

Informational Losses in Rational Trading Panics

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Abstract

I consider a sequential trading model with both trade timing and information acquisition decisions. Traders benefit from waiting to acquire additional information before trading, but waiting is costly because others may then trade before them. I determine the conditions under which traders rationally panic, rushing to trade to avoid adverse price movements. When panics occur, information is forgone, causing asset prices to aggregate only weak information. Counter-intuitively, better quality information can result in less information being aggregated. Panics are shown to occur during times of uncertainty so that, perversely, information is forgone precisely when it is most valuable. Strategic complementarity in timing decisions induces a multiplicity of equilibria, allowing expectations of panic to become self-fulfilling. I present several testable predictions, some of which are consistent with existing empirical evidence.

1 Introduction

“Apple Stock Hits All-Time High As Investors Rush To Get In Early Before iPhone 5” - August 17, 2012, Cult of Mac

“Apple Stock Hit by Panic Selling: ‘Someone Yelled Fire’” - November 15, 2012, CNBC

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Informal commentary abounds with anecdotal evidence of “panic selling” and “panic buying” in financial markets. The two articles cited above claim to provide explanations for both booms and crashes in Apple’s stock.¹ In panic buying, or “buying frenzies”, traders rush to buy a stock for fear of missing out on continued price increases. The first article quotes a fund manager as saying that *“his biggest fear [...] is that he won’t get [the] chance to put all of his money into Apple before the share price skyrockets.”* In the opposite direction, panic selling refers to traders rushing to liquidate their positions before prices fall further. The second article quotes David Greenberg of Greenberg Capital as saying *“Someone yelled fire in the theater [...] and as traders do, they will trample you trying to be first to get to the exit.”* These quotations illustrate the common perception that panics are driven by emotional or irrational investors who trade out of the fear of adverse price movements.

This paper shows that this behavior is not necessarily irrational: perfectly rational investors may optimally “panic”. Although the word “panic” in a financial market setting may have many different connotations, here I associate the term with the general meaning of the word: a sense of urgency to act as soon as possible. I define panics in a market setting as trading at the first opportunity, which does not necessarily imply irrational behavior or price movements in a single direction (which do not arise here). I show that panics are a natural consequence of unconditional correlation of information. Under the standard assumption that private signals are independent conditional on the asset’s true value, signals are *unconditionally* correlated: traders are likely to receive similar news. Consequently, others’ trades are likely to move prices adversely, rationalizing the fear of adverse price movements, and motivating one to preemptively trade. This rational fear of adverse price movements appears in previous papers, specifically Bulow and Klemperer (1994), Brunnermeier and Pedersen (2005), and Pedersen (2009).² These papers have focused on explanations for price crashes, taking information as exogenously given.³ I instead consider the impact of panics on the informational content of trades, a topic that has not been previously addressed. Intuitively, if traders panic, they forgo the opportunity to acquire additional information through time-

¹The first article can be found at: <http://www.cultofmac.com/185279/apple-stock-hits-all-time-high-as-investors-rush-to-get-in-early-before-iphone-5/>. The second is from CNBC: <http://www.cnbc.com/id/49842457>.

²For other papers on rational panics, see Romer (1993), Smith (1997), Lee (1998), and Barlevy and Veronesi (2003).

³In Brunnermeier and Pedersen (2005) and Pedersen (2009), rational panics occur when traders try to front-run each other. In their model, an exogenous shock to a trader’s holdings forces her to liquidate her position. Other traders then try to sell before the distressed trader pushes the price down, resulting in a price crash that may have a ripple effect on other traders. A similar ripple effect occurs here, but it can happen in the absence of an exogenous shock. In Bulow and Klemperer (1994), a single seller sells K objects to $K + L$ bidders through a series of sequential auctions. Bidders trade off waiting for a lower price against the probability that they will not receive a good. As a result, both clustering of trades and price crashes arise.

consuming research. This failure to “do one’s homework” can result in trades that are based on weak information, causing prices to more frequently deviate away from fundamental values and take longer to converge. As other researchers have noted (Vives (1993)), convergence speeds are important because knowing that prices converge to an asset’s fundamental value is not practically useful if convergence is so slow that, by the time it occurs, the value has changed.

I build upon the classic trading model of Glosten and Milgrom (1985) in which privately-informed agents sequentially trade an asset with a market maker.⁴ In the period in which she arrives, a trader may buy or sell short a single unit of an asset (with no restrictions on short sales). To this model I add an information acquisition decision in which traders can wait to obtain additional information before trading. Requiring traders to wait to obtain information reflects the fact that information is fundamentally generated through time-consuming research and must be processed to generate one’s trading decision. It is this feature that sets this paper apart from the literature on information acquisition in financial markets, which instead assumes information can be acquired at a monetary cost. Equilibrium predictions are substantially different when information takes time to acquire: waiting to obtain information cannot be equivalently modeled as paying a monetary, opportunity cost.

To focus on the simplest possible timing decision, traders may trade only once in one of two periods: the period in which they arrive or in the subsequent period. Traders receive a private signal in the period in which they arrive and, should they choose to wait, an additional private signal in the subsequent period.⁵ In order to study preemptive trading, I adopt an overlapping timing structure such that if a trader waits, another trader may front-run her. The option value of waiting therefore depends upon the strategy of a trader’s successor. This payoff interdependence in a social-learning setting typically makes characterizing the set of

⁴In terms of model structure, my paper is related to the financial herding literature that utilizes the Glosten-Milgrom model (and, more broadly, to the herding literature that originated with Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992)). For a survey of the earlier financial herding literature, see Devenow and Welch (1996), and for more recent contributions, see Avery and Zemsky (1998), Park and Sabourian (2011), and Dasgupta and Prat (2007). In these papers, traders may ignore their private information to follow (herd) or trade contrary to their predecessors, which creates inefficiencies in information aggregation. Herding and contrarianism do not arise here, but convergence slows through another channel: not waiting to acquire information. More closely related to my paper are the relatively few papers that allow for trade timing, such as Malinova and Park (2012), and Ostrovsky (2012), who extends the insider trading model of Kyle (1985) to multiple traders. In these papers, information is exogenously given, so information acquisition plays no role.

⁵The substantive assumption is that, should one trade upon arrival, one cannot also trade after receiving the additional signal. This assumption is meant to capture, in a tractable way, the idea that information arrives continuously, but trades are necessarily discrete (trading continuously, even if technically feasible, would be prohibitively costly). Therefore, a trader contemplating a trade at any point in time must decide how much information to obtain before trading. For tractability, I focus on one such decision, which eliminates incentives to manipulate prices or not reveal one’s private information (as in Kyle (1985)).

equilibria difficult.⁶

As in Glosten and Milgrom (1985), all trades are made with a market maker. In the first version of the model (the *expected value* model), I assume that the market maker sets a single price equal to the expected value of the asset conditional on all available public information.⁷ In this model, I abstract from the bid and ask prices that arise from the adverse selection problem in order to focus on the strategic interaction between traders. Under intuitive restrictions on off-equilibrium beliefs and prices, I obtain a complete characterization of all equilibria. I then consider the *zero-profit* model, adopting the standard market microstructure setting in which competitive market makers post separate bid and ask prices, earning zero profits in expectation. Here, a complete characterization of the equilibria is complicated by the additional strategic interaction between the traders and the market maker. However, Section 5 demonstrates that the main insights of the expected value model extend to this case.

Panics (in which all traders buy or sell immediately upon arrival) occur when prices reflect uncertain public beliefs. Interestingly, because information is most valuable when uncertainty is high, this result implies that panics cause information to be forgone precisely when it is most valuable. This counter-intuitive finding results from the fact that the cost of waiting is endogenously determined by the price impact of others' trades, which is largest when uncertainty is high. This result goes against standard intuition from the literature on information acquisition in financial markets. As first noted by Grossman and Stiglitz (1980), there is typically strategic substitutability between traders' decisions to acquire information: when others acquire information, prices have strong informational content so that one has little incentive to do her own research.⁸ Conversely, here there is strategic complementarity: acquiring information is less costly when others are doing so. Intuitively, if others are not in a rush to trade, there is no need for you to rush either.

Strategic complementarity in traders' timing decisions leads to a multiplicity of equilibria for some parameterizations. This multiplicity is reflected in the effects of market commentary on real asset markets, where news of panic can lead to further panic. It may also suggest a role for market intervention. For example, if a trading halt (or circuit-breaker) intervention is sufficient to change expectations of panic, panics can in fact be soothed.⁹ In a related

⁶For examples of papers with interdependent payoffs, see Callander (2007), Ali and Kartik (2012), and Chamley (2007), which is discussed further below.

⁷As in the financial herding literature, prices reflect all past information, so there is no incentive to wait to learn from others' trades as there is in models of common investment opportunities, such as Chamley and Gale (1994), Gul and Lundholm (1995), and Chari and Kehoe (2004).

⁸See Barlevy and Veronesi (2010) for an interesting counterexample.

⁹For an example of a successful trading halt, see <http://www.ft.com/intl/cms/s/0/710240e6-1945-11e2-9b3e-00144feabdc0.html#axzz2NCmc7u66>: "Google shares fell as much as 10 per cent to \$676 before trading

finding, Chamley (2007) considers a Glosten-Milgrom setup in which market participants trade for short-term profit, unwinding their positions in the period after they first trade. His setting also produces strategic complementarity in acquiring information and a multiplicity of equilibria. However, in his model, information is acquired at a monetary cost and trade timing is exogenous, so the nature of the strategic complementarity is much different. He also does not explicitly study the effect of information acquisition on the informational content of prices, instead focusing on its ability to produce discontinuous trader behavior and trading volumes.

Because information is forgone when it is most valuable, price convergence is much different than in sequential trading models with monetary costs of information, such as that of Nikandrova (2012) and Lew (2013). In their models, more information is acquired when uncertainty is high and information is most valuable. There, starting from a price reflecting high uncertainty, prices converge quickly to a price near the true asset value and then convergence slows (or stops completely if there is a fixed cost component), whereas here, convergence slows down immediately. Simulation results show that these slowdowns are quantitatively significant, as much as doubling the time required to converge under certain realistic parameterizations. Panics also cause prices to more frequently deviate *away* from fundamental values.

In a particularly counter-intuitive and perhaps troubling result, I show that increasing the quality of the information immediately available can actually slow convergence overall. This result can contribute to the debate regarding the impacts of high-frequency traders in the market (Blais, Foucault, and Moinas (2011) and Hoffman (2013)). Technical innovations that make better quality information available more quickly can actually reduce the amount of information that is incorporated into asset prices.

The model can capture several existing empirical findings, including that the estimated probability of informed trading (Easley et al. (1996)) and persistent price impacts of trades (Hasbrouck (1991)) are lower when trading volume is higher. Section 6 develops several testable predictions and discusses existing evidence that is suggestive of the novel predicted relationships between the informational content of trades and volume and uncertainty. Motivated by the fact that the information of market participants is often difficult to ascertain, Kendall (2013) tests the predictions of the model in a controlled laboratory setting. There, rational panics arise as predicted, but traders also use a particular trading heuristic that leads to additional panics, resulting in even greater informational losses.

The paper is organized as follows. The expected value model is described in the following section and analyzed in Section 3. The results are used to study several implications of

was halted. Their partial recovery to \$695 when trading resumed ..."

panics in Section 4. Section 5 considers the zero-profit model in which market makers earn zero profits. Section 6 develops several testable implications of the model and relates them to existing empirical evidence. Section 7 concludes.

2 The Expected Value Model

The model is set in discrete time, $t = 1, 2, \dots, T$. A single asset of unknown value, $V \in \{0, 1\}$, may be traded in each period. Its value is realized at T . For convenience, I assume $T \rightarrow \infty$ but the results do not depend on this assumption.¹⁰ The prior belief that $V = 1$ is given by $p_1 \in (0, 1)$. In each period, n new risk-neutral traders enter the market. In the remainder of the model description and the analysis of Section 2, I focus on the case of $n = 1$. In Appendix E, I consider the case of $n > 1$ to establish a comparative static prediction with respect to volume.¹¹

Upon arrival, each trader receives a private, binary signal, $\underline{s}_t \in \{0, 1\}$, which matches V with probability $\underline{q} = Pr(\underline{s}_t = 1|V = 1) = Pr(\underline{s}_t = 0|V = 0) \in (\frac{1}{2}, 1)$. I identify a trader by the period in which she arrives, t . Given her signal, trader t may choose to buy, sell, or not trade, $\underline{a}_t \in \{B, S, NT\}$. If she trades, she leaves the market. If she chooses not to trade, she receives an additional, private, binary signal in period $t + 1$, $\bar{s}_{t+1} \in \{0, 1\}$, which matches V with probability $\bar{q} = Pr(\bar{s}_{t+1} = 1|V = 1) = Pr(\bar{s}_{t+1} = 0|V = 0) \in (\frac{1}{2}, 1)$ and may then trade, $\bar{a}_{t+1} \in \{B, S, NT\}$. All signals are assumed to be independent conditional on V . To allow for trades to occur during the time a trader waits to acquire the additional signal, each period is divided into two sub-periods with trader $t + 1$ trading prior to trader t if they trade in the same period.¹² The timing of possible trades is shown in Figure 1. I abbreviate the time of trade as R for rush ($\underline{a}_t \in \{B, S\}$) and W for wait ($\underline{a}_t = NT$). With this overlapping arrival structure, a trader t that chooses to acquire additional information may face up to two intervening trades: one from trader $t - 1$ (if she chose to wait) and one from trader $t + 1$ (if she chooses to rush). The complete history of trades, timing decisions, and prices, denoted \underline{H}_t and \bar{H}_{t+1} , are observed by all traders.¹³

A risk-neutral market maker posts a single price equal to the public belief about the value

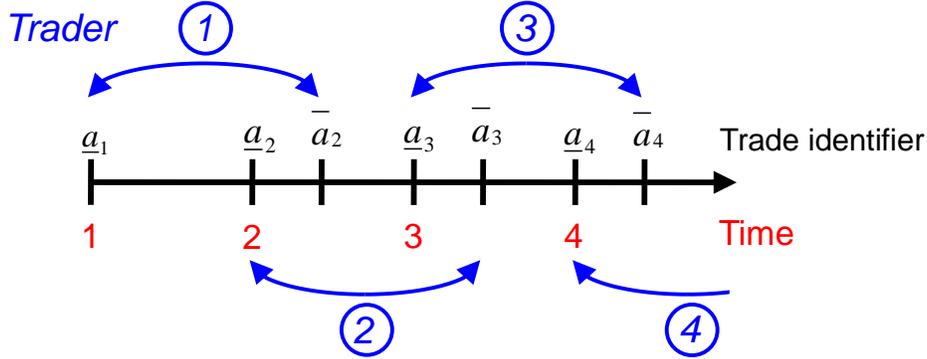
¹⁰For finite T , the strategy of the final trader must be considered separately as no further traders arrive to the market.

¹¹The $n > 1$ results are also used in Section 4 to numerically assess the impacts of panics.

¹² t is used to refer to the period containing the two sub-periods. A lower bar identifies values of variables (signals, actions, prices, etc.) in the first sub-period and an upper bar identifies values in the second sub-period.

¹³Formally, the histories can be defined recursively as $\underline{A}_t = \bar{A}_{t-1} \cup \bar{a}_{t-1}$, $\bar{A}_t = \underline{A}_t \cup \underline{a}_t$, $\underline{P}_t = \bar{P}_{t-1} \cup \bar{p}_{t-1}$, and $\bar{P}_t = \underline{P}_t \cup \underline{p}_t$, for $t = 2, \dots$ where $\underline{A}_1 = \emptyset$, $\bar{A}_1 = \underline{A}_2 = \underline{a}_1$, and $\underline{P}_1 = \bar{P}_1 = \underline{P}_2 = p_1$. $\underline{H}_t = \underline{A}_t \cup \underline{P}_t$ and $\bar{H}_{t+1} = \bar{A}_{t+1} \cup \bar{P}_{t+1}$ denote the joint action and price histories.

Figure 1: Trade Timing



Note: Arrows reflect the two periods in which each trader may trade. Trade identifier subscripts denote period, not trader identity.

of the asset, its expected value based upon all public information, $\underline{p}_t = E[V|\underline{H}_t] = Pr[V = 1|\underline{H}_t]$ or $\bar{p}_{t+1} = E[V|\bar{H}_{t+1}] = Pr[V = 1|\bar{H}_{t+1}]$. The expected payoffs to a trader who buys the asset at t or $t+1$ are then, $E[V|\underline{H}_t, \underline{s}_t] - p_t$ or $E[V|\bar{H}_{t+1}, \underline{s}_t, \bar{s}_{t+1}] - \bar{p}_{t+1}$, respectively. The expected payoffs from selling are identical but with opposite sign. \underline{p}_t plays a more prominent role in the analysis so I will generally refer to it as *the* price at t and denote it p_t , unless a distinction must be made.

I emphasize that the market maker is not a strategic player: prices may be best thought of as being determined by an entity that is more concerned about information aggregation than profits (for example, the owner of a prediction market). Given the strategies of the traders, the market maker can compute the public belief on the equilibrium path, but at off-equilibrium histories some assumption must be made. In particular, if the equilibrium strategy is for traders with both $\underline{s}_t = 0$ and $\underline{s}_t = 1$ to rush, then the price that arises should a trader deviate to wait depends upon what information about \underline{s}_t is assumed to be revealed by the deviation.¹⁴ Fortunately, a natural assumption arises in the model. Lemma 2 shows that, independent of any particular assumptions about off-equilibrium prices, traders with $\underline{s}_t = 0$ and $\underline{s}_t = 1$ choose to wait with the same probability in any equilibrium. As a consequence, after a delayed trade, no information about \underline{s}_t is revealed and therefore the public belief is unchanged. Given this, I assume that if a trader deviates to wait, the public belief and price are also unchanged. I restrict the analysis to equilibria that satisfy this restriction on prices,

¹⁴Other off-equilibrium histories are possible but it turns out that the assumption made about prices in these cases is not critical, so I make no particular assumption.

providing further justification after Lemma 2 in Section 3.2.¹⁵

Being a dynamic game of incomplete information, the appropriate solution concept for the model is sequential equilibrium (Kreps and Wilson, 1982).¹⁶ In addition to the restriction on price formation, I require that strategies be a function of only the payoff-relevant state (i.e. Markov strategies). While the sequence of past trades and/or prices could be used as coordination devices, I ignore such possibilities. The payoff-relevant state for a trader trading at time $t + 1$ is simply the price she faces. But, for a trader trading at time t , the price she faces and whether or not the previous trader, $t - 1$, rushed are both relevant. The price reflects any information revealed by the decision of $t - 1$ to wait, but when $t - 1$ waits, t 's expected profit from waiting is impacted by $t - 1$'s delayed trade. Formally, an equilibrium of the expected value model is defined as follows.

Equilibrium Definition: *An equilibrium of the expected value model consists of a set of behavioral strategies, $\underline{\sigma}_t : (\underline{H}_t, \underline{s}_t) \rightarrow \{B, S, NT\}$ and $\bar{\sigma}_{t+1} : (\bar{H}_t, \underline{s}_t, \bar{s}_{t+1}) \rightarrow \{B, S, NT\}$, a system of beliefs ν and a sequence of prices, \underline{p}_t and \bar{p}_{t+1} , such that:*

1. $\underline{\sigma}_t$ and $\bar{\sigma}_{t+1}$ are sequentially rational given beliefs ν ;
2. *There exist a sequence of completely mixed strategies $\{\underline{\sigma}_t^k\}_{k=1}^\infty$ and $\{\bar{\sigma}_{t+1}^k\}_{k=1}^\infty$ with $\underline{\sigma}_t = \lim_{k \rightarrow \infty} \underline{\sigma}_t^k$ and $\bar{\sigma}_{t+1} = \lim_{k \rightarrow \infty} \bar{\sigma}_{t+1}^k$ such that $\nu = \lim_{k \rightarrow \infty} \nu^k$ where ν^k denotes the beliefs derived from $\underline{\sigma}_t^k$ and $\bar{\sigma}_{t+1}^k$ using Bayes' rule;*
3. *Prices satisfy $\underline{p}_t = E[V|\underline{H}_t]$ and $\bar{p}_{t+1} = E[V|\bar{H}_{t+1}]$ as derived from $\underline{\sigma}_t$ and $\bar{\sigma}_{t+1}$ using Bayes' rule wherever possible;*
4. *At any history: (i) $\underline{\sigma}_t$ must specify the same behavioral strategy for any two traders who face the same price \underline{p}_t , the same timing decision of their immediate predecessor $I(\underline{a}_t = NT)$, and have the same signal \underline{s}_t ; (ii) $\bar{\sigma}_{t+1}$ must specify the same behavioral strategy for any two traders who face the same price \bar{p}_{t+1} , and have the same signals \underline{s}_t and \bar{s}_{t+1} ;*
5. *At an off-equilibrium history \bar{H}_t reached by a trader waiting: (i) beliefs ν are common among all traders and are such that the public belief does not change, $E[V|\bar{H}_t] = E[V|\underline{H}_t]$; (ii) prices are unchanged, $\bar{p}_t = \underline{p}_t$.*

Items 1-2 form the standard definition of sequential equilibrium. Item 3 specifies that

¹⁵The assumption can be thought of as a restriction on off-equilibrium beliefs as far as the other traders are concerned. Because the market maker is essentially a robot, however, formulating the restriction in terms of beliefs may lead to confusion. Under other assumptions about price formation, the general analysis is unaffected, with the main difference being that the cutoff prices in Theorem 1 are more difficult to compute.

