3 Inflation, Interest Rate and Unemployment

Denoting P_t to be GDP deflator at time t , $\pi_t = \ln(P_t/P_{t-1})$ to be inflation, we estimate the responses of inflation to government spending shocks using the following version of the baseline specification (2):

$$\pi_{t+h} = \alpha_h^\pi + \beta_h^\pi shock_t + \psi^\pi(L)y_{t-1} + \epsilon_{t+h}^\pi \text{ for } h = 0, 1, 2, ..$$

where y_t includes not only the standard controls as above but also five-year nominal interest rate.¹⁷ Figure 6 plots the responses of inflation in both normal and ZLB periods. Inflation responds little to a positive government spending shock in both periods. As tabulated in Table 2, an increase in government spending by 1% of GDP leads to a 0.07% increase in inflation in normal period and 0.14% in ZLB period on impact. Inflation increases about 0.20% in one-year horizon in both periods. Overall, the responses of inflation is mild in both periods and the confidence intervals include zero.

The effects of government spending on one quarter ahead expected inflation, denoted by $F_t\pi_{t+1}$ are estimated using the following version of the baseline specification (2):

$$F_{t+h}\pi_{t+1+h} = \alpha_h^f + \beta_h^f shock_t + \psi^f(L)y_{t-1} + \epsilon_{t+h}^f \text{ for } h = 0, 1, 2, \dots$$

where the controls vector y_t includes both the standard controls as above, five-year nominal interest rate, and (lagged) expected inflation. Figure 6 plots the impulse responses of one-quarter ahead inflation expectation to an increase in government spending by one percent of output. Inflation expectation increases on impact in both the normal period and the ZLB period. However, inflation expectation responds slightly more strongly in the ZLB period than in the normal period although not significantly. As reported in Table 2, the one-quarter ahead inflation expectation increases by 0.85% after four quarters in the ZLB period while it is -0.17 in the normal period. This result suggests agents expect monetary policy would react to the inflationary pressure from government spending in the normal period.

Figure 6 plots the impulse responses of the short-term (overnight) interest rate and the five-year interest rate to an increase in government spending by 1% of output, respectively. These responses are estimated using the following specification:

$$i_{t+h} = \alpha_h^i + \beta_h^i shock_t + \psi^i(L)y_{t-1} + \zeta_h^i z_t + \gamma_h^i trend_t + \epsilon_{t+h}^i \text{ for } h = 0, 1, 2, \dots,$$

where i_t is the short-term (or five-year) nominal interest rate, y_{t-1} includes not only the standard controls but also inflation and interest rate i_t , z_t is a vector containing the contemporaneous inflation and output, and $trend_t$ is the trend. We include the trend variable to control for the declining nominal interest rate over time. We report here the result estimated with a quadratic trend, but

¹⁷Due to the limited availability of 10-year nominal interest rate, we use 5-year nominal interest rate. The results do not change if we use other nominal interest rates.

the results do not change if we include a linear trend. In the normal period, the short term interest rate increases to nearly 1% in one year horizon in response to an increase in government spending by one percent of output. The response of five-year interest rate is less significant and only increases after 10 quarters. In the ZLB period, both short and long term interest rates do not react to the government spending shocks, consistent with the idea that the central bank is not responsive to government spending shocks during the ZLB period. These results together with the response of expected inflation suggest that the real interest rate increases more in the normal period than in the ZLB period.

We examine the responses of the labor market to a government spending shock by estimating the following specification for unemployment rate:

$$U_{t+h} - U_{t-1} = \alpha_h^U + \beta_h^U shock_t + \psi^U(L)y_{t-1} + \epsilon_{t+h}^U \text{ for } h = 0, 1, 2, \dots,$$

and y_t includes all the standard controls and unemployment rate. We plot the responses of unemployment rate in Figure 6. During the normal period, unemployment rate decreases in response to an increase in government spending by one percent of GDP. However, the decrease in unemployment rate is small and insignificant. In contrast, in the ZLB period, unemployment rate decreases significantly by 0.1% on impact and further to 0.5% a year after the shock, as shown in Table 2.

To sum up, using Japanese data between 1980Q1 and 2014Q1, we find that:

1. The output multiplier in the ZLB period is larger than one and larger than that in normal period.
2. Government spending crowds in private consumption and investment in the ZLB but it crowds out in the normal period.
3. Inflation responses are small and insignificant in both periods.
4. Unemployment rate decreases in the ZLB more than in the normal period.
5. Nominal interest rate does not increase much in the ZLB period relative to the normal period.
6. Expected inflation responses are mild in both periods.

2.4 Output Multipliers in the ZLB period and in Recessions

Recent papers by [Auerbach and Gorodnichenko \(2012a,b\)](#) finds that the output multiplier is larger than one in recessions while it is smaller than one in expansions using U.S. and OECD data. As

the ZLB period often coincides with recessions, it is important to differentiate evidence from the ZLB period and evidence from recessions. This section shows that our estimated multiplier in the ZLB period may not be attributed to the large effects of government spending in recessions.

We first estimate the multipliers in booms and recessions in Japan between 1980Q1 and 2014Q1 by estimating a state-dependent version of the baseline specification, similarly to [Ramey and Zubairy \(2014\)](#):

$$x_{t+h} = I_{t-1}^{Recession} \times [\alpha_{A,h} + \beta_{A,h} \cdot shock_t + \psi(L)y_{t-1}] \\ + (1 - I_{t-1}^{Recession}) \times [\alpha_{B,h} + \beta_{B,h} \cdot shock_t + \psi(L)y_{t-1}] + \epsilon_{t+h} \text{ for } h = 1, 2, \dots,$$

where $I_{t-1}^{Recession}$ is one if the economy is in recession in period $t-1$ and zero otherwise. The recession indicator is based on the Cabinet Office of Japan classification of trough periods.¹⁸ Figure 7 plots the output multipliers in recessions and expansions and the difference between the two multipliers. The output multiplier on impact in recessions is 2.3 while it is 0.8 in expansions. The difference between the multipliers in recessions and in expansions are smaller at horizons longer than three quarters. The difference between the multipliers in recessions and in expansions is not significant at longer horizons, as reported in Table 3. This result for Japan is qualitatively similar to that for the U.S. in [Auerbach and Gorodnichenko \(2012a\)](#) but weaker in significance.

Although it is possible that the difference in multipliers between the ZLB and normal periods can be driven by the difference in multipliers between recessions and booms, we argue that this is not the case. Japan is not in a recession for the whole ZLB period between 1995Q4 and 2014Q1, as can be seen in Figure 1. The number of quarters in recession are slightly higher in normal period than in the ZLB period: 45% of the quarters in the normal period is recession while it is only 30% in the ZLB period. This implies that the multiplier during the ZLB period should be smaller than the multiplier during the normal period if the only fundamental difference is between the values of the multiplier in recessions and booms.¹⁹

¹⁸In the Cabinet Office of Japan, individual members classify recession in a similar manner to procedure used by the NBER. They then agree on the classification collectively. More information can be found at <http://www.esri.cao.go.jp/jp/stat/di/150724hiduke.html>. We show in Appendix Figure A1 that the results in this section do not change if we use the peak-to-trough classification by the OECD.

¹⁹It is probably possible that the multiplier is bigger in deeper recessions. However, it is not the case that Japan has experienced more severe recessions during the ZLB period than in the normal period.

2.5 Importance of Forecast Data

We now show the importance of controlling for expectations in the identification of government spending shocks. We first examine the forecastability of the government spending shocks identified without forecast data in the standard VAR as in [Blanchard and Perotti \(2002\)](#). To implement this, we estimate the following specification:

$$x_t = \alpha^g + \psi^g(L)y_{t-1} + \epsilon_t^g,$$

for two cases. In the first case, the dependent variable x_t is the realized government spending growth rate, $\Delta \ln G_t$; we obtain the residuals, $\hat{\epsilon}_{1,t}^g$. In the second case, the dependent variable x_t is the one-quarter ahead forecast of government spending, $F_{t-1}\Delta \ln G_t$; the residuals for this case are $\hat{\epsilon}_{2,t}^g$. We then calculate the correlation between $\hat{\epsilon}_{1,t}^g$ and $\hat{\epsilon}_{2,t}^g$. A non-negative correlation implies that some of the government spending shocks identified without forecast data are predictable. The scatter plots of these two residuals along with the correlations in the whole sample, in the normal period and in the ZLB period are shown in [Figure 8](#). For the entire sample between 1980Q1-2014Q1, the correlation between the two residuals are 0.34 and statistically significant, suggesting that there are some forecastability of the government spending shocks $e_{1,t}^g$ identified without forecast data. This correlation is 0.39 in the normal period but it is only 0.11 for the ZLB period between 1995Q4 and 2014Q1. This result suggests that the government spending shocks are less predictable in the ZLB period than in the normal period.

We then compare the baseline estimates of the output multipliers in the normal period and in the ZLB period with those estimated without forecast data. Specifically, in the case without forecast data, $shock_t$ in the baseline specification (2) is proxied by $\hat{\epsilon}_{1,t}^g$. We plot the estimated multiplier without forecast data along with the baseline in [Figure 9](#). Controlling for the information agents have about future government spending tends to make the output multipliers larger in the normal period and to a lesser extent in the ZLB period. This result is similar with the findings for the U.S. reported in [Auerbach and Gorodnichenko \(2012a\)](#). Consistent with the predictability analysis above, forecast data do not change the multiplier in the ZLB period as much as in the normal period. The confidence interval is larger in our baseline estimation than in the case without forecast data. These results suggest that forecast data change the estimated multipliers in a non-trivial way and it is important to control for the expectational effects.

2.6 Other Specifications and Extensions

We investigate the robustness of our baseline results by estimating variants of the baseline specification (2) and using different government spending data series. First, we estimate a version of specification (2) with a quadratic trend since time series estimates can be sensitive to trends. Figure 10 plots the output multipliers in the normal and in the ZLB periods estimated in the baseline along with those estimated with a quadratic trend. We find that the multipliers estimated with a trend is similar to those in the baseline although the output multiplier estimated with a trend in the normal time is somewhat larger in longer horizons than in the baseline.

Second, we perform an alternative transformation of government spending by potential output to calculate the multipliers, similar to Gordon and Krenn (2010). The motivation for this approach is as follows: In our baseline estimation, we convert government spending from the percent changes to dollar changes using the value of the government spending-output ratio at each point in time, rather than using sample averages. A potential problem of the baseline transformation is that the cyclical nature of output can bias the estimated multiplier.²⁰ Figure 10 which plots the output multipliers in both periods, shows that the output multipliers estimated using this transformation are essentially the same as our baseline.

Third, one potential concern with our implementation to identify the effects of unexpected government spending shocks is that we use the residuals $\hat{\epsilon}_t$ of equation (1) to proxy for $shock_t$ without taking into account the uncertainty of the estimates in equation (1). To address this concern, we implement a one-step estimation of the effects of unexpected government spending on output, i.e.

$$x_{t+h} = \alpha_h + \beta_h^x \Delta \ln G_t + \gamma_h^x F_{t-1} \Delta \ln G_t + \psi_h^x(L) y_{t-1} + \epsilon_{t+h}^x \text{ for } h = 0, 1, 2, \dots$$

The multipliers from this estimation are plotted in Figure 10. The multipliers are virtually identical to our baseline estimates. Furthermore, as reported in Table 3, the standard errors of the one-step estimation and the baseline are almost identical. These results show that our two-step estimation approach correctly identifies the unexpected government spending shocks as the one-step estimation.

