

# MEANS OF PAYMENT AND TIMING OF MERGERS AND ACQUISITIONS IN A DYNAMIC ECONOMY\*

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# MEANS OF PAYMENT AND TIMING OF MERGERS AND ACQUISITIONS IN A DYNAMIC ECONOMY

## **Abstract**

We study the interplay between bidders equilibrium timing of acquisitions, means of payment (cash versus stock), and takeover premiums when bidders are cash-constrained. Because of ability to bid in stock, constraints have no effect on bidders' maximum willingness to pay. However, both own and rivals constraints usually make a bidder more reluctant to initiate a bid. The effect of own constraints is opposite for high-growth high-synergy targets. Both fundamentals and cash constraints drive acquisition activity. Positive-but low-synergy targets are either never acquired or acquired after they have grown and using stock, while the opposite is true for high-synergy targets.

*Keywords:* Auctions, financial constraints, mergers and acquisitions, real options, security design.

The decision to acquire a target is one of the most important choices that the firm's management and board of directors face, with the potential to gain or lose millions and billions in profit.<sup>1</sup> It is therefore important to understand how these multifaceted decisions are made and what factors affect them. While according to the neoclassical theory of mergers, the only driver of a merger timing and outcome should be the net total gains created from the deal, it appears that the ability of bidders to pay cash and, more broadly, access to finance is also important.<sup>2</sup>

The link between bidders' cash constraints and their propensity to make acquisitions is not obvious. A simple intuition would suggest that once a bidder and the target find the deal worthwhile, the target can agree to receive the payment in stock of the combined company, in case the bidder is unable to pay cash. This intuition is incomplete because the split of gains from the deal can depend on the means of payment, which, in turn, can affect the timing of the deal. The goal of this paper is provide a theoretical analysis of a bidder's dynamic decision to bid for the target in the presence of a cash constraint. We build a real-options model of acquisitions based on three simple assumptions: (i) a bidder can choose when to approach the target with an offer; (ii) its synergy with the target is this bidder's private information; (iii) its ability to pay cash is limited by a cash constraint. Our analysis has three main insights. First, a bidder's cash constraint as well as cash constraints of other potential acquirers matter for the decision of a bidder to initiate a bid for the target despite their ability to bid in stock. Second, there is heterogeneity across targets and bidders in how a bidder's cash constraint affects initiation. For a typical target, e.g., a company with COMPUSTAT-average growth and volatility of assets, the presence of a bidder's cash constraint results in a delay of deal initiation. However, for high-growth, high-volatility targets, high-synergy bidders who are unable to pay cash have incentives to initiate a deal faster. Finally, the model delivers many implications about the relationship between means of payment in acquisitions, synergies, cash constraints, and the distribution of gains among the contest participants. Many of these implications are consistent with existing empirical evidence, and some have not been looked at yet.

More specifically, we consider a dynamic model in which there are three agents: a target and two potential bidders. The target is a growth firm: its assets and cash flows stochastically grow over time.

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<sup>1</sup>In 2007 alone, the value of world-wide deal volume of M&A deals exceeded \$4.8 trillion.

<sup>2</sup>For neoclassical arguments, see, e.g., Mitchell and Mulherin (1996), Jovanovic and Rousseau (2002), and Lambrecht (2004). For evidence on relation of M&A to the costs of borrowing see Harford (2005).

Both bidders are mature companies: the bidder's assets and cash flows do not grow unless it acquires the target.<sup>3</sup> The bidders have synergies with the target: an acquisition improves productivity of the target in a combined company by a bidder-specific multiple. At any time each bidder can approach the target with an offer. Upon this event, the auction between the first bidder and the competitor is initiated, and the bidder who submits the highest bid wins the auction. A bidder's decision when to approach the target reflects the following trade-off. On one hand, approaching the target earlier leads to an earlier increase in its productivity. On the other hand, a deal involves a cost: If the bidder loses the auction, its post-merger value will diminish, because it will face a stronger competitor: for example, for the competitor, combining assets with the target can lead to cost savings, more innovation, and ultimately a larger market share. In addition, approaching the target today destroys the option to acquire the target in the future. If the bidder's valuation of the target is low, it is optimal to wait until the target grows in size so that the increase in its productivity outweighs the cost of the acquisition.

The second building block of the model is information asymmetry between the target and the bidders. Similarly to the literature on auctions, but unlike the prior literature that considers takeovers in the real-options framework, we assume that potential synergies from acquiring the target are the private information of the bidder. As shown in the literature on securities auctions, this feature makes bids in stock and in cash not equivalent, in contrast to the case when bidders do not have any private information. Specifically, because the value of a winning bid in stock (but not in cash) depends on the winning bidder's private information, it is costlier for a bidder to separate itself from a marginally lower type in a stock auction than in a cash auction. Intuitively, even if both stock bidders offer the same proportion of the combined company to the target's shareholders, the bidder with the higher valuation ends up paying more in cash equivalent. Because of this effect, each bidder wants to bid in cash whenever possible. The ability to do this is, however, limited by the third building block of the model: the financing constraint of the bidder. We model it by assuming that the bidder cannot pay in cash above a certain limit.

We initially solve for the equilibrium initiation strategies and terms of takeovers in three special cases of the model: both bidders are unconstrained and thus bid in cash; both bidders are extremely constrained and thus bid in stock; one bidder is unconstrained, while the other is extremely constrained. These cases are convenient for the analysis of the effects of cash constraints on the timing of acquisitions

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<sup>3</sup>An alternative interpretation of the framework is that the target's assets and cash flows change relative to those of the bidders.

but are limited, because they do not allow for endogenous means of payment. Later, we solve the general model with arbitrary cash constraints.

Our first result concerns the link between a bidder's cash constraint and its decision to initiate a bid. We show that there are two opposite effects. The first, static, effect is that bidding in stock transfers surplus from the winning bidder to the seller. As a result, all else equal, the bidder's expected payoff from the auction is lower if the bidder is more cash constrained. This higher payoff from option exercise leads to an earlier exercise, i.e., an earlier initiation of the auction. The second, dynamic, effect is that the fraction of the total surplus that the winning bidder obtains in a stock auction, all else equal, decreases as the target grows over time. Intuitively, if the target is very small, there is little difference between dollar values of bidders' private synergies and hence between bids in cash and in stock. However, these differences are substantial if the target is large. This dynamic effect has the opposite impact on the bidder's decision to bid for the target: because of it, a more constrained bidder benefits from accelerating the bid. If the target does not grow very quickly or the bidder's synergy is not too high, the first effect always dominates, and cash constraints always make the bidder delay bidding for the target. However, if the target operates in a high-growth or high-volatility industry and the bidder has a high valuation of the target, the second effect may dominate, and constraints can speed up the acquisition. The non-trivial interplay between the static and dynamic effect on acquisition timing is a novel feature of our dynamic model. It cannot be obtained by simply extending the intuition of a static model of security-bid auctions (which would imply only the static effect) into dynamics, nor can it be obtained in a real-options model of acquisitions with perfect information structure.

Second, we show that a bidder's decision to initiate a bid is affected not only by its own cash constraint, but also by the cash constraint of its rival. In the "typical" case when own cash constraints delay acquisitions, cash constraints of both bidders have the same directional effect: a bidder is more reluctant to initiate a bid if the rival bidder is constrained than if it is unconstrained. This result obtains because a bidder learns differently about the valuation of its differently constrained rival by observing that the rival has not initiated the bid for the target yet. When the rival is constrained, it is more reluctant to bid for the target for the same valuation. Hence, observing that the constrained rival has not approached the target yet, the other bidder does not update its estimate of the rival bidder's valuation as much as in the case of the unconstrained rival. In its eyes, the bidder faces a stronger competitor at each date. As a result, this bidder expects to obtain a lower payoff from the

auction with the constrained rival, which also make it delay its initiation.

To further study interactions between the choice of timing and means of payment, we provide solution of the general model with arbitrary cash constraints. We show that there is endogenous selection of deals into cash and non-cash. Despite cash constraints being uncorrelated with bidders' synergies by assumption, high-synergy targets are typically acquired young and for cash, while low-synergy targets are typically acquired old (if at all, despite positive synergies) and for stock. Intuitively, if the bidder expects high synergies, it does not pay off to wait, so the target is acquired when small. As a result, for an acquirer, the required payment is likely to be below the financing constraint, leading to deals done in cash. Because of high synergies, such deals are also likely to result in high takeover premiums (relative to the current value of the target under its current management). Thus, the model predicts that in a sample of deals, cash deals can be associated with higher takeover premiums, despite that stock deals are perceived as more expensive by bidders. This finding is broadly consistent with empirical evidence (e.g., Betton, Eckbo, and Thorburn, 2008). While this evidence can seem inconsistent with predictions of static security-bid auctions literature, it becomes consistent once dynamic selection of targets by bidders into cash and stock deals is taken into account.

The model delivers interesting comparative statics as to which deals are likely to be done in cash versus in stock and when. For example, all else equal, the option to delay approaching the target is more valuable if the value of the target's assets is more volatile. Thus, such targets are acquired later, when the financial constraint of the acquirer is less likely to be satisfied, and hence are more likely to be done in stock. All else equal, stock deals for these targets are also, on average, better than stock deals for lower-risk targets: they have higher average synergies and higher average takeover premiums.

Our paper is related to three strands of research. First, it is related to literature that studies mergers and acquisitions as real options. Lambrecht (2004) studies a setting in which mergers are driven by economies of scale and shows that the merger takes place once the price of the industry output rises to a sufficiently high threshold, thereby providing a rationale for the procyclicality of mergers. Hackbarth and Morellec (2008) apply a similar framework to a setting with incomplete information between the market and the merging firms to study the dynamics of stock returns and risk in M&A. Other papers that study mergers and acquisitions as real-options problems include Morellec and Zhdanov (2005), Alvarez and Stenbacka (2006), Lambrecht and Myers (2007), Margsiri, Mello, and Ruckes (2008), Morellec and Zhdanov (2008), and Hackbarth and Miao (2012). To our knowledge, all prior literature assumes that the target and the potential acquirer have the same information about the value of the

combined company. This assumption makes cash and stock bids equivalent, and thus bidders' ability to pay cash is irrelevant. To make it relevant, we follow the traditional literature on auctions in assuming that bidders have private information about their valuations of the target.

Second, our paper is related to information theories of means of payment in mergers and acquisitions and, more generally, in auctions in which bidders can make bids in securities.<sup>4</sup> These models are static, and do not explore strategic timing in the presence of financing constraints. An exception is Cong (2013) who studies the interplay between post-auction moral hazard and the seller's strategic timing of auctioning the asset in a security-bid auction framework. Cong (2013) does not consider cash constraints of bidders, which is our focus here. Perhaps, the most relevant paper in this literature is Fishman (1989), as it delivers several of our empirical implications for means of payment using a different mechanism, in a static model with a two-sided information asymmetry between bidders and the target.<sup>5</sup> The advantage of a stock bid is that it reduces the adverse selection problem, inducing a more efficient accept/reject decision of the target. A cash bid is, however, used when a bidder has a high enough valuation to preempt competition by signaling a high valuation. In contrast to Fishman (1989), our paper shows that a one-sided information asymmetry in which only bidders have private information is sufficient to capture empirical evidence on means of payment, once dynamic aspects are taken into account. It also explains why stock bids are often perceived as more expensive by bidders, yet look smaller in the data. The way to test the relative importance of the two explanations for the observed means of payment would be to study whether the timing of acquisitions (captured by, e.g., size of the target and its age), dynamic target characteristics (e.g., growth and volatility of assets), and financing constraints of bidders are related to the payment type in the data.

Finally, our paper is related to literature on auctions with cash-constrained bidders. Che and Gale (1998, 2000) and Che, Gale, and Kim (2013) consider buyers with exogenous budget constraints, as we do here. Zheng (2001), Rhodes-Kropf and Viswanathan (2005), Board (2007), and Vladimirov (2012) have bidders that can raise capital in the financial market to finance their cash bids. All these papers restrict bids to be made in cash. Our contributions to this literature are that we allow bidders to time the decision to bid strategically and to make bids in securities.

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<sup>4</sup>Security-bid auctions are studied by Hansen (1985), Rhodes-Kropf and Viswanathan (2000), DeMarzo, Kremer, and Skrzypacz (2005), Gorbenko and Malenko (2011), and Liu (2013). Skrzypacz (2013) provides a review of the literature.

<sup>5</sup>Other static models of means of payment are provided by Hansen (1987), Eckbo, Giammarino, and Heinkel (1990), and Berkovitch and Narayanan (1990). Bhagwat and Dam (2013) relate means of payment to insurance against stock price fluctuations between the announcement of the deal and its completion. Shleifer and Vishny (2003) and Rhodes-Kropf and Viswanathan (2004) develop theories relating means of payment in mergers to merger waves.

The remainder of the paper is organized in the following way. Section I outlines the setup of the model. Section II solves for the equilibrium in the auction with cash-constrained bidders, assuming that the auction has just been initiated. The next two sections endogenize the auction’s timing. Specifically, Section III solves for the full equilibrium of the model in three special cases: when both bidders are unconstrained, when both bidders are extremely constrained, and when one bidder is unconstrained, and the other is extremely constrained. Section IV considers the general case of the model, thereby also endogenizing the means of payment. Section V provides the comparative statics analysis. Section VI studies the properties of the equilibrium and the predictions of the model, and discusses testable hypotheses. Section VII provides discussion of various alternative model assumptions and their impact on the results. Section VIII concludes. All proofs appear in Appendix A. Appendix B contains the details of numerical solutions.

## I Model Setup

We consider a setting in which the risk-neutral target attracts two potential risk-neutral acquirers, or bidders. The roles of the target and the bidders are exogenous. The value of the target as a separate entity at time  $t$  is given by  $X_t$ , where  $X_t$  evolves as a geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dB_t, \quad X_0 = x. \quad (1)$$

Here,  $\mu$  and  $\sigma > 0$  are constant growth rate and volatility, and  $dB_t$  is the increment of a standard Brownian motion. The discount rate is constant at  $r$ . To guarantee finite values, we assume that  $r > \mu$ . Process  $(X_t)_{t>0}$  is a reduced-form specification of the present value of the target’s assets. For example, this value can be obtained by assuming that the target produces cash flow  $(r - \mu) X_t$  per unit of time. We interpret  $X_t$  as the current size of the target. It accounts for all exogenous shocks to its value, such as changes in the price of the final product and inputs, as well as for the endogenous response of the target firm to them.<sup>6</sup> The initial value of each bidder as a separate entity is constant

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<sup>6</sup>In this paper, we focus on fundamental rather than market prices of the target (that is, prices clear of market expectations about the potential acquisition). This is consistent with related empirical studies, in which target prices are typically cleared of pre-acquisition runups.

at  $\Pi_b$ .<sup>7</sup> If bidder  $i$  acquires the target at time  $t$ , the value of the combined firm is

$$\Pi_b + v_i X_t, \tag{2}$$

where  $v_i \in [\underline{v}, \bar{v}]$ ,  $\bar{v} > \underline{v} > 1$  is the multiple that characterizes an improvement in operations of the target due to a change in ownership.<sup>8</sup> We refer to  $v_i$  as bidder  $i$ 's valuation of the target. Importantly, each bidder's valuation is its private information that is known to it before the start of the acquisition process.<sup>9</sup> Each valuation is an i.i.d. draw from distribution with p.d.f.  $f(v) > 0$  on  $[\underline{v}, \bar{v}]$ . Each bidder knows its valuation, but not the valuation of its competitor, except for the distribution. We assume that the distribution of valuations satisfies the restriction that the payoff of the winning bidder monotonically increases in its valuation  $v$  in all specifications.<sup>10</sup> This assumption intuitively means that the direct effect on the winner's payoff of having a higher valuation is stronger than the indirect effect of a higher expected payment.

To have a non-trivial timing of the acquisition, the deal has to entail a cost. We capture this cost by assuming that the losing bidder is also affected by the acquisition: its value changes from  $\Pi_b$  to  $\Pi_o < \Pi_b$ . Intuitively, the acquisition makes the winning bidder a stronger competitor for the losing bidder, resulting in the lower post-acquisition value of the latter.<sup>11</sup> For example, the recent acquisition of Instagram by Facebook made Facebook a stronger competitor for other social network firms. This loss in the losing bidder's value is a source of delay of the acquisition in the model. Of course, other potential sources of delay such as direct costs of initiating the takeover contest are possible too. We denote the value loss of the losing bidder as  $\Delta \equiv \Pi_b - \Pi_o$ .

In practice, acquisitions by strategic buyers are usually initiated by a potential bidder, rather than

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<sup>7</sup>Bidders' values are equal for simplicity of exposition; this assumption does not affect the main trade-offs of the model. This setup captures a situation in which a relatively mature company aims to acquire a growing company. An additional assumption could be that the growth rate of the target decreases as it grows, so that it becomes a more mature company. Although more realistic, this assumption results in less tractability and does not alter the economics behind our results. Similarly, it is possible to extend our setup by allowing bidders to grow over time. Our results hold in this setup as long as the cash balances of each bidder, defined below, do not grow at a faster rate than the target.

<sup>8</sup>Allowing  $\underline{v}$  below one does not enrich the model intuition in any way other than some targets will be sold to uncontested acquirers.

<sup>9</sup>Introducing the additional private information that the bidder can learn at the beginning of the contest does not affect the results of the model qualitatively. It is only the ex-ante private information that defines bidders' strategies to initiate the takeover contest.

<sup>10</sup>For example, in the model of Section II.B this restriction is equivalent to a restriction that  $v - \mathbb{E}[w|w \leq v]$  is a strictly increasing function of  $v$ . An example of distribution that satisfies these restrictions is uniform distribution.

<sup>11</sup>Spiegel and Tookes (2013) quantify this effect at 1.86% of the rival firm value on average. Horizontal mergers also feature an opposite effect, because the losing bidder faces fewer competitors. This effect is not present in our setup, because the target is not a direct competitor of the bidder.

the target (Fidrmuc et al., 2012). To reflect this practice, we assume that each bidder has a real option to approach the target at any time.<sup>12</sup> If a bidder approaches the target at time  $t$ , the takeover contest is initiated and both bidders compete for the target in an open ascending-bid auction, formally defined below. Payments can be in cash, stock of the combined company, or their combination. The ability to submit bids in cash is potentially limited by a bidder’s cash constraint. For simplicity, we assume that bidder  $i$  can pay up to  $C_i$  units of cash, and the cash constraint is infinitely rigid after that.<sup>13</sup>

## I.A The Auction

We extend the formalization of the English (open ascending) auction for bids in combinations of cash and stock. The following definition sets up a formal structure:

**Definition (English auction for bids in combinations of stock and cash).** The auctioneer sets the starting price to zero and gradually raises it. As price  $p$  rises, a bidder confirms its participation until it decides to withdraw from the auction. Confirming participation at price  $p$  means that the bidder commits to pay any combination of  $b \geq 0$  dollars in cash and fraction  $\alpha$  in the stock of the combined company, whose value, evaluated according to the seller’s beliefs, is greater or equal than  $p$ . As soon as only one bidder remains, it is declared the winner. This bidder then chooses a combination  $(b, \alpha)$  to offer to the target. If  $\mathbb{E}[b + \alpha(\Pi_b + vX_t) | \mathcal{I}] \geq p$ , where  $\mathcal{I}$  is the information set of the target at this point, then this bidder acquires the target in exchange for paying  $b$  in cash and fraction  $\alpha$  of the combined company in stock. If  $\mathbb{E}[b + \alpha(\Pi_b + vX_t) | \mathcal{I}] < p$ , the bidder defaults on its commitment, suffers a reputation loss  $R > \Delta$ , and the target remains independent forever.<sup>14</sup>

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<sup>12</sup>Section VII.B extends the model by allowing the target to affect the timing of the acquisition. We show that for typical model parameters, the target does not have incentives to either delay or accelerate initiation. In addition, Section VII.C discusses alternative assumptions about  $\Delta$  and Section VII.D discusses the target’s incentives to restrict bidders to cash-only or stock-only bids.

<sup>13</sup>Modeling the cash constraint with a rigid limit is common in models of auctions with budget-constrained bidders (e.g., see a model of Section 3 in Che and Gale, 1998). A more general model of the cash constraint that is dynamically adjusted can be an interesting extension. One may also consider the traditional case of a budget constraint when bidders are unable to bid in stock, which is the case for, e.g., financial bidders such as private equity funds. Such a model will be similar to the problem of investment under uncertainty with a budget constraint (Boyle and Guthrie, 2003). As we show later, bidding in stock helps alleviate the budget constraint but is expensive, which may lead to a delay of an acquisition. Inability to bid in stock would instead accelerate the acquisition as the bidders try to increase the chance to fit into their budget. See Section VII.A for further discussion of the cash constraint assumption.

<sup>14</sup>The reputation loss ensures that bidders do not default on their commitments. Most of auction theory implicitly makes a similar assumption by assuming that bidders honor their bids. Loss  $R$  can be interpreted as an exogenous termination agreement.

This formalization extends the standard “button” model of an English auction for all-cash bids (Milgrom and Weber, 1982), as well as the analogous model for all-stock bids (Hansen, 1985). The difficulty with extending these models for payments in combinations of stock and cash is that there is no longer a clear notion of a higher bid. Because the value of the cash-stock combination depends on the bidder’s type, the auction has the features of a signaling game with two sequential signals: (i) timing of initiation and identity of the initiating bidder; (ii) offer  $(b, \alpha)$  made by the winning bidder. We impose two sensible restrictions on off-the-equilibrium-path beliefs of the seller. First, the seller’s belief following an off-equilibrium offer  $(b, \alpha)$  must satisfy the D1 criterion, which is common in the security design literature.<sup>15</sup> Intuitively, under the D1 criterion, if the winning bidder makes an “unexpected” offer  $(b, \alpha)$ , the target believes that this offer comes from the type with the strongest incentive to deviate. Second, the seller’s belief following an off-equilibrium offer  $(b, \alpha)$  must satisfy the resetting property of Cho (1990).<sup>16</sup> The appealing feature of this restriction, discussed in Cho (1990), is that the equilibrium does not change drastically if the uninformed party becomes unsure about its belief. For example, if there is an infinitesimal but positive probability that the valuation of each bidder changes after entering the auction but before bidding, then the equilibrium in this modified game will approximate the equilibrium in the game in which bidders’ valuations never change. However, if the resetting property of beliefs is violated, the equilibria may differ drastically. In the next section, we show that the English auction has a symmetric equilibrium, in which each bidder withdraws at price  $p$ , at which it is indifferent between winning and losing, and, upon winning, the bidder pays using as much cash as possible.

