

Costly Contracts and Consumer Credit*

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Abstract

This paper explores the implications of technological progress in consumer lending. The model features households who differ in endowment risk. To offer a lending contract, an intermediary incurs a fixed cost. Each lending contract is comprised of an interest rate, a borrowing limit and a set of eligible borrowers. Technological improvements which lower the cost of offering a contract lead to an increase in the number of contracts offered. This leads to increased risk based pricing and extension of credit to riskier households. This in turn leads to increased defaults and borrowing. We also extend the model to consider the implications of improved credit technology which allow lenders to better differentiate between borrowers with different default risks. To corroborate the predictions of the model, we examine data on the distribution of credit card interest rates reported by households in the Survey of Consumer Finance. We find that the number of different credit card interest rates reported increases over time. Strikingly, the empirical density of credit card interest rates has become much more disperse since 1983.

Keywords: Consumer Credit, Endogenous Financial Contracts, Bankruptcy.

JEL Classifications: E21, E49, G18, K35

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1 Introduction

This paper explores the implications of improvements in consumer lending technology for unsecured consumer borrowing. The past thirty years has witnessed substantial innovations in credit markets and changes in consumer borrowing. The rapid spread and increased usage of credit cards has been accompanied (driven?) by a rapid increase in statistical tools such as credit scoring to price loans. These changes in lending technology have been accompanied by a dramatic increase in personal bankruptcies from 1.4 per thousand of the working age population in 1970 to 8.5 in 2002, with virtually all of the increase occurring between 1980 and 2000. Unsecured consumer borrowing has also increased significantly, with the rise driven largely by a rapid increase in credit card borrowing.

We explore the implications of innovations in financial markets for two reasons. First, there is substantial evidence of significant technological change in consumer credit markets (Barron and Staten (2003), Berger (2003), Evans and Schmalensee (1999)). Second, a number of authors have argued that the diffusion and increased use of credit cards have played a key role in the rise in bankruptcy and unsecured consumer borrowing (White (2007), Ellis (1998)). This argument has been buttressed by the recent findings from quantitative incomplete market models of bankruptcy that changes in the supply of credit appear to have played a significant role in the rise of bankruptcies (Athreya (2004), Livshits, MacGee, and Tertilt (2007a)). Livshits, MacGee, and Tertilt (2007a) argue that a rise in income and expense (such as uninsured medical expenses) plays a small role in accounting for the rise in filings and unsecured credit. Instead, they find that a decline in the cost (stigma) associated with bankruptcy together with a decline in the transactions cost of borrowing can account for both the dramatic increase in consumer bankruptcy filings and increased unsecured borrowing by consumers.

One limitation of the existing literature is that it relies on reduced form ways of modeling financial innovation. For example, Athreya (2004) and Livshits, MacGee,

and Tertilt (2007a) model financial innovation as impacting consumer lending via two adhoc channels: a fall in the cost of bankruptcy and reduced transaction cost of lending. This theory has relatively little to say about how improved information technology may have facilitated the extension of credit to riskier borrowers, or led to more accurate pricing of borrowers default risk (Barron and Staten (2003)). This also means that the existing literature has relatively little to say about how (or if) recent technological changes have impacted different consumers with different income or risk characteristics. Further, it limits the extent to which existing theory can help to address policy issues related to recent suggestion of some policy-makers to legislate tighter lending standards. We thus believe that a closer examination of the nature and implications of recent changes in consumer credit markets is needed to both better understand its role in the rise of unsecured borrowing and bankruptcies as well as to asses the welfare consequences of these innovations.

To address this question, we undertake two (related) tasks in this paper. First, to better understand changes in consumer credit, we assemble extensive data on the number of unsecured consumer credit contracts targeted at specific types (groups) of borrowers. To measure the number and distribution of credit contracts across consumers we look at two specific features of credit contracts: the interest rate and credit limit. We pay particular attention to the distribution of credit card interest rates. Using data from the Survey of Consumer Finance, we document a large increase in the number of different credit card interest rates reported by households since 1983. More strikingly, the empirical density of credit card interest rates has become much “flatter” since 1983. While in 1983 nearly 55% of households reported they faced the same credit rate (18%), by the late 1990s no single credit card interest rate was reported by more than 15% of households. We also document a similar pattern in data on interest rates for 24-month consumer loans and credit cards from surveys of banks conducted by the Board of Governors. These shifts in the distribution of interest rates have also been accompanied by increased lending to lower income households.

The second task we tackle is to develop a simple model so as to analyze the qual-

itative implications of two mechanisms via which improved information technology may have impacted credit markets. The first mechanism is that technological innovation (such as computers) reduced the cost of designing and marketing financial contracts. The underlying story we have in mind is that for each contract (which we take to be an interest rate and a credit limit) lenders must identify the characteristics of households to accept as clients. This fixed cost of creating contracts means that some pooling of different risk types is optimal. Improvements in information technology which lower the cost of designing these contracts should lead to more contracts being offered, each of which is targeted at smaller subsets of the population. The second mechanism we explore is that improved information technology reduced adverse selection problems by improving the ability of lenders to predict prospective borrowers default risk, which facilitated the expansion of credit. Explicitly modeling these channels enables us to both better understand the exact mechanism through which financial progress affects bankruptcies and provides predictions which we can compare to the data to allow a better assessment of the channels.

The model environment features borrowers who differ in their endowment risk. Borrowers live for two periods, and have a stochastic endowment in the second period. To offer a lending contract, an intermediary incurs a fixed cost. Each lending contract is comprised of a bond price (interest rate), a borrowing limit, and a set of consumers who are eligible for the contract. There is free entry into the credit market, so that in equilibrium each contract earns zero profit. We first analyze the special case where intermediaries know the exact type (default risk) of each consumer. We characterize the set of contracts and the set of consumers with access to credit and then analyze how this changes as the fixed cost declines. Next, we relax the assumption that lenders know a borrowers type by introducing a public signal σ_i of a household's type. We assume that with probability α , this signal is accurate and with complementary probability $1 - \alpha$ the signal is a random draw from the distribution of households types. This creates an adverse selection problem. We explore the implications of improvements in the accuracy of this signal for the set of contracts

offered in equilibrium.

We show that this environment generates a finite set of contracts. This is driven by the assumption that there is a fixed cost of contracts which implies that *some* “pooling” is optimal. A pooling contract offers cost savings per borrower, since the fixed cost can be shared. The cost of pooling is that different risk types face the same interest rate and credit limit, which means that the lower risk types cross-subsidize higher risk types by paying a disproportionate share of the fixed cost. With free entry of intermediaries, these two forces lead to a finite set of contracts for any (strictly positive) fixed cost. We also show that this characterization holds when we introduce some private information about households true risk type.

We find that technological improvements which lower the cost of offering a contract lead to an increase in the number of contracts offered. The increase in the number of contracts leads to the extension of credit to riskier households. This generates more unsecured borrowing and an increase in defaults, since the “new” borrowers are more likely to default. Risk based pricing is increased as the measure of households served by each contract shrinks, which reduces the extent of cross-subsidization. We further find that technological improvements which make signals about a borrowers type more accurate also lead to an increase in the number of contracts and to more risk based pricing.

The model also generates an interesting insights into the possible relationship between the risk free interest rate and the average borrowing interest rate. In an influential paper, Ausubel (1991) documented that the decline in the risk-free rate in the U.S. in the 1980s was not accompanied by a decline in the average credit card rate. This led to a debate over whether or not the credit card industry was not competitive. We show in our model that a decline in the risk free rate can sometimes lead to higher average borrowing interest rate. The mechanism is that a decline in the risk free rate makes borrowing more attractive, and can thus lead to an increase in the number of contracts offered in equilibrium. Since new contracts are offered to riskier borrowers, the average borrowing interest can increase if the average risk

premium on borrowing increases by more than the fall in the risk-free rate.

Our theoretical model is closely related to and builds on the classic contribution by Jaffee and Russell (1976) who first examined the problem of existence of equilibrium when people have private information about their type and competitive banks can offer pooling and separating contracts. Our equilibrium concept (which we largely formalize in the timing of the lending game) builds on work by Hellwig (1987), who discusses under what condition (pooling) equilibria exist in environments similar to ours.

The equilibrium model of bankruptcy that we use is related to recent work on equilibrium models of consumer bankruptcy.¹ Both Livshits, MacGee, and Tertilt (2007b) and Chatterjee, Corbae, Nakajima, and Rios-Rull (2005) outline dynamic equilibrium models where interest rates vary with borrowers' characteristics, and show that for reasonable parameter values, these models can match the level of U.S. bankruptcy filings and debt-income ratios. In recent work, Chatterjee, Corbae, and Rios-Rull (2007) and Chatterjee, Corbae, and Rios-Rull (2006) present the first formal model of the role of credit histories and credit scoring in supporting the repayment of unsecured credit. Closely related to the story we explore is work by Narajabad (2006), who also argues that improvements in information technologies have led to an extension of credit to riskier borrowers. He formalizes this mechanism in a model without adverse selection, since he assumes that consumers do not know their own riskiness, while lenders see a noisy signal on a borrowers type. In a relevant empirical contribution, Edelberg (2006) examines PSID and SCF data and finds that the risk-based pricing of consumer loans has increased over the past twenty years.

The remainder of the paper is organized as follows. Section 2 documents technological progress in the financial sector over the last couple decades, while Section 3 examines data on the terms of consumer unsecured borrowing (especially interest rates). Section 4 sets up the general model. In Section 5 we characterize the set of equilibrium contracts, while in Section 6 we show how a decline in the fixed

¹See Athreya (2005) for a more detailed survey.

cost changes the set of contracts. We extend the model to analyze a second type of technological progress in Section 7. Section 8 concludes.

2 Financial Innovation

The past thirty years have witnessed the diffusion and introduction of numerous innovations in consumer credit markets (Mann (2006)). Since most of the expansion of unsecured consumer credit happened in form of revolving credit, the evidence we present in this section focuses mostly on innovations related to credit cards. Crucial innovations in the credit card industry include the following:

- Increased use of computers to process information to facilitate customer acquisition, designing credit cards, marketing, as well as monitoring repayment, and debt collection.
- The development of improved credit-scoring techniques to identify and then monitor creditworthy customers, during the 1970s. The most prominent player here being the FICO score developed by Fair Isaac Cooperation.²
- Increased securitization of credit card debt (starting in 1987).

Many of these changes are related to the rapid improvements in information technology, which has significantly reduced the cost of processing information and led to large increases in information sharing on borrowers between financial intermediaries (Barron and Staten (2003), Berger (2003), Evans and Schmalnsee (1999)). It has been argued that this has increased the analysis of the relationship between borrower characteristics and loan performance by lenders to better price loans (Barron and Staten 2003). However, not all technological progress in the financial sector was

²Fair Isaac started building credit scoring systems as early as the late 1950s. However, the first credit card scoring system was not delivered until 1970. In 1975 Fair Isaac introduced the first behavior scoring system to predict credit risk related to existing customers. In 1981 the Fair Isaac credit bureau scores were introduced. For details see: http://en.wikipedia.org/wiki/Fair_Isaac

directly related to a better assessment of credit risk. Other innovations took place that simply increased the efficiency of designing credit cards, marketing credit cards, and processing accounts. Some of this progress can be thought of a reduction in the cost of credit per account, while other parts may be interpreted as a reduction in the fixed cost of entering this market with a differentiated product. Below we review the evidence on each of these innovations. It should be pointed out that despite a broad descriptive literature on financial innovation, there are very few empirical studies documenting the extent of it quantitatively.³ This might be partly due to the lack of good empirical measures of financial innovation. Also, some financial innovation came in the form of very discrete new ideas which are hard to quantify. Below, we give a survey of the evidence on technological progress in this industry, distinguishing explicitly between information-related progress and other productivity increases. We summarize some facts in Table 1.

Information Technology

There are two direct pieces of evidence that suggest that technological innovations related to shifts in the cost of processing information have diffused and become widely used. First, there was a substantial spread of credit scoring throughout the consumer credit industry during the 1980s and 1990s (McCorkell (2002), Engen (2001), Asher (1994)).⁴ The diffusion of credit scoring is reflected in usage figures reported by the American Banking Association (ABA). The fraction of large banks using credit scoring as a loan approval criteria increased from half in 1988 to nearly seven-eighths in 2000, and the fraction of large banks using fully automated loan processing (for direct loans) increased from 12 percent in 1988 to nearly 29 percent in 2000 (Installment Lending Report 2000). While larger banks are more likely to adopt credit scoring

³Frame and White (2004) in a recent survey of the literature on financial innovation noted that: “A striking feature of this literature [...] is the *relative dearth of empirical studies* that [...] provide a quantitative analysis of financial innovation.”

⁴Credit scoring is the evaluation of the credit risk of loan applicants using historical data and statistical techniques ((Mester 1997)). Credit scores are used both to evaluate initial loan applications, and to adjust the interest rates and credit limits of revolving (credit card) debts (up and down).

than smaller banks (Berger (2003)), banks of any size can access this technology by purchasing scores from other providers.⁵ Mester (1997) cites several case studies which document a decrease in the time and cost required to evaluate loan applications.

Several authors have argued that the development and spread of credit scoring was necessary for the growth of the credit card industry (Evans and Schmalensee (1999), Johnson (1992)). Barron and Staten (2003) argue that credit cards companies during the early 1990s rapidly expanded their use of risk based pricing, which led to substantial declines in interest rates for low risk customers and increased interest rates for higher risk consumers.

The second (related) piece of direct evidence is the rapid increase in information on borrowers that is currently collected by credit bureaus and purchased by lenders. For every credit-using person in the United States, there is at least one (more likely three) credit bureau files (Hunt 2002). The information in these files is widely used by lenders, as more than 2 million credit reports are sold by credit bureaus in the U.S. daily (Riestra (2002)).⁶ The rapid growth in the number of credit reports issued in the U.S. is striking. The number of consumer credit reports increased from 100 million in 1970 to 400 million in 1989, to more than 700 million today. This reflects the widespread adoption of credit scoring to evaluate loan applicants.

Market Segmentation and Product Differentiation

Credit contracts are a differentiated product, where each product is tailored to a specific segment of the market. One reason for observing this market segmentation would be a fixed cost of designing a particular credit product (contract). In fact, there are several reasons to believe that there is a fixed cost in the design and marketing process of credit cards. Moreover, over time, credit contracts have become more

⁵Further support for the significant impact of credit scoring on lending comes from studies of small business lending. Frame et al (2001) find that the adoption of credit scoring by banks to evaluate small business loans led to lending, while Berger, Frame and Miller (2002) found that credit scoring led to the extension of credit to "marginal applicants" at higher interest rates.

⁶In Canada and the U.S., credit bureaus report data on borrowers payment history, the stock of current debt and any public judgments (such as bankruptcy).