Fourth, to demonstrate that we identify a good measure of unexpected government spending shocks in the baseline, we add other variables that may contain prior information about government

²⁰This point was raised by Yuriy Gorodnichenko in his discussion for Ramey and Zubairy (2014) in the NBER.

spending into equation (1) to back out another proxy for $shock_t$ and re-estimate the output multiplier. One variable that can contain the prior information agents have about the future economy is the one-quarter ahead forecast of output. The estimated multipliers in this case together with those in the baseline are plotted in Figure 10. The output multiplier in the normal time is lower than the baseline on impact but is slightly higher in horizons longer than six quarters. The multiplier in the ZLB time is higher than the baseline. However, the difference between this case and the baseline are not so large and the multipliers lie within the confidence intervals of the baseline.

Fifth, as noted in the Data section, we adjust the government spending data for transfers in the baseline estimates. However, previous literature such as Auerbach and Gorodnichenko (2014) uses unadjusted government spending and the Japanese Cabinet office also include actual final government spending data from 1980. We show that our results remain robust to using these different data series. The estimated output multipliers using unadjusted and actual final government spending data are shown in Figure 10. The multipliers are within the confidence interval of the baseline. In fact, using actual final government spending leads to an even higher multiplier in the ZLB period. These results suggest that our results are robust to using different government spending data.

Finally, we extend the baseline specification to estimate output multipliers with a rolling window of 15 years between 1967Q1 and 2014Q1. Figure 11 plots the multiplier in different horizons. The multiplier is time-varying. For the 15 years windows between 1967 and 1984, the cumulative output multiplier is about 1.2 on impact and increases to about 3 in two year horizon. This result suggests that the multiplier may be larger than one in the 1960s and 1970s when the Japanese economy is under the fixed exchange rate regime. After the collapse of the fixed exchange rate regime, the multiplier is below unity for all periods up to 1997. This result is consistent with the finding in Ilzetzki et al. (2013) that the multiplier is larger in the fixed exchange rate regimes than in the flexible regime. The multiplier becomes higher than unity for rolling windows starting in 1995. This rolling multiplier result is consistent with our baseline estimates and suggest that the multiplier is larger in the ZLB period than in the periods up to 1995.

3 A Model of Government Spending

In this section, we examine a standard New Keynesian model in which the ZLB period can occur due to either fundamental shocks or self-fulfilling state of low confidence. We show that a calibrated

model in which fundamental shocks cause the zero bound to bind can match our empirical results well. Furthermore, the model with the ZLB period caused by low confidence can also generate multipliers close to the empirical estimates under an extreme parametrization. The proposed model and analysis build on [Woodford \(2010\)](#), [Christiano et al. \(2011\)](#) and [Mertens and Ravn \(2014\)](#).

3.1 The Model

Households. The economy is populated by a continuum of households. Different households supply different types of labor indexed by i and there are an equal number of households supplying each type of labor. This is the heterogeneous labor supply assumption. A household supplying labor of type i maximizes their utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \chi \frac{L_t(i)^{1+\nu^{-1}}}{1+\nu^{-1}} \right), \quad (3)$$

where C_t is an index of the household's consumption, $L_t(i)$ is the quantity of labor of type i supplied, β denotes the subjective discount factor, ν is the Frisch elasticity of labor supply, and σ is the elasticity of intertemporal substitution.

Consumption C_t is an index given by

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

where $C_t(j)$ denotes consumption of variety j , $\theta > 1$ is the elasticity of substitution between varieties. There is a continuum of measure one of varieties. We denote $P_t(j)$ the price of variety j , and

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

is the corresponding price index.

Household of type i maximizes its utility subject to a flow budget constraint given by

$$\int_0^1 P_t(j) C_t(j) di + \mathbb{E}_t [Q_{t,t+1} B_{t+1}(i)] + T_t \leq B_t(i) + W_t(i) L_t(i) + \int_0^1 \Pi_t(j) dj, \quad (4)$$

together with a no-Ponzi condition. In this equation, $B_{t+1}(i)$ is a state-contingent payoff at the beginning of period $t + 1$ of the financial portfolio of household i , $Q_{t,t+1}$ is the price of Arrow-Debreu securities divided by the conditional probability of the corresponding state, which equals

the unique stochastic discount factor in equilibrium, $W_t(i)$ is the nominal wage received by labor type i in period t , $\Pi_t(j)$ is the nominal profit of the firm that produces variety j in period t , T_t is lump sum taxes.

Government. There is a government that conducts fiscal and monetary policy. Fiscal policy is represented by a government spending G_t and lump sum taxes T_t . The government spending follows an AR(1) process:

$$\log(G_t/\bar{G}) = \rho_g \log(G_{t-1}/\bar{G}) + \epsilon_{g,t},$$

where \bar{G} is the steady state value of government spending, ρ_g is the persistence, and $\epsilon_{g,t}$ is an i.i.d., zero-mean random variable, representing unexpected changes to fiscal policy. Because the Ricardian equivalence holds, the timing of taxes is irrelevant. Government spending G_t has the same CES form as the index of household's consumption:

$$G_t = \left[\int_0^1 G_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

where $G_t(j)$ is government consumption of variety j . The government splits its expenditure $\int_0^1 P_t(j)G_t(j)di$ across varieties to maximize government spending.

Monetary policy is represented by the following Taylor rule:

$$i_t = \max \{0, \bar{r} + \phi_\pi \pi_t + \phi_y \hat{y}_t\}, \quad (5)$$

where hatted variables denote percentage deviations from steady state, the policy instrument i_t is a one-period riskless nominal interest rate, and $\bar{r} = -\log \beta$ is the value of this rate in a steady state with zero inflation, and $\phi_\pi > 1, \phi_y > 0$ are the response coefficients.

Firms. There is a continuum of firms, each of which specializes in the production of differentiated good j out of labor using the technology given by

$$Y_t(j) = A_t f(L_t(j)),$$

where A_t is the aggregate productivity, $f(\cdot)$ is increasing and concave. We follow [Woodford \(2003\)](#) and assume that firm j sets monopolistic price $P_t(j)$ for its output but acts as a price-taker on the

market for labor of type j .²¹ We assume that firms pay a constant employment tax $1 + \tau^L$ so that the nominal total cost of production is $(1 + \tau^L)W_t(j)f^{-1}(Y_t(j)/A_t)$.

Firm j can re-optimize its price with probability $1 - \alpha$. The firm maximizes its value,

$$\mathbb{E}_t \sum_{n=0}^{\infty} Q_{t,t+n} \alpha^n \left[P_t(j) Y_{t+n|t}(j) - (1 + \tau^L) W_t(j) f^{-1} \left(\frac{Y_{t+n|t}(j)}{A_t} \right) \right],$$

where $Y_{t+n|t}(j) = (C_{t+n} + G_{t+n}) \left(\frac{P_t(j)}{P_{t+n}} \right)^{-\theta}$, taking the sequences of $C_t, G_t, P_t, W_t(j), Q_{t,t+n}$ as given.

3.2 Equilibrium Conditions

Household's optimal choice of consumption, labor supply and securities holdings leads to

$$\frac{u_L(C_t, L_t(i))}{u_C(C_t, L_t(i))} = \frac{W_t(i)}{P_t}, \quad (6)$$

$$\beta^n \frac{u_C(C_{t+n}, L_{t+n}(i))}{u_C(C_t, L_t(i))} = \frac{P_{t+n}}{P_t} Q_{t,t+n}, \quad (7)$$

$$C_t(j) = C_t \left(\frac{P_t(j)}{P_t} \right)^{-\theta}, \quad (8)$$

where $u_C(C_t, L_t(i)) = C_t^{-\sigma-1}$ and $u_L(C_t, L_t(i)) = -\chi L_t(i)^{\nu-1}$ are the derivatives of instantaneous utility function with respect to consumption and labor. Equation (6) represents the household labor supply, equation (7) is the consumption Euler equation, and equation (8) is the optimal choice of variety j .

Government demand for variety j is

$$G_t(j) = G_t \left(\frac{P_t(j)}{P_t} \right)^{-\theta}.$$

Firm j optimal price is

$$P_t(j) = \mathbb{E}_t \sum_{n=0}^{\infty} \frac{\alpha^n Q_{t,t+n} Y_{t+n|t}(j)}{\mathbb{E}_t \sum_{n=0}^{\infty} \alpha^n Q_{t,t+n} Y_{t+n|t}(j)} S_{t+n|n}(j),$$

where $S_{t+n|n}(j) = W_{t+n}(i) / (A_{t+n} f' (f^{-1}(Y_{t+n|t}(j)/A_{t+n})))$ is the nominal marginal cost.

²¹More specifically, firms belong to industries. There is a large number of firms in every industry. Each firm in industry x employs labor of type x . In addition, all firms in a particular industry reset their prices at the same time.

3.3 Calibration

We set the values of the parameters unrelated to policy choices as follows. The Frisch elasticity of labor supply ν is 1 which is the standard value used in the macroeconomics literature. The elasticity of intertemporal substitution (IES) σ is set to 1.1 which is within the wide range of the IES values used in the literature. The subjective discount factor β is 0.99. The elasticity of substitution across varieties θ is set to 5 which equals the estimate in [Burstein and Hellwig \(2007\)](#). The production function is $f(L_t(i)) = A_t L_t(i)^a$ with $a = 2/3$. The probability of price adjustment $1 - \alpha$ is 0.25.

We calibrate the fiscal policy parameters as follows. The steady state ratio of government spending over output is 0.18. This number corresponds to the mean of government spending over GDP in Japan during 1980Q1-2014Q1. We set the persistence of government spending disturbances ρ_g to 0.8. This corresponds to the persistence of government spending shocks in our sample in Japan. This persistence has half-life of three quarters.

3.4 Fundamental-driven ZLB Period

We first analyze the effects of government spending in the model in the normal period as well as in the ZLB period caused by fundamental shocks. To do this, we consider two monetary policy specifications: a “Taylor rule policy” and a “fixed nominal-rate policy.” The Taylor rule policy corresponds to the monetary policy in the normal period when the central bank reacts to changes in output and inflation. For this case, we set $\phi_\pi = 1.5$ and $\phi_y = 0.05$.²² The fixed nominal-rate policy assumes that the government does not react to the effects of fiscal policy changes. However, it reacts to any other disturbances in the economy according to the above Taylor rule policy. The fixed nominal-rate policy captures the essence of the reaction of the economy to government spending disturbances under the zero lower bound constraint on the nominal interest rate: the rate does not respond to changes in fiscal policy.²³ It is important to note that the response of the economy to government spending shocks under the fixed nominal-rate policy can be equivalent to the response of the economy under liquidity trap when it is caused by fundamental shocks. The two responses are equivalent if the persistence of government spending shocks equals the persistence of

²²[Aruoba et al. \(2013\)](#) estimate the Taylor rule with the interest rate smoothing term of the form $i_t = \max\{0, (1 - \rho_i)(\bar{r} + \phi_\pi \pi_t + \phi_y \hat{y}_t) + \rho_i i_{t-1}\}$. Using Japanese data, they found $\rho_i = 0.6, \phi_\pi = 1.5, \phi_y = 0.3$. To avoid introducing a state variable i_{t-1} in our analysis, we set $\rho_i = 0$. To approximate their estimates, we set ϕ_y to a lower value of 0.05 to capture a mild response of interest rate to output gap when $i_t > 0$. We set ϕ_π to 1.5 to be consistent with the Taylor principle. Alternative values of ϕ_π lead to only small variation in the multipliers in the normal period.