## I.B Equilibrium Concept

In the auction, we focus on the symmetric equilibrium specified in the next section. Prior to the auction, a strategy of bidder  $i$  at time  $t$  is a mapping from the history of the game  $\mathcal{H}_t$  to a binary action  $a_{i,t} \in \{0, 1\}$ , where  $a_{i,t} = 1$  stands for “initiate a bid” and  $a_{i,t} = 0$  stands for “wait.” If the rival initiates a bid at time  $t$ , it is a weakly dominant strategy for bidder  $i$  to join the auction. Because the game ends once the auction takes place, the history of the game  $\mathcal{H}_t$  can be summarized by a sample

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<sup>15</sup>See, e.g., DeMarzo, Kremer, and Skrzypacz (2005). See Ramey (1996) for the definition of the D1 criterion for the case of a continuum of types.

<sup>16</sup>Specifically, even though on the equilibrium path, as will be shown later, the support of the beliefs about bidder types is truncated as the game proceeds, if the winning bidder makes an “unexpected” offer, the target is not restricted to believe that it only comes from a type in the truncated support.

path of  $\{X(s), s \leq t\}$  and the fact that the auction has not been initiated yet. The equilibrium concept is Markov Perfect Bayesian equilibrium (MPBE), and we look for separating equilibria in continuous threshold strategies. Specifically, we look for equilibria that satisfy the following conditions: (i) the strategy of bidder  $i \in \{1, 2\}$  with valuation  $v$  is to initiate a bid when  $X(t)$  reaches some upper threshold  $\bar{X}_i(v)$  for the first time; (2)  $\bar{X}_i(v_1) = \bar{X}_i(v_2) < \infty$  if and only if  $v_1 = v_2$ ; (3)  $\bar{X}(v)$  is continuous.

Continuity and separation imply that  $\bar{X}(v)$  is strictly monotone in  $v$ . Because types with valuations close enough to  $\underline{v}$  obtain a negative payoff in the auction at any finite  $X(t)$ ,  $\bar{X}(v)$  must be strictly decreasing in  $v$ .

## II Equilibrium in the Auction

In this section, we show that there exists an equilibrium in the auction that is similar to the equilibrium in weakly undominated strategies in the standard English auction.

**Proposition 1.** *Consider the following strategy profile at the auction stage. Bidder  $i \in \{1, 2\}$  with valuation  $v_i$  drops out once the price reaches its valuation of the combined company less its post-auction value as a stand-alone firm:*

$$p(v_i) = v_i X_t + \Delta. \quad (3)$$

*Conditional on winning, a bidder makes a proposal to pay the winning bid  $y$  using as much cash as possible, if  $v_i \geq p^{-1}(y)$ , and as much stock as possible, if  $v_i < p^{-1}(y)$ . Specifically, if  $v_i \geq p^{-1}(y)$ , the proposal of the winning bidder is*

$$(b, \alpha) = \begin{cases} (y, 0), & \text{if } C_i \geq y, \\ \left(C, \frac{y-C}{\Pi_o+y}\right), & \text{if } C_i < y. \end{cases} \quad (4)$$

*If  $v_i < p^{-1}(y)$ , the proposal is  $(b, \alpha) = \left(0, \frac{y}{\Pi_o+y}\right)$ . This strategy profile constitutes an equilibrium. On the equilibrium path, the winning bidder pays the winning bid using as much cash as possible, i.e., (4). The off-the-equilibrium-path beliefs of the target are that if the winning bidder deviates from offering (4), the seller believes that the deviation comes from type  $p^{-1}(y)$ .*

The reason why bidding up to (3) is rational for each bidder is identical to that in the standard cash English auction. At (3), the bidder with valuation  $v_i$  is exactly indifferent between winning the auction and paying  $p(v_i)$  and losing the auction and getting  $\Pi_o$ . Conditional on this valuation, the value of this break-even bid does not depend on the mix of cash and stock. Dropping out below (3) is suboptimal, because it leads to potentially not winning the auction when the payoff from winning is higher than that from losing. Dropping out above (3) leads to potentially winning the auction at a price  $y$  above the break-even level. In Appendix A, we show that in this case, even though the bidder can pay less than  $y$  by making an all-stock bid, it is still better off losing the auction. Thus, bidding up to (3) is optimal for each bidder. The off-the-equilibrium-path beliefs of the seller satisfy the D1 refinement, because type  $v_i = p^{-1}(y)$  benefits the most from substituting equity for cash.

An interesting property is that the break-even bid strategy is independent of the cash position of the bidder. Intuitively, the bidder type that marginally wins the auction is indifferent between paying in stock, cash or combinations. The cash position of a bidder, however, does affect the equilibrium division of the surplus between the target and the winner. To see this, consider a bidder with valuation  $v$  and cash position  $C$ . It wins the auction if and only if the valuation of its competitor  $w$  is below  $v$ . If  $C \geq wX_t + \Delta$ , the winner acquires the target by paying  $wX_t + \Delta$  in cash. Otherwise, it pays  $C$  in cash and fraction  $\alpha(C, wX_t + \Delta)$  in stock. In the former case, the change in the value of the winner relative to its pre-auction value is

$$(v - w) X_t - \Delta. \tag{5}$$

In the latter case, it is

$$(\Pi_o + C) \frac{(v - w) X_t}{\Pi_b + wX_t} - \Delta. \tag{6}$$

Value (5) is strictly higher than (6) for any  $v > w$ , because a stock bid, but not a cash bid, is worth more if the bidder's type is higher, and the type of the winning bidder is higher than the type of the rival (that determines the winning bid).

In the following sections, we will solve for bidders' decision of when to bid for a target. The results there will be driven by two key effects that are evident from the comparison of (5) and (6). The first, *static*, effect is that (5) exceeds (6), and, more generally, (6) is strictly increasing in the amount  $C \leq wX_t + \Delta$  of cash portion in the bid. It implies that all else equal, a less cash-constrained bidder obtains a higher payoff, conditional on winning. The second, *dynamic*, effect is that (5) and (6) change

differently, as the target grows over time. Specifically, when a bidder pays the bid in cash, its payoff from winning is increasing linearly in the size of the target  $X_t$ . However, when the marginal dollar of the bid is paid in stock, the bidder's payoff is increasing in  $X_t$  at a decreasing rate. Specifically, as the target grows relative to the bidder, a lower fraction of the total surplus from the auction remains with the bidder and a higher fraction is transferred to the target. If the target is very small, there is little difference between bids in cash and in stock. However, this difference can be significant if the target is large. Because of the first, static, effect, a cash-constrained bidder benefits from letting the target grow more internally compared to an unconstrained bidder. At the same time, the impact of the second, dynamic, effect is opposite: a cash-constrained bidder benefits from acquiring the target early, because it would retain a smaller share of the combined company if the target were allowed to grow further. Because of the dynamic effect, it can be misleading to simply use a static security-bid auction model to extrapolate to predictions about strategic initiation.

### III Model with No or Extreme Cash Constraints

Before analyzing the general version of the model, we consider special cases of it when each bidder is either unconstrained ( $C_i = \infty$ ) or extremely constrained ( $C_i = 0$ ). In the first case, both bidders are unconstrained, and as a result, always compete in cash bids. In the second case, both bidders are extremely constrained, and as a result, always compete in stock bids. Finally, in the third case, one bidder is constrained and thus competes in cash bids, while the other is extremely constrained. These special cases are useful for developing intuition about how cash constraints, both a bidder's and its rival, affect incentives to initiate a bid. Their limitation is that the means of payment are effectively exogenous with respect to the initiation stage: they only depend on whether the bidder is unconstrained or extremely constrained but not on the bidders' synergies and the size of the target. The model with partial cash constraints studied in the next section fully endogenizes the means of payment.

#### III.A Two Unconstrained Bidders

Consider the case in which both bidders are unconstrained. By Proposition 1, the payment of the winning bidder is always in cash. Suppose that the auction is initiated at time  $\tau$  and both bidders compete for the target in an English auction. If the bidder with valuation  $v$  wins the auction against

the bidder with valuation  $w$ , the change in its value relative to the stand-alone level is given by (5). If, on the other hand, the bidder loses, the corresponding difference is  $-\Delta$ . If  $\tau$  is the first passage time by  $X(t)$  of an upper threshold  $\bar{X}$ , then the present value of a security that pays \$1 at time  $\tau$  equals  $\mathbb{E}[e^{-r\tau}] = \left(\frac{X_0}{\bar{X}}\right)^\beta$ , where  $\beta$  is the positive root of the fundamental quadratic equation  $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$  (e.g., Dixit and Pindyck, 1994):

$$\beta = \frac{1}{\sigma^2} \left[ -\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2} \right] > 1. \quad (7)$$

If the bidder with valuation  $v$  follows the strategy of approaching the target at threshold  $\bar{X}$ , while its rival follows the equilibrium strategy of approaching the target at strictly decreasing threshold  $\bar{X}_c(w)$ , where  $w$  is its type ( $c$  stands for “cash”), then the expected payoff of the bidder at the initial date is

$$\begin{aligned} & \left(\frac{X_0}{\bar{X}}\right)^\beta \int_{\underline{v}}^{\bar{X}_c^{-1}(\bar{X})} (\bar{X} \max\{v - w, 0\} - \Delta) dF(w) \\ & + \int_{\bar{X}_c^{-1}(\bar{X})}^{\bar{v}} \left(\frac{X_0}{\bar{X}_c(w)}\right)^\beta (\bar{X}_c(w) \max\{v - w, 0\} - \Delta) dF(w). \end{aligned} \quad (8)$$

Intuitively, the auction is initiated either by the bidder (if  $\bar{X} < \bar{X}_c(w)$ ) or by its rival (if  $\bar{X} > \bar{X}_c(w)$ ). In the former case, the auction takes place at threshold  $\bar{X}$  and conditional on the rival initiating the auction later, its type must be below  $\bar{X}_c^{-1}(\bar{X})$ . The payoff corresponding to this case is given by the first term in (8). The latter case corresponds to the valuation of the rival bidder being above  $\bar{X}_c^{-1}(\bar{X})$ . If this valuation is  $w$ , the auction occurs when  $X(t)$  reaches threshold  $\bar{X}_c(w)$ . Integrating over all realizations of  $w$  above  $\bar{X}_c^{-1}(\bar{X})$  yields the second term of (8).

Maximizing (8) with respect to  $\bar{X}$  and applying the equilibrium condition that the maximum is reached at  $\bar{X}_c(v)$ , we obtain

$$\bar{X}_c(v) = \frac{\beta}{\beta - 1} \frac{\Delta}{v - \mathbb{E}[w|w \leq v]}. \quad (9)$$

This equation is intuitive. Because of the option to delay approaching the target, a bidder approaches the target only at a point when its expected surplus from initiating the contest exceed the costs by a high enough margin. The increase in the target’s efficiency that is captured by the acquirer in expectation is  $(v - \mathbb{E}[w|w \leq v]) X_t$ , and the cost of approaching the target is  $\Delta$ . The term  $\beta/(\beta - 1) > 1$  captures the degree to which the option to delay approaching the target is important. It is higher if

the target grows faster ( $\mu$  is higher), is more volatile ( $\sigma$  is higher), or if the discount rate  $r$  is lower.

Assume that the distribution of types is such that the expected surplus of the winning bidder,  $v - \mathbb{E}[w|w \leq v]$ , is strictly increasing in its type. This property holds for many distributions. For example, it holds for uniform distribution.<sup>17</sup> Then, there indeed exists a unique equilibrium in separating threshold strategies:

**Proposition 2.** *Assume that  $v - \mathbb{E}[w|w \leq v]$ , is strictly increasing in  $v$ . Then, there exists a unique equilibrium in separating threshold strategies. In this equilibrium, a bidder with valuation  $v$  initiates the auction at threshold  $\bar{X}_c(v)$ , given by (9), provided that no bidder has initiated the auction before.*

The equilibrium has three properties. First, a deal with a higher synergy occur earlier in time, before the target has grown much. Second, among the two potential bidders, the bidder that approaches the target is the bidder with the higher valuation. It follows that in equilibrium, the bidder that approaches the target always wins the auction. This property will not hold if the bidders are asymmetric in their cash constraints.<sup>18</sup> Finally, all bidders with valuations  $v > \underline{v}$  find it optimal to approach the target at some finite  $\bar{X}_c(v)$ . This is because, as (5) shows, there always exists high enough  $X_t$  such that the winning bidder receives a positive surplus for any  $w < v$ .

In the special case of the uniform distribution of  $v$  over  $[\underline{v}, \bar{v}]$ ,  $\mathbb{E}[w|w < v] = (\underline{v} + v)/2$ . Therefore,

$$\bar{X}_c(v) = \frac{\beta}{\beta - 1} \frac{2\Delta}{v - \underline{v}}. \tag{10}$$

It is easy to see that  $\bar{X}_c(v)$  is indeed a decreasing function of  $v$ .

### III.B Two Extremely Constrained Bidders

Now, consider the opposite case: Assume that both bidders are extremely constrained and always make offers in stock. Suppose that the auction is initiated at time  $\tau$ . If the bidder with valuation  $v$  wins the auction against the bidder with valuation  $w$ , the change in its value relative to the stand-alone

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<sup>17</sup>Intuitively, there cannot be too “few” low types.

<sup>18</sup>In a more general setting, in which bidders with symmetric constraints can update their valuations after the contest initiation (e.g., during due diligence), this result would also not hold, but the bidder that initiates the contest would always win with a higher probability than its competitor, provided that the degree of initial information is the same for both bidders.

level is given by (6). If the bidder loses, this difference is  $-\Delta$ . Thus, if the bidder with valuation  $v$  follows the strategy of approaching the target at threshold  $\bar{X}$ , while its rival follows the equilibrium strategy of approaching the target at strictly decreasing threshold  $\bar{X}_s(w)$ , where  $w$  is its type ( $s$  stands for “stock”), then the expected payoff of the bidder at the initial date is

$$\begin{aligned} & \left(\frac{X_0}{\bar{X}}\right)^\beta \int_{\underline{v}}^{\bar{X}_s^{-1}(\bar{X})} \left(\frac{\Pi_o}{\Pi_b + w\bar{X}} \bar{X} \max\{v - w, 0\} - \Delta\right) dF(w) \\ & + \int_{\bar{X}_s^{-1}(\bar{X})}^{\bar{v}} \left(\frac{X_0}{\bar{X}_s(w)}\right)^\beta \left(\frac{\Pi_o}{\Pi_b + w\bar{X}_s(w)} \bar{X}_s(w) \max\{v - w, 0\} - \Delta\right) dF(w), \end{aligned} \quad (11)$$

Similarly to the case of two unconstrained bidders, the first (second) term of (11) reflects the case in which the bidder with valuation  $v$  (its competitor) initiates the auction.

Maximizing (11) with respect to  $\bar{X}$  and applying the equilibrium condition that the maximum is reached at  $\bar{X}_s(v)$ , we obtain

$$\mathbb{E} \left[ \frac{\Pi_o \left(\Pi_b + \frac{\beta}{\beta-1} w \bar{X}_s(v)\right)}{\left(\Pi_b + w \bar{X}_s(v)\right)^2} (v - w) | w \leq v \right] \bar{X}_s(v) = \frac{\beta}{\beta-1} \Delta. \quad (12)$$

The left-hand side is a strictly increasing function of  $\bar{X}$ , which implies that the optimal approaching policy of each bidder is given by the upper trigger  $\bar{X}_s(v)$ . In particular, monotonicity implies that if the trigger exists, it is unique.

However, (12) does not have a solution for some  $v$ . By monotonicity, the highest value of the left-hand side of (12) is

$$\lim_{\bar{X} \rightarrow \infty} \mathbb{E} \left[ \frac{\Pi_o \left(\Pi_b + \frac{\beta}{\beta-1} w \bar{X}\right) \bar{X}}{\left(\Pi_b + w \bar{X}\right)^2} (v - w) | w \leq v \right] = \frac{\beta}{\beta-1} \Pi_o \mathbb{E} \left[ \frac{v - w}{w} | w \leq v \right]. \quad (13)$$

This value decreases in  $v$  and reaches zero when  $v = \underline{v}$ .<sup>19</sup> Thus, once  $v$  decreases to a sufficiently low

<sup>19</sup>To see that the value decreases in  $v$ , differentiate it with respect to  $v$ . The derivative is

$$-\frac{\beta}{\beta-1} \Pi_o \int_{\underline{v}}^v \frac{v-w}{w} \frac{f(w) f(v)}{F(v)^2} dw < 0.$$

level  $v^*$ , given by

$$\mathbb{E} \left[ \frac{v^* - w}{w} | w \leq v^* \right] = \frac{\Delta}{\Pi_o}, \quad (14)$$

no bidder finds it optimal to approach the target, even though it is socially optimal to do so when  $X_t$  is high enough. This result is driven by the dynamic effect of a stock auction discussed in Section II and can be seen from (6). As  $X_t$  increases, for the same  $v$ , the bidder has to give away a larger portion of the combined company to the target. As a result, the expected revenue of the bidder with valuation  $v$  is also limited from above as  $X_t \rightarrow \infty$ . For sufficiently low valuations, it does not exceed the cost of losing the contest,  $\Delta$ , and the bidder never has an incentive to initiate a bid for the target.

The equilibrium is summarized in the following proposition:

**Proposition 3.** *The equilibrium in separating threshold strategies must have the following characterization. If the valuation of a bidder is  $v > v^*$ , where  $v^*$  is defined by (14), then it approaches the target at threshold  $\bar{X}_s(v)$ , given by (12), provided that no bidder has approached the target before. If  $v \leq v^*$ , then a bidder never approaches the target.*

A sufficient condition for existence is that the distribution of types is such that the left-hand side of (9) is strictly increasing in  $v$ . This condition is analogous to that in the case of unconstrained bidders. It holds for many distributions: in particular, for uniform distribution, and more generally, for any distribution with a non-increasing density on its support.

While there is no analytical solution for  $\bar{X}_s(v)$ , it is easy to study its properties. In particular, it is interesting to see how (12) relates to (9). For this purpose, it is convenient to decompose (12) into two parts:

$$\mathbb{E} \left[ \frac{\Pi_o (v - w) \bar{X}}{\Pi_b + w \bar{X}} | w \leq v \right] + \frac{1}{\beta - 1} \mathbb{E} \left[ \frac{\Pi_o (v - w) w \bar{X}^2}{(\Pi_b + w \bar{X})^2} | w \leq v \right] = \frac{\beta}{\beta - 1} \Delta. \quad (15)$$

The left-hand side of (15) consists of two components. The first component is the surplus that the bidder obtains in expectation. It is always below the left-hand side of (9), because separation is costlier in stock than in cash. If this were the only term on the left-hand side of (15), then each bidder would always find it optimal to approach the target later if it bids in stock. However, (15) contains an additional positive second term. It corresponds to the effect that the delay causes the surplus of the bidder to increase at a slower pace when the bidder makes bids in stock. Alternatively, one can think

of this term as a part of the delay cost on the right-hand side of (15): when  $X_t$  is higher, further delay is less costly to the bidder as further increase in  $X_t$  has a negative effect of a smaller magnitude on the bidder revenue. The magnitude of this effect depends on the value of delay parameter  $\beta/(\beta - 1)$ . The following proposition shows that if  $\beta/(\beta - 1)$  is not too high, then the first effect dominates, so bidders approach the target earlier if they are unconstrained:

**Proposition 4.** *Suppose that the measure of the option value of delay,  $\beta/(\beta - 1)$ , is not too high:*

$$\frac{\beta}{\beta - 1} < 2 \frac{\Pi_b}{\Pi_o}. \quad (16)$$

Then,  $\bar{X}_s(v) > \bar{X}_c(v)$  for any  $v$ .

In most calibrations in the literature, the multiplier of the delay option,  $\beta/(\beta - 1)$ , does not exceed 2 for the average US publicly-traded firm. As a consequence, condition (16) is likely to hold for a wide range of firms, so we refer to this case as the standard case. According to Proposition 4, if bidders are unconstrained, they are more likely to undertake an acquisition over any finite time interval  $[0, t]$  than if bidders are extremely constrained.

However, if the target grows very quickly or with very high volatility or if the interest rate is very low, then Proposition 4 no longer applies. Because  $\lim_{v \downarrow v^*} \bar{X}_s(v) = \infty$  and  $\bar{X}_c(v^*) < \infty$ , constraints delay the auction for low enough types even in this case. However, constrained bidders with high valuations may initiate the bid for the target earlier than unconstrained bidders, despite obtaining a lower fraction of the total surplus from the auction. Figure 1 presents two examples: the standard case, in which constraints delay initiation of the auction for all realizations of valuations, and the non-standard case, in which they speed up initiation for high realizations of valuations.

The results of this and the previous subsections highlight that a bidder's cash constraint has a non-trivial effect on its decision to bid for the target. First, while a constraint usually makes a bidder more reluctant to initiate a bid, this is not always so. If the target is a very high-growing or high-volatility company and a bidder has a high valuation, a constraint may make a bidder more willing to bid for the target. Second, constraints make bidders with positive but low synergies never willing to initiate the bid for the target. This leads to some positive-NPV deals never occurring in equilibrium.