¹⁶Sequential equilibrium, as opposed to weak Perfect Bayesian equilibrium, places restrictions on beliefs off-equilibrium which help to pin down the public belief, and therefore price. It is not, however, restrictive enough to completely pin down beliefs after a trader deviates to wait and thus an additional assumption is still necessary.

prices are set equal to the public belief as derived from the traders' strategies. Item 4 is the restriction of strategies to payoff-relevant states. Finally, item 5 is the restriction on prices and beliefs after a trader deviates to wait in the period she arrives. In the definition, I denotes the indicator function so that $I(\underline{a}_{t-1} = NT)$ is 1 if t 's predecessor waited and 0 otherwise.

Without loss of generality, strategies can be decomposed into a trading strategy (buy or sell) for each of the two periods in which a trader may trade and a timing strategy (rush or wait). When strategies are restricted to payoff-relevant states, we can define the probability with which a trader observing a particular first period signal (which I refer to as her type) waits as $\beta_x(p_t, I(\underline{a}_{t-1} = NT)) \equiv Pr(\underline{a}_t = NT | \underline{s}_t = x, p_t, I(\underline{a}_{t-1} = NT))$. When it doesn't lead to confusion, I drop the dependencies of β_x .

3 Analysis of the Expected Value Model

To characterize the equilibria of the expected value model, I first prove several intermediate results that are of interest on their own. The first lemma determines the optimal trading strategy taking the timing strategy as given. The second lemma proves that both types of traders must follow the same timing strategy in any equilibrium. Proposition 1 establishes the intuition motivating the paper: the presence of other traders reduces the expected profit from waiting. Proposition 2 establishes the interesting fact that, as prices become certain, traders wait to obtain additional information.

3.1 Optimal Trading Strategy

Lemma 1 provides the optimal trading strategy of a trader taking as given the timing strategies, $\beta_1(p_t, I(\underline{a}_{t-1} = NT))$ and $\beta_0(p_t, I(\underline{a}_{t-1} = NT))$. All proofs are contained in the Appendix, part C.

Lemma 1: *In any equilibrium:*

1. Any trader who rushes buys if $\underline{s}_t = 1$ and sells if $\underline{s}_t = 0$.
2. With timing strategies $\beta_1(p_t, I(\underline{a}_{t-1} = NT))$ and $\beta_0(p_t, I(\underline{a}_{t-1} = NT))$, at least one of which is strictly positive, for all p_t and $I(\underline{a}_{t-1} = NT)$, any trader who waits:
 - (a) buys if $\underline{s}_t = 1$ and $\bar{s}_{t+1} = 1$; sells if $\underline{s}_t = 0$ and $\bar{s}_{t+1} = 0$
 - (b) buys (sells) if $\underline{s}_t = 0$, $\bar{s}_{t+1} = 1$, and $g_0(\underline{q}, \bar{q}) \equiv (1 - \underline{q})\bar{q}NT_0 - \underline{q}(1 - \bar{q})NT_1 \geq (<)0$
 - (c) buys (sells) if $\underline{s}_t = 1$, $\bar{s}_{t+1} = 0$, and $g_1(\underline{q}, \bar{q}) \equiv \underline{q}(1 - \bar{q})NT_0 - (1 - \underline{q})\bar{q}NT_1 > (\leq)0$
 where $NT_0 = (1 - \underline{q})\beta_1(p_t, I(\underline{a}_{t-1} = NT)) + \underline{q}\beta_0(p_t, I(\underline{a}_{t-1} = NT))$, $NT_1 = \underline{q}\beta_1(p_t, I(\underline{a}_{t-1} = NT)) + (1 - \underline{q})\beta_0(p_t, I(\underline{a}_{t-1} = NT))$ are shorthand for $Pr(\underline{a}_t = NT|V = 0)$ and $Pr(\underline{a}_t = NT|V = 1)$, respectively.

For rushed trades, the result of Lemma 1 is intuitive: traders with good signals buy and those with bad signals sell. For delayed trades, the optimal trading strategy is also determined by comparing the trader's private belief to the price she faces, but the price she faces depends upon the timing strategies of the two types of traders, β_0 and β_1 . If $\beta_0 \neq \beta_1$, some information about \underline{s}_t is revealed to the market when a trader waits and so her optimal trading strategy may depend upon these strategies. If $\beta_0 = \beta_1$, no information is revealed, so that the resulting optimal trading strategy is independent of the timing strategies (as can be seen by setting $\beta_0 = \beta_1$ in Lemma 1). By assumption, a deviation to wait does not reveal information and thus the optimal trading strategy in the continuation game after a deviation to wait corresponds to setting $\beta_0 = \beta_1$ in Lemma 1. Also, note that other traders' strategies do not affect the delayed trading strategy of t because any information revealed by their trades is both known to the trader and reflected in the price.

An implication of Lemma 1 is that, except in the case of indifference when a trader's signals oppose each other, a trader always trades.¹⁷ This simple fact is a consequence of the trader always having an informational advantage and hence a profitable trade.¹⁸ As a result, some private information is revealed by each trade. In Section 4, this fact will be used to establish that prices converge asymptotically to the true asset value.

3.2 The Benefit of Additional Information

To determine the optimal timing strategy of a trader, one must calculate the value of waiting to obtain additional information: the expected profit from trading at $t + 1$ less the profit

¹⁷For concreteness, I assume a trader that is indifferent trades according to \bar{s}_{t+1} but the analysis is qualitatively robust to other assumptions, including that she does not trade.

¹⁸Another implication is that herding and contrarianism are precluded: a trader never ignores her private information and either copies or trades contrary to her predecessors.

from trading at t . When the benefit is positive (negative), a trader's best response is to wait (rush). The benefit from waiting depends upon the timing strategies of t because information may be revealed by her decision to wait. In equilibrium, the timing strategy for each type of trader must be consistent with the sign of the benefit.

The benefit from delaying also depends upon the timing and trading strategies of $t - 1$ and $t + 1$, because any trade that occurs while t waits affects the price she will trade at. Initially, I use generic notation to denote the actions of the other traders because several properties of the benefit to waiting can be established for any possible strategies of others. I denote a generic event that occurs during the time t waits as \hat{a} and the set of possible such events as A , so that $\hat{a} \in A$. I also abbreviate $Pr(\hat{a}|V = y)$ as \hat{a}_y , for $y \in \{0, 1\}$, so that, when summing over all possible events, $\sum_{\hat{a} \in A} \hat{a}_0 = \sum_{\hat{a} \in A} \hat{a}_1 = 1$.¹⁹ In Part A of the Appendix, I derive a general form of the benefit from waiting, which I denote as $B_x(p_t, \beta_0, \beta_1)$ for traders with $\underline{s}_t = x$. It is given by

$$B_x(p_t, \beta_0, \beta_1) = \frac{p_t(1 - p_t)}{Pr(\underline{s}_t = x)} \left[\sum_{\hat{a} \in A} \frac{\hat{a}_0 \hat{a}_1 f(\underline{q}, \bar{q}, \beta_0, \beta_1)}{Pr(\hat{a} \& \underline{a}_t = NT)} - (2\underline{q} - 1) \right] \quad (1)$$

where

$$f(\underline{q}, \bar{q}, \beta_0, \beta_1) = \begin{cases} (2\bar{q} - 1)(\underline{q}NT_0 + (1 - \underline{q})NT_1) & \text{if } \underline{s}_t = 1 \& g_1(\underline{q}, \bar{q}) \leq 0 \\ \underline{q}NT_0 - (1 - \underline{q})NT_1 & \text{if } \underline{s}_t = 1 \& g_1(\underline{q}, \bar{q}) > 0 \\ \underline{q}NT_1 - (1 - \underline{q})NT_0 & \text{if } \underline{s}_t = 0 \& g_0(\underline{q}, \bar{q}) < 0 \\ (2\bar{q} - 1)(\underline{q}NT_1 + (1 - \underline{q})NT_0) & \text{if } \underline{s}_t = 0 \& g_0(\underline{q}, \bar{q}) \geq 0 \end{cases}$$

The formula for the benefit (1) is only defined when at least one type of trader t waits with a positive probability. If both types rush with probability 1, then each of their benefits from deviating to wait depends upon the price set after such an off-equilibrium action. Without imposing any restriction on these off-equilibrium prices, however, Lemma 2 establishes that both traders must follow the same timing strategy in equilibrium.

Lemma 2: *In any equilibrium, traders with $\underline{s}_t = 0$ and $\underline{s}_t = 1$ must follow the same timing strategy, $\beta_1(p_t, I(\underline{a}_{t-1} = NT)) = \beta_0(p_t, I(\underline{a}_{t-1} = NT)) \forall p_t, I(\underline{a}_{t-1} = NT)$.*

The intuition behind Lemma 2 is instructive and the proof by contradiction follows the

¹⁹As an example, if $t - 1$ and $t + 1$ both rush, then $A = \{\underline{a}_{t+1} = B, \underline{a}_{t+1} = S\}$ is the set of possible events. From Lemma 1, $t + 1$ buys if $\underline{s}_{t+1} = 1$ and sells if $\underline{s}_{t+1} = 0$, so $\sum_{\hat{a} \in A} \hat{a}_1 = \sum_{\hat{a} \in A} \hat{a}_0 = \underline{q} + 1 - \underline{q} = 1$. If instead, both $t - 1$ and $t + 1$ trade during the time t waits, then each possible (joint) event is more complex and may consist of, for example, $t - 1$ buying and $t + 1$ waiting, etc.

intuitive argument closely. If, for example, a trader with a good signal were to wait more often than a trader with a bad signal, then the market maker would infer from the decision to wait that the first period signal is more likely to be good and thus raise the posted price. This increase in price benefits the trader with a bad signal (who is more likely to sell) more so than the trader with a good signal (who is more likely to buy). Thus, if the trader with a good signal is waiting with some positive probability, the trader with a bad signal must be waiting with certainty, contradicting the initial assumption that the trader with a good signal waits more often.

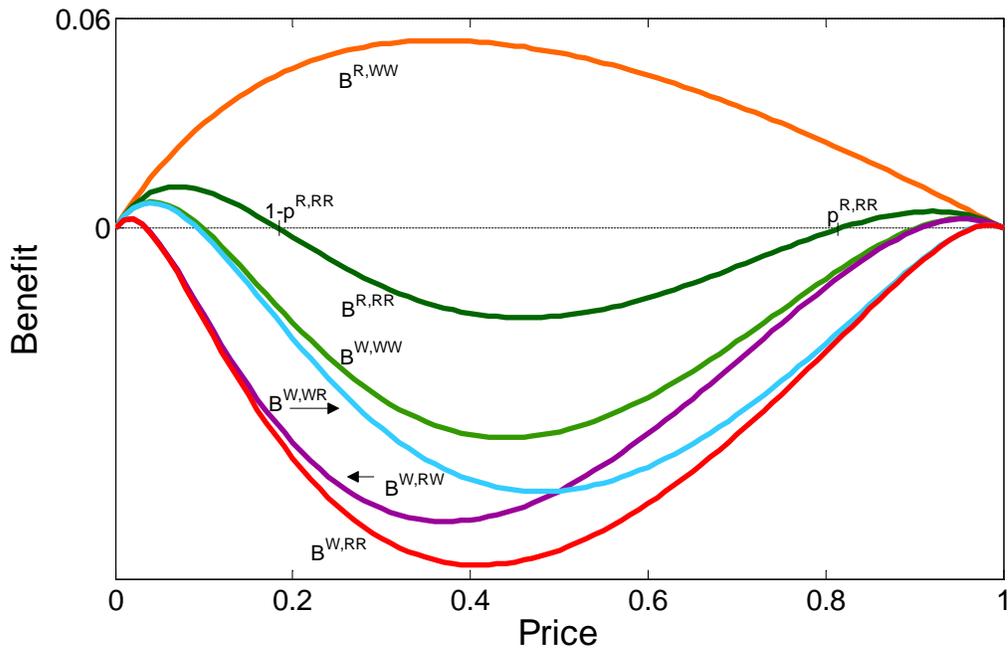
If both types of trader rush with probability 1, then their expected profit from deviating depends upon the information that is assumed to be revealed by the deviation. But, under the assumption made about off-equilibrium price formation, no information is revealed, which corresponds to setting $\beta_0 = \beta_1$ in (1). Therefore, Lemma 2 and this assumption together imply that we can set $\beta_0 = \beta_1 = \beta$ in (1) for the remainder of the analysis.²⁰ As shown in part A of the Appendix, with $\beta_0 = \beta_1$, the benefit function simplifies so that it no longer depends upon the timing strategies of the two types of traders. Therefore, I denote the simplified function $B_x(p_t)$.

Although Lemma 2 does not preclude mixed timing strategies, $\beta_1 = \beta_0 \in (0, 1)$, it proves useful to first consider a trader's benefit when all other traders use pure timing strategies. Because both types of a trader must follow the same timing strategy, a pure timing strategy for a trader can be described simply as rush or wait. There are six possible cases for the (joint) timing strategies of others from trader t 's perspective. First, $t-1$ is observed to either rush or wait. On the other hand, $t+1$'s timing decision must be anticipated by t and t may expect it to depend upon the price $t+1$ is expected to face. If $t-1$ has rushed, t knows that if she waits, $t+1$ faces the same price as t . But, if $t-1$ waits, t may rationally expect $t+1$'s decision to depend upon whether $t-1$ buys or sells (which is observed by $t+1$ but not t). The formulas for the benefit in each case are provided in part A of the Appendix. The benefit is denoted $B_x^{u,v_1v_2}(p_t)$ where the superscript, u, v_1v_2 , signifies the timing strategies, R or W , of the other traders. u corresponds to the observed timing decision of $t-1$. When $u = R$, $v_1 = v_2$ corresponds to $t+1$'s expected timing decision. When $u = W$, $v_1 (v_2)$ corresponds to the expected timing decision of $t+1$ if $t-1$ buys (sells).

To determine the best response of a trader to the strategies of others, it is helpful to

²⁰Note that, were a different assumption to be made about the price and beliefs after a deviation to wait, the expected benefit from deviating would depend upon the beliefs and price assumed and therefore the benefit function would be discontinuous in the timing strategies of the two traders at $\beta_0 = \beta_1 = 0$. A different assumption would also introduce an asymmetry between the two types of trader that Lemma 2 establishes cannot be present on the equilibrium path. These reasons provide further justification for the assumption made.

Figure 2: Example Benefit Functions when $\underline{s}_t = 1$, $\underline{q} = 0.75$, and $\bar{q} = 0.8$



Note: Only the zero-crossings for $B_x^{R,RR}(p_t)$, $1 - \hat{p}^{R,RR}$ and $\hat{p}^{R,RR}$ are labeled to reduce clutter.

study the properties of the six benefit functions as a function of price. To illustrate, Figure 2 plots all six benefit functions for a trader with $\underline{s}_t = 1$, given parameters $\underline{q} = 0.75$ and $\bar{q} = 0.8$. In general, each function is either always negative (not shown) or has one of the two shapes in the example: always positive or positive only near $p_t = \{0, 1\}$. Propositions 1 and 2 prove two of the more interesting properties of the benefit functions, while the remainder are established in Lemma C1 of Appendix C.

Proposition 1 establishes the motivating intuition for the paper: each additional trade that may occur during the time a trader waits reduces her expected benefit from waiting.²¹ The unconditional correlation of signals rationalizes the fear of expected adverse price movements.²²

Proposition 1: *Each additional, conditionally independent, informative, potential trade between t and $t + 1$ strictly reduces the benefit to waiting, $B_x(p_t, \beta_0, \beta_1)$, for all $p_t \in (0, 1)$ and all timing strategies, $\beta_0(p_t, I(\underline{a}_{t-1} = NT))$ and $\beta_1(p_t, I(\underline{a}_{t-1} = NT))$.*

The implications of Proposition 1 can be observed in Figure 2. Specifically, the benefit, $B_x^{R,WW}(p_t)$, is largest because, during t 's waiting period, neither $t - 1$ nor $t + 1$ trade. $B_x^{R,RR}(p_t)$ and $B_x^{W,WW}(p_t)$ are next largest because only one or the other trades. The remaining benefits are smaller because both trade, at least some of the time. From the formula for $B_x^{R,WW}(p_t)$ in Appendix A, we see that it is positive for all $p_t \in (0, 1)$ if and only if $\bar{q} > \underline{q}$. When $\bar{q} \leq \underline{q}$, it is zero for all p_t because the additional signal never changes t 's trading decision and so is of no value. This fact, combined with Proposition 1, immediately implies that all of the benefit functions are (weakly) negative for all p_t when $\bar{q} \leq \underline{q}$. For the more interesting case of $\bar{q} > \underline{q}$, also note that $B_x^{R,RR}(p_t)$ is greater than $B_x^{W,WW}(p_t)$ at all prices due to the weaker information revealed by rushed trades.

Proposition 2 captures a property of the benefit to waiting as prices become certain, $p_t \rightarrow \{0, 1\}$: the value of waiting approaches that of a trader that faces no possible trades between t and $t + 1$. This property arises because any intervening trade has a negligible impact on prices as they become certain. Importantly, however, it is not just the magnitude of the benefit that approaches that of a single trader. Proposition 2 also shows that the two benefits must have the same sign. Together, these properties ensure that the best response of a trader facing other traders approaches that of a trader who is alone in the market.

²¹The focus here is on the potential trades by $t - 1$ and $t + 1$, but the lemma is actually broader in application. Any realization of a random variable, x_i , (for example, a public earnings announcement) that is both informative (takes at least two possible values such that $Pr(x_i|V = 1) \neq Pr(x_i|V = 0)$ for at least one value) and independent (conditional on V) similarly reduces the benefit to waiting.

²²Even when $\underline{q} = \frac{1}{2}$, such that the best prediction of the price at $t + 1$ is the price at t , informative trades by other traders reduce the benefit to waiting because others' signals are unconditionally correlated with \bar{s}_t so that they are likely to trade in the same direction as you.

Defining the benefit function when no other informative intervening trades are possible as $B_x^{ST}(p_t, \beta_0, \beta_1)$, we have:

Proposition 2: *For all timing strategies, $\beta_0(p_t, I(\underline{a}_{t-1} = NT))$ and $\beta_1(p_t, I(\underline{a}_{t-1} = NT))$, the benefit function, $B_x(p_t, \beta_0, \beta_1)$, for any possible intervening informative trades satisfies:*

1. $\lim_{p \rightarrow \{0,1\}} |B_x(p_t, \beta_0, \beta_1) - B_x^{ST}(p_t, \beta_0, \beta_1)| \rightarrow 0$
2. $\lim_{p \rightarrow \{0,1\}} \text{sgn}(B_x(p_t, \beta_0, \beta_1)) = \text{sgn}(B_x^{ST}(p_t, \beta_0, \beta_1))$

When $\beta_1 = \beta_0$ and $\bar{q} > \underline{q}$, I previously established that the benefit for a trader alone in the market, $B_x^{R,WW}(p_t)$, is strictly positive for all $p_t \in (0, 1)$ and so an immediate consequence of Proposition 2 is that there exist prices sufficiently close to 0 and 1 such that all of the benefit functions are strictly positive. Therefore, in any equilibrium of the model with $\bar{q} > \underline{q}$, as prices become certain, panics cease to exist and all traders wait. This feature of the model may be surprising because, building on the intuition of Grossman and Stiglitz (1980), one may think that there would be a tendency to free ride off of the strong public information contained in prices as they become certain. While it is true that the private value of information decreases to zero as prices become certain, the difference here is that the cost of additional information also (endogenously) decreases to zero. As long as public beliefs are not perfectly certain, there remains a strictly positive benefit to obtaining the stronger information of the second signal.

Note that Propositions 1 and 2 did not impose the equilibrium restriction of Lemma 2 that the timing strategies of the two types of trader are the same. This fact becomes important in Section 5 where the robustness of results to the incorporation of bid and ask prices is analyzed.²³

In addition to the properties established by Propositions 1 and 2, Lemma C1 establishes that each benefit function has, other than those at $p_t = \{0, 1\}$, at most two zero crossing points, \hat{p} and $1 - \hat{p}$, which are symmetric with respect to $p_t = \frac{1}{2}$.²⁴ When the additional zero-crossings exist, I denote the one for $B_x^{u,v_1,v_2}(p_t)$ that occurs for $p_t \in [\frac{1}{2}, 1)$ by \hat{p}^{u,v_1v_2} . Under the restriction on prices and public beliefs on off-equilibrium histories, each \hat{p}^{u,v_1v_2} is a function of \underline{q} and \bar{q} only.²⁵ In the following section, I in turn consider each of the price

²³In fact, Lemmas 3 and 4 would continue to hold if the restriction that information is revealed by a deviation to wait was replaced by some other reasonable assumption about what information is revealed by a deviation to wait (i.e. $\underline{s}_t = 0$ deviates with a different probability than $\underline{s}_t = 1$). It is in this sense that the fundamental nature of the equilibria are not altered by different off-equilibrium assumptions.

²⁴The zero-crossings for $B_x^{W,RW}(p_t)$ and $B_x^{W,WR}(p_t)$ are not symmetric, but they do have a symmetry property with respect to each other. If $B_x^{W,RW}(p_t)$ crosses zero at p_t , then $B_x^{W,WR}(p_t)$ crosses zero at $1 - p_t$, and vice versa.

²⁵Under different assumptions, \hat{p}^{u,v_1v_2} would depend upon the assumed off-equilibrium beliefs.

regions delineated by the zero-crossings, \hat{p}^{u,v_1v_2} , to establish the fixed point of best responses within the region. In the discussion, I assume all of the price regions exist, as in Figure 2. However, Theorem 1 also applies when \underline{q} and \bar{q} are such that one or more of the regions does not exist (see footnote 26).

3.3 Equilibrium Trading Behavior

In this section, I use the properties of the benefit functions to determine the fixed point of best responses in timing strategies at any price. For some price regions, the fixed point of best responses is particularly straightforward, while for others it is more complex because it involves considering the prices $t + 1$ is expected to face. Here, I work out two simple cases to provide the intuition, leaving the more complex cases to the proof in Appendix C. The discussion here is for the case of $\bar{q} > \underline{q}$. When $\bar{q} \leq \underline{q}$, the unique equilibrium is for all traders to rush for the same reason that all traders rush at prices, $p_t \in (1 - \hat{p}^{R,RR}, \hat{p}^{R,RR})$, which is established below. First, however, I comment on the role of mixed timing strategies in equilibrium.