Table 1: Measures of Technological Progress in the Financial Sector

Measure of Innovation	then	and now	Source
Credit scoring as loan approval tool*	50% (1988)	85% (2000)	ILR 2000
Credit Reports issues	400 million (1989)	700 million (2001)	Riestra (2002)
Number of Payment Cards	660.6 mill (1991)	1,135.5 mill (2004)	Fed [§]
Credit Card Payments	10.4 billion (1995)	15 billion (2000)	Berger (2003)
Fully automated loan processing*	12% (1988)	29% (2000)	ILR 2000
banks that offer internet banking	37.3% (2000)	49.7% (2001)	Berger (2003)
Mail solicitations	1.1 billion (1990)	5.23 billion (2004)	Synovate**
credit card offer response rate	2.1% (1990)	0.4% (2004)	Synovate**
Securitization as a share of all credit card balances held by banks	26.7% (1991)	48.3% (2005)	Fed [§]

* for large banks

* *Mail Monitor*, Synovate, as cited in Federal Reserve Board (2006).

§ Federal Reserve Board (2006)

and more differentiated (i.e. tailored to finer and finer segments of the population) – as we will show in the next section. One potential reason for this increase in segmentation is a decline in the fixed cost – a hypothesis we are pursuing in our theoretical analysis. Unfortunately, direct evidence on the decline in such a fixed cost is hard to obtain – other than by pointing to general evidence on productivity increases in the credit sector. Below we briefly discuss the fixed costs involved in the consumer credit industry. To the extent that productivity increased in those industries that deliver these services, it seems plausible to believe that these fixed costs have been falling over time.

A product in the consumer loan industry is “a collection of loans or lines of credit governed by *standard terms and conditions*.” (Lawrence and Solomon (2002, p. 23)). Developing such a product is costly. According to Lawrence and Solomon (2002), a prominent industry handbook, the following steps are involved in product develop-

ment: selecting the target market, researching the competition in the target market, designing the terms and conditions of the product, (potentially) testing the product⁷, brand creation through advertisements, point-of-sale promotions and mass mailings, forecasting profitability, preparing a formal documentation of the product, an annual formal review of the product, and providing well-trained customer service tailored specifically to the needs of the product. It should be fairly obvious, that, to a large extent, these costs are fixed for each product, rather than a function of the number of loans. Even after the initial product launch, account maintenance requires additional fixed costs, such as customer data base maintenance, costs involved in changing in the terms of the product, etc.

A similar process is also described by Siddiqi (2006), who explains the development of credit risk *scorecards*. A scorecard is a mapping from individual characteristics to a risk score for a particular subset of the population. There are “generic scorecards” that can be purchased by small issuers which do not have sufficient data to conduct their own statistical analysis. Large issuers develop their own “custom scorecards” based on data from their own customers.⁸ Because of industry change as well as changes to the overall economic environment, scorecards are constantly updated (a scorecard is usually developed on data that’s up to two years old), i.e. there is not one “true” mapping that once developed becomes a public good, as one might have thought. The individual steps (and costs) involved in scorecard development, such as data acquisition, data mining, etc., are described in details in Siddiqi (2006).⁹

⁷This involves the actual testing costs plus the delay induced by testing. Note that a typical testing period in this industry is eighteen month (Lawrence and Solomon 2002).

⁸Some financial firms build their own custom scorecards based on purchased data. Firms that offer market research services to financial companies include CLARITAS Marketing Solutions and HSBC Retail Services.

⁹The book gives an example about a financial company that outsourced scorecard development and purchased ten different cards at an average cost of \$27,000 a card.

3 Unsecured Consumer Credit Facts

The rise in unsecured (especially credit card) consumer borrowing and personal bankruptcies over the past thirty years is well known and widely documented (Athreya (2004), Livshits, MacGee, and Tertilt (2007a)). However, much less is known about whether these changes have been accompanied by significant changes in the distribution of borrowing, consumer credit contracts and access to credit across households. In this section, we tackle this issue and document several changes in the distribution of the terms at which households access consumer credit. The facts we document both motivate and provide the backdrop with which we evaluate the predictions of the theoretical model we develop and analyze in Section 4.

Given our interest in unsecured credit, we focus primarily upon the terms at which consumers can access credit cards. We pay particular attention to the credit card market for several reasons. First, credit card borrowing currently accounts for the majority of unsecured borrowing in the United States and has increased dramatically over the past 30 years. Second, credit cards are a relatively recent innovation which have become widely used over the past thirty years. While the first bank credit cards were issued during the mid 1960s, by the early 1990s more than 6,000 US institutions issued general purpose credit cards (Canner and Lueckert (1992)). Another reason to focus on credit cards is that the cost structure of credit card issuers differs substantially from that of other lenders. Canner and Lueckert (1992) report that operating costs accounted for nearly 60 percent of the costs of credit card operations, but less than 20 percent of mortgage lending. This suggests that technological innovations may have a much larger impact on credit card operations than on other consumer lending.

We examine three dimensions of consumer credit: access (whether a consumer has a credit card), the interest rate and the credit limits offered to borrowers. While credit cards do vary along other dimensions, we abstract from these potential differences both due to data limitations and since we view interest rates and credit limits as the

most important features of credit contracts.¹⁰

Before examining shifts in the distribution across consumers, we review the trends in the mean level of credit card limits, borrowing and interest rates in means from the Survey of Consumer Finances. The SCF asks whether a household has a credit card, and the amount borrowed. The SCF also reports the interest rates for the primary card used to borrow on in 1983, and in 1995 and subsequent surveys. In addition, the SCF also contains data on the outstanding balance and (starting in 1989) on the credit limit on credit cards.

As can be seen from Table 2, credit cards became more widely held over the 1980s and 1990s. This was accompanied by little change in the fraction of people with cards who used their card to borrow funds, as well as an increase in the average credit limit and outstanding balance over time. The data suggests that the growth in the fraction using credit cards and in credit limits relative to income appear to have leveled off since 1998. The data also indicates that the average (nominal) interest rate on credit card borrowing has declined significantly.

Table 2: Mean Values of Limits and Interest Rates Credit Cards, SCF

Variable	1983	1989	1995	1998	2001	2004
Have CC	43%	56%	66%	68%	73%	72%
CC Bal > 0	51%	52%	56%	55%	54%	56%
Credit Limit	NA	7,077	10,366	12,846	13,552	15,424
Credit Limit/Income	NA	0.19	0.34	0.41	0.37	0.41
Balance (all HH)	497	952	1,340	1,695	1,452	1,860
Balance (HH bal > 0)	971	1,828	2,393	3,096	2,706	3,312
Int Rate (all HH)	18.05%	NA	14.51%	14.46%	14.36%	11.49%
Int Rate (HH bal > 0)	18.08%	NA	14.14%	14.48%	14.20%	11.81%

Source: Survey of Consumer Finance. Values are in constant 2004 U.S. \$, deflated using the CPI.

¹⁰The extent to which credit cards provide cash back on purchases, purchase insurance, insurance on car rental, etc, also appear to have varied over time. For example, in the late 1970s, annual fees became common, while during the late 1980s and early 1990s many cards removed or reduced annual fees and introduced benefits such as travel or car rental insurance (Canner and Luckett (1992)).

The aggregate trends may mask significant shifts in the distribution of access to unsecured borrowing across households. To address this, we look at data from the Survey of Consumer Finance on the distribution across households. The distribution of interest rates across lenders should also provide useful information about the distribution of terms of borrowing facing households. This leads us to examine data on interest rates offered by charged by banks and credit card issuers collected by the Federal Reserve Board. Based on these data sources, we document three important facts:

- Increased variety in credit contracts
- More risk-based pricing
- Increased access by lower income households

3.1 Increased Variety in Consumer Credit Contracts

The simplest measure of variety is the number of different products offered. The analog of a product in consumer credit markets is a credit contract, typically characterized by a loan size (or credit line) and an interest rate. The Survey of Consumer Finance asked questions about the interest rate paid on credit card accounts, which we use to count the number of *different* interest rates. The data reported in Table 3 shows a substantial increase in variety, with the number of different rates roughly tripling between 1983 and 2004.¹¹

A more nuanced view of variety can be gained by examining the variance of interest rates across households. Since we are comparing trends in dispersion of variables with different (and changing) means, we compute the coefficient of variation (CV).¹²

¹¹It is worth emphasizing that this measure likely significantly understates the increased variety of credit card contracts, as both Furletti (2003) and Furletti and Ody (2006) argue that credit card providers have made increased use of features such as annual fees, different penalty fees for late payments and other features such as purchase insurance to provide differentiated products.

¹²This is important for two reasons. First, variables such as credit limits should increase over time

Table 3: Number of Different Credit Card Interest Rates, SCF

Year	All Households	Households with Positive Credit Card Debt
1983	78	47
1995	142	118
1998	136	115
2001	222	155
2004	211	145

Source: Survey of Consumer Finance.

Table 4 reports the CV for the interest rate, credit limit, and actual balance for six different waves of the SCF. We find a substantial increase in the variability of credit interest rates across households over time: the CV in interest rates almost triples during the 1983-2004 time period. On the other hand, there is little evidence of a trend in heterogeneity in credit limits and balances over time, although there has been an increase in the dispersion of credit limits relative to income and balances of households who borrow on their credit cards. However, in terms of levels, both credit limits and borrowing are more disperse than interest rates.

Table 4: Coefficient of Variation of Different Credit Card Interest Rates, SCF

Variable	1983	1989	1995	1998	2001	2004
Credit Limit	NA	1.60	1.33	1.45	1.64	1.49
Credit Limit/Income	NA	1.27	1.60	1.85	1.53	1.82
Int Rate (all HH)	0.22	NA	0.30%	0.32%	0.37%	0.56%
Int Rate (HH bal > 0)	0.21	NA	0.32	0.35	0.40	0.56
Balance (all HH)	1.80	2.22	2.28	2.35	2.87	2.29
Balance (HH bal > 0)	1.08	1.45	1.58	1.60	1.99	1.59

Source: Survey of Consumer Finance. Standard deviations are weighted.

The increased dispersion of borrowing interest rates can also be seen from the decline in nominal interest rates due to growth in real GDP. Second, the decline in nominal interest rates has shifted down mean borrowing interest rates, which will show up as a decline in the variance of interest rates.

lenders' side. Data collected by the Board of Governors directly from banks suggests a similar increase in interest rate dispersion.¹³ We use both information on interest rates on 24-month consumer loans from a bank survey as well data on credit card interest rates from a survey of credit card issuers starting in 1990. The data has to be interpreted with caution, since every bank is asked to report only *one* interest rate (the most commonly used one) and hence likely understates the number of loan options faced by consumers.¹⁴

We find a large increase in the dispersion of interest rates. As can be seen from Figure 1, the CV for 24 month consumer loans increases from roughly 1.5 in the early 1970s to about 3.0 by the late 1990s. A similar increase over time also occurs in credit cards. This finding is consistent with increased banks specialization in different segments of the market.

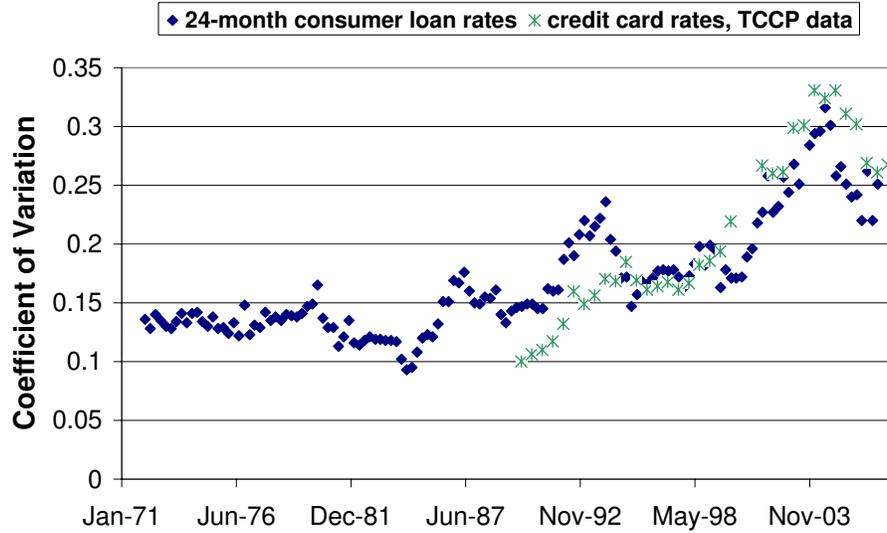
Even more details about shifts in the terms of borrowing across households over time can be gleaned from changes in the empirical distribution of interest rates across households. Figure 2 displays the fraction of households reporting different interest rates in the SCF (essentially, a normalized histogram) for two different years: 1983 and 2001. This figure clearly shows the increase in interest rate dispersion between these two cross-sections. It is striking that in 1983 more than 50% of households faced a rate of *exactly* 18%. The distribution in 2001 is strikingly “flatter” than the 1983 distribution (the comparison with other years is similar). A very similar figure also emerges for the distribution of interest rates offered by different banks (not reported here).

Another feature of the increased dispersion is that although the average (nominal)

¹³We use data from the Quarterly Report of Interest Rates on Selected Direct Consumer Installment Loans (LIRS) and the Terms of Credit Card Plans (TCCP). See the Appendix for a more detailed discussion of these data.

¹⁴To take the most extreme case, each bank could offer a large menu of interest rates and the menu itself could be expanding over time, yet, the most common rate could be identical across banks and unchanging over time. In this case, looking at the variation of interest rates across banks one could erroneously conclude that interest rate variety was constant.

Figure 1: CV Consumer Interest Rates

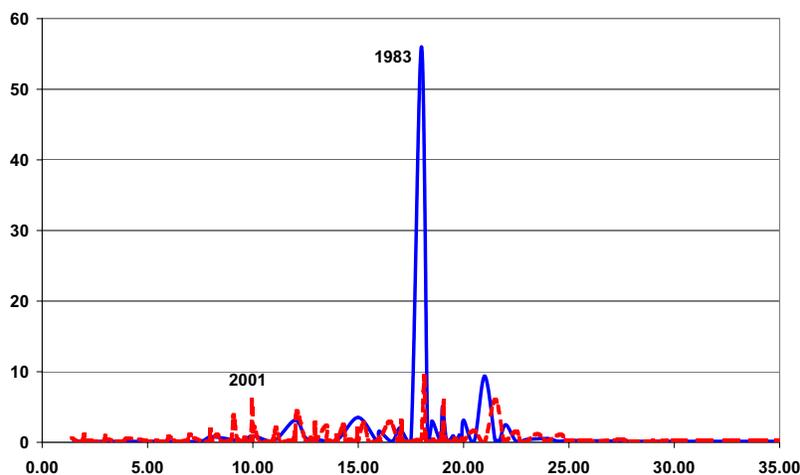


interest rate has declined over time, the maximum rate charged by banks has actually increased, as can be seen from Figure 3. This points towards an expansion of credit to riskier households.