### III.C An Unconstrained vs. An Extremely Constrained Bidder

Finally, consider the case in which one bidder is unconstrained and thus bids in cash, while the other bidder is extremely constrained, and thus always bids in stock. Without loss of generality, we refer to the unconstrained bidder as “bidder 1” and to the constrained bidder as “bidder 2.” Let  $\bar{X}_i(v)$  denote the (possibly infinite) initiation threshold of bidder  $i \in 1, 2$  with valuation  $v$ . We do not make any assumptions about ordering of the two strategies but later provide conditions under which such ordering can be established.

First, if bidder 1 with valuation  $v$  approaches the target at threshold  $\bar{X}$ , its expected payoff at the initial date equals<sup>20</sup>

$$\begin{aligned} & \left(\frac{X_0}{\bar{X}}\right)^\beta \int_{\underline{v}}^{\bar{X}_2^{-1}(\bar{X})} (\bar{X} \max\{v - w, 0\} - \Delta) dF(w) \\ & + \int_{\bar{X}_2^{-1}(\bar{X})}^{\bar{v}} \left(\frac{X_0}{\bar{X}_2(w)}\right)^\beta (\bar{X}_2(w) \max\{v - w, 0\} - \Delta) dF(w). \end{aligned} \quad (17)$$

Intuitively, if valuation of bidder 2 is below  $\bar{X}_2^{-1}(\bar{X})$ , bidder 1 initiates the auction at threshold  $\bar{X}$ . Otherwise, the auction is initiated by bidder 2. If the auction is initiated at some  $X_t$  and valuation of bidder 1,  $v$ , is above valuation of bidder 2,  $w$ , then bidder 1 wins the auction, makes a payment in cash and is left with the revenue equal to  $X_t(v - w) - \Delta$ . If  $v < w$ , it loses the auction and suffers the loss of  $\Delta$ . Maximizing (17) with respect to  $\bar{X}$  and applying the equilibrium condition that the maximum is reached at  $\bar{X}_1(v)$ , we obtain

$$\bar{X}_1(v) = \frac{\beta}{\beta - 1} \frac{\Delta}{v - \mathbb{E}[w | w \leq \Omega_1(v)]} \Psi_1(v), \quad (18)$$

where for bidder  $i$  and its competitor  $-i$ ,  $\Omega_i(v) = \min\{v, \bar{X}_{-i}^{-1}(\bar{X}_i(v))\}$  and  $\Psi_i(v) \equiv \max\left\{1, \frac{F(\bar{X}_{-i}^{-1}(\bar{X}_i(v)))}{F(v)}\right\}$ .

Note that (18) is very similar to (9). To see the intuition for the difference, consider  $\bar{X}_1(v) < \bar{X}_2(v)$ . Then for bidder 1,  $\Omega_1(v) = v$ ,  $\Psi_1(v) \geq 1$ . Consequently, bidder 1 delays approaching the target compared to the case in which it faces another cash bidder:  $\bar{X}_1(v) \leq \bar{X}_c(v)$ . Intuitively, because other things equal bidder 2 with the same valuation approaches the target later than bidder 1, upon approaching bidder 1 faces a stronger competitor than if it faced a cash bidder. Because of this, bidder 1 faces a lower probability of winning the auction, which decreases its expected surplus. Consequently,

<sup>20</sup>Here and hereafter, we use  $\bar{X}_i^{-1}(\bar{X})$ ,  $i = \{1, 2\}$  instead of the more precise  $\min\{\bar{X}_i^{-1}(\bar{X}), \bar{v}\}$  to save on notation.

it further delays approaching the target.

Second, if bidder 2 with valuation  $v$  approaches the target at threshold  $\bar{X}$ , its expected payoff at time 0 is equal to

$$\begin{aligned} & \left(\frac{X_0}{\bar{X}}\right)^\beta \int_v^{\bar{X}_1^{-1}(\bar{X})} \left(\frac{\Pi_0}{\Pi_b + w\bar{X}} \bar{X} \max\{v - w, 0\} - \Delta\right) dF(w) \\ & + \int_{\bar{X}_1^{-1}(\bar{X})}^{\bar{v}} \left(\frac{X_0}{\bar{X}_1(w)}\right)^\beta \left(\frac{\Pi_0}{\Pi_b + w\bar{X}_1(w)} \bar{X}_1(w) \max\{v - w, 0\} - \Delta\right) dF(w). \end{aligned} \quad (19)$$

This expression is similar to (17), with the only difference that bidder 2 pays stock if it wins the contest and is left with its payoff equal to  $\left(\frac{\Pi_0}{\Pi_b + wX_t} X_t \max\{v - w, 0\} - \Delta\right)$ . Maximizing (19) with respect to  $\bar{X}$  and applying the equilibrium condition that the maximum is reached at  $\bar{X}_2(v)$ , we obtain

$$\mathbb{E} \left[ \frac{\Pi_o \left(\Pi_b + \frac{\beta}{\beta-1} w \bar{X}_2(v)\right)}{\left(\Pi_b + w \bar{X}_2(v)\right)^2} (v - w) | w \leq \Omega_2(v) \right] \bar{X}_2(v) = \frac{\beta}{\beta-1} \Delta \Psi_2(v). \quad (20)$$

Note that (20) is very similar to (12). To see the intuition for the difference, again, consider  $\bar{X}_1(v) < \bar{X}_2(v)$ , so that for bidder 2,  $\Omega_2(v) < v$  and  $\Psi_2(v) = 1$ . Because  $w$  takes lower values compared to the case in which bidder 2 faces another stock bidder, bidder 2 accelerates approaching the target:  $\bar{X}_2(v) \geq \bar{X}_s(v)$ . Intuitively, because other things equal bidder 1 with the same valuation approaches the target earlier than bidder 2, upon approaching bidder 2 faces a weaker competitor than if it faced another stock bidder. Because of this, bidder 2 obtains a higher expected surplus from the auction, which accelerates its decision to approach the target.

The equilibrium is summarized in the following proposition:

**Proposition 5.** *The equilibrium in separating threshold strategies must have the following characterization. The initiation strategy of bidder 1 (the unconstrained bidder) with valuation  $v_1$  is to approach the target at threshold  $\bar{X}_1(v_1)$ , given by (18), provided that no bidder has approached the target before. The initiation strategy of bidder 2 (the constrained bidder) with valuation  $v_2 > v_2^*$  is to approach the target at threshold  $\bar{X}_2(v_2)$ , given by (20), provided that no bidder has approached the target before. If  $v_2 \leq v_2^*$ , then bidder 2 never approaches the target first. The boundary type  $v_2^*$  is given by*

$$v_2^* = \frac{\Pi_b}{\Pi_o} \underline{v} > \underline{v}. \quad (21)$$

As in the case of two constrained bidders, expecting low payoff from acquiring the target in stock, the constrained bidder does not initiate the takeover contest for low enough valuations. There is no analytical solution for the jointly determined  $\bar{X}_1(v)$  and  $\bar{X}_2(v)$  but two closed form equations can be obtained for  $\bar{X}_1^{-1}(X)$  and  $\bar{X}_2^{-1}(X)$  which make the numerical analysis of the strategies easy. Appendix B provides more detail.

In the standard case, when the option value of delay is not too high so that financial constraints delay acquisition, Proposition 6 establishes the ordering of strategies in the three cases of the model:

**Proposition 6.** *Suppose that  $\frac{\beta}{\beta-1} < 2\frac{\Pi_b}{\Pi_o}$  and that equilibria in separating threshold strategies exist in all three cases of the model. Then, the equilibrium strategies are ordered:  $\bar{X}_s(v) > \bar{X}_2(v) > \bar{X}_1(v) > \bar{X}_c(v)$  for any  $v$ .*

For the numerical example, we choose the benchmark model parametrization:  $r = 0.05$ ,  $\mu = 0.01$ ,  $\sigma = 0.25$ ,  $\underline{v} = 1.1$ ,  $\bar{v} = 1.5$ ,  $v \sim \text{Uniform}[\underline{v}, \bar{v}]$ ,  $\Pi_b = 100$ ,  $\Pi_o = 95$ . These values are also reported in Table I. Specifically, the benchmark case considers acquisition of a target whose assets grow at the risk-adjusted rate  $\mu$ , typically used in dynamic models of the firm, and that has the average COMPUSTAT asset volatility  $\sigma$ . The losing bidder's profits are 5% below the pre-acquisition levels. The average synergies are equal to 30% of the target's core business. The interest rate is set at 5%. The benchmark parametrization satisfies  $\beta/(\beta-1) < 2$ . The non-standard case features identical parameters except  $\mu = 0.035$ .

Figure 1 shows the four thresholds as functions of bidders' valuations,  $v$ , in the standard and non-standard cases. Consider the standard case. A higher probability of losing the takeover contest makes a constrained bidder that competes against an unconstrained bidder more cautious compared to the case when it competes against another unconstrained bidder. As a result, its initiation threshold increases. The opposite is also true: a lower probability of losing the takeover contest makes a constrained bidder more aggressive when it competes against an unconstrained bidder. As a result, its initiation threshold decreases. In the non-standard case, cash constraints speed up initiation of the bid for bidders with high enough valuations.<sup>21</sup> In either case, constraints of the rival bidder matter. Another interesting

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<sup>21</sup>Note that for a fixed valuation, the ordering of the four thresholds in the non-standard case is either the same as established by Proposition 6 or reverse. This result can be shown formally. The proof follows the lines of Proposition 6 proof and is omitted here for brevity.

result is that competing against an unconstrained bidder also makes constrained bidders with lower valuations willing to initiate in the first place:  $v_2^* < v^*$ .

The main result of this subsection is that a bidder’s decision to initiate a bid depends not only on its own cash constraint but also on the cash constraints of its competitors. This is so despite the fact that the bidding strategy is “myopic” in the sense that it is independent of cash constraints of other bidders. Intuitively, when deciding whether to initiate a bid, a bidder cares about the type of its competitors. Whether the rival is constrained or not impacts its own decision to initiate a bid, and thus indirectly affects the learning of the other bidder. In the normal case, if the rival is constrained, it delays its decision to approach the target for every possible realization of its valuation. Thus, conditional on the rival not initiating a bid, the bidder believes that the rival is more pessimistic about its valuation, if the rival is unconstrained. Therefore, cash constraints of the rival reduce the expected payoff of the other bidder from the auction at any point, and consequently make it reluctant to approach the target. This result also implies that in empirical analysis changes in financial constraints in the economy should be accounted for even if they do not have an effect on a particular bidder.

## IV Model with General Constraints

The special cases of the previous section highlighted the role of bidders’ financial constraints in acquisitions decisions. However, means of payment were uniquely determined by the constraint of the acquirer. In this section, we develop richer implications for means of payment by introducing general cash constraints of bidders: specifically, bidder  $i$  can only bid up to  $C_i \geq 0$  in cash. We show that endogenous timing of an acquisition leads to an interconnection between bidders’ financial constraints, means of payment, and synergies. High-synergy targets are acquired when they are young and small, and are paid for in cash. In contrast, low-synergy targets are acquired (if at all) after they have grown and are paid for using stock. Because of this selection, cash acquisitions can feature a higher average takeover premium despite the fact that bidders perceive acquisitions in stock as more expensive. We also show that in the general model the impact of constraints is non-trivial and can lead to acceleration of acquisition decisions even in the standard case. Throughout the section, we assume that the separating equilibrium in threshold strategies,  $X_1(v)$  and  $X_2(v)$ , exists. This is the case in all of our numerical examples.

Consider the decision of bidder  $i$  with valuation  $v$  to approach the target. If bidder  $i$  approaches

the target at threshold  $\bar{X}$ , its expected payoff at the initial date equals

$$\begin{aligned} & \left(\frac{X_0}{\bar{X}}\right)^\beta \int_v^{\bar{X}_{-i}^{-1}(\bar{X})} \left(\min\left\{\frac{\Pi_o + C_i}{\Pi_b + w\bar{X}}, 1\right\} \bar{X} \max\{v - w, 0\} - \Delta\right) dF(w) \\ & + \int_{\bar{X}_{-i}^{-1}(\bar{X})}^{\bar{v}} \left(\frac{X_0}{\bar{X}_{-i}(w)}\right)^\beta \left(\min\left\{\frac{\Pi_o + C_i}{\Pi_b + w\bar{X}_{-i}(w)}, 1\right\} \bar{X}_{-i}(w) \max\{v - w, 0\} - \Delta\right) dF(w). \end{aligned} \quad (22)$$

Intuitively, if the valuation of the competitor is below  $\bar{X}_{-i}^{-1}(\bar{X})$ , bidder  $i$  approaches the target at threshold  $\bar{X}$ . Otherwise, the competitor approaches the target at equilibrium threshold  $\bar{X}_{-i}(w)$ . In both cases, if  $v > w$ , bidder  $i$  wins the auction and makes a payment either in cash or in a combination of cash and stock. If  $v < w$ , it loses the auction and suffers the loss of  $\Delta$ . Maximizing (22) with respect to  $\bar{X}$  and using the equilibrium condition that the maximum is reached at  $\bar{X}_i(v)$ , we obtain

$$\begin{aligned} & \mathbb{E} \left[ \min\left\{\frac{\Pi_o + C_i}{\Pi_b + w\bar{X}_i(v)}, 1\right\} (v - w) \mid w \leq \Omega_i(v) \right] \bar{X}_i(v) \\ & + \frac{1}{\beta - 1} \int_{\min\left(\frac{C_i - \Delta}{\bar{X}_i(v)}, \Omega_i(v)\right)}^{\Omega_i(v)} (\Pi_o + C_i) \frac{(v - w) w \bar{X}_i(v)^2}{(\Pi_b + w\bar{X}_i(v))^2} \frac{f(w)}{F(\Omega_i(v))} dv \\ & = \frac{\beta}{\beta - 1} \Delta \Psi_i(v), \end{aligned} \quad (23)$$

where  $\Omega_i(v) \equiv \min\{v, \bar{X}_{-i}^{-1}(\bar{X}_i(v))\}$  and  $\Psi_i(v) \equiv \max\left\{1, \frac{F(\bar{X}_{-i}^{-1}(\bar{X}_i(v)))}{F(v)}\right\}$ . The system of equations (23) for bidders 1 and 2 jointly determines equilibrium thresholds  $\bar{X}_1(v)$  and  $\bar{X}_2(v)$ . Note that this solution embeds solutions for three special cases, studied in Section II. The following proposition summarizes the equilibrium:

**Proposition 7.** *The separating threshold equilibrium in the general model must take the following form. Bidder  $i$  with valuation  $v_i > v_i^*$  initiates the auction the at threshold  $\bar{X}_i(v_i)$ , provided that it has not been approached before, where  $\bar{X}_i(v)$  satisfies (23) and  $v_i^*$  is defined in Appendix A, provided that the rival bidder has not initiated the auction yet. If  $v_i \leq v_i^*$ , bidder  $i$  never initiates the auction.*

As long as  $C_1 < \infty$  and  $C_2 < \infty$ , each bidder never approaches the target for valuations equal to or below, correspondingly,  $v_1^*$  and  $v_2^*$ . Appendix B provides more detail on the numerical solution for  $\bar{X}_1(v)$  and  $\bar{X}_2(v)$ .

Figure 2, Panel A shows the four thresholds (cash vs. cash bidders, stock vs. stock bidders, and

bidders with internal cash  $C_1 = 125$  and  $C_2 = 0$  competing against each other) for our benchmark parametrization as a function of bidders' valuations,  $v$ . An interesting new effect compared to the case of exogenous means of payment is that for intermediate valuations, constrained bidders can choose to accelerate initiation even relative to the case of two cash bidders. This happens because they attempt to “fit into” their cash constraints. Consider Figure 2, Panels B and C that show expected bidder revenue from non-cash and cash-only deals. As the valuation of bidder 1 decreases, it initiates contests for a larger target and eventually finds itself unable to complete all deals in cash (the dashed vertical line on the right-hand side of all panels). At this stage, bidder 1 trades off costs of inefficiently early initiation against its benefits (a smaller probability that the deal is non-cash, resulting in a higher expected revenue from the auction). If the latter dominates, bidder 1 can approach a smaller target compared to the case when it is unconstrained ( $C_1 = 0$ ) or even to the case when both bidders are unconstrained. As the valuation of bidder 1 decreases even further (beyond the dashed vertical line on the left-hand side of all panels), any successful contest requires the payment of at least  $C_1$  that makes fitting into cash not possible. Then, bidder 1's initiation threshold increases faster, similarly to an all-stock bidder.

Consider bidder 2 who competes against bidder 1 with  $C_1 < \infty$  instead of  $C_1 \rightarrow \infty$ . Bidder 1 attempts to fit into cash and, for intermediate valuations, accelerates its initiation compared to  $C_1 \rightarrow \infty$ , so bidder 2 becomes a stronger bidder with higher expected revenues. As a result, it is optimal for bidder 2 to also accelerate initiation for intermediate valuations.

## V Comparative Statics

In this section, we investigate the effects of target and bidder characteristics on initiation strategies. Proposition 8 establishes comparative statics results:

**Proposition 8.** *Assume that each bidder is, in any combination, either severely constrained ( $C_i < \Delta$ ) or unconstrained ( $C_i \rightarrow \infty$ ), and that (16) holds. Consider an equilibrium in strictly decreasing initiation strategies  $\bar{X}_i(v)$ . For any  $v$ ,  $\bar{X}_i(v)$ ,  $i \in \{1, 2\}$ :*

1. *increase in  $\mu$ ;*
2. *increase in  $\sigma$ ;*

3. *decrease in  $r$ ;*
4. *increase in  $\Delta$  (keeping  $\Pi_b$  fixed);*
5. *weakly decrease in  $\Pi_b$  (keeping  $\Delta$  fixed).*

The results of Proposition 8 are intuitive. (1) When  $\mu$  is higher, bidders wait longer before approaching the target: the present value of costs associated with losing the deal increases due to  $X_t$  reaching the initiation threshold of a competitor faster, and this increase dominates an increase in the present value of synergies in case of success. (2) For the same reason, when the discount rate  $r$  is lower, the costs of losing the deal loom larger, so the takeover contest is initiated later. (3) Similarly, higher  $\sigma$  implies a higher likelihood of the competitor reaching the initiation threshold fast, which in turn increases costs of losing the deal and leads to delay in initiation. (4) When costs of losing the contest,  $\Delta$ , are high, the winning bidder has to pay more to separate itself from the losing bidder: the value of the winning bidder's outside option (losing) is a negative function of  $\Delta$ . As a result, the bidders' expected payoffs from the contest decrease, so they initiate later. (5) The additional restrictions on constraints here make the motive to fit into cash weak, resulting in monotone comparative statics. The initiation strategies of two unconstrained bidders competing with each other are constant in  $\Pi_b$  keeping  $\Delta$  fixed. For a severely constrained bidder, however, a larger  $\Pi_b$  results in its bidding a smaller portion of the combined company, which leads to earlier initiation, no matter the constraints of the competitor.

In case (5), it is easy to notice that when an unconstrained bidder competes against a severely constrained bidder, its initiation threshold also decreases in  $\Pi_b$ . The reason is that a higher  $\Pi_b$  speeds up initiation by the constrained bidder. Thus, conditional on the constrained bidder not initiating yet, the unconstrained bidder faces, on average, a weaker competitor. As a result, at any hypothetical initiation threshold, the expected payoff of the unconstrained bidder from initiating the contest is higher, leading to a lower initiation threshold.

Figure 3 shows the comparative statics of the four equilibrium initiation strategies corresponding to the model in Sections III.A–III.C. The strategies are built for the benchmark model parametrization, for a bidder with the average valuation,  $v = 1.3$ . The comparative statics are with respect to the five model parameters highlighted in Proposition 8 as well as the dispersion of the bidders' valuations. As the dispersion of the valuations increases, a bidder with valuation  $v$  becomes better separated from bidders with lower valuations, and therefore on average pays less in a successful contest. As a result,

the bidder initiates the auction earlier. The initiation strategies seem to be particularly sensitive to the costs of losing the deal and the dispersion of the bidders' valuations. In fact, when costs of losing the deal (the dispersion of valuations) are sufficiently high (low), the stock bidder with the average valuation never initiates the contest: its valuation is below the threshold  $v^*$  ( $v_2^*$ ) obtained in Proposition 3 (4).

Figure 4 depicts the comparative statics of the four equilibrium initiation strategies (two unconstrained bidders, two extremely constrained bidders ( $C_1 = C_2 = 0$ ), and bidders with internal cash  $C_1 = 125$  and  $C_2 = 0$  competing against each other) for the benchmark model parametrization as a function of the same six model parameters. The strategies are plotted for the bidder with the average valuation,  $v = 1.3$ . Incentives to fit into cash constraints are strong when  $\mu$ ,  $\sigma$  or  $\Pi_b$  are higher, and when  $r$  is smaller. In all these cases, the combined company has a higher expected value. When means of payment are endogenous, the bidders are unwilling to share this highly-valued company with the target and choose to predominantly pay cash at the cost of earlier initialization.

Figure 5 shows the comparative statics of the optimal initiation strategies for the benchmark model parametrization and bidders with cash constraints  $C_1$  and  $C_2 = 0$ , with respect to  $C_1$  for the bidder with the average valuation,  $v = 1.3$ . For intermediate ranges of  $C_1$ , bidder 1 has incentives to fit into cash and bidder 2, recognizing that now it faces a weaker competitor, follows by decreasing its own initiation threshold. For low and high values of  $C_1$ , all deals either require all available cash to be done or are always done in cash only, weakening the motives to fit into cash. As a result, strategies of both cash-constrained bidders lie between the strategies of two unconstrained and two extremely constrained bidders competing against each other.

## VI Analysis

The results obtained in previous sections yield many implications. We discuss them in this section. First, we discuss implications of the model that relate the endogenous timing of the acquisition, synergies, means of payment, and the split of gains between the acquirer and the target. Then, we relate these implications to existing empirical evidence. Finally, we discuss the properties of initiating versus winning bidders.