In order for a trader to mix between rushing and waiting, she must be indifferent between the two strategies. Because the timing decision of $t - 1$ is observed, whether or not she is using a mixed strategy is irrelevant for t . Therefore, if $t + 1$ is playing a pure strategy, t can only mix if p_t happens to be at the zero-crossing of the appropriate benefit function. For generic parameters, however, a price equal to a zero-crossing is never reached. I ignore non-generic mixing possibilities in the analysis and assume that traders wait if indifferent. On the other hand, if $t + 1$ is mixing between rushing and waiting with the appropriate probabilities, t can be induced to mix. When $t + 1$ mixes, the benefit to waiting for t is a linear combination of the benefits that result when $t + 1$ uses each of her pure strategies, with the weights being determined by the mixing probabilities. The possibility of mixed timing strategies in equilibrium is accounted for in what follows. Importantly, I show that one can still proceed by individually considering the price regions that are delineated by the zero-crossings of the pure strategy benefit functions.

The equilibrium timing strategy for a trader that faces a price, $p_t \in (1 - \hat{p}^{R,RR}, \hat{p}^{R,RR})$, nicely illustrates the idea of rational panic. If $t - 1$ waits, t 's benefit is one of $B_x^{W,WW}(p_t)$, $B_x^{W,WR}(p_t)$, $B_x^{W,RW}(p_t)$, or $B_x^{W,RR}(p_t)$, all of which are negative in this price range, so she will rush regardless of what $t + 1$ does. If, instead, $t - 1$ rushes and t were to wait, $t + 1$ would face a price of p_t and would rush because t waited. But, if $t + 1$ rushes, t 's benefit is $B_x^{R,RR}(p_t) < 0$, so t must rush. Thus, we see that all traders rush because of the off-equilibrium threat of the next trader rushing if a trader were to wait. Mixing is not possible because $t + 1$ can

never mix when t waits. Panic ripples throughout the sequence of traders as long as prices remain in this range.

Next, consider $p_t \in (1 - \hat{p}^{W,WW}, 1 - \hat{p}^{R,RR}] \cup [\hat{p}^{R,RR}, \hat{p}^{W,WW})$. This price range differs from the previous one in that $B_x^{R,RR}(p_t) > 0$ so that t best responds by waiting if $t - 1$ rushes, regardless of what $t + 1$ does. But, as in the previous case, if $t - 1$ waits, t 's benefit is negative, so she will rush regardless of what $t + 1$ does. Because t can condition her behavior on the observed timing decision of $t - 1$, we obtain what I call *conditional rushing* as the equilibrium timing strategy in this price range: if $t - 1$ waits, t rushes; if $t - 1$ rushes, t waits. Again, mixing is not possible because a trader's decision does not depend upon what her successor does.

For $p_t \in (0, 1 - \hat{p}^{W,WR}] \cup [\hat{p}^{W,WR}, 1)$, the proof of Theorem 1 establishes that all traders wait, reflecting Proposition 2. For the remaining price range, $p_t \in (1 - \hat{p}^{W,RR}, 1 - \hat{p}^{W,WW}] \cup [\hat{p}^{W,WW}, \hat{p}^{W,RR})$, the reasoning that determines equilibrium timing strategies is more complex. We can rule out rushing when $t - 1$ rushes because, in this case, the benefit is either $B_x^{R,RR}(p_t)$ or $B_x^{R,WW}(p_t)$, both of which are positive in this price range, so t will wait independent of what $t + 1$ does. Therefore, only conditional rushing or waiting are possible equilibrium strategies. When $t - 1$ waits, t 's best response depends upon what she expects $t + 1$ to do, which may in turn depend upon the two possible prices reached after $t - 1$ trades. Although t does not know whether $t - 1$ will buy or sell, she knows the resulting prices after each possible trade because they are functions of p_t and \bar{q} only. I denote the price reached after a buy decision as $p_t^+ = p_{t+1} | \bar{a}_t = B$ and the price reached after a sell decision as $p_t^- = p_{t+1} | \bar{a}_t = S$. If p_t^+ and p_t^- lie in price ranges for which it has previously been determined that $t + 1$ must follow a specific equilibrium timing strategy, then t can easily anticipate what $t + 1$ will do and thus best respond accordingly. However, for any given p_t , p_t^+ and p_t^- don't necessarily lie in any particular price range so there are many cases to consider. Also, if either p_t^+ or p_t^- lie again in a price range for which the best response of $t + 1$ is not known, what t expects $t + 1$ to do depends upon what she expects $t + 1$ expects $t + 2$ to do. Of course, this reasoning may extend indefinitely.

Progress can be made by realizing that $t + 2$ may in fact be in an identical situation to t . If all of $t - 1$, t , and $t + 1$ wait and the trading decisions of $t - 1$ and t turn out to be opposite, $t + 2$ faces an identical situation to t and, given the restriction that strategies must be the same in payoff-equivalent situations, $t + 2$ must follow the same timing strategy as t . Therefore, rather than thinking of the game between a trader and her successor, one can re-frame the problem as a static game between the trader and the "neighboring" traders at prices which can be reached by delayed trades.

To describe this translation of the dynamic game into a static game formally, I introduce

the concept of an unrestricted price chain, $\mathbb{P}(\tilde{p})$: the set of all of possible prices that can be reached from delayed trades starting at some price, $\tilde{p} \in [\frac{1}{2}, \bar{q}]$.

$$\mathbb{C}^U(\tilde{p}) \equiv \left\{ p \mid p = \frac{\tilde{p}\bar{q}^k}{\tilde{p}\bar{q}^k + (1 - \tilde{p})(1 - \bar{q})^k}, k = -\infty \dots \infty \right\}$$

I also define a restricted price chain as the set of prices in $\mathbb{C}^U(\tilde{p})$ for which the equilibrium timing decision is not necessarily unique.

$$\mathbb{C}^R(\tilde{p}) \equiv \mathbb{C}^U(\tilde{p}) \cap ((1 - \hat{p}^{W,WR}, 1 - \hat{p}^{W,WW}) \cup [\hat{p}^{W,WW}, \hat{p}^{W,WR}))$$

If $\hat{p}^{W,WR}$ does not exist, I define the restricted price range to be the null set. The union of restricted price chains for all $\tilde{p} \in [\frac{1}{2}, \bar{q}]$ consists of all prices, $p \in (1 - \hat{p}^{W,WR}, 1 - \hat{p}^{W,WW}) \cup [\hat{p}^{W,WW}, \hat{p}^{W,WR})$ and no two restricted price chains have any price in common. These facts together imply that one can partition this price range into price chains and describe the equilibrium strategies for each price chain.

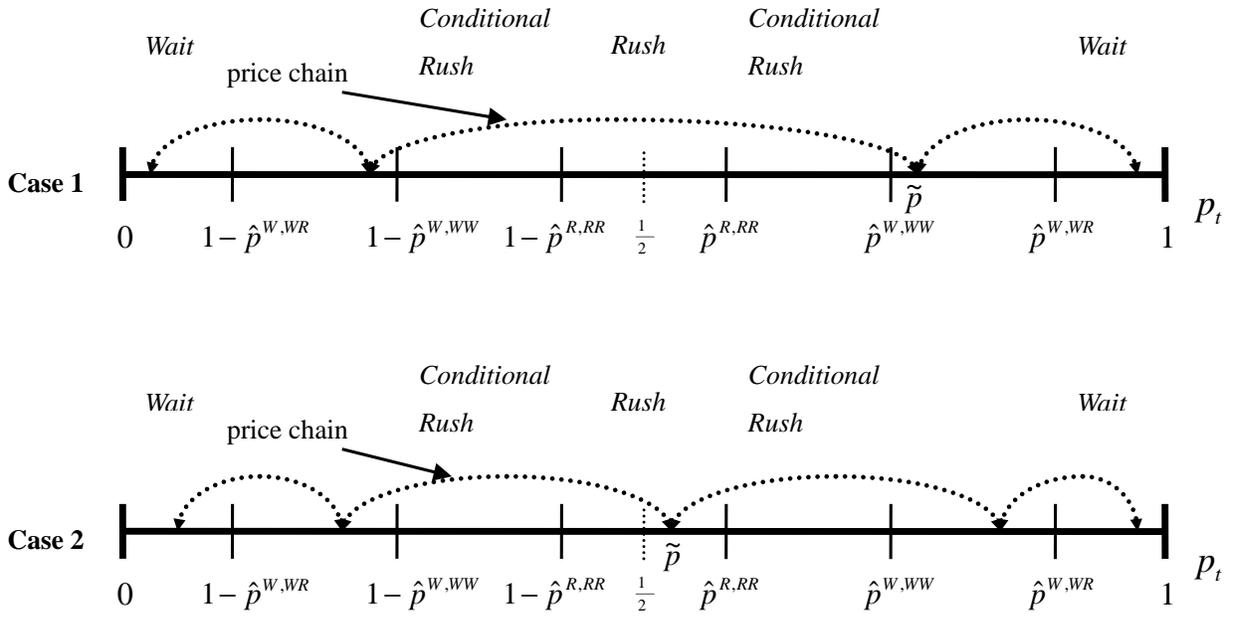
Two examples of price chains are illustrated in Figure 3. The prices in an unrestricted price chain for k greater than some finite \hat{k} (and smaller than some finite $-\hat{k}$) must lie in a price region in which traders always wait due to Proposition 2. I loosely refer to this fact as the unrestricted price chain “ending” in a region in which traders wait, although strictly speaking there is no actual end to a price chain. The equilibrium timing decisions for those regions in which the equilibrium timing decision is unique are labeled accordingly.

In Case 1, the unrestricted price chain does not pass through $p_t \in (1 - \hat{p}^{W,WW}, 1 - \hat{p}^{W,WR})$ and therefore the only prices for which the equilibrium timing strategy is known are at its ends. In this case, multiple possible equilibrium timing strategies exist at prices in the associated restricted price chain. Intuitively, if each of a trader’s neighbors in the chain are waiting, a trader faces $B_x^{W,WW}(p_t)$ and waits. If they are conditionally rushing, a trader faces a strictly negative benefit when $t - 1$ waits and conditionally rushes. The static game is essentially a coordination game between a trader and her neighbors, so as one might expect, there also exists an equilibrium in mixed strategies, as the proof in Appendix C demonstrates.

In Case 2, the unrestricted price chain does pass through $(1 - \hat{p}^{W,WW}, 1 - \hat{p}^{W,WR})$, and the fact that the traders in this interval rush or conditionally rush forces all traders in the chain to conditionally rush. Theorem 1, the main result of the paper, establishes these claims formally, characterizing the equilibria of the trading model.²⁶

²⁶When a price range in Theorem 1 does not exist, the next innermost price range specifies the timing strategies. For example, when $\hat{p}^{R,RR}$ does not exist, part 2b applies to all prices, $p_t \in (1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})$ and when $\hat{p}^{W,WR}$ does not exist, traders at all prices wait.

Figure 3: Equilibrium Timing Strategies and Sample Price Chains ($\bar{q} > \underline{q}$)



Note: The infinite set of prices at each “end” of the price chain are not shown. The cutoff, $\hat{p}^{W,RW}$ (between $\hat{p}^{W,WW}$ and $\hat{p}^{W,WR}$) is not shown because it does not directly play a role in determining the equilibrium timing strategies. Similarly, $1 - \hat{p}^{W,RW}$ (between $1 - \hat{p}^{W,WR}$ and $1 - \hat{p}^{W,WW}$) is also not shown.

Theorem 1: *All equilibria are characterized by the trading strategies of Lemma 1 and the following timing strategies, $\beta_1(p_t, I(\underline{a}_{t-1} = NT)) = \beta_0(p_t, I(\underline{a}_{t-1} = NT)) \equiv \beta(p_t, I(\underline{a}_{t-1} = NT))$.*

1. For $\bar{q} \leq \underline{q}$: $\forall p_t \in (0, 1)$, a trader rushes
2. For $\bar{q} > \underline{q}$:
 - (a) For $p_t \in (1 - \hat{p}^{R,RR}, \hat{p}^{R,RR})$, a trader rushes
 - (b) For $p_t \in (1 - \hat{p}^{W,WW}, 1 - \hat{p}^{R,RR}] \cup [\hat{p}^{R,RR}, \hat{p}^{W,WW})$, a trader conditionally rushes
 - (c) For $p_t \in (1 - \hat{p}^{W,WR}, 1 - \hat{p}^{W,WW}] \cup [\hat{p}^{W,WW}, \hat{p}^{W,WR})$, partition the interval into the restricted price chains $\mathbb{C}^R(\tilde{p})$ for each $\tilde{p} \in [\frac{1}{2}, \bar{q})$. Then,
 - i. $\forall \mathbb{C}^R(\tilde{p})$ such that $\mathbb{C}^U(\tilde{p}) \cap (1 - \hat{p}^{W,WW}, \hat{p}^{W,WW}) \neq \emptyset$, a trader with $p_t \in \mathbb{C}^R(\tilde{p})$ conditionally rushes
 - ii. $\forall \mathbb{C}^R(\tilde{p})$ containing only one price, p_s , and such that $\mathbb{C}^U(\tilde{p}) \cap (1 - \hat{p}^{W,WW}, \hat{p}^{W,WW}) = \emptyset$, a trader at p_s waits
 - iii. $\forall \mathbb{C}^R(\tilde{p})$ not satisfying (i) or (ii), a timing strategy is part of an equilibrium if and only if it is one of: (A) all traders at prices $p_t \in \mathbb{C}^R(\tilde{p})$ wait; (B) all traders at prices $p_t \in \mathbb{C}^R(\tilde{p})$ conditionally rush; (C) two or more traders at prices $p_t \in \mathbb{C}^R(\tilde{p})$ mix between conditionally rush and wait and the remaining traders conditionally rush or wait
 - (d) For $p_t \in (0, 1 - \hat{p}^{W,WR}] \cup [\hat{p}^{W,WR}, 1)$, each trader waits

Here, rush means $\beta(p_t, 0) = \beta(p_t, 1) = 0$, conditionally rush means $\beta(p_t, 1) = 0$ and $\beta(p_t, 0) = 1$, and wait means $\beta(p_t, 0) = \beta(p_t, 1) = 1$.

Theorem 1 establishes that panics occur when uncertainty in the market is high, in accord with intuition. Panics are predicted to be more likely to occur, for example, in hot new technology stocks than in stocks for which the fundamental value of the company is well known, such as well-researched blue chip stocks. This result is driven by the feature of the model that the price impacts of other traders are largest when uncertainty is high. Without this effect, traders would be more likely to acquire more information when uncertainty is high because information is most valuable at this time. But, the fact that panics occur when uncertainty is high also means that their impacts on price convergence are significant because when information is most valuable, it is not acquired. I study price convergence in detail in Section 4.

The multiplicity of Theorem 1 2c, part (iii), allows for expectations to affect optimal behavior and thus opens the door to the possibility that public information that influences expectations (such as media coverage, etc.) may shift equilibria, inducing or calming panics. The multiplicity exists over certain price ranges only, but, for some parameterizations, these

price ranges can span almost the entire range of possible prices.²⁷ On the other hand, for some parameterizations, the equilibrium is unique.²⁸

2c, part (iii), states that mixed timing strategies are possible but does not provide a complete characterization. I don't pursue a complete characterization but, as Appendix B shows, any equilibrium involving mixed strategies is unstable in the pseudo-dynamic sense: any small change in the strategy of one of the traders will tend to lead away from the mixed equilibrium towards one of the pure strategy equilibria. The pure strategy equilibria, on the other hand, are stable in that a small change in strategy will tend to reverse itself. Given their stability, the pure strategy equilibria are the focus in the following analysis of price convergence.

4 Effects of Panics

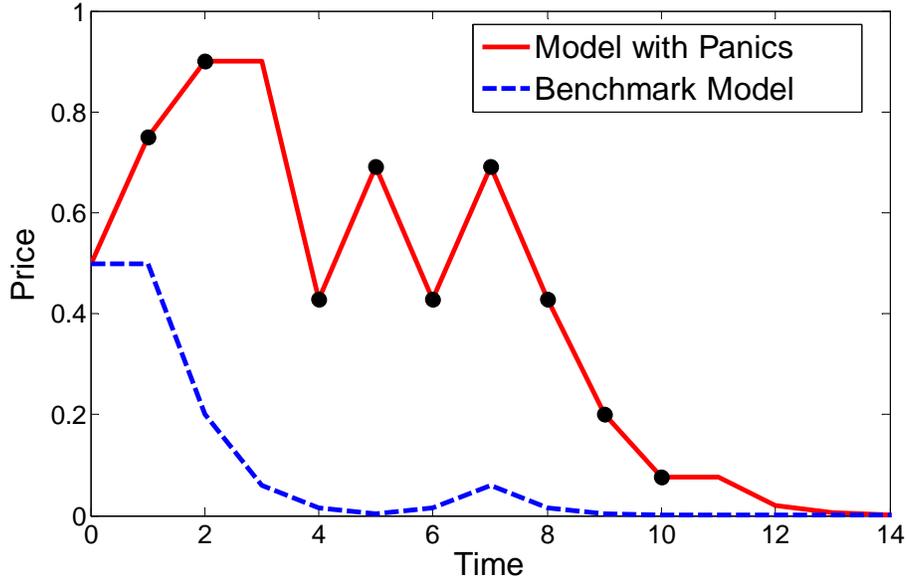
In this section, I discuss the impact of panics on the ability of prices to reflect fundamental values. In addition, I demonstrate the potential for panic cycles.

I begin with an illustrative example using $\underline{q} = 0.75$ and $\bar{q} = 0.80$. Under this parameterization, the equilibrium is to rush for all $p_t \in (0.18, 0.82)$ and conditionally rush for all $p_t \in (0.10, 0.18] \cup [0.82, 0.90)$. Figure 4 plots a randomly generated price path for the case of $V = 0$. For comparison, I consider a benchmark model in which all traders wait, ensuring all potential information is incorporated into prices. Figure 4 illustrates two (related) detrimental effects of panics on asset prices. First, because panics imply trading on weaker information, prices are more likely to diverge away from fundamental values, as seen at $t = 1, 2$. Second, although prices eventually converge to $V = 0$, they converge more slowly than in the benchmark model. While this example only illustrates the possibility of these negative effects, in the following subsections, I show that increasing deviations from fundamentals and longer convergence times also occur in expectation.

²⁷When $\hat{p}^{W,WW}$ doesn't exist and $\bar{q} < \hat{p}^{W,WR}$, then there are no restricted price chains in which the equilibrium timing strategy is unique. Then, the equilibrium timing strategy is only unique for $p_t \in (0, 1 - \hat{p}^{W,WR}] \cup [\hat{p}^{W,WR}, 1)$, which may be very small.

²⁸When $\hat{p}^{W,WW}$ exists and $(\hat{p}^{W,WW})^- > 1 - \hat{p}^{W,WW}$, it is possible to show that all unrestricted price chains contain a price $p \in (1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})$ and therefore 2c, part (ii) ensures conditionally rushing is the unique equilibrium timing strategy for $p_t \in (1 - \hat{p}^{W,WR}, 1 - \hat{p}^{W,WW}) \cup [\hat{p}^{W,WW}, \hat{p}^{W,WR})$ so that the equilibrium timing strategy is unique for all prices. If $(\hat{p}^{W,WR})^- < 1 - \hat{p}^{W,WR}$, the opposite is true: the unique equilibrium has traders waiting at every $p_t \in (1 - \hat{p}^{W,WR}, 1 - \hat{p}^{W,WW}) \cup [\hat{p}^{W,WW}, \hat{p}^{W,WR})$ because all restricted price chains are singletons so 2c, part (i) applies to this entire range of prices. It is also possible to show that there is no parameterization in which both 2c, part(i) and 2c, part(ii) apply.

Figure 4: Sample Price Path for $n = 1$, $\underline{q} = 0.75$, $\bar{q} = 0.80$, $V = 0$



Note: Solid dots correspond to the periods in which a trader optimally panicked.

4.1 Price Convergence

A standard result from the literature, beginning with Glosten and Milgrom (1985), is that prices converge to the true value of the asset. This result is easily shown to extend to the model considered here. From Lemma 1, as long as prices are not equal to 0 or 1, there always exists a trader who is willing to buy and another who is willing to sell, and one of these traders must have a private belief about the value of the asset that is further from the true value than the public belief. These facts, as shown in Avery and Zemsky (1998), Proposition 4, imply that public beliefs, and therefore prices, must converge to the true value of the asset.

Of more interest here is the rate of convergence. Panics cause traders to trade on lower quality information than what they could have obtained, which can slow the rate of price convergence. I focus on the more interesting case of $\bar{q} > \underline{q}$ because when $\bar{q} \leq \underline{q}$, although the unique equilibrium is to panic at every price, panics have no impact on convergence because all trades are based upon \underline{s}_t in both models.²⁹ When $\bar{q} > \underline{q}$, on the other hand, traders that wait trade according to \bar{s}_t , revealing information of quality \bar{q} , but traders that panic trade according to \underline{s}_t , revealing information of weaker quality, \underline{q} .