²³See [Farhi and Werning \(2012\)](#) and [Nakamura and Steinsson \(2014\)](#) for the discussion of this interpretation.

the liquidity trap.

We solve the model by log-linearizing the equilibrium conditions around the targeted zero inflation steady state. The analytic derivation is standard and relegated to Appendix A. We denote steady state variables with bars on top of them, while hatted variables are deviations from the steady state: $\hat{y}_t \equiv \log(Y_t/\bar{Y}) \approx (Y_t - \bar{Y})/\bar{Y}$, $\hat{c}_t \equiv \log(C_t/\bar{C}) \approx (C_t - \bar{C})/\bar{C}$, $\hat{g}_t \equiv (\bar{G}/\bar{Y}) \cdot \log(G_t/\bar{G}) \approx (G_t - \bar{G})/\bar{Y}$. Inflation rate is defined as $\pi_t \equiv \log(P_t/P_{t-1})$.

We define the output multiplier as the impact response of output to a government spending shock as follows

$$\gamma_y \equiv \frac{\log(Y_0/Y_{-1})}{\frac{G_{-1}}{Y_{-1}} \log(G_0/G_{-1})} = \frac{\hat{y}_0 - \hat{y}_{-1}}{\frac{G_{-1}}{Y_{-1}} \cdot \frac{\bar{Y}}{\bar{G}} (\hat{g}_0 - \hat{g}_{-1})}.$$

Assuming that in period $t = -1$, the economy rests in the steady state, the output multiplier is $\gamma_y = \hat{y}_0/\hat{g}_0$. γ_y indicates the units of currency (yens or dollars) output changes when government spending increases by one unit of currency. This definition of the output multiplier in the model is identical to the definition of on-impact output multiplier in our empirical part.²⁴

We also define consumption and inflation multipliers as follows

$$\begin{aligned} \gamma_c &\equiv \frac{\log(C_0/C_{-1})}{\frac{G_{-1}}{C_{-1}} \log(G_0/G_{-1})} = \frac{\hat{c}_0 - \hat{c}_{-1}}{\frac{G_{-1}}{C_{-1}} \cdot \frac{\bar{Y}}{\bar{G}} (\hat{g}_0 - \hat{g}_{-1})} = \frac{\bar{C}}{\bar{Y}} \cdot \frac{\hat{c}_0}{\hat{g}_0}, \\ \gamma_\pi &\equiv \frac{\log(P_0/P_{-1})}{\frac{G_{-1}}{Y_{-1}} \log(G_0/G_{-1})} = \frac{\pi_0}{\frac{G_{-1}}{Y_{-1}} \cdot \frac{\bar{Y}}{\bar{G}} (\hat{g}_0 - \hat{g}_{-1})} = \frac{\pi_0}{\hat{g}_0}, \end{aligned}$$

where the last equalities on both of the last lines use the fact that in $t = -1$ the economy is in the steady state.

Table 4 summarizes the multipliers predicted by the model under the two monetary policies, which correspond to the normal period and the ZLB period caused by fundamental shocks. Under the Taylor rule policy, as shown in the first line of the table, the output multiplier is 0.71: an increase in government spending by one percent of output leads to an increase in output by 0.71%. This number equals our empirical point estimate of 0.71. The consumption multiplier is -0.29 . Private consumption falls as the central bank increases its policy rate.²⁵ The theoretical inflation response is mild and positive: an increase in government consumption by one percent leads to an

²⁴Because we do not have endogenous state variables in this simple model and the model is solved by log-linearization, the longer horizon multipliers would be similar to on-impact multipliers. This is why we compare the multipliers defined in the model to on-impact multipliers estimated in the empirical part.

²⁵The empirical consumption multiplier on impact is positive. However, the consumption multiplier turns negative after one quarter and it becomes significantly negative after four quarters.

increase in inflation by 0.041 percentage points, which is similar to our empirical estimates.

When monetary policy does not react to government spending shocks, the multipliers are substantially larger. The output multiplier is 1.47 which is equal to our empirical estimate of 1.47, the consumption multiplier is 0.47, which is close to our empirical estimate of 0.69. The inflation response is positive and small: an increase in government spending by one percent of output increases inflation by 0.13 percentage points which is close to our point estimate of 0.14.

The mild response of inflation rate under both monetary policy specifications is due to a sufficiently flat Phillips curve in our model. In our model, the heterogeneous labor market assumption increases the degree of complementarity between different firms optimal price choices relative to a model with homogeneous labor market.²⁶ This leads to smaller response of inflation as the firms who currently set prices choose to set prices closer to those firms who do not reset prices. Thus, a given level of inflation response leads to a larger change in real variables.

Another important assumption for the calibrated model to match the high output multiplier in the ZLB period observed in the data is the fixed nominal-rate policy. In other words, the persistence of government spending shocks is equal to that of the liquidity trap. If government spending shocks are more persistent than the liquidity trap, the multiplier in the ZLB period can be smaller, as shown in [Woodford \(2010\)](#).

3.5 Confidence-driven ZLB Period

In this section, we study the effects of government spending shocks in a standard New Keynesian model when the nominal interest rate is at the zero bound due to a self-fulfilling state of low confidence.

[Benhabib et al. \(2001a,b\)](#) show that there are two steady states in the standard New Keynesian model when the policy rate actively responds to inflation but is also constrained by the zero lower bound. The first steady state is the standard targeted inflation steady state. The second steady state features zero nominal interest rate and deflation. We call this steady state “deflationary trap.” Intuitively, if agents believe that there is permanent deflation, the central bank sets the nominal rate to zero. If the level of deflation equals the subjective discount factor, the economy can remain in this equilibrium forever. Thus, the economy may end up in a deflationary steady state with zero nominal interest rate because agents unexpectedly change their beliefs.

²⁶[Woodford \(2003\)](#) discusses this and other assumptions that can lead to a higher degree of complementarity between firms optimal prices.

Deflationary trap provides an alternative explanation for zero nominal rate periods in addition to the liquidity trap driven by fundamental shocks analyzed above. In a recent paper, [Aruoba et al. \(2013\)](#) estimate a New Keynesian model and conclude that Japan is more likely to be at the zero lower bound because it is stuck in a deflationary trap rather than a liquidity trap caused by fundamental shocks. [Mertens and Ravn \(2014\)](#) argue that the output multiplier in a deflationary trap is substantially smaller than the high values possible in a liquidity trap caused by fundamental shocks. Moreover, the multiplier in their calibration in the deflationary trap is even smaller than the output multiplier in normal period.

However, contrary to the previous findings, we show that the output multiplier can be larger than one in a deflationary trap. In particular, if government spending shocks are almost permanent, the model can generate the multipliers similar to our empirical estimates.

3.5.1 Non-fundamental Uncertainty

Since [Benhabib et al. \(2001b\)](#), it is well known that there are an infinite number of equilibrium paths to the deflationary steady state leading to infinitely many responses of the economy to government spending shocks. We follow [Mertens and Ravn \(2014\)](#) and restrict our attention to Markov equilibria in which the only state variables are a non-fundamental random variable, called sunspot, and the price dispersion.²⁷ This ensures a unique path to the deflationary steady state: a sunspot determines a steady state to which the economy approaches, the Markovian structure pins down a unique path when approaching the deflationary trap steady state.

Consider a sunspot, represented by ω_t , that can take on two values $\{\omega_P, \omega_O\}$. We call ω_O a state with “optimistic” expectations and ω_P a state with “pessimistic” expectations because, by assumption, the economy converges to the desirable targeted inflation steady state when $\omega_t = \omega_O$ and to deflationary steady state when $\omega_t = \omega_P$. The optimistic expectations state is absorbing. Formally, $Pr(\omega_t = \omega_O | \omega_{t-1} = \omega_O) = 1$. If the economy starts in state with pessimistic expectations, the probability to remain in this state in the next period is $Pr(\omega_t = \omega_P | \omega_{t-1} = \omega_P) = \mu \in (0, 1)$.

We solve the model by log-linearizing the equilibrium conditions around the zero inflation steady state.²⁸ We consider an unexpected increase in government spending when the economy is in the deflationary steady state. As [Mertens and Ravn \(2014\)](#), we assume that government spending stays

²⁷[Cass and Shell \(1983\)](#) show that sunspots matter in macroeconomic models. See [Benhabib and Farmer \(1999\)](#) for a review of sunspots in macroeconomics.

²⁸The approximation around the zero inflation steady state is valid as long as the values of endogenous variables are sufficiently close in the two steady states.

at the same elevated level \hat{g}_P as long as $\omega_t = \omega_P$ and it reverts back to its steady state after sunspot turns to ω_O . The details of the derivation are standard and presented in Appendix A.2.

3.5.2 Deflationary Trap Results

To numerically evaluate the response of the economy to government spending shocks in deflationary trap, we solve the model setting all the same parameters as in the calibration of the baseline model. We analyze three values of the persistence of pessimistic expectations state $\mu = 0.875; 0.95; 1$. $\mu = 0.89$ is slightly higher than the lowest possible $\mu^c = 0.871$ that ensures the existence of deflationary steady state. $\mu = 0.95$ is as in Aruoba et al. (2013) and the corresponding half-life is 3.6 years. $\mu = 1$ corresponds to the permanent deflationary trap.

Table 5 presents the results. First, the output multiplier is lower than one, and lower than our estimate of 1.5, in all of the cases. The output multiplier is monotonic in μ (we prove this formally in appendix A.2). The output multiplier can be negative if μ is sufficiently low. The consumption and inflation multipliers are below zero in all of the cases. When $\mu = 1$, the output multiplier is exactly equal to the flexible price output multiplier, and inflation does not move in response to a government spending shock.

To understand the logic behind the results in Table 5 we plot the Phillips and the IS curves in (π, \hat{y}) space in Figure 12.²⁹ For the two steady states to exist, the Phillips curve must be steep enough to intersect the IS curve twice. Consider a deflationary trap, represented by the intersection of the IS and Phillips Curve at point A in the figure. The rightward shift of the IS curve due to increase in government spending, a shift from IS to IS' on the figure, necessarily implies that the intersection with the Phillips curve is at an even lower level of inflation. Thus, government spending shock is deflationary. Intuitively, an increase in government spending stimulates aggregate demand. For a given level of real wages, firms would want to increase prices. However, a higher price level induces even a higher aggregate demand. In the deflationary trap, for a given change in price level, aggregate supply responds less than aggregate demand. Therefore, prices have to fall to clear the market. This increases the real interest rate because the nominal rate is at zero, which reduces consumption and output. There is a second effect of increased government spending: lower consumption increases labor supply, which reduces firm's marginal cost leading to a shift of the Phillips curve from PC to PC' . If this second effect is large enough, the output multiplier is

²⁹This figure reproduces Figure 7 in the case of sunspot shocks from Mertens and Ravn (2014).

positive. This is the case in Table 5 for high levels of μ . However, inflation always goes down.³⁰

Note that both in a liquidity trap driven by fundamental shocks and in a deflationary trap, the logic of a New Keynesian model applicable in normal times changes considerably. For example, in liquidity trap driven by fundamentals, negative aggregate supply shocks stimulate the economy (see Eggertsson, 2008). In contrast, in a deflationary trap, it is negative aggregate demand shocks that stimulate the economy.