## VI.A Endogenous Timing, Means of Payment, and Premiums

A1. *Companies acquired in stock are (usually) larger and older than companies acquired in cash.*

Bidders with lower valuations have higher benefits to wait until the target grows, and when the target is larger, the cash constraint of the bidder is more likely to bind. Thus, these targets tend to be acquired with the help of stock. This is always the case for targets with low enough valuations. To see this, note that  $\lim_{v \downarrow v_i^*} \bar{X}_i(v) = \infty$ . Thus, targets with synergies of the winning bidder close to  $\min(v_1^*, v_2^*)$  are always acquired with the help of stock. In contrast, targets with higher synergies can be acquired either in cash or in stock, depending on whether the valuation of the rival bidder is high enough so that the cash constraint of the winning bidder binds. Thus, the probability of a cash deal conditional on the valuation  $v$  of the acquirer always increases in  $v$ , when  $v$  is low. In theory, this probability may be non-monotone in  $v$  for larger  $v$ , because of the countervailing effect: an increase in the strength of the rival, as  $v$  increases, may dominate the effect of the target getting acquired smaller. However, we found it to be monotone in all the numerical specifications we have tried.

This effect can be important for empirical research as it highlights an omitted variable in the link between the size of the target and means of payment. Not only are large companies acquired in non-cash deals because the acquirer does not have sufficient cash to finance a large payment; such companies were allowed to grow large because potential synergies were not high enough for bidders to acquire them small.

Figure 6 shows, for the benchmark parametrization and  $C_1 = 125$ ,  $C_2 = 0$ , probabilities that cash and non-cash deals are completed in years 1, 2–5, 6–10, 11–25, and 26–100<sup>22</sup> as well as

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<sup>22</sup>Formally, for each given realization of the two bidders' valuations,  $v_1$  and  $v_2$ , the conditional probability that a contest is initiated over a finite time horizon  $T$  is

$$\mathbb{P}[\text{acquisition}|v_1, v_2, X_t, T] = \min \left\{ 1, N \left( \frac{-\log \frac{\min\{\bar{X}_1(v_1), \bar{X}_2(v_2)\}}{X_t} + (\mu - \sigma^2/2)T}{\sigma\sqrt{T}} \right) \right. \\ \left. + \exp \left\{ \frac{2(\mu - \sigma^2/2) \log \frac{\min\{\bar{X}_1(v_1), \bar{X}_2(v_2)\}}{X_t}}{\sigma^2} \right\} N \left( \frac{-\log \frac{\min\{\bar{X}_1(v_1), \bar{X}_2(v_2)\}}{X_t} - (\mu - \sigma^2/2)T}{\sigma\sqrt{T}} \right) \right\}.$$

Then, the conditional probability that a contest is initiated over a finite time horizon  $T$  for any  $v_1$  and  $v_2$  is

$$\mathbb{P}[\text{acquisition}|X_t, T] = \mathbb{E}_{v_1, v_2} [\mathbb{I}[v_1 > v_1^*, v_2 > v_2^*] \mathbb{P}[\text{acquisition}|v_1, v_2, X_t, T]],$$

where  $\mathbb{I}[\cdot]$  is the indicator function equal to one if the condition in brackets is satisfied and zero otherwise.

average acquisition size in deals completed by the end of year 1, 5, 10, 25, and 100. The starting value of the target is such that it is on the verge of being acquired by the highest-synergy bidder with the lowest cash constraints:  $X_0 = \bar{X}_1(\bar{v})$ . Cash deals mostly happen within the first five years of the target's life while non-cash deals reach their peak in years 2–5 and continue to be dominant types of acquisition in years 6–10. Cash deals are on average smaller and the gap in average size of cash and non-cash deals increases with the sample horizon as more and more non-cash deals are made for large targets by bidders with the lowest valuations.

A2. *Conditional on its valuation and the size of the target, the acquirer pays a higher takeover premium if the deal is done in stock.*

Conditional on the valuation  $v$  of the winning bidder and size of the target at acquisition  $\bar{X}$ , whether the deal is done in cash or not is driven by the variation in the valuation  $w$  of the rival bidder. The deal is done in cash if  $w < (C - \Delta) / \bar{X}$ , where  $C$  is the cash position of the acquirer, and is done with the help of stock otherwise. Thus, the takeover premium in the stock deal is higher for two reasons: first, the stock bid transfers wealth from the winning bidder to the seller; and, second, the acquirer is more likely to use stock if its rival is stronger.

At the same time, without conditioning on the acquirer's valuation of the target, the average takeover premium can be higher in cash deals than in stock deals:

A3. *For some parameterizations of the model, bidders pay higher takeover premiums to acquire companies in cash.*

Despite the fact that acquirers give away a smaller portion of their valuations in cash deals, they tend to be bidders with higher valuations. They give away a smaller portion of a larger pie. As a result, there exist parameterizations for which the effect of a pie increase dominates the effect of a smaller pie share and cash bidders on average pay higher takeover premiums (as a percentage of the target's value).

Figure 7 shows the average takeover premiums in cash and non-cash deals, both conditional on observing the highest bidder valuation and sample-wide unconditional, where the sample consists of takeover contests that differ only in valuations of participating bidders. As expected, the conditional takeover premiums are higher in non-cash deals for any value of highest valuation. However, in the case when both bidders have non-zero internal cash (Panels B and D), best deals are done exclusively in stock while worst deals are done exclusively in combinations on

cash and stock which leads to an inverse relationship between the sample-wide unconditional average takeover premiums. This result is obtained without assuming either adverse selection about the bidders' assets or private information of the acquirer about its own firm as in the previous literature. It is the takeover timing-determined positive correlation between cash deals and high-synergy deals that is responsible for the result.

An empirical implication of A2–A3 is that, if a good proxy of synergies can be found, then conditional on this proxy, takeover premiums in cash deals should be lower than those in non-cash deals. Conditional on the recovered valuation of the highest bidder, takeover premiums in cash deals should be lower than those in non-cash deals, despite the empirical evidence that they are higher unconditionally.

A4. *Stock bidders receive usually lower acquirer premiums than cash bidders.*

Not only do stock bidders give away a larger portion of their valuations, but also they have lower valuations, so the two effects complement each other.

## VI.B Related Empirical Evidence

While a joint test of above implications is yet to be performed, several empirical findings are consistent with them.<sup>23</sup> The most basic implication of the model is that there is a dynamic selection of targets into size and age groups, whereby firms with high synergies with potential bidders are acquired when they are young and small, while firms with low synergies are acquired, if ever, after they have grown. In addition, high-synergy deals are predominantly done in cash while low-synergy deals usually require stock to complete. In a sample of tender offers, Bhagat et al. (2005) estimate total synergies in acquisitions using two different methodologies. If the estimated improvement is measured as a fraction of the value of the target (i.e., parameter  $v$  in our model), it is positively related to the relative size of the bidder versus the target, as our model predicts. Bhagat et al. (2005) also find that estimated value improvements are significantly higher for cash deals than for stock deals. There is also robust evidence that cash deals are associated with higher combined announcement returns than stock deals (e.g., Andrade, Mitchell, and Stafford, 2001). Finally, largest deals tend to be done mostly in stock, while regular-size deals tend to be done mostly in cash (Bayazitova, Kahl, and Valkanov, 2012).<sup>24</sup>

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<sup>23</sup>See Betton, Eckbo, and Thorburn (2008) for a summary of the literature.

<sup>24</sup>Furthermore, many large deals done in cash are often financed by issuance of securities. As discussed in the next section, there is a difference between acquisitions in cash financed with retained earnings and acquisitions in cash financed

There is broad evidence that acquirer’s announcement stock returns are lower in stock acquisitions than in cash acquisitions.<sup>25</sup> Despite this, in many samples, average takeover premium in cash deals is higher than in stock deals (e.g., Asquith, Bruner, and Mullins, 1987; Eckbo and Langohr, 1989), suggesting that there is indeed selection of better deals into cash and worse deals into stock. Ours is, of course, not the only model that offers a selection mechanism. Hansen (1987), Fishman (1989), Berkovitch and Narayanan (1990), and Eckbo, Giammarino, and Heinkel (1990) develop static models<sup>26</sup>, in which the use of cash in the offer is a signaling device of the bidder’s valuation, so bidders with higher synergies self-select into cash offers. Apart from somewhat different moving forces, our theory has two distinct features. First, with the exception of Berkovitch and Narayanan (1990), these papers assume two-sided private information, in which bidders have private information about their synergies and the target has private information about the value of assets in place. In contrast, in our model selection into cash and non-cash deals occurs on the basis of one-sided private information only, as in conventional auction models. Second, and more importantly, our model endogenizes the size of targets at acquisitions. By doing this, our theory gives a unified explanation for large deals being worse than small deals, for stock deals being worse than cash deals, and for large deals being done using more stock. Of course, our model ignores many other possible determinants of means of payment, so it fails to explain some empirical phenomena, such as the use of stock in certain acquisitions of small targets.<sup>27</sup>

Several papers also look at how cash balances and/or financial constraints of firms are related to their likelihood of making acquisitions as well as to the means of payment in acquisitions. Harford (1999) finds that cash-rich firms are more likely to make acquisitions than cash-poor firms. Relatedly, Harford (2005) finds that the timing of mergers is related to aggregate liquidity, as measured by the spread between interest rates on commercial and industrial loans and the Federal Funds rate. Evidence in Harford (1999, 2005) is in line with our model when the static effect of cash constraints dominates the dynamic effect, which, as discussed above, is reasonable for an average public firm. Financially constrained bidders use stock more often than financially unconstrained bidders as means of payment in their acquisitions (Alshwer, Sibilkov, and Zaiats, 2011), although there is recent evidence that bidders with a recent issuance of securities. In particular, acquisitions in cash financed by stock issuance are conceptually closer to acquisitions in stock in this model than to acquisitions in cash.

<sup>25</sup>Travlos (1987), Asquith, Bruner, and Mullins (1987), Eckbo, Giammarino, and Heinkel (1990), and Servaes (1991).

<sup>26</sup>To be precise, by “static” we mean that these papers do not consider endogenous timing of the auction.

<sup>27</sup>Many such targets are high-growth, high-volatility firms. See Section VII.C for further discussion of how acquisitions of such targets can fit into our model due to the target management’s preference to restrict the type of bids to stock.

with larger internal cash balances are, in fact, less likely to use cash in their acquisitions (Pinkowitz, Sturgess, and Williamson, 2013). These conflicting findings could potentially be reconciled by the fact that firms choose how much cash to hold strategically, so firms with larger cash balances may hold them for precautionary motives, which is also consistent with evidence on credit spreads in Acharya, Davydenko, and Strebulaev (2012). In light of this, one should interpret variable  $C_i$  in the model not as a total cash balance but rather as free cash, i.e., excess cash that the firm can afford to spend on an acquisition without harming other productive uses of cash.

## VI.C Properties of Initiating and Winning Bidders

The model has two interesting implications relating initiating and winning bidders. First, in equilibrium, in initiated contests, the distribution of participating bidders' valuations is determined endogenously and is asymmetric. Conditional on one bidder approaching the target, the distribution of her valuation, which is degenerate in equilibrium, is different from the distribution of the valuation of her rival. This is despite the fact that the unconditional distribution of valuations is the same for the bidders. Second, whether the initiating bidder is the winning bidder or not depends on how her constraints compare with the constraints of her rival.

These properties are illustrated in Figure 8. For the case of  $C_1 = 125$  and  $C_2 = 0$ , it plots the equilibrium identities of the initiating and the winning bidders, and whether the equilibrium payment is all-cash or not, for all possible realizations of valuations of both bidders. Figure 8, left-most dashed line shows valuations of bidders 1 and 2,  $v$  and  $w$ , at which they initiate contest at the same threshold,  $\bar{X}_1(v) = \bar{X}_2(w)$ . In contests initiated by any bidder, the highest possible valuation of the more constrained bidder is higher than that of the less constrained bidder; the less constrained bidder also faces a stronger competitor on average. Interestingly, in the sample of takeovers that differ only in valuations of participating bidders, this result is reversed: because more constrained bidders are less likely to initiate takeover contests in the first place, their average valuation across all initiated contests is lower than that of less constrained bidders. Figure 9 shows how average valuations of the bidders with cash constraints  $C_1 = 125$ ,  $C_2 = 0$  change with respect to the parameters that have the strongest effect on the probability that a contest is never initiated: the value of the losing bidder,  $\Pi_0$ , and the cash constraint of one of the bidders, specifically,  $C_1$ . Lower  $\Pi_0$  and  $C_1$  correspond to a larger gap between  $v_1^*$  and  $v_2^*$  and result in a larger difference between average valuations in the sample of similar takeovers.

Another interesting prediction relates endogenous means of payments and the likelihood of winning an initiated contest. Specifically, under some parameterizations of the model, initiating bidder's offers of cash are less likely to be rejected in favor of a competing bid compared to initiating bidders' offers that include stock. The prediction is consistent with empirical evidence (e.g., Betton, Eckbo, and Thorburn, 2009). This prediction may seem contradictory to the one above relating constraints and the likelihood of winning an initiated deal. However, a less constrained bidder and a bidder who completes the deal in cash are not equivalent. The latter bidder is more likely to have both high cash balances and high valuation so that it approaches the target while the deal can still be sealed in cash. For the benchmark parametrization and  $C_1 = 125$ ,  $C_2 = 0$ , Figure 8, regions (2) and (4) show contests initiated by the less constrained bidder 1 in which the initial bidder bids in combinations of cash and stock. Region (4) shows contests in which such bidder loses to bidder 2 who bids in stock. Region (6) shows contests initiated by bidder 2 who wins in stock. The conditional probability of the initiating non-cash bidder losing the contest is the area of region (4) divided by the combined areas of regions (2), (4), and (6) and is equal to approximately 10%. In contrast, regions (3) and (5) show contests initiated by the less constrained bidder 1 in which the initial bidder bids in cash. Region (5) shows contests in which such bidder loses to bidder 2 who bids in stock. The conditional probability of the initiating cash bidder losing the contest is the area of region (5) divided by the combined area of regions (3) and (5) and is equal to approximately 2.6%. Hence, for a given parametrization, cash bids by the initiating bidder indeed have a smaller probability to be rejected compared to non-cash bids. It is easy to construct an example in which the opposite is true: take  $C_1 \rightarrow \infty$ ,  $C_2 = 0$ . In this case, there is zero correlation between cash bids and cash bidder valuations and only initial cash but not stock bids can be rejected.

## VII Further Discussion

For parsimony, we omitted a number of important considerations from the analysis. Here, we discuss four of them: (i) strategic cash management and issuance of securities by bidders; (ii) ability of the target to delay an acquisition; (iii) dependence of the losing bidder's value on the size of the acquisition and the type of the winning bidder; (iv) target's preference for cash versus stock bids.

## VII.A Issuance of Securities and Cash Management

One important consideration we ignored is the ability of bidders to raise cash by issuing securities and using it to submit bids in cash. It is important to recognize that cash bids financed by issuance of securities are different from cash bids financed by retained earnings. In a recent paper, Vladimirov (2012) shows that if outside investors and the seller have the same information set and under certain restrictions on the space of contracts, bidding with cash raised by issuing securities to outside investors has the same effect on the bidders' payoff as bidding with securities directly. In this respect, cash bids financed by a recent issuance of stock are equivalent to stock bids, so stock bids in our model should be considered more broadly as either bids in stock or in cash financed by a recent stock issuance.

We also assume that the ability of bidders to pay cash is exogenous and constant over time. In practice, it is endogenous and changes over time, because potential acquirers can retain earnings instead of paying them out to investors. An interesting, though challenging, extension would be to incorporate the cash management problem similar to Miller and Orr (1966) into our setting.<sup>28</sup> Such a model will relate the timing and properties of acquisitions to not only bidders' ability to pay cash but also to their ability to accumulate cash.

## VII.B Active Target

In the main specification of the model, we do not allow the target to initiate the acquisition. There are institutional reasons for this restriction: delay of a positive-synergy acquisition by the target's management is often not possible because of the "Revlon Rule"<sup>29</sup>, according to which the target's board of directors would be legally responsible to maximize immediate shareholder value by considering all offers and accepting the highest bid offered provided it exceeds the target's value under the current management. Delay of an acquisition can be shareholder value-destroying as, in our framework, bidders have incentives to withdraw their bids if the target value decreases between the offer and the target's decision to accept it. Acceleration of an acquisition is also unlikely because in a dynamic world, the target would typically be unable to commit to a take-it-or-leave-it offer to sell itself, accepting later bidder offers in case its own offer fails. Bidders would then be unwilling to accept the target's offer at a bidder-suboptimal time in the first place. In Proposition 9, we show formally for the three cases

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<sup>28</sup>See Bolton, Chen, and Wang (2011, 2013) and Hugonnier, Malamud, and Morellec (2013) for models that connect cash management and investment (divisible and lumpy, respectively).

<sup>29</sup>Revlon, Inc. v. MacAndrews & Forbes Holdings, Inc., Supreme Court of Delaware, 1985, 506 A.2d 173.

considered in Section III that when the target is allowed to actively delay or accelerate acquisitions relative to the bidder-optimal choice, it will not have incentives to do so, at least locally in time<sup>30</sup>:

**Proposition 9.** *Assume that each bidder is, in any combination, either extremely constrained ( $C_i = 0$ ) or unconstrained ( $C_i \rightarrow \infty$ ), and that, in the case of bidders with asymmetric constraints, target characteristics satisfy  $\frac{\beta}{\beta-1} < 2\frac{\Pi_b}{\Pi_0}$ . Then in equilibrium, the target never locally delays the auction compared to the bidders' strategies  $\bar{X}_i(v)$  in the absence of the active target. For the case  $C_i \rightarrow \infty$ , the target never globally delays the auction. If the bidders believe that out of equilibrium, no rival type accepts the early target's offer to sell itself then in equilibrium, the target never accelerates the auction compared to the bidders' strategies  $\bar{X}_i(v)$  in the absence of the active target.*

## VII.C Dependence of the Losing Bidder's Value on Acquisition Size and Winning Bidder's Valuation

In the model, the losing bidder's loss of value,  $\Delta$ , is assumed constant. In practice, this loss of value may depend on the size of the acquired target,  $X_t$ , and the valuation of the winning bidder,  $v$ . First, assume that  $\Delta(X_t)$  depends on the size of the target upon acquisition. Suppose that  $\Delta(X_t)$  satisfies the following technical conditions:  $\Delta(X_t)$  is differentiable and  $\Delta'(X_t) > 0$  for any  $X_t$ ;  $\lim_{X_t \rightarrow \infty} \Delta(X_t) < \Pi_b$ ;  $X_t \Delta'(X_t) < \beta \Delta(X_t)$  for any  $X_t$ . In this case, the initiation threshold of an unconstrained bidder competing against another unconstrained bidder is the solution to

$$\bar{X}_c(v) = \frac{\beta \Delta(\bar{X}_c(v)) - \bar{X}_c(v) \Delta'(\bar{X}_c(v))}{(\beta - 1)(v - \mathbb{E}[w|w \leq v])}. \quad (24)$$

Similar equations can be obtained for other combinations of bidders.<sup>31</sup> In this version of the model, initiation is affected by the sensitivity of  $\Delta$  to the change in the target value. Ceteris paribus, a higher sensitivity incentivizes the bidders to initiate an acquisition earlier in expectation of a greater value loss in case initiation is delayed and they lose the contest. This extra effect, however, does not affect the main trade-offs we emphasize.

Second, if  $\Delta(v)$  depends on the valuation of the winning bidder, at any time  $t$  each bidder decides

<sup>30</sup>The same result holds globally numerically for any specification we have tried.

<sup>31</sup>Derivation of the above equation and similar equations for other combinations of bidders, as well as sufficient conditions imposed on  $\Delta(X_t)$  for the existence of an equilibrium are available from the authors upon request.

whether to initiate the auction based not only on his valuation but also on the expected externality suffered in case another bidder acquires the target. Because the upper bound on the possible valuation of another bidder and hence the likelihood that the rival wins the contest depends on the maximum size the target has attained prior to time  $t$ , this expected externality is time-varying. As a result, our model becomes a model of endogenous initiation of auctions with time-varying externalities.<sup>32</sup> While full analysis of such model is beyond the scope of this paper, the main trade-offs we emphasize do not require the absence of externalities.

## VII.D Target's Preference for Cash versus Stock Bids

An influential result of the literature on security-bid auctions (Hansen, 1985; DeMarzo, Kremer, and Skrzypacz, 2005) is that a seller's expected revenues are higher in a stock auction than in a cash auction. In this section, we show that this result relies on an exogenous timing of the auction, and the revenues ranking can reverse in a dynamic setting, when bidders can time an acquisition. Intuitively, even though the seller obtains a higher fraction of the total pie in a stock auction, bidders will be reluctant to approach the seller, which can result in a more suboptimal timing of the auction, reducing the ex-ante expected payoff of the seller. To show this trade-off in a simple way, we assume that both bidders are completely unconstrained, and the target commits to a security design at time zero, which must be time-independent. Our results in this section are related to Cong (2013), who shows that an auctioneer selling a real option, such as a lease to explore an oil well, can prefer the auction in cash over the auction in stock, because of the post-auction moral hazard that affects the timing of the option exercise. Our argument is different, because the timing of actions is reverse: a bidder exercises its option (approaches the target) before the auction takes place.

Figure 10 shows the ratio of present values of target revenues in cash and stock contests as a function of  $\mu$ ,  $\sigma$ , and  $r$ . For realistic parameters, the target prefers not to commit to restricting bids to stock. When  $\mu$  and  $\sigma$  are well above realistic parameters (or  $r$  is very low), auctions in stock start to dominate auctions in cash in terms of the target's payoff. Intuitively, if a target has a higher growth rate or higher volatility of assets (or interest rate is lower), the difference between initiation thresholds of cash and stock bidders is passed quicker (or affects the present value of target revenues less). As a result, the effect of extra delay is less important for the present value of high-growth targets, which leads to their preference for battles in stock.