In trading models with binary signals, no single number defines the rate of convergence

²⁹Panics actually speed convergence through the mechanical effect of all trades being one period earlier in time. I do not emphasize this effect because it relies on the specific assumption of a single period of delay to acquire information and there being an initial period.

because it varies with the public belief. Thus, convergence is typically measured by the expected number of periods it takes the price (or some convenient function of price such as the log-odds ratio) to reach some specified value, conditional on knowing the true value of the asset. As in Glosten-Milgrom (1985), one can use Wald's lemma to derive an analytical expression for the expected number of periods, \tilde{T} , for the log-odds ratio, $\log\left(\frac{p_{\tilde{T}}}{1-p_{\tilde{T}}}\right)$, to reach a particular value in the benchmark model.³⁰ However, in the model with panics, there is no corresponding analytical expression. Knowing only the number of buys and sells is not sufficient to determine the price because the price ranges that are passed through, and therefore which traders rush or wait, depend upon the order of the buy and sell decisions. This complication makes it impossible to derive a closed form expression for the expected price (or log-odds ratio), making it difficult to perform comparative statics on the effects of changes in parameters on the rate of convergence.

An indirect approach allows me to obtain an interesting result: an increase in the first period signal strength, \underline{q} , can actually *slow* convergence. Basic intuition in a model with exogenous information would suggest that providing higher quality information would tend to increase the rate of convergence. But, this intuition does not apply here because increasing the quality of information in the first period induces traders to rush more often through two effects. First, it increases the profit from trading in the first period, making waiting less attractive. In the absence of other traders, this effect on its own is insufficient to cause traders to rush. With the fear of price movements, however, there is a second effect: if one waits and the subsequent trader rushes, her trade has a larger impact on the price, reducing one's profits in expectation. Together these effects may induce a trader to trade on lower quality information. Proposition 3 formalizes this intuition by demonstrating that, for any second period signal strength, one can always find first period signal strengths such that convergence is slower when more information is available.³¹

Proposition 3: *For all $\bar{q} \in (\frac{1}{2}, 1)$ and all $p_1 \in (\frac{1}{2}, 1)$, there exist $\underline{q}_l, \underline{q}_h \in (\frac{1}{2}, \bar{q})$ with $\underline{q}_l < \underline{q}_h$, and a price, $\tilde{p} > p_1$, such that the expected time for prices to converge to any cutoff price $p \geq \tilde{p}$, conditional on $V = 1$, is strictly larger under \underline{q}_h than under \underline{q}_l .*

The reason Proposition 3 only guarantees convergence is slower to prices greater than or equal to some \tilde{p} is because panics initially speed up convergence mechanically by forcing

³⁰Specifically, the expected number of periods for the log-odds ratio to exceed $\log\left(\frac{b}{1-b}\right)$ for some public belief, $b \in (0, 1)$, when the asset is worth $V = 1$ can be shown to be $E[\tilde{T}] = \frac{\log\left(\frac{b}{1-b}\right) - \log\left(\frac{p_1}{1-p_1}\right)}{(2\bar{q}-1)\log\left(\frac{\bar{q}}{1-\bar{q}}\right)}$ where p_1 is the initial public belief.

³¹Proposition 3 is stated in terms of convergence when $V = 1$, but a symmetric proposition is easily proven for the case of $V = 0$.

trades to occur earlier in time. This caveat is relatively innocuous because one is normally interested in the time it takes prices to converge to a value close to the true asset value. The basic idea behind the proof is that, for \underline{q} sufficiently close to $\frac{1}{2}$, all traders wait in the unique equilibrium because all benefit functions are strictly positive. Thus, at this low value of \underline{q} convergence proceeds at the rate of the benchmark model where all trades reveal signals of strength \bar{q} . On the other hand, for \underline{q} sufficiently large, any equilibrium involves price regions for which the unique trading strategy is to rush. Therefore, at least one trade must be based on lower quality information causing prices to take longer to converge in expectation.

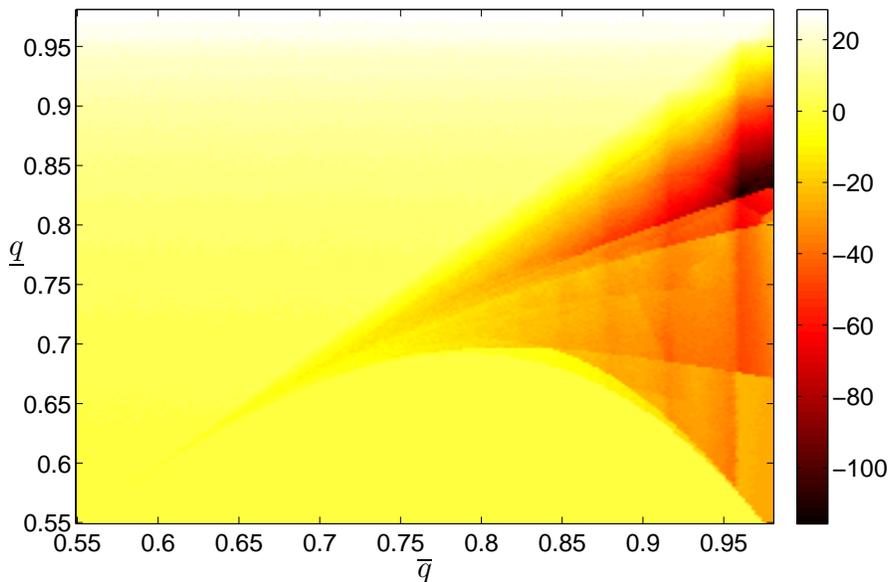
To demonstrate that the quantitative impact of panics on convergence speeds can be substantial, in the absence of a closed-form expression for convergence times, I rely on numerical simulation. Specifically, I simulate the time necessary to reach an average price of 0.99 when the true value of the asset is 1 for both the expected value model and the benchmark model, then calculate the percentage difference relative to the benchmark model.³² To understand how the quantitative effect of panics on price convergence varies with the signal strengths, \underline{q} and \bar{q} , Figure 5 provides a heat map of the simulated percentage slowdown for each combination of parameters, scaled such that black represents the largest slowdown (negative numbers) and white represents the largest speed up (positive numbers).

From Figure 5, we first note that slowdowns in convergence can be dramatic, with the expected value model taking more than twice as long to converge in the neighborhood of $\underline{q} \in [0.82, 0.84]$ and $\bar{q} \in [0.96, 0.98]$. Also, there is clearly a complicated relationship between the parameters and the slowdown in price convergence, suggesting that no simple comparative static results exist. Convergence is noticeably discontinuous in the parameters at certain boundaries due to the fact that a small change in parameters can cause the discrete prices reached by trades to jump across a cutoff price that delineates panicking and waiting. Generally, convergence slows down as \underline{q} increases for fixed \bar{q} , as long as $\underline{q} < \bar{q}$, reflecting Proposition 3. Increasing \bar{q} for a fixed \underline{q} , on the other hand, has a non-monotonic effect. Although increasing \bar{q} intuitively increases the benefit to waiting, if one knows that by waiting, the asset value may be revealed with near certainty because of a delayed trade by your predecessor, it can cause one to rush to avoid getting almost zero profit at $t + 1$. Thus, initial increases in \bar{q} raise the benefit but at higher values, the benefit begins to fall. This non-monotonicity carries over to the size of the ranges of prices for which panics occur and, therefore, the rates of price convergence.

While Figure 5 demonstrates substantial slowdowns in convergence speeds due to panics,

³²To be conservative, in all simulations, I assume all traders wait in the region for which a multiplicity of equilibria exists so that convergence results for the expected value model are actually upper bounds on the speed of convergence. If traders instead rush, the average price would be lower because $E[p_{t+1}|p_t, V = 1]$ is easily shown to be increasing in q and rushing implies trading on a signal of weaker strength, $\underline{q} < \bar{q}$.

Figure 5: Heat Map of Simulated Percentage Slowdowns in Price Convergence ($n = 1$)

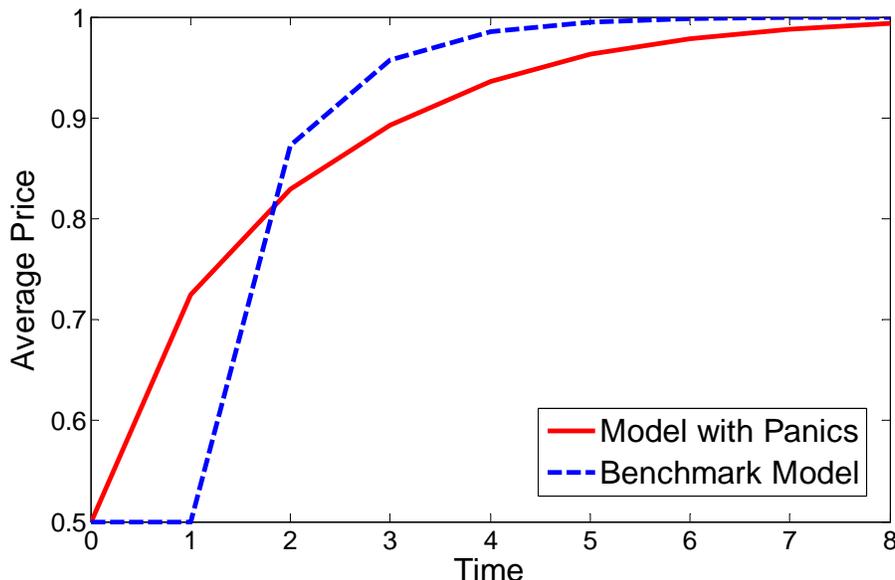


Note: The color reflects the percentage difference in average convergence times (to 99% of the asset's true value) divided by the average convergence time of the benchmark model. Negative numbers correspond to the expected value model converging more slowly. Each average is over 100,000 simulated price paths.

when only a single trader arrives each period ($n = 1$) the most substantial slowdowns are only observed when the second signal is very strong. By increasing n so that multiple traders arrive each period, waiting costs are increased so that traders may be more willing to panic. With $n > 1$, a full equilibrium characterization is difficult to obtain, but in Appendix E I establish an upper bound on the amount of waiting that can occur in any equilibrium. Simulations then provide an upper bound on average prices at each point in time. To illustrate an example with $n > 1$, Figure 2 plots the average price over time when $n = 4$, $\underline{q} = 0.7$, and $\bar{q} = 0.8$. Under this parameterization, traders must rush at all $p_t \in (0.13, 0.87)$ and I assume they wait outside this range.

Figure 6 demonstrates that, although prices initially arise more rapidly in the model with panics due to the mechanical effect of trading earlier, panics cause an overall slowdown in convergence. In the benchmark model, the average price takes about 4.5 periods to reach 99% of the asset's true value, whereas in the expected value model, the same price level is only reached after 7.5 periods, a slow down of about 66%. Thus, by increasing the number of traders arriving in each period, one can demonstrate substantial slowdowns even with what might be considered more reasonable signal strengths. Furthermore, $n > 1$ seems realistic in real markets with many traders.

Figure 6: Simulation of Average Price Paths for $n = 4$, $\underline{q} = 0.7$, $\bar{q} = 0.80$, $V = 1$



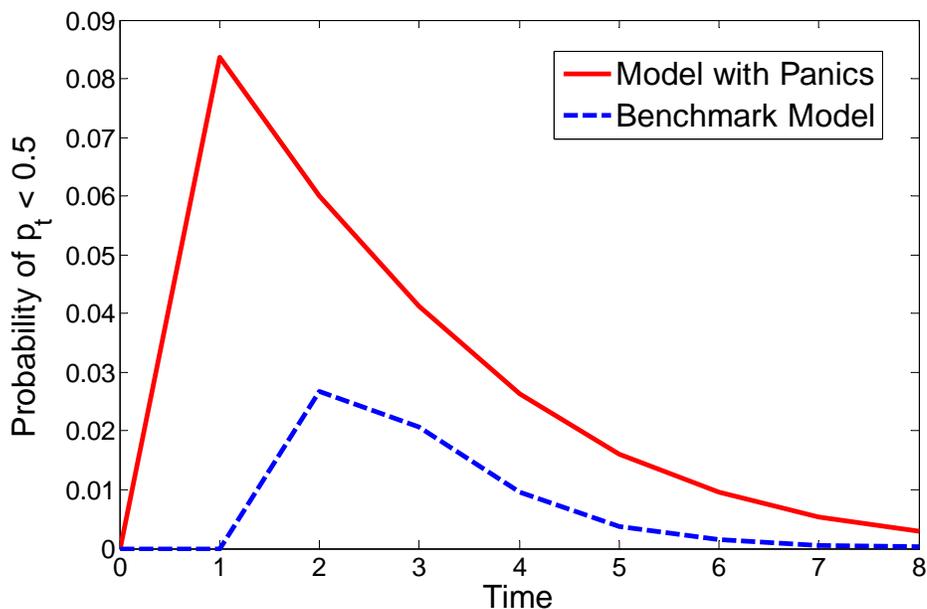
Note: Price paths are determined by averaging over 100,000 simulations. Each price reflects all trades in the period (both rushed and delayed).

4.2 Increases in Mispricing

In addition to longer convergence times, Figure 4 illustrates that trading on weaker information during panics increases the probability that prices move away from fundamentals. Figure 7 plots the probability of observing a price that is farther away from the fundamental value ($V = 1$) than the initial price ($p_1 = 0.5$) for the same parameters for which average convergence was studied in Figure 6.³³ Figure 7 clearly demonstrates that increases in mispricing are much more frequent when traders may panic and that this effect persists over time. In fact, at $t = 9$, the probability of observing $p_t < 0.5$ is more than an order of magnitude higher in the expected value model than in the benchmark model. Widening mispricing is important not only theoretically, but also may impact any arbitrage activity that attempts to correct for it. An arbitrageur that knows the true asset value and buys the asset at $t = 0$ would face considerably larger arbitrage risk under the expected value model where prices have a much higher chance of moving further away from fundamental value before they move towards it. Thus, arbitrageurs may be less active in attempting to correct for the mispricing (Shleifer and Vishny (1997)).

³³The simulated probability of observing a price, $p_t < 0.5$, is a lower bound because the simulations assume the most amount of waiting possible. Because $E[p_{t+1}|p_t, V = 1]$ is increasing in q , if traders actually rush more than assumed, mispricing would be more frequent.

Figure 7: Simulation of Probability of Increase in Mispricing for $n = 4$, $\underline{q} = 0.7$, $\bar{q} = 0.80$, $V = 1$



Note: Probability of $p_t < 0.5$ is determined by averaging over 100,000 simulations.

4.3 Panic Cycles

The illustrative example of Figure 4 also shows that panic cycles can occur in equilibrium. During the initial period of uncertainty about the value of the asset, all traders panic, acting on relatively weak information about the value of the asset. As the value of the asset becomes more certain, a trader optimally waits to obtain additional information ($t = 3$). When a strong signal is obtained, prices fall back into the region of uncertainty, generating further panic and oscillations in prices. Although eventually panics cease to exist, such panic cycles arise naturally during the early life of an asset.

Figure 4 also shows that there is a sense in which price crashes can arise endogenously in the model when prices reach the point at which traders find it valuable to do additional research.³⁴ The price path in the figure could represent, for example, the stock issuance of a new technology firm, the value of which is initially uncertain. Imagine the technology is actually not viable such that the true value of the firm is low. Initial information, based on little research, happens to be favorable, so we observe an initial boom in the price of the stock as traders correctly infer from their positive information that others also have positive information. No one is willing to perform further research due to the rational fear

³⁴By crashes and booms, I simply mean large changes in the price as new (stronger) information is made available to the market. Crashes are often defined to be *discontinuous* changes in prices (Barlevy and Veronesi (2003)). No such discontinuities arise here.

of continued upward price movements. However, as the market becomes more certain that the firm is of good quality, traders are induced to perform further research because prices are no longer increasing as rapidly. When they do further research, they discover the firm's value is actually low, and a large drop in the stock price results. Although it is unlikely that information acquisition is the sole source of price booms and crashes, the model demonstrates how endogenous information quality can play a role in their occurrence.

5 The Zero-Profit Model

In the preceding analysis, I assumed that the price of the asset was given by a single value that does not incorporate the information contained in the current trade order. While this assumption allowed me to focus on the interaction between traders, it leads to a market maker who loses money in expectation. In this section, I show that the main results of the expected value model continue to hold when market makers instead earn zero profits in expectation.³⁵ Formally, I modify the model as follows. I consider only the case of $n = 1$: a single trader arrives each period.

In each period, the trader that arrives at t is an informed trader with probability $\mu \in (0, 1)$ and is an uninformed, noise trader with probability $1 - \mu$.³⁶ If a trader is informed, she receives information as in the expected value model, but if uninformed, she trades for reasons exogenous to the model, such as to meet liquidity needs. Specifically, a noise trader buys or sells in the period she arrives or the next period, with each possible trade having probability $\frac{1}{4}$.³⁷ Because the market maker takes into account the information contained in the current trade order, he posts separate prices for buy and sell orders. For a rushed trade, the ask price at which the market maker is willing to sell the asset is given by $p_t^A = Pr(V = 1 | \underline{H}_t, \underline{a}_t = B)$ and the bid price at which he is willing to buy the asset is given by $p_t^B = Pr(V = 1 | \underline{H}_t, \underline{a}_t = S)$. The bid and ask prices are set similarly for delayed trades. These bid and ask prices are easily shown to be optimal for a market maker that is constrained to earn zero profits.

To analyze the model with noise traders, I proceed in the same manner as in the analysis of the expected value model. The equilibrium definition remains as in Section 2 except that item 5, the restriction on off-equilibrium prices and beliefs, is no longer required because noise traders ensure that beliefs and prices are pinned down after all histories. The main

³⁵This standard zero-profit condition results from the assumption that the market maker faces unmodeled competition (Bertrand competition in prices, for example). For a more detailed discussion, see Glosten and Milgrom (1985).

³⁶Noise traders must be introduced once prices reflect the information contained in the current trade because, without them, the no trade theorem of Milgrom and Stokey (1982) applies.

³⁷Allowing noise traders to not trade in each period with some probability does not affect the qualitative results

difference from the expected value model is that here, a trader's timing strategy affects the prices she faces. Intuitively, if informed traders wait, then delayed trades will have a lot of informational content and the bid/ask spread (the difference between the bid and ask prices) will be large. Any rushed trade then must be due to a noise trader so that the bid/ask spread at that time will be zero. However, this difference in spreads reduces the value to waiting and encourages informed traders to rush. If informed traders always rush, the reasoning is reversed and waiting becomes profitable. Based upon this intuition, one can anticipate that equilibria generally involve mixed timing strategies on the part of the informed traders. The existence of bid/ask spreads also creates several possibilities in terms of which types of traders, based on their signals, will buy or sell when waiting. To focus on only one particular case, I set $\bar{q} = \underline{q} \equiv q$ in this section so that traders with contradictory signals do not trade because their expected value of the asset lies within the bid/ask spread.³⁸

The unique bid and ask prices are easily calculated using Bayes' rule and are provided in Appendix D. For delayed trades, traders with $\underline{s}_t = \bar{s}_{t+1} = 1$ buy, those with $\underline{s}_t = \bar{s}_{t+1} = 0$ sell. For rushed trades, traders with $\underline{s}_t = 1$ buy and those with $\underline{s}_t = 0$ sell. These trading decisions can be shown to be optimal by comparing a trader's private belief to the appropriate price, as in the proof of Lemma 1.

Given the optimal trading decisions, one can derive a general formula for the benefit to waiting for each type of trader just as in the expected value model. The formulas are provided in Appendix D. Unlike in the expected value model, the benefit function for each type of trader, $B_x(p_t, \beta_x)$, depends only on her own timing strategy because a delayed trade reveals both \underline{s}_t and \bar{s}_{t+1} . One can show that each of the benefit functions is strictly decreasing in the timing strategy of the trader so that the equilibrium timing strategy is uniquely determined for fixed strategies of the other traders.

Comparing the formulas in Appendix D for the benefit to waiting in the zero-profit model with those of the expected value model, (1), one can see that the general structures of the benefit functions are very similar. Because of this similarity, Propositions 1 and 2 are easily extended to the zero-profit model, as stated in Corollary 1.

Corollary 1: *Replacing $B_x(p_t, \beta_0, \beta_1)$ with $B_x(p_t, \beta_x)$ for $x \in \{0, 1\}$, the statements of Propositions 1 and 2 hold in the zero-profit model.*

From Corollary 1, we know that the benefit to waiting decreases when others trade during the waiting period and that this effect diminishes as prices become certain, as in the expected value model. There is, however, no counterpart to Lemma 2 of the expected

³⁸Results for $\bar{q} \neq \underline{q}$ are qualitatively similar as long as the probability of a noise trader is not too high because traders with contradictory signals will still have expectations that lie within the bid/ask spread.

value model because equilibria generally involve the two types of traders following different timing strategies. Also, as discussed previously, equilibria generally involve mixing due to the interaction between the market maker and the traders. These complications make determining the equilibria of the model more difficult than in the expected value case and I do not have a complete characterization. However, by solving for the simplest case of a single trader (i.e. a finite time version of the model with $T = 1$) and using Propositions 1 and 2, I am able to derive several necessary properties of the equilibria for $T \rightarrow \infty$.

Proposition 4 describes the equilibrium timing decisions of a single trader facing the market maker. Together with the optimal trading decisions at each point in time discussed previously, it characterizes the unique equilibrium for the single trader case.

Proposition 4: *The unique equilibrium of the zero-profit model with a single trader is characterized by the following timing strategies for all $p_t \in (0, 1)$.*

$$\beta_0^{ST}(p_t) = \frac{p_t(1-q) + (1-p_t)q}{p_t(1-q) + (1-p_t)q + p_t(1-q)^2 + (1-p_t)q^2}$$

$$\beta_1^{ST}(p_t) = \frac{p_tq + (1-p_t)(1-q)}{p_tq + (1-p_t)(1-q) + p_tq^2 + (1-p_t)(1-q)^2}$$

The optimal timing strategies given by Proposition 4 are interior for all $p_t \in (0, 1)$ and all q so that each type of trader mixes in the unique equilibrium. Furthermore, the two types wait with the same probability only at $p_t = \frac{1}{2}$, which is intuitive given the symmetry of the problem at this price. One can also show that $\frac{\partial \beta_1}{\partial p_t} < 0$ and $\frac{\partial \beta_0}{\partial p_t} > 0$ so that, when $p_t > \frac{1}{2}$, the trader with $s_t = 0$ is waiting with a higher probability. Intuitively, when $p_t > \frac{1}{2}$, the trader with $s_t = 0$ has the greater benefit from waiting due to having more unexpected information. By waiting more often, however, negative information about the quality of the asset is revealed, which reduces the price and thus decreases the benefit of type $s_t = 0$ relative to that of $s_t = 1$. In equilibrium, this reduction in price is just sufficient to ensure both types are indifferent between rushing and waiting. Also note that the equilibrium strategies of the traders in Proposition 4 are independent of the probability of an informed trader, μ . An increase in μ increases the bid/ask spreads at both $t = 1$ and $t = 2$ in such a way as to keep each type of trader indifferent between rushing and waiting.