The above results may suggest that the response of the economy to government spending shocks in a deflationary trap cannot match our empirical findings. However, we now show that if government spending shocks are more persistent than the deflationary trap, the multiplier during the ZLB period can be as large as our empirical estimate. Specifically, we continue to assume that the government spending is \hat{g}_P in all of the periods when $\omega_t = \omega_P$. However, when the sunspot turns optimistic, government spending remains elevated with probability λ and reverts back to its steady state level with probability $1 - \lambda$. The detailed derivation of the solution is in Appendix A.2

To illustrate that multipliers can be large even in a deflationary trap, we compute the multipliers for $\mu = 0.875$ with different levels of persistence of government spending shocks λ . The results are presented in Figure 13. The solid curves correspond to the model multipliers while the horizontal dashed lines are our empirical point estimates for the multipliers during the ZLB period in Japan. With higher persistence of government spending shocks, output, consumption, and inflation multipliers increase. When $\lambda = 0.993$, the output multiplier is 1.51, the consumption multiplier is 0.51, and the inflation multiplier is 0.19. All of these values are close to our empirical estimates.

To match the estimated multipliers, government spending shocks have to be extremely persistent. Specifically, the value of $\lambda = 0.993$ implies half-life of government spending shocks to be about 25 years. Recall that we have to also add the duration of the pessimistic expectations state because fiscal policy is active during this state. Thus, to obtain large fiscal multipliers in a deflationary trap, we need to assume almost permanent shocks to government spending.

Why does the output multiplier increase with higher degree of persistence of government spending shocks? Higher government spending crowds out private consumption in the optimistic state because monetary policy responds actively to changes in inflation and output. As a result, marginal utility of consumption increases. In addition, inflation rate can go down. The expectation of these

³⁰See Appendix A.2 for a formal proof. Intuitively, the shift in the Phillips curve is never large enough relative to the shift in the IS curve to make inflation go up.

two effects reduces incentives to consume when the economy is still in the pessimistic state. Thus, in addition to the direct effect that shifts the IS curve to the right, there is a second force from higher government spending. This second force shifts the IS curve to the left. If this second force is large enough, the total effect of increased government spending is a shift of the IS curve to the left. In a deflationary trap, the leftward shift of the IS curve leads to higher inflation which decreases real interest rate and increases output. If the leftward shift of the IS curve is quite large, which is the case in our numerical example, the output multiplier is higher than one.

4 Conclusion

We exploit the rich information about the ZLB period in Japan to estimate the effects of government spending changes on output. We control for the expected government spending to extract its unexpected changes. Our point estimate of the output multiplier is larger than one in the ZLB period, and this output multiplier is larger than that in the normal period. On impact, the output multiplier is 1.47 in the ZLB period and it is 0.70 in the normal period. The difference in the multipliers between the two periods are larger over longer horizons: while the multiplier increases to over two in the ZLB period, it becomes negative in the normal period. Furthermore, government spending crowds in private consumption and investment in the ZLB period, in contrast with the crowding out effects in the normal period. We estimate a mild response of inflation in both periods. Additionally, the ex-ante real interest rate decreases by more in the ZLB period relative to the normal period.

We examine the standard New Keynesian model where the ZLB can occur due to either a fundamental shock or a self-fulfilling state of low confidence. We show that the calibrated version of the model with a fundamental-driven ZLB period is consistent with our empirical estimate. The two key assumptions to help this model to match our empirical estimates are a sufficiently flat Phillips curve and the low persistence of government spending relative to the duration of the ZLB period. Furthermore, we show that in a model where the ZLB occurs due to low confidence, the output multiplier can be larger than one, in contrast to previous literature. This result relies on the assumption that government spending is expected to be more persistent than the ZLB period, which seems unlikely for Japanese data.

References

- Aruoba, S Boragan, Pablo Cuba-Borda, and Frank Schorfheide**, “Macroeconomic dynamics near the ZLB: A tale of two countries,” Technical Report, National Bureau of Economic Research 2013.
- Auerbach, Alan J and Yuriy Gorodnichenko**, “Fiscal multipliers in recession and expansion,” in “Fiscal Policy after the Financial crisis,” University of Chicago press, 2012, pp. 63–98.
- **and** – , “Measuring the output responses to fiscal policy,” *American Economic Journal: Economic Policy*, 2012, *4*, 1–27.
- **and** – , “Fiscal Multipliers in Japan,” Technical Report, National Bureau of Economic Research 2014.
- Benhabib, Jess and R. E. Farmer**, “Indeterminacy and sunspots in macroeconomics,” in J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, Vol. 1, Elsevier, 1999, pp. 387–448.
- , **Stephanie Schmitt-Grohé, and Martin Uribe**, “Monetary policy and multiple equilibria,” *American Economic Review*, 2001, *91* (1), 167–186.
- , – , **and** – , “The perils of Taylor rules,” *Journal of Economic Theory*, 2001, *96* (1), 40–69.
- Blanchard, Olivier and Roberto Perotti**, “An empirical characterization of the dynamic effects of changes in government spending and taxes on output,” *Quarterly Journal of Economics*, 2002, *107* (November), 1329–1368.
- Bruckner, Marcus and Anita Tuladhar**, “Local Government Spending Multipliers and Financial Distress: Evidence from Japanese Prefectures,” *Economic Journal*, 2014, *124*, 1279–1316.
- Burstein, Ariel and Christian Hellwig**, “Prices and Market Shares in a Menu Cost Model,” NBER Working Papers 13455, National Bureau of Economic Research, Inc September 2007.
- Cass, D. and K. Shell**, “Do sunspots matter?,” *Journal of Political Economy*, 1983, *91* (2), 193–227.
- Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo**, “When Is the Government Spending Multiplier Large?,” *Journal of Political Economy*, 2011, *119* (1), 78 – 121.

- Crafts, Nicholas and Terence C. Mills**, “Fiscal Policy in a Depressed Economy: Was there a ‘Free Lunch’ in 1930s’ Britain?,” 2012. Unpublished paper.
- Doi, Takero, Takeo Hoshi, and Tatsuyoshi Okimoto**, “Japanese government debt and sustainability of fiscal policy,” *Journal of the Japanese and International Economies*, December 2011, 25 (4), 414–433.
- Driscoll, John C. and Aart C. Kraay**, “Consistent Covariance Matrix Estimation with Spatially Dependent Panel Data,” *The Review of Economics and Statistics*, 1998, 80 (4), 549–560.
- Dupor, Bill and Rong Li**, “The expected inflation channel of government spending in the postwar US,” *European Economic Review*, 2015, 74, 36–56.
- Eggertsson, Gauti B.**, “Can Tax Cuts Deepen Recessions?,” *Federal Reserve Bank of New York*, December, 2008.
- Eggertsson, Gauti B.**, “What fiscal policy is effective at Zero interest rates?,” *NBER Macroeconomics Annual*, 2011, 25, 59–112.
- Farhi, Emmanuel and Iván Werning**, “Fiscal Multipliers: Liquidity Traps and Currency Unions,” NBER Working Papers 18381, National Bureau of Economic Research, Inc September 2012.
- Gordon, Robert J. and Robert Krenn**, “The End of the Great Depression: VAR Insight on the Roles of Monetary and Fiscal Policy,” *NBER Working paper 16380*, 2010.
- Ilzetzki, Ethan, Enrique G. Mendoza, and Carlos a. Végh**, “How big (small?) are fiscal multipliers?,” *Journal of Monetary Economics*, March 2013, 60 (2), 239–254.
- Jorda, Oscar**, “Estimation and Inference of Impulse Responses by Local Projections,” *American Economic Review*, 2005, 95 (1), 161–182.
- Mertens, K. R. S. M. and M. O. Ravn**, “Fiscal Policy in an Expectations-Driven Liquidity Trap,” *The Review of Economic Studies*, May 2014, 81 (4), 1637–1667.
- Mountford, Andrew and Harald Uhlig**, “What are the effects of fiscal policy shocks?,” *Journal of Applied Econometrics*, 2009, 992 (April), 960–992.

- Nakamura, Emi and Jon Steinsson**, “Fiscal Stimulus in a Monetary Union: Evidence from US Regions,” *American Economic Review*, March 2014, *104* (3), 753–92.
- Ramey, Valerie A.**, “Can government purchases stimulate the economy?,” *Journal of Economic Literature*, 2011, *49* (3), 673–685.
- , “Identifying Government Spending Shocks: It’s all in the Timing,” *The Quarterly Journal of Economics*, March 2011, *126* (1), 1–50.
- **and Sarah Zubairy**, “Government spending multipliers in good times and in bad: Evidence from US historical data,” 2014.
- Stock, J. H. and M. W. Watson**, “Why Has U.S. Inflation Become Harder to Forecast?,” *Journal of Money, Credit and Banking*, 2007, *39* (1), 3–33.
- Wieland, Johannes**, “Are negative supply shocks expansionary at the zero lower bound?,” *University of California–Berkeley, working paper*, 2013.
- Woodford, M.**, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press Princeton, NJ, 2003.
- Woodford, Michael**, “Simple analytics of the government expenditure multiplier,” *American Economic Journal: Macroeconomics*, 2010, *3* (January), 1–35.

Appendices

A Model Solution

A.1 Baseline Model of Government Spending

The log-linearized equilibrium conditions can be summarized by the New-Keynesian IS and the Phillips curves

$$\hat{y}_t - \hat{g}_t = \mathbb{E}_t (\hat{y}_{t+1} - \hat{g}_{t+1}) - \tilde{\sigma} (i_t - \mathbb{E}_t \pi_{t+1} - \bar{r}), \quad (\text{A.1})$$

$$\pi_t = \beta \mathbb{E} \pi_{t+1} + \kappa (\hat{y}_t - \Gamma \hat{g}_t), \quad (\text{A.2})$$

where $\tilde{\sigma} = \sigma(1 - \bar{G}/\bar{Y})$ is the “effective” intertemporal elasticity of substitution, $\kappa = (1 - \alpha)(1 - \alpha\beta)/\alpha \cdot (\tilde{\sigma}^{-1} + \psi_\nu) / (1 + \theta\psi_\nu)$ is the slope of the Phillips curve with $\psi_\nu = (1 - a + \nu^{-1})/a$ being the elasticity of real marginal costs with respect to output, $\Gamma = \tilde{\sigma}^{-1}/(\tilde{\sigma}^{-1} + \psi_\nu) \in (0, 1)$ is the fiscal multiplier under flexible prices. Observe that the results do not depend on disutility of labor parameter χ .

Under the Taylor rule policy, we can find the solution by using the method of undetermined coefficients. We conjecture a solution of the form $\hat{y}_t = \gamma^y \hat{g}_t, \hat{c}_t = \gamma^c \hat{g}_t, \hat{\pi}_t = \gamma^\pi \hat{g}_t, i_t = \bar{r} + \gamma^i \hat{g}_t$. Solving the model we get

$$\begin{aligned}\gamma^y &= \frac{1 - \rho + (\tilde{\psi} - \tilde{\sigma}\phi_y)\Gamma}{1 - \rho + \tilde{\psi}} \in (0, 1) \\ \gamma^c &= (\gamma_y - 1) \frac{\bar{Y}}{\bar{C}} = -\frac{\tilde{\psi}(1 - \Gamma) + \tilde{\sigma}\phi_y\Gamma}{1 - \rho + \tilde{\psi}} \cdot \frac{\bar{Y}}{\bar{C}} < 0, \\ \gamma^\pi &= \frac{\kappa}{1 - \beta\rho} \cdot \frac{(1 - \rho)(1 - \Gamma) - \tilde{\sigma}\phi_y\Gamma}{1 - \rho + \tilde{\psi}} \leq 0,\end{aligned}$$

where $\tilde{\psi} \equiv \tilde{\sigma}(\phi_y + (\phi_\pi - \rho)\frac{\kappa}{1 - \beta\rho})$.