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<sup>32</sup>For a static version of auctions with externalities, see, e.g., Jehiel, Moldovanu, and Stacchetti (1999).

This result suggests that in a dynamic setting, sellers can have aligned incentives with the bidders about cash versus security bids: both the bidders and the target can prefer bids in cash. This is in line with the observation that there are very few (if any) practical cases in which the target attempts to restrict the type of bids. However, a small fraction of firms with either high growth or high volatility of assets can have misaligned incentives with the bidders. If there is any evidence regarding the target's attempts to restrict the type of bids in takeover contests, this analysis suggests that it is likely to be found among high- $\mu$ , high- $\sigma$  targets.

## VIII Concluding Remarks

In this paper, we analyze acquisition timing, means of payment, and other properties of acquisitions in a unified framework. It is based on three ingredients: financial constraints of bidders, private information about their synergies with the target, and the idea that initiating a takeover contest is akin to an exercise of an American call option. We show that the effects of a bidder's financial constraint are more convoluted than one might expect. Because of ability to bid in stock, a cash constraint does not affect a bidder's maximum willingness to pay. Despite this, it affects the desire of a bidder to *initiate* a bid. Furthermore, the total effect is not obvious: there are two effects, one of which favors not approaching the target, whereas the other is the opposite. While the former effect usually dominates, so a cash constraint makes a bidder more reluctant to initiate a bid, the opposite may hold if the target is a very high-growth high-synergy firm.

The analysis has many implications relating financial constraints of bidders, means of payment, deal size, total gains, and their split between the acquirer in the target. In equilibrium, high-synergy targets tend to be approached when they are young and small, and acquired for cash. By contrast, low-synergy targets are acquired after they have grown, and using stock. Thus, the paper provides a reason why simultaneously large deals underperform small deals, stock deals underperform cash deals, acquirers usually pay on average more in cash deals than in stock deals, and, despite this, bidders often perceive stock bids as expensive.

A potential direction of future research is to understand targets' motives to initiate takeover contests by themselves. We abstract from this issue because our focus is on strategic acquisitions, and they are usually bidder-initiated (Fidrmuc et al., 2012). However, target-initiated deals are also common, especially among private equity deals. Another direction for future research is to test predictions of our

model. In particular, it can be interesting to quantify the relative importance of our dynamic selection mechanism and other theories of means of payment (e.g., Fishman, 1989) on takeover outcomes.

## Appendix A Proofs

**Proof of Proposition 1.** First, we prove that conditional on winning, the optimal offer of the bidder is as specified in the proposition. Let  $\alpha(b, y)$  denote the minimum amount of stock that a bidder needs to offer to the target in addition to  $b$  in cash, so that the perceived value of the bid is  $y$  if the target believes that the offer comes from type  $p^{-1}(y)$ . It must satisfy

$$\alpha(b, y) (p^{-1}(y) X_t + \Pi_b) + b = y.$$

Inverting (3) and plugging in,

$$\alpha(b, y) \left( \frac{y - \Delta}{X_t} X_t + \Pi_b \right) + b = y,$$

which yields

$$\alpha(b, y) = \frac{y - b}{\Pi_o + y}.$$

Consider bidder  $i$  with type  $v_i > p^{-1}(y)$  winning the auction at price  $y$ . Paying  $b \leq y$  in cash requires the bidder to pay  $\alpha(b, y)$  in stock to have the seller accept the bid.<sup>33</sup> The value of this payment is

$$\begin{aligned} & \frac{p - b}{\Pi_o + p} (v_i X_t + \Pi_b) + b \\ &= \frac{p(v_i X_t + \Pi_b) + b(p - v_i X_t - \Delta)}{\Pi_o + p} \end{aligned} \tag{A1}$$

The value of the bid is decreasing in  $b$ . Hence, paying (4) is optimal for types  $v_i \geq p^{-1}(y)$ . Consider bidder  $i$  with valuation  $v_i < p^{-1}(y)$  winning the auction at price  $y$ . In this case, (A1) is increasing in  $b$ , so paying all stock  $((b, \alpha) = (0, \frac{y}{\Pi_o + y}))$  is optimal.

Second, we prove that dropping out when  $p$  reaches (3) is individually optimal for each bidder. Consider the decision of a bidder to drop out at a price different from (3). Suppose that it follows the strategy of dropping out at a price above (3). If the bidder wins at price  $y > p(v_i)$  and does not default, its payoff from

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<sup>33</sup>We do not consider deviations to  $(b, \alpha)$  with  $\alpha > \alpha(b, y)$ , as they are dominated by deviations to  $(b, \alpha(b, y))$ .

winning is

$$\begin{aligned}
& \max_{b \leq C_i} \{(1 - \alpha(b, y)) (v_i X_t + \Pi_b) - b\} \\
= & \frac{\Pi_o}{\Pi_o + y} (v_i X_t + \Pi_b) \\
< & \frac{\Pi_o}{\Pi_o + v_i X_t + \Pi_b - \Pi_o} (v_i X_t + \Pi_b) = \Pi_o,
\end{aligned} \tag{A2}$$

where the first equality follows from the optimality of paying in stock when  $y > p(v_i)$ , and the inequality follows from  $y > p(v_i)$ . Similarly, if the bidder wins at price  $y > p(v_i)$  and defaults, its payoff is  $\Pi_b - R < \Pi_o$ .

$$\Pi_b - R < \Pi_o. \tag{A3}$$

Therefore, dropping out at a price above (3) is suboptimal. Similarly, suppose that bidder  $i$  follows the strategy of dropping out at a price  $p$  below (3). Dropping out at (3) instead leads to winning when the other bidder drops out at prices  $y$  between  $p$  and  $p(v_i)$ . The payoff of a bidder from winning in such events is

$$\begin{aligned}
& \max_{b \leq C_i} \{(1 - \alpha(b, y)) (v_i X_t + \Pi_b) - b\} \\
\geq & \frac{\Pi_o}{\Pi_o + y} (v_i X_t + \Pi_b) \\
\geq & \Pi_o.
\end{aligned} \tag{A4}$$

Here, the first inequality holds because paying the bid using only stock is a feasible strategy for the bidder, and the second inequality follows from  $y \leq p(v_i)$ . Therefore, dropping out at a price below (3) is also suboptimal. Thus, it is optimal for a bidder with valuation  $v_i$  to drop out at price (3).

Because (3) is strictly increasing in  $v_i$ , on equilibrium path, the valuation of the winning bidder is always greater or equal than  $p^{-1}(y)$ , where  $y$  is the price at which the other bidder drops out. Hence, in equilibrium  $b = \max\{y, C_i\}$ , i.e., the winning bidder pays as much cash as possible.

We complete the proof by showing that the off-the-equilibrium-path beliefs satisfy the D1 refinement and the resetting property. The resetting property assumes that if the informed agent sends an off-equilibrium signal (submits an off-equilibrium offer  $(b, \alpha)$  while not dropping off before the winning price  $y$ ), then the belief of the uninformed party may take any value of the original support. This is despite the fact that the previous signal (initiation) truncates the support of the distribution to which the target's beliefs belong. The rationale for the resetting property, discussed in Cho (1990), is that although prior actions may reveal the informed party's information, the revelation does not mean that the uninformed party can observe and verify this private information. Let us show that the target's beliefs following an off-equilibrium action  $(b, \alpha, y)$  satisfy

the D1 refinement. By the resetting property, the target's beliefs can be any element in  $[\underline{v}, \bar{v}]$ . Consider type  $v$  winning at  $y$  and submitting an off-equilibrium offer  $(b, \alpha)$ . If the offer gets rejected, the payoff to type  $v$  is  $\Pi_b - R$ . Because  $R > \Delta$ , in this case the bidder's payoff never exceeds the equilibrium payoff, i.e., the set of types that benefit from a deviation conditional on the target rejecting the offer is the empty set.

If the offer gets accepted, the payoff to type  $v$  is  $(1 - \alpha)(\Pi_b + vX_t) - b$ . The bidder's payoff weakly exceeds the equilibrium payoff if

$$(1 - \alpha)(\Pi_b + vX_t) - b \geq \begin{cases} \left(1 - \max\left\{\frac{y-C}{\Pi_o+y}, 0\right\}\right)(\Pi_b + vX_t) - \min\{y, C\}, & \text{if } v \geq p^{-1}(y), \\ \Pi_o, & \text{if } v < p^{-1}(y). \end{cases},$$

If  $(b, \alpha, y)$  satisfy  $(1 - \alpha)(\Pi_b + p^{-1}(y)X_t) - b \leq y$ , then criterion D1 does not restrict posterior beliefs, because the set of types that strictly benefit from the deviation is empty. Consider any  $(b, \alpha, y)$  satisfying  $(1 - \alpha)(\Pi_b + p^{-1}(y)X_t) - b > y$ . If (A5) holds weakly for type  $v < p^{-1}(y)$ , it holds strictly for any type above  $v$ . Thus, criterion D1 restricts the target to place zero posterior weight on any type  $v < p^{-1}(y)$  for any off-equilibrium action  $(\alpha, b, y)$ . For the case  $v \geq p^{-1}(y)$ , the condition is equivalent to

$$\left(\alpha - \max\left\{\frac{y-C}{\Pi_o+y}, 0\right\}\right)(\Pi_b + vX_t) \leq \min\{y, C\} - b. \quad (\text{A5})$$

From (A5), for any  $\alpha > \max\left\{\frac{y-C}{\Pi_o+y}, 0\right\}$ , if (A5) holds weakly for type  $v > p^{-1}(y)$ , it holds strictly for any type below type  $v$ . Thus, following any offer with stock  $\alpha > \max\left\{\frac{y-C}{\Pi_o+y}, 0\right\}$ , criterion D1 restricts the target to place zero posterior weight on any type  $v > p^{-1}(y)$ . For any  $\alpha < \max\left\{\frac{y-C}{\Pi_o+y}, 0\right\}$  and  $b \leq \min\{y, C\}$ , (A5) holds strictly regardless of  $v$ , because the left-hand side is negative, while the right-hand side is non-negative. Thus, criterion D1 does not restrict posterior beliefs following such an action. It remains to consider deviations to  $\alpha < \max\left\{\frac{y-C}{\Pi_o+y}, 0\right\}$  and  $b > \min\{y, C\}$ . If  $\min\{y, C\} = C$ , no deviation to  $b > \min\{y, C\}$  is feasible because of the cash constraint. If  $\min\{y, C\} = y$ , then  $\max\left\{\frac{y-C}{\Pi_o+y}, 0\right\} = 0$ , so the set of deviations to  $\alpha < \max\left\{\frac{y-C}{\Pi_o+y}, 0\right\}$  is empty. Therefore, the beliefs of the target that any off-the-equilibrium-path offer  $(b, \alpha)$  by the winning bidder at price  $y$  is made by type  $p^{-1}(y)$  satisfy the D1 refinement.

**Proof of Proposition 2.** Taking the first-order condition (8) and dividing both sides by  $X_0^\beta$  yields

$$\begin{aligned} 0 = & -\beta \frac{1}{\bar{X}^{\beta+1}} \int_{\underline{v}}^{\bar{X}_c^{-1}(\bar{X})} (\bar{X} \max\{v - w, 0\} - \Delta) dF(w) \\ & + \frac{1}{\bar{X}^\beta} \int_{\underline{v}}^{\bar{X}_c^{-1}(\bar{X})} \max\{v - w, 0\} dF(w). \end{aligned} \quad (\text{A6})$$

In equilibrium, the maximum is reached at  $\bar{X}_c(v)$ . Plugging in and multiplying both sides by  $\bar{X}_c(v)^{\beta+1}$ , we get

$$\bar{X}_c(v) (\beta - 1) \int_{\underline{v}}^v (v - w) dF(w) = \beta \Delta F(v). \quad (\text{A7})$$

Hence,

$$\bar{X}_c(v) = \frac{\beta}{\beta - 1} \frac{\Delta}{v - \mathbb{E}[w|w \leq v]}. \quad (\text{A8})$$

By assumption,  $v - \mathbb{E}[w|w \leq v]$  is increasing in  $v$ . Therefore,  $\bar{X}_c(v)$  is indeed decreasing in  $v$ .

**Proof of Proposition 3.** Taking the first-order condition (11) and dividing both sides by  $X_0^\beta$  yields

$$\begin{aligned} 0 &= -\beta \frac{1}{\bar{X}^{\beta+1}} \int_{\underline{v}}^{\bar{X}_s^{-1}(\bar{X})} \left( \Pi_o \frac{\Pi_b + \bar{X}v}{\Pi_b + \bar{X}w} - \Pi_b \right) dF(w) \\ &\quad + \frac{1}{\bar{X}^\beta} \int_{\underline{v}}^{\bar{X}_s^{-1}(\bar{X})} \Pi_o \left[ \frac{\Pi_b + \bar{X}v}{\Pi_b + \bar{X}w} \right]' dF(w). \end{aligned} \quad (\text{A9})$$

The derivative is equal to

$$\left[ \frac{\Pi_b + \bar{X}v}{\Pi_b + \bar{X}w} \right]' = \frac{(v - w) \Pi_b}{(\Pi_b + \bar{X}w)^2}. \quad (\text{A10})$$

Plugging it into (A9), dividing by  $F(v)$ , and using the fact that in equilibrium the maximum is reached at  $\bar{X}_s(v)$ , we obtain

$$\begin{aligned} 0 &= -\beta \Pi_o \mathbb{E} \left[ \frac{\Pi_b + v \bar{X}_s(v)}{\Pi_b + w \bar{X}_s(v)} | w \leq v \right] + \beta \Pi_b \\ &\quad + \Pi_o \Pi_b \mathbb{E} \left[ \frac{(v - w) \bar{X}_s(v)}{(\Pi_b + w \bar{X}_s(v))^2} | w \leq v \right]. \end{aligned} \quad (\text{A11})$$

Rewriting, we obtain (12).

**Proof of Proposition 4.** We need to compare

$$\mathbb{E}[v - w | w \leq v] \text{ and } \mathbb{E} \left[ \frac{\Pi_o \left( \Pi_b + \frac{\beta}{\beta-1} w \bar{X} \right)}{(\Pi_b + w \bar{X})^2} (v - w) | w \leq v \right]. \quad (\text{A12})$$

Consider the following difference:

$$\begin{aligned}
1 - \frac{\Pi_o \left( \Pi_b + \frac{\beta}{\beta-1} w \bar{X} \right)}{(\Pi_b + w \bar{X})^2} &= \frac{\Pi_b^2 + 2\Pi_b w \bar{X} + w^2 \bar{X}^2 - \Pi_o \Pi_b - \frac{\beta}{\beta-1} \Pi_o w \bar{X}}{(\Pi_b + w \bar{X})^2} \\
&= \frac{\Pi_b (\Pi_b - \Pi_o) + \left( 2\Pi_b - \frac{\beta}{\beta-1} \Pi_o \right) w \bar{X} + w^2 \bar{X}^2}{(\Pi_b + w \bar{X})^2}.
\end{aligned} \tag{A13}$$

The first term in the numerator is positive because  $\Pi_b > \Pi_o$ . The second term in the numerator is positive because of (16). Therefore, (A13) is positive for all  $w$  and  $\bar{X}$ . Consequently,

$$\mathbb{E}[v - w | w \leq v] > \mathbb{E} \left[ \frac{\Pi_o \left( \Pi_b + \frac{\beta}{\beta-1} w \bar{X} \right)}{(\Pi_b + w \bar{X})^2} (v - w) | w \leq v \right]. \tag{A14}$$

Because of this and monotonicity of the left-hand side of (12) with respect to  $\bar{X}$ , the unique solution of (12),  $v > v^*$  is higher than the unique solution of (9).

**Proof of Proposition 5.** First, we maximize (17) with respect to threshold  $\bar{X}$ . Analogously to the proof of proposition 1, we obtain (18). Second, we maximize (19) with respect to threshold  $\bar{X}$ :

$$\begin{aligned}
0 &= -\frac{\beta}{\bar{X}^{\beta+1}} \int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})} \left( \Pi_o \frac{(v-w)\bar{X}}{\Pi_b + w\bar{X}} - \Delta \right) f(w) dw \\
&\quad + \frac{1}{\bar{X}^\beta} \int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})} \Pi_o \left[ \frac{(v-w)\bar{X}}{\Pi_b + w\bar{X}} \right]' f(w) dw.
\end{aligned} \tag{A15}$$

Equivalently,

$$\begin{aligned}
0 &= -\beta \int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})} \Pi_o \frac{(v-w)\bar{X}}{\Pi_b + w\bar{X}} f(w) dw + \beta \Delta F(\bar{X}_1^{-1}(\bar{X})) \\
&\quad + \bar{X} \int_{\underline{v}}^{\bar{X}_1^{-1}(\bar{X})} \Pi_o \frac{(v-w)\Pi_b}{(\Pi_b + w\bar{X})^2} f(w) dw.
\end{aligned} \tag{A16}$$

Dividing by  $F(\bar{X}_1^{-1}(\bar{X}))$ :

$$\begin{aligned}
0 &= -\beta \Pi_o \mathbb{E} \left[ \frac{(v-w)\bar{X}}{\Pi_b + w\bar{X}} | w \leq \bar{X}_1^{-1}(\bar{X}) \right] + \beta \Delta \\
&\quad + \Pi_o \mathbb{E} \left[ \frac{(v-w)\bar{X}\Pi_b}{(\Pi_b + w\bar{X})^2} | w \leq \bar{X}_1^{-1}(\bar{X}) \right].
\end{aligned} \tag{A17}$$

Equivalently,

$$\mathbb{E} \left[ \beta \frac{v-w}{\Pi_b + w\bar{X}} - \Pi_b \frac{v-w}{(\Pi_b + w\bar{X})^2} \mid w \leq \bar{X}_1^{-1}(\bar{X}) \right] \bar{X} = \beta \frac{\Delta}{\Pi_o}. \quad (\text{A18})$$

Rewriting yields (20). Finally, we need to determine valuation  $v^*$  such that bidder 2 never approaches the target if  $v \leq v^*$ . Consider  $\bar{X} \rightarrow \infty$ . Because  $\bar{X}_1(\underline{v})$  is finite as  $\underline{v} > 1$ ,  $\bar{X}_1^{-1}(\bar{X}) = \underline{v}$ . Therefore, the left-hand side of (A18) is

$$\mathbb{E} \left[ \beta \frac{v-w}{w} \mid w \leq \underline{v} \right] = \beta \frac{v-\underline{v}}{\underline{v}}. \quad (\text{A19})$$

Point  $v^*$  is such that

$$\beta \frac{v^* - \underline{v}}{\underline{v}} = \beta \frac{\Delta}{\Pi_o}, \quad (\text{A20})$$

which yields

$$v^* = \frac{\Pi_b}{\Pi_o} \underline{v}. \quad (\text{A21})$$

**Proof of Proposition 6.** Proposition 3 establishes that  $\bar{X}_s(v) > \bar{X}_c(v)$  for all  $v$  when  $\frac{\beta}{\beta-1} < 2\frac{\Pi_b}{\Pi_o}$ . Suppose that  $\bar{X}_1(\tilde{v}) = \bar{X}_2(\tilde{v})$  for some  $\tilde{v}$ . Then,  $\Psi_1(\tilde{v}) = \Psi_2(\tilde{v}) = 1$ ,  $\Omega_1(\tilde{v}) = \Omega_2(\tilde{v}) = v$ . As a result,  $\bar{X}_1(\tilde{v}) = \bar{X}_c(\tilde{v})$ ;  $\bar{X}_2(\tilde{v}) = \bar{X}_s(\tilde{v})$  and, under the assumption  $\bar{X}_1(\tilde{v}) = \bar{X}_2(\tilde{v})$ , all four strategies have to be equal at  $\tilde{v}$  – a contradiction with the result of Proposition 3. Hence  $\bar{X}_2$  and  $\bar{X}_1$  cannot cross.

Assume that  $\bar{X}_1(\hat{v}) > \bar{X}_2(\hat{v})$  for some  $\hat{v}$ . From Proposition 4, as  $v \downarrow v^*$ ,  $\bar{X}_2(v) \rightarrow \infty$  while  $\bar{X}_1(v)$  remains finite. Hence, there exists  $\epsilon > 0$  such that  $\bar{X}_2(v^* + \epsilon) > \bar{X}_1(v^* + \epsilon)$ . This, together with the assumption  $\bar{X}_1(\hat{v}) > \bar{X}_2(\hat{v})$  and continuity of both  $\bar{X}_1(v)$  and  $\bar{X}_2(v)$  in  $v$ , implies that  $\bar{X}_1(\tilde{v}) = \bar{X}_2(\tilde{v})$  for some  $\tilde{v} \in (v^* + \epsilon, \hat{v})$ . By earlier proof, however,  $\bar{X}_2$  and  $\bar{X}_1$  cannot cross. Hence,  $\bar{X}_2(v) > \bar{X}_1(v)$  for all  $v$ .

The final step is to show that  $\bar{X}_s(v) > \bar{X}_2(v)$  and  $\bar{X}_1(v) > \bar{X}_2(v)$  for all  $v$ . Both inequalities follow from the fact that, when  $\bar{X}_2(v) > \bar{X}_1(v)$  for all  $v$ , then  $\Psi_1(v) > 1$ ,  $\Omega_1(v) = v$ ,  $\Psi_2(v) = 1$ , and  $\Omega_2(v) < v$ .