To summarize, the evidence suggests three important changes in credit and interest heterogeneity during the last two decades of the 20th century:

1. An increase in interest rate heterogeneity (both across banks and consumers):
 - (a) the number of different borrowing interest rates has gone up
 - (b) an increase in the dispersion of borrowing interest rates
2. A “flattening” of the distribution of borrowing interest rates (both across banks and consumers).
3. Increased spread between the minimum and maximum borrowing interest rates

Figure 2: Credit Card Interest Rate Distribution

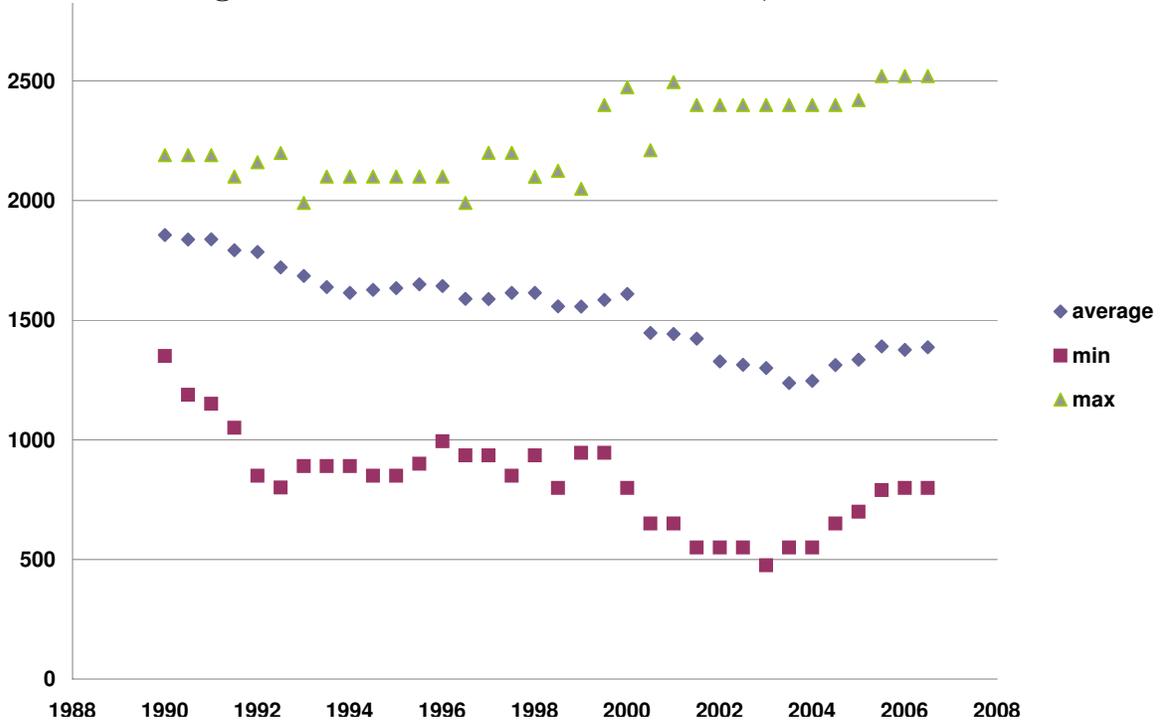


3.2 Risk Based Pricing

One coarse way of seeing whether the dispersion of interest rates is related to increased risk based pricing is to compare the distribution of interest rates of delinquent and non-delinquents. The SCF asks households if they have been delinquent on a debt payment in the past year. Delinquency on debt is positively correlated with the probability of future default, so that delinquent households should be riskier than non-delinquents. As can be seen from Figure 4 in 1983, the distributions for delinquents and non-delinquents was nearly identical. However, by 2001, the delinquent distribution has considerable mass to the right of the non-delinquent interest distribution. This supports the view that the increase in credit card contracts has led to more accurate pricing of borrowers default risk.

We are not the first to document an increase in risk-based pricing. For example, Edelberg (2006) combines data from the PSID and the SCF, and finds that lenders have become better at identifying higher risk borrowers and made increased use of risk based pricing. The timing of the change also coincides with the observation that

Figure 3: Credit Card Interest Rates, TCCP Data

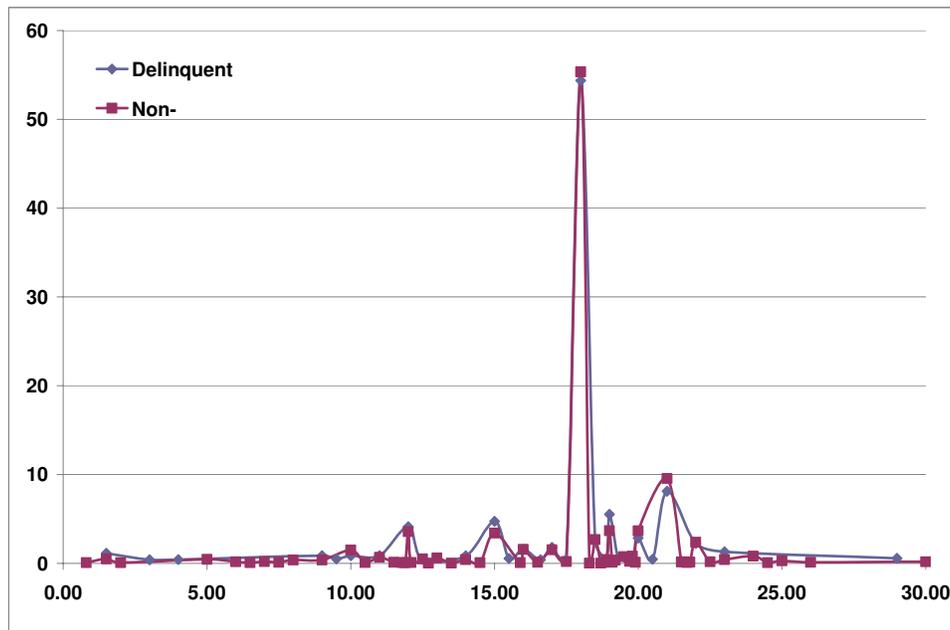


in the late 1980s some credit card banks began to offer more different credit card plans “targeted at selected subsets of consumers, and many charge[d] lower interest rates” (Canner and Luekett (1992)). The rise in risk-based pricing is also consistent with the entry and expansion of monoline lenders such as Capital One base their business plan on targeting specific sub-groups of borrowers with credit card plans priced on their risk characteristics (Mann (2006)). Furletti and Ody (2006) report that credit card issuers make increased use of fees as ways to impose a higher price on riskier borrowers.

3.3 Increased Access/Borrowing on Credit Cards by Lower Income Households

Finally, there is evidence that lower income and riskier households have increased access to unsecured credit. The increased access to unsecured credit of lower income

Figure 4: Delinquency and Credit Card Interest Rates 1983



groups can be seen directly from data on the fraction of each income quintile with a credit card. Table 5 reports the fraction of each income quintile who have a credit card and the fraction of those with a credit card who use them to borrow. The table shows the well known fact that credit card penetration increased most rapidly for lower income households during the 1980s and 1990s.

The increase in the number of lower income borrowers has been accompanied with a significant increase in their share of total credit card debt outstanding. Figure 6 graphs the cdf for the share of total credit card balances held by various percentiles of the earned income distribution. As can be seen from the graph, the fraction of credit debt held by lower income households has increased significantly over the past twenty years. For example, the fraction of debt held by the bottom 30% (50 %) of the earnings distribution nearly doubled from 6.1 % to 11.2% (16.8 % to 26.6%). Given that the value of total credit card debt also increased, this figure implies that lower income household access (and use) of credit card debt has increased significantly.

Figure 5: Delinquency and Credit Card Interest Rates 2001

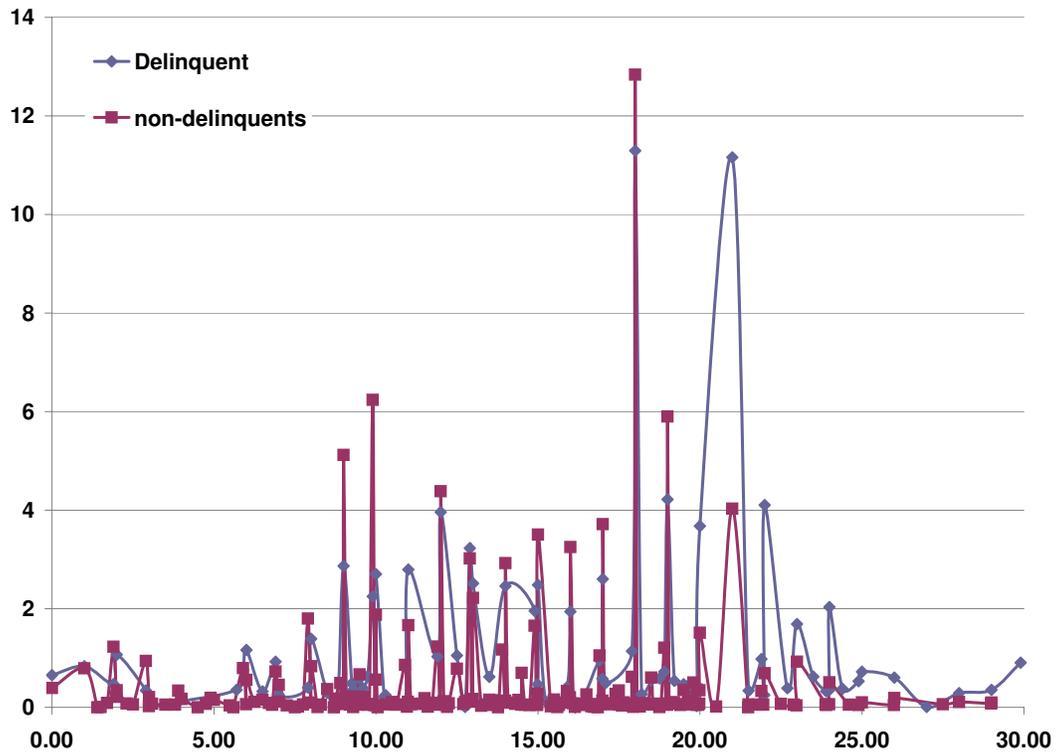


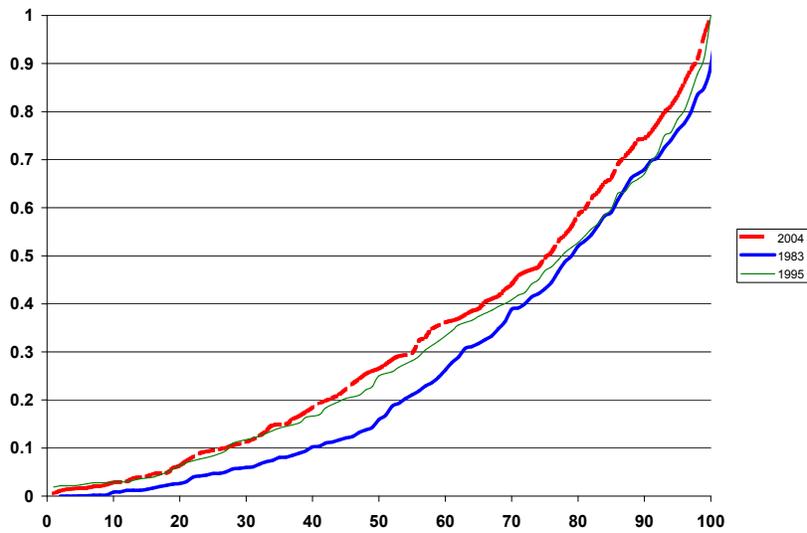
Figure 6 is consistent with the conclusions of numerous papers (for example, see Black and Morgan (1999), Kennickell, Starr-McCluer, and Surette (2000), Durkin (2000)) that the most rapid increase in credit card usage and debt has been among the poorest households. To the extent that lower income groups are riskier, this evidence suggests that borrowing by riskier households has increased over the 1983 - 2004 period.

Table 5: Percent HH with Bank Credit Card, U.S.

Income Quint	1983	1989	1995	1998	2001	2004
Lowest	11%	17%	28%	29%	38%	38%
Balance	40%	43%	57%	59%	60%	61%
2 nd Lowest	27%	36%	54%	58%	65%	61%
Balance	49%	46%	57%	58%	59%	60%
Middle	41%	62%	71%	72%	79%	77%
Balance	58%	56%	58%	58%	61%	64%
2 nd Highest	58%	64%	84%	86%	88%	87%
Balance	55%	60%	60%	60%	55%	57%
Highest	79%	82%	95%	95%	95%	96%
Balance	47%	46%	50%	45%	38%	44%

Source: Survey of Consumer Finance.

Figure 6: CDF Credit Card Borrowing vs Earned Income



4 Model Environment

We analyze a two period small open economy with incomplete markets. The economy is populated by a continuum of borrowers each of whom faces stochastic income in period 2. Markets are incomplete in that only non-contingent contracts can be issued. Borrowers can default on contracts and incur exogenous costs associated with bankruptcy. Financial intermediaries are competitive and have access to funds at an exogenously given (risk-free) interest rate. The creation of each financial contract (characterized by a lending rate, a borrowing limit and eligibility requirement for borrowers) requires the payment of a fixed cost χ .

4.1 People

The economy is populated by a continuum of 2-period lived households. In the benchmark economy, we assume that borrowers are risk-neutral, with preferences represented by:

$$c_1 + \beta E c_2$$

Each household receives the same deterministic endowment of y_1 units of the consumption good in period 1. The second period endowment, y_2^i , is stochastic. The endowment can take on one of two possible values: $y_2^i \in \{y_h, y_l\}$, where $y_h > y_l$. Households differ in their probability ρ_i of receiving the high endowment y_h . The expected value of income of household i is

$$E_i y_2 = (1 - \rho_i) y_l + \rho_i y_h$$

We identify the households with their type ρ_i . ρ is distributed uniformly on $[a, 1]$, where $a \geq 0$. Households know their own type.

4.2 Signals

While each household knows their own type, other agents can only observe a public signals, σ_i , regarding household i 's type. With probability α , this signal is accurate:

$\sigma_i = \rho_i$. With complementary probability $(1 - \alpha)$, the signal is an independent draw from the ρ distribution ($U[a, 1]$). Thus, α is the precision of the public signal.

4.3 Bankruptcy

There is limited commitment by borrowers. We model this as a bankruptcy system, whereby borrowers can declare bankruptcy in period 2. The cost of bankruptcy to a borrower is the loss of fraction γ of the second-period endowment. Lenders do not recover any funds from bankrupt borrowers.

4.4 Financial Market

Financial market for borrowing and lending is competitive. Financial intermediaries can borrow (or save) from the (foreign) market at the exogenously given interest rate r . Financial intermediaries accept deposits from savers and make loans to borrowers. Loans take the form of one period non-contingent bond contracts. However, the bankruptcy option introduces a partial contingency by allowing bankrupts to discharge their debts.

Throughout the paper, we will assume that $\beta < \frac{1}{1+r}$, so that households want to borrow as much as possible (at actuarially fair prices), and never want to save. What limits the households' ability to borrow is their inability to commit to repaying loans.