Under the fixed nominal-rate rule, the multipliers are

$$\gamma^y = \frac{(1 - \rho)(1 - \beta\rho) - \tilde{\sigma}\kappa\rho\Gamma}{(1 - \rho)(1 - \beta\rho) - \tilde{\sigma}\kappa\rho} > 1, \quad (\text{A.3})$$

$$\gamma^c = (\gamma_y - 1) \frac{\bar{Y}}{\bar{C}} = \frac{\tilde{\sigma}\kappa\rho(1 - \Gamma)}{(1 - \rho)(1 - \beta\rho) - \tilde{\sigma}\kappa\rho} \cdot \frac{\bar{Y}}{\bar{C}} > 0,$$

$$\gamma^\pi = \frac{\kappa}{1 - \beta\rho} \cdot \frac{(1 - \rho)(1 - \beta\rho)(1 - \Gamma)}{(1 - \rho)(1 - \beta\rho) - \tilde{\sigma}\kappa\rho} > 0. \quad (\text{A.4})$$

For the unique bounded solution to exist it must be that $(1 - \rho)(1 - \beta\rho) - \tilde{\sigma}\kappa\rho > 0$, otherwise the economy “explodes” (this can be shown by solving forward equations (A.1) and (A.2)). This condition ensures the inequalities in (A.3)-(A.4).

A.2 Deflationary Trap Scenario Solution

In the case of log-linearization around the zero inflation steady state, the price dispersion does not affect linearized equilibrium conditions. Thus sunspot shock remains the only state variable. Because optimistic sunspot state ω_O is absorbing, output gap and inflation are zero in all periods when $\omega_t = \omega_O$. That is, $\hat{y}_N = \pi_N = 0$, where the subscript stands for normal times. Output and inflation in pessimistic state are always the same and are denoted by $\hat{y}_P, \pi_P, \hat{g}_P$.

Given the structure of sunspot and government spending shocks discussed in the text, equations (A.1), (A.2) and the monetary policy in pessimistic state can be written as follows

$$\begin{aligned}(1 - \mu)(\hat{y}_P - \hat{g}_P) &= -\tilde{\sigma}(i_t - \mu\pi_P - \bar{r}), \\ \pi_P &= \frac{\kappa}{1 - \beta\mu}(\hat{y}_P - \Gamma\hat{g}_P), \\ i_P &= \max\{0, \bar{r} + \phi_\pi\pi_P + \phi_y\hat{y}_P\},\end{aligned}$$

When the deflationary trap steady state exists the solution has the following form $i_P = 0, \hat{y}_P = \gamma_P^y \hat{g}_P + \vartheta_P^y \bar{r}, \pi_P = \kappa(1 - \beta\mu)^{-1}[(\gamma_P^y - \Gamma)\hat{g}_P + \vartheta_P^y \bar{r}]$, where

$$\begin{aligned}\gamma_P^y &= \frac{(1 - \mu)(1 - \beta\mu) - \tilde{\sigma}\kappa\mu\Gamma}{(1 - \mu)(1 - \beta\mu) - \tilde{\sigma}\kappa\mu}, \\ \vartheta_P^y &= \frac{\tilde{\sigma}(1 - \beta\mu)}{(1 - \mu)(1 - \beta\mu) - \tilde{\sigma}\kappa\mu},\end{aligned}\tag{A.5}$$

where γ_P^y and ϑ_P^y denote output multipliers with respect to government consumption and \bar{r} respectively. The inflation multipliers are

$$\begin{aligned}\gamma_P^\pi &= \frac{\kappa}{1 - \beta\mu}(\gamma_P^y - \Gamma) = \frac{\kappa}{1 - \beta\mu} \cdot \frac{(1 - \mu)(1 - \beta\mu)(1 - \Gamma)}{(1 - \mu)(1 - \beta\mu) - \tilde{\sigma}\kappa\mu}, \\ \vartheta_P^\pi &= \frac{\kappa}{1 - \beta\mu}\vartheta_P^y.\end{aligned}\tag{A.6}$$

Observe that expressions (A.5) and (A.6) look similar to (A.3) and (A.4) with government spending shocks persistence being ρ in the fixed nominal rate scenario and μ in the deflationary trap scenario. However, there is an important difference. For the deflationary trap equilibrium to exist it must be true that $(1 - \mu)(1 - \beta\mu) - \tilde{\sigma}\kappa\mu < 0$. To see this, consider an equilibrium without fiscal policy shocks. Output and inflation are $\hat{y}_P = \vartheta_P^y \bar{r}, \pi_P = \kappa(1 - \beta\mu)^{-1} \vartheta_P^y \bar{r}$. If the nominal interest rate is zero it must be that output gap \hat{y}_P is negative. This implies that ϑ_P^y is negative, which is equivalent to $(1 - \mu)(1 - \beta\mu) - \tilde{\sigma}\kappa\mu < 0$.

This condition implies that inflation fiscal multiplier γ_P^π is negative. Note that in a special case of permanent deflationary trap, i.e. $\mu = 1$, the inflation response is zero. The sign of fiscal multiplier γ_P^y depends on flexible price multiplier Γ , duration of pessimistic state μ , and $\beta, \tilde{\sigma}, \kappa$. For example, if Γ is close to 1, the output fiscal multiplier is positive. However, if Γ is close to zero, the multiplier is negative.

The output multiplier increases in μ . To see this take a derivative of (A.5) with respect to μ

$$\frac{d \log \gamma_P^y}{d\mu} = \frac{2\mu\beta - \beta - 1 - \tilde{\sigma}\kappa\Gamma}{(1-\mu)(1-\beta\mu) - \tilde{\sigma}\kappa\mu\Gamma} - \frac{2\mu\beta - \beta - 1 - \tilde{\sigma}\kappa}{(1-\mu)(1-\beta\mu) - \tilde{\sigma}\kappa\mu} > 0.$$

Very persistent government spending shocks. Here we solve the model under the assumption that government spending shocks do not revert back to its steady state immediately after pessimistic sunspot subsides, described in the main text.

Output gap and inflation in the optimistic state after government consumption turns to its steady state is $\pi_N = \hat{y}_N = 0$. Denote by π_S and \hat{y}_S inflation and output gap in the optimistic state when the government consumption is still elevated. π_S and \hat{y}_S satisfy the IS and Phillips curves of the following form

$$\begin{aligned}\hat{y}_S - \hat{g}_P &= \lambda(\hat{y}_S - \hat{g}_P) - \tilde{\sigma}(\phi_\pi\pi_S + \phi_y\hat{y}_S - \lambda\pi_S), \\ \pi_S &= \beta\lambda\pi_S + \kappa(\hat{y}_S - \Gamma\hat{g}_P).\end{aligned}$$

Solving the above two equations we get

$$\begin{aligned}\hat{y}_S &= \frac{1 - \lambda + (\tilde{\psi}_\lambda - \tilde{\sigma}\phi_y)\Gamma}{1 - \lambda + \tilde{\psi}_\lambda}\hat{g}_P, \\ \pi_S &= \frac{\kappa}{1 - \beta\lambda} \cdot \frac{(1 - \lambda)(1 - \Gamma) - \tilde{\sigma}\phi_y\Gamma}{1 - \lambda + \tilde{\psi}_\lambda}\hat{g}_P,\end{aligned}$$

where $\tilde{\psi}_\lambda \equiv \tilde{\sigma}\left(\phi_y + (\phi_\pi - \lambda)\frac{\kappa}{1 - \beta\lambda}\right)$. Note that the solution is identical to the solution obtained in Appendix A for the case of government spending shocks decaying according to $AR(1)$ process when the monetary policy follows the Taylor rule. The only difference is that the persistence parameter is now λ .

As before we denote output gap and inflation in the pessimistic state by π_P and \hat{y}_P . They satisfy the following IS and Phillips curves

$$\begin{aligned}\hat{y}_P - \hat{g}_P &= \mu(\hat{y}_P - \hat{g}_P) + (1 - \mu)\lambda(\hat{y}_S - \hat{g}_P) - \tilde{\sigma}[-\mu\pi_P - (1 - \mu)\lambda\pi_S - \bar{r}], \\ \pi_P &= \beta\mu\pi_P + \beta(1 - \mu)\lambda\pi_S + \kappa(\hat{y}_P - \Gamma\hat{g}_P).\end{aligned}$$

The solution is

$$\begin{aligned}\widehat{y}_P &= \frac{(1-\mu)(1-\beta\mu)(1-\lambda) - \Gamma\mu\kappa\tilde{\sigma}}{\Delta}\widehat{g}_P + \frac{\lambda(1-\mu)(1-\beta\mu)}{\Delta}\widehat{y}_S \\ &\quad + \frac{\tilde{\sigma}\lambda(1-\mu)}{\Delta}\pi_S - \frac{\tilde{\sigma}(1-\beta\mu)}{\Delta}\bar{r}, \\ \pi_P &= \frac{\kappa}{1-\beta\mu}(\widehat{y}_P - \Gamma\widehat{g}_P) + \frac{\beta(1-\mu)\lambda}{1-\beta\mu}\pi_S,\end{aligned}$$

where $\Delta \equiv (1-\mu)(1-\beta\mu) - \mu\kappa\tilde{\sigma}$. The output and inflation multipliers can be obtained by taking the full derivative with respect to \widehat{g}_P of the last two expressions, taking into account that π_S and \widehat{y}_S also depend on \widehat{g}_P .

Tables and Figures

Table 1: Multipliers

	Normal	ZLB	Difference	p-value
<u>Output</u>				
On impact	0.71 (0.42)	1.47 (0.55)	0.76 (0.61)	0.19
4 quarter	-0.003 (0.94)	2.38 (1.60)	2.38 (1.84)	0.13
8 quarter	-1.08 (1.03)	2.02 (2.24)	3.1 (2.68)	0.21
<u>Consumption</u>				
On impact	0.24 (0.20)	0.69 (0.32)	0.45 (0.36)	0.12
4 quarter	-0.10 (0.45)	1.55 (0.76)	1.56 (0.87)	0.04
8 quarter	-0.48 (0.44)	1.38 (1.05)	1.86 (1.22)	0.17
<u>Investment</u>				
On impact	-0.06 (0.22)	-0.20 (0.22)	-0.14 (0.29)	0.65
4 quarter	-0.50 (0.64)	0.78 (0.73)	1.28 (0.91)	0.17
8 quarter	-1.25 (0.89)	0.80 (1.07)	2.05 (1.41)	0.17

Notes: The table reports the results of the multipliers on impact and over four- and eight- quarter horizons in the normal period (Normal column) and in the ZLB period (ZLB column). The output multiplier is calculated as the cumulative change of output over the cumulative change of government spending over each horizon. The consumption and investment multipliers are defined analogously. The difference between the multiplier in the normal period and that in the ZLB period is reported in “Difference” column with the corresponding p-value in “p-value” column. All numbers in parentheses are the standard errors.