**Proof of Proposition 7.** The first-order condition of (22) is

$$\begin{aligned} 0 = & -\frac{\beta}{\bar{X}^{\beta+1}} \int_{\underline{v}}^{\bar{X}_i^{-1}(\bar{X})} \left( \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + w\bar{X}}, 1 \right\} \bar{X} \max \{v-w, 0\} - \Delta \right) dF(w) \\ & + \frac{1}{\bar{X}^\beta} \int_{\underline{v}}^{\bar{X}_i^{-1}(\bar{X})} \left[ \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + w\bar{X}}, 1 \right\} \bar{X} \max \{v-w, 0\} \right]' dF(w). \end{aligned} \quad (\text{A22})$$

Equivalently,

$$\begin{aligned}
0 &= -\beta \int_v^{\bar{X}_i^{-1}(\bar{X})} \left( \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + w\bar{X}}, 1 \right\} \bar{X} \max \{v - w, 0\} \right) dF(w) \\
&\quad + \beta \Delta F(\bar{X}_i^{-1}(\bar{X})) + \bar{X} \int_v^{\bar{X}_i^{-1}(\bar{X})} \left[ \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + w\bar{X}}, 1 \right\} \bar{X} \max \{v - w, 0\} \right]' dF(w).
\end{aligned} \tag{A23}$$

Applying the equilibrium condition that the maximum is reached at  $\bar{X}_i(v)$  and dividing by  $F(\Omega_i(v))$  yields

$$\begin{aligned}
&\mathbb{E} \left[ \beta \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + w\bar{X}_i(v)}, 1 \right\} (v - w) \mid w \leq \Omega_i(v) \right] \bar{X}_i(v) \\
&\quad - \mathbb{E} \left[ \left[ \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + w\bar{X}_i(v)}, 1 \right\} (v - w) \bar{X}_i(v) \right]' \mid w \leq \Omega_i(v) \right] \bar{X}_i(v) \\
&= \beta \Delta \Psi_i(v).
\end{aligned} \tag{A24}$$

Let us decompose this expression into two intervals:

- if  $w < (C_i - \Delta) / \bar{X}_i(v)$ , then the expression under the expectation operator is

$$\beta (v - w) - [(v - w) \bar{X}_i(v)]' = (\beta - 1) (v - w); \tag{A25}$$

- if  $w > (C_i - \Delta) / \bar{X}_i(v)$ , then the expression under the expectation operator is

$$\begin{aligned}
&(\Pi_o + C_i) \left( \frac{\beta (v - w)}{\Pi_b + w\bar{X}_i(v)} - \left[ \frac{(v - w) \bar{X}_i(v)}{\Pi_b + w\bar{X}_i(v)} \right]' \right) \\
&= (\Pi_o + C_i) \left( \frac{\beta (v - w)}{\Pi_b + w\bar{X}_i(v)} - \frac{(v - w) \Pi_b}{(\Pi_b + w\bar{X}_i(v))^2} \right) \\
&= (\beta - 1) \frac{(\Pi_o + C_i) (v - w)}{\Pi_b + w\bar{X}_i(v)} + (\beta - 1) \frac{(\Pi_o + C_i) (v - w)}{\Pi_b + w\bar{X}_i(v)} \frac{\frac{1}{\beta - 1} w \bar{X}_i(v)}{\Pi_b + w\bar{X}_i(v)}.
\end{aligned} \tag{A26}$$

Hence, we can rewrite (A24) as (23).

Similar to Section II.B, equations (23) do not have solutions for low enough  $v$ . Let  $v_i^*$  be such that  $\lim_{v \rightarrow v_i^*} \bar{X}_i(v) = \infty$ . Rewriting (23) at this point yields

$$\mathbb{E} \left[ \frac{v_i^* - w}{w} \mid w \leq \Omega_i(v_i^*) \right] = \frac{\Delta \Psi_i(v_i^*)}{\Pi_o + C_i}. \tag{A27}$$

In the case of symmetric cash constraints,  $C_1 = C_2 = C$  and  $v_1^* = v_2^* = v^*$ , given by

$$\mathbb{E} \left[ \frac{v^* - w}{w} | w \leq v^* \right] = \frac{\Delta}{\Pi_o + C_i}. \quad (\text{A28})$$

It is easy to see that in the special cases of  $C \rightarrow \infty$  and  $C = 0$  and , we obtain  $\underline{v}$  and  $v^*$  from Section II.B, respectively.

**Proof of Proposition 8.** Let  $v_i(x) := \bar{X}_i^{-1}(x)$  be the type of bidder  $i \in \{1, 2\}$  that approaches the target at threshold  $x$ . We can re-write (A24) in terms of  $v_1(x)$  and  $v_2(x)$ :

$$\begin{aligned} & \mathbb{E} \left[ \left( \beta \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + wx}, 1 \right\} - \left[ \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + wx}, 1 \right\} x \right]' \right) (v_i(x) - w) | w \leq \min_{j \in \{1, 2\}} v_j(x) \right] x \\ & - \beta \Delta \frac{F(\max_{j \in \{1, 2\}} v_j(x))}{F(v_i(x))} = 0. \end{aligned} \quad (\text{A29})$$

Denote the left-hand side by  $\delta_i(x, v_i, v_{-i}, \Theta)$ , where  $\Theta$  is the set of comparative statics parameters, and where we suppress the dependence of  $v_i$  and  $v_{-i}$  on  $x$  for notational simplicity. The system of equations is thus  $\delta_i(x, v_i(x), v_{-i}(x), \Theta) = 0$ ,  $i \in \{1, 2\}$ .

The following auxiliary result will be useful to prove the proposition.

**Lemma 1.**  $\frac{\partial \delta_1}{\partial v_1} \frac{\partial \delta_2}{\partial v_2} - \frac{\partial \delta_1}{\partial v_2} \frac{\partial \delta_2}{\partial v_1} > 0$  at the equilibrium.

**Proof of Lemma 1.** Taking the full derivatives of these equations around the solution  $x$  everywhere where the derivatives exist yields

$$\frac{\partial \delta_1}{\partial x} + \frac{\partial \delta_1}{\partial v_1} v_1'(x) + \frac{\partial \delta_1}{\partial v_2} v_2'(x) = 0, \quad (\text{A30})$$

$$\frac{\partial \delta_2}{\partial x} + \frac{\partial \delta_2}{\partial v_2} v_2'(x) + \frac{\partial \delta_2}{\partial v_1} v_1'(x) = 0. \quad (\text{A31})$$

Combining these equations, we obtain:

$$\left( \frac{\partial \delta_1}{\partial v_1} \frac{\partial \delta_2}{\partial v_2} - \frac{\partial \delta_1}{\partial v_2} \frac{\partial \delta_2}{\partial v_1} \right) v_i'(x) = \frac{\partial \delta_i}{\partial v_{-i}} \frac{\partial \delta_{-i}}{\partial x} - \frac{\partial \delta_i}{\partial x} \frac{\partial \delta_{-i}}{\partial v_{-i}}, \quad (\text{A32})$$

where  $i \in \{1, 2\}$ . Because  $\bar{X}_i(v)$  maximizes the bidder's value function and not minimizes it,  $\frac{\partial \delta_i(x, v_i, v_{-i}, \Theta)}{\partial x} > 0$ ,  $i \in \{1, 2\}$ .<sup>34</sup>

Fix  $x$ . Without loss of generality, assume  $v_i(x) \geq v_{-i}(x)$ . Then,  $\min_{j \in \{1, 2\}} v_j(x) = v_{-i}(x)$  and

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<sup>34</sup>This follows from the second derivative of the bidder's value function with respect to the threshold at  $\bar{X}_i(v)$  being  $-\frac{\partial \delta(\bar{X}_i(v), v, v_{-i}(\bar{X}_i(v)), \Theta)}{\partial x} / \bar{X}(v)^{\beta+1}$ . It must be negative for any  $v$ .

$\max_{j \in \{1,2\}} v_j(x) = v_i(x)$ . First, consider bidder  $i$ . In the neighborhood of the equilibrium,

$$\delta_i(x, v_i, v_{-i}, \Theta) = \mathbb{E} \left[ \left( \beta \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + wx}, 1 \right\} - \left[ \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + wx}, 1 \right\} x \right]' \right) (v_i - w)x - \beta \Delta | w \leq v_{-i} \right].$$

Hence,

$$\frac{\partial \delta_i}{\partial v_i} = \mathbb{E} \left[ \left( \beta \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + wx}, 1 \right\} - \left[ \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + wx}, 1 \right\} x \right]' \right) | w \leq v_{-i} \right] x > 0.$$

Let

$$d_i(x, v_i, w, \Theta) \equiv \left( \beta \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + wx}, 1 \right\} - \left[ \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + wx}, 1 \right\} x \right]' \right) (v_i - w)x - \beta \Delta.$$

be the integrand under the expectation sign in  $\delta_i(x, v_i, v_{-i}, \Theta)$ . Let us show that  $d_i(x, v_i(x), v_{-i}(x), \Theta) < 0$ .

Consider  $d_i(x, v_i, w, \Theta)$  as a function of  $w$ . Clearly, it is strictly decreasing in  $w$  in the range  $w < (C_i - \Delta)/x$ ,

as  $d_i(x, v_i, w, \Theta) = (\beta - 1)(v_i - w)$ . Consider  $w > (C_i - \Delta)/x$ . Differentiating with respect to  $w$ ,

$$\begin{aligned} \frac{\partial d_i}{\partial w}(x, v_i, w, \Theta) &= -\frac{(\Pi_o + C_i)x}{(\Pi_b + wx)^2} \left( (\Pi_b + xv_i) \left( \beta - \frac{\Pi_b}{\Pi_b + wx} \right) - \frac{\Pi_b x (v_i - w)}{\Pi_b + wx} \right) \\ &< -\frac{(\Pi_o + C_i)x}{(\Pi_b + wx)^2} \left( (\Pi_b + xv_i) \frac{\Pi_b + 2wx}{\Pi_b + wx} - \frac{\Pi_b x (v_i - w)}{\Pi_b + wx} \right) \\ &= -\frac{(\Pi_o + C_i)x}{(\Pi_b + wx)^3} (\Pi_b^2 + 3wx\Pi_b + 2wx^2v_i) < 0, \end{aligned}$$

where the intermediate inequality follows, because  $\frac{\beta}{\beta-1} < 2\frac{\Pi_b}{\Pi_o}$  implies  $\beta > 2$ . Because either  $C_i < \Delta$  or  $C_i \rightarrow \infty$ ,  $d_i(x, v_i, w, \Theta)$  never jumps from one region to the other as  $w$  changes. Therefore,  $d_i(x, v_i, w, \Theta)$  is strictly decreasing in  $w$ . Thus,  $\mathbb{E}[d_i(x, v_i(x), w, \Theta) | w \leq v_{-i}(x)] = 0$  implies  $d_i(x, v_i(x), v_{-i}(x), \Theta) < 0$ .

Therefore,

$$\begin{aligned} \frac{\partial \delta_i}{\partial v_{-i}} &= (d_i(x, v_i, v_{-i}, \Theta) - \delta_i(x, v_i, v_{-i}, \Theta)) \frac{f(v_{-i})}{F(v_{-i})} \\ &= d_i(x, v_i, v_{-i}, \Theta) \frac{f(v_{-i})}{F(v_{-i})} < 0. \end{aligned}$$

Second, consider bidder  $-i$ . In the neighborhood of the equilibrium,

$$\begin{aligned} &\delta_{-i}(x, v_{-i}, v_i, \Theta) \\ &= \mathbb{E} \left[ \left( \beta \min \left\{ \frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1 \right\} - \left[ \min \left\{ \frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1 \right\} x \right]' \right) (v_{-i} - w)x | w \leq v_{-i} \right] - \beta \Delta \frac{F(v_i)}{F(v_{-i})}. \end{aligned}$$

Hence,

$$\frac{\partial \delta_{-i}}{\partial v_i} = -\beta \Delta \frac{f(v_i)}{F(v_{-i})} < 0 \text{ for all } v_i \in [\underline{v}, \bar{v}];$$

$$\begin{aligned}
\frac{\partial \delta_{-i}}{\partial v_{-i}} &= \int_{\underline{v}}^{v_{-i}} \left( \beta \min \left\{ \frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1 \right\} - \left[ \min \left\{ \frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1 \right\} x \right]' \right) x \frac{dF(w)}{F(v_{-i})} - \frac{f(v_{-i})}{F(v_{-i})} \delta_{-i}(x, v_{-i}, v_i, \Theta) \\
&= \int_{\underline{v}}^{v_{-i}} \left( \beta \min \left\{ \frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1 \right\} - \left[ \min \left\{ \frac{\Pi_o + C_{-i}}{\Pi_b + wx}, 1 \right\} x \right]' \right) x \frac{f(w)}{F(v_{-i})} dw > 0.
\end{aligned}$$

Because in the neighborhood of the equilibrium  $\partial \delta_i / \partial x > 0$ ,  $\partial \delta_i / \partial v_i > 0$ , and  $\partial \delta_i / \partial v_{-i} < 0$ , where  $i \in \{1, 2\}$ , the right-hand side of (A32) is negative. Because  $v'_i(x) < 0$  in equilibrium with strictly decreasing strategies,  $\frac{\partial \delta_1}{\partial v_1} \frac{\partial \delta_2}{\partial v_2} - \frac{\partial \delta_1}{\partial v_2} \frac{\partial \delta_2}{\partial v_1} > 0$  at the equilibrium.

Using this lemma, we can prove comparative statics. Consider the derivative of  $\delta_i(x, v_i, v_{-i}, \Theta)$  with respect to  $\theta \in \Theta$  at the equilibrium. Combining the equations for  $i \in \{1, 2\}$ , we obtain:

$$\left( \frac{\partial \delta_1}{\partial v_1} \frac{\partial \delta_2}{\partial v_2} - \frac{\partial \delta_1}{\partial v_2} \frac{\partial \delta_2}{\partial v_1} \right) \frac{\partial v_i}{\partial \theta} = \frac{\partial \delta_i}{\partial v_{-i}} \frac{\partial \delta_{-i}}{\partial \theta} - \frac{\partial \delta_i}{\partial \theta} \frac{\partial \delta_{-i}}{\partial v_{-i}}. \quad (\text{A33})$$

Lemma 1 implies that the sign of  $\partial v_i / \partial \theta$  coincides with the sign of the right-hand side of (A33). As shown above,  $\frac{\partial \delta_i}{\partial v_{-i}} < 0$  and  $\frac{\partial \delta_{-i}}{\partial v_{-i}} > 0$ . In addition, because  $v_i(x)$  is the inverse function of  $\bar{X}_i(v)$  and  $\bar{X}'_i(v) < 0$ , the sign of  $\partial v_i(x) / \partial \theta$  coincides with the sign of  $\partial \bar{X}_i(v) / \partial \theta$ . This can be seen from the full derivative of  $\bar{X}_i(v)$  with respect to  $\theta$ :

$$\bar{X}'_i(v) \frac{\partial v_i}{\partial \theta} + \frac{\partial \bar{X}_i(v)}{\partial \theta} = 0.$$

Therefore, a sufficient condition for  $\partial \bar{X}_i(v) / \partial \theta$  to be positive (negative) is that  $\partial \delta_i / \partial \theta < 0$  ( $\partial \delta_i / \partial \theta > 0$ ) for both  $i \in \{1, 2\}$ .

First, consider  $\theta = \beta$ :

$$\begin{aligned}
\frac{\partial \delta_i(x, v_i, v_{-i}, \Theta)}{\partial \beta} &= \mathbb{E} \left[ \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + wx}, 1 \right\} (v_i - w) \mid w \leq \min_{j \in \{1, 2\}} v_j(x) \right] x - \Delta \frac{F(\max_{j \in \{1, 2\}} v_j(x))}{F(v_i(x))} \\
&= \frac{1}{\beta} \mathbb{E} \left[ \left[ \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + wx}, 1 \right\} x \right]' (v_i(x) - w) \mid w \leq \min_{j \in \{1, 2\}} v_j(x) \right] x > 0,
\end{aligned}$$

where the second equation sign holds by the first-order condition. Hence,  $\partial \bar{X}_i(v) / \partial \beta < 0$ . Because  $\partial \beta / \partial \mu < 0$ ,  $\partial \beta / \partial \sigma < 0$ , and  $\partial \beta / \partial r > 0$ , we obtain  $\partial \bar{X}_i / \partial \mu > 0$ ,  $\partial \bar{X}_i / \partial \sigma > 0$ , and  $\partial \bar{X}_i / \partial r < 0$ .

Second, consider  $\theta = \Delta$ , keeping  $\Pi_b$  fixed. If  $C_i \rightarrow \infty$ ,

$$\frac{\partial \delta_i(x, v_i, v_{-i}, \Theta)}{\partial \Delta} = -\beta \frac{F(\max_{j \in \{1, 2\}} v_j)}{F(v_i)} < 0.$$

If  $C_i < \Delta$ ,

$$\frac{\partial \delta_i(x, v_i, v_{-i}, \Theta)}{\partial \Delta} = -\mathbb{E} \left[ \frac{1}{\Pi_b + wx} \left( \beta - \frac{\Pi_b}{\Pi_b + wx} \right) (v_i - w) \mid w \leq \min_{j \in \{1,2\}} v_j \right] x - \beta \frac{F(\max_{j \in \{1,2\}} v_j)}{F(v_i)} < 0.$$

Hence,  $\partial \bar{X}_i(v) / \partial \Delta > 0$ .

Finally, consider  $\theta = \Pi_b$ , keeping  $\Delta$  fixed. If  $C_i \rightarrow \infty$ ,  $\partial \delta_i(x, v_i, v_{-i}, \Theta) / \partial \Pi_b = 0$ . If  $C_i < \Delta$ ,

$$\begin{aligned} \frac{\partial \delta_i(x, v_i, v_{-i}, \Theta)}{\partial \Pi_b} &= \mathbb{E} \left[ \frac{(wx + \Delta - C_i)(\beta(\Pi_b + wx) - \Pi_b) - wx(\Pi_b - \Delta + C_i)}{(\Pi_b + wx)^3} (v_i - w) \mid w \leq \min_{j \in \{1,2\}} v_j \right] \\ &> \mathbb{E} \left[ \frac{2w^2x^2 + (\Delta - C_i)(\Pi_b + 3wx)}{(\Pi_b + wx)^3} (v_i - w) \mid w \leq \min_{j \in \{1,2\}} v_j \right] > 0, \end{aligned}$$

where the first inequality follows from  $\beta > 2$ . Hence,  $\partial \bar{X}_i(v) / \partial \Pi_b \leq 0$ .

**Proof of Proposition 9.** We consider Markov Perfect equilibria of the initiation game in which, as before, bidders initiate acquisitions at upper thresholds  $\bar{X}_i(v)$  that are strictly decreasing in  $v$ . For all cases of bidders' cash constraints, the target's payoff is then monotonically increasing in  $X_t$ , so the optimal Markov strategy of the target also has an upper threshold structure: offer itself up for sale when  $X_t$  reaches some  $\bar{X}^T$  for the first time. Consider first the case of two unconstrained bidders ( $C_i \rightarrow \infty$ ). Assume that a Markov Perfect equilibrium exists in which the target never delays bidder initiation and a bidder with valuation  $v$  initiates according to  $\bar{X}_c(v)$ . Consider the target's deviations in the class of upper threshold strategies. Suppose that for some  $\bar{X}$ , the target is approached by a bidder but decides to delay the transaction until  $X_t$  reaches the upper threshold  $\bar{X}^T(\bar{X}) > \bar{X}$  (for the case of symmetric cash constraints, we omit the dependence of  $\bar{X}^T$  on bidder identity for brevity). In such deviation, once  $X_t$  reaches  $\bar{X}$  for the first time the target stops updating the upper boundary of bidders' valuations  $v = \bar{X}_c^{-1}(\bar{X})$  because, given bidders' possible synergies, any further decision to finalize the transaction is not bidder-driven. As a result, for the deviation  $\bar{X}^T$  to be profitable, the target needs to have:

$$\mathbb{E}_{\tau, w} [e^{-r\tau} (\Delta + w\bar{X}^T) \mid v = \bar{X}_c^{-1}(\bar{X}), w \leq v] > \mathbb{E}_w [(\Delta + w\bar{X}) \mid v = \bar{X}_c^{-1}(\bar{X}), w \leq v],$$

where  $\tau$  is the random time until  $X_t = \bar{X}$  reaches  $\bar{X}^T$ . Simplifying,

$$\begin{aligned} \left( \frac{\bar{X}}{\bar{X}^T} \right)^\beta \mathbb{E}_w [\Delta + w\bar{X}^T \mid v = \bar{X}_c^{-1}(\bar{X}), w \leq v] &> \mathbb{E}_w [\Delta + w\bar{X} \mid v = \bar{X}_c^{-1}(\bar{X}), w \leq v] \Rightarrow \\ \left( \frac{\bar{X}}{\bar{X}^T} \right)^\beta \frac{\mathbb{E}_w [\Delta + w\bar{X}^T]}{\mathbb{E}_w [\Delta + w\bar{X}]} &> 1. \end{aligned} \tag{A34}$$

Because  $\beta > 1$  and  $\bar{X}^T > \bar{X}$ ,  $\left(\frac{X^\beta}{\Delta+wX}\right)' = \frac{(\beta-1)X^\beta + \beta X^{\beta-1}\Delta}{(\Delta+wX)^2} > 0$ , the inequality (A34) does not hold case by case for every  $w$  so the left hand side of (A34) cannot be above 1. Hence, for any  $\bar{X}$ , the target does not find it profitable to delay initiation by cash bidders to any  $\bar{X}^T$ .