Financial intermediaries must incur a fixed cost χ in order to offer a non-contingent lending contract to (an unlimited number of) households. Endowment-contingent contracts are ruled out (due to un-verifiability of the endowment realization). A contract is characterized by $(L, q, \underline{\sigma})$, where L is the face value of the loan, q is the per-unit price of the loan (so that qL is the amount advanced in period 1 in exchange for promise to pay L in period 2), and $\underline{\sigma}$ is the minimal value of public signal that makes a household eligible for the contract.¹⁵

¹⁵Alternatively, we can specify the contract as just (L, q) and have the eligibility set (characterized by $\underline{\sigma}$) be an equilibrium outcome.

Intermediaries observe the public signal about a household's type, but not the actual type. Households are allowed to accept only one contract, so the intermediaries know the total amount being borrowed. Intermediaries forecast the default probability of loan applicant, and decide to whom to grant loans.

Profit maximization implies that intermediaries never offer loans to types on which they would make negative expected profits, which implies that the expected value of repayments cannot be lower than the cost of the loan to the intermediary.

In equilibrium, free entry implies that intermediaries earn zero expected profits on their *loan portfolio*. The bond price incorporates the fixed cost of offering the contract, so that in equilibrium the operating profits of each contract equal the fixed cost.

4.5 Timing

The timing of events in the financial markets is as follows:¹⁶

- 1.a. Intermediaries pay fixed costs χ of entry and announce their contracts. While this stage can be modelled as simultaneous move game, we prefer to think of it as sequential – the stage does not end until no new intermediary wants to enter (having observed the contracts already being offered).
- 1.b Households observe all offered contracts and choose which one to apply for (realizing that some intermediaries may choose to exit the market).¹⁷
- 1.c Intermediaries, who paid the entry cost, decide whether to stay in the market and advance loans to qualified applicants or to exit the market.¹⁸

¹⁶This timing is necessary for the existence of (partially) pooling equilibria in the environment with imperfect public signals, as in (Hellwig 1987).

¹⁷To simplify the analysis, we could introduce ϵ cost of sending an application, so that each household applies only for a single contract which will be offered in equilibrium.

¹⁸This stage is not necessary in the environment with perfect signals (Section 5) but is essential to ensure existence of equilibria under asymmetric information.

- 1.d Loans are advanced to qualified applicants by lenders who remain in the market.
We can further split this stage into two sub-stages: Successful applicants are notified, and then they make their choice of lenders.
- 2.a Households realize their endowments in period 2, and make their default decisions.
- 2.b Non-defaulting households repay their loans.

4.6 Equilibrium

We defer the definition of equilibrium in this general environment (which involves specifying agents' beliefs on and off the equilibrium path) to Section 7. In the following section, we restrict attention to a simplified environment with perfect information, in which the equilibrium can be defined in a standard (non-game-theoretic) fashion.

5 Perfect Signals ($\alpha = 1$)

We begin by examining the (simpler) environment with complete information regarding households' risk types ($\alpha = 1$). In this environment, the key friction is that each type of lending contract requires a fixed cost χ to create. The number of households of a particular risk type is infinitesimal relative to this fixed cost. Thus, lending contracts have to pool several risk types to recover the fixed cost of creating the contract. This leads to a finite set of contracts being offered in equilibrium.

To save on notation in this section, we will set a , the lower bound on the probability of high income realization, to 0. That is, $\rho \sim U[0, 1]$.

5.1 Definition of Equilibrium

An *equilibrium*¹⁹ is a set of active contracts $\mathcal{K}^* = \{(q_k, L_k, \underline{\sigma}_k)_{k=1, \dots, N}\}$ and consumer contract decision rules $\kappa(\rho, \mathcal{K}) \in \mathcal{K} \cup \{(0, 0, 0)\}$ for each type ρ such that

1. Given $\{(q_k, L_k, \underline{\sigma}_k)_{k \neq j}\}$ and consumer contract decision rules, each (potential) bank j maximizes profits by making the following choice: to enter or not, and if it enters, it chooses contract $(q_j, L_j, \underline{\sigma}_j)$ and incurs fixed cost χ .
2. Given any \mathcal{K} , a consumer of type ρ chooses which contract (if any) to accept so as to maximize expected utility. Note that a consumer of type ρ can choose a contract k only if $\rho \geq \underline{\sigma}_k$.

5.2 Characterizing the Equilibrium

We begin by characterizing the face value of possible equilibrium contracts. In the model, contracts can vary along two key dimensions: the face value L , which the household promises to repay in period 2, and the per-unit price q of the contract.

Given the assumptions on the income process and the nature of contracts, the face values of equilibrium contracts are easily characterized. The key result is that all possible lending contracts are characterized by one of two face values. The risk-free borrowing contract has a face value equal to the cost of bankruptcy in the low income state, so households are always willing to repay this contract in equilibrium. Risky-lending contracts have the maximum face value such that in the high income state borrowers are always willing to repay. Contracts with lower face value are not offered in equilibrium since, if (risk-neutral) households are willing to borrow at a given price, they want to borrow as much as possible at that price. Formally:

Proposition 1: All contracts offered feature either

1. $L = \gamma y_l$ (risk-free)

¹⁹This is a description of a competitive equilibrium that comes out of the (sequential) game specified in section 4.5. For a full description of (sequential) equilibrium, which also includes the set of beliefs of all players (entrants and households) on and off the equilibrium path, see Section 7.

2. or $L = \gamma y_h$ (risky).

Thus, the key dimension along which contracts vary is the bond price. The variation in bond prices is accompanied by variation in the eligibility criteria for borrowers. We first outline the relationship between the terms of contract (q) and these eligibility criteria. Since the eligibility decision is made *after* the fixed cost has been incurred, lenders are willing to accept as clients any household which yields non-negative operating profits. Since households vary in their default probability, each lender offering a risky loan at a price q will have a cut-off rule: it will reject all applicants with risk type lower than the cut-off type $\underline{\rho}(q)$. This cut-off is such that the expected return from accepting the marginal borrower is zero:

$$\underline{\rho}(q)\bar{q}L = qL, \tag{5.1}$$

where $\underline{\rho}(q)\bar{q}L$ is the expected present value of repayment (since households only repay in the good state) and qL is the (present) value advanced to the borrower. This can be easily solved for the cut-off:

$$\underline{\rho}(q) = \frac{q}{\bar{q}} \tag{5.2}$$

Proposition 2: Every lender offering a risky contract at price q rejects an applicant *iff* the expected profit from that applicant is negative: Reject all $\rho < \underline{\rho}(q) = \frac{q}{\bar{q}}$.

This implies that the “riskiest” household accepted by a risky contract makes no contribution to overhead cost χ .

If a risk-free contract is offered in equilibrium, the eligibility set for that contract is unrestricted.

Households may potentially be able to choose from multiple contracts. Given a choice between multiple risky contracts, households always prefer the risky contract with the highest q that they are eligible for. This implies that households’ choice can be characterized as choosing between the best risky contract offered that will accept them, the risk-free contract and autarky (conditional on the risk-free contract and a

risky contract being offered to them in equilibrium). We formalize this acceptance decisions in three household participation constraints.

5.2.1 Household Problem and Participation Constraints

Households can potentially choose between three options: the best available risky contract, a risk-free contract and autarky. Note that for some households this choice is trivial since there may be no risky contracts offered in equilibrium which will accept them. The problem of a consumer of type ρ can be expressed as:

$$\max \{v_\rho(q, L), v_\rho(q_{rf}, L_{rf}), v_\rho(0, 0)\},$$

where the value of an arbitrary risky contract (q, L) is

$$v_\rho(q, L) = qL + \beta(\rho(y_h - L) + (1 - \rho)(1 - \gamma)y_l), \quad (5.3)$$

the value of a risk-free contract (q_{rf}, L_{rf}) is

$$v_\rho(q_{rf}, L_{rf}) = q_{rf}L_{rf} + \beta(\rho y_h + (1 - \rho)y_l - L_{rf}), \quad (5.4)$$

and the value of autarky is

$$v_\rho(0, 0) = \beta(\rho y_h + (1 - \rho)y_l). \quad (5.5)$$

The easiest participation constraint to analyze is that comparing autarky and the risk-free contract. Comparing equations (5.4) and (5.5) shows that, regardless of the loan size, the risk free contract dominates autarky whenever

$$q_{rf} \geq \beta \quad (5.6)$$

This has a straightforward interpretation. The risk-free contract will be accepted whenever the number of people who might accept the risk-free contract (and the value of pledgable income γy_l) is high enough relative to the fixed cost so that the bond price is greater than the discount factor.

We now turn to the two participation constraints involving the risky contract. A household will prefer the risky contract (q, L) to autarky whenever $v_\rho(q, L) \geq v_\rho(0, 0)$. This reduces to

$$q \geq \beta \left(\rho + (1 - \rho) \frac{\gamma y_l}{L} \right). \quad (5.7)$$

For the risk-free household ($\rho = 1$), this collapses to $q \geq \beta$.

If the risk-free contract is offered in equilibrium (and thus preferred to autarky), households face a choice between a risky and the risk-free contracts. The risky contract then has to satisfy $v_\rho(q, L) \geq v_\rho(q_{rf}, L_{rf})$, which reduces to

$$q \geq (q_{rf} - \beta) \frac{\bar{L}_{rf}}{L} + \beta \left(\rho + (1 - \rho) \frac{\gamma y_l}{L} \right) \quad (5.8)$$

These constraints have important implications for the set of equilibrium contracts. First, consider a risky contract (q, L) offered to a set of households in an interval $[\underline{\rho}, \bar{\rho}]$. Recall that the risky contracts have $L > \gamma y_l$ (otherwise the contract would be risk-free), which implies that $\frac{\gamma y_l}{L} < 1$, and the right-hand side of both equation (5.7) and (5.8) are increasing in ρ . As a result, if a participation constraint of a risky contract does not bind for the highest type in the interval, $\bar{\rho}$, then it will not bind for any household in this interval. Hence, we only need to check the participation constraint for the least risky type in an interval covered by a risky contract.

The second point to note is that as one moves from one risky interval to a riskier contract, q decreases. This makes the left-hand side of equations (5.7) and (5.8) smaller, which makes it less likely that the risky contract for that risk bin will be preferred to the risk-free contract or autarky.

5.2.2 The Form of Equilibrium Contracts

As can be seen from Proposition 5.2, all risky contracts have the same size of the loan, $L = \gamma y_h$, and differ only in the price (and eligibility set). We will order these contracts by the degree of riskiness of the clientele served by the contract, from the least to the most risky.

Theorem 5.1. *Finitely many (N) risky contracts are offered. Each contract ($q_n, L = \gamma y_h$) serves borrowers in the interval $\rho \in (\rho_n, \rho_{n-1}]$, where*

$$\begin{aligned}\rho_n &= 1 - n\sqrt{\frac{2\chi}{\gamma y_h \bar{q}}} \\ q_n &= \bar{q}\rho_n\end{aligned}$$

If risk-free contract ($q_{rf}, L_{rf} = \gamma y_l$) is offered, it serves borrowers with $\rho \in [0, \rho_N]$, and

$$q_{rf} = \bar{q} - \frac{\chi}{\gamma y_l \rho_N}. \quad (5.9)$$

This proposition draws upon both the supply and demand side. In equilibrium, the supply side determines the form of the contracts offered, the zero profit condition determines the size of the intervals served by the contracts, while the demand side determines the number of contracts offered.

Proof. The equilibrium idea we exploit in characterizing the set of contracts is as follows. If a contract yields strictly positive profit (in excess of the fixed cost), then a new entrant will enter, offering slightly better price and attracting the best part borrowers away from the existing contract. Such cream-skimming exactly pins down the size of the interval served by a risky contract.

The proof of this theorem proceeds inductively through the set of possible risky contracts, starting with the contract for the best risk types. We begin the proof by constructing the contract offered to households in the interval $(\rho_1, 1]$, where

$$\rho_1 = \frac{q_1}{\bar{q}} \quad (5.10)$$

follows from formula (5.2). The operating profit on this contract is

$$\Pi_1 = \int_{\rho_1}^1 (\rho \bar{q} - q_1) L d\rho = L \left(\frac{1 - (\rho_1)^2}{2} \bar{q} - (1 - \rho_1) q_1 \right)$$

The contract for this group earns zero profits, which implies that the operating profit is equal to the fixed cost of offering the contract ($\Pi_1 = \chi$). Using equation (5.10),

$$L \left(\left(\frac{q_1}{\bar{q}} - 1 \right) q_1 + \frac{1 - \left(\frac{q_1}{\bar{q}} \right)^2}{2} \bar{q} \right) = \chi$$

Expanding and rearranging this expression, one obtains:

$$q_1 = \bar{q} - \sqrt{\frac{2\bar{q}\chi}{L}}$$

This characterizes the first risky contract. We can continue this process down the risk distribution of households. The second contract will cover households in the interval $(\rho_2, \rho_1]$. The operating profit on this contract is

$$\Pi_2 = \int_{\rho_2}^{\rho_1} (\rho\bar{q} - q_2)Ld\rho = L \left(\frac{(\rho_1)^2 - (\rho_2)^2}{2} \bar{q} - (\rho_1 - \rho_2)q_2 \right)$$

where $\rho_2 = \frac{q_2}{\bar{q}}$. Working off the free entry condition, one has that the operating profit is equated to the fixed cost of offering the contract:

$$L \left(\frac{(\frac{q_1}{\bar{q}})^2 - (\frac{q_2}{\bar{q}})^2}{2} \bar{q} - \left(\frac{q_1}{\bar{q}} - \frac{q_2}{\bar{q}} \right) q_2 \right) = \chi$$

Rearranging yields:

$$q_2 = q_1 - \sqrt{\frac{2\bar{q}\chi}{L}}$$

Proceeding in this fashion, we obtain the price of the j^{th} risky contract:

$$q_j = q_{j-1} - \sqrt{\frac{2\bar{q}\chi}{L}}$$

We can also characterize what the risk-free contract will look like if it is offered in equilibrium. Recall from the household participation constraint (5.6) that the choice between autarky and the risk-free contract does not depend upon the households type. Hence, the set of households $\rho \in [0, \rho_N]$ will either all purchase the risk-free contract or none will. The zero profit condition for the risk-free contract, $\Pi_{rf} = \chi$, implies that the bond price of the risk-free contract is:

$$q_{rf} = \bar{q} - \frac{\chi}{\rho_N L_{rf}}$$

□

This characterization does not rely in any way on the households' participation constraints. However, these participation constraints are critical in determining the number of risky offered in equilibrium and whether the risk-free contract is offered in equilibrium.