Table 2: Impulse Responses

	On impact	4 quarters	8 quarters
<u>Inflation</u>			
Normal	0.07 (0.15)	0.20 (0.16)	-0.20 (0.22)
ZLB	0.14 (0.23)	0.20 (0.21)	-0.03 (0.17)
<u>Inflation expectation</u>			
Normal	0.18 (0.23)	-0.17 (0.23)	-0.40 (0.26)
ZLB	0.32 (0.26)	0.85 (0.31)	0.31 (0.35)
<u>Short-term interest rate</u>			
Normal	0.06 (0.29)	0.35 (0.64)	-0.44 (1.17)
ZLB	-0.04 (0.02)	-0.01 (0.09)	0.03 (0.25)
<u>Long-term interest rate</u>			
Normal	-0.40 (0.23)	-0.14 (0.48)	-0.77 (0.86)
ZLB	-0.07 (0.09)	-0.03 (0.18)	-0.22 (0.20)
<u>Unemployment</u>			
Normal	-0.03 (0.04)	-0.05 (0.10)	-0.03 (0.13)
ZLB	-0.09 (0.05)	-0.50 (0.13)	-0.31 (0.27)

Notes: This table reports the impulse responses of inflation, inflation expectation, short-term and long-term nominal interest rates and unemployment rate to an increase in government spending by 1% of output. All numbers in parentheses are the standard errors.

Table 3: Output Multipliers in Different Specifications

	On impact	4 quarter	8 quarter
<u>Slackness</u>			
Expansion	0.82 (0.34)	1.44 (0.72)	1.49 (1.04)
Recession	2.31 (0.71)	3.09 (1.89)	2.50 (2.75)
Difference	1.48 (0.80)	1.65 (1.99)	1.01 (2.90)
<u>No Forecast Data</u>			
Normal	0.47 (0.35)	-0.28 (0.84)	-1.01 (0.96)
ZLB	1.44 (0.55)	2.18 (1.47)	1.97 (1.98)
<u>Quadratic trend</u>			
Normal	0.65 (0.44)	-0.06 (1.03)	-0.13 (1.14)
ZLB	1.49 (0.64)	3.00 (2.37)	2.81 (3.42)
<u>Normalized by potential output</u>			
Normal	0.71 (0.42)	0.00 (0.93)	-1.08 (1.01)
ZLB	1.47 (0.55)	2.39 (1.61)	2.01 (2.26)
<u>One step estimation</u>			
Normal	0.71 (0.42)	-0.007 (0.94)	-1.08 (1.03)
ZLB	1.47 (0.56)	2.32 (1.60)	2.04 (2.11)
<u>Add output forecast</u>			
Normal	0.47 (0.55)	0.02 (1.19)	-0.69 (1.34)
ZLB	1.50 (0.53)	2.72 (2.15)	2.73 (3.31)
<u>Unadjusted government spending</u>			
Normal	0.77 (0.44)	-0.19 (0.91)	-1.30 (0.99)
ZLB	1.33 (0.49)	2.06 (1.20)	1.76 (1.53)
<u>Actual final government spending</u>			
Normal	0.68 (0.46)	-0.05 (0.95)	-1.14 (1.01)
ZLB	1.58 (0.57)	2.68 (1.62)	2.63 (2.44)

Notes: This table reports the output multipliers over several horizons in alternative specifications. “Slackness” reports the multipliers in two regimes: recession and expansion, which is classified based on the Japanese Cabinet Office, and the difference in multipliers. “No Forecast Data” reports the estimates without controlling for expectations of government spending. “Quadratic trend” reports the estimates when we add quadratic trend to the baseline specification. “Normalized by potential output” reports the estimates when the RHS variables in the baseline specification are converted to the same unit by dividing by potential output. “One step estimation” estimates the output multiplier in one regression by adding one-quarter ahead forecast of government spending to the control variables. “Add output forecast” reports the output multiplier when adding additionally one-quarter ahead forecast of output to control for the expectations when identifying unexpected government spending shocks. “Unadjusted government spending” reports the multiplier when we use the published government spending data that include transfer of goods and services to estimate the baseline specification. “Actual final government spending” reports the multiplier when we use the published government spending data that exclude all transfer of goods and services to estimate the baseline specification. All numbers in parentheses are the standard errors.

Table 4: Model Multipliers

	Output Multiplier	Consumption Multiplier	Inflation Multiplier
Taylor Rule	0.71	-0.29	0.04
Fixed Nominal Rate	1.47	0.47	0.13

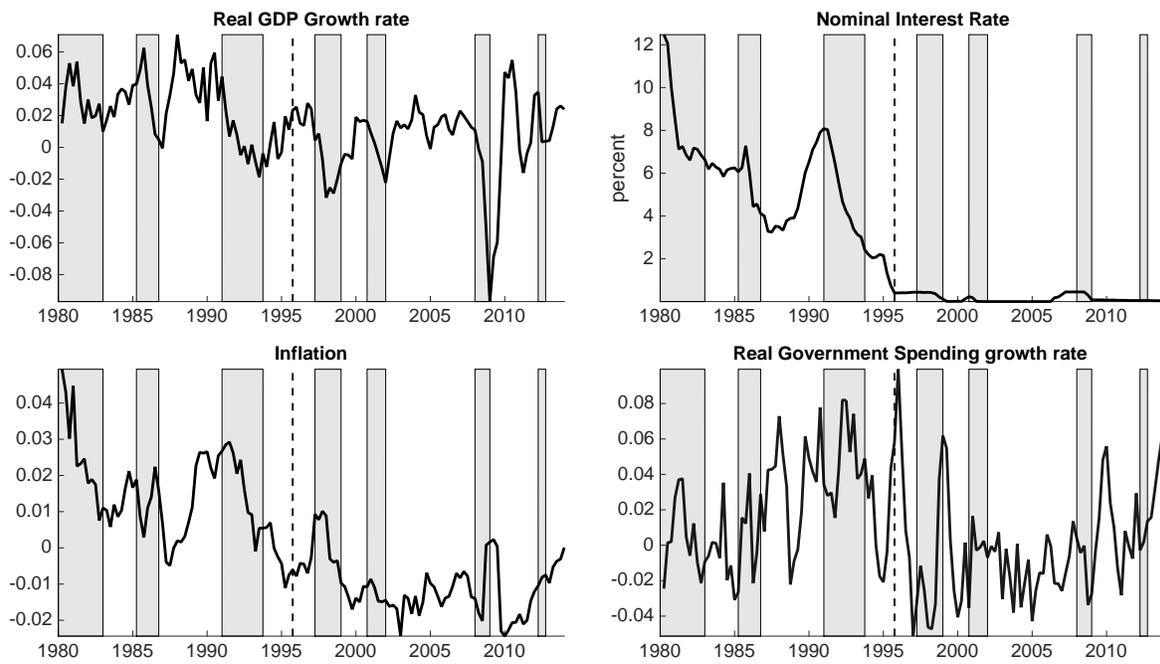
Notes: The table reports output, consumption and inflation fiscal multipliers under two monetary policy specifications. The first row (Taylor Rule) corresponds to active monetary policy response to inflation and output gap. The second row (Fixed Nominal Rate) corresponds to monetary policy that does not respond to government spending shocks.

Table 5: Model Multipliers in Deflationary Trap

μ	Output Multiplier	Consumption Multiplier	Inflation Multiplier
0.875	-4.00	-5.00	-0.79
0.95	0.25	-0.75	-0.04
1	0.36	-0.64	0

Notes: The table reports output, consumption and inflation multipliers in deflationary trap. The first column shows the persistence of the pessimistic expectations state of the economy.

Figure 1: Japanese Macroeconomic Data



Notes: The shaded areas are the recessions.

Figure 2: Government Spending Annual Growth Rate Forecast: 1980Q2-2014Q1

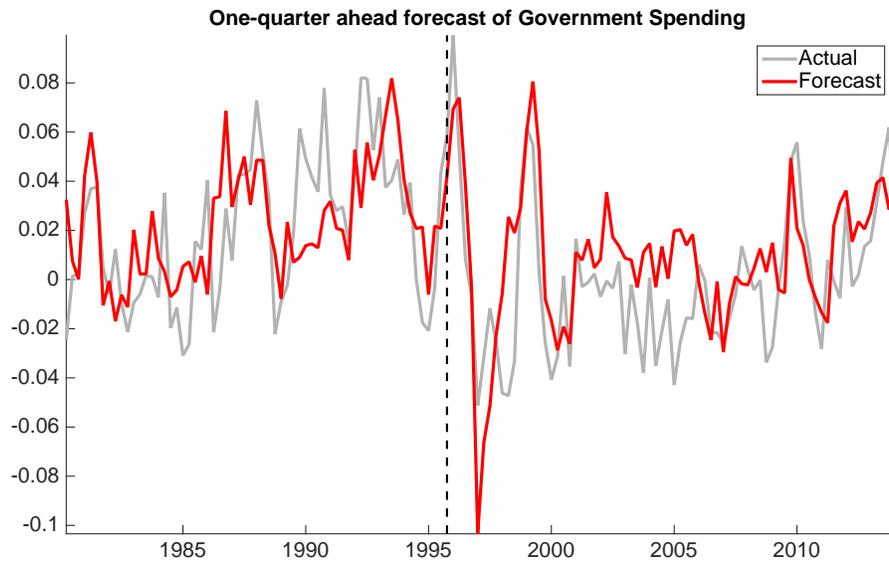
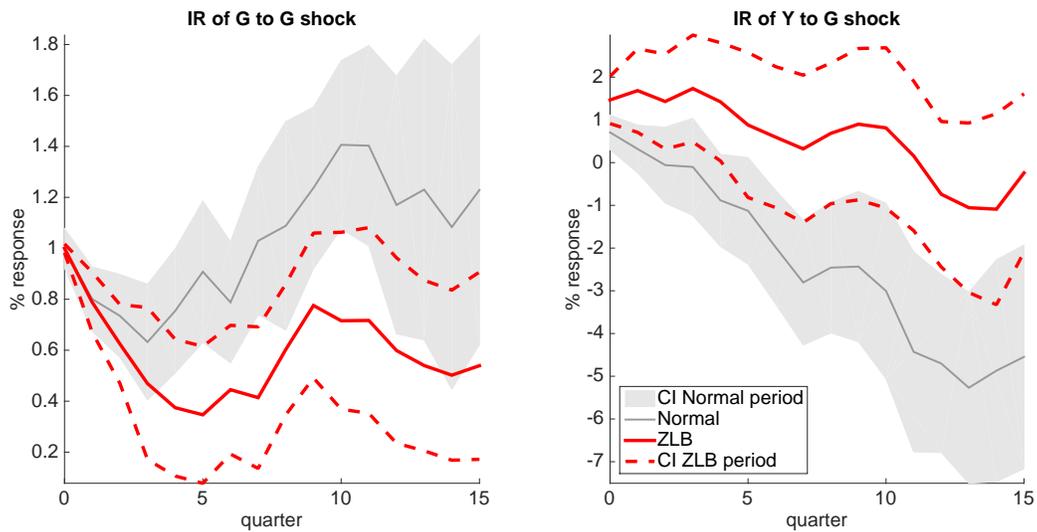


Figure 3: Impulse Responses of Output and Government Spending



Notes: Impulse responses of output and government spending to an unexpected increase in government spending by 1% of output during normal and ZLB periods.