Next, consider the case of stock bidders who approach the target with a bid at  $\bar{X}_s(v)$ . We look for profitable deviations from some  $\bar{X}$  at which the target is approached to  $\bar{X}^T$ . While we cannot show analytically that the target would never delay initiation to arbitrarily large  $\bar{X}^T$ , we show that marginal deviations are not profitable (numerically, for realistic parameters, large deviations are also not profitable). Maximize the target's revenue with respect to  $\bar{X}^T$ :

$$\left(\frac{\bar{X}}{\bar{X}^T}\right)^\beta \int_{\underline{v}}^{\bar{X}_s^{-1}(\bar{X})} \left(1 - \frac{\Pi_0}{\Pi_b + w\bar{X}^T}\right) (\Pi_b + \bar{X}_s^{-1}(\bar{X})\bar{X}^T) dF(w) \rightarrow \max_{\bar{X}^T}.$$

The derivative with respect to  $\bar{X}^T$ :

$$\begin{aligned} & -\beta \bar{X}^\beta (\bar{X}^T)^{-\beta-1} \int_{\underline{v}}^v \left(1 - \frac{\Pi_0}{\Pi_b + w\bar{X}^T}\right) (\Pi_b + v\bar{X}^T) \\ & + \bar{X}^\beta (\bar{X}^T)^{-\beta} \int_{\underline{v}}^v \left(\frac{\Pi_0 w}{(\Pi_b + w\bar{X}^T)^2} (\Pi_b + v\bar{X}^T) + v \left(1 - \frac{\Pi_0}{\Pi_b + w\bar{X}^T}\right)\right) dF(w). \end{aligned}$$

After some algebra, this condition simplifies to

$$\bar{X}^\beta (\bar{X}^T)^{-\beta-1} \left( (-\beta-1)v\bar{X}^T + \beta(\Pi_0 - \Pi_b) \int_{\underline{v}}^v dF(w) + (\beta-1) \int_{\underline{v}}^v \frac{\Pi_0 \left(\Pi_b + \frac{\beta}{\beta-1}w\bar{X}^T\right)}{(\Pi_b + w\bar{X}^T)^2} (v-w) \bar{X}^T dF(w) \right).$$

Calculate the derivative at  $\bar{X}$  using (12):

$$\begin{aligned} & \bar{X}^{-1} ((-\beta-1)v\bar{X} + \beta(\Pi_0 - \Pi_b)) F(v) + \beta(\Pi_b - \Pi_0) F(v) \\ & = -(\beta-1)vF(v) < 0. \end{aligned}$$

Hence, the target does not have incentives to locally delay the stock acquisition.

Finally, consider the case of cash versus stock bidders who approach the target with a bid at  $\bar{X}_1(v)$  and  $\bar{X}_2(v)$  correspondingly. First, if the stock bidder approaches the target with a bid at  $\bar{X}$  and the target considers deviation to  $\bar{X}^T(2, \bar{X}) > \bar{X}$ , where the first argument is the identity of the initiating bidder, its

payoff from delaying is

$$\begin{aligned} & \left( \frac{\bar{X}}{\bar{X}^T} \right)^\beta \int_{\underline{v}}^{\Omega_2(v)} \left( 1 - \frac{\Pi_0}{\Pi_b + w\bar{X}^T} \right) (\Pi_b + v\bar{X}^T) dF(w) \\ & + \left( \frac{\bar{X}}{\bar{X}^T} \right)^\beta \int_{\Omega_2(v)}^{\bar{X}_1^{-1}(\bar{X})} (v\bar{X}^T + \Delta) dF(w), \end{aligned}$$

where  $\Omega_2(v) = \min \{v, \bar{X}_1^{-1}(\bar{X})\}$ . The derivative with respect to  $\bar{X}^T$ :

$$\begin{aligned} & - \beta \bar{X}^\beta (\bar{X}^T)^{-\beta-1} \int_{\underline{v}}^{\Omega_2(v)} \left( 1 - \frac{\Pi_0}{\Pi_b + w\bar{X}^T} \right) (\Pi_b + v\bar{X}^T) \\ & + \bar{X}^\beta (\bar{X}^T)^{-\beta-1} \int_{\underline{v}}^{\Omega_2(v)} \left( \frac{\Pi_0 w}{(\Pi_b + w\bar{X}^T)^2} (\Pi_b + v\bar{X}^T) + v \left( 1 - \frac{\Pi_0}{\Pi_b + w\bar{X}^T} \right) \right) dF(w) \\ & - \beta \bar{X}^\beta (\bar{X}^T)^{-\beta-1} \int_{\Omega_2(v)}^{\bar{X}_1^{-1}(\bar{X})} (v\bar{X}^T + \Delta) dF(w) + \bar{X}^\beta (\bar{X}^T)^{-\beta} \int_{\Omega_2(v)}^{\bar{X}_1^{-1}(\bar{X})} v dF(w) \\ & = \bar{X}^\beta (\bar{X}^T)^{-\beta-1} \left( (-\beta-1)v\bar{X}^T + \beta(\Pi_0 - \Pi_b) \int_{\underline{v}}^{\Omega_2(v)} dF(w) \right. \\ & + (\beta-1) \int_{\underline{v}}^{\Omega_2(v)} \frac{\Pi_0 \left( \Pi_b + \frac{\beta}{\beta-1} w\bar{X}^T \right)}{(\Pi_b + w\bar{X}^T)^2} (v-w) \bar{X}^T dF(w) \\ & \left. + (-\beta-1)v\bar{X}^T + \beta(\Pi_0 - \Pi_b) \int_{\Omega_2(v)}^{\bar{X}_1^{-1}(\bar{X})} dF(w) \right). \end{aligned}$$

Calculate the above expression at  $\bar{X}^T = \bar{X}$  using that from (20),

$$(\beta-1) \int_{\underline{v}}^{\Omega_2(v)} \frac{\Pi_0 \left( \Pi_b + \frac{\beta}{\beta-1} w\bar{X} \right)}{(\Pi_b + w\bar{X})^2} (v-w) \bar{X} dF(w) = \beta \Delta \Psi_2(v) F(\Omega_2(v)),$$

where  $\Psi_2(v) = \max \left\{ 1, \frac{F(\bar{X}_1^{-1}(\bar{X}))}{F(v)} \right\}$ :

$$\bar{X}^{-1} \left( (-\beta-1)v\bar{X} + \beta(\Pi_0 - \Pi_b) F(\bar{X}_1^{-1}(\bar{X})) + \beta(\Pi_b - \Pi_0) \Psi_2(v) F(\Omega_2(v)) \right).$$

Note that if  $\bar{X}_1^{-1}(\bar{X}) < v$  then  $\Omega_2(v) = \bar{X}_1^{-1}(\bar{X})$ ;  $\Psi_2(v) = 1$ ; and  $\Psi_2(v)F(\Omega_2(v)) = F(\bar{X}_1^{-1}(\bar{X}))$ . If  $\bar{X}_1^{-1}(\bar{X}) > v$  then  $\Omega_2(v) = v$ ;  $\Psi_2(v) = \frac{F(\bar{X}_1^{-1}(\bar{X}))}{F(v)}$ ; and, again,  $\Psi_2(v)F(\Omega_2(v)) = F(\bar{X}_1^{-1}(\bar{X}))$ . As a result, the final expression for the derivative is

$$= -(\beta-1)vF(\bar{X}_1^{-1}(\bar{X})) < 0.$$

Hence, the target does not have incentives to locally delay initiation by a stock bidder.

Second, if the cash bidder approaches the target with a bid at  $\bar{X}$  and the target considers a deviation to

$\bar{X}^T(1, \bar{X}) > \bar{X}$ , its payoff from delaying is

$$\begin{aligned} & \left( \frac{\bar{X}}{\bar{X}^T} \right)^\beta \int_{\underline{v}}^{\Omega_1(v)} (w\bar{X}^T + \Delta) dF(w) \\ & + \left( \frac{\bar{X}}{\bar{X}^T} \right)^\beta \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} \left( 1 - \frac{\Pi_0}{\Pi_b + v\bar{X}^T} \right) (\Pi_b + w\bar{X}^T) dF(w), \end{aligned}$$

where  $\Omega_1(v) = \min \{v, \bar{X}_2^{-1}(\bar{X})\}$ . The derivative with respect to  $\bar{X}^T$ :

$$\begin{aligned} & - \beta \bar{X}^\beta (\bar{X}^T)^{-\beta-1} \int_{\underline{v}}^{\Omega_1(v)} (w\bar{X}^T + \Delta) dF(w) + \bar{X}^\beta (\bar{X}^T)^{-\beta} \int_{\underline{v}}^{\Omega_1(v)} w dF(w) \\ & - \beta \bar{X}^\beta (\bar{X}^T)^{-\beta-1} \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} \left( 1 - \frac{\Pi_0}{\Pi_b + v\bar{X}^T} \right) (\Pi_b + w\bar{X}^T) \\ & + \bar{X}^\beta (\bar{X}^T)^{-\beta-1} \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} \left( \frac{\Pi_0 v}{(\Pi_b + v\bar{X}^T)^2} (\Pi_b + w\bar{X}^T) + w \left( 1 - \frac{\Pi_0}{\Pi_b + v\bar{X}^T} \right) \right) dF(w) \\ & = \bar{X}^\beta (\bar{X}^T)^{-\beta-1} \left( -(\beta-1)\bar{X}^T \int_{\underline{v}}^{\Omega_1(v)} w dF(w) + \beta(\Pi_0 - \Pi_b) \int_{\underline{v}}^{\Omega_1(v)} dF(w) \right. \\ & - (\beta-1)\bar{X}^T \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} w dF(w) + \beta(\Pi_0 - \Pi_b) \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} dF(w) \\ & \left. + (\beta-1) \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} \frac{\Pi_0 \left( \Pi_b + \frac{\beta}{\beta-1} v\bar{X}^T \right)}{(\Pi_b + v\bar{X}^T)^2} (w-v)\bar{X}^T dF(w) \right). \end{aligned}$$

Calculate the above expression at  $\bar{X}^T = \bar{X}$  using that from (18),

$$(\beta-1)\bar{X} \int_{\underline{v}}^{\Omega_1(v)} w dF(w) = (\beta-1)v\bar{X}F(\Omega_1(v)) - \beta\Delta\Psi_1(v)F(\Omega_1(v)),$$

where  $\Psi_1(v) = \max \left\{ 1, \frac{F(\bar{X}_2^{-1}(\bar{X}))}{F(v)} \right\}$  (note that, again,  $\Psi_1(v)F(\Omega_1(v)) = \bar{X}_2^{-1}(\bar{X})$ ):

$$\begin{aligned} & \bar{X}^{-1} \left( -(\beta-1)v\bar{X}F(\Omega_1(v)) + \beta(\Pi_b - \Pi_0)F(\bar{X}_2^{-1}(\bar{X})) + \beta(\Pi_0 - \Pi_b)F(\Omega_1(v)) \right) \\ & - (\beta-1)\bar{X} \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} w dF(w) + \beta(\Pi_0 - \Pi_b)(F(\bar{X}_2^{-1}(\bar{X})) - F(\Omega_1(v))) \\ & + (\beta-1) \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} \frac{\Pi_0 \left( \Pi_b + \frac{\beta}{\beta-1} v\bar{X} \right)}{(\Pi_b + v\bar{X})^2} (w-v)\bar{X} dF(w) \\ & = -(\beta-1) \left( vF(\Omega_1(v)) + \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} \left( w - \frac{\Pi_0 \left( \Pi_b + \frac{\beta}{\beta-1} v\bar{X} \right)}{(\Pi_b + v\bar{X})^2} (w-v) \right) dF(w) \right). \end{aligned}$$

If  $\Omega_1(v) = \bar{X}_2^{-1}(\bar{X})$  (the cash bidder initiates later), the derivative is equal to  $-(\beta-1)(vF(\bar{X}_2^{-1}(\bar{X}))) < 0$

and the target has no local incentives to deviate. Consider  $\Omega_1(v) = v$  now. Also, assume that  $\frac{\beta}{\beta-1} < \frac{2\Pi_b}{\Pi_0}$ . Then, the derivative is less than

$$\begin{aligned}
& - (\beta - 1) \left( vF(\Omega_1(v)) + \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} \left( w - \frac{\Pi_0 \left( \Pi_b + \frac{2\Pi_b}{\Pi_0} v \bar{X} \right)}{(\Pi_b + v \bar{X})^2} (w - v) \right) dF(w) \right) \\
& < -(\beta - 1) \left( vF(\Omega_1(v)) + \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} \left( w - \frac{\Pi_0 \left( \Pi_b + \frac{2\Pi_b}{\Pi_0} v \bar{X} \right)}{(\Pi_b + v \bar{X})^2} w \right) dF(w) \right) \\
& = -(\beta - 1) \left( vF(\Omega_1(v)) + \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} \left( \frac{\Pi_b^2 - \Pi_0 \Pi_b + v^2 \bar{X}^2}{(\Pi_b + v \bar{X})^2} \right) w dF(w) \right) \\
& = -(\beta - 1) \left( vF(\Omega_1(v)) + \int_{\Omega_1(v)}^{\bar{X}_2^{-1}(\bar{X})} \left( \frac{\Pi_b(\Pi_b - \Pi_0) + v^2 \bar{X}^2}{(\Pi_b + v \bar{X})^2} \right) w dF(w) \right) < 0.
\end{aligned}$$

Hence, for the standard parametrization  $\frac{\beta}{\beta-1} < \frac{2\Pi_b}{\Pi_0}$ , the target does not find it profitable to locally delay the acquisition.

Finally, we consider whether it is optimal for the target to accelerate the auction at some  $\bar{X}^T$  (where  $\bar{X}^T$  is either an upper threshold or a point upon reaching which for the first time, the target makes a single take-it-or-leave-it offer), conditional on not being approached by any bidder earlier. Suppose first that the target plays a take-it-or-leave-it (TIOLI) strategy and chooses to initiate the contest only once  $X_t$  reaches  $\bar{X}^T$  for the first time. Then there is always an equilibrium in which the bidders, believing that no rival type accepts the target's offer to sell itself, also do not approach it. The proof is by backward induction. Post-TIOLI, if (i) the bidders do not accept it, bidder  $i$  approaches later, and the target rejects this later offer then the target obtains payoff  $X_t$ . If (ii) the bidders do not accept the TIOLI and no bidder approaches then the target also obtains payoff  $X_t$ . If (iii) the bidders do not accept the TIOLI, bidder  $i$  approaches later at some  $\bar{X}$  and the target accepts this later offer then the target obtains payoff  $\mathbb{E}[R^T | v = \bar{X}_i^{-1}(\bar{X}), w \leq v] > X_t$  where  $R^T$  stands for target's payoff conditional on  $v, w$ . Thus for every approaching bidder, the target has ex-post incentives to accept its later offer despite making a TIOLI earlier. Post-TIOLI then, the bidders will play exactly the same initiation strategies as in the case when the option for the target to make a TIOLI is absent.

At the stage of the TIOLI, for the set of initiating bidders ( $v > v_i^*$ ), the game without the TIOLI is exactly the same as the game with TIOLI if they do not accept it and they believe that no rival bidder will accept the TIOLI as well. If  $\bar{X}^T$  is suboptimal, they will delay the initiation. For the set of non-initiating bidders ( $v \leq v_i^*$ ), it is not optimal for them to initiate because initiation and the following contest destroys their value. In the same way, accepting the TIOLI also destroys their value so it is optimal to not accept it. However then, because every type of bidder effectively ignores the TIOLI due to inability of the target to commit to it ex-post, the target does not have incentives to make this offer in the first place and we have

an equilibrium. There may be other equilibria where different beliefs about rival bidders may result in the TIOLI being accepted for some values of bidders' synergies. The proof for the case when the target plays a threshold strategy (initiate the contest as long as  $X_t$  is above  $\bar{X}^T$ ) is similar and omitted here for brevity.

## Appendix B Asymmetric Initiation: Numerical Procedure

For illustrative purposes, consider the case of cash versus stock bidder,  $C_1 \rightarrow \infty$ ,  $C_2 = 0$ . The case of endogenous means of payment is numerically solved in the same fashion, using equations (A24). We use substitution of variables to express the first order conditions for the two asymmetrically constrained bidders in terms of  $\bar{X}_1^{-1}(x)$ ,  $\bar{X}_2^{-1}(x)$  for a given initiation threshold  $x$ . Specifically, let

$$\begin{aligned} x_1 &\equiv \bar{X}_1(v_1) \Rightarrow v_1 = \bar{X}_1^{-1}(x_1), \bar{X}_2^{-1}(\bar{X}_1(v_1)) = \bar{X}_2^{-1}(x_1); \\ x_2 &\equiv \bar{X}_2(v_1) \Rightarrow v_2 = \bar{X}_2^{-1}(x_2), \bar{X}_1^{-1}(\bar{X}_2(v_2)) = \bar{X}_1^{-1}(x_2). \end{aligned} \quad (\text{B1})$$

Then, the system of equations (18), (20) becomes

$$x_1 = \frac{\beta}{\beta - 1} \frac{\Delta}{\bar{X}_1^{-1}(x_1) - \int_{\underline{v}}^{\bar{X}_1^{-1}(x_1)} w \frac{f(w)}{F(\bar{X}_1^{-1}(x_1))} dw} \frac{F(\bar{X}_2^{-1}(x_1))}{F(\bar{X}_1^{-1}(x_1))}, \quad (\text{B2})$$

$$x_2 \int_{\underline{v}}^{\bar{X}_1^{-1}(x_2)} \frac{\Pi_o \left( \Pi_b + \frac{\beta}{\beta - 1} w x_2 \right)}{(\Pi_b + w x_2)^2} (\bar{X}_2^{-1}(x_2) - w) \frac{f(w)}{F(\bar{X}_1^{-1}(x_2))} dw = \frac{\beta}{\beta - 1} \Delta. \quad (\text{B3})$$

We have two equations and four different combinations of functions and arguments as unknowns. We consider the interior case ( $\bar{X}_i^{-1}(x) \in (\underline{v}, \bar{v})$  for  $i \in \{1, 2\}$ ,  $x \in \{x_1, x_2\}$ ). Assume that both boundaries are equal,  $x_1 = x_2 = x$ , for some  $v = \bar{X}_1^{-1}(x)$ ,  $w = \bar{X}_2^{-1}(x)$ . This allows to simplify the system to two non-linear equations and two functions of one argument as unknowns, which can be easily solved with a mathematical package.

Note that the above algorithm does not provide corner solution for  $v > \bar{v} = \bar{X}_1^{-1}(X_2(\bar{v}))$ . Observe, however, that (B2) in this case can be rewritten as

$$x = \frac{\beta}{\beta - 1} \frac{\Delta}{\bar{X}_1^{-1}(x) - \int_{\underline{v}}^{\bar{X}_1^{-1}(x)} w \frac{f(w)}{F(\bar{X}_1^{-1}(x))} dw} \frac{1}{F(\bar{X}_1^{-1}(x))}, \quad (\text{B4})$$

and does not depend on  $\bar{X}_2^{-1}(x)$ . As a result, a single non-linear equation with a single unknown is easily solved numerically. Combinations  $(\bar{X}_1^{-1}(x), x)$  and  $(\bar{X}_2^{-1}(x), x)$  constitute pairs of valuations and equilibrium initiation strategies for the two bidders.

As an example, when bidder valuations are uniformly distributed on  $[\underline{v}, \bar{v}]$ , in the interior case

$$x = \frac{\beta}{\beta - 1} \frac{\Delta}{(\bar{X}_1^{-1}(x) - \underline{v})/2} \frac{\bar{X}_2^{-1}(x) - \underline{v}}{\bar{X}_1^{-1}(x) - \underline{v}}, \quad (\text{B5})$$

$$x \int_{\underline{v}}^{\bar{X}_1^{-1}(x)} \frac{\Pi_o \left( \Pi_b + \frac{\beta}{\beta-1} wx \right)}{(\Pi_b + wx)^2} \frac{\bar{X}_2^{-1}(x) - w}{\bar{X}_1^{-1}(x) - w} dw = \frac{\beta}{\beta - 1} \Delta. \quad (\text{B6})$$

The integral in (B6) has a closed form representation.

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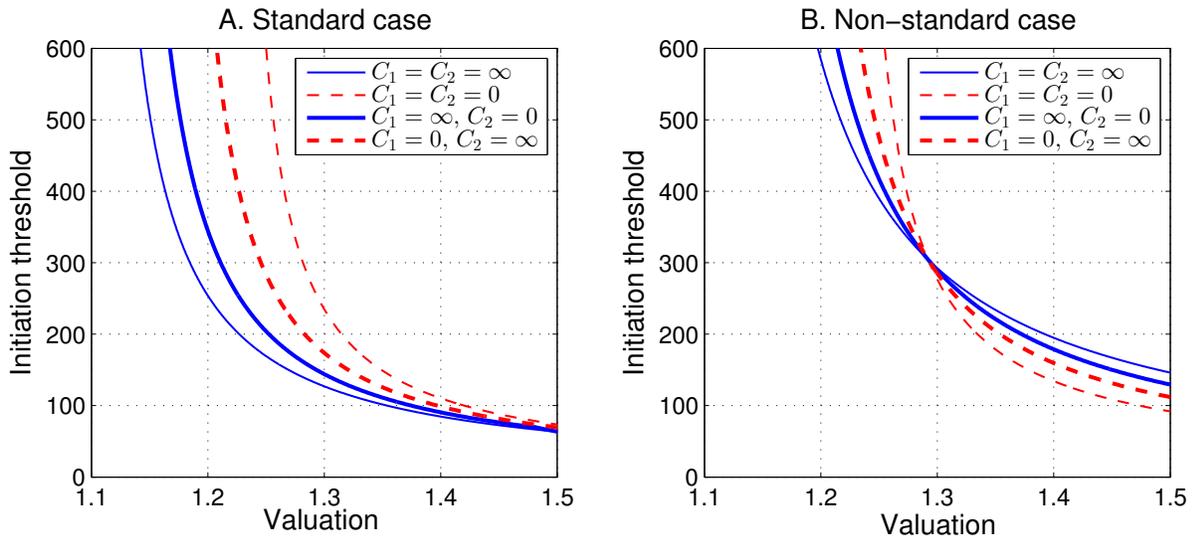
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**Table I: Benchmark model parameters**

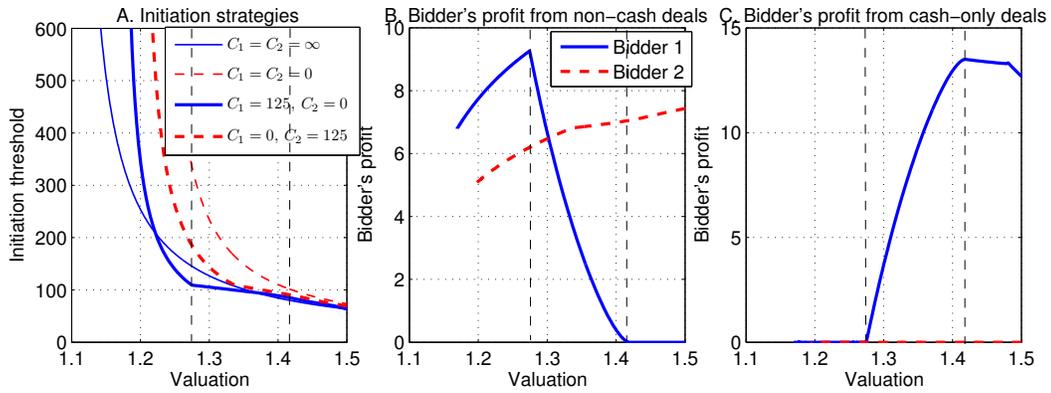
This table reports the benchmark parametrization of the model.