5.2.3 The Set of Equilibrium Contracts: Participation Constraints

To construct an equilibrium, one needs to check the household participation decisions to solve for N and for whether the risk-free contract is offered in equilibrium. There are several facts which make this process straightforward. First, note that for any N it is easy to check if the risk-free contract will be offered. Combining the household participation constraint (5.6) with the expression for the risk-free bond price (5.9), one can solve for the minimum length of the interval (not served by risky contracts) that makes the risk-free contract viable. Letting the upper-cutoff for this interval be denoted by $\bar{\rho}_{rf}$

$$\begin{aligned}\beta = q_{rf} &= \bar{q} - \frac{\chi}{\bar{\rho}_{rf} L_{rf}} \\ \rho_{rf} &= \frac{\chi}{(\bar{q} - \beta) L_{rf}}\end{aligned}\tag{5.11}$$

To solve for N , one has to find the (first) risky contract for which the household participation constraint with respect to either autarky or the risk-free contract is violated. Recall that for any risky contract serving the interval $(\underline{\rho}, \bar{\rho}]$ we only need to check the participation constraint of the individual with the highest type, $\bar{\rho}$. Further, if $\underline{\rho} \geq \bar{\rho}_{rf}$, then we have to check the household $\bar{\rho}$'s participation constraint with respect to the risk-free contract (serving $[0, \underline{\rho}]$). Conversely, if $\underline{\rho} < \bar{\rho}_{rf}$, we check the value of the risky contract against the autarky participation constraint (5.7).

This observation allows us to put an upper bound on the number of risky contracts offered in equilibrium by considering only the autarky participation constraint.

Theorem 5.2. *Assume that $\beta \frac{y_L}{y_H} + \bar{q} > 1$. Taking participation constraints into account, the number of risky contracts offered in equilibrium, N , cannot exceed*

$$\bar{N} = \text{int} \left(\frac{(\bar{q} - \beta) \sqrt{\frac{\gamma y_h \bar{q}}{2\chi}} - \beta \left(1 - \frac{y_L}{y_H}\right)}{\beta \frac{y_L}{y_H} + \bar{q} - 1} \right)$$

Proof. We want to find that largest N that is consistent with firm maximization and household participation. In other words, what is the largest n that satisfies

$$\rho_n = 1 - n \sqrt{\frac{2\chi}{y_h \gamma \bar{q}}} \text{ and } q_n = \bar{q} \rho_n$$

and does not violate the household participation constraint for all types in $(\rho_n, \rho_{n-1}]$? Since the autarky participation constraint (equation 5.7) must always hold (and since when the risk-free contract is offered, the autarky constraint is slacker), checking the autarky constraint will give the upper bound on the number of possible contracts. As discussed above, it suffices to check the participation constraint only for type ρ_{n-1} . Recalling that the autarky-risky participation constraint is $\beta \left(\frac{\gamma y_l (1-\rho)}{L} + \rho \right) \leq q$, we have that contract n will satisfy the participation constraint for ρ_{n-1} if and only if:

$$\beta \left(\frac{\gamma y_l (1 - \rho_{n-1})}{L} + \rho_{n-1} \right) \leq \bar{q} \rho_n$$

Substituting $\rho_n = 1 - n \sqrt{\frac{2\chi}{y_h \gamma \bar{q}}}$ from Theorem 5.1, we get

$$\beta \left(\frac{\gamma y_l \left[1 - \left(1 - (n-1) \sqrt{\frac{2\chi}{y_h \gamma \bar{q}}} \right) \right]}{L} + \left(1 - (n-1) \sqrt{\frac{2\chi}{y_h \gamma \bar{q}}} \right) \right) \leq \bar{q} \left(1 - n \sqrt{\frac{2\chi}{y_h \gamma \bar{q}}} \right)$$

simplifying and collecting terms:

$$n \sqrt{\frac{2\chi}{\gamma y_h \bar{q}}} \left(\beta \frac{y_l}{y_h} - 1 + \bar{q} \right) \leq \bar{q} - \beta - \beta \left(1 - \frac{y_l}{y_h} \right) \sqrt{\frac{2\chi}{\gamma y_h \bar{q}}}$$

Under the condition that $\beta \frac{y_l}{y_h} + \bar{q} > 1$, this inequality can be rewritten as:

$$n \leq \frac{(\bar{q} - \beta) \sqrt{\frac{\gamma y_h \bar{q}}{2\chi}} - \beta \left(1 - \frac{y_l}{y_h} \right)}{\beta \frac{y_l}{y_h} + \bar{q} - 1}$$

The equilibrium number of contracts cannot exceed the largest n that satisfies the inequality above. \square

Clearly, this bound \bar{N} is weakly decreasing in χ .

5.3 Characterizing Equilibria: Aggregates

The rest of the equilibrium variables of interest can be easily computed once one has solved for N and for whether the risk-free contract is offered. Here we briefly define the main aggregates that we are interested in. In section 6, we will examine how these variables vary with fundamentals.

Given the number of risky contracts N and the length of the interval served by each risky contract, the fraction of the population which borrows using the risky contract is $1 - \rho_N$. *Total Defaults* is given by the total number of households who borrowed using the risky contract and experienced low income (y_l) in the second period of life. Given the assumption that households are uniformly distributed over the interval, defaults are

$$\text{Defaults} = \int_{\rho_N}^1 (1 - \rho) d\rho = 1 - \rho_N - \frac{1 - \rho_N^2}{2}$$

The total amount of borrowing is the sum of risky and risk-free borrowing. We choose to report this in terms of the present value of the amount borrowed at date 1, rather than the face value. *Total Risky Borrowing* in units of the period 1 good is given by

$$\text{Total Risky Borrowing} = \sum_{j=1}^N (\rho_{j-1} - \rho_j) q_j L$$

where $\rho_0 = 1$.

Total Borrowing is the sum of risky and risk-free borrowing. If the risk free contract is offered in equilibrium, then

$$\text{Total Borrowing} = \sum_{j=1}^N (\rho_{j-1} - \rho_j) q_j L + \rho^N q_{rf} L_{rf}$$

We define the average *risk premium* as the default rate on a risky contract. This can be expressed in terms of the face value of the debt in period 2. The face value of defaults for a risky contract is:

$$\begin{aligned} \text{Defaults on Contract}_j &= \int_{\rho_j}^{\rho_{j-1}} (1 - \rho) L d\rho \\ \text{Defaults on Contract}_j &= \left(\rho_{j-1} - \rho_j - \frac{(\rho_{j-1})^2 - (\rho_j)^2}{2} \right) L \end{aligned}$$

The fraction of debt not repaid is thus

$$\begin{aligned} \text{Default Rate} &= \frac{\text{Face Value of Defaults}}{\text{Face Value of Debt}} \\ &= \frac{\left(\rho_{j-1} - \rho_j - \frac{(\rho_{j-1})^2 - (\rho_j)^2}{2} \right) L}{(\rho_{j-1} - \rho_j) L} \\ &= 1 - \frac{\rho_{j-1} + \rho_j}{2}. \end{aligned}$$

In our model, bond prices (interest rates) are lower (higher) than the risk-free both due to defaults and due to overhead costs associated with creating and making loans. The *Average Overhead on Contract_j* is measured as a fraction of the amount borrowed:

$$\text{Average Overhead on Contract}_j = \frac{\chi}{(\rho_{j-1} - \rho_j) q_j L}$$

We use the term *average* overhead because different households may pay more or less than the average overhead. The reason is that the low default risks within a contract type effectively cross-subsidize higher default risks by paying a larger share of the overhead costs.

Another measure that can be compared to the data is the *Total Overhead* cost of making loans. The total overhead cost of risky loans is simply the number of contracts times the cost per contract.

$$\text{Total Overhead Risky} = N\chi$$

If the risk-free contract is offered, total overhead costs is $(N + 1)\chi$. To get the total overhead as a fraction of loans, we simply need to divide the Total Overhead by the Total Borrowing defined above.

6 Results: Comparative Statics

The model is a stylized two period model. Given this, we focus on the qualitative effects in the model, and explore the implications of two forces. First, what is the effect of changes in the fixed cost of creating contracts? Second, what is the effect of shifts in the risk-free bond price on the number and price of contracts offered in equilibrium? We provide most comparative statics results in the form of theorems. To better illustrate the results, we also present a numerical example. The parameters used in the example are given in Table 6. Since the example is intended to help illustrate how the qualitative features of the model (i.e. direction of change) matches up with the data, the parametrization is chosen for simplicity rather than to match any quantitative facts.

Table 6: Parameters used in the Numerical Example

β	γ	y_h	y_l	\bar{r}	χ	a
0.75	0.25	3	0.4	4%	[0.0001, 0.002]	0

6.1 Decline in Cost of Contract χ

Suppose that the cost of offering a contract declines.²⁰ Since the equilibrium features a discrete number of contracts, the effects of a change in χ depends upon the size. What is true is that for a *sufficiently large* change in χ , the number of risky contracts offered in equilibrium increases. This follows from Theorem 5.2, which states that the maximum number of contracts is weakly decreasing declines in χ . Figure 7 shows how in our example an increase in the fixed cost leads to a reduction in the number of contracts. The reason for the step function is that for small enough changes in χ , adding a new contract is not profitable.

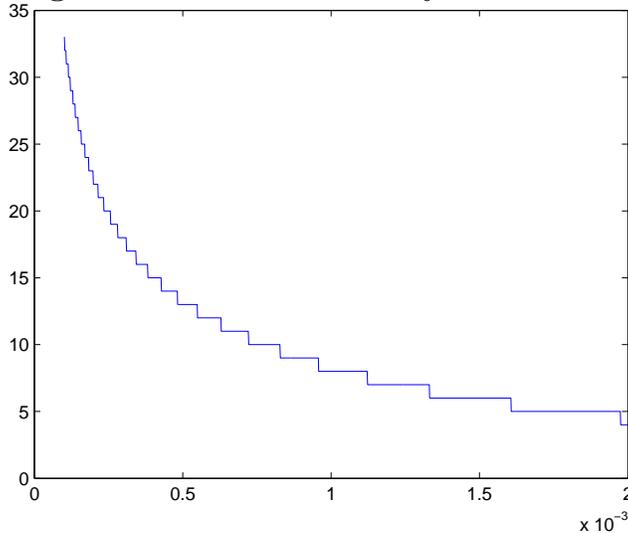
An increase in the number of contracts goes hand in hand with an expansion of credit to more (and riskier) people. This can be seen from Figure 8 where for a high fixed cost only 30% of the population is able to borrow, while for a low fixed cost, about 55% of the population has access to credit.²¹

The length of the interval served by *each* contract increases in χ . Since total borrowing and defaults depend upon the fraction of population covered by a risky contract, $(1 - \rho_N)$, the interaction between shrinking interval length and the increased number of contracts is key. As Figure 8 illustrates, whenever χ declines enough to generate an additional risky contract in equilibrium, the total measure of households

²⁰The rise of information technology has also made it easier for companies to offer contracts to a wider geographical area. For example, large credit card providers such as Citi and MBNA offer cards nationally, whereas early credit cards were offered by regional banks. In this model, this would act as an increase in the market size which has a similar effect to a fall in χ .

²¹Recall that the model also allow for a small loan which is repaid with certainty. These loans are not part of the picture.

Figure 7: Number of Risky Contracts



borrowing via risky contracts increases. However, when χ declines by less than this amount, the only effect is to shrink the length of each interval – which reduces the fraction of the population with risky borrowing. The next theorem provides theoretical bounds for the contract coverage.

Theorem 6.1. *Assume that $\beta \frac{y_L}{y_H} + \bar{q} > 1$. Then the fraction of households served by risky contracts in equilibrium*

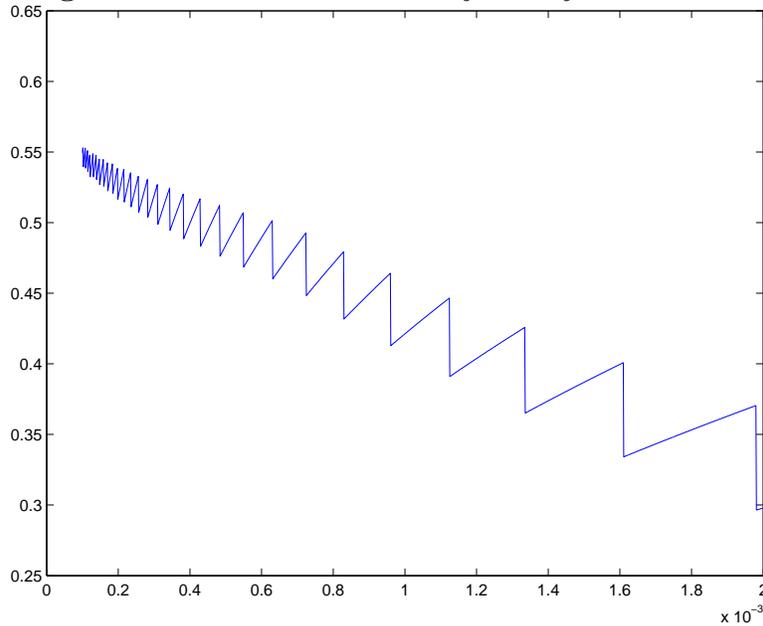
$$(1 - \rho_N) \in \left(\frac{\bar{q} - \beta - (\beta + \bar{q} - 1) \sqrt{\frac{2\chi}{\gamma y_h \bar{q}}}}{\beta \frac{y_L}{y_H} - 1 + \bar{q}}, \frac{\bar{q} - \beta - \beta \left(1 - \frac{y_L}{y_H}\right) \sqrt{\frac{2\chi}{\gamma y_h \bar{q}}}}{\beta \frac{y_L}{y_H} - 1 + \bar{q}} \right].$$

Proof. This follows directly from Theorem 5.2. □

The size of the group is locally increasing in χ almost everywhere, which corresponds to the number of risky contracts remaining the same and the size of a group served by each contract getting larger. However, globally, the fraction served by risky contracts is decreasing in χ as can be seen from the fact that both the upper and the lower bounds of the interval are decreasing in χ .

The number of risky contracts is thus key to how aggregate borrowing and defaults vary with χ . The reason is that the fraction of the population with access to risky

Figure 8: Measure Served by Risky Contracts



contracts is increasing in the number of risky contracts offered. From the definitions of aggregates in Section 5.3, it follows that total defaults and total risky borrowing are increasing in the fraction of the population served by risky contracts. Basically, the previous theorem establishes a weak monotonicity of ρ_N in χ : as contract costs fall enough, the fraction of people covered by risky contracts will increase. Building on this, we can now write several additional results on how aggregates behave as a function of contract coverage, $1 - \rho_N$.

Theorem 6.2. *The number of total defaults strictly decreases in ρ_N .*

Proof. Total defaults are given by

$$\int_{\rho_N}^1 (1 - \rho) d\rho = 1/2 - \rho_N + \frac{\rho_N^2}{2}$$

This is strictly decreasing in ρ_N as long as $\rho_N < 1$.

q.e.d.