Figure 4: Output Multipliers

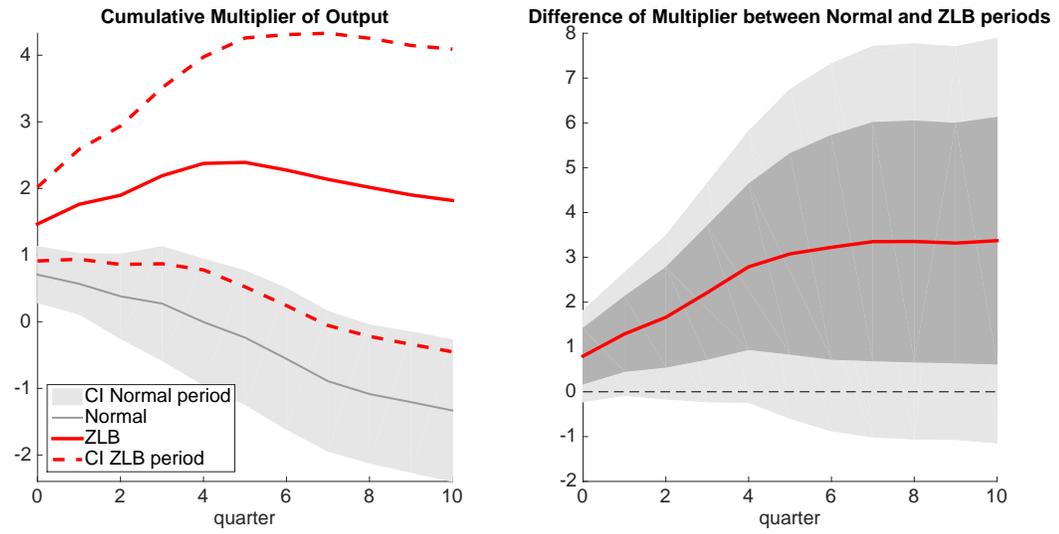
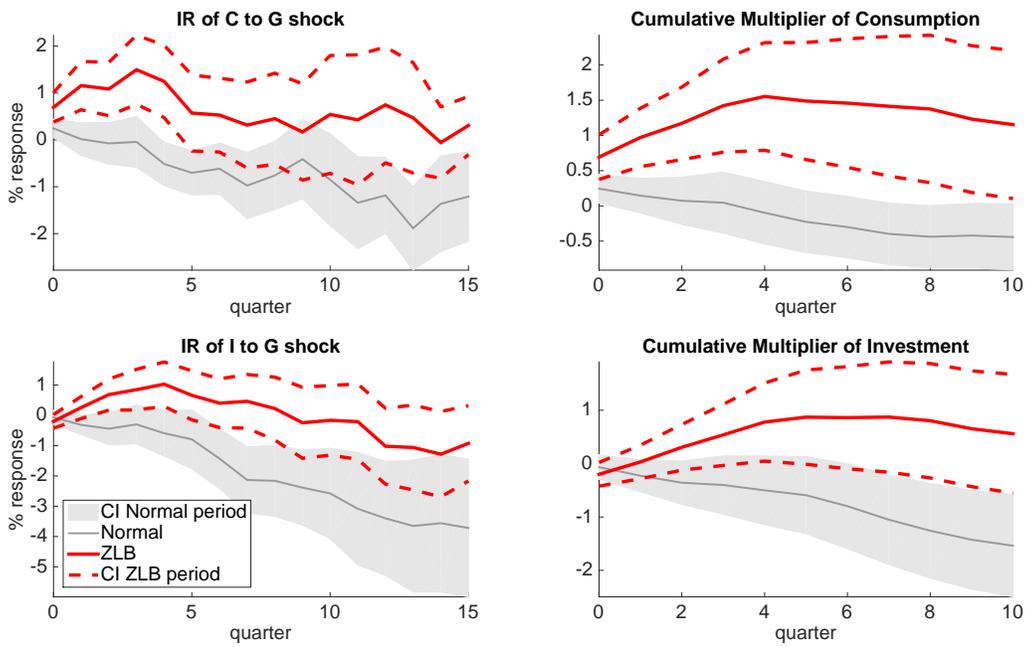
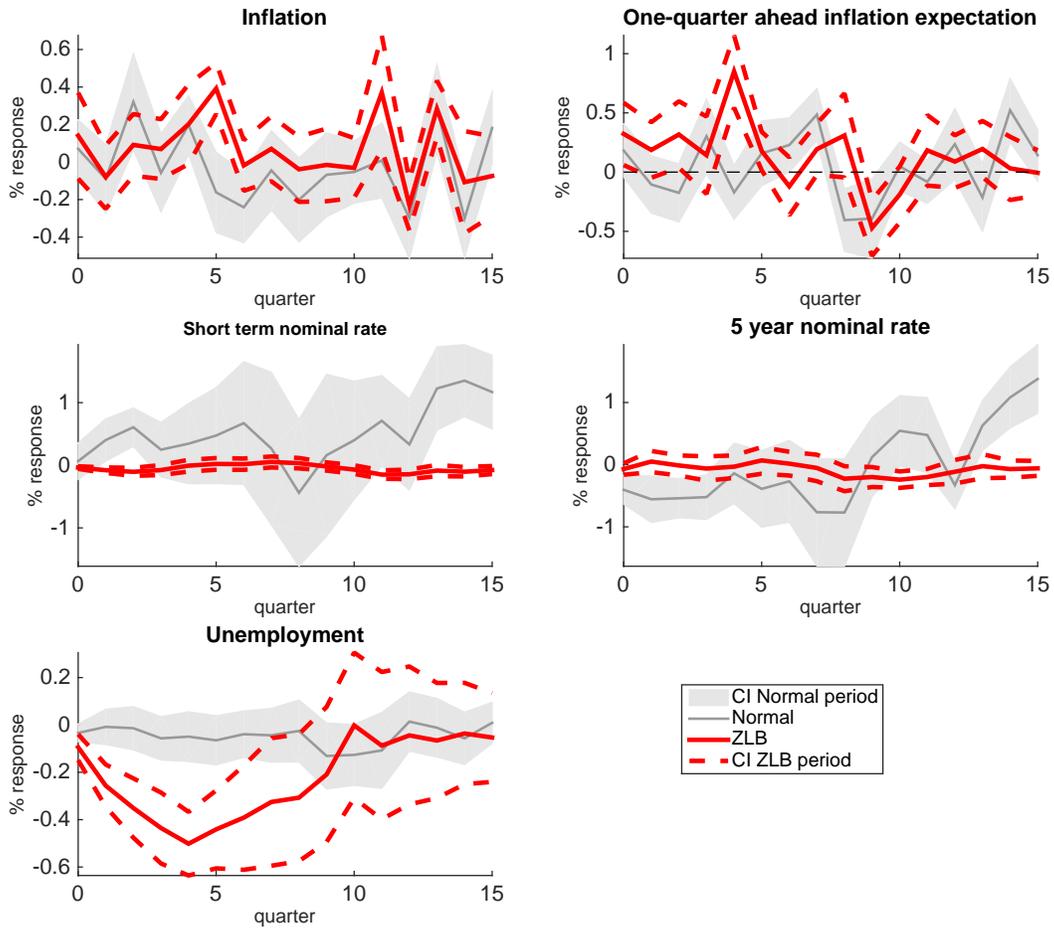


Figure 5: Impulse Responses and Multipliers for Consumption and Investment



Notes: The figure presents the impulse responses of Consumption and Investment to an increase in government spending along with the consumption and investment multipliers in normal period and ZLB period.

Figure 6: Other Impulse Responses



Notes: the figure plots impulse responses to an increase in government spending by 1% of output in normal and ZLB periods.

Figure 7: Output Multiplier during Recessions and Expansions

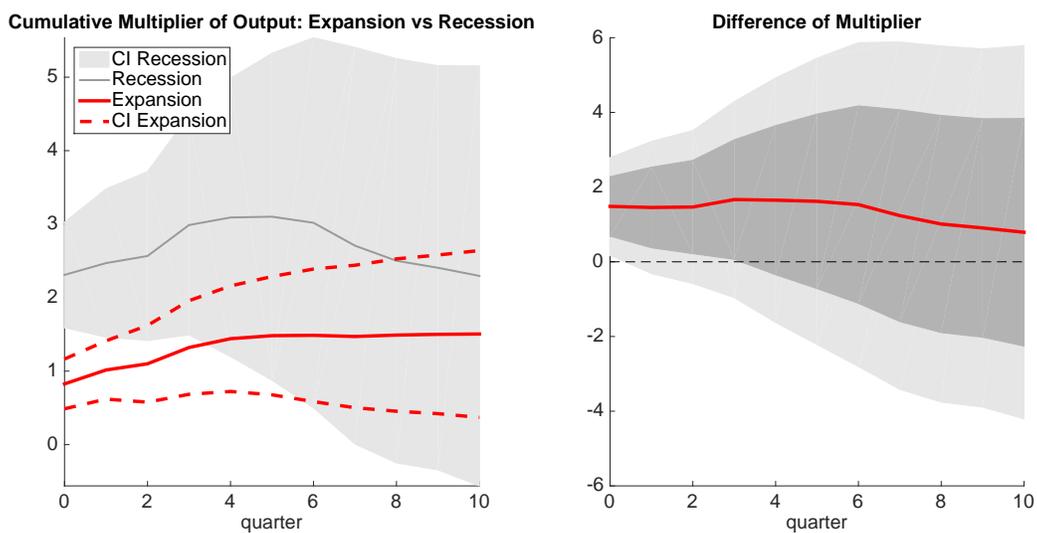
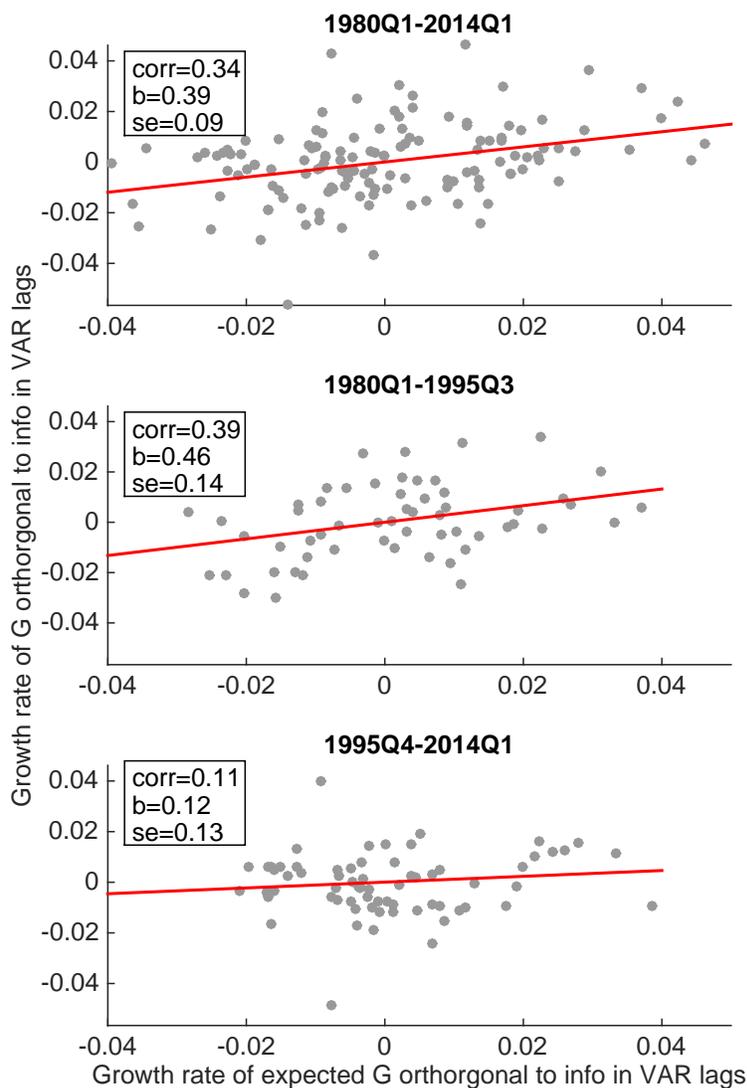


Figure 8: Predictability of Government Spending Shocks without Controlling for Expectations



Notes: The Figure plots residuals from projection of the growth rate of government spending predicted in JCER forecast (horizontal axis) and actual growth rate of government spending (vertical axis) on the information contained in the lags of output, government spending and tax revenues. *corr* denotes the correlation between the two series, *b* is the regression coefficient and *se* is the standard errors of the regression coefficient.