Using Propositions 1 and 4, Proposition 5 shows that the presence of other traders causes all traders to rush more often than they would if they were alone in the market.

Proposition 5: *In any equilibrium of the infinite horizon zero-profit model, each type of each trader waits with probability $\beta_0(p_t, I(\underline{a}_{t-1} = NT)) < \beta_0^{ST}(p_t)$ and $\beta_1(p_t, I(\underline{a}_{t-1} = NT)) < \beta_1^{ST}(p_t)$ for all p_t and $I(\underline{a}_{t-1} = NT)$, where $\beta_0^{ST}(p_t)$ and $\beta_1^{ST}(p_t)$ are given in Proposition 4.*

From Proposition 2, we also know that the equilibrium timing strategies approach those given in Proposition 4 as prices become certain. Therefore, the main insights of the expected value model hold in the zero-profit model: panics cause traders to rush more often when uncertainty is high, but as prices become certain they acquire information as if the other traders were not present.

6 Empirical Implications

Theoretical analysis of the model has developed predictions with respect to the quality of information traders will trade upon relative to time, uncertainty, and volume. Because the quality of information traders possess is typically unobservable, in this section I develop several testable predictions in terms of observables. Some of these predictions have empirical backing and others suggest novel tests that could be used to validate (or falsify) the model.

The extension of the model to $n > 1$ traders arriving each period (see Appendix E) demonstrates that an increase in n leads to more panics and thus trading on lower quality information. An increase in n corresponds to an increase in volume, so we have the following prediction.

Prediction 1: *In either a cross-sectional or time-series analysis, holding the level of uncertainty constant, order flows are more balanced and persistent price impacts are smaller when volume is higher.*

Trading on lower quality information causes more balanced order flows because signal realizations are more likely to be incorrect when information is of lower quality.³⁹ Beginning with Easley et al. (1996), empirical work has used the order flow imbalance to estimate the probability of informed trading (PIN). In the model of Easley et al. (1996), traders are assumed to be either uninformed or perfectly informed, so that more balanced order flows occur when the percentage of uninformed traders is higher (PIN is lower). In their model, the rates of arrival of informed and uninformed traders are exogenous, so any relationship between volume and the PIN can be captured. Here, because a lower estimated PIN results

³⁹Order flow imbalance is defined with respect to those initiating the trade as in Easley et al. (1996).

from more balanced order flows, the correlate of Prediction 1 is that the estimated PIN should be lower when volume is higher.⁴⁰ In a cross-sectional study, Easley et al. (1996) in fact find that the PIN is lower in their sample of high volume stocks than in their sample of low volume stocks, consistent with Prediction 1. They do not control for the level of uncertainty among the stocks, which is important in this context because order flows are predicted to be more balanced when uncertainty is higher, as discussed below. However, unless their sample of high volume stocks happens to be those in which uncertainty is lower, their result can be considered a validation of Prediction 1.

When market participants trade on lower quality information, we would also expect the persistent price impact of trades to be lower. As first suggested by Hasbrouck (1991), informative trades should have longer lasting price impacts than trades for other purposes, such as inventory rebalancing, etc. Using the vector auto-regressive approach developed by Hasbrouck (1991) to measure the persistent price impact of trades, trades of high volume stocks have been found to have smaller price impacts in a sample of NYSE stocks (Engle and Patton (2004)) and in foreign exchange markets (Payne (2003) and Lyons (1996)). These results validate Prediction 1, again assuming that high volume is not directly correlated with low uncertainty. Thus, we see that lower informational content of trades when volume is higher is a robust empirical finding consistent with the model.

Proposition 2 and the equilibrium characterization of Theorem 1 predict that market participants trade on lower quality information when uncertainty is higher. As in Prediction 1, trades on lower quality information are expected to result in more balanced order flows, so we have the following prediction.⁴¹

Prediction 2: *In either a cross-sectional or time-series analysis, holding volume constant, order flows are more balanced when uncertainty is higher.*

⁴⁰Models that have been developed to explain intraday trading patterns in observables also make predictions regarding the relationship between volume and the PIN. In Admati and Pfleiderer (1988) and Malinova and Park (2012), the PIN increases with volume, and in Foster and Viswanathan (1990) either relationship is possible. The informational content of high volume stocks may also be reduced if high volume stocks attract noise traders due to saliency (Barber and Odean (2008)).

⁴¹The price impacts of trades are not necessarily predicted to be lower when uncertainty is higher due to two opposing effects. Holding the quality of information constant, price impacts are larger when uncertainty is higher. On the other hand, because market participants trade on lower quality information when uncertainty is higher, price impacts are lower. Thus, the prediction of the model is only that price impacts due to not necessarily increase as uncertainty increases, which would be the case in models with constant information quality, such as Glosten and Milgrom (1985). Similarly, the model makes no straightforward, monotonic prediction with respect to the relationships between volatility or the bid/ask spread and uncertainty. An important consequence of the model is that the way in which the informational content of prices is measured (PIN, bid/ask spread, persistent price impact) is important: not all measures need necessarily deliver the same results.

Prediction 2 is perhaps the most novel prediction of the model as it is contrary to the predictions of models in which information is exogenous, such as Glosten and Milgrom (1985), and models with monetary costs such as Nikandrova (2012) and Lew (2013). With a fixed quality of information, there should be no relationship between order flows and uncertainty. With fixed monetary costs, when uncertainty is low, trading should stop completely rather than become more informative. Lew (2013) constructs a sequential trading model with increasing monetary costs of acquiring (perfect) information. His model delivers a prediction opposite to Prediction 2: the PIN is higher (and hence order flows are less balanced) when uncertainty is higher. Should Prediction 2 hold in the data, it would provide strong evidence for the mechanism suggested by the model, because it is the endogenous cost of acquiring information that delivers this prediction.⁴²

The relationship between the PIN and various measures of uncertainty has been explored (Kumar (2009) and Aslan et al. (2011)) with mixed results. However, the main difficulty in performing a test of Prediction 2 is in finding a suitable proxy for uncertainty. Uncertainty here refers to a flatter distribution over the possible fundamental values of the asset being traded. The ideal proxy would be uncorrelated with volume and would also not be likely to affect the information traders possess through any other channel. In Kumar (2009), firm age, firm size, monthly volume turnover, and idiosyncratic volatility are used as proxies in a cross-sectional study, but firm age and size are likely to be directly correlated with the quality of information available, monthly volume turnover is closely related to volume, and volatility is an output of the model that is not necessarily related to uncertainty (see footnote 41). In Aslan et al. (2011), PIN is regressed on many accounting variables in a cross-sectional study. While some variables can arguably be interpreted as proxies for uncertainty (industry, volatility, volatility in earnings), these proxies suffer from problems similar to those in Kumar (2009).

Because of the difficulty in finding a suitable proxy for a cross-sectional study, a time-series study in which one can control for firm-specific effects may be better able to tease out the effect of uncertainty on the informational content of trades. Potential proxies that could be used include the implied volatility index of the market (VIX) as a whole or dispersion in analyst forecasts. Alternatively, one could look for particular news events that one can reasonably argue have increased (or decreased) uncertainty about firm valuations (for example,

⁴²The model also predicts more frequent deviations from fundamental values during times of high uncertainty. Limits to arbitrage (see Shleifer and Vishny (1997) and Mitchell et al. (2002)) can also cause mispricing when volatility is high due to increased arbitrage risk. To the extent volatility is related to uncertainty, these two explanations provide similar predictions. However, volatility and uncertainty are not necessarily related (see footnote 41). Furthermore, the explanations can be distinguished by studying the informational content of trades when uncertainty is high.

results of drug trials in the pharmaceutical industry). Certainly there is scope for interesting empirical work in this direction.⁴³

Proposition 3 states that an increase in the initial quality of information can actually reduce the rate of price convergence: the increase, holding the second period information constant, can induce traders to panic. As panics result in more balanced order flows, we have:

Prediction 3: *Access to higher quality initial information, holding the level of uncertainty and volume constant, results in more balanced order flows.*

For evidence related to Prediction 3, in a study related to that of Easley et al. (1996), Yan and Zhang (2012) provide a new algorithm to estimate the PIN. Interestingly, they find that estimates of the PIN on both the NYSE and the American Stock Exchange (AMEX) have significantly declined between the years of 1993 and 2004. They do not put forward a theory as to why this has occurred, but one possibility is that improvements in technology have made better quality information available sooner.⁴⁴

The novel empirical predictions of the model are driven both by the endogenous cost of acquiring information and the fact that weak information can result in misinformed traders that trade in a direction that moves prices away from fundamental values. Empirical work that estimates PIN or the persistence of price impacts assumes (explicitly or implicitly) instead that informed trades always move prices towards fundamental values. Should the predictions of the model be validated, it would suggest the importance of relaxing this assumption.

7 Conclusion

This paper has studied informational losses due to rational panics in a stylized setting. The fear of adverse price movements due to others' trades can induce traders to rush to trade as soon as possible. Because traders panic, they forgo doing research about the asset they are

⁴³For other cross-sectional evidence that is suggestive of less-informed trades when uncertainty is higher, it has been found that underreaction in stock prices is stronger in stocks with higher uncertainty (Zhang (2006) and Jiang et al. (2005)). One interpretation of underreaction, as summarized by Zhang (2006), is that underreaction is "more likely to reflect slow absorption of ambiguous information into stock prices than to reflect missing risk factors". Under this interpretation, the fact the information appears to be more slowly absorbed into stocks of higher uncertainty is consistent with the model. However, strictly speaking, the model does not capture underreaction (prices follow a martingale). Extending the model to capture underreaction is an interesting avenue for future research.

⁴⁴Recent increases in high-frequency trading as documented in Blais et al. (2011) and Hoffman (2013) may have led to what Hoffman calls an "arms race" in which investors invest in better and better technology to obtain faster access to information.

trading, resulting in trades based upon relatively weak information. As a consequence, prices take longer in expectation to converge to fundamental values and more frequent increases in mispricing are predicted. Furthermore, markets are susceptible to panics when uncertainty is high, so that information is forgone when it is most valuable in stark contrast to models with monetary information costs. A multiplicity of equilibria exists due to strategic complementarity between traders in the market, allowing expectations of panics to become self-fulfilling. As such, there may be a role for market intervention to calm fears during times of panic. Finally, technology that makes more information available more quickly can, perversely, cause markets to aggregate less information overall.

This paper can also contribute to the debate on market design.⁴⁵ Rushing to trade is only optimal when there is a rational fear of others trading before one has a chance to acquire information. In markets that are continuously open, this fear can be justified. On the other hand, in markets that are cleared through call auctions, as long as the auctions are not too frequent, such fears are alleviated. Thus, the model suggests that markets that are cleared through call auctions may be less prone to traders trading on weak information and the associated consequences. Similarly, the model justifies the practice of regulatory trading halts on stock exchanges when news is to be released. The temporary trading halt ensures traders have time to process information, which prevents rushing to trade on weak information and, assuming the accumulated information is sufficient, ensures no panics occur when trading resumes because prices will adjust to values near the fundamental value (where not panicking is optimal).⁴⁶

In future work, the ideas presented here may be used to enhance our understanding of volume in markets and its relation to information. A limitation of the current model is that volume is essentially exogenous in the model (other than shifting forward or back one period). Kendall (2013) considers a model in which traders may trade over several periods, but obtain information in only two periods. Rational panics are predicted to cause not only informational losses but also spikes in volume as traders rush to trade before each other. These predictions are then robustly confirmed in laboratory data. In order to detect such behavior in real markets, it will be important to account for the fact that the information possessed by traders is endogenous and may vary in quality. This paper suggests that empirical work may benefit from moving away from the informed-uninformed trader paradigm to instead consider the fact that traders may be misinformed due to low quality information.

⁴⁵For a recent paper in this area that summarizes previous theoretical and empirical work, see Kuo and Li (2011).

⁴⁶In the case of earnings announcements, information is public, but as shown by Kandel and Pearson (1995), there are reasons to believe public information becomes private due to heterogeneous abilities to process the information or different models of a firm's dividend-generating process.

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Appendices

A. Benefit Functions

Here, I show that the general form of the benefit to waiting can be written as (1). From Lemma 1, a trader trading at t with $\underline{s}_t = 1$ always buys and so her profit is given by

$$Pr(V = 1 | \underline{s}_t = 1, \underline{H}_t) - p_t \iff \frac{p_t \underline{q}}{p_t \underline{q} + (1 - p_t)(1 - \underline{q})} - p_t \iff \frac{p_t(1 - p_t)(2\underline{q} - 1)}{p_t \underline{q} + (1 - p_t)(1 - \underline{q})}$$

The profit for a trader with $\underline{s}_t = 0$ is calculated similarly, using the fact that she always sells. The profit for both types of traders can be written $\frac{p_t(1-p_t)(2\underline{q}-1)}{Pr(\underline{s}_t)}$.

The expected profit from waiting depends upon the timing strategies of the two types of trader and the strategies of the other traders. I calculate the profit explicitly for a trader

with $\underline{s}_t = 1$ who buys when receiving $\bar{s}_{t+1} = 1$ and sells when $\bar{s}_{t+1} = 0$. The other cases are calculated similarly and are omitted for brevity. The expected profit is

$$\sum_{\hat{a} \in A} \left\{ Pr(\bar{s}_{t+1} = 1 \& \hat{a} | \underline{s}_t = 1) (Pr(V = 1 | \underline{s}_t = 1, \bar{s}_{t+1} = 1, \hat{a}) - Pr(V = 1 | \underline{a}_t = NT, \hat{a})) \right. \\ \left. + Pr(\bar{s}_{t+1} = 0 \& \hat{a} | \underline{s}_t = 1) (Pr(V = 1 | \underline{a}_t = NT, \hat{a}) - Pr(V = 1 | \underline{s}_t = 1, \bar{s}_{t+1} = 0, \hat{a})) \right\}$$

where all probabilities are also conditional on \underline{H}_t . Here, I sum over each possible generic combination of events that result from the timing and trading strategies of $t - 1$ and $t + 1$. The first term corresponds to the profit from buying the asset after receiving $\bar{s}_{t+1} = 1$ and the second term from selling after receiving $\bar{s}_{t+1} = 0$. Using Bayes' rule and the independence of signals, the above becomes

$$\sum_{\hat{a} \in A} \left\{ \frac{Pr(\bar{s}_{t+1} = 1 \& \underline{s}_t = 1 \& \hat{a})}{Pr(\underline{s}_t = 1)} \left(\frac{p_t \underline{q} \bar{q} Pr(\hat{a} | V = 1)}{Pr(\bar{s}_{t+1} = 1 \& \underline{s}_t = 1 \& \hat{a})} - \frac{p_t NT_1 Pr(\hat{a} | V = 1)}{Pr(\underline{a}_t = NT \& \hat{a})} \right) \right. \\ \left. + \frac{Pr(\bar{s}_{t+1} = 0 \& \underline{s}_t = 1 \& \hat{a})}{Pr(\underline{s}_t = 1)} \left(\frac{p_t NT_1 Pr(\hat{a} | V = 1)}{Pr(\underline{a}_t = NT \& \hat{a})} - \frac{p_t \underline{q} (1 - \bar{q}) Pr(\hat{a} | V = 1)}{Pr(\bar{s}_{t+1} = 0 \& \underline{s}_t = 1 \& \hat{a})} \right) \right\}$$

with NT_0 and NT_1 as in Lemma 1. Combining each of the terms in the first and second expressions, canceling and factoring out common terms gives

$$\frac{p_t(1 - p_t)}{Pr(\underline{s}_t = 1)} \sum_{\hat{a} \in A} \frac{\hat{a}_0 \hat{a}_1}{Pr(\underline{a}_t = NT \& \hat{a})} (\underline{q} \bar{q} NT_0 - NT_1 (1 - \bar{q})(1 - \underline{q}) + NT_1 \bar{q}(1 - \underline{q}) - \underline{q}(1 - \bar{q}) NT_0) \\ \iff \frac{p_t(1 - p_t)}{Pr(\underline{s}_t = 1)} \sum_{\hat{a} \in A} \frac{\hat{a}_0 \hat{a}_1}{Pr(\underline{a}_t = NT \& \hat{a})} ((2\bar{q} - 1)(\underline{q} NT_0 + (1 - \underline{q}) NT_1))$$

Finally, subtracting the profit at t from the expected profit and factoring out common terms gives the benefit formula stated in (1).

As discussed in the main text, we can set, $\beta_0 = \beta_1$, simplifying the benefit function. $f(\underline{q}, \bar{q}, \beta_0, \beta_1)$ simplifies to $\beta(2\bar{q} - 1)$ when $\bar{q} > \underline{q}$ and $\beta(2\underline{q} - 1)$ when $\bar{q} < \underline{q}$, reflecting the fact that a trader follows her stronger signal when they are contradictory. The denominator simplifies to $Pr(\hat{a} \& \underline{a}_t = NT) = \beta Pr(\hat{a})$ so that β cancels in the numerator and denominator. When $\bar{q} = \underline{q}$, a trader with contradictory signals is indifferent between trading or not, but because I assume that such a trader follows her second period signal, $f(\underline{q}, \bar{q}, \beta_0, \beta_1) = \beta(2\bar{q} - 1)$ and β cancels in this case as well. Denoting $q = \max(\underline{q}, \bar{q})$, one can then write the general benefit function as

$$B_x(p_t) = \frac{p_t(1 - p_t)}{Pr(\underline{s}_t = x)} \left[\sum_{\hat{a} \in A} \frac{\hat{a}_0 \hat{a}_1 (2q - 1)}{Pr(\hat{a})} - (2q - 1) \right]$$

where the function no longer depends on β_0 or β_1 .

For specific (pure) strategies of the other traders, one can substitute the information revealed by each possible event into the general formula to obtain a specific formula for those strategies. As shown in Lemma 1, any trader that rushes trades according to her first period

signal and so, for a rushed trade by $t + 1$, $Pr(\underline{a}_{t+1} = B|V = 1) = Pr(\underline{s}_{t+1} = 1|V = 1) = \underline{q}$ and $Pr(\underline{a}_{t+1} = S|V = 1) = 1 - \underline{q}$. For a delayed trade by $t - 1$, $Pr(\bar{a}_t = B|V = 1) = q$ and $Pr(\bar{a}_t = S|V = 1) = 1 - q$ where q denotes $\max(\bar{q}, \underline{q})$. The unconditional probabilities of buy and sell decisions by $t - 1$ are then $Pr(\bar{a}_t = B) = pq + (1 - p)(1 - q)$ and $Pr(\bar{a}_t = S) = p(1 - q) + (1 - p)q$. The reason only the stronger signal is revealed comes from Lemma 1 when $\beta_0 = \beta_1$: if $\bar{q} > \underline{q}$, a trader with $\bar{s}_t = 1$ ($\bar{s}_t = 0$) buys (sells) regardless of \underline{s}_t , so that no information about \underline{s}_t is revealed by her trade. Similarly, if $\underline{q} > \bar{q}$, trades don't depend on \bar{s}_t . If the two signal strengths are the same, since I have assumed a trader follows her second period signal, only it is revealed. Substituting the information revealed, we get the following formulas, where $Pr(\bar{a}_t = x \& \underline{s}_{t+1} = y)$ is abbreviated $Pr(x, y)$, $S_x \equiv \frac{p_t(1-p_t)}{Pr(\underline{s}_t=x)}$ is a scale factor common to all of the benefit functions, $Q \equiv q(1 - q)$, $\underline{Q} \equiv \underline{q}(1 - \underline{q})$, and $\bar{Q} \equiv \bar{q}(1 - \bar{q})$.

$$\begin{aligned}
B_x^{R,WW}(p_t) &= S_x [(2q - 1) - (2\underline{q} - 1)] \tag{2} \\
B_x^{R,RR}(p_t) &= S_x \left[(2q - 1)\underline{Q} \left(\frac{1}{Pr(\underline{s}_{t+1} = 1)} + \frac{1}{Pr(\underline{s}_{t+1} = 0)} \right) - (2\underline{q} - 1) \right] \\
B_x^{W,WW}(p_t) &= S_x \left[(2q - 1)\bar{Q} \left(\frac{1}{Pr(\bar{a}_t = B)} + \frac{1}{Pr(\bar{a}_t = S)} \right) - (2\bar{q} - 1) \right] \\
B_x^{W,RW}(p_t) &= S_x \left[(2q - 1)\bar{Q} \left(\frac{1}{Pr(\bar{a}_t = S)} + q(1 - q) \left(\frac{1}{Pr(B, 1)} + \frac{1}{Pr(B, 0)} \right) \right) - (2\bar{q} - 1) \right] \\
B_x^{W,WR}(p_t) &= S_x \left[(2q - 1)\bar{Q} \left(\frac{1}{Pr(\bar{a}_t = B)} + q(1 - q) \left(\frac{1}{Pr(S, 1)} + \frac{1}{Pr(S, 0)} \right) \right) - (2\bar{q} - 1) \right] \\
B_x^{W,RR}(p_t) &= S_x \left[(2q - 1)\underline{Q}\bar{Q} \left(\frac{1}{Pr(B, 1)} + \frac{1}{Pr(B, 0)} + \frac{1}{Pr(S, 1)} + \frac{1}{Pr(S, 0)} \right) - (2\underline{q} - 1) \right]
\end{aligned}$$

B. Stability of Equilibria

In this section, I argue that any possible mixed strategy equilibrium of the expected value trading model is unstable in the sense that pseudo-dynamics would tend to destabilize it, and that the pure strategy equilibria are instead stable. The discussion is relatively informal, as the purpose is to simply justify a focus on the pure strategy equilibria.