Theorem 6.3. *Aggregate risky borrowing strictly decreases in ρ_N .*

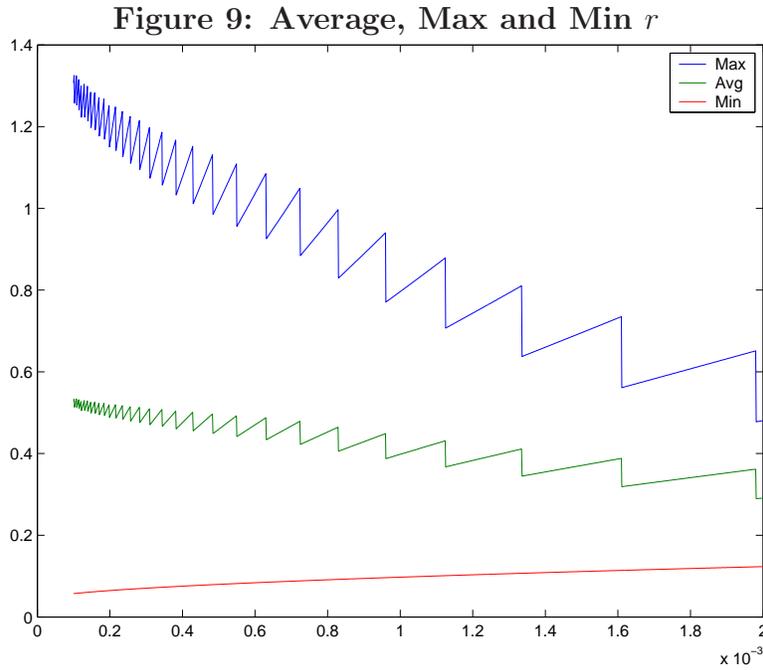
Aggregate risky borrowing (in period 1 dollars) is:

$$\int_{\rho_N}^1 \bar{q}L d\rho = (1 - \rho_N)\bar{q}L$$

This clearly decreases in ρ_N .

q.e.d.

The overall effect on borrowing and defaults is straightforward. Whenever χ decreases enough to increase N , total borrowing and defaults increase. In the model, this also implies that borrowing by lower income (riskier in model) households increases. As a result, defaults increase at a faster rate than debt and the share of risky debt held by lower income households increases.

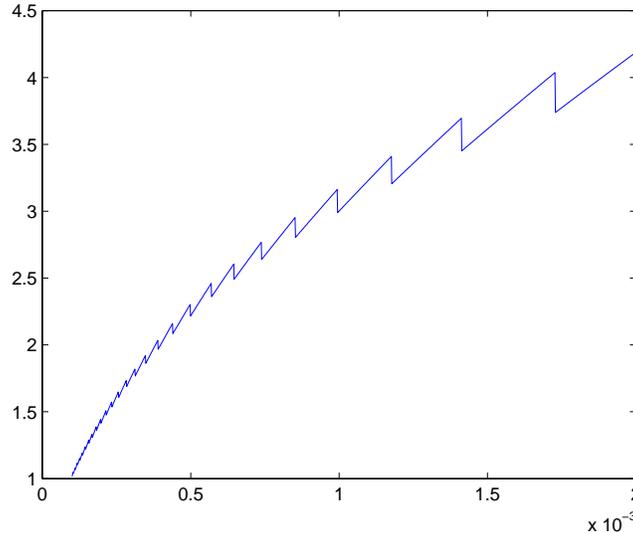


A fall in χ also reduces the extent of cross-subsidization. The reason is that the lowest risk household in each contract interval “subsidizes” the highest risk households in that pool $\rho \in (\rho_n, \rho_{n-1}]$. The shrinking of each contract interval shrinks the amount of cross-subsidization and hence leads to more accurate risk-based pricing. This pattern can be seen by looking at the bond prices in the example. As χ declines and the number of contracts increases, the model generates both more disperse interest

rates and higher average borrowing interest rate. The expansion of credit to higher risk borrowers is accompanied by an increase in the bond price offered to existing borrowers. This can be seen in Figure 9, which plots the average, largest and smallest risky interest rates as a function of χ .

Total overhead costs as a percentage of borrowing are shown in Figure 10. In the example, even though χ falls by a factor of 1:20, total overhead costs (as % of debt) fall only by a factor of 1:4. The reason that a fall in χ lowers overhead costs by less than proportional is that even though fixed costs per contract are falling, fewer borrowers are now “sharing” a contract, so that each borrower has to pay a larger share of the overhead. This suggests that cost of operations of banks (or credit card issuers) might not be a good measure of technological progress in the banking sector.

Figure 10: Overhead Costs as Percent of Borrowing



6.1.1 Model vs. Data

Table 7 summarizes the qualitative implications of a decline in the fixed cost in the model and compares them to the observed changes over time in the data.²² The story

²²We focus here on changes in χ that are large enough to increase borrowing. As discussed before, for small changes, these implications are not always true.

Table 7: Comparison Model vs. Data

	Change in Model (in response to $\chi \downarrow$)	Change in Data (over time)
Bankruptcy		
Defaults/Population	↑	↑
Defaults/Borrowers	↑	?
Borrowing		
Debt/Income	↑	↑
Borrowers/Population	↑	↑
Credit Limit/Income	N/A	↑
Debt/Borrower	N/A	↑
Pricing		
Number of different interest rates	↑	↑
max r	↑	↑
min r	↓	↓
CV (r)	↑	↑

that our model formalizes is that technological innovations have substantially reduced the cost of designing and marketing credit contracts (in particular credit cards) over the 1980s and 1990s. We find that our simple model is qualitatively consistent with almost all of the empirical facts established in Section 3. This suggests that the mechanism explored in this paper might indeed be an important factor in the observed changes in consumer credit markets over the last few decades. We thus believe that exploring this channel in a serious quantitative model would be a promising avenue for future research.

6.2 Decline in Risk Free Rate – Increase in \bar{q}

In an influential paper, Ausubel (1991) documented that the decline in risk-free interest rates in the U.S. in the 1980s were not accompanied by a decline in the average credit card rates reported by the Board of Governors. This led some to claim that the credit card industry was characterized by imperfect competition. In contrast, others such as Evans and Schmalensee (1999) argued that significant measurement issues associated with fixed costs of lending and the expansion of credit to riskier households during the late 1980s implied that Ausubel's observation could be consistent with a competitive credit card industry.

To explore the implications of our model for this debate, we consider the effect of a decline in the risk free interest rate on the number of contracts and average borrowing interest rates. We begin by considering the effect of a rise in \bar{q} on the existing contracts. To see this, recall that from Theorem 5.1 that each of N risky contracts offered in equilibrium $(q_n, \gamma y_h)$ serves borrowers in the interval $\rho \in (\rho_n, \rho_{n-1}]$, where

$$\rho_n = 1 - n\sqrt{\frac{2\chi}{y_h\gamma\bar{q}}} \quad \text{and} \quad q_n = \bar{q}\rho_n$$

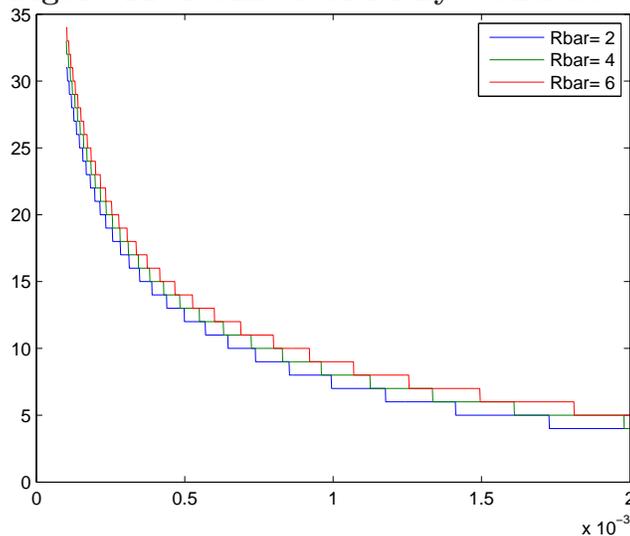
It follows directly that ρ_n is decreasing in \bar{q} and χ , so that an increase in the risk-free bond price will reduce the size of each risky interval. In addition, the risky bond price is increased.

The main question for comparison with the average is what happens to the total number of contracts. As with a change in the fixed cost χ , a sufficiently large rise in the risk free bond price generates an expansion of the number of risky contracts. This leads to an increase in the total measure of risky borrowers, and pulls in borrowers who are riskier than existing borrowers. This generates a rise in defaults. As a result, average borrowing interest rates decline less than proportionally.

To illustrate this, we extend our numerical example and compare the equilibrium associated with three different risk free interest rates $\bar{r} = 2\%, 4\%, 6\%$ (and corresponding bond prices 0.943, 0.962, 0.980). Figure 12 plots the number of risky contracts for value of χ for each \bar{q} . The figure shows that the number of risky contracts is weakly

increasing in the risk free bond price.

Figure 11: Number of Risky Contracts

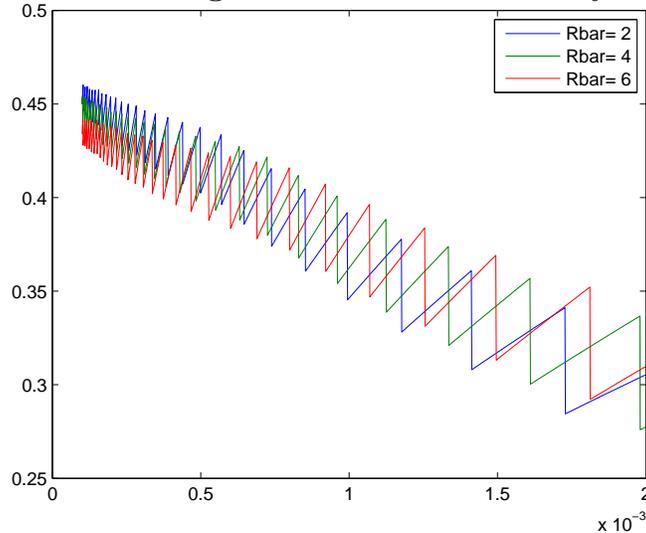


To illustrate the effect of variations in the risk-free rate on the average borrowing interest rate we plot the average risky bond prices for each value of χ . The figure shows that the effect of a shift in the risk free rate on the average borrowing rate depends upon the level of the fixed cost. When the cost of creating contracts is high and there are very few risky contracts offered, the average borrowing bond price tends to be positively related to the risk free rate. In contrast, for lower values of χ , lower risk free bond prices can have higher average borrowing interest rates. The reason is that the extensive margin effect of extending credit to riskier households can dominate the reduction in interest rates for existing borrowers.

7 Imperfect Signals

We now turn to the case where signals are a *noisy* signal of a borrowers type. Formally, there are public signals, σ_i , regarding household i 's type. With probability $\alpha < 1$, the signal is accurate: $\sigma_i = \rho_i$. With complementary probability $(1 - \alpha)$, the signal

Figure 12: Average Interest Rate on Risky Bonds



is an independent draw from the ρ distribution ($U[a, 1]$). Thus, α is the precision of the public signal.

Intermediaries observe the public signal about a household's type, but not the true type. Households can accept only one contract, so intermediaries know the total amount being borrowed. Financial intermediaries must pay a fixed cost χ in order to offer a non-contingent lending contract to (unlimited number of) households. Endowment-contingent contracts are ruled out (due to un-verifiability of the endowment realization). A contract is characterized by $(L, q, \underline{\sigma})$, where L is the maximum face value of the loan, q is the per-unit price of the loan (so that qL is the amount advanced in period 1 in exchange for promise to pay L in period 2), and $\underline{\sigma}$ is the minimal value of public signal that makes a household eligible for the contract. The timing of events in the financial markets is the same as in section 4.5.

7.1 Equilibrium

We focus on a pure strategy equilibrium with pooling within the public types. We begin by characterizing the set of contracts offered in equilibrium. These results

parallel those of the perfect information case.

The first result is that free entry leads to zero profits net of the cost of offering contracts.

Proposition 7.1. *All contracts offered earn exactly χ profits (valued as of period 1).*

Proof. Profits of less than χ preclude entry in the pure strategy equilibrium. Profits of more than χ would generate entry of a competing contract with better terms. \square

The second result is that the face value of all risky contracts is the same. This was trivial in the case when signals were perfect, but is not so obvious here. This result is tied to the fact that in the equilibrium of the game separating contracts are not an equilibrium outcome.

Proposition 7.2. *There are at most two types of contracts offered in equilibrium: risk-free contract with $L = \gamma y_l$ and $\underline{\sigma} = a$, and N risky contracts with $L = \gamma y_h$. Risky contracts are repaid by all households who realize high endowment y_h in period 2.*

Proof. Risk-free contract with $L = \gamma y_l$ dominates all other risk-free contracts (and thus generates the highest possible profit). Risky contracts with $L > \gamma y_h$ cannot be offered as they would never be repaid.

The more interesting result is that there will be no risky contracts with $L' < \gamma y_h$. It is clear that, keeping the price q constant, all households who would apply for (L', q) would prefer $(\gamma y_h, q)$, and it would generate greater profits for the lender. However, there is potential for cream-skimming by offering a smaller loan L' with a slightly better price $q' > q$ (such that $q'L' < qL$) so that (L', q') is preferred to (L, q) by households with high unobservable type ρ . Households with a low ρ would prefer (L, q) , since they are less likely to repay and hence are willing to promise the larger repayment L in exchange for the larger advance qL . Such cream-skimming is never an equilibrium outcome, due to the specific timing introduced in Section 4.5. If an entrant attempts to cream-skim from an existing contract by offering (L', q') that is preferred to the existing contract only by the better types, the equilibrium

of that subgame has incumbent contract quitting the market (having realized that the “good” customers have applied for the new contract). As a result, the “bad” customers (with low unobservable ρ) also apply for the new contract, even though they prefer the terms of the incumbent contract. Thus, “cream-skimming” fails, and the entrant makes lower profit than would the incumbent who was “cream-skimmed” (since both $q' > q$ and $L' < L$). Since the incumbent contract was exactly recovering the fixed cost χ , such an entry is unprofitable. \square

Proposition 7.3. *With α sufficiently high, we can support pooling (within public types) in equilibrium.*

Proof. What we have to show is that there is no profitable deviation (by other intermediaries) which would unravel the pooling equilibrium. Such deviation would offer an alternative contract that is attractive only to “good” (private) types. In our environment, such deviation would include slightly lower face value of the debt L' with slightly (but sufficiently) better price q' . What rules out such deviation is our timing which includes application and exit stages (see 1.b and 1.c in Section 4.5).

If such a deviation were introduced, the households would recognize that the original pooling contract is no longer viable and would not be offered in equilibrium. Thus, both “good” and “bad” private types would apply for the new (deviation) contract, thus making it unprofitable ex-ante. \square

7.1.1 Participation Constraints

Imperfect signals complicate households’ participation decisions. As in the perfect information case, we have to consider the decision of households with accurate signal about their type to participate in the risky and risk-free contracts. In addition, we also have to consider the participation decision of high types (households with a high probability of getting the high income shock in period 2) who have low public signals to participate in risky contracts that they are eligible for.