<b>Variable</b>	<b>Description</b>	<b>Value</b>
$r$	Risk-free rate	0.05
$\mu$	Growth rate of target value	0.01
$\sigma$	Volatility of growth rate of target value	0.25
$\Pi_b$	Initial value of bidders	100
$\Pi_o$	Post-takeover value of the losing bidder	95
$\Delta$	Value loss of the losing bidder	5
$\underline{v}$	Lowest value of the acquired target	110%
$\bar{v}$	Highest value of the acquired target	150%
$F(v)$	Distribution of valuations	Uniform
$D(v)$	Dispersion of valuations*	11.55%

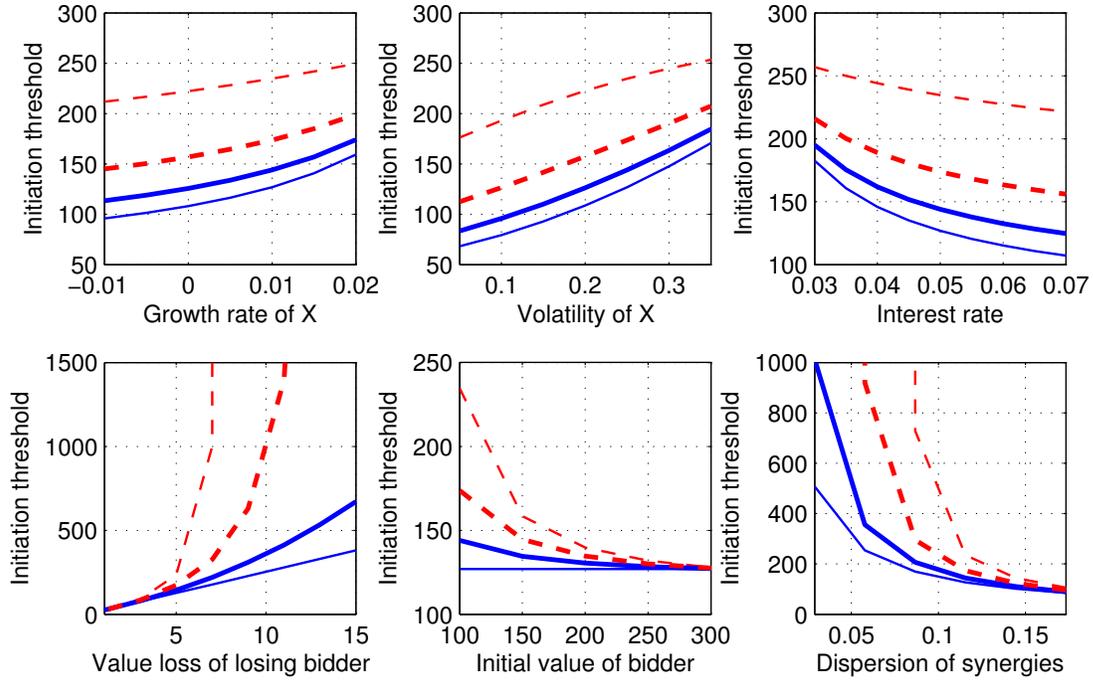
\* Note: Dispersion of valuations for the uniform distribution is  $D(v) = \sqrt{(\bar{v} - \underline{v})^2/12}$ .



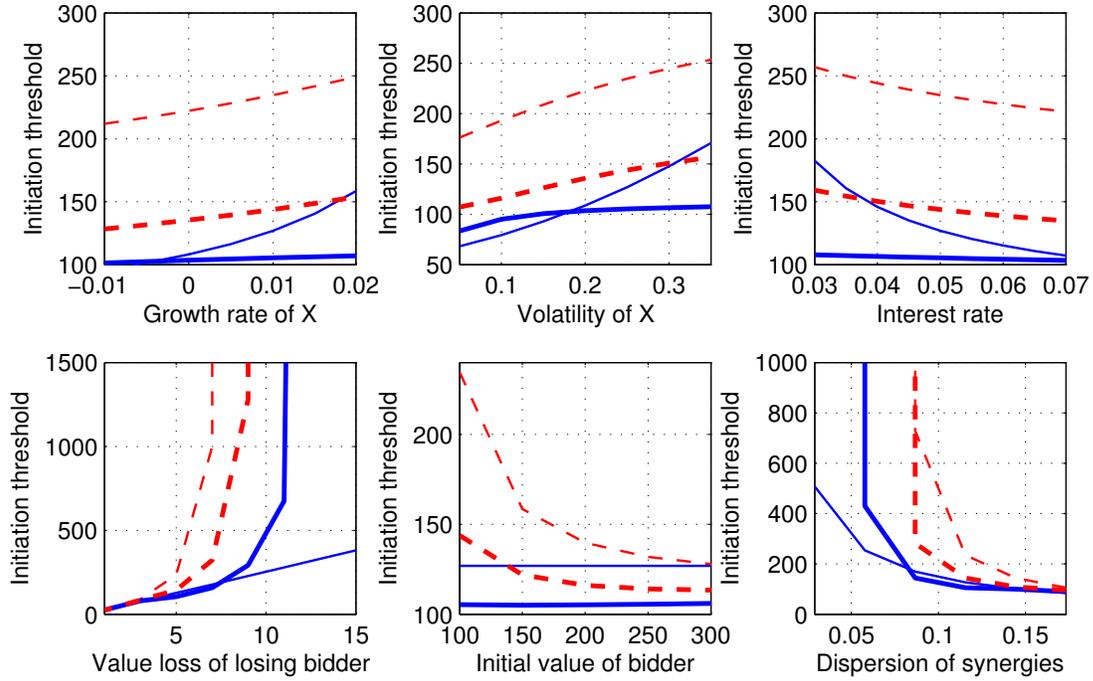
**Figure 1: Initiation strategies of unconstrained and constrained bidders facing different types of competitors.** The figure shows the optimal initiation strategies of bidders as a function of their valuations,  $v$ . The thin solid (thin dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing another unconstrained (extremely constrained) bidder; the thick solid (thick dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing an extremely constrained (unconstrained) bidder.



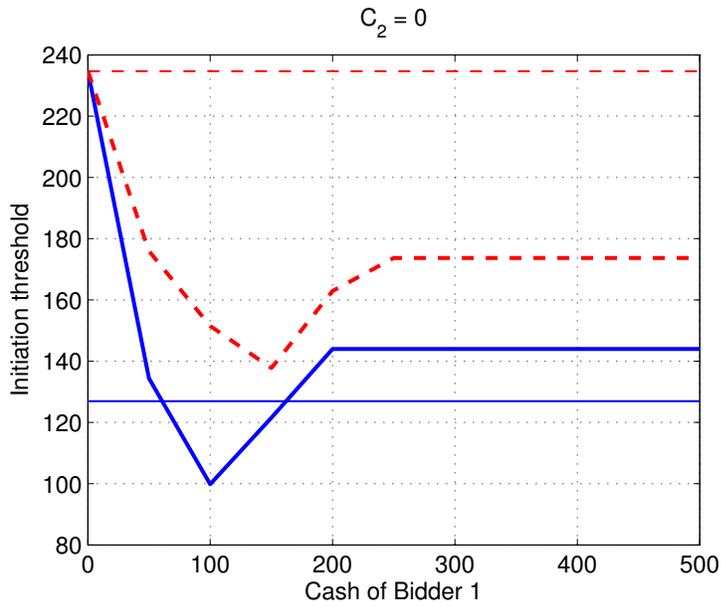
**Figure 2: Initiation strategies of bidders facing different types of competitors.** Panel A shows the equilibrium initiation thresholds of bidders as functions of their valuations,  $v$ . The thin solid (thin dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing another unconstrained (extremely constrained) bidder; the thick solid (thick dashed) line is the strategy of a bidder with cash  $C_1 = 125$  ( $C_1 = 0$ ) facing a bidder with cash  $C_2 = 0$  ( $C_2 = 125$ ). Panel B (C) shows the part of the total expected payoff of a bidder with valuation  $v$  at the date of the auction that comes from non-cash (cash) deals for bidders with internal cash  $C_1 = 125$  and  $C_2 = 0$ .



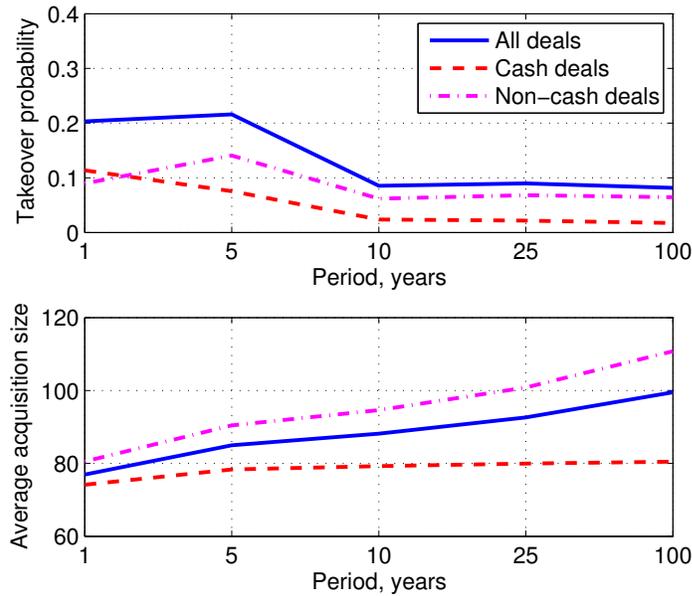
**Figure 3: Initiation strategies of unconstrained and extremely constrained bidders as a function of model parameters.** The figure shows the comparative statics of the four initiation strategies for the benchmark model parametrization (Table I) and a bidder with the average valuation,  $v = 1.3$ . The thin solid (thin dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing another unconstrained (extremely constrained) bidder; the thick solid (thick dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing an extremely constrained (unconstrained) bidder. The comparative statics are with respect to (i) the growth rate of a target's assets,  $\mu$ , (ii) the volatility of a target's assets,  $\sigma$ , (iii) the interest rate,  $r$ , (iv) costs of losing the contest,  $\Delta$ , (v) the initial value of bidders,  $\Pi_b$  (keeping  $\Delta$  fixed), and (vi) the dispersion of the bidders' valuations,  $D(v)$  (keeping the average valuation fixed).



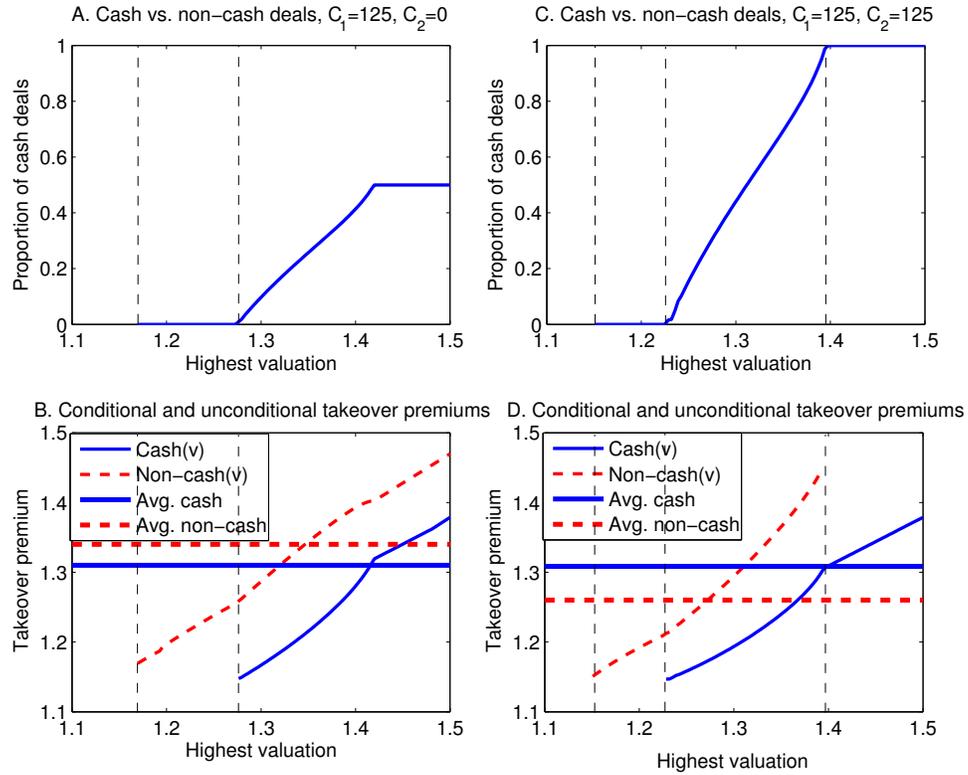
**Figure 4: Initiation strategies of constrained bidders as a function of model parameters.** The figure shows the comparative statics of the four initiation strategies for the benchmark model parametrization (Table I) and a bidder with the average valuation,  $v = 1.3$ . The thin solid (thin dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing another unconstrained (extremely constrained) bidder; the thick solid (thick dashed) line is the strategy of a bidder with cash  $C_1 = 125$  ( $C_1 = 0$ ) facing a bidder with cash  $C_2 = 0$  ( $C_2 = 125$ ). The comparative statics are with respect to (i) the growth rate of a target's assets,  $\mu$ , (ii) the volatility of a target's assets,  $\sigma$ , (iii) the interest rate,  $r$ , (iv) costs of losing the contest,  $\Delta$ , (v) the initial value of bidders,  $\Pi_b$  (keeping  $\Delta$  fixed), and (vi) the dispersion of the bidders' valuations,  $D(v)$  (keeping the average valuation fixed).



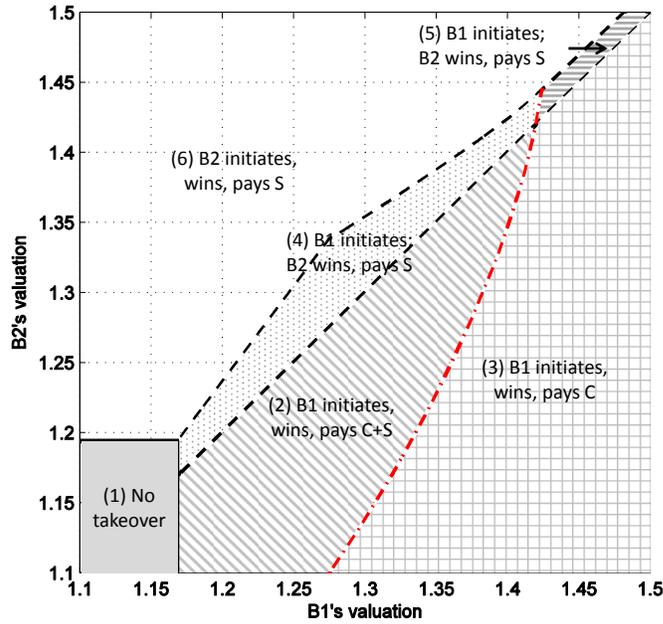
**Figure 5: Initiation strategies of constrained bidders as a function of asymmetries in cash constraints.** The figure shows the comparative statics of the four initiation strategies for the benchmark model parametrization (Table I) as a function of the cash position of bidder 1,  $C_1$ . The comparative statics are calculated for the bidder with the average valuation,  $v = 1.3$ . The thick solid (thick dashed) line is the strategy of a bidder with cash  $C_1$  ( $C_2 = 0$ ) competing against a bidder with cash  $C_2 = 0$  ( $C_1$ ). The thin solid (thin dashed) line is the strategy of an unconstrained (extremely constrained) bidder facing another unconstrained (extremely constrained) bidder.



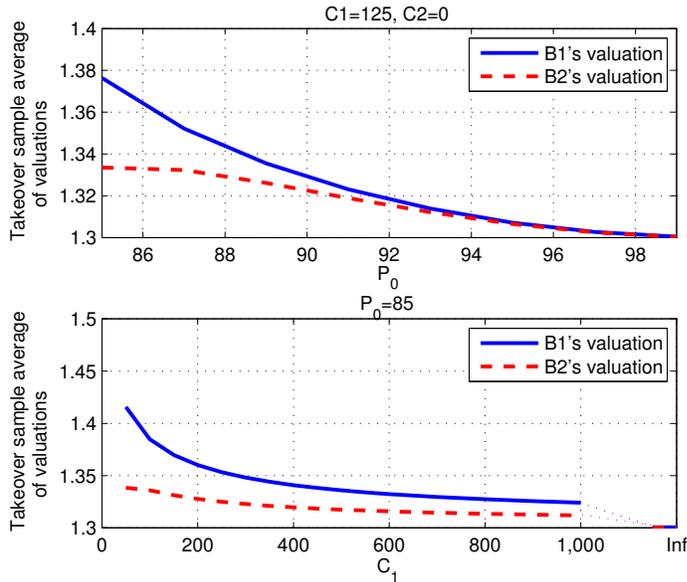
**Figure 6: Takeover probability and average acquisition size in cash and non-cash deals.** The figure corresponds to prediction A1. For the benchmark parametrization (Table I) and  $C_1 = 125$ ,  $C_2 = 0$ , the top panel shows the frequency of takeovers initiated and completed in years 1, 2–5, 6–10, 11–25, and 26–100. The bottom panel shows the average acquisition size in deals completed by the end of years 1, 5, 10, 25, and 100. The starting value of the target is  $X_0 = \bar{X}_1(\bar{v})$ . The solid line corresponds to all types of deals. The dashed (dash-dotted) line corresponds to cash (non-cash) deals.



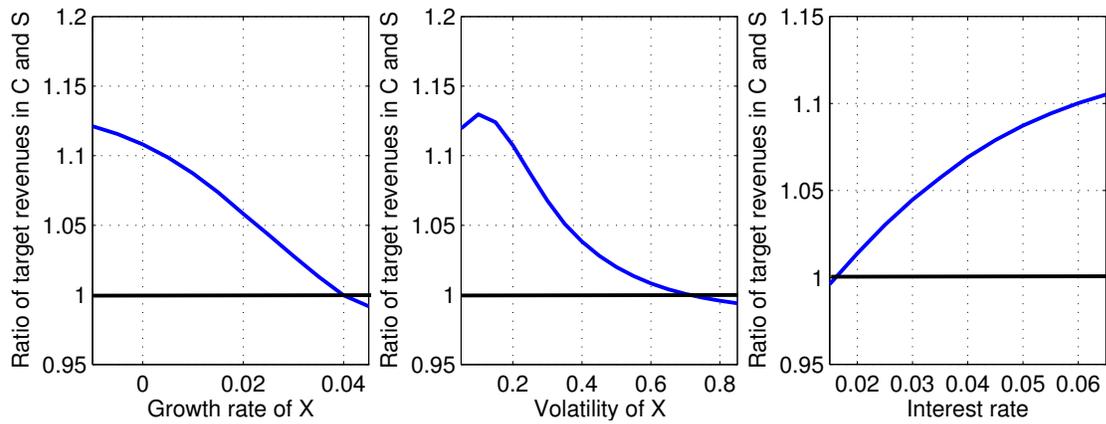
**Figure 7: Conditional and unconditional takeover premiums in cash and non-cash deals.** The figure corresponds to prediction A3. Panels A and C show, for the two cases: (i)  $C_1 = 125, C_2 = 0$ , (ii)  $C_1 = 125, C_2 = 125$ , the probability that a takeover contest is completed in cash as a function of the highest bidder valuation. Panels B and D show, for the same two cases, the average takeover premiums in cash and non-cash deals, both conditional on observing the highest bidder valuation (thick solid and dashed lines) and sample-wide unconditional (extra thick solid and dashed lines).



**Figure 8: Initiation, acquisition and means of payment in takeover contests with cash constrained bidders.** For the benchmark parametrization (Table I) and cash constraints of bidders 1 and 2 equal to  $C_1 = 125$ ,  $C_2 = 0$ , the figure shows regions of valuations for which bidders initiate and win takeover contests, as well as the resulting type of the deal (cash, cash and stock, stock). The dash-dotted line separates the cases in which bidder 1 makes cash and non-cash final bids.



**Figure 9: Average valuations of cash constrained bidders in initiated contests.** The figure shows average valuations of cash constrained bidders for the benchmark parametrization (Table I) as a function of (i) the value of the losing bidder,  $P_0$ , assuming cash constraints  $C_1 = 125$ ,  $C_2 = 0$ , and (ii) cash constraint of bidder 1,  $C_1$ , assuming  $C_2 = 0$  and  $P_0 = 85$ . The solid (dashed) line is the average valuation of bidder 1 (2).



**Figure 10: The ratio of the target revenue (present value) from contests in cash and in stock.** For the benchmark parametrization (Table I), the figure shows the ratio of present values of target revenues in cash and stock deals as a function of (i) the growth rate of the target's assets,  $\mu$ , (ii) the volatility of the target's assets,  $\sigma$ , (iii) the interest rate  $r$ .