Figure 9: Output Multipliers with and without Forecast Data

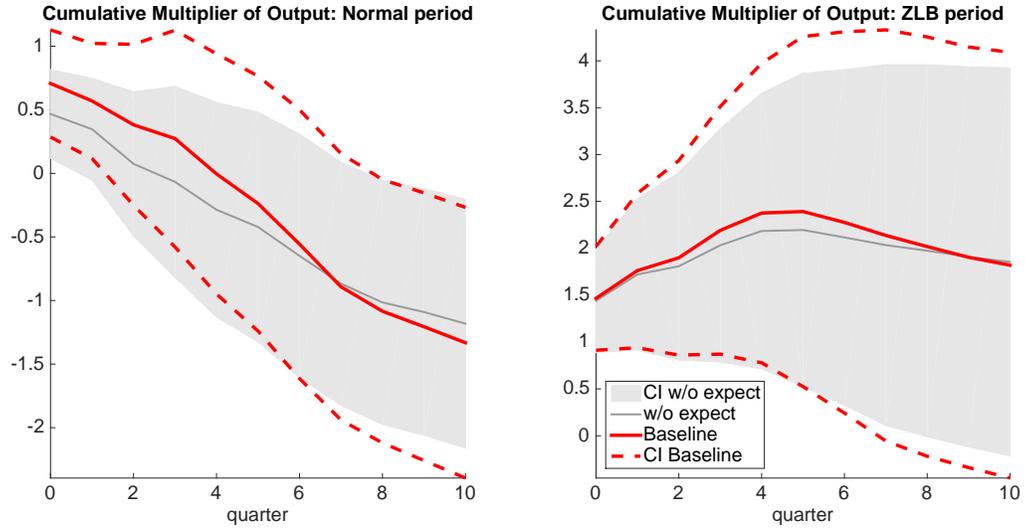


Figure 10: Output Multipliers: Different Specifications

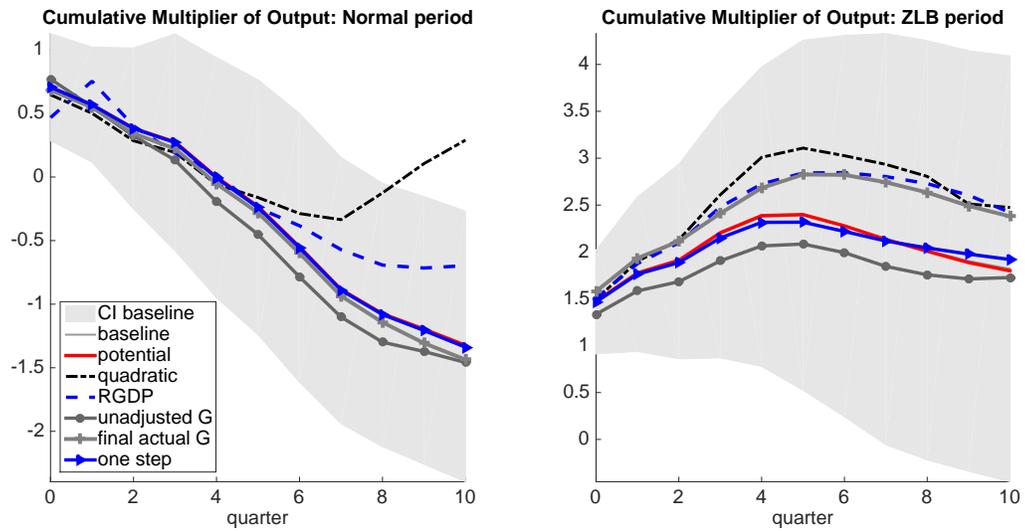
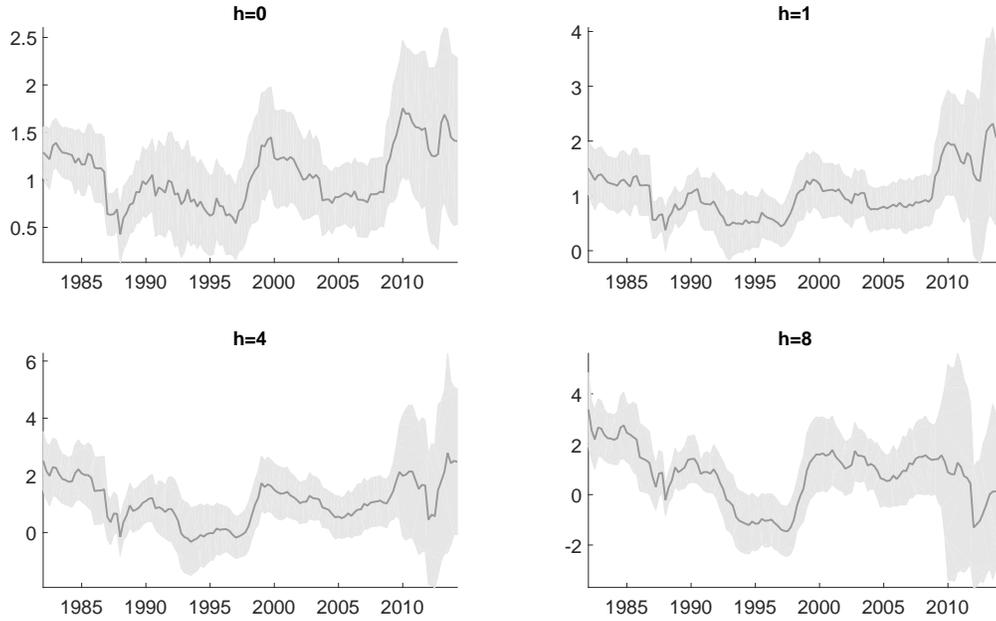


Figure 11: Output Multiplier: Rolling Estimation



Notes: The year of a reported multiplier corresponds to the last year of the 60 quarter window; for example, a multiplier reported for 1990Q1 is estimated over 1975Q1-1990Q1.

Figure 12: Government Spending Increase in Deflationary Trap

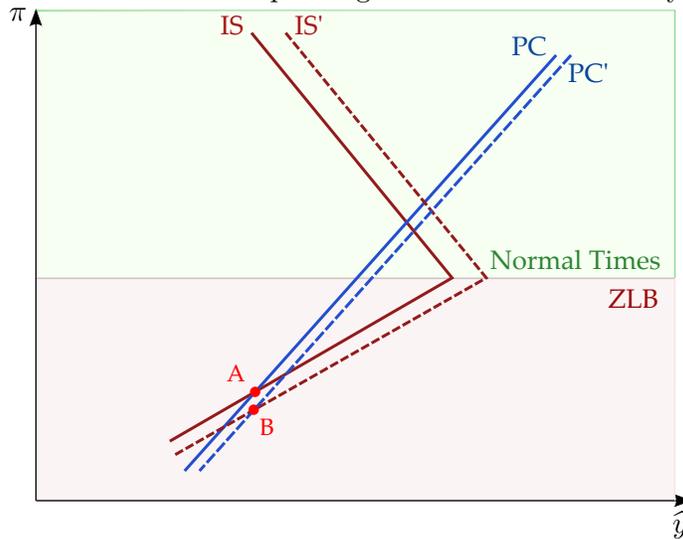
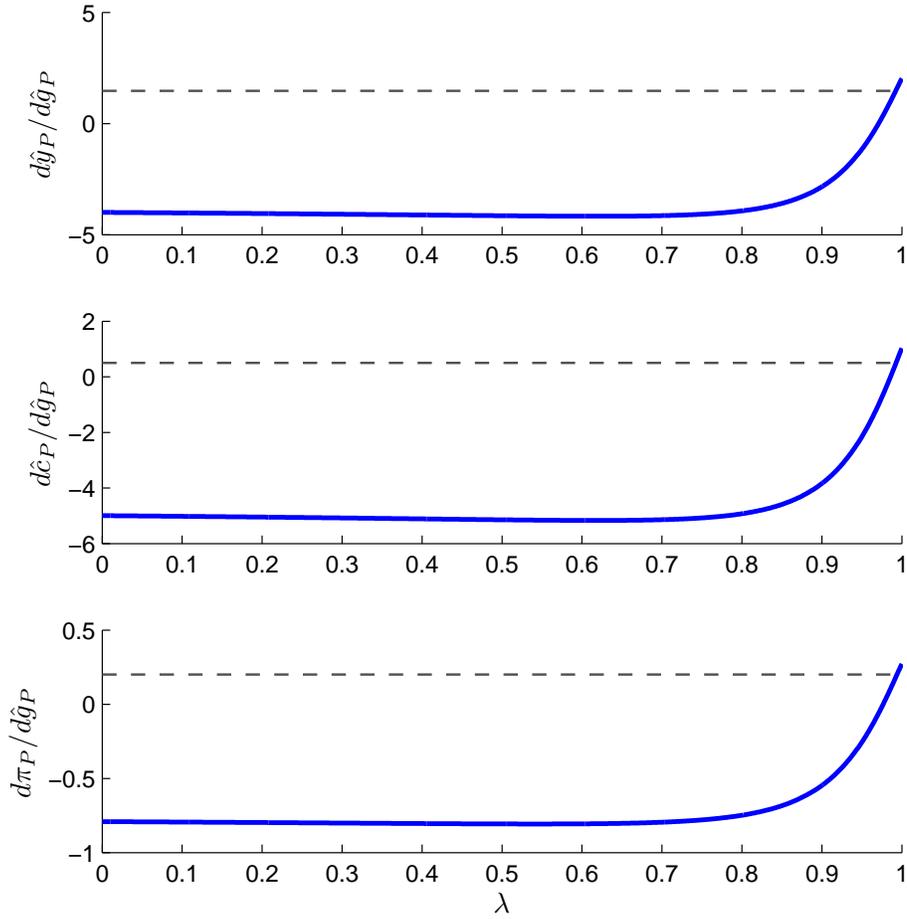


Figure 13: Multipliers in Deflationary Trap for Different Degrees of Persistence λ



Note: The solid lines are the model multipliers. The horizontal dashed lines are our empirical point estimates of the multipliers. The model parameters are as in the calibration of the baseline model. Persistence of pessimistic sunspot is set to 0.875.

Figure A1: Output Multiplier during Recessions and Expansion

Cumulative Multiplier of Output: Different recession dates

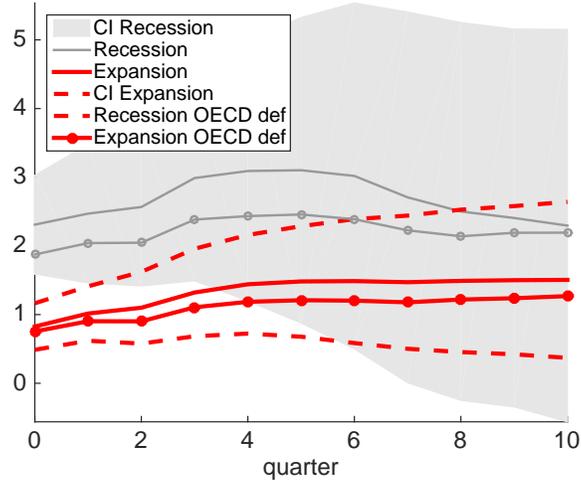


Figure A2: Total Government Spending and its Components in Japan