The participation constraints of households with accurate signals ($\sigma_i = \rho_i$) are the same as in the perfect information case (Section 5.2.1). First, if the risk-free contract

is offered, it has to be preferred to autarky, which is the case whenever

$$q_{rf} \geq \beta. \quad (7.1)$$

Second, every equilibrium risky contract has to be preferred (by the customers it targets) to autarky. That requires

$$q \geq \beta \left(\rho + (1 - \rho) \frac{\gamma y_l}{L} \right) \quad (7.2)$$

for all ρ in the eligibility set who are not eligible for a better risky contract. Lastly, if a risk-free contract is offered in equilibrium, every equilibrium risky contract has to be preferred (by the customers it targets) to the risk-free contract:

$$q \geq (q_{rf} - \beta) \frac{L_{rf}}{L} + \beta \left(\rho + (1 - \rho) \frac{\gamma y_l}{L} \right) \quad (7.3)$$

for all ρ in the eligibility set. As in the perfect information case, if an incentive constraint is satisfied for a household with some $\bar{\rho}$, it is also satisfied for all households with $\rho < \bar{\rho}$.

The added complication in the imperfect information case is the need to consider participation decisions of households with inaccurate signals. Since the right-hand side of the participation constraints above is increasing in ρ , households with signals greater than their true type ($\sigma_i > \rho_i$) will always accept the risky contracts they are eligible for. On the other hand, households with public signals below their true type ($\sigma_i < \rho_i$) may choose to opt out of the risky contracts offered to them and choose autarky or the risk-free contract (if it is offered in equilibrium).

Exploiting the monotonicity of the incentive constraint (7.2) in ρ , we can specify the highest true type that would choose a risky contract (L, q) over autarky:

$$\bar{\rho}_{aut}(q, L) = \frac{qL - \beta\gamma y_l}{\beta(L - \gamma y_l)}. \quad (7.4)$$

Analogously, the highest type that would prefer a risky contract (L, q) to the risk-free contract (L_{rf}, q_{rf}) (where in equilibrium $L_{rf} = \gamma y_l$) is

$$\bar{\rho}_{rf}(q, L) = \frac{qL - (q_{rf} - \beta)L_{rf} - \beta\gamma y_l}{\beta(L - \gamma y_l)} = \frac{qL - q_{rf}\gamma y_l}{\beta(L - \gamma y_l)}.$$

7.1.2 The Set of Equilibrium Contracts

As in the perfect information case, the equilibrium is characterized by a finite number N of risky contracts, which serve consecutive intervals $[\underline{\sigma}_n, \underline{\sigma}_{n-1})$ of public types (which we continue to order from the best to the worst types targeted), and a possible risk-free contract, all of which generate operating profit of exactly χ . What differs from the perfect information case is not just the pricing of these contracts, but also the need to characterize the set of true types that accept a risky contract when their public type falls into the eligibility set for that risky contract. As argued above, this set of true types is $[a, \bar{\rho}(q, L)]$. Furthermore, the set of potential customers for a risk free contract includes not only all the households who are not eligible for risk-free contracts, but also households with mistakenly low public signals who choose not to accept risky contracts they are eligible for.

Proposition 7.4. *The risky contracts are of the form $(q_n, L, \underline{\sigma}_n)$, where $L = \gamma y_h$,*

$$\underline{\sigma}_n = 1 - n \sqrt{\frac{2\chi(1-a)}{\alpha \bar{q} L}}. \quad (7.5)$$

The first contract serves the interval $[\underline{\sigma}_1, 1]$, and each subsequent contract serves the interval $[\underline{\sigma}_n, \underline{\sigma}_{n-1})$. If the participation constraints of borrowers with $\sigma < \rho$ does not bind ($\bar{\rho}(q_n, L) \geq 1$), then $q_n = \bar{q} \left(\alpha \underline{\sigma}_n + (1 - \alpha) \frac{1+a}{2} \right)$. If the participation constraints of borrowers with $\sigma < \rho$ does bind, then

$$\bar{q} \underline{\sigma}_n \alpha = q_n \left(\alpha + (1 - \alpha) \frac{\bar{\rho}_n - a}{1 - a} \right) - \bar{q} (1 - \alpha) \frac{(\bar{\rho}_n)^2 - a^2}{2(1 - a)},$$

where $\bar{\rho}_n = \bar{\rho}(q_n, L)$ is given by equation (7.2) or (7.3).

Proof. (1) We start with the case where the participation constraints of borrowers with $\sigma < \rho$ do not bind. We begin by establishing the equilibrium threshold for accepting applicants (as a function of the price offered). Clearly, intermediaries accept those and only those applicants who deliver non-negative expected profit. That implies that for any contract with price q , the marginal type $\underline{\sigma}$ satisfies

$$\left(\alpha \underline{\sigma} + (1 - \alpha) \frac{1+a}{2} \right) \bar{q} = q. \quad (7.6)$$

We now use the free entry condition to pin down the equilibrium values:

$$\left(\alpha \int_{\underline{\sigma}_n}^{\underline{\sigma}_{n-1}} \frac{\rho}{1-a} d\rho + (1-\alpha) \frac{\underline{\sigma}_{n-1} - \underline{\sigma}_n}{1-a} \frac{1+a}{2} \right) \bar{q} - \frac{\underline{\sigma}_{n-1} - \underline{\sigma}_n}{1-a} q_n = \frac{\chi}{L} \quad (7.7)$$

Using equation (7.6) to eliminate q_n , we obtain

$$\begin{aligned} \frac{\bar{q}}{2(1-a)} [\alpha (\underline{\sigma}_{n-1}^2 - \underline{\sigma}_n^2) + (1-\alpha)(\underline{\sigma}_{n-1} - \underline{\sigma}_n)(1+a)] - \\ \frac{\underline{\sigma}_{n-1} - \underline{\sigma}_n}{1-a} \bar{q} \left(\alpha \underline{\sigma}_n + (1-\alpha) \frac{1+a}{2} \right) = \frac{\chi}{L}. \end{aligned}$$

Rearranging further,

$$\frac{\underline{\sigma}_{n-1} - \underline{\sigma}_n}{2(1-a)} [\alpha (\underline{\sigma}_{n-1} + \underline{\sigma}_n) + (1-\alpha)(1+a) - 2\alpha \underline{\sigma}_n - (1-\alpha)(1+a)] = \frac{\chi}{\bar{q}L},$$

which reduces to

$$\underline{\sigma}_{n-1} - \underline{\sigma}_n = \sqrt{\frac{2\chi(1-a)}{\alpha \bar{q}L}} \quad (7.8)$$

(2) Noisy signals create the possibility of adverse selection. Households with signals $\sigma > \rho$ will always accept a contract that a $\sigma = \rho$ type would. However, high types with $\sigma < \rho$ may choose to not participate. This means that some risky contracts may find that good types with low signals opt for the risk-free contract or autarky.

Note that for the first risky contract (q_1, \bar{L}) , this is not an issue. However, it may matter for some contract (q_n, \bar{L}) with $n \geq 2$. For each contract, we let $\bar{\rho}_n$ denote the cutoff such that all households with $\rho \geq \bar{\rho}_n$ (and $\sigma \in (\underline{\sigma}_n, \underline{\sigma}_{n-1}]$) would not purchase contract (q_n, L) .

Suppose that the participation constraint of high types with low signals does not bind for the first $n-1$ contracts, but binds for contract n . We know that such a possibility always exists whenever at least two risky contracts are offered in equilibrium, since this participation constraint cannot bind for the first contract. The n^{th} contract covers households with public types in the interval $(\underline{\sigma}_n, \underline{\sigma}_{n-1}]$, where the cut-offs are given by:

$$\underline{\sigma}_{n-1} = \frac{1}{\alpha} \left(\frac{q_{n-1}}{\bar{q}} - (1-\alpha) \frac{1+a}{2} \right) \quad (7.9)$$

$$\underline{\sigma}_n = \frac{1}{\alpha} \left[\frac{q_n}{\bar{q}} \left(\alpha + (1-\alpha) \frac{\bar{\rho}_n - a}{1-a} \right) - (1-\alpha) \frac{(\bar{\rho}_n)^2 - a^2}{2(1-a)} \right] \quad (7.10)$$

The operating profit can be decomposed into the contributions from households with accurate and inaccurate signals. The total amount repaid by households with inaccurate signals (taking into account participation) is:

$$(1 - \alpha) \int_a^{\bar{\rho}_n} \frac{\rho}{1 - a} L d\rho = (1 - \alpha) L \frac{(\bar{\rho}_n)^2 - a^2}{2(1 - a)}$$

Hence, the operating profit is given by:

$$\begin{aligned} \Pi_n &= \frac{\bar{q}L}{1 - a} \left(\alpha \frac{(\underline{\sigma}_{n-1})^2 - (\underline{\sigma}_n)^2}{2} + (\underline{\sigma}_{n-1} - \underline{\sigma}_n)(1 - \alpha) \frac{(\bar{\rho}_n)^2 - a^2}{2(1 - a)} \right) \\ &\quad - \frac{q_n L}{1 - a} (\underline{\sigma}_{n-1} - \underline{\sigma}_n) \left(\alpha + (1 - \alpha) \frac{\bar{\rho}_n - a}{1 - a} \right) \end{aligned}$$

Substituting in the zero profit condition $\Pi_n = \chi$ and using equation (7.10) to eliminate q_n , we obtain (after rearranging and canceling terms):

$$\alpha (\underline{\sigma}_{n-1}^2 - \underline{\sigma}_n^2) - (\underline{\sigma}_{n-1} - \underline{\sigma}_n) 2\alpha \underline{\sigma}_n = \frac{\chi}{\bar{q}L},$$

which reduces to

$$\underline{\sigma}_{n-1} - \underline{\sigma}_n = \sqrt{\frac{2\chi(1 - a)}{\alpha \bar{q}L}} \quad (7.11)$$

Note that this is the same as in the case where the participation constraints of the high types with low signals does not bind. Hence, when the incentive constraint of high types with low σ binds, the length of each interval remains unchanged but the bond price of the risky contract decreases. This follows from equation (7.11), and from comparing (7.9) and (7.10). This implies that the effect of adverse selection is to lower the bond price offered from what it would have been had the high types (with mistakenly low signal) chosen to participate. \square

Note that the environment with perfect signals (observable types) is a special case of the environment considered above with $\alpha = 1$.

7.2 Characterizing Equilibria: Aggregates

The rest of the equilibrium variables of interest can be computed once one has solved for N and for whether the risk-free contract is offered. Here we briefly define the

main aggregates we are interested in, and highlight how they differ from the perfect information case as outlined in section 5.3.

The fraction of the population who borrow via risky contracts is no longer simply given by the interval served by public signals $\frac{1-\rho_N}{1-a}$, as some households with $\sigma < \rho$ may choose not to accept the risky contract they are offered. Rather, it is given by

$$\alpha \frac{1-\rho_N}{1-a} + \frac{(1-\alpha)}{1-a} \sum_{j=1}^N [(\sigma_{j-1} - \sigma_j) \frac{(\bar{\rho}_j - a)}{1-a}] \quad (7.12)$$

Note that if the participation constraint of types with lower public signals than their true type never binds (so $\bar{\rho}_j = 1$ for all contracts j), then this collapses back to $\frac{1-\rho_N}{1-a}$.

Similarly, *Total Defaults* is given by

$$\text{Defaults} = \frac{\alpha}{1-a} \left(1 - \rho_N - \frac{1-\rho_N^2}{2} \right) + \frac{(1-\alpha)}{1-a} \sum_{j=1}^N [\sigma_{j-1} - \sigma_j] \frac{1}{1-a} \left(\bar{\rho}_j - a - \frac{(\bar{\rho}_j)^2 - a^2}{2} \right)$$

Total Risky Borrowing in units of the period 1 good is given by²³

$$\text{Total Risky Borrowing} = \sum_{j=1}^N \frac{(\sigma_{j-1} - \sigma_j)}{1-a} q_j L \left(\alpha + (1-\alpha) \left[\frac{(\bar{\rho}_j - a)}{1-a} \right] \right)$$

where $\sigma_0 = 1$. If the risk free contract is offered in equilibrium, then

$$\text{Total Risk-Free Borrowing} = q_{rf} L_{rf} \left[\frac{1}{1-a} - \sum_{j=1}^N \frac{(\sigma_{j-1} - \sigma_j)}{1-a} \left(\alpha + (1-\alpha) \left[\frac{(\bar{\rho}_j - a)}{1-a} \right] \right) \right]$$

The total amount of borrowing is the sum of risky and risk-free borrowing.

We define the average *risk premium* as the default rate on a risky contract. The number of defaults on contract j is

$$\text{Defaults}_j = \frac{\alpha}{1-a} \left(\sigma_{j-1} - \sigma_j - \frac{\sigma_{j-1}^2 - \sigma_j^2}{2} \right) + \frac{(1-\alpha)}{1-a} [\sigma_{j-1} - \sigma_j] \frac{1}{1-a} \left(\bar{\rho}_j - a - \frac{(\bar{\rho}_j)^2 - a^2}{2} \right) \quad (7.13)$$

The face value of defaults on contract j is then the product of $\text{Defaults}_j L$. The fraction of the face value of debt not repaid is thus simply Defaults_j .

²³This is the (present value of) the amount borrowed at date 1, rather than the face value of debt outstanding at $t = 2$.

Table 8: Parameters used in the Numerical Example

β	γ	y_h	y_l	\bar{r}	χ	a	α
0.75	0.25	3	0.4	4%	0.0001	0	[0.75, 0.999]

In our model, bond prices (interest rates) are lower (higher) than the risk-free both due to defaults and due to overhead costs associated with creating and making loans. The *Average Overhead on Contract_j* is measured as a fraction of the amount borrowed:

$$\text{Average Overhead on Contract}_j = \frac{\chi}{\frac{(\sigma_{j-1} - \sigma_j)}{1-a} q_j L \left(\alpha + (1 - \alpha) \left[\frac{(\bar{p}_j - a)}{1-a} \right] \right)}$$

Finally, the *Total Overhead* cost of making risky loans is simply the number of contracts times the cost per contract ($N\chi$).

7.3 Numerical Example

To better illustrate the working of equilibrium with adverse selection, we turn to a numerical example. We use the same parameters as in section 6. However, we now consider comparative statics for variations in the extent of adverse selection by varying α while holding fixed the fixed cost χ .