Consider first an equilibrium that involves mixed strategies. That such an equilibrium can exist was demonstrated in the proof of Theorem 1. In the equilibrium, two neighbors in the restricted price chain are mixing between conditionally rushing and waiting with probabilities such that the other is indifferent. Without loss of generality, label the two traders as the prices they face, p_t and p_t^+ . No assumptions are made about the strategies of the traders at p_t^- and $(p_t^+)^+$: those traders may be mixing or playing pure strategies of either conditional rushing or waiting. p_t^+ is mixing such that p_t has a benefit of zero given the strategy of p_t^- . If p_t^+ changes her strategy to wait slightly more often, the benefit of p_t then becomes strictly positive because her benefit is linear in the probability with which p_t^+ mixes and increases when p_t^+ waits more often. For example, if p_t^- is waiting, then p_t faces a benefit of $B_x^{W,WW}(p_t) > 0$ when p_t^+ waits and $B_x^{W,RW}(p_t) < 0$ when p_t^+ conditionally rushes.

Therefore, her benefit is $\beta B_x^{W,WW}(p_t) + (1 - \beta)B_x^{W,RW}(p_t) = 0$ where β is the equilibrium probability with which p_t^+ waits. When p_t^+ waits more often, p_t 's benefit is strictly positive because more weight is placed upon $B_x^{W,WW}(p_t) > 0$ which will cause her to change her strategy to wait with probability 1. But, when this happens, by the same argument, p_t^+ will be induced to change her strategy to wait with probability 1. The small change to waiting more often by one trader leads both traders to wait with probability 1, so the equilibrium is unstable. A small change to rushing more often leads to both traders (conditionally) rushing with probability 1 by very similar arguments.

Consider instead what happens when the equilibrium is such that all traders in the restricted price chain wait. In this case, each trader faces a strictly positive benefit to waiting (ignoring the non-generic case of the price being exactly equal to one of the cutoff prices). Now, if a trader begins to conditionally rush with some small probability, since the other traders have strictly positive benefits, they will continue to wait. In addition, the trader who conditionally rushes is strictly worse off because any time she conditionally rushes, her profit is reduced given her positive benefit from waiting. Therefore, it is in her best interest to return to waiting with probability 1: the equilibrium is stable. A similar argument establishes that an equilibrium with all traders conditionally rushing is also stable, so that all pure strategy equilibria are stable.

C. Omitted Proofs

Proof of Lemma 1:

The proof when rushing is similar to the proof when waiting, only simpler, so is omitted for brevity. Let \hat{a} denote any information that becomes public due to the decisions of traders other than t between t and $t + 1$ and abbreviate $Pr(\hat{a}|V = y)$ as \hat{a}_y , for $y \in \{0, 1\}$. A trader, t , who waits buys if her expected value of the asset is greater than the price she faces, $\bar{p}_{t+1} = Pr(V = 1|\hat{a}, \underline{a}_t = NT, \underline{H}_t)$:

$$\begin{aligned}
& Pr(V = 1|\underline{s}_t, \bar{s}_{t+1}, \hat{a}, \underline{H}_t) > Pr(V = 1|\hat{a}, \underline{a}_t = NT, \underline{H}_t) \\
\iff & \frac{p_t Pr(\underline{s}_t|V = 1) Pr(\bar{s}_{t+1}|V = 1) \hat{a}_1}{p_t Pr(\underline{s}_t|V = 1) Pr(\bar{s}_{t+1}|V = 1) \hat{a}_1 + (1 - p_t) Pr(\underline{s}_t|V = 0) Pr(\bar{s}_{t+1}|V = 0) \hat{a}_0} \\
& > \frac{p_t \hat{a}_1 Pr(\underline{a}_t = NT|V = 1)}{p_t \hat{a}_1 Pr(\underline{a}_t = NT|V = 1) + (1 - p_t) \hat{a}_0 Pr(\underline{a}_t = NT|V = 0)} \\
\iff & Pr(\underline{s}_t|V = 1) Pr(\bar{s}_{t+1}|V = 1) NT_0 > Pr(\underline{s}_t|V = 0) Pr(\bar{s}_{t+1}|V = 0) NT_1 \quad (3)
\end{aligned}$$

where the first equivalence follows from applying Bayes' rule to each side of the inequality and using the fact that the public belief, $Pr(V = 1|\underline{H}_t) = p_t$. Using (3), a trader with $\underline{s}_t = 1, \bar{s}_{t+1} = 1$ buys if

$$\begin{aligned}
& \underline{q}\bar{q}NT_0 > (1 - \underline{q})(1 - \bar{q})NT_1 \\
\iff & \underline{q}(1 - \underline{q})(2\bar{q} - 1)\beta_1 + (\underline{q}^2\bar{q} - (1 - \underline{q})^2(1 - \bar{q}))\beta_1 > 0
\end{aligned}$$

which is true $\forall \underline{q}, \bar{q} \in (\frac{1}{2}, 1)$. Similarly, a trader with $\underline{s}_t = 0, \bar{s}_t = 0$ always sells. Finally, the

conditions in Lemma 1 under which traders with $\underline{s}_t = 1, \bar{s}_{t+1} = 0$ and $\underline{s}_t = 0, \bar{s}_{t+1} = 1$ buy or sell are easily obtained by substituting for the appropriate probabilities in (3). \square

Proof of Lemma 2:

The proof is by contradiction. Assume there is an equilibrium in which $\beta_1 > \beta_0$ (the case of $\beta_0 > \beta_1$ similarly leads to a contradiction) at some p_t and for some arbitrary strategies of the other traders. I first show that $\beta_1 > \beta_0$ and $B_1(p_t, \beta_0, \beta_1) \geq 0$ together imply $B_0(p_t, \beta_0, \beta_1) > 0$ for all possible strategies of the other traders and for all p_t .

One can see from the general form of the benefit in (1), that the sign of $B_x(p_t, \beta_0, \beta_1)$ is determined by the term in square brackets and that the only difference between the bracketed terms in $B_0(p_t, \beta_0, \beta_1)$ and $B_1(p_t, \beta_0, \beta_1)$ is due to differences in the function, $f(\underline{q}, \bar{q}, \beta_0, \beta_1)$. I show that this function is strictly greater in $B_0(p_t, \beta_0, \beta_1)$ than in $B_1(p_t, \beta_0, \beta_1)$ whenever $\beta_1 > \beta_0$ so that if $B_1(p_t, \beta_0, \beta_1) \geq 0$, we must have $B_0(p_t, \beta_0, \beta_1) > 0$ (because $f(\underline{q}, \bar{q}, \beta_0, \beta_1)$ is always positive). There are four possible cases depending upon the optimal strategies of buying and selling from Lemma 1.

Consider first the case of $g_1(\underline{q}, \bar{q}) \leq 0$ and $g_0(\underline{q}, \bar{q}) \geq 0$. The comparison of $f(\underline{q}, \bar{q}, \beta_0, \beta_1)$ in $B_0(p_t, \beta_0, \beta_1)$ relative to $B_1(p_t, \beta_0, \beta_1)$ is $\underline{q}NT_1 + (1 - \underline{q})NT_0 > \underline{q}NT_0 + (1 - \underline{q})NT_1 \iff (NT_1 - NT_0)(2\underline{q} - 1) > 0 \iff (\beta_1 - \beta_0)(2\underline{q} - 1)^2 > 0$ so $f(\underline{q}, \bar{q}, \beta_0, \beta_1)$ is strictly greater in $B_0(p_t, \beta_0, \beta_1)$. When $g_1(\underline{q}, \bar{q}) > 0$ and $g_0(\underline{q}, \bar{q}) < 0$, we have $\underline{q}NT_1 - (1 - \underline{q})NT_0 > \underline{q}NT_0 - (1 - \underline{q})NT_1 \iff NT_1 - NT_0 > 0 \iff (\beta_1 - \beta_0)(2\underline{q} - 1) > 0$ so again $f(\underline{q}, \bar{q}, \beta_0, \beta_1)$ is strictly greater in $B_0(p_t, \beta_0, \beta_1)$.

When $g_1(\underline{q}, \bar{q}) \leq 0$ and $g_0(\underline{q}, \bar{q}) < 0$, I begin by noting that

$g_0(\underline{q}, \bar{q}) < 0 \implies \underline{q}NT_1 - (1 - \underline{q})NT_0 > (2\bar{q} - 1)(\underline{q}NT_1 + (1 - \underline{q})NT_0)$. This can be seen algebraically or simply by noting that $B_0(p_t, \beta_0, \beta_1)$ must be larger when t follows her optimal trading strategy at $t + 1$ instead of the optimal strategy for the other case, $g_0(\underline{q}, \bar{q}) \geq 0$. But, then we have, $\underline{q}NT_1 - (1 - \underline{q})NT_0 > (2\bar{q} - 1)(\underline{q}NT_1 + (1 - \underline{q})NT_0) > (2\bar{q} - 1)(\underline{q}NT_0 + (1 - \underline{q})NT_1)$ where the second inequality was shown in the first case above, so $f(\underline{q}, \bar{q}, \beta_0, \beta_1)$ is strictly greater in $B_0(p_t, \beta_0, \beta_1)$. Similarly, when $g_1(\underline{q}, \bar{q}) > 0$ and $g_0(\underline{q}, \bar{q}) \geq 0$, $g_0(\underline{q}, \bar{q}) \geq 0$ implies $(2\bar{q} - 1)(\underline{q}NT_1 + (1 - \underline{q})NT_0) > \underline{q}NT_1 - (1 - \underline{q})NT_0$ by optimality of the second period trading decision and $\underline{q}NT_1 - (1 - \underline{q})NT_0 > \underline{q}NT_0 - (1 - \underline{q})NT_1$ as shown in the second case above. Thus, $f(\underline{q}, \bar{q}, \beta_0, \beta_1)$ is strictly greater in $B_0(p_t, \beta_0, \beta_1)$ in all cases and therefore $B_1(p_t, \beta_0, \beta_1) \geq 0$ implies $B_0(p_t, \beta_0, \beta_1) > 0$. Furthermore, with $\beta_1 > \beta_0$, we must have $\beta_1 \in (0, 1]$ and therefore $B_1(p_t, \beta_0, \beta_1) \geq 0$. Then, we must have $B_0(p_t, \beta_0, \beta_1) > 0$ by the established implication. However, we must also have $\beta_{0,t} \in [0, 1)$ and therefore $B_0(p_t, \beta_0, \beta_1) \leq 0$, a contradiction. \square

Proof of Proposition 1:

To establish Proposition 1, I first prove the following mathematical claim:

Claim: *The following inequality holds for any $x, y \in \mathbb{R}^+$, $n \geq 1$, and any $a_i, b_i \in [0, 1] \forall i = 1 \dots n$ satisfying $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i = 1$ and at least one of a_i or b_i greater than zero $\forall i = 1 \dots n$. Furthermore, it holds with equality if and only if $a_i = b_i \neq 0 \forall i = 1 \dots n$.*

$$\sum_{i=1}^n \frac{a_i b_i}{a_i x + b_i y} \leq \frac{1}{x + y}$$

When $n = 1$, we must have $a_1 = b_1 = 1$ due to the constraints that each set of a_i and b_i must sum to one. In this case, the inequality is easily seen to be satisfied with equality.

So, consider $n > 1$. I show that the lhs of the inequality reaches its global maximum of $\frac{1}{x+y}$ when $a_i = b_i \neq 0 \forall i = 1 \dots n$ to show both that the inequality is always satisfied and that it only holds with equality when $a_i = b_i \neq 0 \forall i = 1 \dots n$.

I first show that $f(a_i, b_i) = \frac{a_i b_i}{a_i x + b_i y}$ is concave and use the fact that the sum of any number of concave functions is also concave so that $\sum_{i=1}^n \frac{a_i b_i}{a_i x + b_i y}$ is concave. With simple algebra, we have

$\frac{\partial^2 f(a_i, b_i)}{\partial a_i^2} = \frac{-2b_i^2 xy}{(a_i x + b_i y)^3} \leq 0$, $\frac{\partial^2 f(a_i, b_i)}{\partial b_i^2} = \frac{-2a_i^2 xy}{(a_i x + b_i y)^3} \leq 0$, and $\frac{\partial^2 f(a_i, b_i)}{\partial a_i \partial b_i} = \frac{-2a_i b_i xy}{(a_i x + b_i y)^3}$. Therefore, $\frac{\partial^2 f(a_i, b_i)}{\partial a_i^2} \frac{\partial^2 f(a_i, b_i)}{\partial b_i^2} - \left(\frac{\partial^2 f(a_i, b_i)}{\partial a_i \partial b_i} \right)^2 = 0$. Thus, the Hessian of $f(a_i, b_i)$ is negative semi-definite and therefore $f(a_i, b_i)$ is concave.

If any a_i or b_i is equal to zero, the corresponding term in the summation of the lhs is zero and thus contributes nothing. So, consider the $n^* \leq n$ non-zero terms of the summation. We must have $\sum_{i=1}^{n^*} a_i = v$ and $\sum_{i=1}^{n^*} b_i = w$ for some $v, w \leq 1$, due to the constraints. Consider the unconstrained maximization of the non-zero terms of the lhs of the inequality after using these constraints to substitute out a_{n^*} and b_{n^*} . Because we are maximizing a concave function, the first-order conditions are necessary and sufficient for determining the global maximizer(s) of the function. We have, $\forall i = 1 \dots n^* - 1$, the first-order conditions with respect to a_i :

$$\frac{b_i^2 y}{(a_i x + b_i y)^2} = \frac{b_{n^*}^2 y}{(a_{n^*} x + b_{n^*} y)^2} \iff \frac{b_i}{(a_i x + b_i y)} = \frac{b_{n^*}}{(a_{n^*} x + b_{n^*} y)} \iff \frac{a_i}{b_i} = \frac{a_{n^*}}{b_{n^*}}$$

The first-order conditions with respect to b_i result in the same set of equations. Substituting for each a_i in the constraint $\sum_{i=1}^{n^*} a_i = v$, we have $\frac{a_{n^*}}{b_{n^*}} \sum_{i=1}^{n^*} b_i = v \iff a_{n^*} = b_{n^*} \frac{v}{w}$ which then implies $a_i = b_i \frac{v}{w} \forall i = 1 \dots n^*$. Using this relationship, we have $\sum_{i=1}^{n^*} \frac{a_i b_i}{a_i x + b_i y} = \frac{vw}{vx + wy} \leq \frac{1}{x+y}$, where the inequality holds with equality only when $v = w = 1$ which requires no non-zero terms in the summation and implies $a_i = b_i \neq 0$. Therefore, $\sum_{i=1}^n \frac{a_i b_i}{a_i x + b_i y} \leq \frac{1}{x+y}$ is always satisfied and is satisfied with equality if and only if $a_i = b_i \neq 0 \forall i = 1 \dots n$, as claimed.

Now, to see that any additional informative potential trade reduces the benefit from waiting, one can apply the mathematical claim. In particular, looking at the general structure of the benefit functions, (1), one can see that an additional, independent, trade, c , which results in one of n possible actions, modifies each contribution to the benefit from an event, d_j , of trade d by replacing

$$\frac{d_{j,1} d_{j,0}}{p_t N T_1 d_{j,1} + (1 - p_t) N T_0 d_{j,0}}$$

with

$$\begin{aligned} \sum_{i=1}^n \frac{d_{j,1} d_{j,0} \Pr(c = c_i | V = 1) \Pr(c = c_i | V = 0)}{p_t N T_1 d_{j,1} \Pr(c = c_i | V = 1) + (1 - p_t) N T_0 d_{j,0} \Pr(c = c_i | V = 0)} \\ \leq \frac{d_{j,1} d_{j,0}}{p_t N T_1 d_{j,1} + (1 - p_t) N T_0 d_{j,0}} \end{aligned}$$

where $d_{j,y}$ is shorthand for $\Pr(d = d_j | V = y)$, $y \in \{0, 1\}$. The inequality follows from applying the claim with $a_i = \Pr(c = c_i | V = 1)$, $b_i = \Pr(c = c_i | V = 0)$, $x = p_t N T_1 d_{j,1}$, and

$y = (1 - p_t)NT_0d_{j,0}$. Thus, the additional trade reduces the benefit to waiting, where the inequality is strict if the trade is informative ($Pr(c = c_i|V = 1) \neq Pr(c = c_i|V = 0)$ for some $i \in 1 \dots n$ as long as $p_t \neq \{0, 1\}$).⁴⁷ \square

Proof of Proposition 2:

By continuity of the benefit functions, the limit as $p_t \rightarrow 1$ must be the value of the function evaluated at 1. Therefore, consider the bracketed term in the general formula for the benefit function given in (1), evaluated at $p_t = 1$. We have

$$\begin{aligned}
& \sum_{\hat{a} \in A} \frac{\hat{a}_0 \hat{a}_1 f(\underline{q}, \bar{q}, \beta_0, \beta_1)}{Pr(\hat{a} \& \underline{a}_t = NT)} - (2\underline{q} - 1) \\
&= \sum_{\hat{a} \in A} \frac{\hat{a}_0 \hat{a}_1 f(\underline{q}, \bar{q}, \beta_0, \beta_1)}{p_t \hat{a}_1 NT_1 + (1 - p_t) \hat{a}_0 NT_0} - (2\underline{q} - 1) \\
&= \sum_{\hat{a} \in A} \frac{\hat{a}_0 \hat{a}_1 f(\underline{q}, \bar{q}, \beta_0, \beta_1)}{\hat{a}_1 NT_1} - (2\underline{q} - 1) \\
&= \frac{f(\underline{q}, \bar{q}, \beta_0, \beta_1)}{NT_1} - (2\underline{q} - 1) \tag{4}
\end{aligned}$$

where $q \equiv \max(\bar{q}, \underline{q})$ and using $\sum_{\hat{a} \in A} \hat{a}_0 = 1$. (4) is the benefit of a trader that faces no informative intervening trades, $B_x^{ST}(p_t, \beta_0, \beta_1)$, evaluated at $p_t = 1$ (set $\hat{a}_0 = \hat{a}_1 = 1$ in the general formula for the benefit). At $p_t = 1$, and therefore, by continuity in the limit as $p_t \rightarrow 1$, we have then established that the sign of the benefit, which is determined by the bracketed term, is the same as when no informative intervening trades occur, establishing part 2 of the proposition. For part 1, it is easy to see that the magnitude of the benefit function approaches zero whether there are informative intervening trades or not because the part of the benefit outside of the bracketed term approaches zero as $p_t \rightarrow 1$ and the bracketed term remains finite, as shown above. The proof of both properties for p_t approaching 0 is identical. \square

Proof of Theorem 1:

Before proving Theorem 1, Lemma C1 establishes the remaining properties of the benefit functions exhibited in the example of Figure 2, specifically the number and locations of the zero-crossings. Theorem 1 assumes the restrictions on off-equilibrium public beliefs and prices and thus the simplified benefit functions given in (2) are the relevant ones for the theorem and lemma.

⁴⁷If event d were perfectly informative, the inequality would not be strict, but I have assumed signals (and therefore trades) are not perfectly informative.

Lemma C1: For $\bar{q} \leq \underline{q}$, each of the benefit functions, $B_x^{u,v_1v_2}(p_t)$, is ≤ 0 for all p_t . For $\bar{q} > \underline{q}$, the benefit functions satisfy:

1. $B_x^{u,v_1v_2}(p_t) = 0$ for $p_t \in \{0, 1\}$
2. For $p_t \in (0, 1)$:

- (a) $B_x^{R,WW}(p_t) > 0$
- (b) $B_x^{R,RR}(p_t), B_x^{W,WW}(p_t)$, and $B_x^{W,RR}(p_t)$ are either > 0 over the full range or

$$\begin{cases} < 0 & p_t \in (1 - \hat{p}^{u,v_1v_2}, \hat{p}^{u,v_1v_2}) \\ = 0 & p_t \in \{1 - \hat{p}^{u,v_1v_2}, \hat{p}^{u,v_1v_2}\} \\ > 0 & p_t \in (0, 1 - \hat{p}^{u,v_1v_2}) \cup (\hat{p}^{u,v_1v_2}, 1) \end{cases}$$

- (c) $B_x^{W,RW}(p_t)$ is either > 0 over the full range or

$$\begin{cases} < 0 & p_t \in (1 - \hat{p}^{W,WR}, \hat{p}^{W,RW}) \\ = 0 & p_t \in \{1 - \hat{p}^{W,WR}, \hat{p}^{W,RW}\} \\ > 0 & p_t \in (0, 1 - \hat{p}^{W,WR}) \cup (\hat{p}^{W,RW}, 1) \end{cases}$$

- (d) $B_x^{W,WR}(p_t)$ is either > 0 over the full range or

$$\begin{cases} < 0 & p_t \in (1 - \hat{p}^{W,RW}, \hat{p}^{W,WR}) \\ = 0 & p_t \in \{1 - \hat{p}^{W,RW}, \hat{p}^{W,WR}\} \\ > 0 & p_t \in (0, 1 - \hat{p}^{W,RW}) \cup (\hat{p}^{W,WR}, 1) \end{cases}$$

3. $1 - \hat{p}^{W,RR} < 1 - \hat{p}^{W,WR} < 1 - \hat{p}^{W,RW} < 1 - \hat{p}^{W,WW} < 1 - \hat{p}^{R,RR} < \hat{p}^{R,RR} < \hat{p}^{W,WW} < \hat{p}^{W,RW} < \hat{p}^{W,WR} < \hat{p}^{W,RR}$ where the zero crossings nearer to $p_t = \{0, 1\}$ exist if the next innermost zero crossing exists (but not necessarily the converse).

Proof of Lemma C1:

With $\underline{q} \leq \bar{q}$, the fact that all benefit functions are weakly less than zero follows from the fact that $B_x^{R,WW}(p_t) = 0$ for all p_t and Proposition 1. So, consider $\bar{q} > \underline{q}$. In this case, $Pr(\bar{a}_t = B) = Pr(\bar{s}_t = 1)$ and $Pr(\bar{a}_t = S) = Pr(\bar{s}_t = 0)$, so these substitutions can be made in equations (2).

Part 1 follows from evaluation of the benefit functions at $p_t = \{0, 1\}$.

Part 2a follows from the fact that $B_x^{R,WW}(p_t) > 0 \iff \bar{q} > \underline{q}$.

For part 2b, Proposition 2 ensures $B_x^{R,RR}(p_t), B_x^{W,WW}(p_t), B_x^{W,RR}(p_t) > 0$ as prices become close to 0 and 1 because each of these functions must have the same sign as $B_x^{R,WW}(p_t)$, which is positive from 2a. It remains to be shown that these functions cross zero at at most two additional prices, \hat{p}^{u,v_1v_2} and $1 - \hat{p}^{u,v_1v_2}$. Considerable algebraic manipulation allows each of

the three functions to be written

$$\begin{aligned}
B_x^{R,RR}(p_t) &= S_x \left[\frac{2(\bar{q} - \underline{q})\underline{Q} - (2\underline{q} - 1)^3 P_t}{Pr(\underline{s}_{t+1} = 1)Pr(\underline{s}_{t+1} = 0)} \right] \\
B_x^{W,WW}(p_t) &= S_x \left[\frac{2(\bar{q} - \underline{q})\bar{Q} - (2\underline{q} - 1)(2\bar{q} - 1)^2 P_t}{Pr(\bar{s}_t = 1)Pr(\bar{s}_t = 0)} \right] \\
B_x^{W,RR}(p_t) &= S_x \left[\frac{(2\bar{q} - 1)\bar{Q}QA - (2\underline{q} - 1)D}{D} \right]
\end{aligned}$$

where $A = \bar{Q}\underline{Q} + P_t(\bar{Q} + \underline{Q} - 8\bar{Q}\underline{Q})$, $D = P_t^2(\bar{Q} - \underline{Q})^2 + P_t\bar{Q}\underline{Q}(1 - 2\bar{Q} - 2\underline{Q}) + (\bar{Q}\underline{Q})^2$, and defining $\bar{P}_t \equiv p_t(1 - p_t)$.

Given that the zeros of each of the three benefit functions can be written as a function of P_t , it follows immediately that any zero, \hat{p}^{u,v_1v_2} must be paired with another zero, $1 - \hat{p}^{u,v_1v_2}$.⁴⁸ For $B_x^{R,RR}(p_t)$ and $B_x^{W,WW}(p_t)$, we have a linear function of P_t and thus there is at most one additional pair of zeros. Furthermore, we can see that there are cases in which such a pair of zeros exist by, for example, taking $\bar{q} \rightarrow 1$. For $B_x^{W,RR}(p_t)$, the additional zeros are determined by a quadratic function of P_t , so that there may exist two additional pairs of zeros. But I now show that one of the pairs always lies outside $p_t \in (0, 1)$ by showing that one of the solutions to the quadratic is always negative, $P_t^* < 0$, implying $p_t^* < 0$ or $p_t^* > 1$. The quadratic which determines the additional zeros of $B_x^{W,RR}(p_t)$ can be algebraically manipulated to obtain

$$\begin{aligned}
&(2\bar{q} - 1)\bar{Q}QA - (2\underline{q} - 1)D = 0 \\
\iff &-(2\underline{q} - 1)P_t^2(\bar{Q}^2 - \underline{Q}^2)^2 + P_t f(\underline{q}, \bar{q}) + 2(\bar{q} - \underline{q})(\bar{Q}\underline{Q})^2 = 0
\end{aligned}$$

where $f(\underline{q}, \bar{q})$ is a function which proves to be inconsequential. The solutions to the quadratic are, $P_t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a = -(2\underline{q} - 1)(\bar{Q}^2 - \underline{Q}^2)^2$, $b = f(\underline{q}, \bar{q})$, and $c = 2(\bar{q} - \underline{q})(\bar{Q}\underline{Q})^2$. Now, $a < 0$ and $c > 0$ for $\bar{q} > \underline{q}$ and therefore $-4ac > 0$. But, this means that $\sqrt{b^2 - 4ac} > |b|$ and therefore the root $-b - \sqrt{b^2 - 4ac}$ is always strictly negative. Thus, there are at most two additional zeros of $B_x^{W,RR}(p_t)$ and furthermore we know that there are exactly two roots in some cases because, by Proposition 1, $B_x^{W,RR}(p_t) < B_x^{W,WW}(p_t)$ and I have already established that $B_x^{W,WW}(p_t) < 0$ for $p_t \in (1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})$, for some parameterizations.

For parts 2c and 2d, I first note that Proposition 1 implies that

$$B_x^{W,RR}(p_t) < B_x^{W,RW}(p_t), B_x^{W,WR}(p_t) < B_x^{W,WW}(p_t)$$

for all $p_t \in (0, 1)$. Because of this fact and the fact that the equation that determines the additional zeros of $B_x^{W,RW}(p_t)$ and $B_x^{W,WR}(p_t)$ (beyond those that occur at $p_t = \{0, 1\}$) is cubic, one can easily see graphically that at most two of the additional zeros lie within $p_t \in (0, 1)$. That is, it is not possible to have either of these functions cross zero three times within $p_t \in (0, 1)$ and still have it remain within the envelope of $B_x^{W,RR}(p_t)$ and $B_x^{W,WW}(p_t)$, given their established properties. It also implies that, for some parameter

⁴⁸For specific parameters, the two zeros can be equal, $\hat{p}^{u,v_1v_2} = 1 - \hat{p}^{u,v_1v_2} = \frac{1}{2}$.

ranges these functions are always positive and for others they must cross zero exactly twice.⁴⁹ Therefore, what remains to be shown is that if $\hat{p}^{W,WR}$ and $\hat{p}^{W,RW}$ are zeros for $B_x^{W,WR}(p_t)$ and $B_x^{W,RW}(p_t)$, respectively, then $1 - \hat{p}^{W,WR}$ and $1 - \hat{p}^{W,RW}$ are zeros for $B_x^{W,RW}(p_t)$ and $B_x^{W,WR}(p_t)$, respectively, as claimed in the lemma.

Combining terms in $B_x^{W,RW}(p_t)$, we see that for it to be zero at $\hat{p}^{W,RW}$, we must have

$$(2\bar{q} - 1)\bar{Q} (Pr(1,1)Pr(1,0) + \underline{Q} (Pr(1,0) + Pr(1,1)) (\hat{p}^{W,RW}(1 - \bar{q}) + (1 - \hat{p}^{W,RW})\bar{q})) \\ - (2\underline{q} - 1)Pr(1,1)Pr(1,0) (\hat{p}^{W,RW}(1 - \bar{q}) + (1 - \hat{p}^{W,RW})\bar{q}) = 0$$

where $Pr(1,1) = Pr(\bar{s}_t = 1 \& \underline{s}_{t+1} = 1)$ and $Pr(1,0) = Pr(\bar{s}_t = 1 \& \underline{s}_{t+1} = 0)$ are understood to be evaluated at the price, $\hat{p}^{W,RW}$. We want to show that $1 - \hat{p}^{W,RW}$ is a zero for $B_x^{W,WR}(p_t)$. Combining terms in $B_x^{W,WR}(p_t)$, we see that for it to be zero at $1 - \hat{p}^{W,RW}$ requires

$$(2\bar{q} - 1)\bar{Q} (Pr(0,0)Pr(0,1) + \underline{q}(1 - \bar{q}) (Pr(0,0) + Pr(0,1)) ((1 - \hat{p}^{W,RW})\bar{q} + \hat{p}^{W,RW}(1 - \bar{q}))) \\ - (2\underline{q} - 1)Pr(0,1)Pr(0,0) ((1 - \hat{p}^{W,RW})\bar{q} + \hat{p}^{W,RW}(1 - \bar{q})) = 0$$

where, here, $Pr(0,0) = Pr(\bar{s}_t = 0 \& \underline{s}_{t+1} = 0)$ and $Pr(0,1) = Pr(\bar{s}_t = 0 \& \underline{s}_{t+1} = 1)$ are understood to be evaluated at the price, $1 - \hat{p}^{W,RW}$. Now, note that $Pr(0,0)$ evaluated at $1 - \hat{p}^{W,RW}$ is equal to $Pr(1,1)$ evaluated at $\hat{p}^{W,RW}$ and $Pr(0,1)$ evaluated at $1 - \hat{p}^{W,RW}$ is equal to $Pr(1,0)$ evaluated at $\hat{p}^{W,RW}$ so the two conditions are identical. Therefore, if $\hat{p}^{W,RW}$ is a zero of $B_x^{W,RW}(p_t)$, $1 - \hat{p}^{W,RW}$ is a zero of $B_x^{W,WR}(p_t)$. The reverse is shown in the same manner.

For part 3, the properties established in parts 2b-d and Proposition 1 ensure that

$$\hat{p}^{R,RR}, \hat{p}^{W,WW} < \hat{p}^{W,RW}, \hat{p}^{W,WR} < \hat{p}^{W,RR}$$

so it remains to be shown that $\hat{p}^{R,RR} < \hat{p}^{W,WW}$ and $\hat{p}^{W,RW} < \hat{p}^{W,WR}$. The fact that

$$1 - \hat{p}^{W,RR} < 1 - \hat{p}^{W,WR} < 1 - \hat{p}^{W,RW} < 1 - \hat{p}^{W,WW} < 1 - \hat{p}^{R,RR}$$

then follows by the established symmetry properties of the zero-crossings.

To show $\hat{p}^{R,RR} < \hat{p}^{W,WW}$, I show $B_x^{R,RR}(p_t) > B_x^{W,WW}(p_t)$ for all $p_t \in (0, 1)$. For $\bar{q} > \underline{q}$, this inequality is equivalent to

$$\frac{\underline{q}(1 - \underline{q})}{Pr(\underline{s}_{t+1} = 1)} + \frac{\underline{q}(1 - \underline{q})}{Pr(\underline{s}_{t+1} = 0)} > \frac{\bar{q}(1 - \bar{q})}{Pr(\bar{s}_t = 1)} + \frac{\bar{q}(1 - \bar{q})}{Pr(\bar{s}_t = 0)}$$

Substituting for the probabilities of each signal and simple algebra shows that this inequality is equivalent to $(\bar{q} - \underline{q})(1 - \underline{q} - \bar{q}) < 0$ which holds for $\bar{q} > \underline{q}$ and $\bar{q}, \underline{q} > \frac{1}{2}$.

To show $\hat{p}^{W,RW} < \hat{p}^{W,WR}$, I show that $B_x^{W,RW}(p_t) > B_x^{W,WR}(p_t)$ for $p_t > \frac{1}{2}$. This comparison

⁴⁹For specific parameters, the two zeros may be degenerate with both equal to $\frac{1}{2}$.

simplifies to

$$\begin{aligned}
& \frac{Q}{Pr(\bar{s}_t = 0)} + \underline{Q}\underline{Q} \left(\frac{1}{Pr(1,1)} + \frac{1}{Pr(1,0)} \right) > \frac{Q}{Pr(\bar{s}_t = 1)} + \underline{Q}\underline{Q} \left(\frac{1}{Pr(0,1)} + \frac{1}{Pr(0,0)} \right) \\
& \iff \frac{Pr(\bar{s}_t = 1) - Pr(\bar{s}_t = 0)}{Pr(\bar{s}_t = 0)Pr(\bar{s}_t = 1)} + \underline{Q} \left(\frac{Pr(\bar{s}_t = 1)}{Pr(1,1)Pr(1,0)} - \frac{Pr(\bar{s}_t = 0)}{Pr(0,1)Pr(0,0)} \right) > 0 \\
& \iff (Pr(\bar{s}_t = 1) - Pr(\bar{s}_t = 0)) Pr(1,1)Pr(1,0)Pr(0,1)Pr(0,0) \\
& + \underline{q}(1 - \underline{q})Pr(\bar{s}_t = 0)Pr(\bar{s}_t = 1) (Pr(\bar{s}_t = 1)Pr(0,1)Pr(0,0) - Pr(\bar{s}_t = 0)Pr(1,1)Pr(1,0)) > 0
\end{aligned}$$

Considerable algebraic manipulation of the second term allows one to factor out $Pr(\bar{s}_t = 1) - Pr(\bar{s}_t = 0)$, resulting in this difference multiplied by a term which can be shown to be strictly positive. Thus, $B_x^{W,RW}(p_t) > B_x^{W,WR}(p_t) \iff Pr(\bar{s}_t = 1) - Pr(\bar{s}_t = 0) = (2\bar{q} - 1)(2p_t - 1)$ which is strictly positive for all $p_t > \frac{1}{2}$. \square

Given Lemma C1, I now prove Theorem 1 by considering each of the price ranges in turn. The proofs for the cases of $\bar{q} \leq \underline{q}$ and for $p_t \in (1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})$ have already been established in the main text, so here the focus is on the case of $\bar{q} > \underline{q}$ and the remaining price ranges (parts 2c and 2d of the theorem). Also, as discussed in the main text, for the remaining price ranges, any equilibrium timing strategy involves waiting when $t - 1$ rushes, so I consider only the best response to $t - 1$ waiting here.

2c, part (i). When the unrestricted price chain passes through $(1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})$, if there exist any traders in the associated restricted price chain and also in the price range, $[\hat{p}^{W,WW}, \hat{p}^{W,WR})$, then one of them, p , must be such that the trader at p^- faces a price in $(1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})$. Then, because the trader at p^- rushes or conditionally rushes, p must conditionally rush because her benefit is either $B_x^{W,WR}(p_t)$, or $B_x^{W,RR}(p_t)$, both of which are negative (ruling out p mixing as well). But then any trader in the restricted price chain at prices greater than p must also conditionally rush for the same reason: her neighbor at the next lowest price in the price chain conditionally rushes. Similarly, any trader in the restricted price chain and also in the price range, $(1 - \hat{p}^{W,WR}, 1 - \hat{p}^{W,WW}]$, must conditionally rush because her neighbor at p^+ rushes or conditionally rushes, so her benefit is either $B_x^{W,RW}(p_t)$ or $B_x^{W,RR}(p_t)$, which are both negative in this price range.

2c, part (ii). When the unrestricted price chain does not pass through $(1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})$ and the restricted price chain contains only one price, p_s , then, by the definition of the restricted price chain, p_s^+ and p_s^- must both be such that the trader at those prices waits. Thus, the trader at p_s has a benefit of $B_x^{W,WW}(p_t) > 0$ and must wait.⁵⁰

2c, part (iii). When a restricted price chain contains more than one price and its associated unrestricted price chain does not pass through $(1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})$, multiple possible timing strategies can be sustained in equilibrium. Note that only the two traders nearest the ends of the restricted price chain have neighbors that lie outside it and these neighbors

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The conditions for this part of the theorem are only satisfied if $(\hat{p}^{W,WW})^- < 1 - \hat{p}^{W,WR}$. In this case, it can be shown that no non-empty restricted price chain has an associated unrestricted price chain with a price contained in $(1 - \hat{p}^{W,WW}, \hat{p}^{W,WW})$. Therefore, there is no parameterization for which 2c, parts (i) and (ii) both apply.

wait. Consider first case A in which all traders facing prices in the restricted price chain wait. Then, each trader in the price chain has a benefit of $B_x^{W,WW}(p_t) > 0$ so that waiting is a best response and thus an equilibrium.

I next show that if any trader in the restricted price chain conditionally rushes and all use pure strategies, then they must all do so. This fact simultaneously proves that all of the traders conditionally rushing (case B) can be sustained in equilibrium and that no other combination of pure strategies is possible.

First, note that any trader in $(1 - \hat{p}^{W,RW}, 1 - \hat{p}^{W,WW}) \cup [\hat{p}^{W,WW}, \hat{p}^{W,RW})$ must conditionally rush if either of her neighbors in the restricted price chain conditionally rushes because she then faces either $B_x^{W,RW}(p_t)$ or $B_x^{W,WR}(p_t)$, both of which are negative in this price range. Second, any trader in $[\hat{p}^{W,RW}, \hat{p}^{W,WR})$ must follow the same timing strategy as her neighbor at the next lowest price because if this neighbor conditionally rushes, she faces either $B_x^{W,WR}(p_t)$ or $B_x^{W,RR}(p_t)$ which are both negative and, if this neighbor waits, she faces either $B_x^{W,RW}(p_t)$ or $B_x^{W,WW}(p_t)$ which are both positive. Similarly, any trader in $(1 - \hat{p}^{W,WR}, 1 - \hat{p}^{W,RW}]$ must follow the same timing strategy as her neighbor at the next highest price. Therefore, it is immediate that if any trader in $(1 - \hat{p}^{W,RW}, 1 - \hat{p}^{W,WW}) \cup [\hat{p}^{W,WW}, \hat{p}^{W,RW})$ conditionally rushes, then all traders in the price chain must conditionally rush and that this set of timing strategies can be sustained in equilibrium. The remaining possibility is that a trader in either $(1 - \hat{p}^{W,WR}, 1 - \hat{p}^{W,RW}]$ or $[\hat{p}^{W,RW}, \hat{p}^{W,WR})$ conditionally rushes but some other trader in the restricted price chain waits. I now show this is impossible. Assume that the trader that conditionally rushes is in $[\hat{p}^{W,RW}, \hat{p}^{W,WR})$. Then, by the second property above, all traders in the restricted price chain at higher prices must conditionally rush too, so the only remaining possibility is that a trader at a lower price in the restricted price chain waits. But, by the second property, her neighbor at the next lowest price must be conditionally rushing, otherwise she herself would wait. This argument can be repeated until the neighbor at the next lowest price is at a price lower than $\hat{p}^{W,RW}$. This neighbor must be conditionally rushing. If this neighbor lies within $(1 - \hat{p}^{W,RW}, 1 - \hat{p}^{W,WW}) \cup [\hat{p}^{W,WW}, \hat{p}^{W,RW})$ then again, all traders must be conditionally rushing. If this neighbor is instead at a price less than $1 - \hat{p}^{W,RW}$, then all traders less than $1 - \hat{p}^{W,RW}$ must conditionally rush because they must all follow the same timing strategy as their neighbor at the next highest price. Finally, if the neighbor at the next lowest price is outside the restricted price chain, she can't be conditionally rushing which creates a contradiction. In all cases then, all traders in the restricted price chain must be conditionally rushing if any trader in $[\hat{p}^{W,RW}, \hat{p}^{W,WR})$ conditionally rushes. A symmetric argument establishes that if any trader in $(1 - \hat{p}^{W,WR}, 1 - \hat{p}^{W,RW}]$ conditionally rushes, then all must do so.

To demonstrate the possibility of mixed timing strategies in equilibrium (case C), consider the simplest case of only two prices in the restricted price chain, $p_1 < \frac{1}{2}$ and $p_2 > \frac{1}{2}$. The trader facing p_1 mixes between conditionally rushing and waiting such that the trader at p_2 has a benefit which is a linear combination of $B_x^{W,WR}(p_2) < 0$ and $B_x^{W,WW}(p_2) > 0$. The trader facing p_2 mixes such that the trader at p_1 has a benefit which is a linear combination of $B_x^{W,RW}(p_1) < 0$ and $B_x^{W,WW}(p_1) > 0$. It is easy to see that such mixing probabilities can always be found and therefore, there exists an equilibrium in which traders at both prices mix.

2d. For $p_t \in (0, 1 - \hat{p}^{W,RR}) \cup [\hat{p}^{W,RR}, 1)$, no matter what $t - 1$ and $t + 1$ do, t has a dominant strategy to wait because all of the benefit functions are strictly positive. Thus,

the only equilibrium timing strategy in this range is for all traders to wait (mixing is again precluded). Also note that if $\hat{p}^{W,RR}$ does not exist, then all benefit functions are strictly positive so that the unique equilibrium consists of all traders waiting at every price.

Next, consider $p_t \in [\hat{p}^{W,WR}, \hat{p}^{W,RR})$. For prices in this range, when $t - 1$ waits, t 's best response is to rush if $t + 1$ rushes after both a buy and sell by $t - 1$ and to wait otherwise. Thus, knowing that $t + 1$ waits if $t - 1$ buys is sufficient to ensure t 's best response is to wait. Mixing is also precluded in this case because even if $t + 1$ mixes after a sell decision, t faces a linear combination of $B_x^{W,WR}(p_t)$ and $B_x^{W,W}(p_t)$ which is always strictly positive in this price range, so t won't mix. Now, note that when $t - 1$ buys, the price increases so that, for some $p_t \in [\hat{p}^{W,WR}, \hat{p}^{W,RR})$, p_{t+1} exceeds $\hat{p}^{W,RR}$ so that $t + 1$ waits, as already established. Thus, the trader at this p_t waits. The following algorithm extends this reasoning to prove that the unique equilibrium timing strategy for $p_t \geq \hat{p}^{W,WR}$ is to always wait.