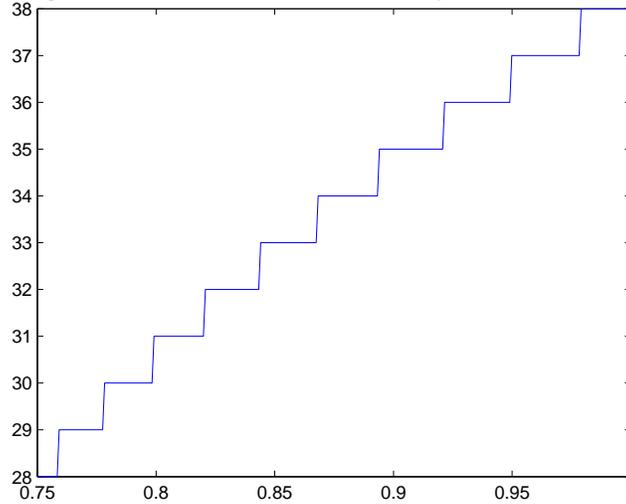
The equilibrium set of contracts can be solved explicitly by considering several cases. The procedure is slightly more involved with the possibility of adverse selection, since this influences the bond prices defined by equation (7.10). As a result, one has to simultaneously solve a system of non-linear equations for the cut-offs for the risk-free participation and the bond prices for each risky contract as well as for the risk-free bond price.²⁴

Improvements in the accuracy of the signal has a similar effect on the number of contracts to a decline in the fixed cost of creating contracts. Since the equilibrium

²⁴See the appendix for more details on the numerical computation of equilibria here.

features a discrete number of contracts, the effects of a change in α depends upon the size. For a *sufficiently large* change in α , the number of risky contracts offered in equilibrium increases. Figure 13 shows that an increase in signal accuracy leads to an increase in the number of contracts.

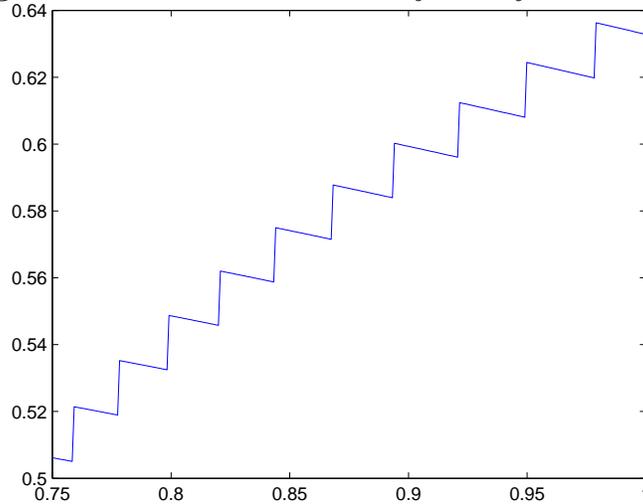
Figure 13: Number of Risky Contracts



The length of the interval served by *each* contract decreases in α . The interaction between shrinking interval length and the increased number of contracts determines the total fraction of the population with *access* to risky borrowing. As Figure 14 illustrates, whenever α declines enough to generate an additional risky contract in equilibrium, the total measure of households borrowing via risky contracts increases. However, when α declines by less than this amount, the length of each contract interval shrinks— which reduces the fraction of the population with access to risky borrowing. This effect is partially offset by the fact that an increase in signal accuracy leads to an increase in the acceptance rate for each contract by reducing the adverse selection problem. This is driven by the fact that an increase in the signal accuracy means that each risky interval has fewer misclassified types. This lowers the risk premium, and pulls in a larger fraction of low risk types with higher risk public types.

An increase in the number of contracts goes hand in hand with an expansion of

Figure 14: Measure Served by Risky Contracts



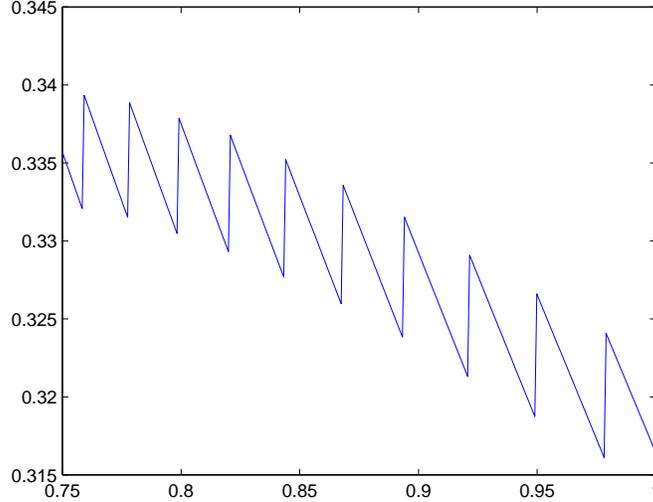
credit to more people. This can be seen from Figure 14 where for a low signal accuracy only 50% of the population is able to borrow, while for perfect signals over 63 % of the population has access to credit.²⁵ In turn, this expansion of credit leads to an increase in borrowing and defaults. However, the rise in defaults is partially muted by the fact that improvements in signal accuracy lead to a reduction in the number of high risk people who borrow on terms offered to low risk borrowers. As a result, the default rate (and the default premium) of contracts offered to existing borrowers decreases with improvements in signal accuracy.

The overall effect of these forces can be seen by examining the average default rate of borrowers. The expansion of credit to more borrowers in this environment involves the extension of credit to public types with higher default risk than existing borrowers. This tends to increase the average default rate of all borrowers. This effect can be seen in Figure 15, as the average default rate spikes up whenever a new contract is introduced. However, unlike in the case of decline in the fixed cost of creating a contract, improvements in signal quality also tend to lower the average default rate

²⁵Recall that the model also allow for a small loan which is repaid with certainty. These loans are not part of the picture.

by reducing the number of high risk households whose public type classifies them as low risk. In this example, this effect dominates so that improvements in signal quality leads to a slightly lower average default rate of all borrowers.

Figure 15: Average Default Rate on Risky Contracts



8 Conclusion

The past thirty years have witnessed substantial changes in access to unsecured credit as well as to the terms at which households can borrow. These changes have led to a dramatic increase in the fraction of lower income households with access to credit card borrowing. In addition, there appears to have been significant increase in the variety of borrowing contracts. This increase in the number of contracts has been accompanied by more diversity in the terms of borrowing, as the empirical density of credit card interest rates has become much “flatter” since 1983.

This paper shows that the qualitative implications of two mechanisms via which improved information technology may impact credit markets are consistent with these empirical observations. Technological innovation (such as computers) which reduced

the cost of designing and marketing financial contracts or improved information technology reduced adverse selection problems by improving the ability of lenders to predict prospective borrowers default risk both imply an increase in the number of contracts offered. In addition, both of these channels imply that each contract is targeted to a smaller subsets of the population, with terms that more accurately reflect the default risk of borrowers. The predictions of both these stories are also qualitatively consistent with the increase in unsecured borrowing and defaults.

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A Data Appendix

The Survey of Consumer Finance asked questions on the credit card interest rate of respondents. The questions asked were for the card with the largest balance (1995 - 2004), while the 1983 survey asked for the best guess of the average annualized interest respondent would pay on the bank or store card he uses most often if the full amount was not paid. One issue that affects the number of different rates reported is that in the later years the SCF imputed values for respondents who did not report an interest rate. To count the number of different interest rates, we drop imputed values. The sample size for the various years does increase, but by much less than the reported number of different interest rates.

Year	HH Count	HH with non-imputed non-imputed LOC R	HH with non-imputed CC R	HH with positive CC Balance
1983	4103	-	2196	768
1989	3143	263	-	-
1992	3906	282	-	-
1995	4299	251	2458	1072
1998	305	279	2386	1029
2001	4442	265	2523	1085
2004	4519	440	2458	1185

When computing the CDF of credit card debt held by percentiles of the earned income distribution, we define earned income to be the sum of Wages + Salaries + Professional Practice, Business, Limited Partnership, Farm + Unemployment or Worker's Compensation.

A.1 Board of Governor Interest Rate Data

We use data collected by the Board of Governors directly from banks on the interest rate for various financial contracts. The Board of Governors conducts a bank survey asking banks for information on interest rates charged to consumers. In principle,

interest rate information is available for general consumer loans (both 12 and 24 months), automobile loans, credit cards, and mobile home loans. However, not all interest rates are available over a long time series and in several instances the wording of the question on the bank questionnaire has changed over time which makes the data difficult to compare over time. The longest consistent time series that is available is the interest rate charged on 24-months consumer loans. The data is collected in a quarterly survey (in February, May, August and November) and is available from February 1972 until February 2007.²⁶ Precisely, the survey asks about the most common rate (annual percentage rate) charged on “other loans for consumer goods and personal expenditures (24-month).” It includes loans for goods other than automobiles or mobile homes whether or not the loan is secured. These loans are typically used for consolidation of debts, medical attention, taxes, vacations, and general personal and family expenditures, including student loans currently being repaid. It excludes all home improvement loans, and all loans secured primarily by real estate.

The data has to be interpreted with caution since every bank is asked to report only *one* interest rate (the most commonly used one) and hence does not necessarily represent an accurate picture of all loan options faced by consumers. For example, taking the most extreme case, each bank could offer a large menu of interest rates and the menu itself could be expanding over time, yet, the most common rate could be identical across banks and unchanging over time.

A.2 Numerical Algorithm: Imperfect Signals

The equilibrium set of contracts can be solved explicitly by considering several cases. The procedure is slightly more involved with the possibility of adverse selection, since this influences the bond prices defined by equation (7.10). Here we briefly outline some results one can use to compute the equilibrium.

²⁶This information is collected on form FR 2835. The data is coded as item LIRS7808 by the Board.

We now have to solve for the bond price taking into account participation. We start with the case where the autarky participation constraint is the one which binds. In this case, for the marginal type $\underline{\sigma}_n$, we have:

$$\bar{q}\underline{\sigma}_n\alpha = q_n(\alpha + (1 - \alpha)\left[\frac{(\rho_n^\alpha) - \bar{a}}{1 - \bar{a}}\right]) - \bar{q}\frac{(1 - \alpha)}{2}\left[\frac{(\rho_n^\alpha)^2 - \bar{a}^2}{1 - \bar{a}}\right] \quad (\text{A.1})$$

We also have that the cut-off type in terms of the bond price is given by:

$$\rho_{n,\text{aut}}^\alpha = \frac{[q_n\bar{L} - \beta\gamma y_L]}{\beta(\bar{L} - \gamma y_L)} \quad (\text{A.2})$$

Combining these equations we obtain a quadratic in the risky bond price:

$$\bar{q}\underline{\sigma}_n\alpha = q_n(\alpha + (1 - \alpha)\left[\frac{(\frac{[q_n\bar{L} - \beta\gamma y_L]}{\beta(\bar{L} - \gamma y_L)}) - \bar{a}}{1 - \bar{a}}\right]) - \bar{q}\frac{(1 - \alpha)}{2}\left[\frac{(\frac{[q_n\bar{L} - \beta\gamma y_L]}{\beta(\bar{L} - \gamma y_L)})^2 - \bar{a}^2}{1 - \bar{a}}\right] \quad (\text{A.3})$$

Expanding this expression and rearranging:

$$\begin{aligned} 0 &= q_n^2(1 - \alpha) \left[\left(\frac{[\bar{L}]}{\beta(\bar{L} - \gamma y_L)} \right) - \frac{\bar{q}}{2} \left[\left(\frac{[\bar{L}^2]}{(\beta(\bar{L} - \gamma y_L))^2} \right) \right] \right] \\ &+ q_n \left[\alpha - \bar{a} + (1 - \alpha) \left(\left[\left(\frac{[-\beta\gamma y_L]}{\beta(\bar{L} - \gamma y_L)} \right) \right] + \bar{q} \left[\left(\frac{[\bar{L}\beta\gamma y_L]}{(\beta(\bar{L} - \gamma y_L))^2} \right) \right] \right) \right] \\ &- \bar{q}\frac{(1 - \alpha)}{2} \left[\left(\frac{[(\beta\gamma y_L)^2]}{(\beta(\bar{L} - \gamma y_L))^2} \right) - \bar{a}^2 \right] - \bar{q}\underline{\sigma}_n\alpha(1 - \bar{a}) \end{aligned} \quad (\text{A.4})$$

Using the expression for q_n **if** this participation constraint does not bind:

$$\beta\left(\frac{\gamma(1-i)y_L}{\bar{L}} + i\right) \leq \bar{q} \left(\alpha\underline{\sigma}_n + (1 - \alpha)\frac{1+a}{2} \right) \quad (\text{A.5})$$

If this constraint binds, then for the cut-off household this holds with equality

$$\begin{aligned} \beta\left(\frac{\gamma(1-i)y_L}{\bar{L}} + i\right) &= \bar{q} \left(\alpha\underline{\sigma}_n + (1 - \alpha)\frac{1+a}{2} \right) \\ (i\bar{L} - \gamma i y_L) &= \frac{\bar{L}}{\beta} [\bar{q} (\alpha\underline{\sigma}_n + (1 - \alpha)\frac{1+a}{2})] - \gamma y_L \\ i &= \frac{1}{L - \gamma y_L} \left[\frac{\bar{L}}{\beta} [\bar{q} (\alpha\underline{\sigma}_n + (1 - \alpha)\frac{1+a}{2})] - \gamma y_L \right] \end{aligned} \quad (\text{A.6})$$

Note that once this equation binds, the pricing formula for any contracts that follow will no longer hold. Also, one should note that this constraint is stricter than that of the highest accurately priced household in any contract interval. It is also worth

noting that this equation is likely to bind for reasonable parameters for at least some potentially risky contracts.

An alternative way to look at this is to ask at what cut-off for the riskiest contract offered in equilibrium will this first bind. To solve or this, set the LHS = 1:

$$\begin{aligned}\beta\left(\frac{\gamma(1-\alpha)y_l}{L} + 1\right) &= \bar{q} \left(\alpha\underline{\sigma}_n + (1-\alpha)\frac{1+a}{2}\right) \\ \frac{1}{\alpha}\left[\frac{\beta}{\bar{q}} - (1-\alpha)\frac{1+a}{2}\right] &= \underline{\sigma}_n\end{aligned}\tag{A.7}$$

If the risk-free contract $(q_{rf}, \gamma y_l)$ is offered, it serves borrowers with $\rho \in [\bar{a}, \sigma_N]$ as well as all of the households with mistakenly low public signals who opt out of the risky contracts they are eligible for. The size of this group is $\frac{\Delta}{1-a}$, where Δ is given by $\Delta = \sum_{n=1}^N (1-\alpha)(\sigma_{n-1} - \sigma_n) \max\{(1 - \bar{p}_{rf}(q_n, L)), 0\}$.

$$q_{rf} = \bar{q} - \frac{\chi(1-a)}{L_{rf}(\rho_N - \bar{a} + \Delta)}$$

Substituting in for the bond prices

$$\bar{q} \left(\alpha\underline{\sigma}_n + (1-\alpha)\frac{1+a}{2}\right) - \beta(1-\rho^i)\gamma y_l - \beta\rho^i\bar{L} \leq \left(\bar{q} - \frac{\chi}{L_{rf}(\rho_N - \bar{a})} - \beta\right)\bar{L}_{rf}\tag{A.8}$$

Note that in general $\rho_N \leq \underline{\sigma}_n$.

This gives us a system of equations. To solve this system, we have to simultaneously solve for the cut-offs for the risk-free participation and the bond prices for each risky contract as well as for the risk-free bond price.