1. Define \hat{p}_i such that $\hat{p}_i^+ = \hat{p}_{i-1}$ for $i = 1 \dots k$ where $k \geq 1$ is the smallest integer for which $\hat{p}_k < \hat{p}^{W,WR}$. Also, define $\hat{p}_0 = \hat{p}^{W,RR}$.
2. If $k = 1$, jump to step 4. Otherwise, when $i = 1$, we have $p_t \geq \hat{p}_1$ is such that $p_t^+ \geq \hat{p}_0 = \hat{p}^{W,RR}$ so that $t + 1$ waits after a buy by $t - 1$. Therefore, t must wait in equilibrium $\forall p_t \in [\hat{p}_1, \hat{p}_0]$.
3. Repeating step 2 for $i = 2 \dots k - 1$, at each step we establish that waiting is the unique equilibrium $\forall p_t \in [\hat{p}_i, \hat{p}_{i-1}]$ because for $p_t \geq \hat{p}_i$ we know from the previous step that $t + 1$ waits when $t - 1$ buys ($p_t^+ \geq \hat{p}_{i-1}$).
4. At step k , for $p_t \geq \hat{p}^{W,WR}$, t must wait because $p_t^+ \geq \hat{p}_{k-1}$ and step $k - 1$ established $t + 1$ waits for $p_{t+1} \geq \hat{p}_{k-1}$. Thus, after k steps, we have established that each trader facing $p_t \in [\hat{p}^{W,WR}, \hat{p}^{W,RR})$ must wait.

The fact that k is finite follows from the fact a finite number of price increases are sufficient to move the price from $\hat{p}^{W,WR}$ to $\hat{p}^{W,RR}$. For $p_t < \hat{p}^{W,WR}$, the argument no longer goes through because knowing that $t + 1$ waits after a buy by $t - 1$ is not sufficient for t to wait. A symmetric argument for $p_t \in (1 - \hat{p}^{W,RR}, 1 - \hat{p}^{W,WR}]$ proves that the unique equilibrium timing strategy in this price range is also for t to wait. Here, a sufficient condition for t to wait is to know that $t + 1$ will wait after a sell decision by $t - 1$ because $B_x^{W,RW}(p_t) > 0$ for $p_t < 1 - \hat{p}^{W,WR}$. Finally, note that if $\hat{p}^{W,WR}$ does not exist, the argument above can be applied to all $p_t \in (1 - \hat{p}^{W,RR}, \hat{p}^{W,RR})$ so that the timing strategy in any equilibrium must be to wait over this range. \square

Proof of Proposition 3:

The proof consists of three parts. In the first part, I establish that, for all $\bar{q} \in (\frac{1}{2}, 1)$, there exists a $\underline{q}_l \in (\frac{1}{2}, \bar{q})$ such that the unique equilibrium involves traders waiting at all prices. In the second part, I show that there exists a $\underline{q}_h \in (\frac{1}{2}, \bar{q})$ such that at least some traders rush for any $p_1 \in (\frac{1}{2}, 1)$. Finally, using these facts, I show that there must exist a $\tilde{p} > p_1$ such that prices take strictly longer to converge in expectation to \tilde{p} under \underline{q}_h than under \underline{q}_l .

Fix \bar{q} to be any value in $(\frac{1}{2}, 1)$. By Theorem 1, part 2d, when the parameters are such that $\hat{p}^{W,WR}$ does not exist, then all traders must wait in any equilibrium. Given the shape of $B_x^{W,WR}(p_t)$ as determined by Lemma C1, $\hat{p}^{W,WR}$ does not exist if $B_x^{W,WR}(\frac{1}{2}) > 0$. So, it suffices to show that for any \bar{q} we can find a value of \underline{q} such that $B_x^{W,WR}(\frac{1}{2}) > 0$. From the

formula for $B_x^{W,WR}$ in (2), one can easily show that $B_x^{W,WR}(\frac{1}{2}) > 0$ at $\underline{q} = \frac{1}{2}$ for any $\bar{q} \in (\frac{1}{2}, 1)$. Also, Lemma C1 established that $B_x^{W,WR}(p_t) < 0$ at $\underline{q} = \bar{q}$ for any $p_t \in (0, 1)$ which includes $p_t = \frac{1}{2}$. Therefore, because $B_x^{W,WR}(\frac{1}{2})$ is continuous in \underline{q} , there must exist a $\underline{q}_l \in (\frac{1}{2}, \bar{q})$ such that $B_x^{W,WR}(\underline{q}_l) > 0$.

A sufficient condition for at least one trader to rush is $B_x^{R,RR}(p_1) < 0$ so that the first trader rushes in any equilibrium. From Lemma C1, $B_x^{R,RR}(p_t) < 0$ at $\underline{q} = \bar{q}$ for any $p_t \in (0, 1)$, including $p_t = p_1$. Therefore, because $B_x^{R,RR}(p_1)$ is continuous in \underline{q} , there must exist a $\underline{q}_h \in (\frac{1}{2}, \bar{q})$ such that $B_x^{R,RR}(p_1) < 0$. Furthermore, \underline{q}_h must be strictly greater than \underline{q}_l identified in the previous step. To see this, we have $B_x^{R,RR}(\frac{1}{2}) > B_x^{W,WR}(\frac{1}{2}) > 0$ when the benefits are evaluated at \underline{q}_l and the first inequality follows from Lemma 3. We also have $B_x^{R,RR}(\frac{1}{2}) < 0$ when the benefit is evaluated at \underline{q}_h . But, $B_x^{R,RR}(\frac{1}{2})$ is monotonically decreasing in \underline{q} so \underline{q}_h must be strictly greater than \underline{q}_l :

$$\begin{aligned} B_x^{R,RR}(\frac{1}{2}) &= \frac{1}{2} [4(2\bar{q} - 1)\underline{q}(1 - \underline{q}) - (2\underline{q} - 1)] \\ \implies \frac{\partial B_x^{R,RR}}{\partial \underline{q}}(\frac{1}{2}) &= \frac{1}{2} [4(2\bar{q} - 1)(1 - 2\underline{q}) - 2] < 0 \end{aligned}$$

where I have used $\underline{q} < \bar{q}$ in evaluating the benefit function and its derivative.

At the value of \underline{q}_l , we have established that all traders wait and therefore convergence is as in the benchmark model. At the value of \underline{q}_h , (at least) the first trader rushes, implying that the expected value of the price after the first trade is strictly less than the in the benchmark case because it is easily shown that $E[p_{t+1}|p_t, V = 1]$ is increasing in q . Therefore, if the number of trades were equal after some amount of time, the expected price would be lower with \underline{q}_h than with \underline{q}_l . However, due to the first trader rushing, there is initially one more trade with \underline{q}_h than with \underline{q}_l and so the expected price is initially higher with \underline{q}_h . But, if we set $\tilde{p} > p_1$ sufficiently close to 1, traders must begin to wait before reaching \tilde{p} because Lemma 4 established that, for $\underline{q} < \bar{q}$, at sufficiently high prices, all traders wait. Once traders wait, the number of trades becomes equal under \underline{q}_l and \underline{q}_h for any realized signals. Thus, there is no price path reaching \tilde{p} for which the number of trades is higher under \underline{q}_h and so the effect of any trades being earlier disappears. But then convergence to \tilde{p} must be slower in expectation under \underline{q}_h than \underline{q}_l . \square

Proof of Proposition 4:

I prove the proposition for the case of $\underline{s}_t = 1$. The other case is proven similarly. With only a single trader in the market, her benefit can be written (set $\hat{a}_0 = \hat{a}_1 = 1$ in the general form of the benefit function):

$$B_1(p_t) = \frac{p_t(1 - p_t)(2q - 1)(\frac{1-\mu}{4})}{Pr(\underline{s}_t = 1)} \left[\frac{1}{Pr(\bar{a}_{t+1} = B)} - \frac{1}{Pr(\underline{a}_t = B)} \right]$$

The term outside the brackets is always strictly positive, so the bracketed term determines the sign of the benefit. Cross-multiplying the terms inside the brackets, we see that that

sign is determined by

$$\begin{aligned}
& Pr(\underline{a}_t = B) - Pr(\bar{a}_{t+1} = B) \\
& \iff p_t \left(\frac{1-\mu}{4} + \mu q(1-\beta_1) \right) + (1-p_t) \left(\frac{1-\mu}{4} + \mu(1-q)(1-\beta_1) \right) \\
& \quad - p_t \left(\frac{1-\mu}{4} + \mu q^2 \beta_1 \right) + (1-p_t) \left(\frac{1-\mu}{4} + \mu(1-q)^2 \beta_1 \right) \\
& \iff \mu \left((p_t q + (1-p_t)(1-q)) - \beta_1 (p_t q^2 + (1-p_t)(1-q)^2 + p_t q + (1-p_t)(1-q)) \right)
\end{aligned}$$

From the above expression, we see that the benefit is strictly positive when $\beta_1 = 0$ and strictly negative when $\beta_1 = 1$ so that neither corner may be an equilibrium. The unique mixed strategy equilibrium is then obtained by setting the expression to zero, which corresponds to the strategy given in Proposition 4. \square

Proof of Corollary 1:

The proof of Proposition 1 in the case of the zero-profit model is identical to that in the expected value model except that, in applying the mathematical claim, $NT_x = Pr(\underline{a}_t = NT|V = x)$ is replaced with either $Pr(\bar{a}_{t+1} = B|V = x)$ or $Pr(\bar{a}_{t+1} = S|V = x)$, depending on which type of trader's benefit is being considered.

The proof of Proposition 2 in the case of the zero-profit model is identical to that in the expected value model. \square

Proof of Proposition 5:

By the application of Proposition 1 to the zero-profit model, if there is any possibility of either type of trader $t-1$ or $t+1$ trading between t and $t+1$, then each type of t 's benefit must be strictly less than when alone in the market, $B_x(p_t, \beta_x) < B_x^{ST}(p_t, \beta_x) \forall \beta_x \in [0, 1]$ and $\forall p_t$. But, given that $B_x(p_t)$ is strictly decreasing in β_x , this implies that t must be waiting with $\beta_x(p_t, I(\underline{a}_{t-1} = NT)) < \beta_x^{ST}(p_t)$ in equilibrium. The inequality is strict because $\beta_x^{ST}(p_t)$ is interior for all p_t . It then follows immediately that $\beta_x(p_t, 1) < \beta_x^{ST}(p_t)$ for both types of t because, in this case $t-1$ has waited and will potentially trade between t and $t+1$. Therefore, it remains only to show that $\beta_x(p_t, 0) = \beta_x^{ST}(p_t)$ is impossible for either type of t and, by the above argument, it is only possible if both types of $t+1$ always wait. But, if we assume $\beta_x(p_t, 0) = \beta_x^{ST}(p_t)$ for either type of t and both types of $t+1$ always wait, then we arrive at a contradiction: the type of t that is using $\beta_x(p_t, 0) = \beta_x^{ST}(p_t)$ is waiting with strictly positive probability and, when she does, each type of $t+1$ must rush with positive probability by the same argument as above, so both cannot be always waiting. \square

D. Formulas for the Zero-Profit Model

The unique bid and ask prices for delayed trades are:

$$p_{t+1}^A = Pr(V = 1 | \bar{H}_t, \bar{a}_{t+1} = B) = \frac{p_t \left(\frac{1-\mu}{4} + \mu q^2 \beta_1 \right)}{p_t \left(\frac{1-\mu}{4} + \mu q^2 \beta_1 \right) + (1-p_t) \left(\frac{1-\mu}{4} + \mu(1-q)^2 \beta_1 \right)}$$

$$p_{t+1}^B = Pr(V = 1 | \bar{H}_t, \bar{a}_{t+1} = S) = \frac{p_t \left(\frac{1-\mu}{4} + \mu(1-q)^2 \beta_0 \right)}{p_t \left(\frac{1-\mu}{4} + \mu(1-q)^2 \beta_0 \right) + (1-p_t) \left(\frac{1-\mu}{4} + \mu q^2 \beta_0 \right)}$$

For rushed trades, the prices are:

$$p_t^A = Pr(V = 1 | \underline{H}_t, \underline{a}_t = B) = \frac{p_t \left(\frac{1-\mu}{4} + \mu q(1-\beta_1) \right)}{p_t \left(\frac{1-\mu}{4} + \mu q(1-\beta_1) \right) + (1-p_t) \left(\frac{1-\mu}{4} + \mu(1-q)(1-\beta_1) \right)}$$

$$p_t^B = Pr(V = 1 | \underline{H}_t, \underline{a}_t = S) = \frac{p_t \left(\frac{1-\mu}{4} + \mu(1-q)(1-\beta_0) \right)}{p_t \left(\frac{1-\mu}{4} + \mu(1-q)(1-\beta_0) \right) + (1-p_t) \left(\frac{1-\mu}{4} + \mu q(1-\beta_0) \right)}$$

The benefit functions for generic actions of the other traders are given by:

$$B_1(p_t, \beta_1) = \frac{p_t(1-p_t)(2q-1)\left(\frac{1-\mu}{4}\right)}{Pr(\underline{s}_t = 1)} \left[\sum_{\hat{a} \in A} \frac{\hat{a}_0 \hat{a}_1}{Pr(\hat{a} \& \bar{a}_{t+1} = B)} - \frac{1}{Pr(\underline{a}_t = B)} \right]$$

$$B_0(p_t, \beta_0) = \frac{p_t(1-p_t)(2q-1)\left(\frac{1-\mu}{4}\right)}{Pr(\underline{s}_t = 0)} \left[\sum_{\hat{a} \in A} \frac{\hat{a}_0 \hat{a}_1}{Pr(\hat{a} \& \bar{a}_{t+1} = S)} - \frac{1}{Pr(\underline{a}_t = S)} \right]$$

The timing strategies of the informed traders, β_0 and β_1 , enter the benefit functions through the probabilities of observing a buy and sell decision at each point in time:

$$Pr(\underline{a}_t = B) = p_t \left(\frac{1-\mu}{4} + \mu q(1-\beta_1) \right) + (1-p_t) \left(\frac{1-\mu}{4} + \mu(1-q)(1-\beta_1) \right)$$

$$Pr(\underline{a}_t = S) = p_t \left(\frac{1-\mu}{4} + \mu(1-q)(1-\beta_0) \right) + (1-p_t) \left(\frac{1-\mu}{4} + \mu q(1-\beta_0) \right)$$

$$Pr(\hat{a} \& \bar{a}_{t+1} = B) = p_t \hat{a}_1 \left(\frac{1-\mu}{4} + \mu q^2 \beta_1 \right) + (1-p_t) \hat{a}_0 \left(\frac{1-\mu}{4} + \mu(1-q)^2 \beta_1 \right)$$

$$Pr(\hat{a} \& \bar{a}_{t+1} = S) = p_t \hat{a}_1 \left(\frac{1-\mu}{4} + \mu(1-q)^2 \beta_0 \right) + (1-p_t) \hat{a}_0 \left(\frac{1-\mu}{4} + \mu q^2 \beta_0 \right)$$

E. Multiple Arrival ($n > 1$)

In this section, I extend the expected value model to one in which n traders arrive in each period. Each trader receives an independent signal in the period they arrive and in the next period if they choose to wait. With multiple traders arriving each period, waiting costs are increased such that traders may rush to trade with even greater differences in signal quality. This extension is used in the numerical simulations of Section 4 and to derive the comparative static prediction with respect to volume of Section 6. A full equilibrium characterization is

difficult to obtain and I do not attempt to derive one here. Instead, I rely on a combination of theoretical results and numerical analysis to determine an upper bound on the average price reached and a lower bound on the probability of increasing mispricing as functions of time. Because numerical analysis is used, I focus on the specific case of $n = 4$, $\underline{q} = 0.7$ and $\bar{q} = 0.8$ used in Section 4. However, most of the analysis is general in nature and applies to a wide range of parameters.

I look for a symmetric equilibrium in which traders with the same signals follow the same strategies. Because traders who trade at a time t do not affect the price each other faces, the optimal trading strategies are identical to the $n = 1$ case. Furthermore, the general form of the benefit function, (1) continues to apply because it was derived for any possible combination of trades while one waits. Therefore, analogous statements to Lemma 2 and Propositions 1 and 2 can be easily derived. In particular, Lemma 2 implies that both types of traders must follow the same timing strategy so that the simplified benefit function of Appendix A applies. With the addition of the symmetry assumption, we then have that all traders arriving in the same period must follow the same timing strategy.

The main difficulty in obtaining a complete characterization of the equilibrium for $n > 1$ is that when the $t - 1$ traders wait, there are many possible prices that the traders at $t + 1$ may face and thus many possible benefit functions a trader at t may face. Thus, the approach used for the $n = 1$ case, looking at each of the price ranges determined by the zero-crossings of each of the benefit functions, becomes extremely tedious. My approach is to instead characterize the equilibrium within a specific price range and assume all traders wait outside this range. I can then simulate an upper bound on the average price reached and a lower bound on the probability of widening mispricing as argued in Section 4.

More specifically, for the parameters given, I claim that there exists a price, $\hat{p}_n^{R,R}$, such that any equilibrium involves all traders rushing when $p_t \in (1 - \hat{p}_n^{R,R}, \hat{p}_n^{R,R})$. In particular, $1 - \hat{p}_n^{R,R}$ and $\hat{p}_n^{R,R}$ are the prices at which the benefit to waiting when all $t - 1$ and $t + 1$ traders rush, $B_n^{R,R}(p_t)$, is zero. The analytic formula for this benefit is easily obtained using the simplified general form of the benefit in Appendix A. However, establishing the properties corresponding to those of Lemma C1 is more difficult so I rely on numerical simulation to show that it in fact crosses zero at exactly two prices which are symmetric around $p = \frac{1}{2}$, $1 - \hat{p}_n^{R,R}$ and $\hat{p}_n^{R,R}$, where $\hat{p}_n^{R,R} \approx 0.866$. The benefit function is positive near $p = 0, 1$ and negative otherwise. The benefit function when all $t - 1$ and $t + 1$ traders wait, $B_n^{W,W}(p_t)$ also plays a role. Because $\bar{q} > \underline{q}$, the price impact of a $t - 1$ trader is greater than that of a t trader, so $B_n^{W,W}(p_t) < B_n^{R,R}(p_t) < 0$ over $p_t \in (1 - \hat{p}_n^{R,R}, \hat{p}_n^{R,R})$.⁵¹ To establish the claim, I show that there exists an equilibrium in which all traders rush, independent of the number of $t - 1$ traders that wait (with n traders, the number of $t - 1$ traders observed to have waited is payoff-relevant) and that there does not exist an equilibrium in which all traders wait on the equilibrium path. The proof is as follows.

Consider first the claim that there does not exist an equilibrium in which all traders wait on the equilibrium path. If all $t - 1$ traders rush and the traders at t were to all wait at some $p_t \in (1 - \hat{p}_n^{R,R}, \hat{p}_n^{R,R})$, then the traders at $t + 1$ face the same price with a benefit

⁵¹It can be generally shown that one's benefit is strictly decreasing in the signal quality of any trade that occurs while one waits (proof available upon request). This fact ensures that a trade by $t - 1$ impacts t more than a trade by $t + 1$ when $\bar{q} > \underline{q}$.

of $B_n^{W,W}(p_t) < 0$ or less (depending on the strategies of the $t + 2$ traders: by Proposition 1, if any $t + 2$ trader is rushing, $t + 1$'s benefit would be reduced from $B_n^{W,W}(p_t)$) and so each would have an incentive to deviate to rush, a contradiction. Alternatively, if all $t - 1$ traders wait, then all t traders have a benefit of $B_n^{W,W}(p_t) < 0$ or less, and so would rush, a contradiction. Thus, the only possible symmetric equilibria are those in which all traders rush on the equilibrium path. So, consider the strategy profile in which all traders rush regardless of the number of $t - 1$ traders that wait. One must show that no single trader has an incentive to deviate to wait under this strategy profile. There are two possible cases.

In the first case, all $t - 1$ traders were observed to rush. In this case, if a single trader at t deviates to wait, the $t + 1$ traders face $p_t \in (1 - \hat{p}_n^{R,R}, \hat{p}_n^{R,R})$ and therefore rush under the specified strategy profile. Thus, the trader at t that is considering deviating faces the price impacts of $2n - 1$ traders. Given that the price impact of n traders gives her a benefit of $B_n^{R,R}(p_t) < 0$ and that Proposition 1 ensures additional traders reduce her benefit further, she will not deviate.

In the second case, one or more $t - 1$ traders were observed to wait. In this case, if t deviates, nothing can be said about the timing decisions of the $t + 1$ traders because the trades by the $t - 1$ traders may drive the price outside of $p_t \in (1 - \hat{p}_n^{R,R}, \hat{p}_n^{R,R})$. However, t still faces the price impacts of the $n - 1$ other t traders as well as those of the one or more $t - 1$ traders. Because $\bar{q} > \underline{q}$, the price impact of a $t - 1$ trader is greater than that of a t trader, so t 's benefit from waiting is strictly less than $B_n^{R,R}(p_t) < 0$, so she will not deviate.

In summary, there exists an equilibrium in which all traders rush and there does not exist an equilibrium in which all traders wait for all $p_t \in (1 - \hat{p}_n^{R,R}, \hat{p}_n^{R,R})$. Having established these facts, I can then simulate the model under the additional assumption that all traders wait outside $p_t \in (1 - \hat{p}_n^{R,R}, \hat{p}_n^{R,R})$. This simulation provides an upper bound for the average price reached and a lower bound for the probability of increases in mispricing. If in fact traders rush more often than assumed, the average price would in fact be lower and the probability of increases in mispricing higher. Simulation results are presented in Section 4.

Finally, the analysis for $n > 1$ justifies the claim in Section 6 that higher volume leads to more panics and therefore less informational content in trades. Proposition 1 guarantees that an increase in n decreases the benefit to waiting and thus makes the interval over which traders must panic, $(1 - \hat{p}_n^{R,R}, \hat{p}_n^{R,R})$, larger. It is in this sense that an increase in volume leads to more panics. As an example, when $\underline{q} = 0.7$ and $\bar{q} = 0.8$, going from $n = 1$ to $n = 4$ changes the interval from not existing to about 73% of the entire price range.